Chapter 1
Physics: An Introduction

Conceptual Questions

1.1 The meter (SI unit for length) is defined by the distance light travels in a vacuum in a tiny fraction (1/299792458) of a second. The second (SI unit for time) is defined as the time it takes for 9,192,631,770 periods of the transition between the two split levels of the ground state of the cesium-133 atom. The kelvin (SI unit for temperature) is defined in terms of the conditions under which water can exist as ice, liquid, and gas simultaneously. The kilogram (SI unit for mass) is defined by the mass of a carefully protected prototype block made of platinum and iridium that was manufactured in 1889.

1.2 Yes, it is possible to define a system of units where length is not one of the fundamental properties. For example, a system that has fundamental properties of speed and time can derive the quantity of length.

1.3 Use of the metric prefixes makes any numerical calculation much easier to follow. Instead of an obscure conversion (12 in/ft, 1760 yards/mi, 5280 ft/mi), simple powers of 10 make the transformations (10 mm/cm, 1000 m/km, 10^{-6} m/μm).

1.4 The answer should be written as 55.0. When dividing quantities, the number with the fewest significant figures dictates the number of significant figures in the answer. In this case 3411 has four significant figures and 62.0 has three significant figures, which means we are allowed three significant figures in our answer.

1.5 To be a useful standard of measurement, an object, system, or process should be unchanging, replicable, and possible to measure precisely so that errors in its measurement do not carry over into calibration errors in every other measurement.

1.6 Yes, two physical quantities must have not only the same dimensions but also the same units. It is meaningless to add 7 seconds to 5 kilograms, for example. Adding 3 kilometers to 2 kilometers gives a different answer than adding 3 kilometers to 2 meters, even though both quantities are lengths. However, we are allowed to divide (and multiply) two physical quantities regardless of their dimensions or units. Speed, density, and various conversion factors are good examples of this.

1.7 No. The equation "3 meters = 70 meters" has consistent units but it is false. The same goes for "1 = 2," which consistently has no units.

1.8 The fewest number of significant figures in 61,000 is two—the "6" and the "1." If the period is acting as a decimal point, then the trailing zeros are significant and the quantity 61,000. would have five significant figures. When numbers are written in scientific notation, all of the digits before the power of 10 are significant. Therefore, 6.10×10^4 has three significant figures.

1.9 The SI unit for length is the meter; the SI unit for time is the second. Therefore, the SI units for acceleration are meters/(second)^2, or m/s^2.

Multiple-Choice Questions

1.10 **B** (length). Mass density, area, and resistance are all derived quantities.

1.11 **E** (1 m). It is easiest to answer this question by first converting all of the choices into meters:

A) $10 \text{ nm} \times \dfrac{1 \text{ m}}{10^9 \text{ nm}} = 10^{-8} \text{ m}$

B) $10 \text{ cm} \times \dfrac{1 \text{ m}}{10^2 \text{ cm}} = 10^{-1} \text{ m}$

C) $10^2 \text{ mm} \times \dfrac{1 \text{ m}}{10^3 \text{ mm}} = 10^{-1} \text{ m}$

D) 10^{-2} m

E) 1 m

1.12 **C** (10^{-9}s). The prefix *nano*- means 10^{-9}.

1.13 **C** (10^4).

$$1 \text{ m}^2 \times \frac{100 \text{ cm}}{1 \text{ m}} \times \frac{100 \text{ cm}}{1 \text{ m}} = \boxed{10^4 \text{ cm}^2}$$

1.14 **E** (10^{-6}).

$$1 \text{ cm}^3 \times \left(\frac{1 \text{ m}}{100 \text{ cm}}\right)^3 = \boxed{10^{-6} \text{ m}^3}$$

1.15 **E** (32). When adding or subtracting quantities, the quantity with the fewest *decimal places* (<u>not</u> significant figures) dictates the number of decimal places in the final answer, which is 15 in this case.

1.16 **D** (2.5). When dividing quantities, the quantity with the fewest significant figures dictates the number of significant figures in the final answer, which is 0.28 in this case.

1.17 **C** (1810). Both 25.8 and 70.0 have three significant figures. When multiplying quantities, the quantity with the fewest significant figures dictates the number of significant figures in the final answer. Multiplying 25.8 by 70.0 gives 1806, which has four significant figures. Our final answer must have three significant figures, so we round 1806 to 1810.

1.18 **B** (v/t). Acceleration has dimensions of $\frac{[\mathrm{L}]}{[\mathrm{T}]^2}$. To answer this question, we should first determine the dimensions of each of the choices:

A) $\frac{[\mathrm{L}]^2}{[\mathrm{T}]^2} \cdot \frac{1}{[\mathrm{T}]} = \frac{[\mathrm{L}]^2}{[\mathrm{T}]^3}$

B) $\frac{[\mathrm{L}]}{[\mathrm{T}]} \cdot \frac{1}{[\mathrm{T}]} = \frac{[\mathrm{L}]}{[\mathrm{T}]^2}$

C) $\frac{[\mathrm{L}]}{[\mathrm{T}]} \cdot \frac{1}{[\mathrm{T}]^2} = \frac{[\mathrm{L}]}{[\mathrm{T}]^3}$

D) $\frac{[\mathrm{L}]}{[\mathrm{T}]} \cdot \frac{1}{[\mathrm{L}]^2} = \frac{1}{[\mathrm{L}][\mathrm{T}]}$

E) $\frac{[\mathrm{L}]^2}{[\mathrm{T}]^2} \cdot \frac{1}{[\mathrm{L}]^2} = \frac{1}{[\mathrm{T}]^2}$

1.19 **B** (have dimensions of 1/T). An exponent must be dimensionless, so the product of λ and t must be dimensionless. The dimension of t is T. Therefore, λ has dimensions of 1/T.

Estimation Questions

1.20 There is no one answer to this question. When estimating, keep in mind that 1 meter is a little more than 3 feet.

1.21 We can model Mt. Everest as a 45° triangular pyramid—three identical triangles angled at 45° from an equilateral triangle base. The volume of a triangular pyramid is $\frac{1}{3}$(area of base)(height of pyramid). The base of Mt. Everest is 4500 m above sea level, and its peak is 8800 m above sea level, so its height is 4300 m. We can calculate (from geometry) the length of each side of the base of the equilateral triangle; each side is $\frac{12{,}900}{\sqrt{3}}$. Therefore, the volume of Mt. Everest is approximately $\frac{1}{3}\left(\frac{1}{2}\right)\left(\frac{12{,}900}{\sqrt{3}}\ \mathrm{m}\right)(6450\ \mathrm{m})(4300\ \mathrm{m}) \approx 3 \times 10^{10}\ \mathrm{m}^3$. The density of rock is about $2750\ \mathrm{kg/m^3}$. The mass of Mt. Everest is then $m_{\mathrm{Everest}} = \rho_{\mathrm{rock}} V_{\mathrm{Everest}} = (3 \times 10^{10}\ \mathrm{m}^3) \times \left(\frac{2750\ \mathrm{kg}}{\mathrm{m}^3}\right) \approx \boxed{10^{14}\ \mathrm{kg}}$.

1.22 The distance from home plate to the center field fence is about 100 m. A well-hit ball leaves the bat at around 100 mph, or 45 m/s. Assuming the ball comes off the bat horizontal to the ground, this gives an estimate of $100\ \mathrm{m} \times \frac{1\ \mathrm{s}}{45\ \mathrm{m}} = \boxed{2.2\ \mathrm{s}}$. This is probably a little low but is still reasonable.

1.23 Because laptop computers are very common nowadays, most people (students, faculty, etc.) have at least one laptop. Various departments (for example, academic departments, campus IT) also have laptops available, so there is probably about one laptop per person on campus.

1.24 There is no one answer to this question. The Environmental Protection Agency estimated that an average American produced about 2 kg (4.6 lb) of garbage a day in 2006 (http://www.epa.gov/wastes/nonhaz/).

1.25 We can split an average student's daily water use into four categories: showering, cooking/drinking/hand-washing, flushing the toilet, and doing laundry. A person uses about 100 L of water when showering, about 10 L for cooking/drinking/hand-washing, about 24 L when flushing the toilet, and about 40 L when doing two loads of laundry. This works out to about 150–200 L of water per day.

1.26 There is no one answer to this question. When estimating, keep in mind that one storey is about 10 feet.

1.27 The footprint of Chicago is around 600 km^2. A city block of intermediate size is around 100 m by 100 m. Sidewalk all around the perimeter would translate to 400 m of sidewalk per city block. In each square kilometer, there would be about 100 city blocks. The length of the sidewalks is

$$600\text{ km}^2 \times \frac{100\text{ blocks}}{1\text{ km}^2} \times \frac{400\text{ m sidewalk}}{1\text{ block}} \times \frac{1\text{ km}}{1000\text{ m}} = \boxed{24{,}000\text{ km}}.$$

1.28 In order to estimate the volume flow rate of air that fills your lungs as you take a deep breath, we need to estimate the volume of your lungs and the time it takes to take a deep breath. Let's assume that your lungs fill your rib cage, which has dimensions of 10 in $\times$ 6 in $\times$ 4 in. This is a volume of 240 in^3, or 3.9×10^{-3} m^3. We can time how long it takes to take a deep breath; it is about 4 s. Putting these estimates together, we find that the volume flow rate when taking a deep breath is about $\boxed{10^{-3}\text{ m}^3/\text{s}}$.

1.29 We can estimate the number of cells in the human body by determining the mass of a cell and comparing it to the mass of a human. An average human male has a mass of 80 kg. A person is mainly water, so we can approximate the density of a human body (and its cells) as 1000 kg/m^3. We are told that the volume of a cell is the same as a sphere with a radius of 10^{-5} m, or approximately 4×10^{-15} m^3; the mass of a single cell is $4 \times 10^{-15}\text{ m}^3 \times \dfrac{1000\text{ kg}}{1\text{ m}^3} = 4 \times 10^{-12}$ kg. The number of cells in the body is then $n_{\text{cell}} = \dfrac{m_{\text{body}}}{m_{\text{cell}}} = \dfrac{80\text{ kg}}{4 \times 10^{-12}\text{ kg}} = \boxed{2 \times 10^{13}}$.

Problems

1.30

SET UP

Scientific notation is a simple, compact way of expressing large and small numbers. The numbers are written as a coefficient multiplied by a power of 10. The coefficient should contain all of the significant figures in the quantity and be written as a nonzero digit in the ones place, a decimal place, and then the remaining significant digits.

SOLVE

A) 2.37×10^{2}	E) 1.487×10^{4}
B) 2.23×10^{-3}	F) 2.1478×10^{2}
C) 4.51×10^{1}	G) 4.42×10^{-6}
D) 1.115×10^{3}	H) 1.2345678×10^{7}

REFLECT

Scientific notation easily shows the significant figures in a quantity.

1.31

SET UP

We are given eight numbers written using a power of 10 and asked to write them as decimals. For numbers smaller than one, it is customary to include the zero before the decimal place.

SOLVE

A) 0.00442	E) 456,000
B) 0.00000709	F) 0.0224
C) 828	G) 0.0000375
D) 6,020,000	H) 0.000138

REFLECT

The answers to this problem show the advantage scientific notation offers in easily reading numbers.

1.32

SET UP

A list of metric prefix symbols is given. We are asked to write the power of 10 associated with each prefix. We can use Table 1-3 in the text to determine the correct factor associated with each metric prefix.

SOLVE

A) pico $= 10^{-12}$	E) femto $= 10^{-15}$
B) milli $= 10^{-3}$	F) giga $= 10^{9}$
C) mega $= 10^{6}$	G) tera $= 10^{12}$
D) micro $= 10^{-6}$	H) centi $= 10^{-2}$

REFLECT

It will be useful to memorize some of the more common prefixes, such as *milli-*, *mega-*, *micro-*, and *centi-*.

1.33

SET UP

A list of powers of 10 is given. We can use Table 1-3 in the text to determine the correct metric prefix associated with each factor. Eventually, knowing some of the more common prefixes will become second nature.

SOLVE

A) kilo (k)

B) giga (G)

C) mega (M)

D) tera (T)

E) milli (m)

F) pico (p)

G) micro (μ)

H) nano (n)

REFLECT

Whether or not the prefix is capitalized is important. *Mega-* and *milli-* both use the letter "m," but *mega-* is "M" and *milli-* is "m." Confusing these two will introduce an error of 10^9!

1.34

SET UP

This problem provides practice converting between some of the most common metric units. The prefixes we will need are *centi-* (10^{-2}), *kilo-* (10^3), and *milli-* (10^{-3}). Don't forget to convert *each* factor when dealing with areas and volumes.

SOLVE

A) $$125 \text{ cm} \times \frac{1 \text{ m}}{100 \text{ cm}} = \boxed{1.25 \text{ m}}$$

B) $$233 \text{ g} \times \frac{1 \text{ kg}}{1000 \text{ g}} = \boxed{0.233 \text{ kg}}$$

C) $$786 \text{ ms} \times \frac{1 \text{ s}}{1000 \text{ ms}} = \boxed{0.786 \text{ s}}$$

D) $$454 \text{ kg} \times \frac{10^6 \text{ mg}}{1 \text{ kg}} = \boxed{4.54 \times 10^8 \text{ mg}}$$

E) $$208 \text{ cm}^2 \times \left(\frac{1 \text{ m}}{100 \text{ cm}}\right)^2 = \boxed{0.0208 \text{ m}^2}$$

F) $$444 \text{ m}^2 \times \left(\frac{100 \text{ cm}}{1 \text{ m}}\right)^2 = \boxed{4.44 \times 10^6 \text{ cm}^2}$$

G) $$12.5 \text{ cm}^3 \times \left(\frac{1 \text{ m}}{100 \text{ cm}}\right)^3 = \boxed{1.25 \times 10^{-5} \text{ m}^3}$$

H) $$144 \text{ m}^3 \times \left(\frac{100 \text{ cm}}{1 \text{ m}}\right)^3 = \boxed{1.44 \times 10^8 \text{ cm}^3}$$

REFLECT

Although it is an extra step to convert from, say, kilograms to grams to milligrams, it is easier (and more useful) to memorize how many grams are in a kilogram and milligrams in a gram than to memorize how many milligrams are in a kilogram.

1.35

SET UP

We are asked to convert a list of quantities from U.S. units to metric units. The conversions we will need are 1 in = 2.54 cm, 1 L = 33.8 oz, 1 kg = 2.205 lb, and 1 mi = 1.609344 km. We will write each answer with the correct number of significant figures.

SOLVE

$$\text{A) } 238 \text{ ft} \times \frac{12 \text{ in}}{1 \text{ ft}} \times \frac{2.54 \text{ cm}}{1 \text{ in}} \times \frac{1 \text{ m}}{100 \text{ cm}} = \boxed{72.5 \text{ m}}$$

$$\text{B) } 772 \text{ in} \times \frac{2.54 \text{ cm}}{1 \text{ in}} = \boxed{1960 \text{ cm}}$$

$$\text{C) } 1220 \text{ in}^2 \times \left(\frac{2.54 \text{ cm}}{1 \text{ in}}\right)^2 = \boxed{7870 \text{ cm}^2}$$

$$\text{D) } 559 \text{ oz} \times \frac{1 \text{ L}}{33.8 \text{ oz}} = \boxed{16.5 \text{ L}}$$

$$\text{E) } 973 \text{ lb} \times \frac{1 \text{ kg}}{2.205 \text{ lb}} \times \frac{1000 \text{ kg}}{1 \text{ kg}} = \boxed{4.41 \times 10^5 \text{ g}}$$

$$\text{F) } 122 \text{ ft}^2 \times \left(\frac{12 \text{ in}}{1 \text{ ft}}\right)^3 \times \left(\frac{2.54 \text{ cm}}{1 \text{ in}}\right)^3 \times \left(\frac{1 \text{ m}}{100 \text{ cm}}\right)^3 = \boxed{3.45 \text{ m}^3}$$

$$\text{G) } 1.28 \text{ mi}^2 \times \left(\frac{1.609344 \text{ km}}{1 \text{ mi}}\right)^2 = \boxed{3.32 \text{ km}^2}$$

$$\text{H) } 442 \text{ in}^3 \times \left(\frac{2.54 \text{ cm}}{1 \text{ in}}\right)^3 = \boxed{7240 \text{ cm}^3}$$

REFLECT

Learning some common conversions between U.S. units and metric units will help you determine whether an answer you calculate is reasonable. For example, most Americans have a better handle on 3 feet versus 1 meter. Some simple conversions are 1 in = 2.54 cm, 1 kg = 2.2 lb, and 1 mi = 1.6 km.

1.36

SET UP

We are asked to perform some unit conversions. Some conversion factors we will need are 1 in = 2.54 cm, 1 min = 60 s, and 1 hr = 60 min.

SOLVE

$$\text{A) } 125\text{ cm} \times \frac{1\text{ in}}{2.54\text{ cm}} = \boxed{49.2\text{ in}}$$

$$\text{B) } 233\text{ in} \times \frac{2.54\text{ cm}}{1\text{ in}} = \boxed{592\text{ cm}}$$

$$\text{C) } 553\text{ ms} \times \frac{1\text{ s}}{1000\text{ ms}} \times \frac{1\text{ min}}{60\text{ s}} \times \frac{1\text{ hr}}{60\text{ min}} = \boxed{1.54 \times 10^{-4}\text{ hr}}$$

$$\text{D) } 454\text{ in} \times \frac{2.54\text{ cm}}{1\text{ in}} \times \frac{1\text{ m}}{100\text{ cm}} = \boxed{11.5\text{ m}}$$

$$\text{E) } 355\text{ cm}^2 \times \left(\frac{1\text{ in}}{2.54\text{ cm}}\right)^2 = \boxed{55.0\text{ in}^2}$$

$$\text{F) } 333\text{ m}^2 \times \left(\frac{100\text{ cm}}{1\text{ m}}\right)^2 \times \left(\frac{1\text{ in}}{2.54\text{ cm}}\right)^2 \times \left(\frac{1\text{ ft}}{12\text{ in}}\right)^2 = \boxed{3580\text{ ft}^2}$$

$$\text{G) } 424\text{ in}^3 \times \left(\frac{2.54\text{ cm}}{1\text{ in}}\right)^3 = \boxed{6950\text{ cm}^3}$$

$$\text{H) } 172\text{ m}^3 \times \left(\frac{100\text{ cm}}{1\text{ m}}\right)^3 \times \left(\frac{1\text{ in}}{2.54\text{ cm}}\right)^3 = \boxed{1.05 \times 10^7\text{ in}^3}$$

REFLECT

Don't forget to convert *each* factor when dealing with areas and volumes.

1.37

SET UP

This problem provides practice with metric unit conversions. We need to look up (and/or memorize) various conversions between SI units and hectares and liters. One hectare is equal to 10^4 m^2, and 1000 L is equal to 1 m^3. Another useful conversion is that 1 mL equals 1 cm^3.

SOLVE

$$\text{A) } 328\text{ cm}^3 \times \frac{1\text{ mL}}{1\text{ cm}^3} \times \frac{1\text{ L}}{1000\text{ mL}} = \boxed{0.328\text{ L}}$$

$$\text{B) } 112\text{ L} \times \frac{1\text{ m}^3}{1000\text{ L}} = \boxed{0.112\text{ m}^3}$$

$$\text{C) } 220\text{ hectares} \times \frac{10^4\text{ m}^2}{1\text{ hectare}} = \boxed{2.2 \times 10^6\text{ m}^2}$$

D) $44300\ \text{m}^2 \times \dfrac{1\ \text{hectare}}{10^4\ \text{m}^2} = \boxed{4.43\ \text{hectares}}$

E) $225\ \text{L} \times \dfrac{1\ \text{m}^3}{1000\ \text{L}} = \boxed{0.225\ \text{m}^3}$

F) $17.2\ \text{hectare} \cdot \text{m} \times \dfrac{10^4\ \text{m}^2}{1\ \text{hectare}} \times \dfrac{10^3\ \text{L}}{1\ \text{m}^3} = \boxed{1.72 \times 10^8\ \text{L}}$

G) $2.253 \times 10^5\ \text{L} \times \dfrac{1\ \text{m}^3}{10^3\ \text{L}} \times \dfrac{1\ \text{hectare}}{10^4\ \text{m}^2} = \boxed{2.253 \times 10^{-2}\ \text{hectare} \cdot \text{m}}$

H) $2000\ \text{m}^3 \times \dfrac{1000\ \text{L}}{1\ \text{m}^3} \times \dfrac{1000\ \text{mL}}{1\ \text{L}} = \boxed{2 \times 10^9\ \text{mL}}$

REFLECT

Rewriting the measurements and conversions in scientific notation makes the calculations simpler and helps give some physical intuition.

1.38

SET UP

We are asked to perform some volume unit conversions. Some useful conversion factors are 1 gal = 231 in^3 = 3.785 L, 1 L = 33.8 fl oz, and 1 pint = 0.4732 L.

SOLVE

A) $118\ \text{gal} \times \dfrac{231\ \text{in}^3}{1\ \text{gal}} \times \left(\dfrac{1\ \text{ft}}{12\ \text{in}}\right)^3 = \boxed{15.8\ \text{ft}^3}$

B) $1.3\ \text{ft}^3 \times \left(\dfrac{12\ \text{in}}{1\ \text{ft}}\right)^3 \times \dfrac{1\ \text{gal}}{231\ \text{in}^3} = \boxed{9.7\ \text{gal}}$

C) $14{,}400\ \text{fl oz} \times \dfrac{1\ \text{L}}{33.8\ \text{fl oz}} = \boxed{426\ \text{L}}$

D) $128\ \text{fl oz} \times \dfrac{1\ \text{L}}{33.8\ \text{fl oz}} \times \dfrac{1\ \text{m}^3}{1000\ \text{L}} = \boxed{3.79 \times 10^{-3}\ \text{m}^3}$

E) $487\ \text{in}^3 \times \left(\dfrac{2.54\ \text{cm}}{1\ \text{in}}\right)^3 = \boxed{7980\ \text{cm}^3}$

F) $0.0032\ \text{gal} \times \dfrac{3.785\ \text{L}}{1\ \text{gal}} = \boxed{0.012\ \text{L}}$

G) $129\ \text{in}^3 \times \left(\dfrac{2.54\ \text{cm}}{1\ \text{in}}\right)^3 \times \left(\dfrac{1\ \text{m}}{100\ \text{cm}}\right)^3 = \boxed{2.11 \times 10^{-3}\ \text{m}^3}$

H) $324\ \text{pint} \times \dfrac{0.4732\ \text{L}}{1\ \text{pint}} = \boxed{153\ \text{L}}$

REFLECT

Don't forget to convert *each* factor when dealing with volume units.

1.39

SET UP

We are asked to perform some unit conversions. Some useful conversion factors for this problem are 1 gal = 3.785 L, 1 acre = 43,560 ft^2, 1 ft = 0.3048 m, 1 L = 33.814 fl oz, 1 cm^3 = 1 mL, 1 cup = 8 fl oz, 1 hectare = 10^4 m^2, 1 pint = 473.2 mL, and 1 quart = 2 pints.

SOLVE

$$\text{A) } 33.5 \text{ gal} \times \frac{3.785 \text{ L}}{1 \text{ gal}} = \boxed{127 \text{ L}}.$$

$$\text{B) } 62.8 \text{ L} \times \frac{1 \text{ gal}}{3.785 \text{ L}} = \boxed{16.6 \text{ L}}$$

$$\text{C) } 216 \text{ acre} \cdot \text{ft} \times \frac{43{,}560 \text{ ft}^2}{1 \text{ acre}} \times \left(\frac{0.3048 \text{ m}}{1 \text{ ft}}\right)^2 \times \frac{1000 \text{ L}}{1 \text{ m}^3} = \boxed{2.66 \times 10^8 \text{ L}}$$

$$\text{D) } 1770 \text{ gal} \times \frac{3.785 \text{ L}}{1 \text{ gal}} \times \frac{1 \text{ m}^3}{1000 \text{ L}} = \boxed{6.70 \text{ m}^3}$$

$$\text{E) } 22.8 \text{ fl oz} \times \frac{1 \text{ L}}{33.814 \text{ fl oz}} \times \frac{1000 \text{ mL}}{1 \text{ L}} \times \frac{1 \text{ cm}^3}{1 \text{ mL}} = \boxed{674 \text{ cm}^3}$$

$$\text{F) } 54.2 \text{ cm}^3 \times \frac{1 \text{ mL}}{1 \text{ cm}^3} \times \frac{1 \text{ L}}{1000 \text{ mL}} \times \frac{33.814 \text{ fl oz}}{1 \text{ L}} \times \frac{1 \text{ cup}}{8 \text{ fl oz}} = \boxed{0.229 \text{ cups}}$$

$$\text{G) } 1.25 \text{ hectares} \times \frac{10^4 \text{ m}^2}{1 \text{ hectare}} \times \left(\frac{1 \text{ ft}}{0.3048 \text{ m}}\right)^2 \times \frac{1 \text{ acre}}{43{,}560 \text{ ft}^2} = \boxed{3.09 \text{ acre}}$$

$$\text{H) } 644 \text{ mm}^3 \times \left(\frac{1 \text{ cm}}{10 \text{ mm}}\right)^3 \times \frac{1 \text{ mL}}{1 \text{ cm}^3} \times \frac{1 \text{ pint}}{473.2 \text{ mL}} \times \frac{1 \text{ quart}}{2 \text{ pints}} = \boxed{7.02 \times 10^{-4} \text{ qt}}$$

REFLECT

The conversion 1 cm^3 = 1 mL is very useful to know.

1.40

SET UP

We are given three quantities and asked to rewrite them in scientific notation without prefixes. The prefixes correspond to *kilo*- (10^3), *micro*- (10^{-6}), and *giga*- (10^9), respectively. Be sure to include all of the significant figures in the quantity.

SOLVE

A) $300 \text{ km} = 300 \times 10^3 \text{ m} = \boxed{3 \times 10^5 \text{ m}}$

B) $33.7\ \mu\text{m} = 33.7 \times 10^{-6} \text{ m} = \boxed{3.37 \times 10^{-5} \text{ m}}$

C) $77.5 \text{ GW} = 77.5 \times 10^9 \text{ W} = \boxed{7.75 \times 10^{10} \text{ W}}$

REFLECT

Writing numbers using powers of 10 allows you to perform calculations quickly and without a calculator.

1.41

SET UP

We are given quantities in scientific notation and asked to rewrite these quantities using metric prefixes. Since the metric prefixes correspond to factors of 10^3, it is easiest to first change the scientific notation to a power of 10^3 and then replace it with a prefix.

SOLVE

A) $3.45 \times 10^{-4} \text{ s} = 345 \times 10^{-6} \text{ s} = \boxed{345\ \mu\text{s}}$

B) $2.00 \times 10^{-11} \text{ W} = 20.0 \times 10^{-12} \text{ W} = \boxed{20.0 \text{ pW}}$

C) $2.337 \times 10^8 \text{ m} = 233.7 \times 10^6 \text{ m} = \boxed{233.7 \text{ Mm}}$

D) $6.54 \times 10^4 \text{ g} = 65.4 \times 10^3 \text{ g} = \boxed{65.4 \text{ kg}}$

REFLECT

There are actually many different answers for each part. For example, we can rewrite 345 μs as 0.345 ms.

1.42

SET UP

A ticket roll consists of a string of 1000 tickets wrapped around a circular core that is 3 cm in diameter, or 1.5 cm in radius. Each ticket is 2 in (5.08 cm) long and 0.22 mm (2.2×10^{-2} cm) thick. The area of the entire roll is equal to the area of the circular core ($A_{\text{core}} = \pi R_{\text{core}}^2$) plus the area of the tickets, A_{tickets}. If we unravel all 1000 tickets from the roll, we know that the area still needs to be equal to A_{tickets}. Rather than a spiral shape, the tickets form a rectangle that is 2000 in (5080 cm) long and 0.22 mm (2.2×10^{-2} cm) wide. The area of the tickets is now just the area of this rectangle. Adding these two areas together gives A_{roll}, which is also equal to $A_{\text{roll}} = \pi R_{\text{roll}}^2$. This lets us find the radius and, therefore, the diameter of the roll.

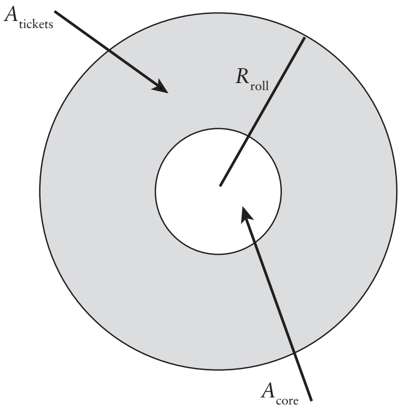


Figure 1-1 Problem 42

SOLVE

Total area of a string of 1000 tickets:

$$A_{\text{tickets}} = (5080\text{ cm})(2.2 \times 10^{-2}\text{ cm}) = 111.76\text{ cm}^2$$

Total area of the roll of tickets:

$$A_{\text{roll}} = A_{\text{core}} + A_{\text{tickets}}$$

$$\pi R_{\text{roll}}^2 = \pi R_{\text{core}}^2 + A_{\text{tickets}}$$

$$R_{\text{roll}} = \sqrt{\frac{\pi R_{\text{core}}^2 + A_{\text{tickets}}}{\pi}} = \sqrt{\frac{\pi(1.5\text{ cm})^2 + (111.76\text{ cm}^2)}{\pi}} = 6.2\text{ cm}$$

$$d_{\text{roll}} = 2R_{\text{roll}} = 2(6.2\text{ cm}) = \boxed{12\text{ cm}}$$

REFLECT

A diameter of 12 cm is approximately 5 in, which is a reasonable size for a spool of tickets.

1.43

SET UP

In the United States, fuel efficiency is reported in miles per gallon (or mpg). We are given fuel efficiency in kilometers per kilogram of fuel. We can use the conversions listed in the problem to convert this into mpg.

SOLVE

$$\frac{7.6\text{ km}}{\text{kg}} \times \frac{0.729\text{ kg}}{1\text{ L}} \times \frac{3.785\text{ L}}{1\text{ gal}} \times \frac{1\text{ mi}}{1.609\text{ km}} = \boxed{13\,\frac{\text{mi}}{\text{gal}}} \text{ or } \boxed{13\text{ mpg}}$$

REFLECT

This would be the gas mileage for a cargo van or large SUV. A hybrid sedan would have a gas mileage of around 40 mpg.

1.44

SET UP

We need to determine the number of significant figures in each number. Every nonzero digit is considered significant. A zero between nonzero digits is significant. Leading zeros are not significant. Trailing zeros are significant as long as there is a decimal point. The rules for significant figures are outlined in Physicist's Toolbox 1-2.

SOLVE

A) 112.4 has four significant figures.

B) 10 has one significant figure.

C) 3.14159 has six significant figures.

D) 700 has one significant figure.

E) 1204.0 has five significant figures.

F) 0.0030 has two significant figures.

G) 9.33×10^3 has three significant figures.

H) 0.02240 has four significant figures.

REFLECT

Be careful when determining the significance of zeroes. Scientific notation is the clearest way of determining which digits are significant.

1.45

SET UP

The smallest divisions on a standard meter stick are millimeters. Accordingly, a millimeter is the smallest measurement we can reliably make. The largest measurement is a meter. The reading error of the meter stick will determine the number of significant figures in a measurement.

SOLVE

A measurement made using a standard meter stick that has mm as its smallest division can have one to four significant figures (for example, 2 mm, 2.1 cm, 28.7 cm, 1.000 m).

REFLECT

Significant digits are related to the uncertainty in a measurement, which in this case is the reading error associated with the meter stick.

1.46

SET UP

The smallest divisions on a given thermometer are 1°C apart. Accordingly, we can read the thermometer to the closest 1°C. The largest measurement we can make is 100°C. The reading error of the thermometer will determine the number of significant figures in the measurement.

SOLVE

A measurement made using a thermometer that is marked from 0–100°C, subdivided into 1°C increments, can have one to three significant figures (1°C, 12°C, 100°C).

REFLECT

Significant digits are related to the uncertainty in a measurement, which in this case is the reading error associated with the thermometer.

1.47

SET UP

The smallest divisions on a given thermometer are 0.1°C apart. Accordingly, we can read the thermometer to the closest 0.1°C. The largest measurement we can make is 10.0°C. The reading error of the thermometer will determine the number of significant figures in the measurement.

SOLVE

A measurement made using a thermometer that is marked from 0–10°C, subdivided into 0.1°C increments, can have one to three significant figures (0.3°C, 1.8°C, 10.0°C).

REFLECT

Significant digits are related to the uncertainty in a measurement, which in this case is the reading error associated with the thermometer. Note that this is the same answer as 1.46.

1.48

SET UP

We are asked to calculate products and quotients with the correct number of significant figures. When multiplying or dividing quantities, the quantity with the fewest significant figures dictates the number of significant figures in the final answer. If necessary, we will need to round our answer to the correct number of significant figures.

SOLVE

A) $5.36 \times 2.0 = \boxed{11}$

B) $14.2 \div 2 = \boxed{7}$

C) $2 \times 3.14159 = \boxed{6}$

D) $4.040 \times 5.55 = \boxed{22.4}$

E) $4.444 \times 3.33 = \boxed{14.8}$

F) $1000 \div 333.3 = \boxed{3}$

G) $2.244 \times 88.66 = \boxed{199.0}$

H) $133 \times 2.000 = \boxed{266}$

REFLECT

Be careful when determining whether or not a zero is significant, especially when there is no decimal point.

1.49

SET UP

We are asked to calculate sums and differences with the correct number of significant figures. When adding or subtracting quantities, remember that the quantity with the fewest *decimal places* (<u>not</u> significant figures) dictates the number of decimal places in the final answer. If necessary, we will need to round our answer to the correct number of decimal places.

SOLVE

A) $4.55 + 21.6 = \boxed{26.2}$

B) $80.00 - 112.3 = \boxed{-32.3}$

C) $71.1 + 3.70 = \boxed{74.8}$

D) $200 + 33.7 = \boxed{200}$

REFLECT

The answer to part D may seem weird, but 200 only has one significant figure—the "2." The hundreds place is, therefore, the smallest decimal place we are allowed.

1.50

SET UP

A girl mows a lawn that is shaped like a parallelogram with a base $b = 25$ m and height $h = 20$ m. The area of a parallelogram is $A = bh$. Once we have the area, we can divide her total pay (\$125) by the area we calculated to determine her pay rate.

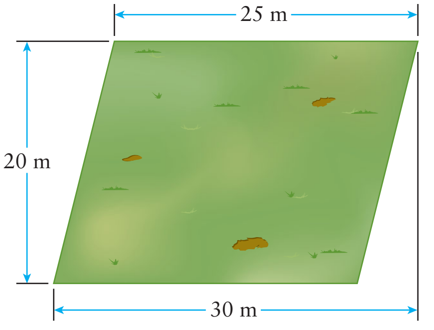


Figure 1-2 Problem 50

SOLVE

Part a)

$$A_Y = bh = (25\text{ m})(20\text{ m}) = \boxed{500\text{ m}^2}$$

Part b)

$$\text{Pay rate} = \frac{\text{Total pay}}{\text{Total area}} = \frac{\$125}{500\text{ m}^2} = \boxed{\frac{\$0.25}{\text{m}^2}} \text{ or } \boxed{25\ \frac{\text{cents}}{\text{m}^2}}.$$

REFLECT

If you did not recognize that the shape was a parallelogram or did not know the area of a parallelogram, you could treat the lawn as a rectangle measuring 30 m × 20 m with two triangles (or small 5 m × 20 m rectangle) missing. The area in this case would be $A = (30\text{ m})(20\text{ m}) - (5\text{ m})(20\text{ m}) = 500\text{ m}^2$, which is the same as above.

1.51

SET UP

We are asked to find the ratio of the volumes of two spheres. The first sphere has a radius $R_1 = 5$ cm, and the second sphere has a radius $R_2 = 10$ cm. The volume of a sphere is equal to $V = \frac{4}{3}\pi R^3$.

SOLVE

$$\frac{V_2}{V_1} = \frac{\frac{4}{3}\pi R_2^3}{\frac{4}{3}\pi R_1^3} = \frac{R_2^3}{R_1^3} = \left(\frac{R_2}{R_1}\right)^3 = \left(\frac{10\text{ cm}}{5\text{ cm}}\right)^3 = 2^3 = \boxed{8}$$

The ratio is not equal to 2 because the volume depends on the cube of the sphere's radius.

REFLECT

Make sure the radii are given in the same units when comparing them or else you will introduce spurious factors of 10.

1.52

SET UP

We need to find the mass of an aluminum spacer. We are given the dimensions of cylindrically shaped spacer—$R_{\text{out}} = 6$ cm, $R_{\text{in}} = 1$ cm, $h = 1$ cm—and the density of aluminum, $\rho = 2700\ \text{kg/m}^3$. The volume of a cylinder is the cross-sectional area multiplied by the height. In this case, there is a small hole in the cylinder, so the cross-sectional area is $A = (\pi R_{\text{out}}^2 - \pi R_{\text{in}}^2)$. Once we calculate the volume, we can multiply it by the density to get the mass of the spacer.

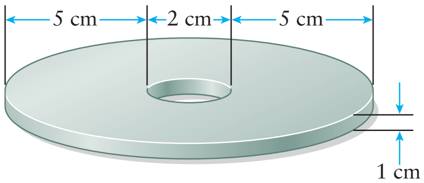


Figure 1-3 Problem 52

SOLVE

$$V = Ah = (\pi R_{\text{out}}^2 - \pi R_{\text{in}}^2)h = \pi((6\text{ cm})^2 - (1\text{ cm})^2)(1\text{ cm}) = \pi(35\text{ cm}^2)(1\text{ cm}) = 35\pi\text{ cm}^3$$

$$= 35\pi\text{ cm}^3 \times \left(\frac{1\text{ m}}{10^2\text{ cm}}\right)^3 = 35\pi \times 10^{-6}\text{ m}^3$$

$$m = \rho V = \left(2700\ \frac{\text{kg}}{\text{m}^3}\right)(35\pi \times 10^{-6}\text{ m}^3) = \boxed{0.297\text{ kg}}$$

REFLECT

The volume of the spacer can also be found by subtracting the volume of the "missing" cylinder from the volume of the larger cylinder: $V = V_{\text{out}} - V_{\text{in}} = \pi R_{\text{out}}^2 h - \pi R_{\text{in}}^2 h = 35\pi\text{ cm}^3$.

1.53

SET UP

A string of length S is stretched into a rectangle with width W and length L. We need to find the ratio of W to L that yields the largest area. The area of the rectangle is $A = LW$. There is a constraint on the system: The perimeter of the rectangle, $(2L + 2W)$, must equal S. This lets us eliminate one of the variables, say, L. To find the value of W that maximizes the area A, we need to set the derivative of A with respect to W equal to zero and solve for W_{max}. Once we have W_{max}, we can solve for L_{max} and find the ratio between them.

SOLVE

$$2L + 2W = S,\text{ so } L = \frac{S}{2} - W$$

$$A = LW = \left(\frac{S}{2} - W\right)W = \frac{S}{2}W - W^2$$

Maximizing the area:

$$\left(\frac{dA}{dW}\right)_{W_{\text{max}}} = 0 = \frac{S}{2} - 2W_{\text{max}}$$

$$W_{\text{max}} = \frac{S}{4}$$

Finding L_{max}:

$$2L_{max} + 2\left(\frac{S}{4}\right) = S$$

$$L_{max} = \frac{S}{4}$$

$$\frac{W_{max}}{L_{max}} = \frac{\left(\frac{S}{4}\right)}{\left(\frac{S}{4}\right)} = \boxed{1}$$

REFLECT

A square is a rectangle where $L = W$. A square is known to maximize the area for a fixed perimeter, which is what we explicitly calculated.

1.54

SET UP

We are asked to find the volume of a triangular prism with a width $w = 5$ cm, a height $h = 15$ cm, and a thickness $t = 8$ cm. The volume of a triangular prism is $V_{\substack{\text{triangular}\\\text{prism}}} = \frac{1}{2}wht$.

SOLVE

$$V_{\substack{\text{triangular}\\\text{prism}}} = \frac{1}{2}wht = \frac{1}{2}(5\text{ cm})(15\text{ cm})(8\text{ cm}) = \boxed{300\text{ cm}^3}$$

REFLECT

The volume of a triangular prism is the area of the triangular face $\left(\frac{1}{2}(\text{base})(\text{height})\right)$ multiplied by the thickness of the solid.

1.55

SET UP

A spherical planet has a radius $R = 5000$ km. The circumference about the equator is the same as the circumference of a circle of radius $R = 5000$ km. The circumference of a circle $C = 2\pi R$.

SOLVE

$$C = 2\pi R = 2\pi(500\text{ km}) = \boxed{30{,}000\text{ km}}$$

REFLECT

The radius has one significant figure, which is why our final answer only has one significant figure.

1.56

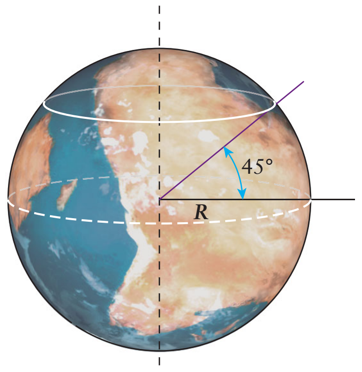


Figure 1-4 Problem 56

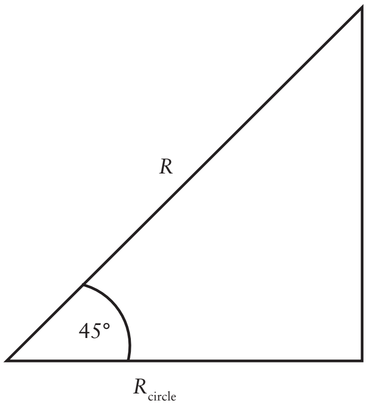


Figure 1-5 Problem 56

SET UP

A spherical planet has a radius $R = 5000$ km. We are asked to find the circumference of a path that follows a great arc at a latitude of 45°. We can relate the radius of that circle, R_{circle}, to the radius R by drawing a triangle and noticing that $\sin(45°) = \frac{R_{\text{circle}}}{R}$. The circumference is then equal to $2\pi R_{\text{circle}}$.

SOLVE

$$R_{\text{circle}} = R\sin(45°) = \frac{5000}{\sqrt{2}}\text{ km}$$

$$C = 2\pi R_{\text{circle}} = 2\pi\left(\frac{500}{\sqrt{2}}\text{ km}\right) = \boxed{20{,}000\text{ km}}$$

REFLECT

This distance is smaller than the circumference about the equator, which is what we expected. Also, we reported our answer with only one significant figure because the radius has one significant figure.

1.57

SET UP

We are given an equation that describes the motion of an object and asked if it is dimensionally consistent; we need to make sure the dimensions on the left side equal the dimensions on the right side. The equation contains terms related to position, speed, and time. Position has dimensions of length; speed has dimensions of length per time; and time has dimensions of, well, time.

SOLVE

$$x = vt + x_0$$

$$[\text{L}] \stackrel{?}{=} \frac{[\text{L}]}{[\text{T}]}[\text{T}] + [\text{L}]$$

$$[\text{L}] = [\text{L}] + [\text{L}]$$

REFLECT
Dimensions are general (for example, length), while units are specific (for example, meters, inches, miles, furlongs).

1.58

SET UP
We are given an equation that describes the motion of an object and asked if it is dimensionally consistent; we need to make sure the dimensions on the left side equal the dimensions on the right side. The equation contains terms related to position, acceleration, time, and speed. Position has dimensions of length; acceleration has dimensions of length per time squared; time has dimensions of time, and speed has dimensions of length per time.

SOLVE

$$x = \frac{1}{2}at^2 + v_0t + x_0$$

$$[\mathrm{L}] \stackrel{?}{=} \frac{[\mathrm{L}]}{[\mathrm{T}]^2}[\mathrm{T}]^2 + \frac{[\mathrm{L}]}{[\mathrm{T}]}[\mathrm{T}] + [\mathrm{L}]$$

$$[\mathrm{L}] = [\mathrm{L}] + [\mathrm{L}] + [\mathrm{L}]$$

REFLECT
As expected, the equation is dimensionally correct. It is always a good idea to check the dimensions and units throughout a calculation.

1.59

SET UP
The kinetic energy K of a moving particle is equal to $\frac{1}{2}mv^2$, where m is the mass of the particle and v is its speed. The SI unit of energy is the joule (J). Since the equation needs to be dimensionally correct, we know that a joule will be related to the SI units for mass (kg) and speed (m/s). Therefore, we can represent a joule in these fundamental units.

SOLVE

$$K = \frac{1}{2}mv^2$$

$$[K] = [m][v^2]$$

In SI units:

$$\mathrm{J} = (\mathrm{kg})\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2 = \frac{\mathrm{kg} \cdot \mathrm{m}^2}{\mathrm{s}^2}$$

REFLECT
A joule is also equal to a newton · meter.

1.60

SET UP

The motion of a vibrating system (for example, a mass attached to a spring) is described by $y(x, t) = A_0 e^{-\alpha t} \sin(kx - \omega t)$. The arguments of both the exponent and the sine need to be dimensionless, that is, dimensions of 1. This allows us to determine the dimensions of α, k and ω. Once we know the dimensions of these quantities, we can select the appropriate SI unit(s).

SOLVE

Dimensions of α:

$$[\alpha][t] = 1$$

$$[\alpha] = \frac{1}{[t]} = \frac{1}{[\mathrm{T}]}, \text{ which means the } \boxed{\text{SI unit is s}^{-1}}$$

Dimensions of k:

$$[k][x] = 1$$

$$[k] = \frac{1}{[x]} = \frac{1}{[\mathrm{L}]}, \text{ which means the } \boxed{\text{SI unit is m}^{-1}}$$

Dimensions of ω:

$$[\omega][t] = 1$$

$$[\omega] = \frac{1}{[t]} = \frac{1}{[\mathrm{T}]}, \text{ which means the } \boxed{\text{SI unit is s}^{-1}}$$

REFLECT

Usually k and ω are written as rad/m and rad/s, respectively, to emphasize that they are angular quantities.

1.61

SET UP

We need to check that the dimensions of the expected quantity match the dimensions of the units reported. Remember that dimensions should be given in terms of the fundamental quantity (see Table 1-1 in the text). For example, volume has dimensions of (length)^3.

SOLVE

A) Volume flow rate has dimensions of volume per time, or $\frac{[\mathrm{L}]^3}{[\mathrm{T}]}$, so $\frac{\mathrm{m}^3}{\mathrm{s}}$ is $\boxed{\text{correct}}$.

B) Height has dimensions of [L]. Units of m^2 are $\boxed{\text{not correct}}$.

C) A fortnight has dimensions of [T], while m/s are the units of speed. This statement is $\boxed{\text{not correct}}$.

D) Speed has dimensions of $\frac{[\mathrm{L}]}{[\mathrm{T}]}$, but $\frac{\mathrm{m}}{\mathrm{s}^2}$ are units of acceleration. This statement is $\boxed{\text{not correct}}$.

E) Weight is a force and has dimensions of $\frac{[\mathrm{M}][\mathrm{L}]}{[\mathrm{T}]^2}$, and lb is an appropriate unit for force. This is $\boxed{\text{correct}}$.

F) Density has dimensions of mass per volume, or $\frac{[\mathrm{M}]}{[\mathrm{L}]^3}$, so $\frac{\mathrm{kg}}{\mathrm{m}^2}$ is $\boxed{\text{not correct}}$.

REFLECT

Only statements A and E are correct. Making sure your answer has the correction dimensions and units is an important last step in solving every problem.

1.62

SET UP

The period T of a pendulum is the time it takes to complete one full oscillation. It is related to the length of the pendulum, L, and the acceleration due to gravity, g: $T = 2\pi\sqrt{\frac{L}{g}}$. We are asked to check that the equation is dimensionally correct. The period has dimensions of time; the length has dimensions of length; and the acceleration due to gravity has dimensions of length per time squared.

SOLVE

$$[T] \stackrel{?}{=} \sqrt{\frac{[L]}{[g]}}$$

$$[\mathrm{T}] \stackrel{?}{=} \sqrt{\frac{[\mathrm{L}]}{\left(\frac{[\mathrm{L}]}{[\mathrm{T}]^2}\right)}} = \sqrt{[T]^2}$$

$$[\mathrm{T}] = [\mathrm{T}]$$

REFLECT

The equation is dimensionally correct, as expected. Precisely measuring the period of a pendulum of known length is one way of calculating the acceleration due to gravity.

1.63

SET UP

We are told that it takes light 37.1 μs to travel 11.12 km. The speed of light can be calculated by dividing the length it traveled by the time it took. First we will need to convert from kilometers to meters (1 km = 1000 m) and from microseconds to seconds (10^6 μs = 1 s). We need to report the speed with the correct number of significant figures. When dividing quantities, the number with the fewest significant figures dictates the number of significant figures in the answer. In this case 11.12 has four significant figures and 37.1 has three significant figures, which means we are allowed three significant figures in our answer.

SOLVE

$$v = \frac{\Delta x}{\Delta t} = \frac{11.12 \text{ km}}{37.1\ \mu\text{s}} = \frac{1.112 \times 10^4 \text{ m}}{3.71 \times 10^{-5} \text{ s}} = \boxed{3.00 \times 10^8 \frac{\text{m}}{\text{s}}}$$

REFLECT

Our answer is consistent with the speed of light to three significant figures.

1.64

SET UP

We are asked to convert a prostate-specific antigen (PSA) concentration given in nanograms per milliliter into other units of concentration. We will need to know the following prefixes: *nano*- (10^{-9}), *milli*- (10^{-3}), *centi*- (10^{-2}), *kilo*- (10^3), and *micro*- (10^{-6}). The conversion between milliliters and cubic centimeters ($1 \text{ mL} = 1 \text{ cm}^3$) will also be useful.

SOLVE

Part a)

$$1.7 \frac{\text{ng}}{\text{mL}} \times \frac{10^3 \text{ mL}}{1 \text{ L}} \times \frac{1 \text{ g}}{10^9 \text{ ng}} = \boxed{1.7 \times 10^{-6} \frac{\text{g}}{\text{L}}}$$

Part b)

$$1.7 \frac{\text{ng}}{\text{mL}} \times \frac{1 \text{ mL}}{1 \text{ cm}^3} \times \left(\frac{10^2 \text{ cm}}{1 \text{ m}}\right)^3 \times \frac{1 \text{ g}}{10^9 \text{ ng}} \times \frac{1 \text{ kg}}{10^3 \text{ g}} = \boxed{1.7 \times 10^{-6} \frac{\text{kg}}{\text{m}^3}}$$

Part c)

$$1.7 \frac{\text{ng}}{\text{mL}} \times \frac{10^3 \text{ mL}}{1 \text{ L}} \times \frac{1\ \mu\text{g}}{10^3 \text{ ng}} = \boxed{1.7 \frac{\mu\text{g}}{\text{L}}}$$

REFLECT

The equality $1 \text{ mL} = 1 \text{ cm}^3$ is a handy volume conversion to memorize.

1.65

SET UP

This problem consists of many different unit conversions. We will need to know the following prefixes: *milli*- (10^{-3}), *kilo*- (10^3), *micro*- (10^{-6}), and *centi*- (10^{-2}). To speed up your calculations, write the conversions in exponential form (that is, 10^3 rather than 1000); this allows you to perform mental arithmetic faster. The conversion between milliliters and cubic centimeters ($1 \text{ mL} = 1 \text{ cm}^3$) and the volume of a sphere, $V = \frac{4}{3}\pi R^3$, will also be useful.

SOLVE

Part a)

$$2500 \text{ mg} \times \frac{1 \text{ g}}{10^3 \text{ mg}} \times \frac{1 \text{ kg}}{10^3 \text{ g}} = \boxed{2.5 \times 10^{-3} \text{ kg}}$$

Part b)

$$\frac{35\text{ mg}}{240\text{ mL}} = 0.15\,\frac{\text{mg}}{\text{mL}} \times \frac{1\text{ g}}{10^3\text{ mg}} \times \frac{1\text{ kg}}{10^3\text{ g}} \times \frac{1\text{ mL}}{1\text{ cm}^3} \times \left(\frac{10^2\text{ cm}}{1\text{ m}}\right)^3 = \boxed{0.15\,\frac{\text{kg}}{\text{m}^3}}$$

Part c)

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 = \frac{4}{3}\pi\left(\frac{10\ \mu\text{m}}{2}\right)^3 = \frac{4}{3}\pi(5 \times 10^{-6}\text{ m})^3 = \boxed{5 \times 10^{-16}\text{ m}^3}$$

Part d)

One tablet:

$$81\text{ mg} \times \frac{1\text{ g}}{10^3\text{ mg}} \times \frac{1\text{ kg}}{10^3\text{ g}} = 8.1 \times 10^{-5}\text{ kg}$$

One bottle:

$$(100)(8.1 \times 10^{-5}\text{ kg}) = \boxed{8.1 \times 10^{-3}\text{ kg}}$$

Part e)

$$1.2\,\frac{\text{mL}}{\text{min}} \times \frac{1\text{ cm}^3}{1\text{ mL}} \times \left(\frac{1\text{ m}}{10^2\text{ cm}}\right)^3 \times \frac{1\text{ min}}{60\text{ s}} = \boxed{2.0 \times 10^{-8}\,\frac{\text{m}^3}{\text{s}}}$$

Part f)

$$1.4\,\frac{\text{g}}{\text{cm}^3} \times \left(\frac{10^2\text{ cm}}{1\text{ m}}\right)^3 \times \frac{1\text{ kg}}{10^3\text{ g}} = \boxed{1400\,\frac{\text{kg}}{\text{m}^3}}$$

REFLECT

Nonstandard SI units (for example, milliliters versus cubic meters) allow us to report values that are close to 1. For example, in part c, a volume of 500 μm^3 is more useful than $5 \times 10^{-16}\text{ m}^3$ when discussing cellular volumes.

1.66

SET UP

The horizontal distance R that a projectile travels is given by $R = \frac{v_0^2}{g}\sin(2\theta)$. We can find the launch angle θ that maximizes R by setting the derivative of R with respect to θ equal to zero and solving for θ.

SOLVE

$$R = \frac{v_0^2}{g}\sin(2\theta)$$

$$\frac{dR}{d\theta} = \frac{d}{d\theta}\left(\frac{v_0^2}{g}\sin(2\theta)\right) = \frac{v_0^2}{g}\frac{d}{d\theta}(\sin(2\theta)) = \frac{v_0^2}{g}(2\cos(2\theta)) = 0$$

$$\cos(2\theta) = 0$$

$$\theta = \frac{\arccos(0)}{2} = \frac{90^\circ}{2} = \boxed{45^\circ}$$

REFLECT
A launch angle of 45° maximizes the horizontal distance a projectile travels, which is reasonable and what we expected.

1.67

SET UP
A 2-m-tall woman stands in front of a flagpole of height h. The top of the woman's shadow and the top of the flagpole's shadow overlap. The woman's shadow is 10 m long, while the flagpole's shadow is 22 m long. The woman and the flagpole each create a triangle with their shadows. These triangles are similar to one another. Therefore, we know that the ratio of the woman's height to the size of her shadow is equal to the ratio of the flagpole's height h and the size of its shadow.

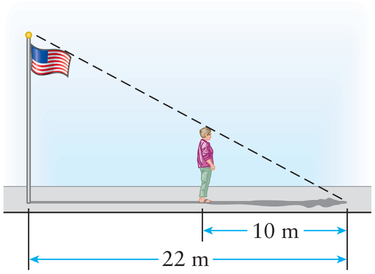


Figure 1-6 Problem 67

SOLVE

$$\frac{2\ \text{m}}{10\ \text{m}} = \frac{h}{22\ \text{m}}$$

$$h = \left(\frac{2\ \text{m}}{10\ \text{m}}\right)(22\ \text{m}) = \boxed{4.4\ \text{m}}$$

REFLECT
This works out to a little over 14 feet, which is a reasonable height for a flagpole.

1.68

SET UP
We are given a function, $x(t)$, and asked to find its third derivative with respect to time and evaluate it at $t = 4$ s. Because the function is a polynomial, we can use the power rule to determine the derivatives. Once we find the functional form of the third derivative, we can evaluate it at $t = 4$ s.

SOLVE

$$x(t) = 4t^4 + 6t^3 + 12t^2 + 5t$$

$$\frac{dx}{dt} = 16t^3 + 18t^2 + 24t + 5$$

$$\frac{d^2x}{dt^2} = 48t^2 + 36t + 24$$

$$\frac{d^3x}{dt^3} = 96t + 36$$

$$\left(\frac{d^3x}{dt^3}\right)_{t=4\ \text{s}} = 96(4) + 36 = \boxed{420}$$

REFLECT

It is easier to take each derivative separately rather than trying to do them all at once. Be sure to plug in the value of t at the very end of the calculation, not the beginning.

1.69

SET UP

We are given the acceleration g of a falling object near a planet in terms of the planet's mass M, the distance from the planet's center R, and the gravitational constant G. All of the quantities are given in SI units: g is in m/s^2, M is in kg, and R is in m. The fact that an equation needs to be dimensionally correct allows us to find the SI units associated with G.

SOLVE

$$g = \frac{GM}{R^2}$$

$$\frac{\text{m}}{\text{s}^2} = [G]\frac{\text{kg}}{\text{m}^2}$$

$$[G] = \frac{\text{m}}{\text{s}^2}\frac{\text{m}^2}{\text{kg}} = \boxed{\frac{\text{m}^3}{\text{kg}\cdot\text{s}^2}}$$

REFLECT

Numerically, the gravitational constant $G \approx 6.674 \times 10^{-11}\,\frac{\text{m}^3}{\text{kg}\cdot\text{s}^2}$. For Earth, $M_{\text{Earth}} = 5.97 \times 10^{24}$ kg and $R_{\text{Earth}} = 6.378 \times 10^6$ m, which means

$$g = \frac{\left(6.674 \times 10^{-11}\,\frac{\text{m}^3}{\text{kg}\cdot\text{s}^2}\right)(5.97 \times 10^{24}\text{ kg})}{(6.378 \times 10^6\text{ m})^2} = 9.8\,\frac{\text{m}}{\text{s}^2}$$

1.70

SET UP

A cell of radius $R_{\text{cell}} = 5\ \mu\text{m}$ is surrounded by a membrane of thickness $d = 5.0$ nm. By modeling the cell as a sphere, we can easily calculate its volume. As a reminder, the volume of a sphere is $\frac{4}{3}\pi R^3$. The entire cell has a radius of $(R_{\text{cell}} + d)$. The volume of the cell membrane can be found by subtracting the volume of the "inner" cell from the volume of the total cell. Because the membrane thickness is so much smaller—three orders of magnitude—than the radius of the inner part of the cell, we should expect the volume of the membrane to contribute very little to the total volume of the cell.

SOLVE

Part a)

$$V_{\text{total cell}} = \frac{4}{3}\pi(R_{\text{cell}} + d_{\text{membrane}})^3 = \frac{4}{3}\pi((5\ \mu\text{m}) + (5.0 \times 10^{-3}\ \mu\text{m}))^3 = \boxed{525\ \mu\text{m}^3}$$

Part b)

$$V_{\text{membrane}} = V_{\text{total cell}} - V_{\text{cell}} = \frac{4}{3}\pi(R_{\text{cell}} + d_{\text{membrane}})^3 - \frac{4}{3}\pi R_{\text{cell}}^3$$

$$= \frac{4}{3}\pi(5.005\ \mu\text{m})^3 - \frac{4}{3}\pi(5\ \mu\text{m})^3 = \boxed{1.6\ \mu\text{m}^3}$$

Part c)

$$\frac{V_{\text{membrane}}}{V_{\text{total cell}}} = \frac{1.6\ \mu\text{m}^3}{525\ \mu\text{m}^3} = 0.003 = \boxed{0.3\%}$$

REFLECT

Although extremely biologically important, the membrane does not affect the total volume of the cell by much. Therefore, in volume calculations, we can ignore it and assume that the volume of the total cell is just the volume of the inner portion of the cell.

1.71

SET UP

It takes about 1 min for all of the blood in a person's body to circulate through the heart. When the heart beats at a rate of 75 beats per minute, it pumps about 70 mL of blood per beat. Blood has a density of 1060 kg/m^3. We can use each of these relationships as a unit conversion to determine the total volume of blood in the body and the mass of blood pumped per beat. For the volume calculation, we will start with the fact that all of the blood takes 1 min to circulate through the body. Because we are interested in the mass per beat, we can start with 1 beat in part b.

SOLVE

Part a)

$$1\ \text{min} \times \frac{75\ \text{beats}}{1\ \text{min}} \times \frac{70\ \text{mL}}{1\ \text{beat}} \times \frac{1\ \text{L}}{1000\ \text{mL}} = \boxed{5.25\ \text{L}} \times \frac{1\ \text{m}^3}{1000\ \text{L}} = \boxed{5.25 \times 10^{-3}\ \text{m}^3}$$

Part b)

$$1\ \text{beat} \times \frac{70\ \text{mL}}{1\ \text{beat}} \times \frac{1\ \text{L}}{1000\ \text{mL}} \times \frac{1\ \text{m}^3}{1000\ \text{L}} \times \frac{1060\ \text{kg}}{1\ \text{m}^3} = \boxed{0.074\ \text{kg}} \times \frac{1000\ \text{g}}{1\ \text{kg}} = \boxed{74\ \text{g}}$$

REFLECT

A volume of 5.25 L is about 11 pints, which is about average. For comparison, they take 1 pint when you donate blood.

1.72

SET UP

A cone has a radius $R = 2.25$ m and a height $h = 3.75$ m. The volume of a cone is $V_{\text{cone}} = \frac{1}{3}\pi R^2 h$.

SOLVE

$$V_{\text{cone}} = \frac{1}{3}\pi R^2 h = \frac{1}{3}\pi(2.25\text{ m})^2(3.75\text{ m}) = \boxed{19.9\text{ m}^3}$$

REFLECT

Each quantity has three significant figures, so our answer should have three significant figures.

1.73

SET UP

A typical prostate gland has a mass of about 20 g. We can model it as a sphere of diameter $d = 4.50$ cm. Dividing the mass of the prostate by its volume will give the density of the prostate, which we can compare to the density of water (1000 kg/m^3). A cylindrical sample of diameter $d_{\text{cyl}} = 0.100$ mm and length $h = 28.0$ mm is removed from the prostate. By calculating its volume and multiplying the volume by the density, we can find the total mass removed from the prostate and then compare it to the original mass of the prostate.

SOLVE

Part a)

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 = \frac{4}{3}\pi\left(\frac{4.50\text{ cm}}{2}\right)^3 = 47.7\text{ cm}^3$$

$$\rho = \frac{m}{V} = \frac{20\text{ g}}{47.7\text{ cm}^3} = \boxed{0.4\ \frac{\text{g}}{\text{cm}^3}} \times \frac{1\text{ kg}}{1000\text{ g}} \times \left(\frac{100\text{ cm}}{1\text{ m}}\right)^3 = \boxed{400\ \frac{\text{kg}}{\text{m}^3}}$$

Part b) The density of water is 1000 kg/m^3, so the density of the prostate is $\boxed{\text{40\% of the density of water}}$.

Part c)

$$V_{\text{cylinder}} = \pi R_{\text{cyl}}^2 h = \pi\left(\frac{d_{\text{cyl}}}{2}\right)^2 h = \pi\left(\frac{0.100\text{ mm}}{2}\right)^2(28.0\text{ mm}) = 0.220\text{ mm}^3 \times \left(\frac{1\text{ cm}}{10\text{ mm}}\right)^3$$

$$= 2.20 \times 10^{-4}\text{ cm}^3$$

$$m = \rho V_{\text{cyl}} = \left(0.4\ \frac{\text{g}}{\text{cm}^3}\right)(2.20 \times 10^{-4}\text{ cm}^3) = \boxed{9 \times 10^{-5}\text{ g}}$$

Part d)

$$\frac{m_{\text{cyl}}}{m_{\text{prostate}}} = \frac{9 \times 10^{-5}\text{ g}}{20\text{ g}} = 5 \times 10^{-6} = \boxed{5 \times 10^{-4}\%}$$

REFLECT

The mass removed from the prostate is negligible compared to the mass of the prostate.

1.74

SET UP

Body mass index (BMI) is defined as the person's mass m in kilograms divided by the square of the person's height h in meters. To convert between SI units and U.S. units (pounds per square inch), we can use the following unit conversions: 1 in = 2.54 cm, 1 kg = 2.205 lb. Multiplying all of the conversion factors together will give the desired coefficient, 703. A height of 5′11″ is equal to 71 inches or 1.80 meters. We can rearrange the BMI equation to solve for the mass, plug in the two ends of the BMI range, and solve for the two corresponding masses.

SOLVE

Part a)

$$\boxed{\text{BMI} = \frac{m}{h^2}} \text{ (in SI units)}$$

Part b)

BMI in SI units = (conversion factor) · (BMI in U.S. units)

$$\frac{\text{kg}}{\text{m}^2} = \frac{\text{lb}}{\text{in}^2} \times \left(\frac{1 \text{ in}}{2.54 \text{ cm}}\right)^2 \times \left(\frac{100 \text{ cm}}{1 \text{ m}}\right)^2 \times \frac{1 \text{ kg}}{2.205 \text{ lb}} = \boxed{703\left(\frac{\text{lb}}{\text{in}^2}\right)}$$

Part c)

$$m = (\text{BMI})h^2$$

$$h = 5'11'' = 71.0 \text{ in} \times \frac{2.54 \text{ cm}}{1 \text{ in}} = 180 \text{ cm} = 1.80 \text{ m}$$

Low end of the range:

$$m_{\text{low}} = (25.0)(1.80)^2 = \boxed{81.0 \text{ kg}}$$

High end of the range:

$$m_{\text{high}} = (30.0)(1.80)^2 = \boxed{97.2 \text{ kg}}$$

REFLECT

This corresponds to a range of weights from 180 to 215 lb.

Chapter 2
Linear Motion

Conceptual Questions

2.1 An object will slow down when its acceleration vector points in the opposite direction to its velocity vector. Recall that acceleration is the change in velocity over the change in time.

2.2 A ball is thrown straight up, stops in midair, and then falls back toward your hand. The velocity of the ball when it leaves your hand is large and points upward. The ball's speed decreases until it reaches its highest point. At this spot, its velocity is zero. The velocity of the ball then increases and points downward on its trip back to your hand. Since the ball is undergoing free fall, its acceleration is constant—it has a magnitude of $g = 9.8\text{ m/s}^2$ and points downward.

2.3 Average velocity is a vector quantity—the *displacement* over the time interval. Average speed is a scalar quantity—the *distance* over the time interval.

2.4 The magnitude of the displacement and the distance traveled will be the same when an object travels in one direction in a straight line or when the object is stationary. They will be different in all other cases.

2.5 Speed and velocity have the same SI units (m/s). Speed is the magnitude of the velocity vector. If the velocity points completely in the positive direction, then the two can be interchanged. If the velocity is not fixed in direction, then the correctly signed component of the velocity will need to be used to avoid confusion.

2.6 We need to convert these into the same unit system (say, m/s) in order to determine which is largest.

$$1\frac{\text{km}}{\text{hr}} \times \frac{1\text{ hr}}{3600\text{ s}} \times \frac{1000\text{ m}}{1\text{ km}} = 0.28\frac{\text{m}}{\text{s}}$$

$$1\frac{\text{mi}}{\text{hr}} \times \frac{1\text{ hr}}{3600\text{ s}} \times \frac{1.61\text{ km}}{1\text{ mi}} \times \frac{1000\text{ m}}{1\text{ km}} = 0.45\frac{\text{m}}{\text{s}}$$

The largest speed—1 m/s—will give the largest displacement in a fixed time.

2.7 One advantage is that people in countries that use the metric system will have a chance of understanding the distance part of the unit. The disadvantages are that no country lists speed limits in m/s, so it is hard to figure out how long it takes to get to highway speeds. (For comparison, 65 mph = 105 km/hr = 29 m/s.) Further, the raw numbers will be smaller than if another unit system were used, so careless readers not comparing units will get the impression that the car accelerates slowly. It would be better to use km/hr/s,

as speed limits around the world are generally expressed in km/hr. The best plan would be to tailor the units to the individual market.

2.8 The acceleration due to gravity is constant in both magnitude (g) and direction (down). When a ball is thrown straight up, its acceleration vector points in the opposite direction to its velocity vector, which means it slows down and eventually stops. Assuming the braking acceleration of the car is constant in magnitude (g) and points opposite to the car's velocity vector, it, too, will slow down and eventually stop. If the ball and the car start at the same initial speed and have the same acceleration, the ball and the car will take the same amount of time to come to rest.

2.9 The average velocity of a moving object will be the same as the instantaneous velocity if the object is moving at a constant velocity (both magnitude and direction). Also, if the average velocity is taken over a period of constant acceleration, the instantaneous velocity will match it for one moment in the middle of that period.

2.10 Yes, an object can be accelerating if its speed is constant. Acceleration is the change in velocity (a vector) over the time interval. Although the speed (magnitude) is constant, a moving object that changes its direction is accelerating.

2.11 No, there is no way to tell whether the video is being played in reverse. An object being thrown up in the air will undergo the same acceleration, time of flight, and so on as an object falling from its maximum height.

2.12 The acceleration of a ball thrown straight up in the air is constant because it is under the influence of Earth's gravity.

2.13 Assuming the initial speed of the ball is the same in both cases, the velocity of the second ball will be the same as the velocity of the first ball. The upward trajectory of the first ball involves the ball going up and reversing itself back toward its initial location. At that point, the trajectories of the two balls are identical as the balls hit the ground.

2.14 Yes, the units for average velocity are the same as the units for instantaneous velocity. Both quantities are length divided by time.

2.15 The second derivative with respect to time is denoted by $\frac{d^2}{dt^2}$. The acceleration, which has dimensions of length per time squared, is the second derivative of position with respect to time: $\frac{d^2x}{dt^2}$. We can also think about it as the first derivative of the speed: $\frac{d}{dt}\left(\frac{dx}{dt}\right)$.

2.16 It's always a good idea to include the units of every quantity throughout your calculation. Eventually, once they become more comfortable, most people will convert all of their values into SI units and stop including them in the intermediate steps of the calculation. Using SI units in all calculations minimizes calculation errors and ensures the answer will be in SI units as well.

2.17 Graphs are reported as "y-axis versus x-axis," so the SI units for each slope will be the SI units of the y-axis divided by the SI units of the x-axis:

A) Displacement versus time: m/s.

B) Velocity versus time: m/s^2.

C) Distance versus time: m/s.

2.18 There is no reason why "up" cannot be labeled as "negative" or "left" as "positive." Usually, *up* and *right* are chosen as positive since these correspond to positive x and y in a standard Cartesian coordinate system.

2.19 Part a) If the police car starts at rest and eventually catches up to a car traveling at a constant speed, the police car must be traveling faster than the speeding car. Therefore, at the point the police car overtakes the speeding car, the police car's speed is greater than that of the speeding car.

Part b) The displacement of the police car is the same as the displacement of the speeding car. If both cars start at the location of the police car, travel along the same path, and end at the same location, the displacements of the two cars will be the same.

Part c) The acceleration of the police car will be greater than the acceleration of the speeding car. The police car starts at rest and must accelerate up to some final speed in order to catch up with the speeding car. The speeding car is traveling at a constant speed, so its acceleration is zero, assuming it's traveling in a straight line.

2.20 A step stool is about 0.33 m tall. A painter falling from rest would hit the ground at a speed of

$$v^2 - v_0^2 = 2a(\Delta y)$$

$$v = \sqrt{v_0^2 + 2a(\Delta y)} = \sqrt{0^2 + 2\left(-9.8\frac{\text{m}}{\text{s}^2}\right)(-0.33\text{ m})} = 2.5\frac{\text{m}}{\text{s}}$$

If he lands with his knees locked, he comes to rest much faster than if he were to bend his knees while landing. Let's say he comes to rest in 0.1 s when his knees are locked but 1 s when his knees are bent. The magnitudes of the acceleration in both cases are

$$a_{\text{locked}} = \frac{\Delta v}{\Delta t} = \frac{2.5\frac{\text{m}}{\text{s}}}{0.1\text{ s}} = \boxed{25\frac{\text{m}}{\text{s}^2}}$$

$$a_{\text{bent}} = \frac{\Delta v}{\Delta t} = \frac{2.5\frac{\text{m}}{\text{s}}}{1\text{ s}} = \boxed{2.5\frac{\text{m}}{\text{s}^2}}$$

Multiple-Choice Questions

2.21 **B** ($\leftarrow$).

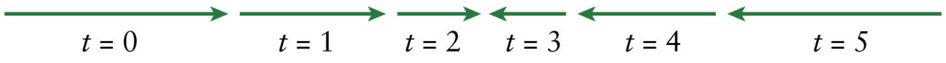


Figure 2-1 Problem 21

The velocity vector starts out pointing to the right and then decreases in magnitude. At $t = 3$, the velocity starts to point toward the left and then increases in magnitude. The velocity vector is always changing in the same manner. We also know that an object slows down if its velocity and acceleration vectors point in opposite directions and that an object speeds up if its velocity and acceleration vectors point in the same direction. This means the acceleration vector is always pointing to the left.

2.22 **B** (the object's instantaneous velocity at that point). The tangent is equal to the derivative at that point. The first derivative of a position versus time plot gives the instantaneous velocity.

2.23 **D**.

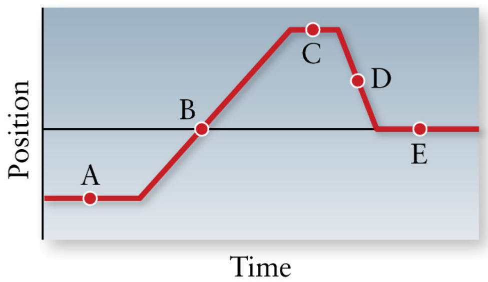


Figure 2-2 Problem 23

The slope of a position versus time plot gives information regarding the speed of the object. Point D has the largest slope, which means the object is moving the fastest there.

2.24 **B**. The slope of a position versus time plot gives information regarding the speed of the object. Point B has a slope of zero, which means the object is momentarily stationary. When determining if an object is "faster" or "slower" than another object, we only need to consider the magnitude of the slope, *not* its sign.

2.25 **A** (increasing). If the car's velocity and acceleration vectors point in the same direction, the car will speed up.

2.26 **E** (decreasing but then increasing). Initially the velocity and acceleration of the car point in opposite directions, which means the car will slow down. Eventually the car will come to a stop, turn around, and start to move faster and faster in this direction. Therefore, the speed decreases and then increases. Remember that the speed is the magnitude of the velocity vector.

2.27 **A** (increasing). If the car's velocity and acceleration vectors point in the same direction, the car will speed up. Because the magnitude of the acceleration is decreasing with time (but always pointing in the same direction), the *rate at which the car speeds up* will decrease.

2.28 **E** (decreasing but then increasing). Initially the velocity and acceleration of the car point in opposite directions, which means the car will slow down. Eventually the car will come to a stop, turn around, and start to move faster and faster in this direction. Because the magnitude of the acceleration is decreasing with time (but always pointing in the same direction), the *rate at which the car's velocity changes* will decrease.

2.29 **C** (the two balls hit the ground at the same time). Both balls are undergoing free fall, which means they both accelerate at the same rate equal to the acceleration due to gravity. The mass of the object does not factor into this calculation.

2.30 **C** ($v = 0$, but $a = 9.8\ \text{m/s}^2$). Right before the ball reaches its highest point, it is moving upward. Right after, it is moving downward. Because the ball changes its direction, its velocity must equal zero at that point. The ball is always under the influence of Earth's gravity, so its acceleration is equal to $9.8\ \text{m/s}^2$.

Estimation Questions

2.31 The average car takes about 10–15 s to reach highway speeds. Cars can brake faster than this, say, 5–10 s.

2.32 A marathon is just over 26 miles in length, and a runner completes it in 5 hours. The average speed of the runner is equal to the total distance she covered divided by the time it took to do so:

$$v_{\text{average}} = \frac{26\ \text{mi}}{5\ \text{hr}} = \boxed{5.2\ \text{mph}} \times \frac{1\ \text{hr}}{3600\ \text{s}} \times \frac{1600\ \text{m}}{1\ \text{mi}} = \boxed{2.3\frac{\text{m}}{\text{s}}}$$

2.33 A swimmer completes a 50-m lap in 100 s. His average speed is equal to

$$v_{\text{average}} = \frac{50\ \text{m}}{100\ \text{s}} \times \frac{3600\ \text{s}}{1\ \text{hr}} \times \frac{1\ \text{km}}{1000\ \text{m}} = \boxed{1.8\frac{\text{km}}{\text{hr}}}$$

2.34 A 1000-km airline trip takes 3 hr in total. Of those 3 hr, the plane is airborne for 2.5 hr. The average speed of the airplane is

$$v_{\text{average}} = \frac{1000\ \text{km}}{2.5\ \text{hr}} = \boxed{400\frac{\text{km}}{\text{hr}}}$$

2.35 The fastest time for the 100-m race for men was set by Usain Bolt (9.58 s) and for women by Florence Griffith-Joyner (10.49 s). These times correspond to top speeds of 10.4 m/s and 9.5 m/s, respectively. A runner who falls in the mud will take much longer to come to rest than one who falls on a running track. Let's assume it takes 2 s to come to rest in the mud but only 0.5 s on a track. The magnitudes of the acceleration for each case are

$$a_{\text{Bolt, mud}} = \frac{\Delta v}{\Delta t} = \frac{10.4\frac{\text{m}}{\text{s}}}{2\ \text{s}} = \boxed{5.2\frac{\text{m}}{\text{s}^2}}$$

$$a_{\text{FloJo, mud}} = \frac{\Delta v}{\Delta t} = \frac{9.5\frac{\text{m}}{\text{s}}}{2\text{ s}} = \boxed{4.8\frac{\text{m}}{\text{s}^2}}$$

$$a_{\text{Bolt, track}} = \frac{\Delta v}{\Delta t} = \frac{10.4\frac{\text{m}}{\text{s}}}{0.5\text{ s}} = \boxed{21\frac{\text{m}}{\text{s}^2}}$$

$$a_{\text{FloJo, mud}} = \frac{\Delta v}{\Delta t} = \frac{9.5\frac{\text{m}}{\text{s}}}{0.5\text{ s}} = \boxed{19\frac{\text{m}}{\text{s}^2}}$$

2.36 We first need to determine the speed with which the cat leaves the floor. Say the cat just lands on a 1-m-tall countertop. Its initial speed was

$$v^2 - v_0^2 = 2a(\Delta y)$$

$$(0)^2 - v_0^2 = 2(-g)(\Delta y)$$

$$v_0 = \sqrt{2\left(9.8\frac{\text{m}}{\text{s}^2}\right)(1\text{ m})} = 4.5\frac{\text{m}}{\text{s}}$$

It takes a cat about 0.5 s to accelerate from rest to just before it leaves the ground, which makes its acceleration

$$a = \frac{\Delta v}{\Delta t} = \frac{4.5\frac{\text{m}}{\text{s}} - 0}{0.5\text{ s}} = \boxed{9\frac{\text{m}}{\text{s}^2}}$$

2.37 On the open sea, a cruise ship travels at a speed of approximately 10 m/s. We need to estimate the time it takes the ship to reach its cruising speed from rest. We should expect that it takes more than a few minutes but less than a full hour; let's estimate the time to be 0.5 hr. The magnitude of the average acceleration of the cruise ship is the change in the ship's speed divided by the time interval over which that change occurs:

$$a_{\text{average}} = \frac{\Delta v}{\Delta t} = \frac{10\frac{\text{m}}{\text{s}}}{0.5\text{ hr}} \times \frac{1\text{ hr}}{3600\text{ s}} = \boxed{0.006\frac{\text{m}}{\text{s}^2}}$$

2.38 Earth is 1.5×10^8 km from the Sun. Assuming Earth's orbit is circular, the total distance Earth travels in one rotation is $2\pi(1.5 \times 10^8\text{ km}) \approx 1 \times 10^{10}$ km. Earth completes one rotation around the Sun in 1 year $\left(1\text{ yr} \times \frac{365.25\text{ day}}{1\text{ yr}} \times \frac{24\text{ hr}}{1\text{ day}} = 8766\text{ hr}\right)$.
We can assume that the speed of Earth is constant, which means the instantaneous speed is equal to the average speed. The average speed of Earth is

$$v_{\text{average}} = \frac{1 \times 10^{10}\text{ km}}{8766\text{ hr}} = \boxed{1.1 \times 10^6\frac{\text{km}}{\text{hr}}}.$$

2.39 Displacement is a vector quantity. Chances are a swimmer will swim across the length of the pool and then back to her starting point. If so, the displacement is equal to zero. At best, her displacement will be the length of the pool. Distance, on the other hand, is a scalar quantity. An Olympic-size swimming pool is 50 m long. If a swimmer completes 100 laps (a single lap is from one side of the pool to the other), she travels a distance of 5 km.

2.40 A pitcher can throw a fastball around 90 mph, which is around 40 m/s. His windup takes a few seconds, say, 3 s. Therefore, the acceleration of the baseball is

$$a = \frac{\Delta v}{\Delta t} = \frac{40\frac{\text{m}}{\text{s}} - 0}{3\ \text{s}} = \boxed{13\frac{\text{m}}{\text{s}^2}}.$$

The speed of a soccer ball is much less than a fastball, but it reaches its top speed in a much shorter time. We'll say a soccer ball has a speed of 10 m/s and attains that speed from rest in 0.1 s: $a = \dfrac{\Delta v}{\Delta t} = \dfrac{10\frac{\text{m}}{\text{s}} - 0}{0.1\ \text{s}} = \boxed{100\frac{\text{m}}{\text{s}^2}}.$

2.41 We can make a plot of position versus time and determine the equations for each region by manually calculating the slope and y intercepts or by using a computer program to fit the data in each time interval.

t(s)	x(m)
0	−12
1	−6
2	0
3	6
4	12
5	15
6	15
7	15
8	15
9	18
10	24
11	33
12	45
13	60
14	65
15	70
16	75
17	80
18	85
19	90
20	95
21	100
22	90
23	80
24	70
25	70

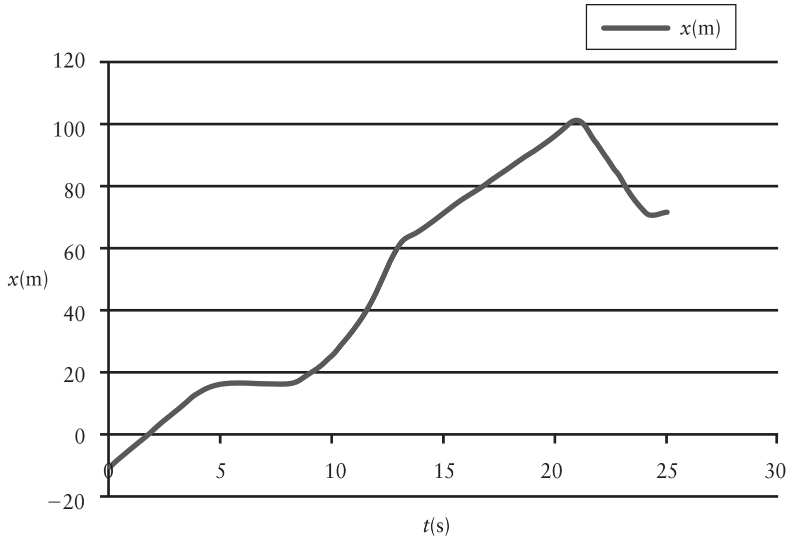


Figure 2-3 Problem 41

Between $t = 0$ s and $t = 4.5$ s:

$$x(t) = \frac{12\text{ m} - (-12\text{ m})}{4\text{ s}}(t - (2\text{ s})) = \left(6\frac{\text{m}}{\text{s}}\right)(t - (2\text{ s}))$$

Between $t = 4.5$ s and $t = 8$ s: $x(t) = 15$ m.

Between $t = 8$ s and $t = 13$ s:

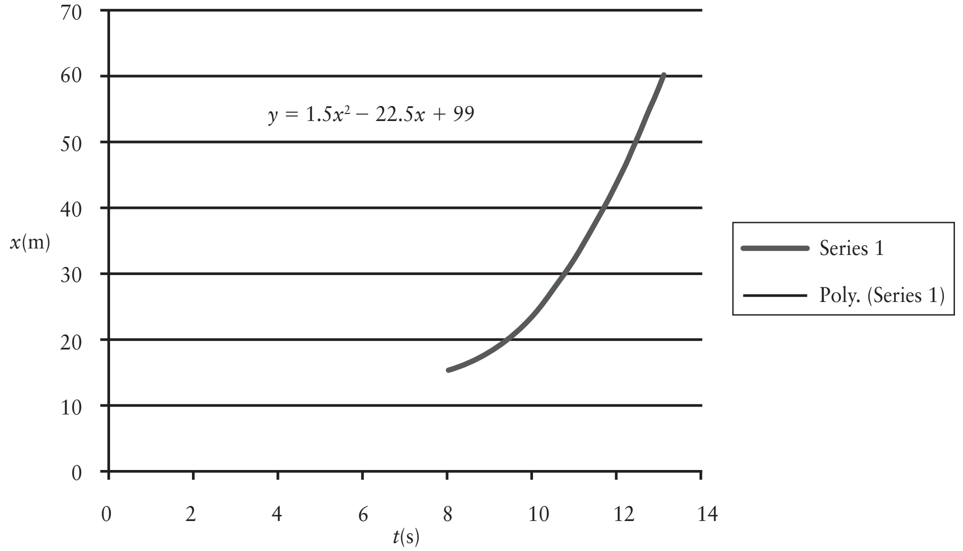


Figure 2-4 Problem 41

We can fit the data to a parabola in order to get position versus time in this region:

$$x(t) = \left(1.5\frac{\text{m}}{\text{s}^2}\right)t^2 - \left(22.5\frac{\text{m}}{\text{s}}\right)t + (99\text{ m}) = \left(1.5\frac{\text{m}}{\text{s}^2}\right)(t - 7.5\text{ s})^2 + (14.6\text{ m})$$

Between $t = 13$ s and $t = 21$ s:

$$x(t) = \frac{100\text{ m} - 60\text{ m}}{8\text{ s}}(t - 13\text{ s}) + (60\text{ m}) = \left(5\frac{\text{m}}{\text{s}}\right)(t - 13\text{ s}) + (60\text{ m})$$

Between $t = 21$ s and $t = 24$ s:

$$x(t) = \frac{70\text{ m} - 100\text{ m}}{3\text{ s}}(t - 21\text{ s}) + (100\text{ m}) = \left(-10\frac{\text{m}}{\text{s}}\right)(t - 21\text{ s}) + (100\text{ m})$$

Between $t = 24$ s and $t = 25$ s: $x(t) = 70$ m.

2.42 The acceleration of the motion is equal to the second derivative of the position versus time plot. The plot of position versus time looks like a parabola that opens upward, which means the acceleration will be positive and constant. Upon closer inspection, some data points are marked:

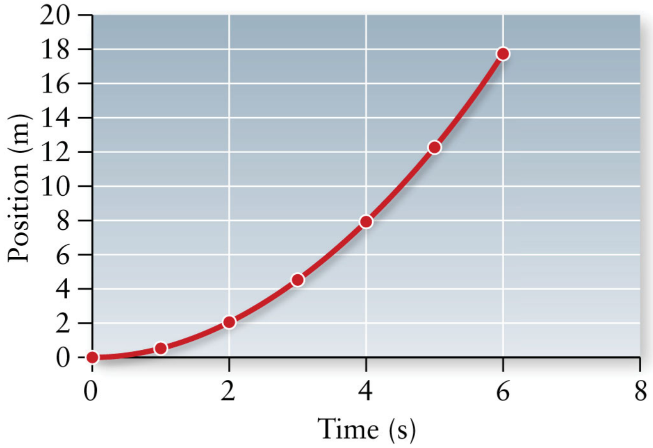


Figure 2-5 Problem 42

t(s)	x(m)
0	0
1	0.5
2	2
3	4.5
4	8
5	12
6	18

These values are consistent with the function $x(t) = \frac{1}{2}t^2$. Therefore, the acceleration is $a = \frac{d^2x}{dt^2} = \frac{d^2}{dt^2}\left(\frac{1}{2}t^2\right) = \frac{d}{dt}(t) = \boxed{1\frac{\text{m}}{\text{s}^2}}$.

Problems

2.43

SET UP

A list of speeds is given in various units. We need to convert between common units of speed. Some of the units (for example, m/s, km/hr, mi/hr) are more common units of speed than others (for example, mi/min). The conversions 1 mi = 1.61 km and 1 mi = 5280 ft will be useful.

SOLVE

A) $$30\frac{\text{m}}{\text{s}} \times \frac{3600\text{ s}}{1\text{ hr}} \times \frac{1\text{ km}}{1000\text{ m}} = \boxed{108\frac{\text{km}}{\text{hr}}}$$

B) $$14\frac{\text{mi}}{\text{hr}} \times \frac{1\text{ hr}}{60\text{ min}} = \boxed{\frac{7\text{ mi}}{30\text{ min}} \approx 0.23\frac{\text{mi}}{\text{min}}}$$

C) $$90\frac{\text{km}}{\text{s}} \times \frac{3600\text{ s}}{1\text{ hr}} \times \frac{1\text{ mi}}{1.61\text{ km}} = \boxed{2 \times 10^5\frac{\text{mi}}{\text{hr}}}$$

D) $$88\frac{\text{ft}}{\text{s}} \times \frac{1\text{ mi}}{5280\text{ ft}} \times \frac{3600\text{ s}}{1\text{ hr}} = \boxed{60\frac{\text{mi}}{\text{hr}}}$$

E) $$100\frac{\text{mi}}{\text{hr}} \times \frac{1\text{ hr}}{3600\text{ s}} \times \frac{1.61\text{ km}}{1\text{ mi}} \times \frac{1000\text{ m}}{1\text{ km}} = \boxed{40\frac{\text{m}}{\text{s}}} \text{ (to one significant figure)}$$

REFLECT

A good conversion to know: To convert from m/s into mi/hr multiply by 2.2. This will help build intuition when working with speeds in SI units.

2.44

SET UP

A bowling ball starts at a position of $x_1 = 3.5$ cm and ends at $x_2 = -4.7$ cm. It takes the ball 2.5 s for it to move from x_1 to x_2. The average velocity of the ball is equal to the displacement of the ball divided by the time interval. The displacement is the (final position) – (initial position). The ball only moves in one dimension, so we can use a positive or negative sign to denote the direction of the velocity.

SOLVE

$$v_{\text{average}} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{(-4.7\text{ cm}) - (3.5\text{ cm})}{(5.5\text{ s}) - (3.0\text{ s})} = \frac{-8.2\text{ cm}}{2.5\text{ s}} = \boxed{-3.3\frac{\text{cm}}{\text{s}}}$$

REFLECT

The bowling ball moves toward negative x, which is consistent with the sign of the average velocity.

2.45

SET UP

A jogger runs 13 km in 3.25 hr. We can calculate his average speed directly from these data: the distance he covered divided by the time it took him.

SOLVE

$$v_{\text{average}} = \frac{13\text{ km}}{3.25\text{ hr}} = \boxed{4.0\frac{\text{km}}{\text{hr}}}$$

REFLECT

This corresponds to about 2.5 mph, which is a little slow for a jogging pace.

2.46

SET UP

The Olympic record for the marathon, which is 26.2 mi long, is 2 hr, 6 min, and 32 s. After converting the time into hours and the distance into kilometers, we can divide the two to find the runner's average speed.

SOLVE

$$6 \text{ min} \times \frac{1 \text{ hr}}{60 \text{ min}} = 0.1 \text{ hr}$$

$$32 \text{ s} \times \frac{1 \text{ hr}}{3600 \text{ s}} = 0.0089 \text{ hr}$$

Total time elapsed:

$$(2.00 \text{ hr}) + (0.10 \text{ hr}) + (0.0089 \text{ hr}) = 2.11 \text{ hr}$$

$$v_{\text{average}} = \frac{26.2 \text{ mi}}{2.11 \text{ hr}} \times \frac{1.61 \text{ km}}{1 \text{ mi}} = \boxed{20.0\frac{\text{km}}{\text{hr}}}$$

REFLECT

This is little more than 12 mph. This is an extremely fast running pace, which makes sense since it is the Olympic record pace! At that pace, the runner would complete 1 mi in about 5 min.

2.47

SET UP

Kevin swims 4000 m in 1 hr. Because he ends at the same location as he starts, his total displacement is 0 m. This means his average velocity, which is his displacement divided by the time interval, is also zero. His average *speed*, on the other hand, is nonzero. The average speed takes the total distance covered into account.

SOLVE

Part a) Displacement = 0 m, so his $\boxed{\text{average velocity} = 0}$.

Part b)

$$v_{\text{average}} = \frac{4000 \text{ m}}{1 \text{ hr}} \times \frac{1 \text{ km}}{1000 \text{ m}} = \boxed{4\frac{\text{km}}{\text{hr}}}$$

Part c)

$$v_{\text{average}} = \frac{25 \text{ m}}{9.27 \text{ s}} \times \frac{1 \text{ km}}{1000 \text{ m}} \times \frac{3600 \text{ s}}{1 \text{ hr}} = \boxed{9.7\frac{\text{km}}{\text{hr}} = 2.7\frac{\text{m}}{\text{s}}}$$

REFLECT

Usually the terms "velocity" and "speed" are used interchangeably in everyday English, but the distinction between the two is important in physics. Kevin's average speed of 4 km/hr is about 2.5 mph. For comparison, Michael Phelps's record in the 100-m butterfly is 49.82 s, which gives him an average speed of 7.2 km/hr or 4.5 mph.

2.48

SET UP

The distance between the lecture hall and the student's house is 12.2 km. It takes the student 21 min to bike from campus to her house and 13 min to bike from her house back to campus. The displacement for her round trip is zero because she starts and ends at the same location (the lecture hall). This means her average velocity for the round trip is zero, since velocity depends on the displacement (rather than the distance). Since we are asked to calculate the average velocity for each leg of her trip, we need to decide on a positive sense of direction. Let's call the direction from the lecture hall toward her house positive; this means the trip from her house to the lecture hall is negative. The average speed of her entire trip is equal to the total distance she covers divided by the time it takes.

SOLVE

Part a) Displacement = 0 m; therefore, $\boxed{\text{average velocity} = 0}$.

Part b)

$$v_{\text{average}} = \frac{12.2\text{ km}}{21\text{ min}} \times \frac{60\text{ min}}{1\text{ hr}} = \boxed{35\frac{\text{km}}{\text{hr}}}$$

Part c)

$$v_{\text{average}} = \frac{12.2\text{ km}}{13\text{ min}} \times \frac{60\text{ min}}{1\text{ hr}} = \boxed{-56\frac{\text{km}}{\text{hr}}}$$

Part d)

$$v_{\text{average}} = \frac{24.4\text{ km}}{34\text{ min}} \times \frac{60\text{ min}}{1\text{ hr}} = \boxed{43\frac{\text{km}}{\text{hr}}}$$

REFLECT

Average velocity takes the direction of the motion into account, while the average speed does not.

2.49

SET UP

The magnitude of a school bus's average velocity is 56 km/hr, and it takes 0.7 hr to arrive at school. We can use the definition of average velocity to determine the bus's displacement.

SOLVE

$$\Delta x = (v_{\text{average}})(\Delta t) = \left(56\frac{\text{km}}{\text{hr}}\right)(0.70\text{ hr}) = \boxed{39\text{ km}}$$

REFLECT

A speed of 56 km/hr is about 35 mph.

2.50

SET UP

A car is traveling at 80 km/hr and is 1500 m (1.5 km) behind a truck traveling at 70 km/hr. We can consider the relative motion of the car with respect to the truck; this is the same, albeit easier, question as the original one. The car is moving 10 km/hr faster than the truck, which means we can consider the truck to be stationary and the car traveling at a speed of 10 km/hr. Now we can calculate the time it takes for a car traveling at a speed of 10 km/hr to cover 1.5 km.

SOLVE

$$v = \frac{\Delta x}{\Delta t}$$

$$\Delta t = \frac{\Delta x}{v_{\text{relative}}} = \frac{1.5\text{ km}}{10\frac{\text{km}}{\text{hr}}} = 0.15\text{ hr} \times \frac{60\text{ min}}{1\text{ hr}} = \boxed{9\text{ min}}$$

REFLECT

We could have solved this question algebraically. Treating the car as the origin, the position of the car as a function of time is $x_{\text{car}} = \left(80\frac{\text{km}}{\text{hr}}\right)t$ and the position of the truck is $x_{\text{truck}} = \left(70\frac{\text{km}}{\text{hr}}\right)t + (1.5\text{ km})$. The two positions are equal to one another when the car overtakes the truck. Solving for t will give the time it takes for the car to reach the truck.

2.51

SET UP

It takes a sober driver 0.32 s to hit the brakes, while a drunk driver takes 1.0 s to hit the brakes. In both cases the car is initially traveling at 90 km/hr. Assuming it takes the same distance to come to a stop once the brakes are applied, the drunk driver travels for an extra $(1.0 - 0.32)\text{ s} = 0.68\text{ s}$ at 90 km/hr before hitting the brakes. We can use the definition of average speed to calculate the extra distance the drunk driver travels.

SOLVE

$$\Delta x = (v_{\text{average}})(\Delta t) = \left(90\frac{\text{km}}{\text{hr}} \times \frac{1\text{ hr}}{3600\text{ s}} \times \frac{1000\text{ m}}{1\text{ km}}\right)(0.68\text{ s}) = \boxed{17\text{ m}}$$

REFLECT

The impaired driver travels an extra distance of over 55 ft before applying the brakes.

2.52

SET UP

A jet travels 4100 km from San Francisco to Montreal. It waits 1 hr and then flies back to San Francisco. The entire trip (flight + layover) is 11 hr, 52 min, or 11.867 hr. The westbound trip (from Montreal to San Francisco) takes 48 min (0.8 hr) longer than the eastbound trip. We'll call the duration of the westbound trip t_{WB} and the duration of the eastbound trip t_{EB}. We know that the sum of these two legs is equal to the flight time, which is the total trip time minus the layover time, or 10.867 hr. Now we have two equations and two unknowns and can solve for the duration of each leg.

The average speed of the overall trip is the total distance traveled, which is 8200 km, divided by the total time.

SOLVE

Calculating the time for each leg:

$$t_{WB} + t_{EB} = \left(10\frac{52}{60}\text{ hr}\right) = 10.867\text{ hr}$$

$$t_{WB} = t_{EB} + \left(\frac{48}{60}\text{ hr}\right) = t_{EB} + (0.8\text{ hr})$$

$$(t_{EB} + 0.8) + t_{EB} = 10.867$$

$$t_{EB} = \frac{10.067}{2} = \boxed{5.03\text{ hr}}$$

$$t_{WB} = t_{EB} + (0.8\text{ hr}) = (5.03\text{ hr}) + (0.8\text{ hr}) = \boxed{5.83\text{ hr}}$$

Average speed for the overall trip:

$$v = \frac{2(4100\text{ km})}{11.867\text{ hr}} = \boxed{691\frac{\text{km}}{\text{hr}}}$$

Average speed, not including the layover:

$$v = \frac{2(4100\text{ km})}{10.867\text{ hr}} = \boxed{755\frac{\text{km}}{\text{hr}}}$$

REFLECT

It makes sense that the average speed of the plane is higher when we don't include the layover time in our calculation.

2.53

SET UP

A list of position data as a function of time is given. We need to make a plot with position on the y-axis and time on the x-axis. The average speed of the horse over the given intervals is equal to the distance the horse traveled divided by the time interval.

SOLVE

Plot of position versus time:

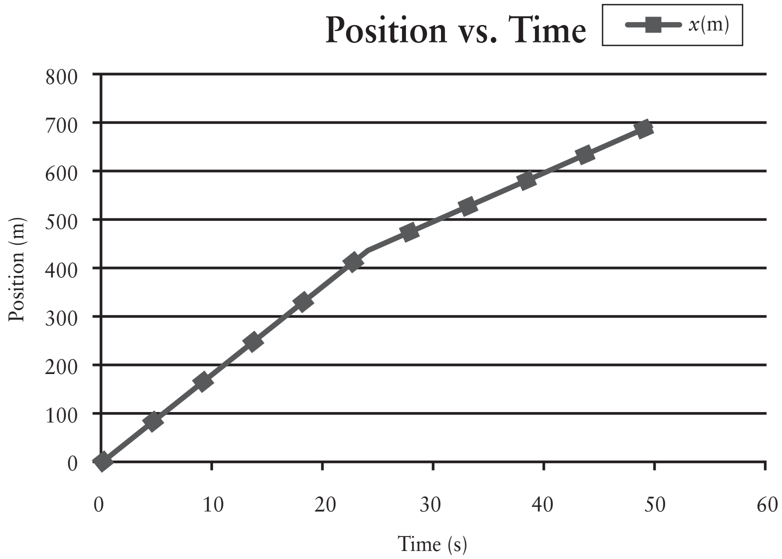


Figure 2-6 Problem 53

Part a)

$$v_{\text{average}} = \frac{x_{10\text{ s}} - x_{0\text{ s}}}{t_{10\text{ s}} - t_{0\text{ s}}} = \frac{180\text{ m}}{10\text{ s}} = \boxed{18\frac{\text{m}}{\text{s}}}$$

Part b)

$$v_{\text{average}} = \frac{x_{30\text{ s}} - x_{10\text{ s}}}{t_{30\text{ s}} - t_{10\text{ s}}} = \frac{(500\text{ m}) - (180\text{ m})}{(30\text{ s}) - (10\text{ s})} = \frac{320\text{ m}}{20\text{ s}} = \boxed{16\frac{\text{m}}{\text{s}}}$$

Part c)

$$v_{\text{average}} = \frac{x_{50\text{ s}} - x_{0\text{ s}}}{t_{50\text{ s}} - t_{0\text{ s}}} = \frac{700\text{ m}}{50\text{ s}} = \boxed{14\frac{\text{m}}{\text{s}}}$$

REFLECT

Remember that graphs are described as "y-axis" versus "x-axis." The speed of the horse is given by the slope of the position versus time plot. The horse's speed is constant over two intervals: 0–25 s and 25–50 s. We can easily see from the plot that the horse was slower in the second half compared to the first half.

2.54

SET UP

A plot of a red blood cell's position versus time is provided and we are asked to determine the instantaneous velocity of the blood cell at $t = 10$ ms. The instantaneous velocity is equal to the slope of a position versus time graph. It is difficult to estimate the slope of a tangent line. Fortunately, the slope of the graph looks to be reasonably constant between $t = 8$ ms and $t = 11.25$ s. We can determine the slope over this region and use that to approximate the instantaneous velocity at $t = 10$ s.

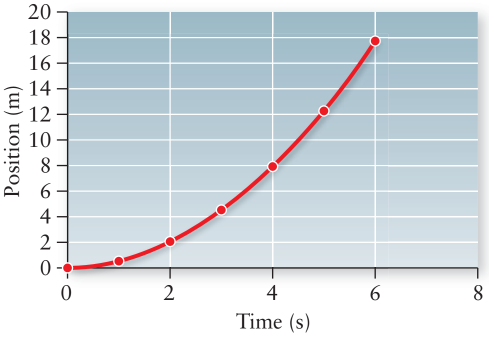


Figure 2-7 Problem 54

SOLVE

$$v = \frac{x_{11.25\text{ ms}} - x_{8\text{ ms}}}{t_{11.25\text{ ms}} - t_{8\text{ ms}}} = \frac{(5\text{ mm}) - (3.25\text{ mm})}{(11.25\text{ ms}) - (8\text{ ms})} = \frac{1.75\text{ mm}}{3.25\text{ ms}} = \boxed{0.54\frac{\text{mm}}{\text{ms}} = 0.54\frac{\text{m}}{\text{s}}}$$

REFLECT

The red blood cell is moving toward positive x as time goes on, which corresponds to the positive sign of the instantaneous velocity. The ratio between millimeters and milliseconds is the same as the ratio between meters and seconds.

2.55

SET UP

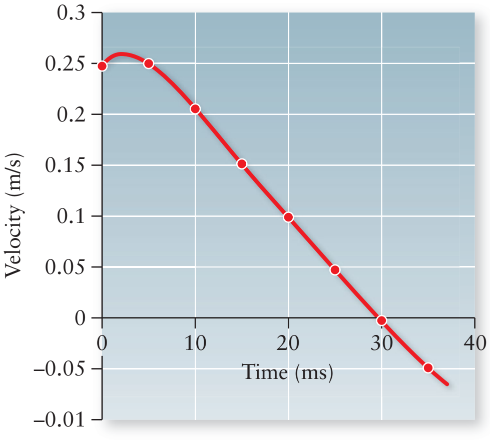


Figure 2-8 Problem 55

We are given a plot of velocity versus time for a kangaroo rat. The displacement of the rat is related to the area under the velocity versus time curve. We can split the graph into simple, geometric shapes in order to easily approximate the area for the given time intervals. Area below the x-axis is considered to be negative.

SOLVE

Part a) 0 to 5 s

We will model this area as a rectangle:

$$\Delta x_{0-5\text{ s}} = \left(0.25\frac{\text{m}}{\text{s}}\right)(5\text{ s}) = \boxed{1.25\text{ m}}$$

Part b) 0 to 10 s

We already know the area from 0–5 s. We just need to calculate the area from 5–10 s, which we can model as a trapezoid:

$$\Delta x_{0-10\text{ s}} = \Delta x_{0-5\text{ s}} + \Delta x_{5-10\text{ s}} = (1.25\text{ m}) + \frac{1}{2}((0.25\text{ m}) + (0.2\text{ m}))(5\text{ s})$$

$$= (1.25\text{ m}) + (1.13\text{ m}) = \boxed{2.4\text{ m}}$$

Part c) 10 to 25 s

We will model this as a trapezoid:

$$\Delta x_{10-25\text{ s}} = \frac{1}{2}((0.2\text{ m}) + (0.05\text{ m}))(15\text{ s}) = \boxed{1.9\text{ m}}$$

Part d) 0 to 35 s

Parts (b) and (c) give us the area from 0–25 s. We can model the areas from 25–30 s and 30–35 s as triangles of equal area. The area from 30–35 s is negative.

$$\Delta x_{0-35\text{ s}} = \Delta x_{0-10\text{ s}} + \Delta x_{10-25\text{ s}} + \Delta x_{25-30\text{ s}} + \Delta x_{30-35\text{ s}}$$

$$= (2.4\text{ m}) + (1.9\text{ m}) + \frac{1}{2}\left(0.05\frac{\text{m}}{\text{s}}\right)(5\text{ s}) - \frac{1}{2}\left(0.05\frac{\text{m}}{\text{s}}\right)(5\text{ s}) = \boxed{4.3\text{ m}}$$

REFLECT

Remember that areas below the x-axis are negative. In this case, the negative area corresponds to a negative displacement, which means the kangaroo rat is moving back toward its initial position. This makes sense because the velocity of the rat in this region is negative.

2.56

SET UP

We are given two polynomials and asked to differentiate them with respect to t. We can use the power rule for polynomials $\left(\frac{d}{dx}x^n = nx^{n-1}\right)$ to find the derivative.

SOLVE

Part a)

$$\frac{d}{dt}(5t^2 + 4t + 3) = \boxed{10t + 4}$$

Part b)

$$\frac{d}{dt}(t^2 - 4t - 8) = \boxed{2t - 4}$$

REFLECT

Being able to take derivatives of basic functions is a very useful skill to have when solving physics problems.

2.57

SET UP

We are given two polynomials and asked to differentiate them with respect to t and evaluate the derivative at $t = 2$. We can use the power rule for polynomials $\left(\frac{d}{dx}x^n = nx^{n-1}\right)$ to find the derivative.

SOLVE

Part a)

$$\frac{d}{dt}(t^2 + t + 1)_{t=2} = (2t + 1)_{t=2} = \boxed{5}$$

Part b)

$$\frac{d}{dt}(2t^3 - 4t^2 - 4)_{t=2} = (6t^2 - 8t)_{t=2} = (6 \cdot 2^2 - 8 \cdot 2) = \boxed{8}$$

REFLECT

Be sure to take the derivative of the function first and then evaluate it at the given value.

2.58

SET UP

We are given two polynomials and asked to integrate them with respect to t.

SOLVE

Part a)

$$\int 6t\, dt = \boxed{3t^2}$$

Part b)

$$\int (5t^4 + 3t^2 + 2t)dt = \boxed{t^5 + t^3 + t^2}$$

REFLECT

Normally indefinite integrals have a constant of integration associated with them (for example, $\int 6t\, dt = 3t^2 + C$), but we were told to ignore them.

2.59

SET UP
We are asked to determine the definite integrals of two polynomials.

SOLVE
Part a)

$$\int_0^2 (12t^2+5t+4)dt = \left[4t^3+\frac{5}{2}t^2+4t\right]_0^2 = \left[\left(4\cdot 2^3+\frac{5}{2}\cdot 2^2+4\cdot 2\right)-0\right] = \boxed{50}$$

Part b)

$$\int_{-2}^2 (4t^3+2t+1)dt = [t^4+t^2+t]_{-2}^2 = [(2^4+2^2+2)-((-2)^4+(-2)^2-2)] = \boxed{4}$$

REFLECT
Be careful to include the correct signs when evaluating definite integrals.

2.60

SET UP
The position of a rabbit as a function of time can be modeled by $x(t) = 50 + 2t^2$. The rabbit's displacement over the first 20 s is the difference in its position from $x(t = 0\text{ s})$ to $x(t = 20\text{ s})$. The instantaneous velocity of the rabbit at $t = 6$ s is calculated by differentiating the position function with respect to t and evaluating the derivative at $t = 6$ s. The average velocity of the rabbit is found by dividing its displacement by the time interval.

SOLVE
Part a)

$$\Delta x_{0-20\text{ s}} = x(20) - x(0) = (50 + 2\cdot(20)^2 - 50) = \boxed{800\text{ m}}$$

Part b)

$$v(t) = \frac{d}{dt}x(t) = \frac{d}{dt}(50+2t^2) = 4t$$

$$v(6) = 4\cdot 6 = \boxed{24\frac{\text{m}}{\text{s}}}$$

Part c)

$$v_{\text{average}} = \frac{x(8)-x(4)}{(8)-(4)} = \frac{(50+2\cdot(8)^2-50-2\cdot(4)^2)}{4} = \frac{96}{4} = \boxed{24\frac{\text{m}}{\text{s}}}$$

REFLECT
The function $x(t)$ is always increasing, so it makes sense that the displacement during the first 20 s is positive. The slope of the tangent line at $x(t = 6\text{ s})$ is equal to the slope of the line between $x(t = 4\text{ s})$ and $x(t = 8\text{ s})$. The velocity is positive because the rabbit is moving toward positive x.

2.61

SET UP

A runner starts from rest and reaches a top speed of 8.97 m/s. Her acceleration is 9.77 m/s^2, which is a constant. We know her initial speed, her final speed, and her acceleration, and we are interested in the time it takes for her to reach that speed. We can rearrange $v_f = v_0 + at$ and solve for t.

SOLVE

$$v_f = v_0 + at$$

$$t = \frac{v_f - v_0}{a}$$

$$t = \frac{8.97\frac{\text{m}}{\text{s}} - 0}{9.77\frac{\text{m}}{\text{s}^2}} = \boxed{0.918\text{ s}}$$

REFLECT

An acceleration of 9.77 m/s^2 means that her speed changes by 9.77 m/s every second. In this problem her speed changed a little less than that, so we expect the time elapsed to be a little less than a second.

2.62

SET UP

We are asked to compare the acceleration and displacement of a car over two 5-s time intervals. During the first interval, the car starts at 35 km/hr and accelerates up to 45 km/hr. In the second interval, the car starts at 65 km/hr and accelerates up to 75 km/hr. We are told the accelerations are constant, which means the average acceleration is equal to the instantaneous acceleration. The average acceleration is the change in the velocity over the time interval. Once we have the acceleration for each interval, we can use the constant acceleration equation $\Delta x = v_0 t + \frac{1}{2}at^2$ to calculate the displacement of the car in each interval.

SOLVE

Acceleration:

$$a_1 = \frac{\Delta v}{\Delta t} = \frac{\left(45\frac{\text{km}}{\text{hr}} - 35\frac{\text{km}}{\text{hr}}\right) \times \frac{1\text{ hr}}{3600\text{ s}} \times \frac{1000\text{ m}}{1\text{ km}}}{5\text{ s}} = \frac{2.78\frac{\text{m}}{\text{s}}}{5\text{ s}} = \boxed{0.55\frac{\text{m}}{\text{s}^2}}$$

$$a_2 = \frac{\Delta v}{\Delta t} = \frac{\left(75\frac{\text{km}}{\text{hr}} - 65\frac{\text{km}}{\text{hr}}\right) \times \frac{1\text{ hr}}{3600\text{ s}} \times \frac{1000\text{ m}}{1\text{ km}}}{5\text{ s}} = \frac{2.78\frac{\text{m}}{\text{s}}}{5\text{ s}} = \boxed{0.55\frac{\text{m}}{\text{s}^2}}$$

Displacement:

$$\Delta x_1 = v_{0_1}t + \frac{1}{2}a_1 t^2 = \left(35\frac{\text{km}}{\text{hr}} \times \frac{1\text{ hr}}{3600\text{ s}} \times \frac{1000\text{ m}}{1\text{ km}}\right)(5\text{ s}) + \frac{1}{2}\left(0.55\frac{\text{m}}{\text{s}^2}\right)(5\text{ s})^2$$

$$= (48.6\text{ m}) + (6.88\text{ m}) = \boxed{55.5\text{ m}}$$

$$\Delta x_2 = v_{0_2}t + \frac{1}{2}a_2t^2 = \left(65\ \frac{\text{km}}{\text{hr}} \times \frac{1\ \text{hr}}{3600\ \text{s}} \times \frac{1000\ \text{m}}{1\ \text{km}}\right)(5\ \text{s}) + \frac{1}{2}\left(0.55\ \frac{\text{m}}{\text{s}^2}\right)(5\ \text{s})^2$$
$$= (90.3\ \text{m}) + (6.88\ \text{m}) = \boxed{97.2\ \text{m}}$$

REFLECT

The accelerations are equal for the two intervals, which makes sense since the speeds increase by 10 km/hr in 5 s for both cases. The car moves farther in the second interval because the car is moving at higher speeds.

2.63

SET UP

A car starts at rest and reaches a speed of 34 m/s in 12 s. The average acceleration is the change in the velocity over the time interval.

SOLVE

$$a_{\text{average}} = \frac{\Delta v}{\Delta t} = \frac{\left(34\frac{\text{m}}{\text{s}} - 0\right)}{12\ \text{s}} = \boxed{2.8\frac{\text{m}}{\text{s}^2}}$$

REFLECT

A speed of 34 m/s is equal to about 76 mph, so it takes the car 12 s to go from 0 to 76 mph.

2.64

SET UP

A list of sports cars and the time it takes each one to accelerate to 60 mph from rest is given. First, we need to convert 60 mph into km/hr and then into m/s. Assuming the acceleration of each car is constant, we can calculate each acceleration using the change in the velocity divided by the time interval.

SOLVE

Converting 60 mi/hr into km/hr and m/s:

$$60\frac{\text{mi}}{\text{hr}} \times \frac{1.61\ \text{km}}{1\ \text{mi}} = \boxed{97\frac{\text{km}}{\text{hr}}} \times \frac{1\ \text{hr}}{3600\ \text{s}} \times \frac{1000\ \text{m}}{1\ \text{km}} = \boxed{27\frac{\text{m}}{\text{s}}}$$

Accelerations:

$$a_{\text{Bugatti}} = \frac{27\frac{\text{m}}{\text{s}} - 0}{2.4\ \text{s}} = \boxed{11.3\frac{\text{m}}{\text{s}^2}}$$

$$a_{\text{Caparo}} = \frac{27\frac{\text{m}}{\text{s}} - 0}{2.5\ \text{s}} = \boxed{10.8\frac{\text{m}}{\text{s}^2}}$$

$$a_{\text{Ultimo}} = \frac{27\frac{\text{m}}{\text{s}} - 0}{2.6\ \text{s}} = \boxed{10.4\frac{\text{m}}{\text{s}^2}}$$

$$a_{SSC} = \frac{27\frac{m}{s} - 0}{2.7\ s} = \boxed{10.0\frac{m}{s^2}}$$

$$a_{Saleen} = \frac{27\frac{m}{s} - 0}{2.8\ s} = \boxed{9.6\frac{m}{s^2}}$$

REFLECT

The Bugatti Veyron Super Sport is the fastest street-legal car in the world. Its top speed of just over 267 mph earned it a spot in the Guinness World Book of Records for the "Fastest Production Car" (http://www.guinnessworldrecords.com/records-1/fastest-production-car/).

2.65

SET UP

A Bugatti Veyron and Saleen S7 can accelerate from 0 to 60 mph in 2.4 s and 2.8 s, respectively. Assuming the acceleration of each car is constant, not only over that time interval but also for *any* time interval, we can approximate the acceleration of each car as the average acceleration. We first need to convert 60 mph into m/s in order to keep the units consistent. The acceleration is equal to the change in velocity divided by the change in time. We are interested in the distance each car travels when it accelerates from rest to 90 km/hr. We know the initial speed, final speed, and the acceleration, so we can use $v^2 - v_0^2 = 2a(\Delta x)$ to calculate Δx for each car. (See Problem 2.67 for a derivation of this equation.)

SOLVE

Finding the acceleration:

$$60\frac{mi}{hr} \times \frac{1.61\ km}{1\ mi} \times \frac{1\ hr}{3600\ s} \times \frac{1000\ m}{1\ km} = 27\frac{m}{s}$$

$$a_{Bugatti} = \frac{27\frac{m}{s} - 0}{2.4\ s} = 11.3\frac{m}{s^2}$$

$$a_{Saleen} = \frac{27\frac{m}{s} - 0}{2.8\ s} = 9.6\frac{m}{s^2}$$

Finding the distance:

$$v^2 - v_0^2 = 2a(\Delta x)$$

$$\Delta x = \frac{v^2 - v_0^2}{2a}$$

$$\Delta x_{Bugatti} = \frac{\left(90\frac{km}{hr} \times \frac{1000\ m}{1\ km} \times \frac{1\ hr}{3600\ s}\right)^2 - (0)^2}{2\left(11.3\frac{m}{s^2}\right)} = \boxed{28\ m}$$

$$\Delta x_{\text{Saleen}} = \frac{\left(90\frac{\text{km}}{\text{hr}} \times \frac{1000\text{ m}}{1\text{ km}} \times \frac{1\text{ hr}}{3600\text{ s}}\right)^2 - (0)^2}{2\left(9.6\frac{\text{m}}{\text{s}^2}\right)} = \boxed{33\text{ m}}$$

REFLECT

These values are around 92 ft and 108 ft, respectively. A speed limit of 90 km/hr is around 55 mph. Most people don't drive expensive sports cars on the highway every day, so these distances are (obviously) much smaller than we should expect for most everyday cars.

2.66

SET UP

A horse is traveling at a constant speed of 16 m/s. The time it takes to travel 4 m can be calculated directly from the definition of average speed.

SOLVE

$$v = \frac{\Delta x}{\Delta t}$$

$$\Delta t = \frac{\Delta x}{v} = \frac{4\text{ m}}{16\frac{\text{m}}{\text{s}}} = \boxed{\frac{1}{4}\text{ s}}$$

REFLECT

The horse is traveling at a constant speed in a straight line, so its acceleration is zero.

2.67

SET UP

We are asked to derive a constant acceleration equation that relates speed, position, and acceleration and eliminates time. Solving for time in the definition of acceleration, we can plug that expression into $x = x_0 + v_0 t + \frac{1}{2}at^2$ and rearrange.

SOLVE

Solving for t in the definition of acceleration:

$$a = \frac{v - v_0}{t}$$

$$t = \frac{v - v_0}{a}$$

Plugging in t:

$$x = x_0 + v_0 t + \frac{1}{2}at^2 = x_0 + v_0\left(\frac{v - v_0}{a}\right) + \frac{1}{2}a\left(\frac{v - v_0}{a}\right)^2$$

$$xa = x_0 a + v_0(v - v_0) + \frac{1}{2}(v - v_0)^2$$

$$(x - x_0)a = v_0 v - v_0^2 + \frac{1}{2}(v^2 - 2v_0 v + v_0^2)$$

$$2(x - x_0)a = 2v_0 v - 2v_0^2 + v^2 - 2v_0 v + v_0^2$$

$$\boxed{2(x - x_0)a = v^2 - v_0^2}$$

REFLECT

This equation is useful if we don't know or don't care about how long something is traveling.

2.68

SET UP

A car is traveling at an initial speed of 30 m/s and needs to come to a complete stop within 80 m. We can calculate the acceleration necessary to produce this motion by assuming the car's acceleration is constant.

SOLVE

$$v^2 - v_0^2 = 2a(\Delta x)$$

$$a = \frac{v^2 - v_0^2}{2(\Delta x)} = \frac{0^2 - \left(30\frac{\text{m}}{\text{s}}\right)^2}{2(80\text{ m})} = \boxed{-5.6\frac{\text{m}}{\text{s}^2}}$$

REFLECT

The negative sign associated with the acceleration implies the car is slowing down. At this acceleration, it would take the car 5.4 s to come to a stop.

2.69

SET UP

A sperm whale has an initial speed of 1 m/s and accelerates up to a final speed of 2.25 m/s at a constant rate of 0.1 m/s^2. Because we know the initial speed, final speed, and the acceleration, we can calculate the distance over which the whale travels by rearranging $v^2 - v_0^2 = 2a(\Delta x)$.

SOLVE

$$v^2 - v_0^2 = 2a(\Delta x)$$

$$\Delta x = \frac{v^2 - v_0^2}{2a} = \frac{\left(2.25\frac{\text{m}}{\text{s}}\right)^2 - \left(1\frac{\text{m}}{\text{s}}\right)^2}{2\left(0.1\frac{\text{m}}{\text{s}^2}\right)} = \boxed{20\text{ m}}$$

REFLECT

This is a little longer than the average size of an adult male sperm whale (about 16 m). It will take the whale 12.5 s to speed up from 1 m/s to 2.25 m/s.

2.70

SET UP

Paola starts from rest and launches herself off the ground with a speed of 4.43 m/s. The distance over which she accomplishes this acceleration is 20 cm (0.2 m). Assuming her acceleration is constant, we can calculate the acceleration by rearranging $v^2 - v_0^2 = 2a(\Delta x)$.

SOLVE

$$v^2 - v_0^2 = 2a(\Delta x)$$

$$a = \frac{v^2 - v_0^2}{2(\Delta x)} = \frac{\left(4.43\frac{\text{m}}{\text{s}}\right)^2 - (0)^2}{2(0.2\text{ m})} = \boxed{49\frac{\text{m}}{\text{s}^2}}$$

REFLECT

We can quickly estimate the expected acceleration in order to double-check our solution. The final speed is 4.43 m/s, which we need to square. The square of 4 is 16 and the square of 5 is 25. The square of 4.43 should be in between those values but closer to 16; let's choose 20. (The actual square of 4.43 is 19.6, so this is a good assumption.) The denominator is 4×10^{-1}, so 20 divided by 4 is 5, which we divide by 10^{-1} (which is the same as multiplying by 10^1). This gives an estimate of 50 m/s^2, which is consistent with our calculation.

2.71

SET UP

The position of an object as a function of time is described by $x(t) = 12 - 6t + 3.2t^2$ in SI units. To calculate the displacement between $t = 4$ s and $t = 8$ s, we first need to find the position at each of those times and then calculate $\Delta x = x(t = 8\text{ s}) - x(t = 4\text{ s})$. The velocity as a function of time is the first derivative of the position with respect to time. Evaluating the derivative at $t = 3$ s will give the velocity at that time. After differentiating $x(t)$, we can set $v(t)$ equal to zero and solve for t. The acceleration is the first derivative of the velocity with respect to time or the second derivative of the position with respect to time.

SOLVE

Part a)

$$x(t = 8\text{ s}) = 12 - 6(8) + 3.8(8)^2 = 169\text{ m}$$

$$x(t = 4\text{ s}) = 12 - 6(4) + 3.2(4)^2 = 39\text{ m}$$

$$\Delta x = x(t = 8\text{ s}) - x(t = 4\text{ s}) = (169\text{ m}) - (39\text{ m}) = \boxed{130\text{ m}}$$

Part b)

$$v(t) = \frac{d}{dt}x(t) = \frac{d}{dx}(12 - 6t + 3.2t^2) = \boxed{-6 + 6.4t}\text{ (SI units)}$$

$$v(t = 3\text{ s}) = -6 + 6.4(3) = \boxed{13.2\frac{\text{m}}{\text{s}}}$$

Part c)

$$v(t) = -6 + 6.4t = 0$$

$$\boxed{t = 0.94\text{ s}}$$

Part d)

$$a(t) = \frac{d}{dt}v(t) = \frac{d}{dx}(-6 + 6.4t) = \boxed{6.4\frac{\text{m}}{\text{s}^2}}$$

REFLECT

The object has a velocity of zero at $t = 0.94$ s, which means it changes direction at this point. Before this time, the object is moving toward negative x, stops, and then moves toward positive x. Its initial position, $x(0)$, is 12 m, while its position at $t = 0.94$ s is $x(0.94) = 9.2$ m and its position at $t = 2$ s is 12.8 m.

2.72

SET UP

The acceleration as a function of time for a given object is $a(t) = \left(6\frac{\text{m}}{\text{s}^2}\right) + \left(0.75\frac{\text{m}}{\text{s}^3}\right)t$. We can find the velocity as a function of time by integrating $a(t)$ over time. We are told that the velocity at $t = 0$ s is equal to zero, which allows us to solve for the constant of integration. We can find the velocity at $t = 5$ s once we have a functional form for the velocity. The displacement of the object from $t = 0$ s to $t = 5$ s is equal to the definite integral of the velocity over that time interval.

SOLVE

Part a)

$$a(t) = \left(6\frac{\text{m}}{\text{s}^2}\right) + \left(0.75\frac{\text{m}}{\text{s}^3}\right)t$$

$$v(t) = \int a(t)dt = \int\left(\left(6\frac{\text{m}}{\text{s}^2}\right) + \left(0.75\frac{\text{m}}{\text{s}^3}\right)t\right)dt = \left(6\frac{\text{m}}{\text{s}^2}\right)t + \left(\frac{0.75}{2}\frac{\text{m}}{\text{s}^3}\right)t^2 + C$$

We know that $v(0) = 0$, so $C = 0$ and $\boxed{v(t) = \left(6\frac{\text{m}}{\text{s}^2}\right)t + \left(\frac{0.75}{2}\frac{\text{m}}{\text{s}^3}\right)t^2}$.

Part b)

$$v(5\text{ s}) = \left(6\frac{\text{m}}{\text{s}^2}\right)(5\text{ s}) + \left(\frac{0.75}{2}\frac{\text{m}}{\text{s}^3}\right)(5\text{ s})^2 = \boxed{39\frac{\text{m}}{\text{s}}}$$

Part c)

$$\Delta x = \int_0^{5\text{ s}} v(t)dt = \int_0^{5\text{ s}}\left(\left(6\frac{\text{m}}{\text{s}^2}\right)t + \left(\frac{0.75}{2}\frac{\text{m}}{\text{s}^3}\right)t^2\right)dt = \left[\left(3\frac{\text{m}}{\text{s}^2}\right)t^2 + \left(\frac{0.75}{6}\frac{\text{m}}{\text{s}^3}\right)t^3\right]_0^{5\text{ s}}$$

$$= \left(3\frac{\text{m}}{\text{s}^2}\right)(5\text{ s})^2 + \left(\frac{0.75}{6}\frac{\text{m}}{\text{s}^3}\right)(5\text{ s})^3 = \boxed{91\text{ m}}$$

REFLECT
This is an example of a *non-constant* acceleration because the acceleration depends on the time. In part (a) we took an indefinite integral because we were interested in a *function* for an answer. We needed to use the condition that the initial velocity was equal to zero in order to eliminate the constant of integration. We took a definite integral in part (c) because we were interested in a *number* for an answer.

2.73

SET UP
A ball is dropped from an initial height $y_0 = 25$ m above the ground, which we'll call $y = 0$, and undergoes free fall. The initial speed of the ball is zero ($v_0 = 0$). Part (a) asks the speed of the ball when it is at a final position of $y_f = 10$ m. We know the initial speed, the initial location, the final location, and the acceleration of the ball ($a = -g$) and are interested in the final speed. We can use $v^2 - v_0^2 = 2a(\Delta y)$ to solve for v. Part (b) asks about the time it takes the ball to fall from its initial position to the ground. Here we know the initial speed, the initial location, the final location, and the acceleration of the ball and are interested in the total time. This suggests that we use $\Delta y = v_0 t + \frac{1}{2}at^2 = 0 + \frac{1}{2}at^2$.

SOLVE

Part a)

$$v^2 - v_0^2 = 2a(\Delta y)$$

$$v = \sqrt{v_0^2 + 2a(\Delta y)} = \sqrt{0^2 + 2\left(-9.8\frac{\text{m}}{\text{s}^2}\right)((10\text{ m}) - (25\text{ m}))} = \boxed{17\frac{\text{m}}{\text{s}}}$$

Part b)

$$\Delta y = v_0 t + \frac{1}{2}at^2 = 0 + \frac{1}{2}at^2$$

$$t = \sqrt{\frac{2\Delta y}{a}} = \sqrt{\frac{2(0 - 25\text{ m})}{\left(-9.8\frac{\text{m}}{\text{s}^2}\right)}} = \boxed{2.3\text{ s}}$$

REFLECT
Remember that value of g is positive and equal to 9.8 m/s^2 and that the acceleration due to gravity points downward, which is usually negative y. A height of 25 m is around 82 ft, so a total time of 2.3 s seems reasonable.

2.74

SET UP
A kiwi is dropped from the roof of a building. We are told that the location of the roof is $y = 0$ and that the kiwi starts from rest. Once the kiwi is released it undergoes free fall and has a constant acceleration $a = -g = -9.8$ m/s^2. We can calculate the position of the kiwi

as a function of time from $y(t) = y_0 + v_0 t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}(-g)t^2$ and its velocity from $v(t) = v_0 + at = 0 + (-g)t$.

SOLVE

t(s)	y(m)	v(m/s)	a(m/s^2)
0	0	0	0
1	−4.9	−9.8	−9.8
2	−19.6	−19.6	−9.8
3	−44.1	−29.4	−9.8
4	−78.4	−39.2	−9.8
5	−122.5	−49	−9.8
10	−490	−98	−9.8

REFLECT

Because the roof is chosen to be $y = 0$, the position of the kiwi will always be negative as it falls toward the ground. The velocity is always negative because the kiwi is moving downward.

2.75

SET UP

Alex throws a ball straight down (toward $-y$) with an initial speed of $v_{0,\mathrm{G}} = 4$ m/s from the top of a 50-m-tall tree. At the same instant, Gary throws a ball straight up (toward $+y$) with an initial speed of $v_{0,\mathrm{G}}$ at a height of 1.5 m off the ground. (We will consider the ground to be $y = 0$, which makes the initial position of Alex's ball $y_{0,\mathrm{A}} = 50$ m and the initial position of Gary's ball $y_{0,\mathrm{G}} = 1.5$ m). Once Alex and Gary throw their respective balls, their accelerations will be equal to $a = -g$. We want to know how fast Gary has to throw his ball such that the two balls cross paths at $y_\mathrm{f} = 25$ m at the same time.

We are given more information about Alex's throw than Gary's so that seems to be a reasonable place to start. First, we need to know the time at which Alex's ball is at $y_\mathrm{f} = 25$ m. We can use $y_\mathrm{f} = y_0 + v_0 t + \frac{1}{2}at^2$ with Alex's information to accomplish this. We know that, at this time, Gary's ball *also* has to be at $y_\mathrm{f} = 25$ m. We can plug this time into $y_\mathrm{f} = y_0 + v_0 t + \frac{1}{2}at^2$ again—this time with Gary's information—to calculate the initial speed of Gary's ball.

SOLVE

Time it takes Alex's ball to reach $y = 25$ m:

$$y_\mathrm{f} = y_{0,\mathrm{A}} + v_{0,\mathrm{A}}t + \frac{1}{2}at^2 = y_{0,\mathrm{A}} + v_{0,\mathrm{A}}t + \frac{1}{2}(-g)t^2$$

$$-\frac{1}{2}gt^2 + v_{0,\mathrm{A}}t + (y_{0,\mathrm{A}} - y_\mathrm{f}) = 0$$

This is a quadratic equation, so we will use the quadratic formula to find t:

$$t = \frac{-v_{0,\mathrm{A}} \pm \sqrt{(v_{0,\mathrm{A}})^2 - 4\left(-\frac{g}{2}\right)(y_{0,\mathrm{A}} - y_{\mathrm{f}})}}{2\left(-\frac{g}{2}\right)} = \frac{-v_{0,\mathrm{A}} \pm \sqrt{(v_{0,\mathrm{A}})^2 + 4\left(\frac{g}{2}\right)(y_{0,\mathrm{A}} - y_{\mathrm{f}})}}{-g}$$

$$= \frac{-\left(-4\frac{\mathrm{m}}{\mathrm{s}}\right) \pm \sqrt{\left(-4\frac{\mathrm{m}}{\mathrm{s}}\right)^2 + 4\left(\frac{9.8\frac{\mathrm{m}}{\mathrm{s}^2}}{2}\right)((50\ \mathrm{m}) - (25\ \mathrm{m}))}}{-\left(9.8\frac{\mathrm{m}}{\mathrm{s}^2}\right)} = \left(\frac{4 \pm 22.5}{-9.8}\right)\mathrm{s}$$

The minus sign in the numerator gives the only physical answer:

$$t = \left(\frac{4 - 22.5}{-9.8}\right)\mathrm{s} = 1.9\ \mathrm{s}$$

Speed Gary needs to launch the ball in order for it to be at $y = 25$ m at $t = 1.9$ s:

$$y_{\mathrm{f}} = y_{0,\,\mathrm{G}} + v_{0,\,\mathrm{G}}t + \frac{1}{2}at^2 = y_{0,\,\mathrm{G}} + v_{0,\,\mathrm{G}}t + \frac{1}{2}(-g)t^2$$

$$v_{0,\,\mathrm{G}} = \frac{y_{\mathrm{f}} - y_{0,\,\mathrm{G}} + \frac{1}{2}gt^2}{t} = \frac{(25\ \mathrm{m}) - (1.5\ \mathrm{m}) + \frac{1}{2}\left(9.8\frac{\mathrm{m}}{\mathrm{s}^2}\right)(1.9\ \mathrm{s})^2}{1.9\ \mathrm{s}} = \boxed{21.7\frac{\mathrm{m}}{\mathrm{s}}}$$

REFLECT

Gary needs to throw his ball almost 50 mph in order for it to cross paths with Alex's at $y = 25$ m! The logic behind the way we solved this problem is much more important than the algebra used to solve this problem. Be sure every step makes logical sense; the crux of the argument is that the two balls have the same position at the same time.

2.76

SET UP

A fox jumps straight up into the air and reaches a maximum height of 85 cm (0.85 m) and then comes back down to the ground. The fox's speed is zero at its maximum height. Because we know the total distance the fox travels, its final speed, and its acceleration ($a = -g$), we can use $v^2 - v_0^2 = 2a(\Delta y)$ to calculate the fox's initial speed. The total time the fox is in the air is twice the time it takes for the fox to jump from the ground to $y_{\mathrm{f}} = 0.85$ m, since it takes the same amount of time to come back down to the ground.

SOLVE

Part a)

$$v^2 - v_0^2 = 2a(\Delta y) = 2(-g)(\Delta y)$$

$$v_0 = \sqrt{v^2 + 2g(\Delta y)} = \sqrt{0 + 2\left(9.8\frac{\mathrm{m}}{\mathrm{s}^2}\right)(0.85\ \mathrm{m})} = \boxed{4.1\frac{\mathrm{m}}{\mathrm{s}}}$$

Part b)

$$v = v_0 + at = v_0 + (-g)t$$

$$t = \frac{v - v_0}{-g} = \frac{0 - \left(4.1\frac{\text{m}}{\text{s}}\right)}{-9.8\frac{\text{m}}{\text{s}^2}} = 0.416 \text{ s}$$

This is the time it takes for the fox to jump 0.85 m in the air from the ground. The total amount of time the fox is in the air is $2t = \boxed{0.83 \text{ s}}$.

REFLECT

Because the fox started and landed at the same height (the ground, in this case), the total time it was in the air is equal to twice the time it takes for half of the motion. If the fox landed at a different height, then this would not be the case.

2.77

SET UP

A person falls from a height of 6 ft off of the ground. We will call the ground $y = 0$. His initial speed is zero and acceleration is $-g$ because he is undergoing free fall. Using the conversion 1 ft = 0.3048 m and $v^2 = v_0^2 + 2a(\Delta y)$, we can calculate the speed at which he hits the ground.

SOLVE

Converting 6 ft into m:

$$6 \text{ ft} \times \frac{0.3048 \text{ m}}{1 \text{ ft}} = 1.8 \text{ m}$$

Solving for the final speed:

$$v^2 = v_0^2 + 2a(\Delta y) = v_0^2 + 2(-g)(\Delta y)$$

$$v = \sqrt{v_0^2 - 2g(\Delta y)} = \sqrt{0^2 - 2\left(9.8\frac{\text{m}}{\text{s}^2}\right)(0 - 1.8 \text{ m})} = \boxed{6\frac{\text{m}}{\text{s}}}$$

REFLECT

A speed of 6 m/s is about 13 mph. In addition to the speed consideration, it's much easier to topple from the top step than the lower ones, hence, the warning.

2.78

SET UP

Wes and Lindsay start at the same initial height, y_0, and each throw a projectile toward the ground. Wes's projectile starts with an initial velocity of $v_{0,\text{W}} = 0$ and it takes t_W s for it to reach the ground ($y_\text{f} = 0$). Lindsay waits 1.25 s and then throws hers with an initial velocity $v_{0,\text{L}} = -28$ m/s. Her projectile hits at the same time as Wes's, which means $t_\text{L} = t_\text{W} - 1.25$ s because she waited. We can first solve for the initial position y_0 in terms of t_W. Because Lindsay starts at the same height, we can plug in y_0 in terms of t_W and t_L in terms of t_W in

order to solve for t_W. Once we know the time it takes Wes's projectile to hit the ground, we can plug it into our original expression to solve for the initial height.

SOLVE

Initial position in terms of Wes's time:

$$y_f - y_0 = v_{0,W}t_W + \frac{1}{2}at_W^2$$

$$-y_0 = 0 + \frac{1}{2}(-g)t_W^2 = -\frac{g}{2}t_W^2$$

$$y_0 = \frac{g}{2}t_W^2$$

Determining t_W from Lindsay's data:

$$y_f - y_0 = v_{0,L}t_L + \frac{1}{2}at_L^2, \text{ but } t_L = t_W - 1.25$$

$$-y_0 = v_{0,L}(t_W - 1.25) + \frac{1}{2}(-g)(t_W - 1.25)^2$$

$$-\left(\frac{g}{2}t_W^2\right) = v_{0,L}(t_W - 1.25) - \frac{g}{2}(t_W^2 + (1.25)^2 - 2(1.25)t_W)$$

$$t_W^2 = \left(-\frac{2}{g}\right)(v_{0,L}(t_W - 1.25)) + (t_W^2(1.25)^2 - 2(1.25)t_W)$$

$$0 = \left(-\frac{2}{g}\right)(v_{0,L}(t_W - 1.25)) + (1.25)^2 - 2(1.25)t_W = \left(-\frac{2}{g}\right)(-28)(t_W - 1.25) + (1.25)^2 - 2(1.25)t_W$$

$$t_W = \frac{\frac{70}{g} - (1.25)^2}{\frac{56}{g} - 2.5} = 1.74 \text{ s}$$

Plugging the time back into Wes's equation:

$$y_0 = \frac{g}{2}t_W^2 = \frac{9.8\frac{\text{m}}{\text{s}^2}}{2}(1.74 \text{ s})^2 = \boxed{15 \text{ m}}$$

REFLECT

We can double-check this answer by solving Lindsay's equation of motion for her initial speed, which we were given. Lindsay's time is equal to 1.74 s − 1.25 s = 0.49 s. The initial speed is then:

$$v_{0,L} = \frac{y_f - y_0 - \frac{1}{2}at_L^2}{t_L} = \frac{0 - (15 \text{ m}) - \frac{1}{2}\left(-9.8\frac{\text{m}}{\text{s}^2}\right)(0.49 \text{ s})^2}{0.49 \text{ s}} = 28\frac{\text{m}}{\text{s}}, \text{ as expected.}$$

2.79

SET UP

A ball is thrown up with an initial speed of 18 m/s. We will consider *up* to be positive motion. We can calculate the velocity of the ball using $v(t) = v_0 + at = v_0 - gt$. The ball has a speed of zero at its maximum height.

SOLVE

$$v(t) = v_0 + at = v_0 - gt$$

Part a)

$$v(t = 1\text{ s}) = \left(18\frac{\text{m}}{\text{s}}\right) - \left(9.8\frac{\text{m}}{\text{s}^2}\right)(1\text{ s}) = \boxed{8.2\frac{\text{m}}{\text{s}}}$$

Part b)

$$v(t = 2\text{ s}) = \left(18\frac{\text{m}}{\text{s}}\right) - \left(9.8\frac{\text{m}}{\text{s}^2}\right)(2\text{ s}) = \boxed{-1.6\frac{\text{m}}{\text{s}}}$$

Part c)

$$v(t = 5\text{ s}) = \left(18\frac{\text{m}}{\text{s}}\right) - \left(9.8\frac{\text{m}}{\text{s}^2}\right)(5\text{ s}) = \boxed{-31\frac{\text{m}}{\text{s}}}$$

Part d)

$$v(t) = 0 = v_0 - gt_{\text{max}}$$

$$t_{\text{max}} = \frac{-v_0}{-g} = \frac{18\frac{\text{m}}{\text{s}}}{9.8\frac{\text{m}}{\text{s}^2}} = \boxed{1.8\text{ s}}$$

REFLECT

A negative velocity in this case refers to the ball coming back down to Earth.

2.80

SET UP

A tennis ball has an initial velocity of 20 m/s straight up. The ball starts at the top of a 30-m-tall cliff, moves upward some unspecified amount, comes back through its initial position, and then hits the ground. When the ball passes back through its initial height on its way to the ground, it will have the same speed as it initially did because the motion is symmetric. (You can explicitly calculate this from $v^2 - v_0^2 = 2a(\Delta y)$ with $\Delta y = 0$.) The ball's path consists of two legs: #1) traveling from its initial position to its maximum height and back to its initial position and #2) traveling from its initial position to the bottom of the cliff. The time and distance the ball travels during leg #1 are twice what it takes to travel from the initial position to the maximum height. At the maximum height, the ball's speed is zero. The time and distance associated with leg #2 can be calculated knowing the ball's initial velocity

is 20 m/s straight down and it travels 30 m straight down. We will need to use the quadratic equation to determine the time for leg #2.

SOLVE

Part a) $\boxed{20\text{ m/s}}$

Part b)

Initial position → maximum height

$$v = v_0 + at = v + (-g)t$$

$$t = \frac{v - v_0}{-g} = \frac{0 - 20\frac{\text{m}}{\text{s}}}{-9.8\frac{\text{m}}{\text{s}^2}} = 2.04\text{ s}$$

Maximum height → initial position will also take 2.04 s due to symmetry.

Initial position → bottom of cliff

$$y_f = y_0 + v_0 t + \frac{1}{2}at^2 = y_0 + v_0 t + \frac{1}{2}(-g)t^2$$

$$\frac{g}{2}t^2 - v_0 t + (y_f - y_0) = 0$$

$$t = \frac{-(-v_0) \pm \sqrt{v_0^2 - 4\left(\frac{g}{2}\right)(y_f - y_0)}}{2\left(\frac{g}{2}\right)} = \frac{\left(-20\frac{\text{m}}{\text{s}}\right) \pm \sqrt{\left(20\frac{\text{m}}{\text{s}}\right)^2 - 4\left(\frac{9.8\frac{\text{m}}{\text{s}^2}}{2}\right)(0 - 30\text{ m})}}{9.8\frac{\text{m}}{\text{s}^2}}$$

$$= \frac{-20 \pm 31.43}{9.8} = \frac{-20 + 31.43}{9.8} = 1.17\text{ s. (Only the positive root is physically sound.)}$$

Total time to reach the ground = 2*(2.04 s) + (1.17 s) = $\boxed{5.3\text{ s}}$.

Part c)

Initial position → maximum height

$$v^2 - v_0^2 = 2a(\Delta y) = 2(-g)(\Delta y)$$

$$\Delta y = \frac{v^2 - v_0^2}{-2g} = \frac{0 - \left(20\frac{\text{m}}{\text{s}}\right)^2}{-2\left(9.8\frac{\text{m}}{\text{s}^2}\right)} = 20.41\text{ m}$$

Maximum height → initial position will also take 20.41 m due to symmetry.

Initial position → bottom of cliff is 30 m.

Total distance the ball travels = 2*(20.41 m) + (30 m) = $\boxed{71\text{ m}}$.

REFLECT

Thinking about the problem first rather than blindly searching for equations will save you time in the long run. For example, we didn't have to calculate the time or distance the ball travels from its maximum height back to its initial height because we used the symmetry of the problem instead.

2.81

SET UP

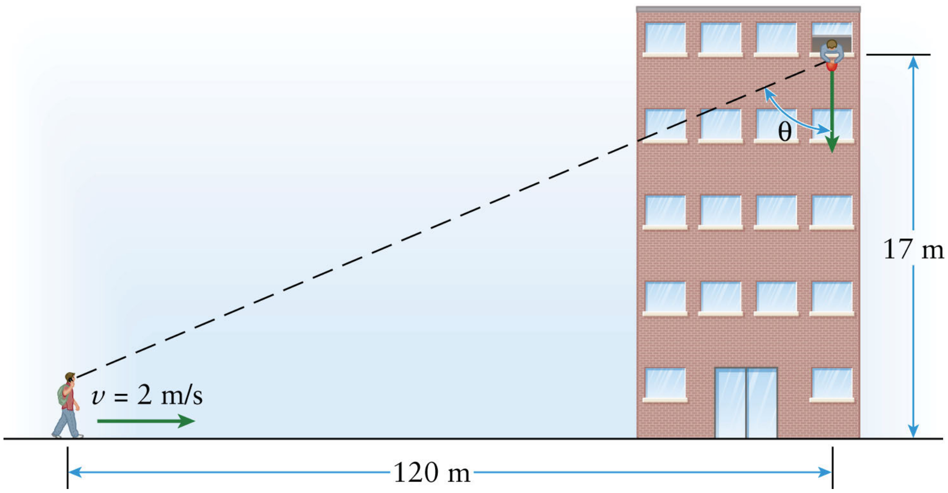


Figure 2-9 Problem 81

Mary is planning on dropping an apple out of her 17-m-high window to Bill. Bill is walking at a velocity of 2 m/s toward Mary's building and starts 120 m from directly below her window. Mary wants Bill to catch the apple, which means Bill and the apple need to be at the same location at the same time. We can use the constant acceleration equation for free fall to calculate the time it takes the apple to drop from an initial location of $y_0 = 17$ m to a final location of $y_f = 1.75$ m (presumably, Bill's height). Assume the apple has an initial velocity of zero. We can compare this to the time it takes Bill to walk 120 m in order to determine how long Mary should wait to drop the apple. Once we know how long Mary waits, we can find the distance Bill is from Mary since he is walking at a constant speed of 2 m/s. Bill's horizontal distance from Mary and the height Mary is in the air are two legs of a right triangle; the angle theta is related to these legs by the tangent.

SOLVE

Part a)

Time it takes Mary's apple to fall to a height of 1.75 m off the ground:

$$y_f = y_0 + v_0 t + \frac{1}{2}at^2 = y_0 + 0 + \frac{1}{2}(-g)t^2$$

$$t = \sqrt{\frac{2(y_f - y_0)}{-g}} = \sqrt{\frac{2((1.75\text{ m}) - (17\text{ m}))}{-\left(9.8\frac{\text{m}}{\text{s}^2}\right)}} = 1.76\text{ s}$$

Time it takes Bill to walk 120 m:

$$(120\text{ m})\left(\frac{1}{2\frac{\text{m}}{\text{s}}}\right) = 60\text{ s}$$

Mary should wait $(60\text{ s}) - (1.76\text{ s}) = \boxed{58\text{ s}}$ to drop her apple.

Part b) In 58 s, Bill travels $(58\text{ s})\left(2\frac{\text{m}}{\text{s}}\right) = 116\text{ m}$, which means he is $\boxed{4\text{ m}}$ horizontally from the window.

Part c)

$$\tan(\theta) = \frac{4\text{ m}}{17\text{ m}}$$

$$\theta = \arctan\left(\frac{4}{17}\right) = \boxed{0.23\text{ rad} = 13°}$$

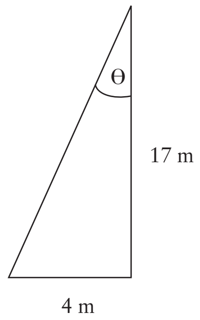


Figure 2-10 Problem 81

REFLECT

As you would expect, Bill has to be reasonably close to Mary in order for him to catch the apple. If Mary throws the apple (that is, nonzero initial velocity), Bill would need to be even closer in order to catch the apple.

2.82

SET UP

A car is driving at 40 km/hr when the traffic signal changes from green to yellow. It takes the driver 0.75 s before hitting the brake. The car then accelerates at -5.5 m/s^2 until it stops. The distance the car travels during this process can be split into two parts: the distance traveled before the brake is applied and the distance traveled after the brake is applied. Before the brake is applied, the car is traveling at a constant speed for 0.75 s. While the brake is applied, the car is undergoing constant acceleration so the equation $v^2 = v_0^2 + 2a(\Delta x)$ can be used to calculate the distance. The minimum distance necessary to come to a stop is the sum of these two.

SOLVE

Converting from km/hr to m/s:

$$40\frac{\text{km}}{\text{hr}} \times \frac{1\text{ hr}}{3600\text{ s}} \times \frac{1000\text{ m}}{1\text{ km}} = 11.1\frac{\text{m}}{\text{s}}$$

Distance car travels before applying the brake:

$$(0.75\ \mathrm{s})\left(11.1\frac{\mathrm{m}}{\mathrm{s}}\right) = 8.33\ \mathrm{m}$$

Distance car travels while braking:

$$v^2 = v_0^2 + 2a(\Delta x)$$

$$\Delta x = \frac{v^2 - v_0^2}{2a} = \frac{0^2 - \left(11.1\frac{\mathrm{m}}{\mathrm{s}}\right)^2}{2\left(-5.5\frac{\mathrm{m}}{\mathrm{s}^2}\right)} = 11.2\ \mathrm{m}$$

Minimum distance necessary for the car to stop: $8.33\ \mathrm{m} + 11.2\ \mathrm{m} = \boxed{19.5\ \mathrm{m}}$.

REFLECT

The car requires at least 19.5 m (64 ft) to come to a stop from a speed of 40 km/hr (25 mph). It would take about 2.75 s from the time the light turned yellow until the car stopped.

2.83

SET UP

Two trains are 300 m apart and traveling toward each other. Train 1 has an initial speed of 98 km/hr and an acceleration of $-3.5\ \mathrm{m/s^2}$. Train 2 has an initial speed of 120 km/hr and an acceleration of $-4.2\ \mathrm{m/s^2}$. To determine whether or not the trains collide, we can calculate the stopping distance required for each train, add them together, and see if it is less than 300 m. If so, the trains are safe; if not, the trains crash. We know the trains' initial speeds, final speeds, and acceleration, and we are interested in finding the distance, which suggests we use $v^2 = v_0^2 + 2a(\Delta x)$.

SOLVE

Stopping distance for train 1:

$$v^2 = v_0^2 + 2a(\Delta x)$$

$$\Delta x = \frac{v^2 - v_0^2}{2a} = \frac{0^2 - \left(98\frac{\mathrm{km}}{\mathrm{hr}} \times \frac{1\ \mathrm{hr}}{3600\ \mathrm{s}} \times \frac{1000\ \mathrm{m}}{1\ \mathrm{km}}\right)^2}{2\left(-3.5\frac{\mathrm{m}}{\mathrm{s}^2}\right)} = 105.8\ \mathrm{m}$$

Stopping distance for train 2:

$$v^2 = v_0^2 + 2a(\Delta x)$$

$$\Delta x = \frac{v^2 - v_0^2}{2a} = \frac{0^2 - \left(120\frac{\mathrm{km}}{\mathrm{hr}} \times \frac{1\ \mathrm{hr}}{3600\ \mathrm{s}} \times \frac{1000\ \mathrm{m}}{1\ \mathrm{km}}\right)^2}{2\left(-4.2\frac{\mathrm{m}}{\mathrm{s}^2}\right)} = 132\ \mathrm{m}$$

Total distance required to stop both trains: 105.8 m + 132 m = 237.8 m. This is less than the 300 m separating the trains, which means the $\boxed{\text{trains will not collide}}$. The distance separating the trains is 300 m − 237.8 m = $\boxed{62\text{ m}}$.

REFLECT

As long as the trains are at least 237 m apart, they will not collide.

2.84

SET UP

A cheetah can reach a top speed of 29 m/s, which we can convert to miles per hour using 1 mi = 1.6 km. The cheetah can accelerate from rest to 20 m/s in 2.5 s. We can divide the change in speed by the change in time to find the acceleration because we are assuming the acceleration is constant. Knowing the acceleration and the initial and final speeds allows us to calculate the time it takes the cheetah to reach its top speed and how far it travels in that time. Finally, the cheetah only accelerates until it reaches its top speed and then continues running at a constant speed. We can calculate how long it takes the cheetah to run 120 m with our answers from part (b). After it accelerates to its top speed and covers the distance we calculated in part (b), it runs at a constant speed for the remainder of the 120 m.

SOLVE

Part a)

$$29\frac{\text{m}}{\text{s}} \times \frac{3600\text{ s}}{1\text{ hr}} \times \frac{1\text{ km}}{1000\text{ m}} \times \frac{1\text{ mi}}{1.61\text{ km}} = \boxed{65\frac{\text{mi}}{\text{hr}}}$$

Part b)

$$a = \frac{\Delta v}{\Delta t} = \frac{20\frac{\text{m}}{\text{s}}}{2.5\text{ s}} = 8\frac{\text{m}}{\text{s}^2}$$

$$v = v_0 + at$$

$$t = \frac{v - v_0}{a} = \frac{29\frac{\text{m}}{\text{s}} - 0}{8\frac{\text{m}}{\text{s}^2}} = \boxed{3.6\text{ s}}$$

$$v^2 - v_0^2 = 2a(\Delta x)$$

$$\Delta x = \frac{v^2 - v_0^2}{2a} = \frac{\left(29\frac{\text{m}}{\text{s}}\right)^2 - 0}{2\left(8\frac{\text{m}}{\text{s}^2}\right)} = \boxed{53\text{ m}}$$

Part c) While the cheetah is accelerating to its top speed, it covers 53 m in 3.6 s. During the remaining 120 m − 53 m = 67 m, the cheetah runs at a constant speed:

$$\frac{67\text{ m}}{\left(29\frac{\text{m}}{\text{s}}\right)} = 2.3\text{ s}$$

The total time it takes the cheetah to run 120 m is 3.6 s + 2.3 s = $\boxed{5.9\text{ s}}$.

REFLECT

Be sure to reread the problem statement as you solve the various parts of the problem. It is easy to miss an important assumption or piece of data that is invaluable later on. For example, in this problem, we were told that the cheetah's acceleration drops to zero once it reaches its top speed. We didn't need to use this information until part (c) of the problem.

2.85

SET UP

The severity index (SI) is defined as SI = $a^{5/2}t$, where a is the acceleration in multiples of g and t is the time the acceleration lasts. We are given a change in speed and acceleration, so we can use the definition of average acceleration to calculate the time. We need to divide the given acceleration by g before plugging it into the SI equation. After calculating the time and converting the initial speed of 5.0 km/hr into m/s, we can find the distance the person moves due to the collision.

SOLVE

Part a)

$$\Delta v = 15\frac{\text{km}}{\text{hr}} \times \frac{1\text{ hr}}{3600\text{ s}} \times \frac{1000\text{ m}}{1\text{ km}} = 4.17\frac{\text{m}}{\text{s}}$$

$$a = \frac{\Delta v}{\Delta t},\text{ so } \Delta t = \frac{\Delta v}{a} = \frac{4.17\frac{\text{m}}{\text{s}}}{35\frac{\text{m}}{\text{s}^2}} = 0.119\text{ s}$$

$$\text{SI} = a^{5/2}t = \left(\frac{35\frac{\text{m}}{\text{s}^2}}{g}\right)^{5/2}(0.119\text{ s}) = \left(\frac{35\frac{\text{m}}{\text{s}^2}}{9.8\frac{\text{m}}{\text{s}^2}}\right)^{5/2}(0.119\text{ s}) = \boxed{2.9\text{ s}}$$

Part b)

$$v_0 = 5\frac{\text{km}}{\text{hr}} \times \frac{1\text{ hr}}{3600\text{ s}} \times \frac{1000\text{ m}}{1\text{ km}} = 1.39\frac{\text{m}}{\text{s}}$$

$$\Delta x = v_0 t + \frac{1}{2}at^2 = \left(1.39\frac{\text{m}}{\text{s}}\right)(0.119\text{ s}) + \frac{1}{2}\left(35\frac{\text{m}}{\text{s}^2}\right)(0.119\text{ s})^2 = \boxed{0.41\text{ m}}$$

REFLECT

An acceleration of 35 m/s^2 is around 3.5g, which is a similar g-force to what astronauts on the space shuttle experience upon reentry. A head impact with an SI of around 400 can result in a concussion or unconsciousness, and there is a 50% chance of death for an impact with an SI of 1000.

2.86

SET UP

The velocity as a function of time of the wall of a beating heart is given in SI units: $v(t) = 0.023\sin(7t)$. The sine function itself oscillates between $+1$ and -1, so the maximum value of the speed is the absolute value of the coefficient in front (which is the amplitude of the sine). The position as a function of time is the integral of the velocity with respect to time. Knowing that the heart wall is at its maximum displacement at $t = 0$ allows us to eliminate the constant of integration. To determine the total distance the heart wall moves in 45 s, we must first determine how far the wall moves in one cycle. In one cycle the heart travels four times the amplitude of the motion. We can calculate the period T of one oscillation (that is, how long it takes for the heart wall to complete one cycle) from the angular frequency of the motion $\left(T = \dfrac{2\pi}{7\frac{\text{rad}}{\text{s}}}\right)$. Since we know how long one cycle takes and how far the heart moves in one cycle, we can now determine how far the heart moves in 45 s.

SOLVE

Part a) The maximum of the sine function is 1, which means the maximum speed of the heart during one contraction is $\boxed{0.023\ \text{m/s}}$.

Part b)

$$x(t) = \int v(t)dt = \int(0.023\sin(7t))dt = 0.023\int\sin(7t)dt = 0.023\left[-\frac{1}{7}\cos(7t)\right] + C$$

$$= -\frac{0.023}{7}\cos(7t) + C$$

We are told that the maximum displacement occurs at $t = 0$, which means $C = 0$. Therefore, the position of the heart's wall as a function of time is $\boxed{x(t) = -\dfrac{0.023}{7}\cos(7t) = -(0.0033)\cos(7t)}$. The term in front of the cosine is the maximum displacement of the heart's wall in meters. The cosine is used to model the contractions of the heart as a function of time. The 7 (which has units of radians/second) is related to the frequency of the oscillation of the heart wall.

Part c) The function $\cos(7t)$ repeats every $\frac{2\pi}{7}$ s $= 0.898$ s. Within that time, the heart wall moves $4(0.0033 \text{ m}) = 0.0132$ m. Over 45 s the heart completes $\frac{45 \text{ s}}{0.898 \text{ s}} = 50.1$ cycles for a total distance of $(50.1)(0.0132 \text{ m}) = \boxed{0.66 \text{ m}}$.

REFLECT

A beat frequency of 7 rad/s corresponds to a pulse of 67 beats per minute, which is a reasonable resting heart rate.

2.87

SET UP

The velocity of a rocket is given as a piecewise function. The SI units of velocity are m/s, so 125 and 116.1 have units of m/s. An exponent must be dimensionless, which means 0.12 has SI units of s^{-1}. The speed at any time is found by evaluating the function at that time; keep in mind that a piecewise function is defined differently over different time intervals. The displacement of the rocket is related to the integral of the velocity over the given time interval.

SOLVE

$$v(t) = \begin{Bmatrix} 125(1 - e^{-0.12t}) & 0 < t < 22 \text{ s} \\ 116.1 & t > 22 \text{ s} \end{Bmatrix}$$

Part a) The numbers 125 and 116.1 both have SI units of $\boxed{\text{m/s}}$ because those are the SI units for velocity. The exponent of e needs to be dimensionless; t has dimensions of time, which means 0.12 has dimensions of reciprocal time. The SI unit associated with 0.12 is, therefore, $\boxed{\text{s}^{-1}}$.

Part b)

Speed at $t = 2.2$ s:

$$v(t = 2.2 \text{ s}) = 125(1 - e^{-0.12(2.2)}) = \boxed{29\frac{\text{m}}{\text{s}}}$$

Speed at $t = 25$ s:

$$v(t = 25 \text{ s}) = \boxed{116.1\frac{\text{m}}{\text{s}}}$$

Part c)

$$\Delta x = \int_0^{10 \text{ s}} v(t)dt = \int_0^{10 \text{ s}} (125(1 - e^{-0.12t}))dt = \left[125\left(t + \frac{1}{0.12}e^{-0.12t}\right)\right]_0^{10 \text{ s}}$$

$$= 125\left[10 + \frac{1}{0.12}e^{-0.12(10)} - 0 - \frac{1}{0.12}e^{-0.12(0)}\right] = 125\left[10 + \frac{1}{0.12}e^{-1.2} - \frac{1}{0.12}\right] = \boxed{520 \text{ m}}$$

Part d)

$$\Delta x = \int_0^{30 \text{ s}} v(t)dt = \int_0^{22 \text{ s}} (125(1 - e^{-0.12t}))dt + \int_{22 \text{ s}}^{30 \text{ s}} 116.1dt = \left[125\left(t + \frac{1}{0.12}e^{-0.12t}\right)\right]_0^{22 \text{ s}} + [116.1t]_{22 \text{ s}}^{30 \text{ s}}$$

$$= 125\left[22 + \frac{1}{0.12}e^{-0.12(22)} - 0 - \frac{1}{0.12}e^{-0.12(0)}\right] + [116.1(30) - 116.1(22)]$$

$$= 125\left[22 + \frac{1}{0.12}e^{-2.64} - \frac{1}{0.12}\right] + [928.8] = \boxed{2700\text{ m}}$$

REFLECT

Be aware of the time intervals when working with piecewise functions. The velocity function for the rocket changes at $t = 22$ s, which is why we had to split the integral in part (d) into two parts.

2.88

SET UP

Starting from the front of a stopped train, a man walks 12 steps that are each 82 cm (0.82 m) in length. The train then begins to accelerate at a rate of 0.4 m/s^2. After 10 s, the man has walked another 20 steps and the end of the moving train has passed him. The total length of the train is the sum of these three distances: the distance the man walked while the train was stopped, the distance the accelerating train covers in 10 s, and the distance the man walked while the train was moving. Because the train's acceleration is constant, we can calculate the distance it covers in 10 s starting from rest using $\Delta x = v_0 t + \frac{1}{2}at^2$.

SOLVE

Distance the man covered before the train starts to move: (0.82 m)(12) = 9.84 m
Distance the train covers in 10 s starting from rest:

$$\Delta x = v_0 t + \frac{1}{2}at^2 = 0 + \frac{1}{2}\left(0.4\frac{\text{m}}{\text{s}^2}\right)(10\text{ s})^2 = 20\text{ m}$$

Distance the man covers while train is moving: (0.82 m)(20) = 16.4 m

Total distance covered $= 9.84\text{ m} + 20\text{ m} + 16.4\text{ m} = \boxed{46\text{ m}}$.

REFLECT

This corresponds to around 150 ft. An average train car is around 50 ft in length, so this particular train would have three cars in it, which is small, but reasonable.

2.89

SET UP

Blythe and Geoff are running a 1-km race with different strategies. Blythe runs the first 600 m at a constant speed of 4 m/s. She accelerates at a constant rate for 1 min (60 s) until she reaches her top speed of 7.5 m/s. We can use the definition of average acceleration and $\Delta x = v_0 t + \frac{1}{2}at^2$ to calculate how much distance she covers in that minute. She runs the remainder of the 1 km at 7.5 m/s. Geoff, on the other hand, starts from rest and accelerates up to his top speed of 8 m/s in 3 min (180 s). As with Blythe, we can use the definition of average acceleration and $\Delta x = v_0 t + \frac{1}{2}at^2$ to calculate how much distance he covers in 3 min. He runs

the remainder of the 1 km at 8 m/s. To determine who wins the race, we need to calculate who runs 1 km in the shorter amount of time.

SOLVE

Blythe

Distance covered while accelerating:

$$a = \frac{7.5\frac{\text{m}}{\text{s}} - 4\frac{\text{m}}{\text{s}}}{60\text{ s}} = 0.0583\frac{\text{m}}{\text{s}^2}$$

$$\Delta x = v_0 t + \frac{1}{2}at^2 = \left(4\frac{\text{m}}{\text{s}}\right)(60\text{ s}) + \frac{1}{2}\left(0.0583\frac{\text{m}}{\text{s}^2}\right)(60\text{ s})^2 = 345\text{ m}$$

Distance covered while running at her top speed: $1000\text{ m} - 600\text{ m} - 345\text{ m} = 55\text{ m}$.

Total time of Blythe's run:

$$t_{\text{Blythe}} = \frac{600\text{ m}}{4\frac{\text{m}}{\text{s}}} + 60\text{ s} + \frac{55\text{ m}}{7.5\frac{\text{m}}{\text{s}}} = \boxed{217\text{ s}}$$

Geoff

Distance covered while accelerating:

$$a = \frac{8\frac{\text{m}}{\text{s}} - 0}{180\text{ s}} = 0.044\frac{\text{m}}{\text{s}^2}$$

$$\Delta x = v_0 t + \frac{1}{2}at^2 = 0 + \frac{1}{2}\left(0.044\frac{\text{m}}{\text{s}^2}\right)(180\text{ s})^2 = 720\text{ m}$$

Distance covered while running at his top speed: $1000\text{ m} - 720\text{ m} = 280\text{ m}$.

Total time of Geoff's run:

$$t_{\text{Geoff}} = 180\text{ s} + \frac{280\text{ m}}{8\frac{\text{m}}{\text{s}}} = \boxed{215\text{ s}}$$

Since he runs 1 km in less time, $\boxed{\text{Geoff wins the race}}$.

REFLECT

Geoff's time is about 3.5 min and his average speed is 10.4 mi/hr; Blythe's average speed is 10.3 mi/hr. These are respectable times to run a kilometer. Remember that 1 km is around 0.63 mi.

2.90

SET UP

We are given the position of a lizard as a function of time. The lizard's velocity as a function of time $v(t)$ is the first derivative of the position with respect to time. Once we have a functional form for the velocity, we can evaluate it at the given times to find those velocities or set it equal to zero and solve for the time t when the lizard is at rest. We should be able to use

our previous answers to determine when the lizard is moving toward positive or negative x. Remember that a function changes sign when it passes through zero. The lizard's acceleration as a function of time $a(t)$ is the first derivative of the velocity with respect to time. After taking the derivative, we can set $a(t)$ equal to zero and solve for t. The total distance the lizard travels in the first 10 s is equal to the distance it travels toward negative x plus the distance it travels toward positive x. The lizard is initially running toward negative x and stops at the time calculated in part (c). The lizard runs toward positive x from that time up to 10 s. We can calculate these distances using the original function $x(t)$.

SOLVE

$$x(t) = \left(0.20\frac{\mathrm{m}}{\mathrm{s}^3}\right)t^3 - \left(0.40\frac{\mathrm{m}}{\mathrm{s}^2}\right)t^2 - \left(0.65\frac{\mathrm{m}}{\mathrm{s}}\right)t$$

Part a)

$$v(t) = \frac{dx}{dt} = \frac{d}{dt}\left(\left(0.20\frac{\mathrm{m}}{\mathrm{s}^3}\right)t^3 - \left(0.40\frac{\mathrm{m}}{\mathrm{s}^2}\right)t^2 - \left(0.65\frac{\mathrm{m}}{\mathrm{s}}\right)t\right)$$
$$= \boxed{\left(0.60\frac{\mathrm{m}}{\mathrm{s}^3}\right)t^2 - \left(0.80\frac{\mathrm{m}}{\mathrm{s}^2}\right)t - \left(0.65\frac{\mathrm{m}}{\mathrm{s}}\right)}$$

Part b)

$$v(t = 2\ \mathrm{s}) = \left(0.60\frac{\mathrm{m}}{\mathrm{s}^3}\right)(2\ \mathrm{s})^2 - \left(0.80\frac{\mathrm{m}}{\mathrm{s}^2}\right)(2\ \mathrm{s}) - \left(0.65\frac{\mathrm{m}}{\mathrm{s}}\right) = \boxed{0.15\frac{\mathrm{m}}{\mathrm{s}}}$$

$$v(t = 4\ \mathrm{s}) = \left(0.60\frac{\mathrm{m}}{\mathrm{s}^3}\right)(4\ \mathrm{s})^2 - \left(0.80\frac{\mathrm{m}}{\mathrm{s}^2}\right)(4\ \mathrm{s}) - \left(0.65\frac{\mathrm{m}}{\mathrm{s}}\right) = \boxed{5.8\frac{\mathrm{m}}{\mathrm{s}}}$$

$$v(t = 10\ \mathrm{s}) = \left(0.60\frac{\mathrm{m}}{\mathrm{s}^3}\right)(10\ \mathrm{s})^2 - \left(0.80\frac{\mathrm{m}}{\mathrm{s}^2}\right)(10\ \mathrm{s}) - \left(0.65\frac{\mathrm{m}}{\mathrm{s}}\right) = \boxed{51\frac{\mathrm{m}}{\mathrm{s}}}$$

Part c)

$$v(t) = 0 = \left(0.60\frac{\mathrm{m}}{\mathrm{s}^3}\right)t^2 - \left(0.80\frac{\mathrm{m}}{\mathrm{s}^2}\right)t - \left(0.65\frac{\mathrm{m}}{\mathrm{s}}\right)$$

$$t = \frac{-(-0.80) \pm \sqrt{(-0.80)^2 - 4(0.60)(-0.65)}}{2(0.60)} = \frac{0.80 \pm 1.483}{1.2}$$

Taking the positive root:

$$t = \frac{0.80 + 1.483}{1.2} = \boxed{1.9\ \mathrm{s}}$$

Part d) Using our answers from parts (b) and (c), the lizard is moving in the positive x direction for $\boxed{t > 1.9\ \mathrm{s}}$.

Part e) Since $v(t = 1.9\text{ s}) = 0$ and the lizard is moving toward positive x for $t > 1.9$ s, the lizard must be moving toward negative x for $\boxed{t < 1.9\text{ s}}$.

Part f)

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}\left(\left(0.60\frac{\text{m}}{\text{s}^3}\right)t^2 - \left(0.80\frac{\text{m}}{\text{s}^2}\right)t - \left(0.65\frac{\text{m}}{\text{s}}\right)\right) = \left(1.20\frac{\text{m}}{\text{s}^3}\right)t - \left(0.80\frac{\text{m}}{\text{s}^2}\right) = 0$$

$$t = \frac{0.80\frac{\text{m}}{\text{s}^2}}{1.20\frac{\text{m}}{\text{s}^3}} = \boxed{0.67\text{ s}}$$

Part g)

$$\Delta x_{\text{total}} = |\Delta x_{0\to 1.9\text{ s}}| + |\Delta x_{1.9\text{ s}\to 10\text{ s}}|$$

$$= |(0.2)(1.9)^3 - (0.4)(1.9)^2 - (0.65)(1.9) - 0 + 0 + 0| + |(0.2)(10)^3$$
$$- (0.4)(10)^2 - (0.65)(10) - (0.2)(1.9)^3 + (0.4)(1.9)^2 + (0.65)(1.9)|$$

$$= |-1.3| + |153.5 - (-1.3)| = 1.3 + 154.8 = \boxed{156\text{ m}}$$

REFLECT

We could have integrated $v(t)$ in part (g) and arrived at the same answer: $\Delta x_{\text{total}} = \left|\int_0^{1.9\text{ s}} v(t)dt\right| + \left|\int_{1.9\text{ s}}^{10\text{ s}} v(t)dt\right| = 156$ m. Note that the acceleration is *not* constant in this case, so we were unable to use the constant acceleration equations.

2.91

SET UP

A rocket starts from rest and has an initial acceleration of 10.0 m/s^2. The acceleration decreases linearly to zero in 8.00 s and then remains at zero. A sketch of the acceleration versus time should have a positive y intercept, have a negative slope, touch the x-axis toward the end of the time interval, and remain zero. A sketch of the speed as a function of time should start at zero, increase parabolically until ~8.00 s, and then remain constant. Because the slope of the acceleration curve is negative, the parabola should open downward. Given the acceleration data, we can calculate the slope of the acceleration for the first 8.00 s and write the piecewise function for $a(t)$. We can calculate $v(t)$ by integrating $a(t)$ with respect to time. Evaluating $v(t)$ at $t = 8.00$ s will give us the speed of the rocket after the acceleration ceases. The speed at $t = 10.0$ s is equal to the speed at $t = 8.00$ s because the acceleration is zero over this region. The displacement of the rocket is found by taking the definite integral of $v(t)$ from $t = 0$ s to $t = 10.0$ s.

SOLVE

Part a)

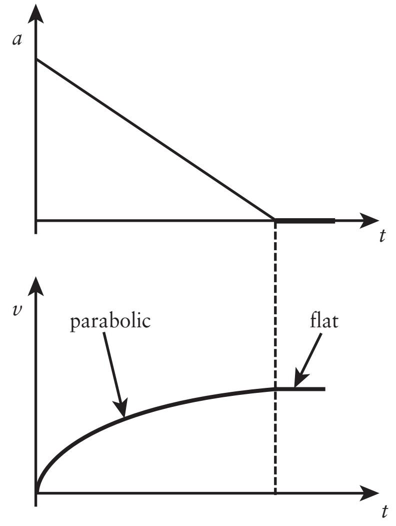


Figure 2-11 Problem 91

Part b)

$$\text{slope} = \frac{0 - 10.0\frac{\text{m}}{\text{s}^2}}{8.00\text{ s} - 0} = -1.25\frac{\text{m}}{\text{s}^3}$$

$$\boxed{a(t) = \begin{cases} \left(-1.25\frac{\text{m}}{\text{s}^3}\right)t + \left(10.0\frac{\text{m}}{\text{s}^2}\right) & 0 < t < 8\text{ s} \\ 0 & 8 < t < 10\text{ s} \end{cases}}$$

Part c, i)

$$v(t) = \int a(t)dt = \int\left(\left(-1.25\frac{\text{m}}{\text{s}^3}\right)t + \left(10.0\frac{\text{m}}{\text{s}^2}\right)\right)dt = \left(-\frac{1.25}{2}\frac{\text{m}}{\text{s}^3}\right)t^2 + \left(10.0\frac{\text{m}}{\text{s}^2}\right)t + C$$

But $v(0) = 0$, so $C = 0$ and $v(t) = \left(-\frac{1.25}{2}\frac{\text{m}}{\text{s}^3}\right)t^2 + \left(10.0\frac{\text{m}}{\text{s}^2}\right)t$.

The acceleration ceases at $t = 8.00$ s, so

$$v(t = 8.00\text{ s}) = \left(-\frac{1.25}{2}\frac{\text{m}}{\text{s}^3}\right)(8.00\text{ s})^2 + \left(10.0\frac{\text{m}}{\text{s}^2}\right)(8.00\text{ s}) = \boxed{40.0\frac{\text{m}}{\text{s}}}$$

Part c, ii) The acceleration is zero from $t = 8.00$ s to $t = 10$ s, which means the velocity is constant. Therefore, the speed at $v(t = 10.0\text{ s}) = v(t = 8.00\text{ s}) = \boxed{40.0\text{ m/s}}$.

Part d)

Distance traveled in the first 8.00 s:

$$\Delta x = \int_0^{8.00\text{ s}}\left(\left(-\frac{1.25}{2}\frac{\text{m}}{\text{s}^3}\right)t^2 + \left(10.0\frac{\text{m}}{\text{s}^2}\right)t\right)dt = \left[\left(-\frac{1.25}{6}\frac{\text{m}}{\text{s}^3}\right)t^3 + \left(\frac{10.0}{2}\frac{\text{m}}{\text{s}^2}\right)t^2\right]_0^{8.00\text{ s}}$$

$$= \left[\left(-\frac{1.25}{6}\frac{\text{m}}{\text{s}^3}\right)(8.00\text{ s})^3 + \left(5.0\frac{\text{m}}{\text{s}^2}\right)(8.00\text{ s})^2\right] = 213\text{ m}$$

Distance traveled in the last 2.00 s:

$$\Delta x = \int_{8.00\text{ s}}^{10.0\text{ s}} \left(40.0\frac{\text{m}}{\text{s}}\right)dt = \left[\left(40.0\frac{\text{m}}{\text{s}}\right)t\right]_{8.00\text{ s}}^{10.0\text{ s}}$$

$$= \left[\left(40.0\frac{\text{m}}{\text{s}}\right)(10.0\text{ s}) - \left(40.0\frac{\text{m}}{\text{s}}\right)(8.00\text{ s})\right] = 80.0\text{ m}$$

Total distance traveled: 213 m + 80.0 m = $\boxed{293\text{ m}}$.

REFLECT
Remember that we are working with piecewise functions, so all of the integrals need to be split up into the correct ranges. Once you calculate $v(t)$ in part (c), you should go back and double-check your answer to part (a) to make sure it is consistent. If you were unsure which way the parabola should open, the negative sign in front of the t^2 term of the velocity tells you that it opens downward. In part (d) we could have approximated the integral as the area of a rectangle of dimensions (40 m/s) by (2 s).

2.92

SET UP
A plot of velocity versus time is given. The instantaneous acceleration at a specific time is equal to the slope of the tangent line to the v versus t plot. Because the velocity appears linear over the various intervals, we can approximate the tangent to the curve as the slope of the line.

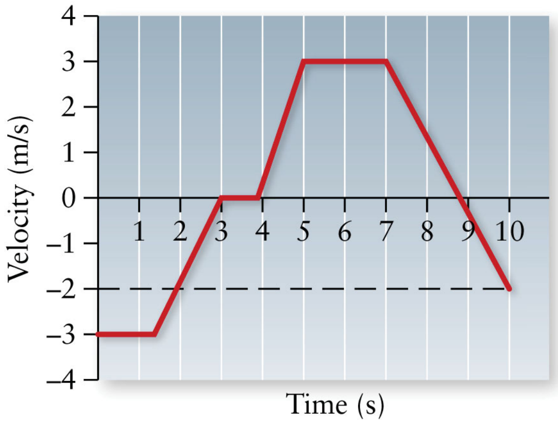


Figure 2-12 Problem 92

SOLVE
Acceleration at t = 2 s:

$$a(2\text{ s}) = \frac{0\frac{\text{m}}{\text{s}} - \left(-3\frac{\text{m}}{\text{s}}\right)}{3\text{ s} - 1.5\text{ s}} = \boxed{2\frac{\text{m}}{\text{s}^2}}$$

Acceleration at t = 4.5 s:

$$a(4.5\text{ s}) = \frac{3\frac{\text{m}}{\text{s}} - 0\frac{\text{m}}{\text{s}}}{5\text{ s} - 4\text{ s}} = \boxed{3\frac{\text{m}}{\text{s}^2}}$$

Acceleration at t = 6 s is $\boxed{0}$.

Acceleration at t = 8 s:

$$a(8\text{ s}) = \frac{\left(-2\frac{\text{m}}{\text{s}}\right) - 3\frac{\text{m}}{\text{s}}}{10\text{ s} - 7\text{ s}} = \boxed{1.7\frac{\text{m}}{\text{s}^2}}$$

REFLECT

Because the acceleration was constant over the various time intervals, the instantaneous acceleration is equal to the average acceleration.

2.93

SET UP

A ball is dropped from an unknown height above your window. You observe that the ball takes 0.18 s to traverse the length of your window, which is 1.5 m. We can calculate the velocity of the ball when it is at the top of the window from the length of the window, the acceleration due to gravity, and the time it takes to pass by the window. Once we have the velocity at that point, we can determine the height the ball needed to fall to achieve that speed, assuming that its initial velocity was zero. Throughout the problem, we will define *down* to be negative.

SOLVE

Speed of ball at top of window:

$$\Delta y = v_0 t + \frac{1}{2}at^2 = v_0 t + \frac{1}{2}(-g)t^2$$

$$v_0 = \frac{\Delta y + \frac{1}{2}gt^2}{t} = \frac{(-1.5\text{ m}) + \frac{1}{2}\left(9.8\frac{\text{m}}{\text{s}^2}\right)(0.18\text{ s})^2}{(0.18\text{ s})} = -7.45\frac{\text{m}}{\text{s}}$$

Distance the ball drops to achieve a speed of 7.45 m/s:

$$v^2 - v_0^2 = 2a(\Delta y) = 2(-g)(\Delta y)$$

$$\Delta y = \frac{v^2 - v_0^2}{2(-g)} = \frac{\left(-7.45\frac{\text{m}}{\text{s}}\right)^2 - 0}{2\left(-9.8\frac{\text{m}}{\text{s}^2}\right)} = -2.8\text{ m}$$

The ball started at a distance of $\boxed{2.8\text{ m}}$ above your window.

REFLECT

Be careful with the signs of Δy, a, and v in this problem.

2.94

SET UP

A lemon is tossed straight up into the air at an initial speed of 15 m/s. Because the lemon is under the influence of gravity, we can calculate the time(s) at which the lemon is 5 m and 7 m above its release point using $\Delta y = v_0 t + \frac{1}{2}at^2$. The equation $v^2 - v_0^2 = 2a(\Delta y)$ will let us find the speed of the lemon 7 m above its release point.

SOLVE

Part a)

$$\Delta y = v_0 t + \frac{1}{2}at^2 = v_0 t + \frac{1}{2}(-g)t^2$$

$$\frac{-g}{2}t^2 + v_0 t - \Delta y = 0$$

$$t = \frac{-v_0 \pm \sqrt{v_0^2 - 4\left(-\frac{g}{2}\right)(-\Delta y)}}{2\left(-\frac{g}{2}\right)} = \frac{-v_0 \pm \sqrt{v_0^2 - 2g(\Delta y)}}{-g}$$

$$= \frac{-\left(15\frac{\text{m}}{\text{s}}\right) \pm \sqrt{\left(15\frac{\text{m}}{\text{s}}\right)^2 - 2\left(9.8\frac{\text{m}}{\text{s}^2}\right)(5\text{ m})}}{-\left(9.8\frac{\text{m}}{\text{s}^2}\right)} = \frac{-\left(15\frac{\text{m}}{\text{s}}\right) \pm \left(11.3\frac{\text{m}}{\text{s}}\right)}{-\left(9.8\frac{\text{m}}{\text{s}^2}\right)} = \boxed{0.38\text{ s and }2.7\text{ s}}$$

Part b)

$$v^2 - v_0^2 = 2a(\Delta y) = 2(-g)(\Delta y)$$

$$|v| = \left|\sqrt{v_0^2 - 2g(\Delta y)}\right| = \left|\sqrt{\left(15\frac{\text{m}}{\text{s}}\right)^2 - 2\left(9.8\frac{\text{m}}{\text{s}^2}\right)(7\text{ m})}\right| = \boxed{9.4\frac{\text{m}}{\text{s}}}$$

Part c)

$$\Delta y = v_0 t + \frac{1}{2}at^2 = v_0 t + \frac{1}{2}(-g)t^2$$

$$\frac{-g}{2}t^2 + v_0 t - \Delta y = 0$$

$$t = \frac{-v_0 \pm \sqrt{v_0^2 - 4\left(-\frac{g}{2}\right)(-\Delta y)}}{2\left(-\frac{g}{2}\right)} = \frac{-v_0 \pm \sqrt{v_0^2 - 2g(\Delta y)}}{-g}$$

$$= \frac{-\left(15\frac{\text{m}}{\text{s}}\right) \pm \sqrt{\left(15\frac{\text{m}}{\text{s}}\right)^2 - 2\left(9.8\frac{\text{m}}{\text{s}^2}\right)(7\text{ m})}}{-\left(9.8\frac{\text{m}}{\text{s}^2}\right)} = \frac{-\left(15\frac{\text{m}}{\text{s}}\right) \pm \left(9.4\frac{\text{m}}{\text{s}}\right)}{-\left(9.8\frac{\text{m}}{\text{s}^2}\right)} = \boxed{0.57\text{ s and }2.5\text{ s}}$$

REFLECT

There are two answers to parts (a) and (c) because the lemon passes through those points on both its way up into the air and its way back down. There is only one answer to part (b) because we are asked about the lemon's *speed*, not its velocity.

2.95

SET UP

Coraline drops a rock into a well of unknown depth, which we will call Δy. After dropping it, she hears the sound of the rock hitting the bottom $t_{\text{total}} = 5.5$ s later. We can split this time up into two parts: #1) the rock falling a distance Δy in a time $t_{\text{free fall}}$ and #2) the sounds traveling

a distance Δy in a time t_{sound}. We can write each time in terms of the common Δy and add them together to give t_{total}. From this we can solve for the depth of the well.

SOLVE

$$t_{\text{total}} = t_{\text{free fall}} + t_{\text{sound}} = 5.5\ \text{s}$$

Free fall:

$$\Delta y = v_0 t_{\text{free fall}} + \frac{1}{2} a t_{\text{free fall}}^2 = 0 + \frac{1}{2}(-g) t_{\text{free fall}}^2$$

$$t_{\text{free fall}} = \sqrt{\left|\frac{2(\Delta y)}{-g}\right|}$$

(The absolute value sign is there since we are only interested in the distance and not the displacement.)

Sound:

$$v_{\text{sound}} = \frac{\Delta y}{t_{\text{sound}}}$$

$$t_{\text{sound}} = \frac{\Delta y}{v_{\text{sound}}}$$

Solving for Δy:

$$t_{\text{total}} = t_{\text{free fall}} + t_{\text{sound}} = \sqrt{\frac{2(\Delta y)}{g}} + \frac{\Delta y}{v_{\text{sound}}}$$

This is a quadratic equation in terms of the variable $\sqrt{(\Delta y)}$:

$$\frac{\Delta y}{v_{\text{sound}}} + \sqrt{\frac{2(\Delta y)}{g}} - t_{\text{total}} = 0 = \frac{1}{v_{\text{sound}}}(\sqrt{\Delta y})^2 + \sqrt{\frac{2}{g}}\sqrt{(\Delta y)} - t_{\text{total}}$$

Using the quadratic formula:

$$\sqrt{(\Delta y)} = \frac{-\sqrt{\frac{2}{g}} \pm \sqrt{\frac{2}{g} - 4\left(\frac{1}{v_{\text{sound}}}\right)(-t_{\text{total}})}}{2\left(\frac{1}{v_{\text{sound}}}\right)} = \frac{-\sqrt{\frac{2}{g}} \pm \sqrt{\frac{2}{g} + \frac{4t_{\text{total}}}{v_{\text{sound}}}}}{\frac{2}{v_{\text{sound}}}}$$

$$= \frac{-\sqrt{\frac{2}{\left(9.8\frac{\text{m}}{\text{s}^2}\right)}} \pm \sqrt{\frac{2}{\left(9.8\frac{\text{m}}{\text{s}^2}\right)} + \frac{4(5.5\ \text{s})}{\left(340\frac{\text{m}}{\text{s}}\right)}}}{\frac{2}{\left(340\frac{\text{m}}{\text{s}}\right)}} = \frac{-0.452 \pm 0.518}{0.00588}\ \text{m}^{1/2}$$

Taking the positive root,

$$\sqrt{(\Delta y)} = \frac{-0.452 + 0.518}{0.00588}\ \text{m}^{1/2} = 11.2\ \text{m}^{1/2}$$

We need to square this to get Δy:

$$\Delta y = (11.2 \text{ m}^{1/2})^2 = \boxed{130 \text{ m}}$$

REFLECT

Rather than solving the quadratic formula for $\sqrt{(\Delta y)}$, you could have squared the expression to get rid of the square root first. The majority of the time is due to the rock falling. It takes 5.1 s for the rock to fall to the water and only 0.4 s for the sound to reach Coraline's ears.

2.96

SET UP

Ten washers are tied to a long string at various locations. Washer #1 is 10 cm off the ground. When the string is dropped, the washers hit the floor with equal time intervals. This means that it takes t_1 s for washer #1 to fall to the ground, $t_2 = t_1 + t_1 = 2t_1$ for washer #2 to fall, and so on. Because all of the washers are undergoing free fall, we can relate the height of each washer above the floor to the time it takes it to hit the floor using $y_f - y_0 = v_0 t + \frac{1}{2}at^2$.

The distance y_n between neighboring washers is equal to the difference in the heights of the neighboring washers.

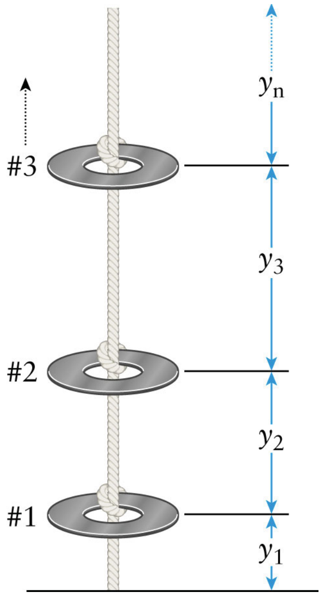


Figure 2-13 Problem 96

SOLVE

Let t_n be the time it takes the nth washer to hit the ground from a height h_n off the ground. Since all of the washers start at rest, we can relate h_n and t_n by:

$$y_f - y_0 = v_0 t + \frac{1}{2}at^2$$

$$0 - h_n = 0 + \frac{1}{2}(-g)t_n^2$$

$$h_n = \frac{1}{2}gt_n^2$$

We know that the washers hit the floor in equal time intervals. This means that it takes t_1 for washer #1 to fall to the ground, $t_2 = t_1 + t_1 = 2t_1$ for washer #2 to fall, and so on. In general, the nth washer will take nt_1 seconds to fall from a height h_n.

Plugging this information into our general height relationship:

$$h_n = \frac{1}{2}gt_n^2 = \frac{1}{2}g(nt_1)^2 = n^2\left(\frac{1}{2}gt_1^2\right) = n^2 h_1, \text{ where } h_1 = 10 \text{ cm}$$

We are interested in the distance, y_n, between the nth washer and the washer below it ("nth $-$ 1"). This distance is related to their heights above the ground:

$$y_n = h_n - h_{n-1} = n^2 h_1 - (n-1)^2 h_1 = n^2 h_1 - (n^2 - 2n + 1)h_1 = \boxed{(2n-1)h_1}$$

REFLECT

It is always a good idea to work through some simple cases to make sure your generalized answer is correct. For example, we can calculate y_1, the distance washer #1 is off the ground:

$y_1 = (2 \cdot 1 - 1)h_1 = (1)h_1 = h_1 = 10$ cm, which is true. We can also calculate the time it takes washers #1 and #2 to fall and make sure the time intervals are equal:

$$h_1 = \frac{1}{2}gt_1^2$$

$$t_1 = \sqrt{\frac{2h_1}{g}} = 0.143 \text{ s}$$

$$h_2 = (2)^2h_1 = 4h_1 = \frac{1}{2}gt_2^2$$

$$t_2 = \sqrt{\frac{8h_1}{g}} = 0.286 \text{ s}$$

Washer #1 takes 0.143 s to fall and washer #2 takes twice that time, which is consistent with the problem statement.

2.97

SET UP

A rocket with two stages of rocket fuel is launched straight up into the air from rest. Stage 1 lasts 10.0 s and provides a net upward acceleration of 15 m/s^2. Stage 2 lasts 5.0 s and provides a net upward acceleration of 12 m/s^2. After Stage 2 finishes, the rocket continues to travel upward under the influence of gravity alone until it reaches its maximum height and then falls back toward Earth. We can split the rocket's flight into four parts: (1) Stage 1, (2) Stage 2, (3) between the end of Stage 2 and reaching the maximum height, and (4) falling from the maximum height back to Earth's surface. The acceleration of the rocket is constant over each of these legs, so we can use the constant acceleration equations to determine the total distance covered in each leg and the duration of each leg. The initial speed for legs #1 and #4 is zero, but we will need to calculate the initial speeds for legs #2 and #3. The maximum altitude is equal to the distance covered in legs #1–#3; the time required for the rocket to return to Earth is equal to the total duration of its flight, which is the sum of the durations of legs #1–#4.

SOLVE

Maximum altitude

Distance traveled in Stage 1:

$$\Delta y = v_0t + \frac{1}{2}at^2$$

$$\Delta y = 0 + \frac{1}{2}\left(15\frac{\text{m}}{\text{s}^2}\right)(10.0 \text{ s})^2 = 750 \text{ m}$$

Speed after Stage 1:

$$v = v_0 + at = 0 + \left(15\frac{\text{m}}{\text{s}^2}\right)(10.0 \text{ s}) = 150\frac{\text{m}}{\text{s}}$$

Distance traveled in Stage 2:

$$\Delta y = v_0t + \frac{1}{2}at^2$$

$$\Delta y = \left(150\frac{\text{m}}{\text{s}}\right)(5.0\text{ s}) + \frac{1}{2}\left(12\frac{\text{m}}{\text{s}^2}\right)(5.0\text{ s})^2 = 900\text{ m}$$

Speed after Stage 2:

$$v = v_0 + at = \left(150\frac{\text{m}}{\text{s}}\right) + \left(12\frac{\text{m}}{\text{s}^2}\right)(5.0\text{ s}) = 210\frac{\text{m}}{\text{s}}$$

Distance traveled after Stage 2:

$$v^2 - v_0^2 = 2a(\Delta y)$$

$$\Delta y = \frac{v^2 - v_0^2}{2a} = \frac{0 - \left(210\frac{\text{m}}{\text{s}}\right)^2}{2\left(-9.8\frac{\text{m}}{\text{s}^2}\right)} = 2250\text{ m}$$

Maximum altitude: 750 m + 900 m + 2250 m = $\boxed{3900\text{ m}}$.

Time required to return to the surface
Time after Stage 2:

$$v = v_0 + at$$

$$t = \frac{v - v_0}{a} = \frac{0 - \left(210\frac{\text{m}}{\text{s}}\right)}{\left(-9.8\frac{\text{m}}{\text{s}^2}\right)} = 21.4\text{ s}$$

Time from maximum height to the ground:

$$\Delta y = v_0 t + \frac{1}{2}at^2 = 0 + \frac{1}{2}(-g)t^2$$

$$t = \sqrt{\frac{2(\Delta y)}{-g}} = \sqrt{\frac{2(-3900\text{ m})}{-\left(9.8\frac{\text{m}}{\text{s}^2}\right)}} = 28.2\text{ s}$$

Total time of flight: 10.0 s + 5.0 s + 21.4 s + 28.2 s = $\boxed{64.6\text{ s}}$.

REFLECT
An altitude of 3900 m is around 2.5 mi. The simplest way of solving this problem was to split it up into smaller, more manageable calculations and then put all of the information together.

2.98

SET UP
A lacrosse ball has an initial velocity of v_0 straight up when it leaves a stick that is 2.00 m above the ground. It travels past a 1.25-m-high window in 0.4 s. The base of the window is 13 m above the ground, or 11 m above the stick. From the information about the window, we can calculate the speed of the ball when it is at the base of the window. Once we know the speed of the ball at the base of the window, we can calculate the necessary initial launch speed of the ball.

SOLVE
Speed at the base of the window:

$$\Delta y = v_0 t + \frac{1}{2}at^2$$

$$v_0 = \frac{\Delta y - \frac{1}{2}at^2}{t} = \frac{(1.25\text{ m}) - \frac{1}{2}\left(-9.8\frac{\text{m}}{\text{s}^2}\right)(0.4\text{ s})^2}{(0.4\text{ s})} = 5.1\frac{\text{m}}{\text{s}}$$

Initial speed:

$$v^2 - v_0^2 = 2a(\Delta y)$$

$$v_0 = \sqrt{v^2 - 2a(\Delta y)} = \sqrt{\left(5.1\frac{\text{m}}{\text{s}}\right)^2 - 2\left(-9.8\frac{\text{m}}{\text{s}^2}\right)(11\text{ m})} = \boxed{15.5\frac{\text{m}}{\text{s}}}$$

REFLECT
A speed of 15.5 m/s is around 35 mph. Some lacrosse shots on goal can be 100 mph!

2.99

SET UP
A black mamba snake has a length of 4.3 m and a top speed of 8.9 m/s. It starts at rest nose-to-nose with a mongoose. The snake accelerates straight forward from rest at a constant rate of 18 m/s^2. We can calculate the time it takes the snake to reach its top speed using $v = v_0 + at$ and then the distance the snake travels during this time using $\Delta x = v_0 t + \frac{1}{2}at^2$. Comparing this distance traveled to the length of the snake will tell us whether or not the entire snake has traveled past the mongoose.

SOLVE

Part a)

$$v = v_0 + at$$

$$t = \frac{v - v_0}{a} = \frac{8.9\frac{\text{m}}{\text{s}} - 0}{18\frac{\text{m}}{\text{s}^2}} = \boxed{0.49\text{ s}}$$

Part b)

$$\Delta x = v_0 t + \frac{1}{2}at^2 = 0 + \frac{1}{2}\left(18\frac{\text{m}}{\text{s}^2}\right)(0.49\text{ s})^2 = \boxed{2.2\text{ m}}$$

Part c) The snake is 4.3 m long and has only traveled 2.2 m at this point. The $\boxed{\text{mongoose has a chance}}$ to catch the snake.

REFLECT
The black mamba's top speed is nearly 20 mph!

2.100

SET UP

Kharissia wants to complete a 1000-m race with an average speed of 8 m/s. She has already completed 750 m with an average speed of 7.2 m/s. We can use a weighted average to determine how fast she needs to run the final 250 m in order to have an overall average speed of 8 m/s. She completed 75% of the race with an average speed of 7.2 m/s. We can then solve for the average speed v at which she needs to run the remaining 25% of the race.

SOLVE

$$(0.75)\left(7.2\frac{\text{m}}{\text{s}}\right) + (0.25)v = 8\frac{\text{m}}{\text{s}}$$

$$v = \frac{8\frac{\text{m}}{\text{s}} - (0.75)\left(7.2\frac{\text{m}}{\text{s}}\right)}{(0.25)} = \boxed{10\frac{\text{m}}{\text{s}}}$$

REFLECT

Kharissia needs to run 2.8 m/s faster, on average, for the last 250 m of the race in order to achieve her goal overall speed.

2.101

SET UP

Steve Prefontaine ("Pre") completed a 10-km race in 27 min, 43.6 s. He ran the first 9 km in 25 min, which means he ran the final 1 km in 2 min, 43.6 s (= 163.6 s). During the final 1000 m, he accelerates for 60 s and maintains that increased speed for the remainder of the race (that is, 103.6 s). We are told that his speed at the 9-km mark is equal to the average speed over the first 9 km. We can use the constant acceleration equations to write the total distance of the final leg in terms of the acceleration a. He accelerates over a distance of Δx_1 for $t_1 = 60$ s and then runs at his final constant speed over a distance of Δx_2 for $t_2 = 103.6$ s. These distances must sum to 1000 m.

SOLVE

Average speed for the first 9 km:

$$v_{\text{first 9 km}} = \frac{9\text{ km}}{25\text{ min}} \times \frac{1000\text{ m}}{1\text{ km}} \times \frac{1\text{ min}}{60\text{ s}} = 6.0\frac{\text{m}}{\text{s}}$$

Time it takes to run the last 1 km:

$$27\text{ min} \times \frac{60\text{ s}}{1\text{ min}} = 1620\text{ s}$$

Total time = 1620 s + 43.6 s = 1663.6 s.

$$25\text{ min} \times \frac{60\text{ s}}{1\text{ min}} = 1500\text{ s}$$

Time for last 1 km = 1663.6 s − 1500 s = 163.6 s.

Calculating his acceleration:

$$\Delta x_{\text{total}} = 1000 \text{ m} = \Delta x_1 + \Delta x_2$$

$$\Delta x_1 = v_0 t_1 + \frac{1}{2} a t_1^2$$

$$\Delta x_2 = v_1 t_2 = (v_0 + a t_1) t_2$$

$$\Delta x_{\text{total}} = 1000 \text{ m} = \left(v_0 t_1 + \frac{1}{2} a t_1^2\right) + (v_0 + a t_1) t_2$$

$$= \left(\left(6.0 \frac{\text{m}}{\text{s}}\right)(60 \text{ s}) + \frac{1}{2} a (60 \text{ s})^2\right) + \left(\left(6.0 \frac{\text{m}}{\text{s}}\right) + a(60 \text{ s})\right)(103.6 \text{ s})$$

$$1000 \text{ m} = (981.6 \text{ m}) + (8016 \text{ s}^2) a$$

$$\boxed{a = 0.0023 \frac{\text{m}}{\text{s}^2}}$$

REFLECT

During the last portion of the race, Pre travels

$\Delta x_1 = \left(6.0 \frac{\text{m}}{\text{s}}\right)(60 \text{ s}) + \frac{1}{2}\left(0.0023 \frac{\text{m}}{\text{s}^2}\right)(60 \text{ s})^2 = 364 \text{ m}$ while accelerating up to a final speed of $v_1 = \left(6.0 \frac{\text{m}}{\text{s}}\right) + \left(0.0023 \frac{\text{m}}{\text{s}^2}\right)(60 \text{ s}) = 6.138 \frac{\text{m}}{\text{s}}$. At this speed he travels a total distance of $\Delta x_2 = \left(6.138 \frac{\text{m}}{\text{s}}\right)(103.6 \text{ s}) = 636 \text{ m}$. These two final distances do indeed add up to 1000 m: $364 \text{ m} + 636 \text{ m} = 1000 \text{ m}$.

2.102

SET UP

An egg is thrown straight down at a speed of 1.5 m/s from the top of a tall tower. After 2 s a second egg is thrown up at a speed of 4.0 m/s from the same starting position. Since both eggs are undergoing free fall, we can calculate the distance each egg is relative to the top of the tower as a function of time. The first egg is in flight for 6 s, while the second egg is in flight for 4 s. In order to calculate the minimum separation between the eggs, we first need to calculate the position of the first egg when the second one is thrown. This will give us insight into what the minimum separation will be.

SOLVE

Distance apart 4 s after second egg is thrown:

$$\Delta y_1 = v_{0,1} t_1 + \frac{1}{2} a_y t_1^2 = \left(-1.5 \frac{\text{m}}{\text{s}}\right)(6 \text{ s}) + \frac{1}{2}\left(-9.8 \frac{\text{m}}{\text{s}^2}\right)(6 \text{ s})^2 = -185.4 \text{ m}$$

$$\Delta y_2 = v_{0,2} t_2 + \frac{1}{2} a_y t_2^2 = \left(4.0 \frac{\text{m}}{\text{s}}\right)(4 \text{ s}) + \frac{1}{2}\left(-9.8 \frac{\text{m}}{\text{s}^2}\right)(4 \text{ s})^2 = -62.4 \text{ m}$$

$$\Delta y_2 - \Delta y_1 = (-62.4\text{ m}) - (-185.4\text{ m}) = \boxed{123.0\text{ m}}$$

Location of the first egg when the second egg is thrown:

$$\Delta y_1 = v_{0,1}t_1 + \frac{1}{2}a_y t_1^2 = \left(-1.5\frac{\text{m}}{\text{s}}\right)(2\text{ s}) + \frac{1}{2}\left(-9.8\frac{\text{m}}{\text{s}^2}\right)(2\text{ s})^2 = -22.6\text{ m}$$

This is already below the starting position, so the minimum separation between the eggs occurs at the $\boxed{\text{instant the second egg is thrown}}$; the minimum separation between the eggs is $\boxed{22.6\text{ m}}$.

REFLECT

We did not need to know the exact height of the tower in order to find the relative separation of the eggs because they started from a common location.

2.103

SET UP

A rocket with two stages of rocket fuel is launched straight up into the air from rest. Stage 1 lasts 15.0 s and provides a net upward acceleration of 2.00 m/s^2. Stage 2 lasts 12.0 s and provides a net upward acceleration of 3.00 m/s^2. After Stage 2 finishes, the rocket continues to travel upward under the influence of gravity alone until it reaches its maximum height and then falls back toward Earth. We can split the rocket's flight into four parts: (1) Stage 1, (2) Stage 2, (3) between the end of Stage 2 and reaching the maximum height, and (4) falling from the maximum height back to Earth's surface. The acceleration of the rocket is constant over each of these legs, so we can use the constant acceleration equations to determine the total distance covered in each leg and the duration of each leg. The initial speed for legs #1 and #4 is zero, but we will need to calculate the initial speeds for legs #2 and #3. The maximum altitude is equal to the distance covered in legs #1–#3. The average speed is the total distance covered by the rocket divided by the total time of the flight. Because the rocket starts and ends at the same position, its displacement and, therefore, average velocity are both equal to zero.

SOLVE

Part a)

Distance traveled in Stage 1:

$$\Delta y = v_0 t + \frac{1}{2}at^2$$

$$\Delta y = 0 + \frac{1}{2}\left(2.00\frac{\text{m}}{\text{s}^2}\right)(15.0\text{ s})^2 = 225\text{ m}$$

Speed after Stage 1:

$$v = v_0 + at = 0 + \left(2.00\frac{\text{m}}{\text{s}^2}\right)(15.0\text{ s}) = 30.0\frac{\text{m}}{\text{s}}$$

Distance traveled in Stage 2:

$$\Delta y = v_0 t + \frac{1}{2}at^2$$

$$\Delta y = \left(30.0\frac{\text{m}}{\text{s}}\right)(12.0\text{ s}) + \frac{1}{2}\left(3.00\frac{\text{m}}{\text{s}^2}\right)(12.0\text{ s})^2 = 576\text{ m}$$

Speed after Stage 2:

$$v = v_0 + at = \left(30.0\frac{\text{m}}{\text{s}}\right) + \left(3.00\frac{\text{m}}{\text{s}^2}\right)(12.0\text{ s}) = 66.0\frac{\text{m}}{\text{s}}$$

Distance traveled after Stage 2:

$$v^2 - v_0^2 = 2a(\Delta y)$$

$$\Delta y = \frac{v^2 - v_0^2}{2a} = \frac{0 - \left(66.0\frac{\text{m}}{\text{s}}\right)^2}{2\left(-9.8\frac{\text{m}}{\text{s}^2}\right)} = 222\text{ m}$$

Maximum altitude: 225 m + 576 m + 222 m = $\boxed{1023\text{ m}}$.

Part b)

Time after Stage 2:

$$v = v_0 + at$$

$$t = \frac{v - v_0}{a} = \frac{0 - \left(66.0\frac{\text{m}}{\text{s}}\right)}{\left(-9.8\frac{\text{m}}{\text{s}^2}\right)} = 6.7\text{ s}$$

Time from maximum height to the ground:

$$\Delta y = v_0 t + \frac{1}{2}at^2 = 0 + \frac{1}{2}(-g)t^2$$

$$t = \sqrt{\frac{2(\Delta y)}{-g}} = \sqrt{\frac{2(-1023\text{ m})}{-\left(9.8\frac{\text{m}}{\text{s}^2}\right)}} = 14.4\text{ s}$$

Total time of flight: 15.0 s + 12.0 s + 6.7 s + 14.4 s = 48.1 s.

i) Average speed:

$$v_{\text{average}} = \frac{2046\text{ m}}{48.1\text{ s}} = \boxed{42.5\frac{\text{m}}{\text{s}}}$$

ii) Average velocity = $\boxed{0}$ because the rocket's displacement is zero.

REFLECT

Although they are used interchangeably in everyday speech, speed and velocity have different meanings in physics. Average speed is the total *distance* traveled per time, while average velocity is the total *displacement* traveled per time.

2.104

SET UP

Acceleration is the first derivative of the velocity with respect to time. We are given the acceleration of a falling object as a function of velocity, which means this is a differential equation. Specifically, since the velocity is to the first power, this is a first-order linear ordinary differential equation. We can solve (that is, find $v(t)$) this type of differential equation by solving for the multiplicative factor μ, such that $\frac{d}{dt}(\mu v) = \mu g$. We can determine the constant of integration from the initial condition that the object is at rest at $t = 0$. The terminal velocity of the object is equal to $v(t)$ in the limit as $t \to \infty$.

SOLVE

Part a)

$$a = \frac{dv}{dt} = g - \alpha v$$

$$\frac{dv}{dt} + \alpha v = g$$

Solving the first-order linear differential equation:

$$\mu = e^{\int \alpha dt} = e^{\alpha t}$$

$$\mu\left(\frac{dv}{dt} + \alpha v = g\right)$$

$$e^{\alpha t}\frac{dv}{dt} + \alpha e^{\alpha t} v = g e^{\alpha t}$$

$$\frac{d}{dt}(e^{\alpha t} v) = g e^{\alpha t}$$

$$e^{\alpha t} v = \int g e^{\alpha t} dt = g\left[\frac{1}{\alpha} e^{\alpha t}\right] + C$$

$$\boxed{v(t) = \frac{g}{\alpha} + Ce^{-\alpha t}}$$

Part b)

$v(0) = 0$, so

$$v(0) = \frac{g}{\alpha} + Ce^{-\alpha(0)} = \frac{g}{\alpha} + C = 0, \text{ or } C = -\frac{g}{\alpha}$$

$$v(t) = \frac{g}{\alpha} - \frac{g}{\alpha} e^{-\alpha t}$$

$$v_{\text{term}} = \lim_{t \to \infty} v(t) = \lim_{t \to \infty}\left(\frac{g}{\alpha} - \frac{g}{\alpha} e^{-\alpha t}\right) = \boxed{\frac{g}{\alpha}}$$

REFLECT

We can double-check that our function $v(t)$ satisfies the original differential equation:

$$v(t) = \frac{g}{\alpha} - \frac{g}{\alpha}e^{-\alpha t}$$

$$\frac{dv}{dt} = \frac{d}{dt}\left(\frac{g}{\alpha} - \frac{g}{\alpha}e^{-\alpha t}\right) = 0 - \frac{g}{\alpha}(-\alpha e^{-\alpha t}) = ge^{-\alpha t}$$

$$a = \frac{dv}{dt} = ge^{-\alpha t} \stackrel{?}{=} g - \alpha v = g - \alpha\left(\frac{g}{\alpha} - \frac{g}{\alpha}e^{-\alpha t}\right)$$

$$ge^{-\alpha t} \stackrel{?}{=} g - g + ge^{-\alpha t}$$

$$ge^{-\alpha t} \stackrel{?}{=} ge^{-\alpha t} \checkmark$$

2.105

SET UP

Acceleration is the first derivative of the velocity with respect to time. We are given the acceleration of a falling object as a function of velocity, which means this is a differential equation. We can solve (that is, find $v(t)$) this type of differential equation by separation of variables. We can determine the constant of integration from the initial condition that the object is at rest at $t = 0$. The terminal velocity of the object is equal to $v(t)$ in the limit as $t \to \infty$.

SOLVE

Part a)

$$a = \frac{dv}{dt} = g - \beta v^2$$

Solving the differential equation:

$$\frac{dv}{g - \beta v^2} = dt$$

$$\int \frac{dv}{\frac{g}{\beta} - v^2} = \int \beta dt$$

$$\frac{1}{\left(\sqrt{\frac{g}{\beta}}\right)}\operatorname{arctanh}\left(v\sqrt{\frac{\beta}{g}}\right) = \beta t + C$$

$$\operatorname{arctanh}\left(v\sqrt{\frac{\beta}{g}}\right) = t\sqrt{g\beta} + C$$

$$\tanh(t\sqrt{g\beta} + C) = v\sqrt{\frac{\beta}{g}}$$

$$v(t) = \sqrt{\frac{g}{\beta}}\tanh(t\sqrt{g\beta} + C)$$

Applying the initial condition that $v(0) = 0$:

$$v(0) = \sqrt{\frac{g}{\beta}}\tanh((0)\sqrt{g\beta} + C) = 0$$

$$C = \operatorname{arctanh}(0) = 0, \text{ so } \boxed{v(t) = \sqrt{\frac{g}{\beta}}\tanh(t\sqrt{g\beta})}$$

Part b)

$$v_{\text{term}} = \lim_{t\to\infty} v(t) = \lim_{t\to\infty}\left(\sqrt{\frac{g}{\beta}}\tanh(t\sqrt{g\beta})\right) = \boxed{\sqrt{\frac{g}{\beta}}}$$

(The limit of $\tanh(x)$ as x approaches infinity is 1.)

REFLECT

We can double-check that our function $v(t)$ satisfies the original differential equation. (We will need the derivative of $\tanh(x)$: $\dfrac{d\tanh(x)}{dx} = 1 - \tanh^2(x)$.)

$$v(t) = \sqrt{\frac{g}{\beta}}\tanh(t\sqrt{g\beta})$$

$$\frac{dv}{dt} = \frac{d}{dt}\left(\sqrt{\frac{g}{\beta}}\tanh(t\sqrt{g\beta})\right) = \sqrt{\frac{g}{\beta}}(1 - \tanh^2(t\sqrt{g\beta}))(\sqrt{g\beta})$$

$$a = \frac{dv}{dt} = \sqrt{\frac{g}{\beta}}(1 - \tanh^2(t\sqrt{g\beta}))(\sqrt{g\beta}) \stackrel{?}{=} g - \beta v^2 = g - \beta\left(\sqrt{\frac{g}{\beta}}\tanh(t\sqrt{g\beta})\right)^2$$

$$g(1 - \tanh^2(t\sqrt{g\beta})) \stackrel{?}{=} g - \beta\left(\frac{g}{\beta}\tanh^2(t\sqrt{g\beta})\right)$$

$$g(1 - \tanh^2(t\sqrt{g\beta})) \stackrel{?}{=} g(1 - \tanh^2(t\sqrt{g\beta})) \quad \checkmark$$

Chapter 3
Motion in Two Dimensions

Conceptual Questions

3.1 Part a) No, the sum of two vectors that have different magnitudes can never equal zero. The only time two vectors can add together to have zero magnitude is when they point in opposite directions and have the same magnitude.

Part b) Yes, the sum of three (or more) vectors with different magnitudes can be equal to zero.

3.2 A scalar is a number, whereas a vector has a direction associated with a number. Distance and speed are examples of scalar quantities, whereas displacement and velocity are examples of vectors.

3.3 If an object is moving at a constant velocity (that is, its speed *and* direction are constant), then the average velocity and instantaneous velocity are equal. If the object is *not* moving at a constant velocity (that is, its speed is changing, its direction is changing, or both), then its average velocity will not equal its instantaneous velocity.

3.4 Part a) An object undergoing uniform circular motion will follow a circular trajectory at a constant speed, whereas an object undergoing projectile motion will follow a parabolic path at a varying speed.

Part b) The magnitude of the acceleration in each case is constant (v^2/R for uniform circular motion, g for projectile motion). The velocity is constantly changing in each case as well.

3.5 Air resistance can be modeled as a force that depends on the velocity. The greater the speed, the greater the magnitude of the drag force. Also, the drag force points in the direction opposite to the direction of motion. Therefore, when the effects of air resistance are taken into account, the projectile's speed in the vertical direction will decrease more rapidly as it ascends and increase less rapidly when it comes down compared to when the effects of air resistance are ignored. When effects of air resistance are ignored, a projectile experiences no acceleration in the horizontal direction and, therefore, travels at constant speed horizontally. Due to the drag force resulting from air resistance, however, a projectile experiences a horizontal acceleration that causes its horizontal speed to decrease.

3.6 Gravity on the Moon is about one-sixth that on Earth. If you were playing tennis on the Moon, you would have to hit the ball more softly than on Earth in order for it to stay in bounds. The trajectories should still look parabolic, though.

3.7 The magnitude refers to the size or quantity of a vector.

3.8 During the motion of a projectile, only v_x, a_x (= 0), and a_y (= $-g$, if up is positive) are constant. The position (x, y) of the projectile is constantly changing while it is moving. The acceleration in the y direction is nonzero, which means v_y is changing.

3.9 Part a) Maximum range is achieved at a launch angle of 45 degrees.

Part b) The longest time of flight is achieved by launching the projectile at an angle of 90 degrees (that is, straight up). We can see this from Equation 3-23: $t_{\text{peak}} = \dfrac{v_0 \sin(\theta)}{g}$.

Part c) The greatest height is achieved by launching the projectile at an angle of 90 degrees (that is, straight up). We can see this from Equation 3-24:
$(y - y_0)_{\text{peak}} = \dfrac{1}{2}\dfrac{v_0^2 \sin^2(\theta)}{g}$.

3.10 The rock's speed will be greater than the speed with which it was thrown. Gravity is accelerating the rock in the y direction, so its speed in the y direction will increase.

3.11 Yes, the point at the top of the soccer ball's trajectory is where the velocity and acceleration vectors are perpendicular. Acceleration (due to gravity) is always down; the top of the trajectory is where the vertical component of the ball's velocity passes from up to down. At that moment the only velocity component is horizontal, so they are perpendicular.

3.12 No; a long jumper should take off at an angle of 45 degrees in order to achieve the maximum range.

3.13 Yes, the ape is accelerating at the bottom of its swing. The force from the vine is accelerating the ape upward since it is at the bottom of the swing that it changes from traveling downward to traveling upward.

3.14 No, the cyclist's acceleration vector is not zero if he rides around a flat, circular track at constant speed. Although his speed is constant, his velocity is constantly changing since he is constantly changing direction.

3.15 Yes, you are accelerating because the direction of the velocity vector is changing. The acceleration vector in this case points east.

Multiple-Choice Questions

3.16 **C** (distance). Distance is not a vector.

3.17 **B** (45 degrees). $\theta = \arctan\left(\dfrac{A_y}{A_x}\right) = \arctan(1) = 45°$.

3.18 **B** $(-2.0, 3.5)$.

The vector points toward $-x$ and $+y$, which means the x component of this vector is negative and its y component is positive:

$$A_x = -(4.0)\sin(30°) = \boxed{-2.0}$$

$$A_y = (4.0)\cos(30°) = \boxed{3.5}$$

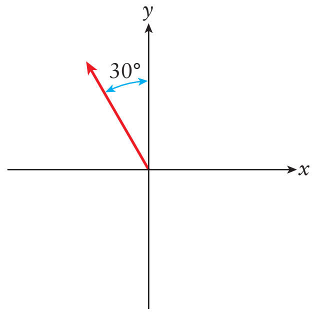


Figure 3-1 Problem 18

3.19 **D** (is vertically downward). Gravity acts downward, and the acceleration in the horizontal direction is zero when air resistance is negligible.

3.20 **C** (they both hit the ground at the same time). The acceleration due to gravity is identical in both cases. The acceleration in the horizontal direction is zero.

3.21 **D** $(d_2 = 4d_1)$.

$$d_1 = \frac{2v_0^2\sin(\theta)\cos(\theta)}{g}$$

$$d_2 = \frac{2(2v_0)^2\sin(\theta)\cos(\theta)}{g} = 4\frac{2v_0^2\sin(\theta)\cos(\theta)}{g} = 4d_1$$

3.22 **A** (aim straight at the monkey). The acceleration due to gravity is identical in both cases.

3.23 **D** (points toward the center of the circle). In uniform circular motion, the speed of the particle is constant, but its direction is constantly changing.

3.24 **A** (2).

$$a_1 = \frac{v^2}{R_1}$$

$$a_2 = \frac{v^2}{2R_1} = \frac{1}{2}a_1$$

3.25 **B** (at the highest point in the flight). The y component of the ball's velocity is equal to zero at the top of its flight.

Estimation Questions

3.26

$$r_x = -24\cos(36°) = 19.4$$

$$r_x = -24\sin(36°) = 14.1$$

Be sure that the vector and angle are to scale when drawing on graph paper.

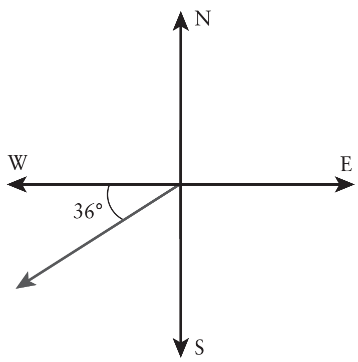


Figure 3-2 Problem 26

3.27

$$r_x = -r\cos(30°) = -r\left(\frac{\sqrt{3}}{2}\right) = -0.87r$$

$$r_y = r\sin(30°) = 0.5r$$

The x component is 87% as long as the full vector. The y component is 50% as long as the full vector. Be sure that the vector and angle are to scale when drawing on graph paper.

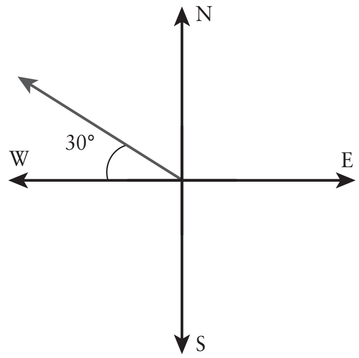


Figure 3-3 Problem 27

3.28

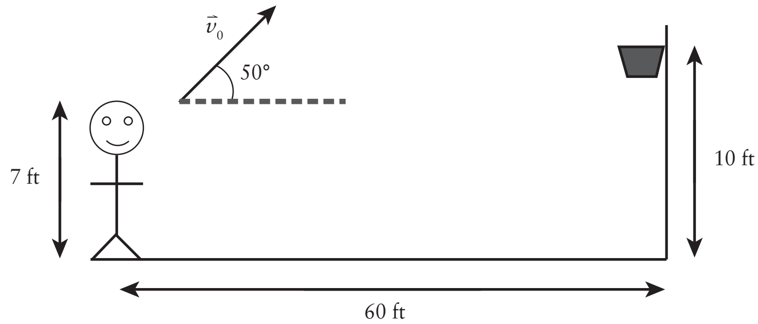


Figure 3-4 Problem 28

We can start by setting up our Know/Don't Know table. We will take West's feet to be the origin, right to be the positive x direction, and up to be the positive y direction.

	X		Y
x_0	0 ft	y_0	7 ft
x_f	60 ft	y_f	10 ft
v_{0x}	WANT	v_{0y}	WANT
v_x	?	v_y	?
a_x	0	a_y	-32 ft/s^2
t	?	t	?

where $v_{0,x} = v_0\cos(50°)$ and $v_{0,y} = v_0\sin(50°)$.

First we will use the x component information to solve for t in terms of known quantities and then use the y component information to calculate the initial speed.

$$\Delta x = v_{0,x}t + \frac{1}{2}a_x t^2 = v_{0,x}t + 0$$

$$t = \frac{\Delta x}{v_{0,x}}$$

$$\Delta y = v_{0,y}t + \frac{1}{2}a_y t^2 = v_{0,y}\left(\frac{\Delta x}{v_{0,x}}\right) + \frac{1}{2}a_y\left(\frac{\Delta x}{v_{0,x}}\right)^2 = (v_0\sin(50°))\left(\frac{\Delta x}{v_0\cos(50°)}\right) + \frac{1}{2}a_y\left(\frac{\Delta x}{v_0\cos(50°)}\right)^2$$

$$= \Delta x\tan(50°) + \frac{a_y(\Delta x)^2}{2\cos^2(50°)}\left(\frac{1}{v_0^2}\right)$$

$$v_0 = \sqrt{\frac{a_y(\Delta x)^2}{2\cos^2(50°)(\Delta y - \Delta x\tan(50°))}} = \sqrt{\frac{\left(-32\frac{\text{ft}}{\text{s}^2}\right)(60\text{ ft})^2}{2\cos^2(50°)((3\text{ ft}) - (60\text{ ft})\tan(50°))}}$$

$$v_0 = \boxed{46\frac{\text{ft}}{\text{s}}} \times \frac{0.3048\text{ m}}{1\text{ ft}} = \boxed{14\frac{\text{m}}{\text{s}}}$$

3.29

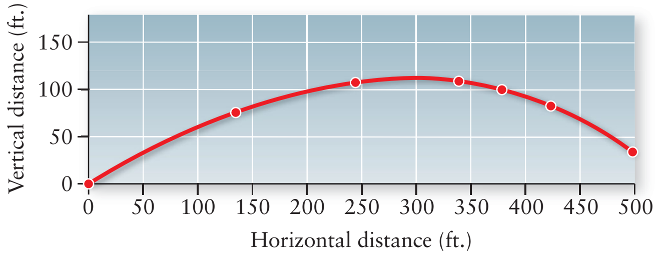


Figure 3-5 Problem 29

The ball reached a maximum height of $\Delta y = 110$ ft:

$$v_y^2 - v_{0,y}^2 = 2a_y(\Delta y)$$

$$v_{0,y} = \sqrt{-2a_y(\Delta y)} = \sqrt{-2\left(-32\frac{\text{ft}}{\text{s}^2}\right)(110\text{ ft})} = 26.5\frac{\text{ft}}{\text{s}}$$

Estimating the initial launch angle of the ball from the graph to be 30 degrees:

$$v_{0,y} = v_0\sin(30°) = \frac{v_0}{2}$$

$$v_0 = 2v_{0,y} = 2\left(26.5\frac{\text{ft}}{2}\right) = 53\frac{\text{ft}}{\text{s}}$$

But the air resistance is not completely negligible here. How much is there? The peak occurs at a horizontal distance of 300 ft and the ball hits Earth at around 540 ft. Without air resistance, the ball should have landed at around 600 ft, so 20% of the forward progress is lost. Therefore, a reasonable estimate of the initial speed is (1.2)(53 ft/s) = 64 ft/s.

3.30 Assume that the object starts from the origin and travels in the positive direction for both x and y. The ground is at $y = 0$.

a) A plot of v_x versus time is constant and greater than zero for all time.

b) A plot of v_y versus time is linear, starts at a maximum $+v_{0,y}$, crosses zero when the ball is at its highest point, and has a constant slope of -9.8 m/s^2. The ball hits the ground when $v_y = -v_{0,y}$.

c) A plot of a_x versus time is equal to zero for all time.

d) A plot of a_y versus time is equal to -9.8 m/s^2 for all time.

Problems

3.31

SET UP

Three vectors are given: $\vec{A} = (6\hat{x} + 9\hat{y})$, $\vec{B} = (7\hat{x} - 3\hat{y})$, and $\vec{C} = (0\hat{x} - 6\hat{y})$. To find the requested sums and differences, we add and subtract the individual components. A vector multiplied by a factor means that each component of the vector is multiplied by that factor.

SOLVE

Part a) $\vec{A} + \vec{B} = (6\hat{x} + 9\hat{y}) + (7\hat{x} - 3\hat{y}) = \boxed{13\hat{x} + 6\hat{y}}$.

Part b) $\vec{A} - 2\vec{C} = (6\hat{x} + 9\hat{y}) - 2(0\hat{x} - 6\hat{y}) = \boxed{6\hat{x} + 21\hat{y}}$.

Part c) $\vec{A} + \vec{B} - \vec{C} = (6\hat{x} + 9\hat{y}) + (7\hat{x} - 3\hat{y}) - (0\hat{x} - 6\hat{y}) = \boxed{13\hat{x} + 12\hat{y}}$.

Part d) $\vec{A} + \frac{1}{2}\vec{B} - 3\vec{C} = (6\hat{x} + 9\hat{y}) + \frac{1}{2}(7\hat{x} - 3\hat{y}) - 3(0\hat{x} - 6\hat{y}) = \boxed{9.5\hat{x} + 25.5\hat{y}}$.

REFLECT

The scaling factor should be multiplied through first before adding the vectors together.

3.32

SET UP

The components of a vector are given. We can find the vector's magnitude by the Pythagorean theorem $\left(r = \sqrt{r_x^2 + r_y^2}\right)$ and the angle it makes with the x-axis from the tangent $\left(\theta = \arctan\left(\frac{r_y}{r_x}\right)\right)$.

SOLVE

$$r = \sqrt{r_x^2 + r_y^2} = \sqrt{(5.0)^2 + (2.5)^2} = \boxed{5.6}$$

$$\theta = \arctan\left(\frac{r_y}{r_x}\right) = \arctan\left(\frac{2.5}{5.0}\right) = \arctan\left(\frac{1}{2}\right) = \boxed{0.46\text{ rad}}$$

REFLECT

An angle of 0.46 radian is about 27 degrees.

3.33

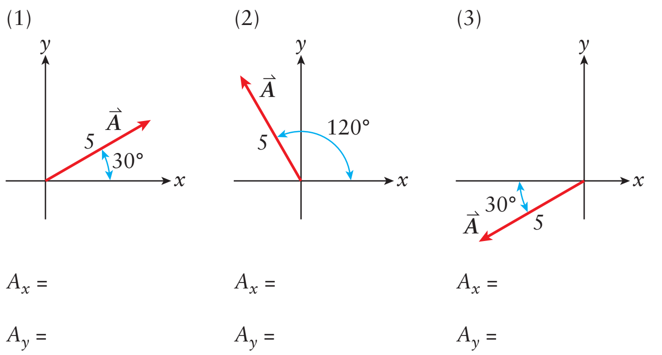


Figure 3-6 Problem 33

SET UP
A vector has a magnitude of 5 and is shown in three different orientations. Since we are given the angles in each case, we can calculate the components directly using $A_x = A\cos(\theta)$ and $A_y = A\sin(\theta)$.

SOLVE
(1)
$$A_x = 5\cos(30°) = \boxed{4.3}$$
$$A_y = 5\sin(30°) = \boxed{2.5}$$

(2)
$$A_x = 5\cos(120°) = -5\cos(60°) = \boxed{-2.5}$$
$$A_y = 5\sin(120°) = 5\sin(60°) = \boxed{4.3}$$

(3)
$$A_x = 5\cos(210°) = -5\cos(30°) = \boxed{-4.3}$$
$$A_y = 5\sin(210°) = -5\sin(30°) = \boxed{2.5}$$

REFLECT
It is a good idea to redraw the right triangle each time to ensure you are taking the sine or cosine of the correct angle.

3.34

SET UP
Three vectors are given in terms of their components. The magnitudes can be calculated using the Pythagorean theorem $(A = \sqrt{A_x^2 + A_y^2})$, and the angles they make with the $+x$-axis can be calculated from the tangent $\left(\theta = \arctan\left(\frac{A_y}{A_x}\right)\right)$.

SOLVE
Part a)
$$\vec{A} = 3\hat{x} - 2\hat{y}$$
$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(3)^2 + (-2)^2} = \boxed{\sqrt{13}}$$
$$\theta = \arctan\left(\frac{A_y}{A_x}\right) = \arctan\left(\frac{-2}{3}\right) = \boxed{5.7 \text{ rad}}$$

Part b)
$$\vec{A} = -2\hat{x} + 2\hat{y}$$
$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(-2)^2 + (2)^2} = \boxed{\sqrt{8}}$$
$$\theta = \arctan\left(\frac{A_y}{A_x}\right) = \arctan\left(\frac{2}{-2}\right) = \boxed{2.4 \text{ rad}}$$

Part c)
$$\vec{A} = -2\hat{y}$$
$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(0)^2 + (-2)^2} = \boxed{2}$$
$$\theta = \arctan\left(\frac{A_y}{A_x}\right) = \arctan\left(\frac{-2}{0}\right) = \boxed{4.7 \text{ rad}}$$

REFLECT

To ensure you have the correct angle, draw out the vector in each case.

3.35

SET UP

The magnitude and direction of vectors $\vec{A}$ and $\vec{B}$ are given. Vector $\vec{A}$ has a magnitude of 66.0 m and makes an angle of 28 degrees with respect to the positive x-axis. Vector $\vec{B}$ has a magnitude of 40.0 m and makes an angle of 56 degrees with respect to the negative x-axis. Recall that a positive angle is measured counterclockwise from the axis, which means vector $\vec{B}$ is in quadrant III. We are asked to find the sum of these vectors, $\vec{C} = \vec{A} + \vec{B}$. In order to add them, we first need to find the components of these vectors, add the components together, and then determine the magnitude and direction of $\vec{C}$.

SOLVE

$$A_x = (66.0\text{ m})\cos(28°) = 58.2\text{ m}$$

$$A_y = (66.0\text{ m})\sin(28°) = 31.0\text{ m}$$

$$\vec{A} = (58.2\text{ m})\hat{x} + (31.0\text{ m})\hat{y}$$

$$B_x = -(40.0\text{ m})\cos(56°) = -22.4\text{ m}$$

$$B_y = -(40.0\text{ m})\sin(56°) = -33.2\text{ m}$$

$$\vec{B} = (-22.4\text{ m})\hat{x} + (-33.2\text{ m})\hat{y}$$

$$\vec{C} = \vec{A} + \vec{B} = ((58.2\text{ m})\hat{x} + (31.0\text{ m})\hat{y}) + ((-22.4\text{ m})\hat{x} + (-33.2\text{ m})\hat{y})$$

$$= (35.8\text{ m})\hat{x} + (-2.2\text{ m})\hat{y}$$

$$C = \sqrt{(35.8\text{ m})^2 + (-2.2\text{ m})^2} = \boxed{35.9\text{ m}}$$

$$\theta = \arctan\left(\frac{C_y}{C_x}\right) = \arctan\left(\frac{-2.2\text{ m}}{35.8\text{ m}}\right) = \boxed{6.22\text{ rad}}$$

REFLECT

An angle of 6.22 radians is equal to 356.5 degrees or −3.5 degrees, so $\vec{C}$ lies in quadrant IV.

3.36

SET UP

Two vectors are given. We can calculate their sum by adding their components and then determining the magnitude and angle using the Pythagorean theorem and the tangent, respectively.

SOLVE

$$\vec{A} = (2.00)\hat{x} + (6.00)\hat{y}$$

$$\vec{B} = (3.00)\hat{x} - (2.00)\hat{y}$$

$$\vec{C} = \vec{A} + \vec{B} = ((2.00)\hat{x} + (6.00)\hat{y}) + ((3.00)\hat{x} - (2.00)\hat{y}) = (5.00)\hat{x} + (4.00)\hat{y}$$

$$C = \sqrt{(5.00)^2 + (4.00)^2} = \boxed{6.40}$$

$$\theta = \arctan\left(\frac{4.00}{5.00}\right) = \boxed{0.675}$$

REFLECT

This angle is in quadrant I, which is reasonable if we sketch out vectors $\vec{A}$ and $\vec{B}$.

3.37

SET UP

Two vectors are given. We can calculate their difference by subtracting their components and then determining the magnitude and angle using the Pythagorean theorem and the tangent, respectively.

SOLVE

$$\vec{A} = (2.00)\hat{x} + (6.00)\hat{y}$$

$$\vec{B} = (3.00)\hat{x} - (2.00)\hat{y}$$

$$\vec{D} = \vec{A} - \vec{B} = ((2.00)\hat{x} + (6.00)\hat{y}) - ((3.00)\hat{x} - (2.00)\hat{y}) = (-1.00)\hat{x} + (8.00)\hat{y}$$

$$D = \sqrt{(-1.00)^2 + (8.00)^2} = \boxed{8.06}$$

$$\theta = \arctan\left(\frac{8.00}{-1.00}\right) = \boxed{1.70}$$

REFLECT

An angle of 1.70 radians is equal to 97.1 degrees. The difference $\vec{A} - \vec{B}$ is the same as the sum $\vec{A} + (-1)\vec{B}$.

3.38

SET UP

Two vectors are given and we are asked to calculate sums and differences. Vector $\vec{A}$ has a magnitude of 30 m/s and makes an angle of 45 degrees with respect to the positive x-axis. Vector $\vec{B}$ has a magnitude of 40 m/s and makes an angle of 90 degrees with respect to the positive x-axis.

SOLVE

$$A_x = \left(30\frac{\text{m}}{\text{s}}\right)\cos(45°) = 21\frac{\text{m}}{\text{s}}$$

$$A_y = \left(30\frac{\text{m}}{\text{s}}\right)\sin(45°) = 21\frac{\text{m}}{\text{s}}$$

$$\vec{A} = \left(21\frac{\text{m}}{\text{s}}\right)\hat{x} + \left(21\frac{\text{m}}{\text{s}}\right)\hat{y}$$

$$B_x = \left(40\frac{\text{m}}{\text{s}}\right)\cos(90°) = 0$$

$$B_y = \left(40\frac{\text{m}}{\text{s}}\right)\sin(90°) = 40\frac{\text{m}}{\text{s}}$$

$$\vec{B} = \left(40\frac{\text{m}}{\text{s}}\right)\hat{y}$$

Part a)

$$\vec{C} = \vec{A} + \vec{B} = \left(\left(21\frac{\text{m}}{\text{s}}\right)\hat{x} + \left(21\frac{\text{m}}{\text{s}}\right)\hat{y}\right) + \left(\left(40\frac{\text{m}}{\text{s}}\right)\hat{y}\right) = \boxed{\left(21\frac{\text{m}}{\text{s}}\right)\hat{x} + \left(61\frac{\text{m}}{\text{s}}\right)\hat{y}}$$

Part b)

$$\vec{D} = \vec{A} - \vec{B} = \left(\left(21\frac{\text{m}}{\text{s}}\right)\hat{x} + \left(21\frac{\text{m}}{\text{s}}\right)\hat{y}\right) - \left(\left(40\frac{\text{m}}{\text{s}}\right)\hat{y}\right) = \boxed{\left(21\frac{\text{m}}{\text{s}}\right)\hat{x} + \left(-19\frac{\text{m}}{\text{s}}\right)\hat{y}}$$

Part c)

$$\vec{E} = 2\vec{A} + \vec{B} = 2\left(\left(21\frac{\text{m}}{\text{s}}\right)\hat{x} + \left(21\frac{\text{m}}{\text{s}}\right)\hat{y}\right) + \left(\left(40\frac{\text{m}}{\text{s}}\right)\hat{y}\right) = \boxed{\left(42\frac{\text{m}}{\text{s}}\right)\hat{x} + \left(82\frac{\text{m}}{\text{s}}\right)\hat{y}}$$

REFLECT

You could have also reported these vectors in terms of a magnitude and an angle. Either way—magnitude/angle or components—is a valid way of reporting a vector.

3.39

SET UP

We are given an initial velocity vector and a final velocity vector and asked to find the change in velocity. We will consider east to be positive x and north to be positive y. The initial velocity is 30 m/s south or $\vec{v}_0 = -\left(30\frac{\text{m}}{\text{s}}\right)\hat{y}$. The final velocity is 40 m/s west or $\vec{v}_\text{f} = -\left(40\frac{\text{m}}{\text{s}}\right)\hat{x}$. Once we calculate the difference between these two vectors, we need to calculate the resulting magnitude and angle.

SOLVE

$$\Delta\vec{v} = \vec{v}_{\text{f}} - \vec{v}_0 = -\left(40\frac{\text{m}}{\text{s}}\right)\hat{x} - \left(-\left(30\frac{\text{m}}{\text{s}}\right)\hat{y}\right) = -\left(40\frac{\text{m}}{\text{s}}\right)\hat{x} + \left(30\frac{\text{m}}{\text{s}}\right)\hat{y}$$

$$\Delta v = \sqrt{\left(-40\frac{\text{m}}{\text{s}}\right)^2 + \left(30\frac{\text{m}}{\text{s}}\right)^2} = \boxed{50\frac{\text{m}}{\text{s}}}$$

$$\theta = \arctan\left(\frac{30}{-40}\right) = \boxed{2.5\ \text{rad}}$$

REFLECT
The resulting vector points up and to the left at an angle of 143 degrees.

3.40

SET UP
Two vectors are given. Nathan says that the magnitude of the resultant (sum) vector is 7 and makes an angle of 37 degrees in the northeasterly direction. We need to explicitly calculate the magnitude and angle to determine if Nathan was correct.

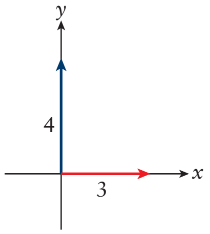


Figure 3-7 Problem 40

SOLVE
Both the magnitude and the direction Nathan gives are wrong. The actual magnitude of the resultant vector is $\sqrt{3^2 + 4^2} = 5$.

The resultant vector makes an angle of $\theta = \arctan\left(\frac{4}{3}\right) = 0.93\ \text{rad} = 53°$ from the positive x-axis.

REFLECT
The angle Nathan gives is the angle the resultant vector makes with the y-axis. By convention, positive angles are measured counterclockwise from the positive x-axis.

3.41

SET UP
The initial and final velocity vectors of an object are given. We can calculate the average acceleration vector of the object by calculating the change in velocity, dividing this new vector by 5 s, and then finding its magnitude. We are not given the intermediate velocity vectors, so we cannot determine if the acceleration was uniform.

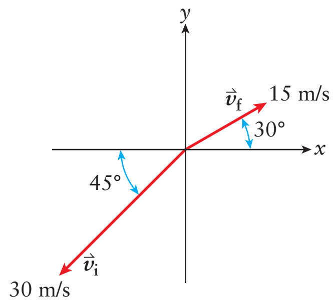


Figure 3-8 Problem 41

SOLVE

$$v_{\text{i},x} = -\left(30\frac{\text{m}}{\text{s}}\right)\cos(45°) = -21\frac{\text{m}}{\text{s}}$$

$$v_{\text{i},y} = -\left(30\frac{\text{m}}{\text{s}}\right)\sin(45°) = -21\frac{\text{m}}{\text{s}}$$

$$\vec{v}_{\text{i}} = -\left(21\frac{\text{m}}{\text{s}}\right)\hat{x} - \left(21\frac{\text{m}}{\text{s}}\right)\hat{y}$$

$$v_{f,x} = \left(15\frac{\text{m}}{\text{s}}\right)\cos(30°) = 13\frac{\text{m}}{\text{s}}$$

$$v_{f,y} = \left(15\frac{\text{m}}{\text{s}}\right)\sin(30°) = 7.5\frac{\text{m}}{\text{s}}$$

$$\vec{v}_f = \left(13\frac{\text{m}}{\text{s}}\right)\hat{x} + \left(7.5\frac{\text{m}}{\text{s}}\right)\hat{y}$$

$$\Delta\vec{v} = \vec{v}_f - \vec{v}_i = \left(\left(13\frac{\text{m}}{\text{s}}\right)\hat{x} + \left(7.5\frac{\text{m}}{\text{s}}\right)\hat{y}\right) - \left(-\left(21\frac{\text{m}}{\text{s}}\right)\hat{x} - \left(21\frac{\text{m}}{\text{s}}\right)\hat{y}\right)$$

$$= \left(34\frac{\text{m}}{\text{s}}\right)\hat{x} + \left(28.5\frac{\text{m}}{\text{s}}\right)\hat{y}$$

$$\vec{a}_{\text{average}} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\left(34\frac{\text{m}}{\text{s}}\right)\hat{x} + \left(28.5\frac{\text{m}}{\text{s}}\right)\hat{y}}{5\ \text{s}} = \left(6.8\frac{\text{m}}{\text{s}^2}\right)\hat{x} + \left(5.7\frac{\text{m}}{\text{s}^2}\right)\hat{y}$$

$$a_{\text{average}} = \sqrt{\left(6.8\frac{\text{m}}{\text{s}^2}\right)^2 + \left(5.7\frac{\text{m}}{\text{s}^2}\right)^2} = \boxed{8.9\frac{\text{m}}{\text{s}^2}}$$

No, it's not possible to know whether the acceleration was uniform from the information given. We would need to know the acceleration as a function of time in order to answer that question.

REFLECT

The average acceleration vector points in the same direction as the change in the velocity. Its magnitude is scaled by the time interval.

3.42

SET UP

An object travels with a constant acceleration for 10 s. The initial and final velocity vectors for the object are given. First we must calculate the change in the velocity by subtracting the vectors from one another and then divide it by 10 s to get the acceleration vector. With the initial velocity vector and acceleration vector, we can generate a data table and a plot of the x velocity, y velocity, and y acceleration versus time.

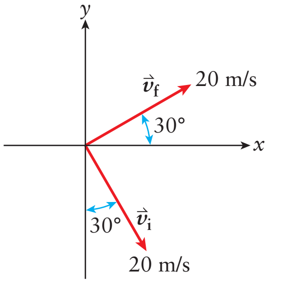


Figure 3-9 Problem 42

SOLVE

Acceleration:

$$v_{i,x} = \left(20\frac{\text{m}}{\text{s}}\right)\cos(60°) = 10\frac{\text{m}}{\text{s}}$$

$$v_{i,y} = -\left(20\frac{\text{m}}{\text{s}}\right)\sin(60°) = -17\frac{\text{m}}{\text{s}}$$

$$\vec{v}_i = \left(10\frac{\text{m}}{\text{s}}\right)\hat{x} - \left(17\frac{\text{m}}{\text{s}}\right)\hat{y}$$

$$v_{f,x} = \left(20\frac{\text{m}}{\text{s}}\right)\cos(30°) = 17\frac{\text{m}}{\text{s}}$$

$$v_{f,y} = \left(20\frac{\text{m}}{\text{s}}\right)\sin(30°) = 10\frac{\text{m}}{\text{s}}$$

$$\vec{v}_f = \left(17\frac{\text{m}}{\text{s}}\right)\hat{x} + \left(10\frac{\text{m}}{\text{s}}\right)\hat{y}$$

$$\Delta\vec{v} = \vec{v}_f - \vec{v}_i = \left(\left(17\frac{\text{m}}{\text{s}}\right)\hat{x} + \left(10\frac{\text{m}}{\text{s}}\right)\hat{y}\right) - \left(\left(10\frac{\text{m}}{\text{s}}\right)\hat{x} - \left(17\frac{\text{m}}{\text{s}}\right)\hat{y}\right)$$

$$= \left(7\frac{\text{m}}{\text{s}}\right)\hat{x} + \left(27\frac{\text{m}}{\text{s}}\right)\hat{y}$$

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\left(7\frac{\text{m}}{\text{s}}\right)\hat{x} + \left(27\frac{\text{m}}{\text{s}}\right)\hat{y}}{10\text{ s}} = \left(0.7\frac{\text{m}}{\text{s}^2}\right)\hat{x} + \left(2.7\frac{\text{m}}{\text{s}^2}\right)\hat{y}$$

Plots:

t(s)	v_x(m/s)	v_y(m/s)	a_y(m/s^2)
0	10	−17	2.7
1	10.7	−14.3	2.7
2	11.4	−11.6	2.7
3	12.1	−8.9	2.7
4	12.8	−6.2	2.7
5	13.5	−3.5	2.7
6	14.2	−0.8	2.7
7	14.9	1.9	2.7
8	15.6	4.6	2.7
9	16.3	7.3	2.7
10	17	10	2.7

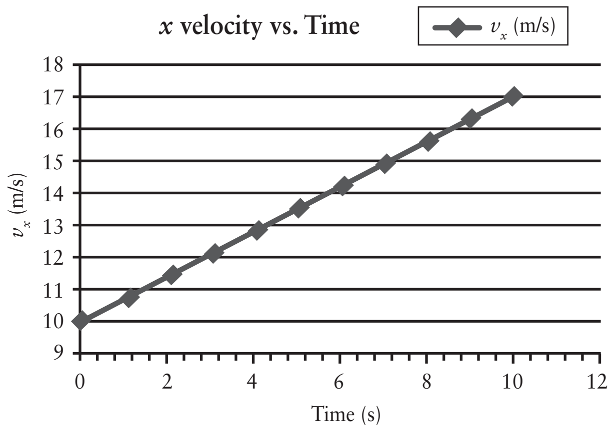


Figure 3-10 Problem 42

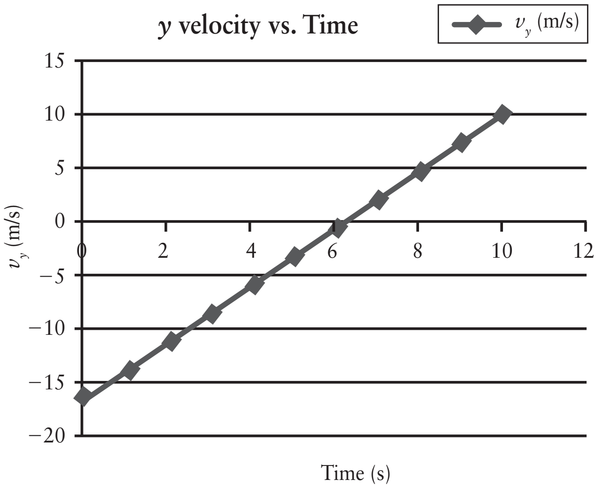


Figure 3-11 Problem 42

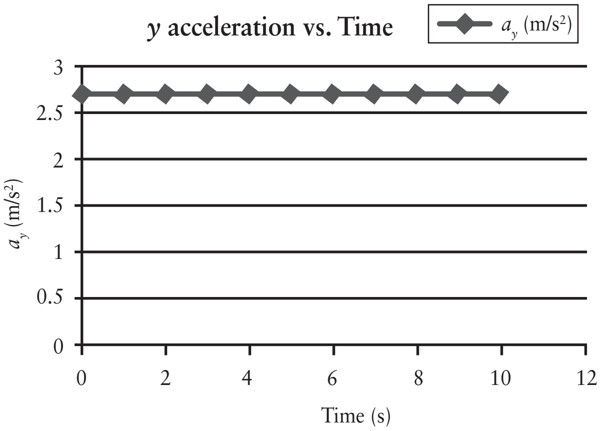


Figure 3-12 Problem 42

REFLECT

The plot of acceleration versus time is constant, as we were told in the problem. The x and y components of the velocity increase linearly over the 10 s, which is consistent with constant acceleration.

3.43

SET UP

An object experiences a constant acceleration of $\vec{a} = -\left(2.0\frac{\text{m}}{\text{s}^2}\right)\hat{x}$ for 2.7 s. Its final velocity has a magnitude of 16 m/s and points at an angle of 45 degrees from the positive x-axis. We can use the definition of average acceleration to calculate the initial velocity of the object.

SOLVE

$$\vec{a} = \frac{\vec{v}_\text{f} - \vec{v}_\text{i}}{\Delta t}$$

$$\vec{v}_\text{i} = \vec{v}_\text{f} - (\vec{a})(\Delta t) = \left(\left(\left(16\frac{\text{m}}{\text{s}}\right)\cos(45°)\right)\hat{x} + \left(\left(16\frac{\text{m}}{\text{s}}\right)\sin(45°)\right)\hat{y}\right) - \left(\left(-2.0\frac{\text{m}}{\text{s}^2}\right)\hat{x}\right)(2.7\text{ s})$$

$$= \left(\left(11.3\frac{\text{m}}{\text{s}}\right)\hat{x} + \left(11.3\frac{\text{m}}{\text{s}}\right)\hat{y}\right) + \left(\left(5.4\frac{\text{m}}{\text{s}}\right)\hat{x}\right) = \boxed{\left(17\frac{\text{m}}{\text{s}}\right)\hat{x} + \left(11\frac{\text{m}}{\text{s}}\right)\hat{y}}$$

REFLECT

The acceleration only acts along the x-axis, which means the y component of the velocity is constant.

3.44

SET UP

Cody starts at a point 6 km to the east and 4 km to the south of the origin. He ends at a point 10 km to the west and 6 km to the north. We can calculate Cody's displacement by subtracting his initial position vector from his final position vector. If it takes him 4 hr to complete the trip, his average velocity is equal to his displacement divided by 4 hr. Marcus also completes the same trip in the same amount of time. Because Marcus's displacement is equal to Cody's, their average velocities will also be equal.

SOLVE

We will consider east to point toward positive x and north to point toward positive y.

$$\vec{r}_\text{i} = (6\text{ km})\hat{x} - (4\text{ km})\hat{y}$$

$$\vec{r}_\text{f} = -(10\text{ km})\hat{x} + (6\text{ km})\hat{y}$$

Part a)

$$\vec{v}_\text{average} = \frac{\Delta\vec{r}}{\Delta t} = \frac{(-(10\text{ km})\hat{x} + (6\text{ km})\hat{y}) - ((6\text{ km})\hat{x} - (4\text{ km})\hat{y})}{4\text{ hr}} = \frac{(-16\text{ km})\hat{x} + (10\text{ km})\hat{y}}{4\text{ hr}}$$

$$= \boxed{\left(-4\frac{\text{km}}{\text{hr}}\right)\hat{x} + \left(2.5\frac{\text{km}}{\text{hr}}\right)\hat{y}}$$

Part b) Cody's and Marcus's $\boxed{\text{average velocities are the same}}$ because they cover the same displacement in the same amount of time.

REFLECT

Although Cody's and Marcus's average velocities are the same, their instantaneous velocities are not the same. Marcus jogs, walks, runs, and rests, so his speed is constantly changing.

3.45

SET UP

A plot of the trajectory of an object undergoing projectile motion is shown. At each of the marked points we need to draw (to scale) the x component of the velocity, the y component of the velocity, and the acceleration of the object. The acceleration always points down with a constant magnitude of g. Because the acceleration is only in one direction, the x component of the velocity remains constant for all time. The y component of the velocity is initially large and pointing up. The magnitude decreases until it reaches zero at the object's maximum height and then begins to point down and increase in size. The magnitude (but not the direction) of the y component is symmetric about the maximum height.

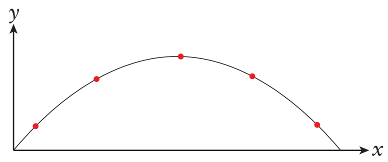


Figure 3-13 Problem 45

SOLVE

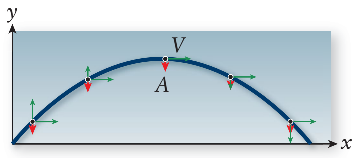


Figure 3-14 Problem 45

REFLECT

Remember that g is a positive number.

3.46

SET UP

An object undergoing parabolic motion travels 100 m before returning to its starting height. The object was launched at an angle of 30 degrees with an initial speed v_0. We can start by setting up our Know/Don't Know table. We will take the object's initial position to be the origin, right to be the positive x direction, and up to be the positive y direction. We can use the x component information to solve for t in terms of known quantities and then use the y component information to calculate the initial speed. Once we have the initial speed we can calculate the x and y components of the initial velocity.

SOLVE

Know/Don't Know table:

	X		Y
x_0	0 m	y_0	0 m
x_f	100 m	y_f	0 m
v_{0x}	WANT	v_{0y}	WANT
v_x	?	v_y	?
a_x	0	a_y	$-9.81\ m/s^2$
t	?	t	?

where $v_{0,x} = v_0\cos(30°)$ and $v_{0,y} = v_0\sin(30°)$.

Solving for the initial speed:

$$\Delta x = v_{0,x}t + \frac{1}{2}a_x t^2 = v_{0,x}t + 0$$

$$t = \frac{\Delta x}{v_{0,x}}$$

$$\Delta y = v_{0,y}t + \frac{1}{2}a_y t^2 = v_{0,y}\left(\frac{\Delta x}{v_{0,x}}\right) + \frac{1}{2}a_y\left(\frac{\Delta x}{v_{0,x}}\right)^2 = (v_0\sin(30°))\left(\frac{\Delta x}{v_0\cos(30°)}\right) + \frac{1}{2}a_y\left(\frac{\Delta x}{v_0\cos(30°)}\right)^2$$

$$= (\Delta x)\tan(30°) + \frac{a_y(\Delta x)^2}{2\cos^2(30°)}\left(\frac{1}{v_0^2}\right)$$

$$v_0 = \sqrt{\frac{a_y(\Delta x)^2}{2\cos^2(30°)((\Delta y) - (\Delta x)\tan(30°))}} = \sqrt{\frac{\left(-9.8\frac{\text{m}}{\text{s}^2}\right)(100\text{ m})^2}{2\cos^2(30°)(0 - (100\text{ m})\tan(30°))}} = 33.6\frac{\text{m}}{\text{s}}$$

$$v_{0,x} = v_0\cos(30°) = \left(33.6\frac{\text{m}}{\text{s}}\right)\cos(30°) = \boxed{29.1\frac{\text{m}}{\text{s}}}$$

$$v_{0,y} = v_0\sin(30°) = \left(33.6\frac{\text{m}}{\text{s}}\right)\sin(30°) = \boxed{16.8\frac{\text{m}}{\text{s}}}$$

REFLECT

It is helpful to use the time when solving projectile motion problems because t is the same for both the x and y components. Doing so allows you to eliminate a variable.

3.47

SET UP

Five balls are thrown off the edge of a cliff with the same speed but at different angles. We can use intuition and the constant acceleration equations to determine which ball travels the farthest horizontal distance, which ball takes the longest to hit the ground, and which ball has the greatest speed when landing.

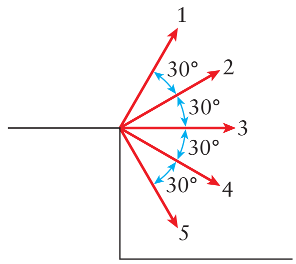


Figure 3-15 Problem 47

SOLVE

Part a) Balls 1 and 2 should travel the farthest since they are thrown upward. Balls 4 and 5 should travel the shortest distance since they are thrown downward. The initial x velocity for ball 2 is larger than ball 1, so it should travel farther horizontally before hitting the ground. Therefore, ball 2 travels the farthest followed by balls 1, 3, 4, and 5.

Part b) Ball 1 should take the longest to hit the ground because it has the largest, positive initial y velocity, followed by balls 2, 3, 4, and 5.

Part c)

$$v^2 - v_0^2 = 2a_y(\Delta y)$$

$$v = \sqrt{v_0^2 + 2a_y(\Delta y)}$$

The initial speed, the acceleration, and the height traveled are all the same for the five cases. Therefore, all five balls will land with the same speed.

REFLECT

Be careful when searching through the text for equations to use. Some of them will only work for specific cases and not in general. For example, Equation 3-26 $\left((x - x_0)_{\text{range}} = \frac{2v_0^2 \sin(\theta)\cos(\theta)}{g}\right)$ can only be used for a projectile returning to its initial launch height.

3.48

SET UP

A salmon jumps out of the water with a speed of 6.3 m/s at an angle of 40 degrees. We can start by setting up our Know/Don't Know table. We will take the object's initial position to be the origin and up to be the positive y direction. We will assume that the fish is moving in the positive x direction. We can use the y component information to solve for t in terms of known quantities and then solve for the displacement in the x direction.

SOLVE

Know/Don't Know table:

	X		Y
x_0	0 m	y_0	0 m
x_f	WANT	y_f	0 m
v_{0x}	60cos(40°)	v_{0y}	60cos(40°)
v_x	?	v_y	?
a_x	0	a_y	-9.81 m/s^2
t	?	t	?

Solving for horizontal distance:

$$\Delta y = v_{0,y}t + \frac{1}{2}a_y t^2 = 0$$

$$t = 0 \text{ and } t = \frac{-2v_{0,y}}{a_y}$$

Using $t = \frac{-2v_{0,y}}{a_y}$:

$$\Delta x = v_{0,x}t = v_{0,x}\left(\frac{-2v_{0,y}}{a_y}\right) = -\frac{2v_0^2\sin(40°)\cos(40°)}{a_y} = -\frac{2\left(6.3\frac{\text{m}}{\text{s}}\right)^2\sin(40°)\cos(40°)}{\left(-9.8\frac{\text{m}}{\text{s}^2}\right)} = \boxed{4.0\text{ m}}$$

REFLECT

It is helpful to use the time when solving projectile motion problems because t is the same for both the x and y components. Doing so allows you to eliminate a variable.

3.49

SET UP

A tiger leaps out of a tree that is 4.00 m tall. He lands 5.00 m from the base of the tree. The tiger's initial velocity is completely in the horizontal direction, which means the y component of his initial velocity is zero. We will choose the tiger's starting point to be the origin and up to be positive y. We can assume the tiger is jumping in the positive x direction. We can use the y component information to solve for t in terms of known quantities and then solve for the initial speed.

SOLVE

Know/Don't Know table:

	X		Y
x_0	0 m	y_0	0 m
x_f	5.000 m	y_f	−4.00 m
v_{0x}	WANT	v_{0y}	0
v_x	?	v_y	?
a_x	0	a_y	−9.8 m/s^2
t	?	t	?

Solving for initial speed:

$$\Delta y = v_{0,\,y}t + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}a_y t^2$$

$$t = \sqrt{\frac{2(\Delta y)}{a_y}}$$

$$\Delta x = v_{0,\,x}t + \frac{1}{2}a_x t^2 = v_{0,\,x}t + 0 = v_{0,\,x}\left(\sqrt{\frac{2(\Delta y)}{a_y}}\right)$$

$$v_{0,\,x} = \Delta x\left(\sqrt{\frac{a_y}{2(\Delta y)}}\right) = (5.00\text{ m})\left(\sqrt{\frac{-9.81\frac{\text{m}}{\text{s}^2}}{2(-4.00\text{ m})}}\right) = \boxed{5.53\frac{\text{m}}{\text{s}}}$$

REFLECT

We could have chosen the base of the tree to be the origin rather than the tiger's initial location. This would not have affected the sign or magnitude of the y displacement.

3.50

SET UP

A football has an initial speed of 25.0 m/s at an angle of 30 degrees. It is launched 1.00 m above the ground. We are asked to find the velocity vector of the football at a point 5.00 m

above the ground. We will assume the football is traveling in the positive x direction and take up to be the positive y direction. Since the acceleration in the x direction is zero, the x component of the velocity is constant. We need to consider the acceleration due to gravity when calculating the y component of the velocity at a point 5.00 m above the ground.

SOLVE

The x component is constant because $a_x = 0$, so

$$v_x = v_{0,x} = v_0 \cos(30°) = \left(25.0\frac{\text{m}}{\text{s}}\right)\cos(30°) = 21.7\frac{\text{m}}{\text{s}}$$

The y component is

$$v_y^2 - v_{0,y}^2 = 2a_y(\Delta y)$$

$$v_y = \pm\sqrt{v_{0,y}^2 + 2a_y(\Delta y)} = \pm\sqrt{v_0^2 \sin^2(30°) + 2a_y(\Delta y)}$$

$$= \pm\sqrt{\left(25.0\frac{\text{m}}{\text{s}}\right)^2 \sin^2(30°) + 2\left(-9.8\frac{\text{m}}{\text{s}^2}\right)(4.00\text{ m})} = \pm 8.82\frac{\text{m}}{\text{s}}$$

Velocity vector at $y = 5.00$ m:

$$\boxed{v = \left(21.7\frac{\text{m}}{\text{s}}\right)\hat{x} \pm \left(8.82\frac{\text{m}}{\text{s}}\right)\hat{y}}$$

REFLECT

There are two answers to this question because the football passes through a point 5.00 m above the ground twice—once on its way up and once on its way down.

3.51

SET UP

A dart is thrown at a dartboard 2.37 m away. The dart lands in 0.447 s at the same height that it is released. We will take the origin to be the dart's initial position. We are asked to find the dart's launch angle θ. Although we don't know the dart's initial speed v_0, we can relate the x and y components of the initial velocity to the initial speed and launch angle. The time t is a common variable to both the x and y motion, which allows us to eliminate the initial speed.

SOLVE

Know/Don't Know table:

	X		Y
x_0	0 m	y_0	0 m
x_f	2.37 m	y_f	0 m
v_{0x}	WANT	v_{0y}	WANT
v_x	?	v_y	?
a_x	0	a_y	-9.8 m/s^2
t	0.447 s	t	0.447 s

Solving for the launch angle:

$$\Delta y = v_{0,y}t + \frac{1}{2}a_y t^2 = 0$$

$$0 = v_0 \sin(\theta)t + \frac{1}{2}a_y t^2$$

$$v_0 = \frac{-a_y t}{2\sin(\theta)}$$

$$\Delta x = v_{0,x}t = v_0\cos(\theta)t = \left(\frac{-a_y t}{2\sin(\theta)}\right)\cos(\theta)t = \frac{-a_y t^2}{2\tan(\theta)}$$

$$\tan(\theta) = \frac{-a_y t^2}{2(\Delta x)}$$

$$\theta = \arctan\left(\frac{-a_y t^2}{2(\Delta x)}\right) = \arctan\left(\frac{-\left(-9.8\frac{\text{m}}{\text{s}^2}\right)(0.447\text{ s})^2}{2(2.37\text{ m})}\right) = \boxed{22.4°}$$

REFLECT

If you are not given a quantity you think you need in order to solve a problem, just introduce a variable and carry it through your calculation. More often than not, it will not factor into your solution. For example, in this problem, we weren't explicitly given the initial speed. We called it v_0, carried it through our calculation, and it eventually canceled out. If your created variable *does* show up in your final answer, perhaps there's a way of determining that quantity or another way of solving the problem.

3.52

SET UP

A ball is swinging at a speed of $v = 2.25$ m/s in a circle of radius $R = 1.25$ m. The speed of the ball is the total distance it travels divided by the time it takes. Since the ball is moving in a circle, this is equal to the circumference of the circle divided by the period.

SOLVE

$$v = \frac{2\pi R}{T}$$

$$T = \frac{2\pi R}{v} = \frac{2\pi(1.25\text{ m})}{\left(2.25\frac{\text{m}}{\text{s}}\right)} = \boxed{3.49\text{ s}}$$

REFLECT

We can quickly estimate the period from these values. The radius is equal to 5/4 m and the speed is 9/4 m/s. The period will then be $\frac{10}{9}\pi$ s, which is a little more than $(1.1)(3) = 3.4$ s.

3.53

SET UP

A ball is attached to a 0.870-m-long string and is moving in a circle with a constant speed of 3.36 m/s. The ball is undergoing uniform circular motion, so its acceleration has a magnitude of $a = \frac{v^2}{R}$ and points toward the center of the circle.

SOLVE

$$a = \frac{v^2}{R} = \frac{\left(3.36\frac{\text{m}}{\text{s}}\right)^2}{0.870\ \text{m}} = \boxed{13.0\frac{\text{m}}{\text{s}^2}\ \text{inward}}$$

REFLECT

Centripetal acceleration always points toward the center of the circle.

3.54

SET UP

A car is traveling around a circular track at a constant speed v. A change in only the magnitude of car's average velocity or the magnitude of the car's average acceleration would change the car's average speed. The car is constantly changing the direction of its velocity as it moves around the circle. A change in the direction of the car's acceleration would also change the car's average speed since there is now a component of the acceleration vector tangential to the car's path.

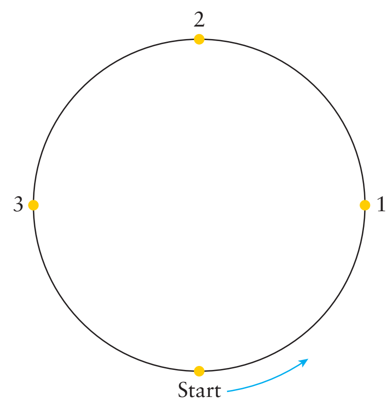


Figure 3-16 Problem 54

SOLVE

Part a)

B: The car gradually speeds up from rest to a speed v while traveling from the starting position to point 1.

D: The car gradually slows down to a speed of $v/2$ while traveling from the starting position to point 3.

Part b)

A: The car maintains a constant speed $2v$ while traveling from the starting position to point 1.

C: The car maintains a constant speed.

Part c)

B: The car gradually speeds up from rest to a speed v while traveling from the starting position to point 1.

D: The car gradually slows down to a speed of $v/2$ while traveling from the starting position to point 3.

Part d)

B: The car gradually speeds up from rest to a speed v while traveling from the starting position to point 1.

D: The car gradually slows down to a speed of $v/2$ while traveling from the starting position to point 3.

REFLECT

Nonuniform circular motion refers to motion in which the acceleration vector has a component that points tangentially to the car's path in addition to a component that points toward the center of the circle. The tangential acceleration will affect the car's speed. Uniform circular motion requires that the car's speed stay constant.

3.55

SET UP

A washing machine drum has a diameter $d = 80.0$ cm. It starts from rest and accelerates at a constant rate to a speed of 1200 rev/min in $t = 22$ s. The acceleration of the drum has two components: a radial component due to the rotation and a tangential component due to the increasing speed. The net acceleration vector of a point on the drum is the resultant of these two components. The tangential component is equal to the change in the speed of the drum divided by the time. This will allow us to calculate the speed of the drum after 1.00 s has elapsed. The radial component is equal to $a = \frac{v^2}{R}$, where v is the speed of the drum at $t = 1.00$ s.

SOLVE

Final speed of the drum:

$$1200\frac{\text{rev}}{\text{min}} \times \frac{1\text{ min}}{60\text{ s}} \times \frac{\pi(0.8\text{ m})}{1\text{ rev}} = 50.3\frac{\text{m}}{\text{s}}$$

Tangential acceleration:

$$a_{\tan} = \frac{\Delta v}{\Delta t} = \frac{\left(50.3\frac{\text{m}}{\text{s}}\right) - 0}{22\text{ s}} = 2.3\frac{\text{m}}{\text{s}^2}$$

Speed after 1.00 s:

$$v = v_0 + a_{\tan}t = 0 + \left(2.3\frac{\text{m}}{\text{s}^2}\right)(1.00\text{ s}) = 2.3\frac{\text{m}}{\text{s}}$$

Radial acceleration:

$$a_{\text{rad}} = \frac{v^2}{R} = \frac{\left(2.3\frac{\text{m}}{\text{s}}\right)^2}{0.4\text{ m}} = 13\frac{\text{m}}{\text{s}^2}$$

Net acceleration:

$$a = \sqrt{a_{\text{rad}}^2 + a_{\tan}^2} = \sqrt{\left(13\frac{\text{m}}{\text{s}^2}\right)^2 + \left(2.3\frac{\text{m}}{\text{s}^2}\right)^2} = \boxed{13.2\frac{\text{m}}{\text{s}^2}}$$

$$\theta = \arctan\left(\frac{a_{tan}}{a_{rad}}\right) = \arctan\left(\frac{2.3\frac{m}{s^2}}{13\frac{m}{s^2}}\right) = \boxed{10° \text{ forward of straight inward}}$$

REFLECT

We are given the *diameter* of the drum, not the *radius*. The radial acceleration uses the radius, so be sure to divide by 2 when performing the calculation.

3.56

SET UP

A quarter-inch-diameter drill bit accelerates from rest to 800 rev/min in 4.33 s. The acceleration of the bit has two components: a radial component due to the rotation and a tangential component due to the increasing speed. The net acceleration vector of a point on the edge of the bit is the resultant of these two components. The tangential component is equal to the change in the speed of the bit divided by the time. The radial component is equal to $a_{rad} = \frac{v^2}{R}$, where v is the final speed of the bit.

SOLVE

Final speed of the bit:

$$800\frac{rev}{min} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{\pi(0.25 \text{ in})}{1 \text{ rev}} \times \frac{2.54 \text{ cm}}{1 \text{ in}} \times \frac{1 \text{ m}}{100 \text{ cm}} = 0.266\frac{m}{s}$$

Tangential acceleration:

$$a_{tan} = \frac{\Delta v}{\Delta t} = \frac{\left(0.266\frac{m}{s}\right) - 0}{4.33 \text{ s}} = 0.0614\frac{m}{s^2}$$

Radial acceleration:

$$a_{rad} = \frac{v^2}{R} = \frac{\left(0.266\frac{m}{s}\right)^2}{0.125 \text{ in}} \times \frac{1 \text{ in}}{2.54 \text{ cm}} \times \frac{100 \text{ cm}}{1 \text{ m}} = 22.3\frac{m}{s^2}$$

Net acceleration:

$$a = \sqrt{a_{rad}^2 + a_{tan}^2} = \sqrt{\left(0.0614\frac{m}{s^2}\right)^2 + \left(22.3\frac{m}{s^2}\right)^2} = \boxed{22.3\frac{m}{s^2}}$$

$$\theta = \arctan\left(\frac{a_{tan}}{a_{rad}}\right) = \arctan\left(\frac{0.0614\frac{m}{s^2}}{22.3\frac{m}{s^2}}\right) = \boxed{0.16° \text{ forward of straight inward}}$$

REFLECT

Because the tangential acceleration is over two orders of magnitude larger than the radial acceleration, we can approximate this as uniform circular motion.

3.57

SET UP

A Ferris wheel is 76 m in diameter and completes a revolution in 20 min, which means a point on the rim will travel a length of one circumference in 20 min. We will assume the Ferris wheel is undergoing uniform circular motion, so its acceleration is equal to $a = \frac{v^2}{R}$.

SOLVE

$$v = \frac{1 \text{ rev}}{20 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{\pi(76 \text{ m})}{1 \text{ rev}} = 0.199\frac{\text{m}}{\text{s}}$$

$$a = \frac{v^2}{R} = \frac{\left(0.199\frac{\text{m}}{\text{s}}\right)^2}{\left(\frac{76 \text{ m}}{2}\right)} = \boxed{1.0 \times 10^{-3}\frac{\text{m}}{\text{s}^2}}$$

REFLECT

We are given the *diameter* of the wheel, not the *radius*. The radial acceleration uses the radius, so be sure to divide by 2 when performing the calculation.

3.58

SET UP

A Ferris wheel is 16 m in diameter and has a radial acceleration of 2.0 m/s^2. We can calculate the speed of the Ferris wheel directly from $a = \frac{v^2}{R}$. Be careful to use the radius, not the diameter.

SOLVE

$$a = \frac{v^2}{R} = \frac{v^2}{\left(\frac{d}{2}\right)}$$

$$v = \sqrt{\frac{ad}{2}} = \sqrt{\frac{\left(2.0\frac{\text{m}}{\text{s}^2}\right)(16 \text{ m})}{2}} = \boxed{4.0\frac{\text{m}}{\text{s}}}$$

REFLECT

This speed is approximately 5 revolutions per minute.

3.59

SET UP

A racecar is driving at a constant speed of $v = 330$ km/hr around a circular track of diameter $d = 1.00$ km. The car's radial acceleration can be calculated directly from $a = \frac{v^2}{R}$.

SOLVE

$$v = 330\frac{\text{km}}{\text{hr}} \times \frac{1\text{ hr}}{3600\text{ s}} \times \frac{1000\text{ m}}{1\text{ km}} = 91.7\frac{\text{m}}{\text{s}}$$

$$a = \frac{v^2}{R} = \frac{v^2}{\left(\frac{d}{2}\right)} = \frac{\left(91.7\frac{\text{m}}{\text{s}}\right)}{\left(\frac{1000\text{ m}}{2}\right)} = \boxed{16.8\frac{\text{m}}{\text{s}^2}\text{ toward the center of the track}}$$

REFLECT

A speed of 330 km/hr is around 205 mi/hr, so it makes sense that the acceleration is so large (almost $2g$!).

3.60

SET UP

Mary and Kelly want to run side by side around a track. We'll assume the track is circular, Mary's lane is at a radius of R_M, Kelly's lane is at a radius R_K, and $R_M < R_K$. Each runner is undergoing uniform circular motion with an acceleration equal to her speed squared divided by the radius of her lane. Each speed is the circumference divided by the time. Although the radii and the speeds are different, the time each runner completes a lap is the same as long as they stay side by side.

SOLVE

$$\frac{a_{\text{Kelly}}}{a_{\text{Mary}}} = \frac{\left(\frac{v_K^2}{R_K}\right)}{\left(\frac{v_M^2}{R_M}\right)} = \frac{\left(\frac{\left(\frac{2\pi R_K}{\Delta t}\right)^2}{R_K}\right)}{\left(\frac{\left(\frac{2\pi R_M}{\Delta t}\right)^2}{R_M}\right)} = \boxed{\frac{R_K}{R_M}}$$

REFLECT

Even though they are running side by side around the track, Mary and Kelly are not running at the same speed. Kelly must run faster than Mary because she needs to cover more distance in the same amount of time.

3.61

SET UP

The space shuttle is in an orbit about 300 km above the surface of Earth, which has a radius of 6.38×10^6 m. We will assume the orbit is circular and that the space shuttle is moving with a constant speed. The shuttle sweeps out a circle of radius $R = R_{\text{Earth}} + R_{\text{orbit}}$ and covers a distance of $2\pi R$ in one period T. The acceleration of the shuttle is $a = \frac{v^2}{R}$ and points toward the center of Earth.

SOLVE

$$v = \frac{2\pi(R_{\text{Earth}} + R_{\text{orbit}})}{T} = \frac{2\pi((6.38 \times 10^6 \text{ m}) + (3.00 \times 10^5 \text{ m}))}{(5.43 \times 10^3 \text{ s})} = 7730\frac{\text{m}}{\text{s}}$$

$$a = \frac{v^2}{R_{\text{Earth}} + R_{\text{orbit}}} = \frac{\left(7730\frac{\text{m}}{\text{s}}\right)^2}{(6.38 \times 10^6 \text{ m}) + (3.00 \times 10^5 \text{ m})}$$

$$= \boxed{8.9\frac{\text{m}}{\text{s}^2} \text{ toward the center of Earth}}$$

REFLECT

This is about 90% of the acceleration due to gravity at Earth's surface.

3.62

SET UP

The Moon is located 3.84×10^8 m from the center of Earth. We will assume the Moon's orbit is circular and that it is moving with a constant speed. The Moon covers a distance of $2\pi R$ in one period T. The acceleration of the shuttle is $a = \frac{v^2}{R}$ and points toward the center of Earth.

SOLVE

$$v = \frac{2\pi R}{T} = \frac{2\pi(3.84 \times 10^8 \text{ m})}{27.3 \text{ days}} \times \frac{1 \text{ day}}{24 \text{ hr}} \times \frac{1 \text{ hr}}{3600 \text{ s}} = 1023\frac{\text{m}}{\text{s}}$$

$$a = \frac{v^2}{R} = \frac{\left(1023\frac{\text{m}}{\text{s}}\right)^2}{3.84 \times 10^8 \text{ m}} = \boxed{2.72 \times 10^{-3}\frac{\text{m}}{\text{s}^2} \text{ toward the center of Earth}}$$

REFLECT

Remember that acceleration is a vector; you need to give both its magnitude and its direction.

3.63

SET UP

We are asked to calculate and compare various centripetal accelerations. For each, we need to know the speed of the object and the radius of the circle it makes. The speed is equal to the circumference of the circle divided by the period of the motion. The distance from Earth to the Sun is 1.5×10^{11} m; the radius of Earth is 6.38×10^6 m; the radius of the Sun is 7×10^8 m; and the distance from the center of Earth to the center of the Moon is 3.84×10^8 m (see Problem 3.62). Earth orbits the Sun once in 365.25 days (1 year); the Moon orbits Earth once every 27.3 days. The satellite has a period of 1.5 hr, or 5400 s.

SOLVE

Part a) Earth as it orbits the Sun

Since the distance from Earth to the Sun is much larger than the radius of either Earth or the Sun, we can ignore those distances in our calculation.

$$v = \frac{2\pi R}{T} = \frac{2\pi(1.5 \times 10^{11}\text{ m})}{365.25\text{ days}} \times \frac{1\text{ day}}{24\text{ hr}} \times \frac{1\text{ hr}}{3600\text{ s}} = 2.987 \times 10^4 \frac{\text{m}}{\text{s}}$$

$$a = \frac{v^2}{R} = \frac{\left(2.987 \times 10^4 \frac{\text{m}}{\text{s}}\right)^2}{1.5 \times 10^{11}\text{ m}} = \boxed{5.9 \times 10^{-3}\frac{\text{m}}{\text{s}^2}\text{ toward the center of the Sun}}$$

Part b) An artificial satellite as it orbits 320 km above Earth's surface

$$v = \frac{2\pi(R_{\text{Earth}} + R_{\text{orbit}})}{T} = \frac{2\pi((6.38 \times 10^6\text{ m}) + (3.20 \times 10^5\text{ m}))}{5400\text{ s}} = 7796\frac{\text{m}}{\text{s}}$$

$$a = \frac{v^2}{R_{\text{Earth}} + R_{\text{orbit}}} = \frac{\left(7796\frac{\text{m}}{\text{s}}\right)^2}{((6.38 \times 10^6\text{ m}) + (3.20 \times 10^5\text{ m}))}$$

$$= \boxed{9.07\frac{\text{m}}{\text{s}^2}\text{ towards the center of Earth}}$$

Part c) The Moon as it orbits Earth

$$v = \frac{2\pi R}{T} = \frac{2\pi(3.84 \times 10^8\text{ m})}{27.3\text{ days}} \times \frac{1\text{ day}}{24\text{ hr}} \times \frac{1\text{ hr}}{3600\text{ s}} = 1023\frac{\text{m}}{\text{s}}.$$

$$a = \frac{v^2}{R} = \frac{\left(1023\frac{\text{m}}{\text{s}}\right)^2}{3.84 \times 10^8\text{ m}} = \boxed{2.72 \times 10^{-3}\frac{\text{m}}{\text{s}^2}\text{ toward the center of Earth}}$$

Part d) A car traveling along a circular path that has a radius of 50 m and a speed of 20 m/s.

$$a = \frac{v^2}{R} = \frac{\left(20\frac{\text{m}}{\text{s}}\right)^2}{50\text{ m}} = \boxed{8\frac{\text{m}}{\text{s}^2}\text{ toward the center of the circle}}$$

The satellite's acceleration is the smallest in magnitude. The acceleration of Earth as it orbits the sun is 22,000 times larger than the satellite's acceleration. The acceleration of the Moon as it orbits Earth is 10,000 times larger than the satellite's acceleration. The car's acceleration is 3×10^7 times larger than the satellite's acceleration.

REFLECT

We could assume the radius of Earth and the radius of the Sun were negligible compared to the distance between Earth and the Sun because $1.5 \times 10^{11}\text{ m} + 6.38 \times 10^6\text{ m} + 7 \times 10^8\text{ m}$ is approximately equal to 1.5×10^{11} m. That is not the case with the satellite, which is why we included both distances.

3.64

SET UP

An ultracentrifuge spins at 100,000 revolutions per minute (rpm) and experiences an acceleration of 800,000 g. We first need to convert the speed of the centrifuge from rpm to m/s. For now, we will write the speed in terms of R, the radius of the centrifuge R. Using $a = \frac{v^2}{R}$, we can first calculate R and then use R to calculate the linear speed of a sample chamber along the rim of the device.

SOLVE

Speed:

$$v = 100{,}000\frac{\text{rev}}{\text{min}} \times \frac{2\pi R}{1\text{ rev}} \times \frac{1\text{ min}}{60\text{ s}} = 10{,}472R\ (\text{SI units}).$$

$$a = 800{,}000g = \frac{v^2}{R} = \frac{(10{,}472R)^2}{R} = (10{,}472)^2 R$$

$$R = \frac{800{,}000(9.81)}{10{,}472^2} = \boxed{0.072\text{ m}}$$

$$v = 10{,}472(0.072) = \boxed{750\frac{\text{m}}{\text{s}}}$$

REFLECT

This speed seems reasonable given the acceleration and rotation speed of the centrifuge.

3.65

SET UP

A peregrine falcon pulls out of a dive and into a circular arc at a speed of 100 m/s. It experiences an acceleration of $0.6g$, where $g = 9.8\text{ m/s}^2$. Assuming the bird's speed is constant, we can calculate the radius of the turn from the centripetal acceleration.

SOLVE

$$a = \frac{v^2}{R}$$

$$R = \frac{v^2}{a} = \frac{\left(100\frac{\text{m}}{\text{s}}\right)^2}{(0.6)\left(9.8\frac{\text{m}}{\text{s}^2}\right)} = 1700\text{ m} = \boxed{1.7\text{ km}}$$

REFLECT

Peregrine falcons are known to be one of the fastest animals on Earth. A speed of 100 m/s is roughly 220 mph!

3.66

SET UP

Two cars, initially separated by 24 m, are traveling in a straight line. The blue car, which is in the lead, is traveling at a speed of 28 m/s, while the red car is traveling at a speed of 34 m/s. We can calculate the time it takes for the red car to catch up with the blue car by realizing the red car is traveling at a speed of 6 m/s relative to the blue car. The time is equal to the separation distance divided by this relative speed. Once we calculate this time, we can multiply it by the red car's actual speed of 34 m/s to determine the distance it covered. If the red car accelerated from an initial relative speed of 6 m/s at a rate of $a = (4/3)\ \mathrm{m/s^2}$ instead, we can calculate the time it would take the car to cover a distance of 24 m using the constant acceleration equations.

SOLVE

Part a)

$$t = \frac{24\ \mathrm{m}}{6\dfrac{\mathrm{m}}{\mathrm{s}}} = \boxed{4\ \mathrm{s}}$$

Part b)

$$\Delta x_{\mathrm{red}} = (4\ \mathrm{s})\left(34\frac{\mathrm{m}}{\mathrm{s}}\right) = \boxed{136\ \mathrm{m}}$$

Part c)

$$\Delta x = v_{0,x}t + \frac{1}{2}a_x t^2$$

$$\frac{1}{2}a_x t^2 + v_{0,x}t - \Delta x = 0$$

$$t = \frac{-v_{0,x} \pm \sqrt{v_{0,x}^2 - 4\left(\frac{1}{2}a_x\right)(-\Delta x)}}{a_x} = \frac{-v_{0,x} \pm \sqrt{v_{0,x}^2 + 2a_x(\Delta x)}}{a_x}$$

$$= \frac{-\left(6\dfrac{\mathrm{m}}{\mathrm{s}}\right) \pm \sqrt{\left(6\dfrac{\mathrm{m}}{\mathrm{s}}\right)^2 + 2\left(\dfrac{4\,\mathrm{m}}{3\,\mathrm{s}^2}\right)(24\ \mathrm{m})}}{\left(\dfrac{4\,\mathrm{m}}{3\,\mathrm{s}^2}\right)} = \frac{-\left(6\dfrac{\mathrm{m}}{\mathrm{s}}\right) \pm \left(10\dfrac{\mathrm{m}}{\mathrm{s}}\right)}{\left(\dfrac{4\,\mathrm{m}}{3\,\mathrm{s}^2}\right)}$$

Taking the positive root, $t = 3$ s.

REFLECT

Because the car is accelerating in part (c), it makes sense that the time should be less than in part (a). We can double-check our answers by calculating the distance the blue car travels in 4 s: $\Delta x_{\mathrm{blue}} = (4\ \mathrm{s})\left(28\dfrac{\mathrm{m}}{\mathrm{s}}\right) = 112\ \mathrm{m}$. The blue car travels 24 m less than the red car, which is exactly the original distance separating them.

3.67

SET UP

Two sensors, separated by a height H, are attached to a tall tower. A small ball is launched from the bottom of the tower at an unknown speed. The sensors measure the time that elapses between the ball going up past the sensor and the ball coming back down past the sensor. It takes the ball $2t_1$ to come back to the lower sensor and $2t_2$ to return to the higher sensor. Because these times are measured with respect to the same position, the total displacement the ball travels in those intervals is zero. We can use this fact to solve for the speed of the ball when it passes by each sensor. Once we know the speeds, we can relate them to both g and H.

SOLVE

Part a)

Speed at the lower sensor:

$$\Delta y = 0 = v_1(2t_1) + \frac{1}{2}(-g)(2t_1)^2$$

$$v_1 = gt_1$$

Speed at the upper sensor:

$$\Delta y = 0 = v_2(2t_2) + \frac{1}{2}(-g)(2t_2)^2$$

$$v_2 = gt_2$$

Relating the speeds to the distance H:

$$v_2^2 - v_1^2 = 2(-g)H$$

$$(gt_2)^2 - (gt_1)^2 = 2(-g)H$$

$$\boxed{g = \frac{2H}{t_1^2 - t_2^2}}$$

Part b)

$$g = \frac{2H}{t_1^2 - t_2^2} = \frac{2(2.5\text{ m})}{(3\text{ s})^2 - (2\text{ s})^2} = \boxed{10\frac{\text{m}}{\text{s}^2}}$$

REFLECT

Our answer to part (a) is dimensionally correct: g has dimensions of length per time squared, H has dimensions of length, and t^2 has dimensions of time squared. It also has the correct sign (positive) since we expect the ball to take a longer time to return to the lower sensor (related to t_1) than the upper sensor (related to t_2). The value we calculated for g is only 2% higher than the accepted value of 9.8 m/s^2.

3.68

SET UP

A rock falls from rest from the top of a building. We'll set the top of the building as the origin and define up to be positive. The rock's acceleration is equal to $-g$ in this case. We can calculate the distance the rock has fallen using $\Delta y = v_{0,y}t + \frac{1}{2}a_y t^2$ and the rock's speed from $v^2 = v_0^2 + 2a_y(\Delta y)$. To calculate how far the top of the window is from the roof, we first need to calculate the rock's speed when it passes by the top of the window. Once we know this, we can determine how far the rock had to fall to achieve that speed.

SOLVE

Part a)

$$\Delta y = v_{0,y}t + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}\left(-9.81\frac{\text{m}}{\text{s}^2}\right)(2.50\text{ s}) = -12.3\text{ m}$$

The rock has fallen $\boxed{12.3\text{ m}}$.

Part b)

$$v^2 = v_0^2 + 2a_y(\Delta y) = 0 + 2\left(-9.81\frac{\text{m}}{\text{s}^2}\right)(-11.0\text{ m}) = 215.82\frac{\text{m}^2}{\text{s}^2}$$

$$|v| = 14.7\frac{\text{m}}{\text{s}}$$

The velocity of the rock is $\boxed{14.7\text{ m/s downward}}$.

Part c)

$$\Delta y_{\text{window}} = v_{\text{top of window}}t + \frac{1}{2}a_y t^2$$

$$v_{\text{top of window}} = \frac{\Delta y_{\text{window}} - \frac{1}{2}a_y t^2}{t} = \frac{(-2.00\text{ m}) - \frac{1}{2}\left(-9.81\frac{\text{m}}{\text{s}^2}\right)(0.117\text{ s})^2}{0.117\text{ s}} = -16.5\frac{\text{m}}{\text{s}}$$

$$v_{\text{top of window}}^2 - v_0^2 = 2a_y(\Delta y)$$

$$\Delta y = \frac{v_{\text{top of window}}^2 - v_0^2}{2a_y} = \frac{\left(-16.5\frac{\text{m}}{\text{s}}\right)^2 - 0}{2\left(-9.8\frac{\text{m}}{\text{s}^2}\right)} = -13.9\text{ m}$$

The top of the window is 13.9 m below the top of the building.

REFLECT

In part (c) we found that the rock had a higher speed than in part (b), which means we should expect the window to be farther than 11.0 m from the top of the building. We calculated the window to be 13.9 m below the roof, so our answers are consistent and make sense.

3.69

SET UP

Steve wants to throw a football to Jerry at an initial speed of 15.0 m/s at an angle of 45 degrees. Jerry runs past Steve at a speed of 8.00 m/s in the direction the football will be thrown. In order for Jerry to catch the football, Jerry and the football need to be at the same place at the same time. We can calculate the distance the football will travel and the time it takes for it to travel that distance. Because we are told that the football is caught at the same height from which it is released, we can use the maximum horizontal range equation (Equation 3-26). We can compare this time with the time it takes Jerry to run the same distance. The difference between these times is how long Steve should wait to throw the ball.

In part (b), Steve starts to run with the football at a speed of 1.50 m/s down the field as well. This means we need to use the speeds for the football and for Jerry relative to Steve. The football will still cover 23.0 m since its speed *relative to the ground* is still 15.0 m/s.

SOLVE

Part a)

Distance the football travels:

$$(x - x_0)_{\max} = \frac{2v_0^2 \sin(\theta)\cos(\theta)}{g} = \frac{2\left(15.0\frac{\mathrm{m}}{\mathrm{s}}\right)^2 \sin(45°)\cos(45°)}{9.81\frac{\mathrm{m}}{\mathrm{s}^2}} = 23.0\ \mathrm{m}$$

Time it takes the football to travel 23.0 m:

$$\Delta x = v_{0,x}t = v_0\cos(45°)t$$

$$t = \frac{\Delta x}{v_0\cos(45°)} = \frac{23.0\ \mathrm{m}}{\left(15.0\frac{\mathrm{m}}{\mathrm{s}}\right)\cos(45°)} = 2.17\ \mathrm{s}$$

Time it takes Jerry to run 23.0 m:

$$v_{\mathrm{J}} = \frac{\Delta x}{\Delta t}$$

$$\Delta t = \frac{\Delta x}{v_{\mathrm{J}}} = \frac{23.0\ \mathrm{m}}{8.00\frac{\mathrm{m}}{\mathrm{s}}} = 2.88\ \mathrm{s}$$

Steve should wait 2.88 s − 2.17 s = $\boxed{0.71\ \mathrm{s}}$ to throw the football.

Part b)

Time it takes the football to travel 23.0 m at a relative speed of 13.5 m/s:

$$\Delta x = v_{0,x}t = v_0\cos(45°)t$$

$$t = \frac{\Delta x}{v_0 \cos(45°)} = \frac{23.0 \text{ m}}{\left(13.5 \frac{\text{m}}{\text{s}}\right)\cos(45°)} = 2.41 \text{ s}$$

Time it takes Jerry to run 23.0 m at a relative speed of 6.5 m/s:

$$v_{\text{J}} = \frac{\Delta x}{\Delta t}$$

$$\Delta t = \frac{\Delta x}{v_{\text{J}}} = \frac{23.0 \text{ m}}{6.50 \frac{\text{m}}{\text{s}}} = 3.53 \text{ s}$$

Steve should wait 3.53 s − 2.41 s = $\boxed{1.12 \text{ s}}$ to throw the football.

REFLECT

It makes sense that Steve should wait longer to release the football when he is running rather than standing still.

3.70

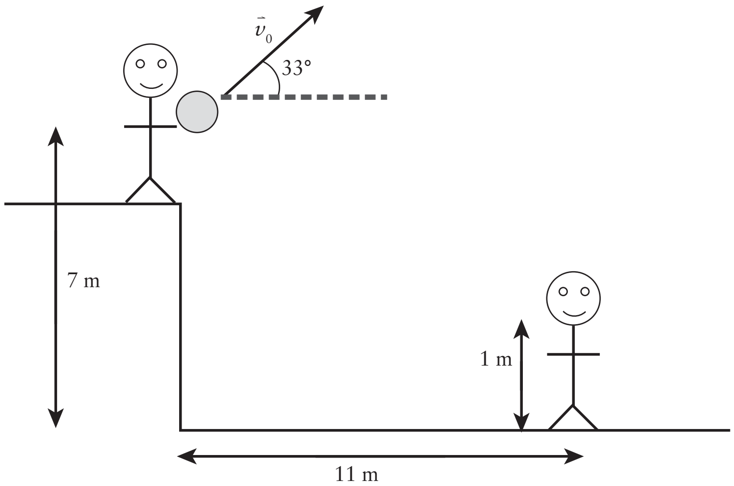


Figure 3-17 Problem 70

SET UP

You throw a ball with a speed of $v_0 = 9$ m/s and an angle of 33 degrees above the horizontal from a balcony 7 m in the air toward the ground. A friend of yours is standing 11 m from the point directly beneath you. She can accelerate from rest at a rate of 1.8 m/s^2 in order to catch the ball 1 m in the air. We need to determine how long your friend must wait before she starts running to catch the ball. First we need to calculate how long it takes for the ball to reach a height of 1 m above the ground and how far the ball moves horizontally in that time. We can compare that horizontal distance with the initial position of your friend to determine the distance she needs to run. Since she accelerates at a constant rate from rest, we can use $\Delta x = v_{0,x}t + \frac{1}{2}a_x t^2$ to determine the time it takes her to run that distance. The time she should wait is the difference between this time and the time necessary for the ball to reach that same position.

SOLVE

Know/Don't Know table for the basketball:

	X		Y
x_0	0 m	y_0	7 m
x_f	?	y_f	1 m
v_{0x}	(9 m/s)cos(33°)	v_{0y}	(9 m/s)sin(33°)
v_x	?	v_y	?
a_x	0	a_y	-9.81 m/s^2
t	?	t	?

Horizontal distance the basketball travels:

$$\Delta y = v_{0,\,y}t + \frac{1}{2}a_y t^2$$

$$\frac{1}{2}a_y t^2 + v_{0,\,y}t - \Delta y = 0$$

$$t = \frac{-v_{0,\,y} \pm \sqrt{v_{0,\,y}^2 - 4\left(\frac{1}{2}a_y\right)(-\Delta y)}}{2\left(\frac{1}{2}a_y\right)} = \frac{-v_0 \sin(33°) \pm \sqrt{v_0^2 \sin^2(33°) + 2a_y(\Delta y)}}{a_y}$$

$$= \frac{-\left(9\frac{\text{m}}{\text{s}}\right)\sin(33°) \pm \sqrt{\left(9\frac{\text{m}}{\text{s}}\right)\sin^2(33°) + 2\left(-9.8\frac{\text{m}}{\text{s}^2}\right)(-6 \text{ m})}}{-9.8\frac{\text{m}}{\text{s}^2}} = \frac{-4.9 \pm 11.9}{-9.8} \text{ s}$$

Taking the negative root:

$$t = 1.71 \text{ s}$$

$$\Delta x = v_{0,\,x}t + \frac{1}{2}a_x t^2 = v_0 \cos(33°)t + 0 = \left(9\frac{\text{m}}{\text{s}}\right)\cos(33°)(1.71 \text{ s}) = 12.91 \text{ m}$$

This distance is 12.91 m − 11 m = 1.91 m from your friend.

Time it takes your friend to run 1.91 m starting from rest:

$$\Delta x = v_{0,\,x}t + \frac{1}{2}a_x t^2 = 0 + \frac{1}{2}a_x t^2$$

$$t = \sqrt{\frac{2(\Delta x)}{a_x}} = \sqrt{\frac{2(1.91 \text{ m})}{1.8\frac{\text{m}}{\text{s}^2}}} = 1.46 \text{ s}$$

Your friend should wait 1.71 s − 1.46 s = $\boxed{0.25 \text{ s}}$ to start running.

REFLECT

Rather than trying to solve the problem all at once, split it up into easier, more manageable steps. Think about what you need to solve the problem before starting in order to better organize your thoughts.

3.71

SET UP

Marcus and Cody want to hike to a destination 12 km directly north of their starting point. Marcus walks 10 km in a direction that is 30 degrees north of east. Cody walks 15 km in a direction that is 45 degrees north of west. We need to determine the direction each hiker needs to walk to arrive at the destination. We'll take their starting point as the origin, east to point toward positive x, and north to point toward positive y. We first need to represent the first leg of each hike as a vector. We can calculate the vector representing the second leg because the two legs of each journey need to add vectorially to $(12\text{ km})\hat{y}$. The distance that each needs to walk is the magnitude of the second-leg vector.

SOLVE

Marcus:

$$\vec{r}_{\text{M},1} = ((10\text{ km})\cos(30°))\hat{x} + ((10\text{ km})\sin(30°))\hat{y}$$

$$\vec{r}_{\text{M},2} = x_{\text{M},2}\hat{x} + y_{\text{M},2}\hat{y}$$

$$(10\text{ km})\cos(30°) + x_{\text{M},2} = 0$$

$$x_{\text{M},2} = -(10\text{ km})\cos(30°) = -8.67\text{ km}$$

$$(10\text{ km})\sin(30°) + y_{\text{M},2} = 12\text{ km}$$

$$y_{\text{M},2} = (12\text{ km}) - (10\text{ km})\sin(30°) = 7\text{ km}$$

$$\vec{r}_{\text{M},2} = (-8.67\text{ km})\hat{x} + (7\text{ km})\hat{y}$$

$$r_{\text{M},2} = \sqrt{(-8.67\text{ km})^2 + (7\text{ km})^2} = \boxed{11.1\text{ km}}$$

Cody:

$$\vec{r}_{\text{C},1} = (-(15\text{ km})\cos(45°))\hat{x} + ((15\text{ km})\sin(45°))\hat{y}$$

$$\vec{r}_{\text{C},2} = x_{\text{C},2}\hat{x} + y_{\text{C},2}\hat{y}$$

$$-(15\text{ km})\cos(45°) + x_{\text{C},2} = 0$$

$$x_{\text{C},2} = (15\text{ km})\cos(45°) = 10.6\text{ km}$$

$$(15\text{ km})\sin(45°) + y_{\text{C},2} = 12\text{ km}$$

$$y_{\text{C},2} = (12\text{ km}) - (15\text{ km})\sin(45°) = 1.4\text{ km}$$

$$\vec{r}_{\text{C},2} = (10.6\text{ km})\hat{x} + (1.4\text{ km})\hat{y}$$

$$r_{\text{C},2} = \sqrt{(10.6\text{ km})^2 + (1.4\text{ km})^2} = \boxed{10.7\text{ km}}$$

REFLECT

The question ("How much farther…") only asks about the distance each hiker must walk. If we were looking for a vector, the question would read "In what direction…".

3.72

SET UP

Nathan walks a distance x_{east} due east and a distance x_{south} due south and ends up a distance of 15 km from his starting position. We are also told that $x_{\text{south}} = 2x_{\text{east}}$. These distances make a right triangle, so we can use the Pythagorean theorem to find x_{east} and x_{south}.

SOLVE

$$15\text{ km} = \sqrt{x_{\text{east}}^2 + x_{\text{south}}^2} = \sqrt{x_{\text{east}}^2 + (2x_{\text{east}})^2} = \sqrt{5x_{\text{east}}^2}$$

$$x_{\text{east}} = \frac{15\text{ km}}{\sqrt{5}} = \boxed{6.7\text{ km}}$$

$$x_{\text{south}} = 2x_{\text{east}} = \frac{2(15\text{ km})}{\sqrt{5}} = \boxed{13.4\text{ km}}$$

REFLECT

Nathan walks at an angle of 64 degrees south of east.

3.73

SET UP

A group of campers needs to find the quickest way to reach a campsite. The first route is a straight-line path that is 10.6 mi long. The campers can hike this route at a speed of 4 mi/hr. The second route is to take a canoe down a river and then hike uphill for 6.6 mi at a speed of 0.5 mi/hr. We can calculate the time necessary to hike route 1 and compare it to the time necessary to hike the uphill portion of route 2 to determine the time required for the canoe leg.

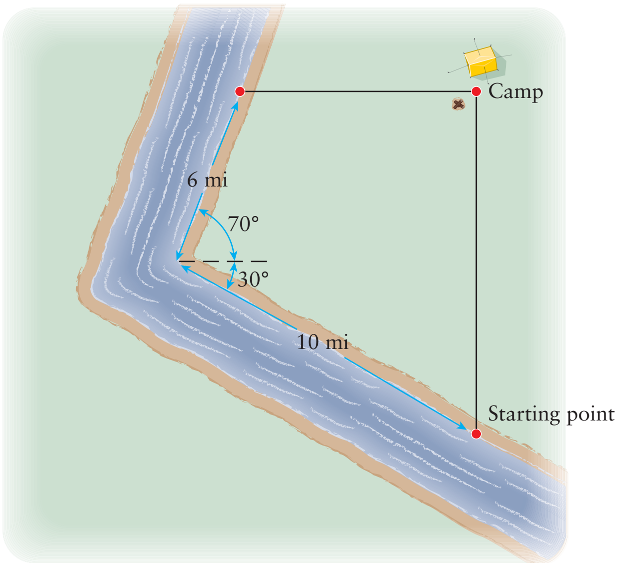


Figure 3-18 Problem 73

SOLVE

Time to hike straight path:

$$t_{\text{straight line}} = 10.6\text{ mi} \times \frac{1\text{ hr}}{4\text{ mi}} = 2.65\text{ hr}$$

Time to hike uphill:

$$t_{\text{uphill}} = 6.6\text{ mi} \times \frac{1\text{ hr}}{0.5\text{ mi}} = 13.2\text{ hr}$$

There is $\boxed{\text{no way for the second route to take less time}}$ than the first.

REFLECT

Walking straight is faster than just the uphill portion of the second trip, so if the canoe were infinitely fast it would still be slower to take the canoe.

3.74

SET UP

While standing on top of a 75.0-m-tall dam, you throw a rock with a speed of 25.0 m/s at an angle of 65.0 degrees above the horizontal. The rock is undergoing projectile motion during its trip, so we can use the constant acceleration equations to calculate the time it takes for you to see the rock hit the water and the time it takes to hear the rock hit the water. Because the speed of light is so fast, we can ignore the time it takes the light to travel back to your eye. Therefore, the time necessary to see the rock hit the water is equal to the time it takes the rock to travel 75.0 m downward. The total time before hearing the rock splash is equal to the time it takes the rock to hit the water (which was our answer to part (a) of this problem) plus the time it takes the sound to travel back up to your ear. The distance d that the sound travels is equal to the hypotenuse of the right triangle made by the x and y distances the rock traveled on its way down. The time it takes the sound to travel back to your ear is d divided by the speed of sound (344 m/s).

SOLVE

Part a)

$$\Delta y = v_{0,y}t + \frac{1}{2}a_y t^2 = v_0 \sin(65.0°)t - \frac{1}{2}gt^2$$

$$-\frac{1}{2}gt^2 + v_0\sin(65.0°)t - \Delta y = 0$$

$$t = \frac{-v_0\sin(65.0°) \pm \sqrt{(v_0\sin(65.0°))^2 - 4\left(-\frac{1}{2}g\right)(-\Delta y)}}{2\left(-\frac{1}{2}g\right)}$$

$$= \frac{-v_0\sin(65.0°) \pm \sqrt{(v_0\sin(65.0°))^2 - 2g(\Delta y)}}{-g}$$

$$= \frac{-\left(25.0\frac{\text{m}}{\text{s}}\right)\sin(65.0°) \pm \sqrt{\left(\left(25.0\frac{\text{m}}{\text{s}}\right)\sin(65.0°)\right)^2 - 2\left(9.8\frac{\text{m}}{\text{s}^2}\right)(-75.0)}}{-\left(9.8\frac{\text{m}}{\text{s}^2}\right)}$$

Taking the negative root,

$$t = \frac{-\left(25.0\frac{\text{m}}{\text{s}}\right)\sin(65.0°) - \sqrt{\left(\left(25.0\frac{\text{m}}{\text{s}}\right)\sin(65.0°)\right)^2 - 2\left(9.8\frac{\text{m}}{\text{s}^2}\right)(-75.0)}}{-\left(9.8\frac{\text{m}}{\text{s}^2}\right)} = \boxed{6.86\text{ s}}$$

Part b)
Horizontal distance the rock travels as it falls:

$$x = v_{0,x}t = v_0\cos(65.0°)t = \left(25.0\frac{\text{m}}{\text{s}}\right)\cos(65.0°)(6.86\text{ s}) = 72.5\text{ m}$$

Distance from the top of the dam to the location that the rock hits water:

$$d = \sqrt{(x^2 + y^2)} = \sqrt{((72.5\text{ m})^2 + (75.0\text{ m})^2)} = 104.3\text{ m}$$

Time for sound to travel this distance:

$$t_{\text{sound}} = \frac{d}{v_{\text{sound}}} = \frac{(104.3\text{ m})}{\left(344\frac{\text{m}}{\text{s}}\right)} = 0.303\text{ s}$$

Total time after throwing the rock:

$$t = (6.86\text{ s}) + (0.303\text{ s}) = \boxed{7.16\text{ s}}$$

REFLECT

You will hear the rock splash 0.3 s after watching it, which seems reasonable.

3.75

SET UP

A water balloon is thrown horizontally at an initial speed of 2.00 m/s from the top of a 6.00-m-tall building. At the same time, a second water balloon is thrown straight down from the same height at an initial speed of 2.00 m/s. To determine which balloon hits the ground first, we need to calculate the time it takes each balloon to hit the ground; whichever balloon has the shorter time will hit the ground first. Since the balloons start with the same initial speed and fall the same distance, they will hit the ground with the same final speed.

SOLVE

Know/Don't Know table for balloon 1:

	X		Y
x_0	0 m	y_0	6.00 m
x_f	?	y_f	0
v_{0x}	2.00 m/s	v_{0y}	0
v_x	?	v_y	?
a_x	0	a_y	-9.81 m/s^2
t	?	t	?

Time for balloon 1 to hit the ground:

$$\Delta y = v_{0,y}t + \frac{1}{2}a_yt^2 = 0 + \frac{1}{2}a_yt^2$$

$$t = \sqrt{\frac{2(\Delta y)}{a_y}} = \sqrt{\frac{2(-6.00\text{ m})}{\left(-9.81\frac{\text{m}}{\text{s}^2}\right)}} = 1.106\text{ s}$$

Know/Don't Know table for balloon 2:

	Y
y_0	6.00 m
y_f	0
v_{0y}	−2.00 m/s
v_y	?
a_y	−9.81 m/s^2
t	?

Time for balloon 2 to hit the ground:

$$\frac{1}{2}a_y t^2 + v_{0,\,y}t - \Delta y = 0$$

$$t = \frac{-v_{0,\,y} \pm \sqrt{v_{0,\,y}^2 - 4\left(\frac{1}{2}a_y\right)(-\Delta y)}}{2\left(\frac{1}{2}a_y\right)} = \frac{-\left(-2.00\frac{\text{m}}{\text{s}}\right) \pm \sqrt{\left(-2.00\frac{\text{m}}{\text{s}}\right)^2 + 2\left(-9.81\frac{\text{m}}{\text{s}^2}\right)(-6.00\text{ m})}}{\left(-9.81\frac{\text{m}}{\text{s}^2}\right)}$$

$$= \frac{-2.00 \pm 11.0}{-9.81}\text{ s}$$

Taking the negative root: $t = 0.917$ s.

$\boxed{\text{Balloon 2 lands 0.189 s before balloon 1}}$.

Final speed of the balloons:

$$v^2 - v_0^2 = 2a_y(\Delta y)$$

$$v = \sqrt{v_0^2 + 2a_y(\Delta y)}$$

Since the balloons start with the same initial speed and fall the same distance, the balloons will $\boxed{\text{land with the same speed}}$.

REFLECT

It will save you time and frustration in the long run if you solve each problem algebraically first and then plug in numbers at the end. For example, we could easily see the balloons have the same final speed from the general, algebraic answer in one calculation, rather than two.

3.76

SET UP

An airplane releases a ball as it flies parallel to the ground at a constant speed at a height of 255 m. The ball lands on the ground 255 m from the release point. Since it is dropped from the plane, the ball's initial velocity is equal to the plane's initial velocity. The initial velocity is solely in the horizontal direction. We will set the origin to the plane's initial position, left (in the figure) to be positive x, and up to be positive y.

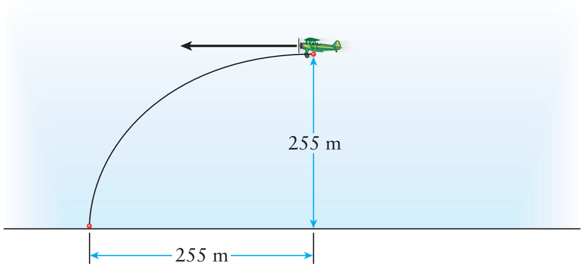


Figure 3-19 Problem 76

SOLVE

Know/Don't Know table:

	X		Y
x_0	0 m	y_0	6.00 m
x_f	255 m	y_f	−255 m
v_{0x}	WANT	v_{0y}	0 m/s
v_x	?	v_y	?
a_x	0	a_y	−9.8 m/s^2
t	?	t	?

$$\Delta y = v_{0,y}t + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}a_y t^2$$

$$t = \sqrt{\frac{2(\Delta y)}{a_y}}$$

$$\Delta x = v_{0,x}t = v_0\left(\sqrt{\frac{2(\Delta y)}{a_y}}\right)$$

$$v_0 = \frac{\Delta x}{\sqrt{\frac{2(\Delta y)}{a_y}}} = \frac{(255\ \text{m})}{\sqrt{\frac{2(-255\ \text{m})}{\left(-9.8\frac{\text{m}}{\text{s}^2}\right)}}} = \boxed{35.3\frac{\text{m}}{\text{s}}}$$

REFLECT

Before the ball is dropped out of the plane, the ball and plane are moving at the same speed in the same direction, which is why their initial velocities are equal.

3.77

SET UP

An airplane releases a ball as it flies upward at a constant speed of 35.3 m/s at an angle of 30 degrees. The ball is released at a height of 255 m. We will set the origin to the plane's initial position, the direction of the horizontal motion to be positive x, and up to be positive y. The ball will undergo parabolic motion once it leaves the plane. Since it is dropped from the plane, the ball's initial velocity is equal to the plane's initial velocity. We can solve the x and y component equations to determine the time the ball takes to reach the ground, the maximum height it attains, and the horizontal distance it travels.

SOLVE

Know/Don't Know table:

	X		Y
x_0	0 m	y_0	0 m
x_f	?	y_f	−255 m
v_{0x}	$v_0\cos(30°)$	v_{0y}	$v_0\sin(30°)$
v_x	?	v_y	?
a_x	0	a_y	−9.8 m/s^2
t	?	t	?

Part a)

$$\Delta y = v_{0,y}t + \frac{1}{2}a_y t^2$$

$$\frac{1}{2}a_y t^2 + v_{0,y}t - \Delta y = 0$$

$$t = \frac{-v_{0,y} \pm \sqrt{v_{0,y}^2 - 4\left(\frac{1}{2}a_y\right)(-\Delta y)}}{2\left(\frac{1}{2}a_y\right)} = \frac{-v_0\sin(30°) \pm \sqrt{v_0^2\sin^2(30°) + 2a_y(\Delta y)}}{a_y}$$

$$= \frac{-\left(35.3\frac{\text{m}}{\text{s}}\right)\sin(30°) \pm \sqrt{\left(35.3\frac{\text{m}}{\text{s}}\right)\sin^2(30°) + 2\left(-9.8\frac{\text{m}}{\text{s2}}\right)(-255\text{ m})}}{-9.81\frac{\text{m}}{\text{s}^2}} = \frac{-17.65 \pm 72.87}{-9.81}\text{ s}$$

Taking the negative root, $\boxed{t = 9.24\text{ s}}$.

Part b)

$$v^2 - v_0^2 = 2a_y(\Delta y)$$

$$\Delta y = \frac{v^2 - v_0^2}{2a_y} = \frac{0 - \left(35.3\frac{\text{m}}{\text{s}}\right)^2\sin^2(30°)}{2\left(-9.8\frac{\text{m}}{\text{s}^2}\right)} = 16\text{ m}$$

The ball's maximum height is 16 m above its initial position, or 255 m + 16 m = $\boxed{271\text{ m}}$.

Part c)

$$\Delta x = v_{0,x}t = \left(35.3\frac{\text{m}}{\text{s}}\right)\cos(30°)(9.24\text{ s}) = \boxed{282.5\text{ m}}$$

REFLECT

This question is similar to Problem 3.76. Both airplanes have the same initial speed (35.3 m/s), but the plane in Problem 3.76 flies horizontally, whereas this one flies upward. It makes sense that the ball from the plane flying horizontally should travel a smaller horizontal distance (255 m) compared to this one (282.5 m).

3.78

SET UP

Javier Sotomayor set a high jump record of 2.45 m. He took off at an angle of 65 degrees from the ground at a speed of v_0. We will take Javier's takeoff location to be the origin, the positive x direction to be the direction he was running, and up to be the positive y direction. The bar is located 1.5 m from his takeoff location. We are told to treat Javier as a point mass located at half of his height; this means his initial location above the ground is y_0 = (1.93 m)/2 = 0.965 m. We can use the x information to solve for t and plug this into the y equation to solve for the initial speed.

SOLVE

Know/Don't Know table:

	X		Y
x_0	0 m	y_0	0.965 m
x_f	1.5 m	y_f	2.45 m
v_{0x}	$v_0\cos(65°)$	v_{0y}	$v_0\sin(65°)$
v_x	?	v_y	?
a_x	0	a_y	-9.8 m/s^2
t	?	t	?

$$\Delta x = v_{0,x}t + \frac{1}{2}a_x t^2 = v_0\cos(65°)t + 0$$

$$t = \frac{\Delta x}{v_0\cos(65°)}$$

$$\Delta y = v_{0,y}t + \frac{1}{2}a_y t^2 = v_0\sin(65°)\left(\frac{\Delta x}{v_0\cos(65°)}\right) + \frac{1}{2}a_y\left(\frac{\Delta x}{v_0\cos(65°)}\right)^2$$

$$= (\Delta x)\tan(65°) + \frac{a_y}{2}\left(\frac{\Delta x}{v_0\cos(65°)}\right)^2$$

$$v_0 = \sqrt{\frac{a_y(\Delta x)^2}{2\cos^2(65°)((\Delta y) - (\Delta x)\tan(65°))}} = \sqrt{\frac{\left(-9.8\frac{\text{m}}{\text{s}^2}\right)(1.5\text{ m})^2}{2\cos^2(65°)((1.49\text{ m}) - (1.5\text{ m})\tan(65°))}}$$

$$\boxed{v_0 = 6.0\frac{\text{m}}{\text{s}}}$$

REFLECT

This is a reasonable running speed for an athlete.

3.79

SET UP

Gabriele Reinsch threw a discus 76.80 m. She launched it an angle of 45 degrees from a height of 2.0 m above the ground. We will take Gabriele's feet to be the origin, the positive x direction to be the direction she threw the discus, and up to be the positive y direction. We can use the x information to solve for t and plug this into the y equation to solve for her initial speed. Remember that $\cos(45°) = \sin(45°) = \frac{1}{\sqrt{2}}$; this will help simplify the algebra.

SOLVE

Know/Don't Know table:

	X		Y
x_0	0 m	y_0	2.0 m
x_f	76.80 m	y_f	0 m
v_{0x}	$v_0\cos(45°)$	v_{0y}	$v_0\sin(45°)$
v_x	?	v_y	?
a_x	0	a_y	-9.81 m/s^2
t	?	t	?

$$\Delta x = v_{0,x}t + \frac{1}{2}a_x t^2 = v_0\cos(45°)t + 0$$

$$t = \frac{\Delta x}{v_0\cos(45°)}$$

$$\Delta y = v_{0,y}t + \frac{1}{2}a_y t^2 = v_0\sin(45°)\left(\frac{\Delta x}{v_0\cos(45°)}\right) + \frac{1}{2}a_y\left(\frac{\Delta x}{v_0\cos(45°)}\right)^2 = \Delta x + a_y\left(\frac{\Delta x}{v_0}\right)^2$$

$$v_0 = \sqrt{\frac{-a_y(\Delta x)^2}{(\Delta x) - (\Delta y)}} = \sqrt{\frac{-\left(-9.81\frac{\text{m}}{\text{s}^2}\right)(76.80\text{ m})^2}{(76.80\text{ m})-(-2.0\text{ m})}} = \boxed{27.1\frac{\text{m}}{\text{s}}}$$

REFLECT

We cannot use the maximum horizontal range formula from the text because the discus does not land at the same height it was released.

3.80

SET UP

A boy is on a diving platform 10.0 m above the surface of the water. He runs straight off the platform at 5.00 m/s. We will set the origin to be at the surface of the water directly beneath his starting position. The positive x direction is the same direction as his initial velocity and up will be the positive y direction. Once the boy leaves the water, he will undergo parabolic motion. His initial velocity is solely in the x direction. We can fill in our Know/Don't Know table with all of this information and solve for the boy's speed at the surface of the water, the time it takes him to fall 10.0 m, and how far he moves horizontally before hitting the water.

SOLVE

Know/Don't Know table:

	X		Y
x_0	0 m	y_0	10.0 m
x_f	?	y_f	0 m
v_{0x}	5.00 m/s	v_{0y}	0 m/s
v_x	?	v_y	?
a_x	0	a_y	-9.81 m/s^2
t	?	t	?

Part a)

$$v_y^2 = v_{0,y}^2 + 2a_y(\Delta y) = 0 + 2a_y(\Delta y)$$

$$v_y = \sqrt{2a_y(\Delta y)} = \sqrt{2\left(-9.81\frac{\text{m}}{\text{s}^2}\right)(-10.0\text{ m})} = 14.0\frac{\text{m}}{\text{s}}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{\left(5.00\frac{\text{m}}{\text{s}}\right)^2 + \left(14.0\frac{\text{m}}{\text{s}}\right)^2} = \boxed{14.9\frac{\text{m}}{\text{s}}}$$

Part b)

$$\Delta y = v_{0,y}t + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}a_y t^2$$

$$t = \sqrt{\frac{2(\Delta y)}{a_y}} = \sqrt{\frac{2(-10.0\text{ m})}{\left(-9.81\frac{\text{m}}{\text{s}^2}\right)}} = \boxed{1.43\text{ s}}$$

Part c)

$$\Delta x = v_{0,x}t + \frac{1}{2}a_x t^2 = \left(5.00\frac{\text{m}}{\text{s}}\right)(1.43\text{ s}) + 0 = \boxed{7.15\text{ m}}$$

REFLECT

The x component of the boy's velocity remains constant since the acceleration in the x direction is zero.

3.81

SET UP

On Earth, a froghopper can jump to a maximum height of $h_E = h$ and a maximum horizontal range $R_E = R$. We are asked to determine the maximum height h_M and range R_M of a froghopper on the Moon, where $g = 1.62\text{ m/s}^2$. We can use Equations 3-24 and 3-26 from the text to write h and R in terms of the initial speed, the launch angle, and the gravitational acceleration. Since the initial speed and launch angle will be the same on both Earth and the Moon, we can take the ratios of h_M/h_E and R_M/R_E to solve for the Moon quantities in terms of Earth quantities.

SOLVE

Maximum height:

$$(y - y_0)_{\text{max}} = h = \frac{1}{2}\frac{v_0^2\sin^2(\theta)}{g}.$$

$$\frac{h_M}{h_E} = \frac{\left(\frac{1}{2}\frac{v_0^2\sin^2(\theta)}{g_M}\right)}{\left(\frac{1}{2}\frac{v_0^2\sin^2(\theta)}{g_E}\right)} = \frac{g_E}{g_M}$$

$$h_M = \left(\frac{g_E}{g_M}\right)h_E = \left(\frac{9.8\frac{\text{m}}{\text{s}^2}}{1.62\frac{\text{m}}{\text{s}^2}}\right)h = \boxed{6.06h}$$

Maximum range:

$$(x - x_0)_{\text{max}} = R = \frac{2v_0^2\sin(\theta)\cos(\theta)}{g}$$

$$\frac{R_M}{R_E} = \frac{\left(\frac{2v_0^2\sin(\theta)\cos(\theta)}{g_M}\right)}{\left(\frac{2v_0^2\sin(\theta)\cos(\theta)}{g_E}\right)} = \frac{g_E}{g_M}$$

$$R_M = \left(\frac{g_E}{g_M}\right)R_E = \left(\frac{9.8\frac{\text{m}}{\text{s}^2}}{1.62\frac{\text{m}}{\text{s}^2}}\right)R = 6.06R$$

REFLECT

Both the maximum height and the range are inversely proportional to the gravitational acceleration, which is why we get the same coefficient.

3.82

SET UP

Jason Elam kicked a football 63.0 yd (= 189 ft) in order to score a field goal. The goal posts are 10 ft in the air. He launched the ball at an angle of 40 degrees from the ground. We will take the football's initial location to be the origin, the positive x direction to be the direction he kicked the ball, and up to be the positive y direction. We can use the x information to solve for t and plug this into the y equation to solve for the initial speed. Once we have the initial speed, we can then plug it into our relationship for t that we used earlier to find the time it took the ball to travel 189 ft.

SOLVE

Know/Don't Know table:

	X		Y
x_0	0 ft	y_0	
x_f	189 ft	y_f	10 ft
v_{0x}	$v_0\cos(40°)$	v_{0y}	$v_0\sin(40°)$
v_x	?	v_y	?
a_x	0	a_y	-32 ft/s^2
t	?	t	?

Part a)

$$\Delta x = v_{0,x}t + \frac{1}{2}a_x t^2 = v_0\cos(40.0°)t + 0$$

$$t = \frac{\Delta x}{v_0\cos(40.0°)}$$

$$\Delta y = v_{0,y}t + \frac{1}{2}a_y t^2 = v_0\sin(40.0°)\left(\frac{\Delta x}{v_0\cos(40.0°)}\right) + \frac{1}{2}a_y\left(\frac{\Delta x}{v_0\cos(40.0°)}\right)^2$$

$$= (\Delta x)\tan(40.0°) + \frac{a_y}{2}\left(\frac{\Delta x}{v_0\cos(40.0°)}\right)^2$$

$$v_0 = \sqrt{\frac{a_y(\Delta x)^2}{2\cos^2(40.0°)((\Delta y) - (\Delta x)\tan(40.0°))}} = \sqrt{\frac{\left(-32\frac{\text{ft}}{\text{s}^2}\right)(189\text{ ft})^2}{2\cos^2(40.0°)((10\text{ ft}) - (189\text{ ft})\tan(40.0°))}}$$

$$\boxed{v_0 = 81\frac{\text{ft}}{\text{s}}}$$

Part b)

$$t = \frac{\Delta x}{v_0\cos(40.0°)} = \frac{189\text{ ft}}{\left(81\frac{\text{ft}}{\text{s}}\right)\cos(40.0°)} = \boxed{3.0\text{ s}}$$

REFLECT

A speed of 81 ft/s is equivalent to 55 mph or 25 m/s, both of which are reasonable speeds for a football that is kicked very hard. Three seconds is a reasonable amount of time for a ball to travel 63 yd down a football field.

3.83

SET UP

You want to build golf courses on the Moon and on Mars that play just like a golf course on Earth. Because the effects of gravity are different at the three locations, the dimensions of each hole for the course need to change relative to Earth. The acceleration due to gravity on the Moon is $g_{\text{Moon}} \approx \frac{1}{6}g_{\text{Earth}}$, and the acceleration due to gravity on Mars is 3.75 m/s^2, or $g_{\text{Mars}} \approx 0.38g_{\text{Earth}}$. Since these are smaller than Earth's acceleration due to gravity, we will need to increase the size of each course by some factor. We can compare the range (Equation 3-26) on each satellite to the range on Earth in order to determine this factor.

SOLVE

$$(x - x_0)_{\text{range}} = \frac{2v_0^2\sin(\theta)\cos(\theta)}{g}$$

Moon:

$$\frac{(x - x_0)_{\text{Moon}}}{(x - x_0)_{\text{Earth}}} = \frac{\left(\frac{2v_0^2\sin(\theta)\cos(\theta)}{g_{\text{Moon}}}\right)}{\left(\frac{2v_0^2\sin(\theta)\cos(\theta)}{g_{\text{Earth}}}\right)} = \frac{g_{\text{Earth}}}{g_{\text{Moon}}} = \frac{g_{\text{Earth}}}{\left(\frac{1}{6}g_{\text{Earth}}\right)} = 6$$

The dimensions of each green should be $\boxed{\text{increased by a factor of 6}}$ for the Moon.

Mars:

$$\frac{(x - x_0)_{\text{Mars}}}{(x - x_0)_{\text{Earth}}} = \frac{\left(\frac{2v_0^2\sin(\theta)\cos(\theta)}{g_{\text{Mars}}}\right)}{\left(\frac{2v_0^2\sin(\theta)\cos(\theta)}{g_{\text{Earth}}}\right)} = \frac{g_{\text{Earth}}}{g_{\text{Mars}}} = \frac{g_{\text{Earth}}}{(0.38g_{\text{Earth}})} = 2.6$$

The dimensions of each green should be $\boxed{\text{increased by a factor of 2.6}}$ for Mars.

REFLECT

The acceleration due to gravity is smaller on Mars and the Moon, so we would expect the ball to go farther there than on Earth. Therefore, it makes sense that the golf course would need to be larger than on Earth. The fact that people hit the ball 63% of the distance on Earth is irrelevant to solving the problem.

3.84

SET UP

Pilots, who were swung in a circle 13.4 m in diameter, blacked out when the rotation speed was increased to 30.6 rpm. The pilot traverses a distance equal to the circumference of the circle in one revolution. Assuming the speed is constant, we can calculate the radial acceleration directly from the linear speed and the radius of the device. If we want to decrease the acceleration, we need to increase the amount of time it takes to complete one revolution, T, known as the period. The new speed is the circumference of the circle, which remains the same, divided by T_{new}. We can calculate T_{old} for the original speed and then compare the two values.

SOLVE

Part a)

$$v = 30.6\frac{\mathrm{rev}}{\mathrm{min}} \times \frac{1\ \mathrm{min}}{60\ \mathrm{s}} \times \frac{2\pi\left(\frac{13.4\ \mathrm{m}}{2}\right)}{1\ \mathrm{rev}} = 21.47\frac{\mathrm{m}}{\mathrm{s}}$$

$$a = \frac{v^2}{R} = \frac{\left(21.47\frac{\mathrm{m}}{\mathrm{s}}\right)^2}{\left(\frac{13.4\ \mathrm{m}}{2}\right)} = \boxed{68.8\frac{\mathrm{m}}{\mathrm{s}^2} = 7g}$$

Part b)

$$(0.75)a = (0.75)\left(68.8\frac{\mathrm{m}}{\mathrm{s}^2}\right) = 51.6\frac{\mathrm{m}}{\mathrm{s}^2}$$

$$a = \frac{v^2}{R} = \frac{\left(\frac{2\pi R}{T}\right)^2}{R} = \frac{4\pi^2 R}{T^2}$$

$$T_{\mathrm{new}} = \sqrt{\frac{4\pi^2 R}{a}} = \sqrt{\frac{4\pi^2\left(\frac{13.4\ \mathrm{m}}{2}\right)}{\left(51.6\frac{\mathrm{m}}{\mathrm{s}^2}\right)}} = 2.26\ \mathrm{s}$$

T_{old}:

$$1\ \mathrm{rev} \times \frac{1\ \mathrm{min}}{30.6\ \mathrm{rev}} \times \frac{60\ \mathrm{s}}{1\ \mathrm{min}} = 1.96\ \mathrm{s}$$

$$\frac{T_{\mathrm{new}}}{T_{\mathrm{old}}} = \frac{2.26\ \mathrm{s}}{1.96\ \mathrm{s}} = 1.15$$

The time for the pilot to make one spin must be $\boxed{\text{increased by 15\%}}$.

REFLECT

The acceleration is inversely proportional to the period, so the period must increase if the acceleration is decreased.

3.85

SET UP

A sample in a centrifuge initially experiences a radial acceleration of $a_1 = 100g$. The acceleration then increases by a factor of 8, $a_2 = 800g$. We can determine the new rotation speed in terms of the original rotation speed by looking at the ratio of the two radial accelerations. The radius is the same in both cases, since the size of the centrifuge doesn't change.

SOLVE

$$a_1 = 100g = \frac{v_1^2}{R}$$

$$a_2 = 800g = \frac{v_2^2}{R}$$

$$\frac{a_2}{a_1} = \frac{800g}{100g} = 8 = \frac{\left(\frac{v_1^2}{R}\right)}{\left(\frac{v_2^2}{R}\right)} = \frac{v_1^2}{v_2^2}$$

$$\boxed{v_2 = v_1\sqrt{8} \approx 2.83v_1}$$

REFLECT

If a problem asks you to find the factor by which a quantity increases or decreases, first try taking a ratio of the quantities. Usually some terms will remain constant over the process and will cancel out in your ratio.

3.86

SET UP

Modern pilots can survival radial accelerations of up to 88 m/s^2. We can calculate the centripetal acceleration of a pilot flying at a constant speed of 500 m/s in a circle of radius $R = 4400$ m and compare it to 88 m/s^2. The pilot will survive as long as he experiences an accelerationless than 88 m/s^2.

SOLVE

$$a = \frac{v^2}{R} = \frac{\left(500\frac{\text{m}}{\text{s}}\right)^2}{4400 \text{ m}} = \boxed{56.8\frac{\text{m}}{\text{s}^2}}$$

$\boxed{\text{Yes, the pilot will survive}}$.

REFLECT

The pilot experiences an acceleration of $5.8g$.

3.87

SET UP

A softball player rotates her entire arm about her shoulder when pitching. She sweeps out a circle with a radius R equal to the length of her arm. She releases the ball at a speed of 31.3 m/s. Right before release, the ball experiences a radial acceleration of 1630 m/s^2. We can calculate the length of her arm directly from the definition of the radial acceleration.

SOLVE

$$a = \frac{v^2}{R}$$

$$R = \frac{v^2}{a} = \frac{\left(31.3\frac{\text{m}}{\text{s}}\right)^2}{1630\frac{\text{m}}{\text{s}^2}} = \boxed{0.601 \text{ m}}$$

REFLECT

This is approximately 2 ft, which is a reasonable length for a person's entire arm.

3.88

SET UP

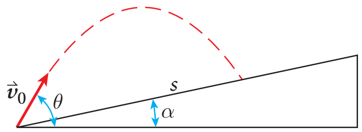


Figure 3-20 Problem 88

A ball is thrown above an inclined plane with an initial speed of v_0 at an angle of θ. The incline is at an angle of α. We'll take the initial position of the ball to be the origin, right (as in the figure) to be positive x, and up to be positive y in our coordinate system. We'll call the x and y positions of the ball when it lands x_{f} and y_{f}. These are related to s and α through the cosine and sine, respectively. We can use the x information to solve for t and plug this into the y equation and simplify in order to get the desired result. In order to find the value of θ that maximizes s, we differentiate s with respect to θ, set it equal to zero, and solve for θ. We will need to use the product rule for derivatives in addition to some trigonometric identities, specifically $\cos(-\theta) = \cos(\theta)$, $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$, and $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$.

SOLVE

Part a)

$$\Delta x = x_{\text{f}} - 0 = v_{0,x}t + \frac{1}{2}a_x t^2 = v_0 \cos(\theta) t + 0$$

$$t = \frac{x_{\text{f}}}{v_0 \cos(\theta)}$$

$$\Delta y = y_{\text{f}} - 0 = v_{0,y}t + \frac{1}{2}a_y t^2 = v_0 \sin(\theta)\left(\frac{x_{\text{f}}}{v_0 \cos(\theta)}\right) + \frac{1}{2}(-g)\left(\frac{x_{\text{f}}}{v_0 \cos(\theta)}\right)^2$$

$$(x_{\text{f}})\frac{\sin(\theta)}{\cos(\theta)} - \frac{g x_{\text{f}}^2}{2v_0^2 \cos^2(\theta)} - y_{\text{f}} = -\frac{g x_{\text{f}}^2}{2v_0^2 \cos(\theta)} + x_{\text{f}} \sin(\theta) - y_{\text{f}} \cos(\theta) = 0$$

But $y_f = s\sin(\alpha)$ and $x_f = s\cos(\alpha)$, so

$$-\frac{gs^2\cos^2(\alpha)}{2v_0^2\cos(\theta)} + s\cos(\alpha)\sin(\theta) - s\sin(\alpha)\cos(\theta) = 0, \text{ which means } s = 0 \text{ or}$$

$$-\frac{gs\cos^2(\alpha)}{2v_0^2\cos(\theta)} + \cos(\alpha)\sin(\theta) - \sin(\alpha)\cos(\theta) = 0$$

$$-\frac{gs\cos^2(\alpha)}{2v_0^2\cos(\theta)} = -\cos(\alpha)\sin(\theta) + \sin(\alpha)\cos(\theta)$$

$$s = \frac{2v_0^2\cos(\theta)}{g\cos^2(\alpha)}(\cos(\alpha)\sin(\theta) - \sin(\alpha)\cos(\theta)) = \boxed{\frac{2v_0^2\cos(\theta)}{g\cos^2(\alpha)}(\sin(\theta - \alpha))}$$

Part b)

$$s = \frac{2v_0^2\cos(\theta)}{g\cos^2(\alpha)}(\sin(\theta - \alpha)) = \frac{2v_0^2}{g\cos^2(\alpha)}\cos(\theta)[\sin(\theta)\cos(\alpha) - \sin(\alpha)\cos(\theta)]$$

$$\frac{ds}{d\theta} = 0 = \frac{d}{d\theta}\left[\frac{2v_0^2}{g\cos^2(\alpha)}\cos(\theta)[\sin(\theta)\cos(\alpha) - \sin(\alpha)\cos(\theta)]\right]$$

$$= \frac{2v_0^2}{g\cos^2(\alpha)}[-\sin(\theta)[\sin(\theta)\cos(\alpha) - \sin(\alpha)\cos(\theta)] + \cos(\theta)[\cos(\theta)\cos(\alpha) + \sin(\alpha)\sin(\theta)]]$$

$$0 = -\sin^2(\theta)\cos(\alpha) + \sin(\theta)\sin(\alpha)\cos(\theta) + \cos^2(\theta)\cos(\alpha) + \sin(\alpha)\sin(\theta)\cos(\theta)$$

$$0 = \cos(\alpha)[\cos^2(\theta) - \sin^2(\theta)] + \sin(\alpha)[2\sin(\theta)\cos(\theta)]$$

$$0 = \cos(\alpha)\cos(2\theta) + \sin(\alpha)\sin(2\theta)$$

$$0 = \cos(2\theta - \alpha)$$

$$2\theta - \alpha = 90°$$

$$\boxed{\theta = \frac{90° + \alpha}{2}}$$

REFLECT

When dealing with level ground, the maximum range occurs at an angle of 45 degrees. For an inclined plane, it makes sense that the maximum angle should be 45 degrees plus some small correction for the angle of the plane, which is what we found.

Chapter 4
Newton's Laws of Motion

Conceptual Questions

4.1 Yes, the direction of the net force will determine the direction of the acceleration vector. Mass is a scalar quantity and will not affect the direction.

4.2 No, it does not necessarily imply the object is at rest. It implies that the object is traveling at a constant velocity. An object at rest has a constant velocity, but so does an object traveling in a straight line at a constant speed.

4.3 The SI units for force are the units of mass multiplied by the units of acceleration: $\text{kg}\cdot\frac{\text{m}}{\text{s}^2}=\text{N}$. The units of force are the same for the differential form:

$$\frac{\text{kg}\cdot\frac{\text{m}}{\text{s}}}{\text{s}}=\text{kg}\cdot\frac{\text{m}}{\text{s}^2}=\text{N}.$$

4.4 A gymnast will come to rest more slowly on a padded floor than on concrete, which means her acceleration and, therefore, the force due to the floor will be smaller.

4.5 This definition agrees with the one given in the book. Each equal quantity of matter (by the measure of mass) resists acceleration to the same degree, so it has the same inertia.

4.6 If the motion of an object is only along one axis, then the acceleration is zero along the other axes. The net force along those axes must, therefore, equal zero as well.

4.7 You can double the rope up. Each of the two strands pulls only 425 N.

4.8 The boxer's claim that both his jaw and his opponent's fist experience the same force is correct. However, the fist is relatively hard while the jaw and the surrounding tissue are not. So, one would imagine that it's more painful to take a blow to the face than one to the hand.

4.9 Yes, both the Sun and Earth pull each other and make up a force pair associated with the gravitational interaction.

4.10 The fisherman can double up his line so that each strand pulls 2.5 lb.

4.11 (1) Electrical wires hung on utility poles have tension on them. (2) Keys on a long cord will produce tension in the cord (especially when "swung"). (3) When you walk a dog on a leash, there is a tension present in the leash. (4) There is tension in the straps of

a backpack that is hanging off your shoulders. (5) There is a tension in your shoelaces when you tie them.

4.12 It is called a normal force because it acts normal (or perpendicular) to the surface.

4.13 As the elevator accelerates up, the scale reading increases to $mg + ma$. While ascending at constant speed, the scale again reads mg. As the elevator slows to a stop, the scale reads $mg - ma$. As the elevator accelerates down, the scale again reads $mg - ma$. For the constant speed portion of the trip the scale reading is mg. As the elevator slows down to a stop, the scale reads $mg + ma$. Once the elevator is at rest, the scale again reads mg.

4.14 The forces acting on a can at rest on a desk are gravity acting downward and the normal force from the desk acting upward. The net force is zero since the can is at rest.

4.15 The net force on the bathroom scale is zero since it is at rest.

4.16 (a) The spring balance will read $mg + ma$. (b) The pan balance will read mg.

4.17 The minimum value for the sum of these forces is 40 N and occurs when the forces act in opposite directions. The maximum value is 100 N and occurs when the forces act in the same direction.

4.18 If the box moves at a constant speed when a 60-N force is applied, the floor must oppose the motion with a force of 60 N, so that the net force and, therefore, the acceleration are zero. When a force of 80 N is applied, the net force on the box is, therefore, 20 N, which results in a nonzero acceleration.

4.19 It's easier to lift a truck on the Moon. Weight is due to gravity. The Moon has less gravity than Earth.

4.20 Seatbelts increase the time over which passengers come to rest, which decreases the magnitude of their accelerations and, correspondingly, magnitude of the net force acting on them.

4.21 In a car wreck, the car slows down extremely fast while the people inside it continue moving forward (due to their inertia). Seat restraints minimize the damage by slowing the people as soon as possible, not letting a large difference in speed build up between the car and its occupants. Conventional restraints can still crush or cause whiplash, especially to children. A rear-facing restraint is tightly attached to the car's frame and will start slowing the infant immediately with a force distributed evenly over its whole body, minimizing all of these risks.

4.22 When taking a step, you push back and down on the Earth. By Newton's third law, the Earth pushes forward and up on you, propelling you forward.

4.23 A bird begins flying by pushing with its wings backward against the air. The air then exerts an equal and opposite force forward on the bird, and it is this force on the bird that propels it forward.

Multiple-Choice Questions

4.24 **A** (frontal portion of the brain). When the car stops suddenly, the driver and his brain are still moving forward until either the seatbelt or windshield stops him. At that point, the front part of his brain will slam into his skull, stopping the brain.

4.25 **B** (rear). The car accelerates forward quickly due to the rear-end collision, while the person stays in place.

4.26 **C** (continue at constant velocity). A net force of zero corresponds to an acceleration of zero, which means the velocity of the object does not change.

4.27 **B** (a newton of gold on the Moon). The gravity of the Moon is smaller than the gravity of Earth, so more gold is necessary in order to achieve a weight of 1 newton on the Moon.

4.28 **B** (inversely proportional to the object's mass). The produce of the mass and the acceleration is equal to the net force on the object.

4.29 **D** (is accelerating). The direction of the object's acceleration is the same as the direction of the net force.

4.30 **C** (accelerating). A net force is required for an acceleration.

4.31 **D** (at rest or in motion with a constant velocity). A net force of zero means the acceleration is also zero.

4.32 **D** (smaller than; larger than). As the elevator slows to a stop when traveling upward, the scale reads $mg - ma$. As the elevator slows down to a stop when traveling downward, the scale reads $mg + ma$.

4.33 **C** (less than the weight of the block). The normal force opposes only a fraction of the weight due to the incline.

4.34 **A** (greater than its acceleration in case (a)). The net force in part (a) is less than 10 N.

Estimation Questions

4.35 An apple is about 1 N. A penny is about 0.03 N. A textbook is about 25 N. A calculator is about 5 N. A liter of water is around 10 N.

4.36 The normal force acting on an apple at rest on a flat surface will be equal in magnitude to the apple's weight (about 1 N) and point straight up. If the surface were tilted at an incline of 30 degrees, the normal force magnitude would be $mg\cos(30°) = 0.87$ N and point perpendicular to the surface.

4.37 A baseball throw can be in the range of 40 m/s (about 89 mph). A baseball has a mass of around 0.15 kg. The powerful part of the throw lasts about 0.05 s. This gives

$$F = m\frac{\Delta v}{\Delta t} = (0.15\text{ kg})\frac{\left(40\frac{\text{m}}{\text{s}}\right)}{0.05\text{ s}} \approx 120\text{ N}.$$

4.38 The weight capacity of a typical elevator is around a ton. To make the math easier, we'll say 2200 pounds, which is around 1000 kg or 10,000 N. An elevator accelerates a little less than 1 m/s^2 when starting and stopping. Therefore, the maximum tension in the cable is around 10,000 N + (1000 kg)(1 m/s^2) = 11,000 N. For safety considerations, the cable should be able to hold much more than this.

4.39 Tennis balls weigh around 57 g. A typical ball launcher sends the ball at 12 m/s. This process takes around 0.1 s. The force is about $F = m\frac{\Delta v}{\Delta t} = (0.057\text{ kg})\frac{\left(12\frac{\text{m}}{\text{s}}\right)}{0.1\text{ s}} \approx 7$ N.

4.40 The tension in the rope is equal in magnitude to the pull of the skier. The force an average person can pull is around his or her weight, or 700 N.

4.41 The "jaws of life"must be able to rip open a car door, so we expect the forces involved to be large, say, 2000–50,000 N.

4.42 A light switch is easy to flip, so it should take less than 1 N to flip it.

4.43 It's relatively easy to snap your fingers. The forces necessary should be small, say, 1–5 N.

4.44

Time (s)	Force (mN)
0	0
1	2.28
2	5.73
3	11.28
4	12.27
5	12.54
6	12.38
7	14.11
8	16.79
9	21.08
10	28.51

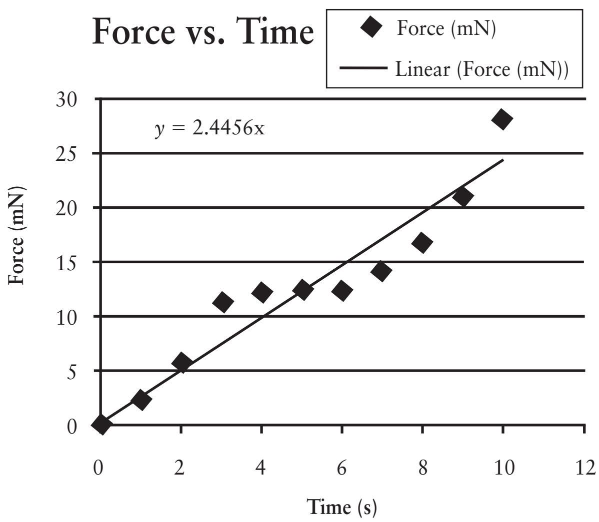


Figure 4-1 Problem 44

The force at a time of 20 s will be $F = \left(2.4456\frac{\text{mN}}{\text{s}}\right)(20\text{ s}) = \boxed{50\text{ mN}}$.

4.45 The weight of the rider will reach its maximum value during the ascent from the ground floor to the fourth floor. This is because the weight of the rider will be added to the force required to accelerate from 0 m/s to 10 m/s.

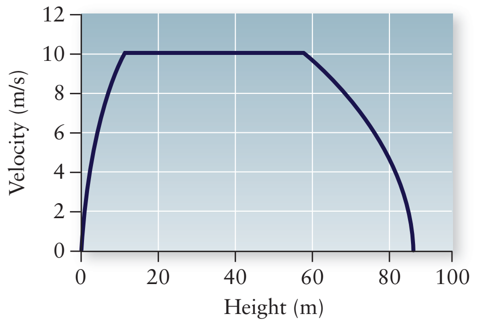


Figure 4-2 Problem 45

4.46

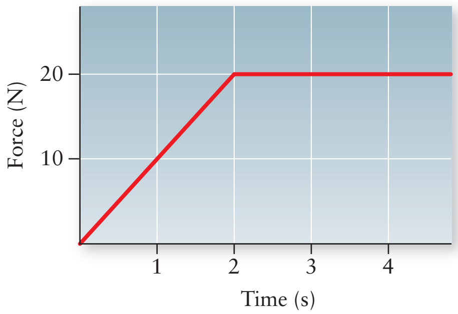


Figure 4-3 Problem 46

The area under the force versus time curve is equal to $m\Delta v$.

$$\text{Area from } t = 0 \text{ s to } t = 1 \text{ s} = \frac{1}{2}(1 \text{ s})(10 \text{ N}) = 5\frac{\text{kg} \cdot \text{m}}{\text{s}} = m(v_{t=1\text{ s}} - v_{t=0\text{ s}}) = mv_{t=1\text{ s}}$$

$$v_{t=1\text{ s}} = \frac{\left(5\frac{\text{kg} \cdot \text{m}}{\text{s}}\right)}{2.5 \text{ kg}} = \boxed{2\frac{\text{m}}{\text{s}}}$$

$$\text{Area from } t = 0 \text{ s to } t = 4 \text{ s} = \frac{1}{2}(2 \text{ s})(20 \text{ N}) + (2 \text{ s})(20 \text{ N})$$

$$= 60\frac{\text{kg} \cdot \text{m}}{\text{s}} = m(v_{t=4\text{ s}} - v_{t=0\text{ s}}) = mv_{t=4\text{ s}}$$

$$v_{t=4\text{ s}} = \frac{\left(60\frac{\text{kg} \cdot \text{m}}{\text{s}}\right)}{2.5 \text{ kg}} = \boxed{24\frac{\text{m}}{\text{s}}}$$

Problems

4.47

SET UP

The net force on a 2000-kg car is 4000 N. We can use Newton's second law to calculate the magnitude of the acceleration of the car.

SOLVE

$$\sum F = ma$$

$$a = \frac{\sum F}{m} = \frac{4000\ \text{N}}{2000\ \text{kg}} = \boxed{2\frac{\text{m}}{\text{s}^2}}$$

REFLECT

The car will accelerate in the same direction as the net force.

4.48

SET UP

A 2000-kg car is accelerating at a rate of 2 m/s^2. We can use Newton's second law to calculate the magnitude of the net force necessary to achieve this acceleration.

SOLVE

$$\sum F = ma = (2000\ \text{kg})\left(2\frac{\text{m}}{\text{s}^2}\right) = \boxed{4000\ \text{N}}$$

REFLECT

The net force will point in the same direction as the acceleration.

4.49

SET UP

The engine of a 1250-kg car can deliver a maximum force of 15,000 N. Since this is the only force acting on the car, we can set this force equal to the mass times the acceleration and solve for the magnitude of the acceleration.

SOLVE

$$\sum F = ma$$

$$a = \frac{\sum F}{m} = \frac{15{,}000\ \text{N}}{1250\ \text{kg}} = \boxed{12\frac{\text{m}}{\text{s}^2}}$$

REFLECT

Because there is only one force acting on the car, this force is equal to the net force.

4.50

SET UP

Applying a known constant force to an object of known mass causes it to accelerate at 10 m/s^2. Assuming that this is the only force acting on the object, we can relate it to the acceleration by Newton's second law: $\sum F = ma$. We can use proportional reasoning to determine what will happen to one parameter if we change the others.

SOLVE

$$\sum F = ma$$

Part a) If the force is doubled while the mass remains constant, the acceleration will $\boxed{\text{double to } 20 \text{ m/s}^2}$.

Part b) If the mass is halved while the net force remains constant, the acceleration will $\boxed{\text{double to } 20 \text{ m/s}^2}$.

Part c) If the net force and mass are doubled, the acceleration will $\boxed{\text{remain constant at } 10 \text{ m/s}^2}$.

Part d) If the net force is doubled and the mass is halved, the acceleration will $\boxed{\text{quadruple to } 40 \text{ m/s}^2}$.

Part e) If the net force is halved and the mass remains constant, the acceleration will be $\boxed{\text{cut in half to } 5 \text{ m/s}^2}$.

Part f) If the mass is doubled while the net force remains constant, the acceleration will be $\boxed{\text{cut in half to } 5 \text{ m/s}^2}$.

Part g) If the net force and mass are halved, the acceleration will $\boxed{\text{remain constant at } 10 \text{ m/s}^2}$.

Part h) If the net force is halved and the mass is doubled, the acceleration will go $\boxed{\text{down by a factor of four to } 2.5 \text{ m/s}^2}$.

REFLECT

When performing proportional reasoning, first determine what remains constant in each case. This makes it easier to see how a change in one parameter affects the remaining parameters.

4.51

SET UP

A wrestler has a mass of 120 kg. His weight is equal to the magnitude of the force of gravity on him, which is equal to his mass multiplied by the acceleration due to gravity.

SOLVE

$$W = mg = (120 \text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right) = \boxed{1176 \text{ N}}$$

REFLECT

This is a weight of around 265 lb.

4.52

SET UP

A wrestler has a mass of 120 kg. Since mass is an intrinsic property of an object, the wrestler will still have a mass of 120 kg on the Moon. His weight, however, will change since the gravity of the Moon is less than the gravity of Earth.

SOLVE

Part a) His mass on the moon is still $\boxed{120 \text{ kg}}$.

Part b) His weight on the moon is $W_{\text{Moon}} = mg_{\text{Moon}} = (120 \text{ kg})\left(1.62\dfrac{\text{m}}{\text{s}^2}\right) = \boxed{194 \text{ N}}$.

REFLECT

Since g on the Moon is about one-sixth the value on Earth, the wrestler on the Moon will weigh one-sixth what he does on Earth.

4.53

SET UP

A tuna has a mass of 250 kg. Its weight is equal to the magnitude of the force of gravity on it, which is equal to the fish's mass multiplied by the acceleration due to gravity.

SOLVE

$$W = mg = (250 \text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right) = \boxed{2450 \text{ N}}$$

REFLECT

This is a weight of 550 lb.

4.54

SET UP

Mass is an intrinsic property of an object, so an object has the same mass regardless of its location. The acceleration due to gravity decreases with height. Mexico City is at a higher altitude than Los Angeles, which means your weight is greater in Los Angeles.

SOLVE

Part a) Your mass in Los Angeles will be the $\boxed{\text{same}}$ as measured in Mexico City.

Part b) $\boxed{\text{Los Angeles}}$.

REFLECT

Mass and weight are not interchangeable. Weight is the force due to gravity on an object.

4.55

SET UP

An astronaut has a mass of 80 kg. The surface gravity on Mars is 38% that on Earth. The weight of the astronaut on Mars is equal to the magnitude of the force of gravity on it, which is equal to the astronaut's mass multiplied by the acceleration due to gravity on Mars.

SOLVE

$$W_{\text{Mars}} = mg_{\text{Mars}} = m(0.38g_{\text{Earth}}) = (0.38)(80 \text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right) = \boxed{298 \text{ N}}.$$

REFLECT

Since the surface gravity on Mars is 38% that on Earth, the weight of the astronaut on Mars (298 N) will be 38% that on Earth (784 N).

4.56

SET UP

A steel cable is lowering a heavy crate straight down at a constant speed. The forces acting on the crate are the tension due to the cable and gravity—the tension acts straight up, while the force due to gravity acts straight down. Because the crate is moving at a constant speed, these forces must be equal in magnitude.

SOLVE

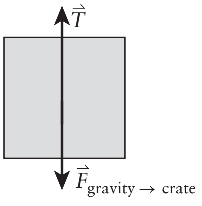


Figure 4-4 Problem 56

REFLECT

An object traveling at a constant speed has an acceleration of zero, which means the net force acting on the object is zero.

4.57

SET UP

A person is pushing a box across a smooth floor at a steadily increasing speed, which means the box is accelerating along that direction. The force due to the man on the box acts horizontally, while the normal force and the force due to gravity act vertically.

SOLVE

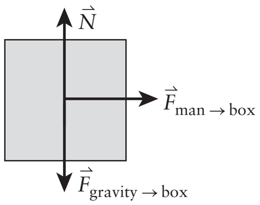


Figure 4-5 Problem 57

REFLECT

The normal force and force due to gravity are equal in magnitude because the box is not accelerating in the vertical direction.

4.58

SET UP

A bicycle is rolling down a hill. The forces acting on the bike are the normal force, gravity, and air resistance. The normal force acts perpendicular to the hill; air resistance acts antiparallel to the motion of the bicycle; and the force due to gravity acts straight down.

SOLVE

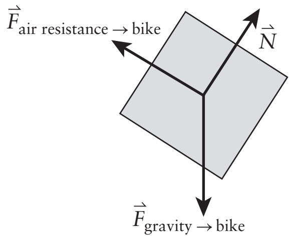


Figure 4-6 Problem 58

REFLECT

As the bicycle is accelerating down the hill, the magnitude of the air resistance will increase because it is dependent on the speed.

4.59

SET UP

A tugboat uses its winch to pick up a 200-kg sailboat. The forces acting on the sailboat are the force of the winch on the boat, drag, and the force of gravity. The force due to the winch acts straight up with a magnitude of 4500 N. The force due to gravity points straight down and has a magnitude equal to the mass of the boat multiplied by g. The drag force acts opposite to the boat's motion with a magnitude of 2000 N. Because the winch is pulling the boat straight up, the drag force acts straight down. We can use Newton's second law to calculate the acceleration of the sailboat. We'll take up to be the positive y direction.

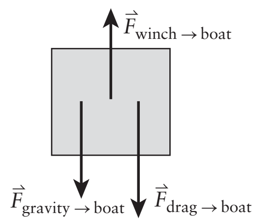


Figure 4-7 Problem 59

SOLVE

$$F_{\text{gravity}} = mg = (200\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right) = 1960\text{ N}$$

$$\sum F_y = F_{\text{winch}} - F_{\text{drag}} - F_{\text{gravity}} = ma_y$$

$$a_y = \frac{F_{\text{winch}} - F_{\text{drag}} - F_{\text{gravity}}}{m} = \frac{(4500\text{ N}) - (2000\text{ N}) - (1960\text{ N})}{200\text{ kg}} = \frac{540\text{ N}}{200\text{ kg}} = \boxed{2.7\frac{\text{m}}{\text{s}^2}}$$

The sailboat is moving upward at an acceleration of 2.7 m/s^2.

REFLECT

If you are unsure of the direction in which a force acts, just pick one to get started. If you are wrong, your answer will end up with a negative sign, which means the force acts in the opposite direction.

4.60

SET UP

Two forces are applied to a block of mass $M = 14$ kg sitting on a horizontal, frictionless surface. One force $\vec{F}_1$ has a magnitude of 10 N and points at an angle $\theta_1 = 37$ degrees. We need to determine the x and y components of the second force such that the block has an x acceleration of $+0.5\ \text{m/s}^2$ and a y acceleration of 0. We can use Newton's second law to calculate the components of $\vec{F}_2$. Once we have the components, we can calculate the magnitude and direction of the second force in order to draw it on the figure.

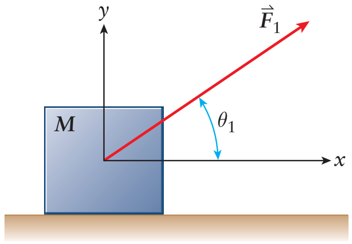


Figure 4-8 Problem 60

SOLVE

Part a)

$$\sum F_x = F_{1,x} + F_{2,x} = F_1\cos(\theta_1) + F_{2,x} = Ma_x$$

$$F_{2,x} = Ma_x - F_1\cos(\theta_1) = (14\text{ kg})\left(0.5\frac{\text{m}}{\text{s}^2}\right) - (10\text{ N})\cos(37°) = \boxed{-0.99\text{ N}}$$

$$\sum F_y = F_{1,y} + F_{2,y} = F_1\sin(\theta_1) + F_{2,y} = Ma_y = 0$$

$$F_{2,y} = -F_1\sin(\theta_1) = -(10\text{ N})\sin(37°) = \boxed{-6.0\text{ N}}$$

Part b)

$$F_2 = \sqrt{F_{2,x}^2 + F_{2,y}^2} = \sqrt{(-0.99\text{ N})^2 + (-6.0\text{ N})^2} = 6.1\text{ N}$$

$$\theta_2 = \arctan\left(\frac{F_{2,y}}{F_{2,x}}\right) = \arctan\left(\frac{-6.0\text{ N}}{-0.99\text{ N}}\right) = 260°$$

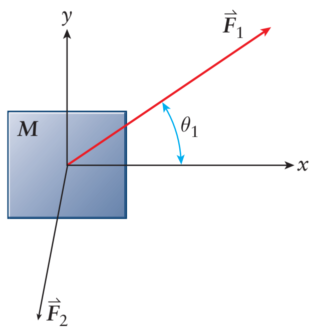


Figure 4-9 Problem 60

REFLECT

Since there are numerous solutions when taking the arctangent, use the components of the vector to determine the desired angle.

4.61

SET UP

Three rugby players are pulling horizontally on ropes attached to a stationary box. Player 1 pulls with a force of 100 N at an angle of -60 degrees. Player 2 pulls with a force of 200 N at an angle of $+37$ degrees. We can use Newton's second law to calculate the components of Player 3's force. Since the box is at rest, the acceleration is equal to zero. Once we know the components of Player 3's force, we can calculate its magnitude and direction and draw it on the figure.

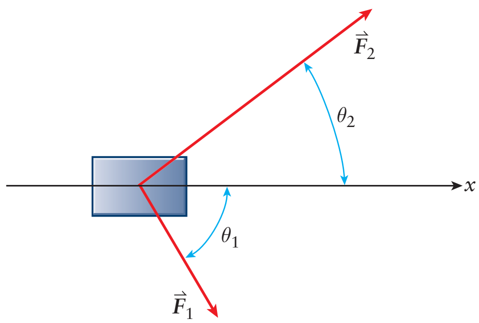


Figure 4-10 Problem 61

Player 3's rope breaks, which means $F_3 = 0$ N, and Player 2 compensates for this by pulling with a force of only 150 N. The direction of the box's resulting acceleration is equal to the direction of the net force due to Players 1 and 2. We can calculate the mass of the box from Newton's second law.

SOLVE

Part a)

$$\sum F_x = F_{1,x} + F_{2,x} + F_{3,x} = F_1\cos(\theta_1) + F_2\cos(\theta_2) + F_{3,x} = ma_x = 0$$

$$F_{3,x} = -F_1\cos(\theta_1) - F_2\cos(\theta_2) = -(100\text{ N})\cos(60°) - (200\text{ N})\cos(37°) = \boxed{-210\text{ N}}$$

$$\sum F_y = -F_{1,y} + F_{2,y} + F_{3,y} = -F_1\sin(\theta_1) + F_2\sin(\theta_2) + F_{3,y} = ma_y = 0$$

$$F_{3,y} = F_1\sin(\theta_1) - F_2\sin(\theta_2) = (100\text{ N})\sin(60°) - (200\text{ N})\sin(37°) = \boxed{-34\text{ N}}$$

Part b)

$$F_3 = \sqrt{F_{3,x}^2 + F_{3,y}^2} = \sqrt{(-210\text{ N})^2 + (-34\text{ N})^2} = 213\text{ N}$$

$$\theta_3 = \arctan\left(\frac{F_{3,y}}{F_{3,x}}\right) = \arctan\left(\frac{-34\text{ N}}{-210\text{ N}}\right) = 189°$$

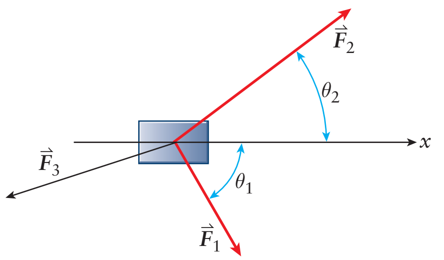


Figure 4-11 Problem 61

Part c)

$$\sum F_x = F_{1,x} + F_{2,x} = F_1\cos(\theta_1) + F_2\cos(\theta_2) = ma_x$$

$$F_1\cos(\theta_1) + F_2\cos(\theta_2) = (100\text{ N})\cos(60°) + (150\text{ N})\cos(37°) = 170\text{ N} = ma_x$$

$$\sum F_\perp = -F_{1,\perp} + F_{2,\perp} = -F_1\sin(\theta_1) + F_2\sin(\theta_2) = ma_\perp$$

$$\sum F_y = -F_{1,y} + F_{2,y} = -F_1\sin(\theta_1) + F_2\sin(\theta_2) = ma_y$$

$$-F_1\sin(\theta_1) + F_2\sin(\theta_2) = -(100\text{ N})\sin(60°) + (150\text{ N})\sin(37°) = ma_y$$

$$\theta = \arctan\left(\frac{3.7\text{ N}}{170\text{ N}}\right) = \boxed{1.2°}$$

Part d)

$$a_x = a\cos(\theta) = \left(10\frac{\text{m}}{\text{s}^2}\right)\cos(1.2°) = 10\frac{\text{m}}{\text{s}^2}$$

$$m = \frac{170\text{ N}}{a_x} = \frac{170\text{ N}}{10\frac{\text{m}}{\text{s}^2}} = \boxed{17\text{ kg}}$$

REFLECT

Players 1 and 2 pull toward $+x$, so Player 3 must pull toward $-x$ if the box is to remain stationary. If Player 3 stops pulling, the box must accelerate toward $+x$.

4.62

SET UP

Three forces that act on a 2-kg object are shown, and we need to determine the magnitudes of two of them. Force 1 has a magnitude of 1.0 N and acts at an angle of 40 degrees. Force 2 acts at an angle of -60 degrees, and force 3 acts at angle of 20 degrees. We are told the acceleration of the object is 1.5 m/s^2 in the $+x$ direction. This means the acceleration in the y direction is equal to zero. We can use Newton's second law in component form in order to calculate the magnitudes of forces 2 and 3. Since the acceleration in the y direction is zero, it will be easier to start with this equation in order to solve the algebra.

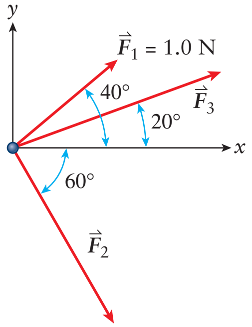


Figure 4-12 Problem 62

SOLVE

$$\sum F_y = F_{1,y} - F_{2,y} + F_{3,y} = 0$$

$$F_1\sin(40°) - F_2\sin(60°) + F_3\sin(20°) = 0$$

$$F_3 = \frac{F_2\sin(60°) - F_1\sin(40°)}{\sin(20°)}$$

$$\sum F_x = F_{1,x} + F_{2,x} + F_{3,x} = ma_x$$

$$F_1\cos(40°) + F_2\cos(60°) + F_3\cos(20°) = (2\text{ kg})\left(1.5\frac{\text{m}}{\text{s}^2}\right) = 3\text{ N}$$

$$F_2\cos(60°) + \left(\frac{F_2\sin(60°) - F_1\sin(40°)}{\sin(20°)}\right)\cos(20°) = (3\text{ N}) - F_1\cos(40°)$$

$$F_2\cos(60°) + \left(\frac{F_2\sin(60°) - (1.0\text{ N})\sin(40°)}{\sin(20°)}\right)\cos(20°) = (3\text{ N}) - (1.0\text{ N})\cos(40°)$$

$$\boxed{F_2 = 1.4\text{ N}}$$

$$F_3 = \frac{F_2\sin(60°) - F_1\sin(40°)}{\sin(20°)} = \frac{(1.4\text{ N})\sin(60°) - (1.0\text{ N})\sin(40°)}{\sin(20°)} = \boxed{1.6\text{ N}}$$

REFLECT

It is a good idea to check your answers by plugging them back in:

$$(1.0\text{ N})\sin(40°) - (1.4\text{ N})\sin(60°) + (1.6\text{ N})\sin(20°) \stackrel{?}{=} 0$$

$$(0.64\text{ N}) - (1.2\text{ N}) + (0.56\text{ N}) = 0 \quad ✓$$

$$(1.0\text{ N})\cos(40°) + (1.4\text{ N})\cos(60°) + (1.6\text{ N})\cos(20°) \stackrel{?}{=} 3\text{ N}$$

$$(0.766\text{ N}) + (0.695\text{ N}) + (1.54\text{ N}) = 3\text{ N} \quad ✓$$

4.63

SET UP

Box A is connected to box B by a rope that is draped over a pulley. Box A weighs 80 N. We are asked to find the force the table exerts on box A for different weights of box B. The forces acting on box A are the tension in the rope, the force due to gravity, and the force the table exerts on box A (also known as the normal force). The tension in the rope is equal to the weight of box B when box B is at rest. If the weight of box B is larger than the weight of box A, both boxes should start to move—box A moves up, while box B should move down.

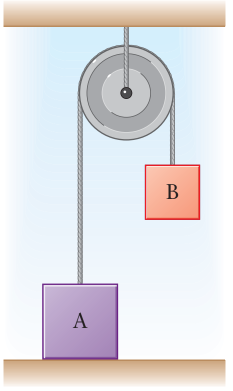


Figure 4-13 Problem 63

SOLVE

Free-body diagram of box A:

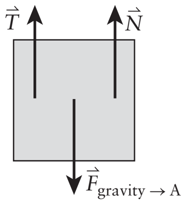


Figure 4-14 Problem 63

If box A remains at rest, then

$$\sum F_{\text{A},\,y} = T + N - F_{\text{gravity}\to\text{A}} = ma_y = 0$$

$$N = F_{\text{gravity}\to\text{A}} - T$$

Part a)
If box B weighs 35 N, then the tension $T = 35$ N, since box B remains at rest.
$N = F_{\text{gravity}\to\text{A}} = (80\text{ N}) - (35\text{ N}) = \boxed{45\text{ N}}$ and $\boxed{\text{points up}}$.

Part b)
If box B weight 70 N, then the tension $T = 70$ N, since box B remains at rest.
$N = F_{\text{gravity}\to\text{A}} = (80\text{ N}) - (70\text{ N}) = \boxed{10\text{ N}}$ and $\boxed{\text{points up}}$.

Part c) If box B weighs more than box A, then we would expect box B to fall down and lift box A off the table. Since box A is no longer touching the table, the normal force goes to $\boxed{0\text{ N}}$.

REFLECT

Remember that the magnitude of a vector can never be negative.

4.64

SET UP

Two ropes are attached to an object of mass M. Rope 1 applies a force with magnitude F_1 at an angle of 30 degrees. We need to determine the magnitude and direction of rope 2's force, $\vec{F}_2$. We are told that the box accelerates only in the x direction at a rate of 7 m/s^2, which means the acceleration in the y direction is equal to 0. We can use Newton's second law in component form in order to calculate the components and, therefore, the magnitude and direction of force 2.

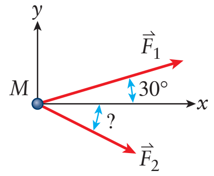


Figure 4-15 Problem 64

SOLVE

Part a)

$$\sum F_x = F_{1,x} + F_{2,x} = F_1\cos(30°) + F_{2,x} = M\left(7\frac{\text{m}}{\text{s}^2}\right)$$

$$F_{2,x} = 7M - F_1\cos(30°)$$

$$\sum F_y = F_{1,y} + F_{2,y} = F_1\sin(30°) + F_{2,y} = 0$$

$$F_{2,y} = -F_1\sin(30°)$$

$$F_2 = \sqrt{F_{2,x}^2 + F_{2,y}^2} = \sqrt{(7M - F_1\cos(30°))^2 + (-F_1\sin(30°))^2}$$

$$= \sqrt{49M^2 - 14MF_1\cos(30°) + F_1^2\cos^2(30°) + F_1^2\sin^2(30°)}$$

$$= \boxed{\sqrt{49M^2 - 14MF_1\cos(30°) + F_1^2}}$$

Part b)

$$F_2 = \sqrt{49(3\text{ kg})^2 - 14(3\text{ kg})(20\text{ N})\cos(30°) + (20\text{ N})^2} = 11\text{ N}$$

$$F_{2,x} = 7M - F_1\cos(30°) = 7(3) - (20)\cos(30°) = 3.7\text{ N}$$

$$F_{2,y} = -F_1\sin(30°) = -(20\text{ N})\sin(30°) = -(10\text{ N})$$

$$\theta_2 = \arctan\left(\frac{F_{2,y}}{F_{2,x}}\right) = \arctan\left(\frac{-10\text{ N}}{3.7\text{ N}}\right) = -70° = 290°$$

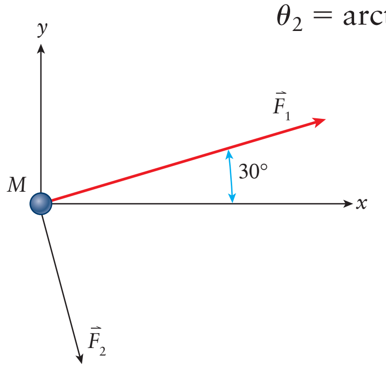


Figure 4-16 Problem 64

REFLECT

Since there is no acceleration in the y direction, the y components of the two forces *must* be equal and opposite. The magnitude of force 2 is smaller than force 1, which means more of it must lie along the y-axis in order to satisfy $a_y = 0$.

4.65

SET UP

Two ropes support a 100-kg streetlight. One rope pulls up and to the left at an angle of 40 degrees above the horizontal with a tension T_1. The other rope pulls up and to the right at an angle of 40 degrees above the horizontal with a tension T_2. The forces acting on the streetlight are the tension in the first rope, the tension in the second rope, and the force of gravity pointing straight down. The streetlight is at rest, which means its acceleration is equal to zero in all directions. We can solve for the tension in each rope by solving Newton's second law in component form. The tension in each rope is equal to the magnitude T_1 or T_2.

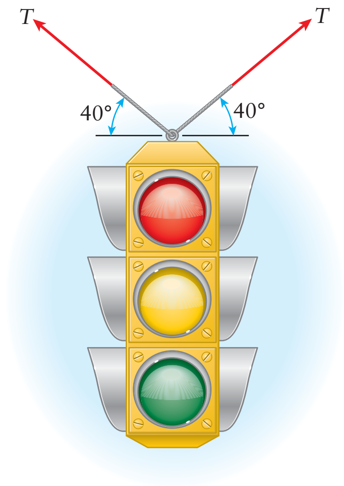


Figure 4-17 Problem 65

SOLVE

$$\sum F_x = T_{1,x} + T_{2,x} = -T_1\cos(40°) + T_2\cos(40°) = 0$$

$$T_1 = T_2 \equiv T$$

$$\sum F_y = T_{1,y} + T_{2,y} + F_{g,y} = T_1\sin(40°) + T_2\sin(40°) - mg = 0$$

$$2T\sin(40°) = mg$$

$$T = \frac{mg}{2\sin(40°)} = \frac{(100\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)}{2\sin(40°)} = \boxed{762\text{ N}}$$

REFLECT

Because the ropes are symmetric, they must have the same tension.

4.66

SET UP

Two ropes support a 200-N sign. One rope pulls up and to the left at an angle of 45 degrees above the horizontal with a tension T_L. The other rope pulls up and to the right at an angle of 30 degrees above the horizontal with a tension T_R. The sign is at rest, which means its acceleration is equal to zero in all directions. The forces acting on the sign are the tension in the left rope, the tension in the right rope, and the force of gravity pointing straight down. We can solve for the tension in each rope by solving Newton's second law in component form. The tension in each rope is equal to the magnitude T_L or T_R.

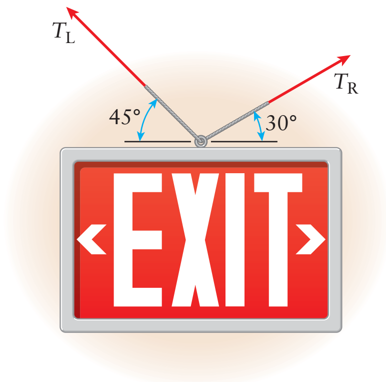


Figure 4-18 Problem 66

SOLVE

$$\sum F_x = T_{L,x} + T_{R,x} = -T_L\cos(45°) + T_R\cos(30°) = 0$$

$$T_L = T_R\frac{\cos(30°)}{\cos(45°)} = T_R\sqrt{\frac{3}{2}}$$

$$\sum F_y = T_{L,y} + T_{R,y} + F_{g,y} = T_L\sin(45°) + T_R\sin(30°) - (200\ \text{N}) = 0$$

$$T_L\sin(45°) + T_R\sin(30°) = (200\ \text{N})$$

$$\left(T_R\sqrt{\frac{3}{2}}\right)\left(\frac{\sqrt{2}}{2}\right) + \frac{T_R}{2} = T_R\left(\frac{\sqrt{3}}{2}\right) + \frac{T_R}{2} = T_R\left(\frac{1+\sqrt{3}}{2}\right) = 200\ \text{N}$$

$$T_R = \frac{2(200\ \text{N})}{1+\sqrt{3}} = \boxed{146\ \text{N}}$$

$$T_L = T_R\sqrt{\frac{3}{2}} = (146\ \text{N})\sqrt{\frac{3}{2}} = 178\ \text{N}$$

REFLECT

It makes sense that T_L is larger than T_R since the left rope is at a steeper angle than the right one.

4.67

SET UP

A telephone wire is stretched between two poles that are 50 m apart. A 0.50-kg bird lands on the wire midway between the poles and the wire sags 0.15 m down. In order to calculate the resulting tension in the wire, we first need to calculate the angle the sagging wire makes with the horizontal. The forces acting on the bird are the tension due to the wire pulling to one or the either side and the force of gravity pointing straight down. The bird is at rest, which means its acceleration is equal to zero in all directions. We can solve for the tension in the wire by solving Newton's second law in component form.

SOLVE

Free-body diagram of the bird:

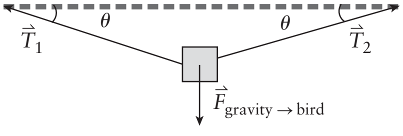


Figure 4-19 Problem 67

Finding theta:

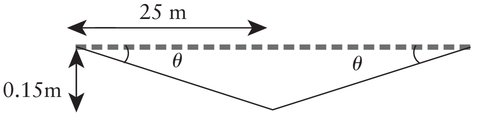


Figure 4-20 Problem 67

$$\sin(\theta) = \frac{0.15\ \text{m}}{25\ \text{m}}, \quad \text{so} \quad \theta = \arcsin\left(\frac{0.15\ \text{m}}{25\ \text{m}}\right) = 0.006\ \ \text{rad} = 0.34°$$

Solving for the tension:

$$\sum F_x = T_{1,x} + T_{2,x} = -T_1\cos(\theta) + T_2\cos(\theta) = 0$$

$$T_1 = T_2 \equiv T$$

$$\sum F_y = T_{1,y} + T_{2,y} + F_{g,y} = T_1\sin(\theta) + T_2\sin(\theta) - mg = 0$$

$$2T\sin(\theta) = mg$$

$$T = \frac{mg}{2\sin(0.34°)} = \frac{(0.5\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)}{2\sin(0.34°)} = \boxed{408\text{ N}}$$

REFLECT

The tension in the wire is so large because the angle the wire sags is so small. We could have also used the small angle approximation for sine: $\sin(\theta) \approx \theta$. This only works if theta is given in radians.

4.68

SET UP

A ball has a weight of 40 N and is resting on a floor. The forces acting on the ball are the force due to gravity and the normal force, which is the contact force of the floor on the ball. The normal force on the ball is the third law partner to the force of the ball on the floor. We can apply Newton's second law to the ball to calculate the magnitude of the normal force. Since the normal force on the ball points upward, the contact force of the ball on the floor will point downward.

SOLVE

Free-body diagram of ball:

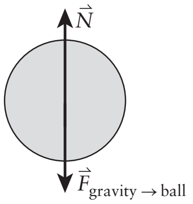


Figure 4-21 Problem 68

Newton's second law for the ball:

$$\sum F_y = N - F_{\text{gravity}\to\text{ball}} = ma_y = 0$$

$$N = F_{\text{gravity}\to\text{ball}} = 40\text{ N}$$

The normal force (that is, the force of the floor on the ball) is the third law partner to the force of the ball on the floor. Therefore, the force of the ball on the floor has a magnitude of $\boxed{\text{40 N and points downward}}$.

REFLECT

Although this force has the same magnitude as the weight of the ball, the two forces (the force of the ball on the floor and the weight) are not third law pairs. The weight is the gravitational force of the Earth on the ball, so its third law partner is the gravitational force of the ball on the Earth.

4.69

SET UP

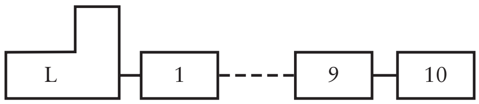


Figure 4-22 Problem 69

A train is made up of a locomotive and 10 identical freight cars, each of mass M. The magnitude of the train's acceleration is 2 m/s^2. The magnitude of the force between the locomotive and the first car is 100,000 N. If we treat all 10 freight cars as the "train," the force between the locomotive and the "train" is equal to the force between the locomotive and the first car. We can draw a free-body diagram of the "train" in order to calculate the mass of one of the cars. We can then draw the free-body diagram of the tenth car and apply Newton's second law in order to calculate the magnitude of the force between the ninth and tenth cars. We'll define positive x as pointing to the right in the above figure.

SOLVE

Free-body diagram of the "train":

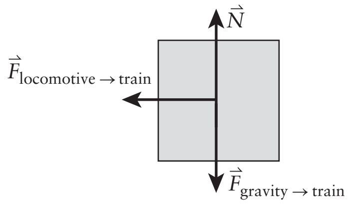


Figure 4-23 Problem 69

Newton's second law for the "train":

$$\sum F_x = -F_{\text{locomotive}\to\text{train}} = (10M)a_x$$

$$M = \frac{-F_{\text{locomotive}\to\text{train}}}{10a_x}$$

Free-body diagram of car 10:

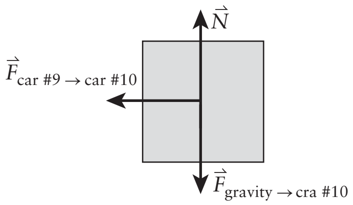


Figure 4-24 Problem 69

Newton's second law for car 10:

$$\sum F_x = -F_{\text{car 9}\to\text{car 10}} = Ma_x = \left(\frac{-F_{\text{locomotive}\to\text{train}}}{10a_x}\right)a_x$$

$$F_{\text{car 9}\to\text{car 10}} = \frac{F_{\text{locomotive}\to\text{train}}}{10} = \frac{100{,}000 \text{ N}}{10} = \boxed{10{,}000 \text{ N}}$$

REFLECT

Rather than treating all 10 cars as one object called the "train," we could have drawn 10 different free-body diagrams—one for each freight car. The force of car 1 on car 2 is equal in magnitude and opposite in direction to the force of car 2 on car 1. Applying Newton's second and third laws for each car in the series will give you the same answer as above.

4.70

SET UP

A train is made up of a locomotive and 10 identical freight cars, each of mass M. The magnitude of the train's acceleration is 2 m/s^2. We can apply Newton's second law in order to calculate the magnitude of the force between the third and fourth cars and the magnitude of the force between the seventh and eighth cars. The third car has seven train cars behind it, so it is effectively carrying a car of mass $7M$. The seventh car has three train cars behind it, so it is effectively carrying a car of mass $3M$. Since the train cars are all attached and moving together, the acceleration is the same in both cases.

SOLVE

Force between cars 3 and 4:

$$\sum F_x = F_{\text{car } 3\to\text{car } 4} = (7M)a_x$$

Force between cars 7 and 8:

$$\sum F_x = F_{\text{car } 7\to\text{car } 8} = (3M)a_x$$

Ratio between the two forces:

$$\frac{F_{\text{car } 3\to\text{car } 4}}{F_{\text{car } 7\to\text{car } 8}} = \frac{(3M)a_x}{(7M)a_x} = \boxed{\frac{3}{7}}$$

REFLECT

It makes sense that the force of car 3 on car 4 is going to be larger than the force of car 7 on car 8. Treating the trailing train cars as a single object allows us to treat all of the other contact forces as internal to the system and thus irrelevant to our Newton's second law calculations.

4.71

SET UP

The net force on the apple is related to its acceleration. Since the apple is at rest, its acceleration is zero, which means the net force on the apple is also zero.

SOLVE

The net force on an apple at rest is $\boxed{0\text{ N}}$.

REFLECT

Newton's second law relates the *net* force to the acceleration of the object.

4.72

SET UP

A 0.01-kg block is attached to a 2-kg block by a rope. A student holds the 2-kg block and lets the 0.01-kg block hang below it. He then releases the 2-kg block and both blocks undergo free fall. The forces acting on the 0.01-kg block are the tension from the rope pulling upward and the force of gravity acting downward. We can use Newton's second law to solve for the magnitude of the tension since we know the acceleration of the block is equal to g downward. We'll take up to point toward positive y.

SOLVE

Free-body diagram for the 0.01-kg block:

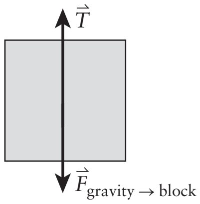


Figure 4-25 Problem 72

Newton's second law for the 0.01-kg block:

$$\sum F_y = T - F_{\text{gravity}\to\text{block}} = ma_y$$

$$T = F_{\text{gravity}\to\text{block}} + ma_y = mg + m(-g) = \boxed{0}$$

REFLECT

Since the two blocks are falling with the same acceleration, the tension in the rope must be 0. We will get the same result if you solve Newton's second law for the 2-kg block.

4.73

SET UP

We are shown three forces acting on an object and asked to calculate the acceleration of the object. The acceleration of the object is related to the net force acting on the object through Newton's second law. We can read the components of each force directly from the graph, add them with the correct sign, and solve for the components of the acceleration.

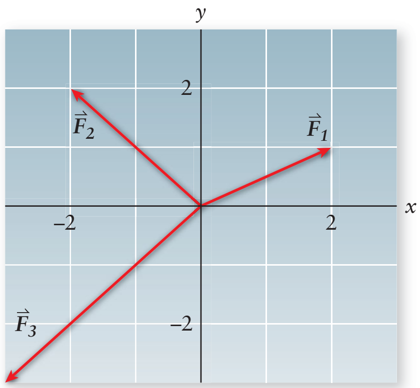


Figure 4-26 Problem 73

SOLVE

x component:

$$\sum F_x = F_{1,x} + F_{2,x} + F_{3,x} = (2\text{ N}) - (2\text{ N}) - (3\text{ N}) = -3\text{ N} = (2\text{ kg})a_x$$

$$a_x = -1.5\frac{\text{m}}{\text{s}^2}$$

y component:

$$\sum F_y = F_{1,y} + F_{2,y} + F_{3,y} = (1\text{ N}) + (2\text{ N}) - (3\text{ N}) = 0\text{ N} = (2\text{ kg})a_y$$

$$a_y = 0$$

$$\boxed{\vec{a} = -\left(1.5\frac{\text{m}}{\text{s}^2}\right)\hat{x}}$$

REFLECT

Remember, when calculating the net force, to split it up into components and solve each equation separately.

4.74

SET UP

A 66-kg person is parachuting and experiences a net downward acceleration of 2.5 m/s^2. The forces acting on the person are the contact force from the parachute on the person straight up and the force of gravity straight down. We can use Newton's second law to calculate the magnitude of the force of the parachute on the person. This is related to the force from the person on the parachute by Newton's third law; these forces will have the same magnitude, but the force from the person on the parachute points downward.

SOLVE

Free-body diagram of the person:

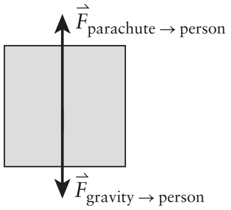


Figure 4-27 Problem 74

Solving Newton's second law in the y direction:

$$\sum F_y = F_{\text{parachute}\to\text{person}} - F_{\text{gravity}\to\text{person}} = ma_y$$

$$F_{\text{parachute}\to\text{person}} = F_{\text{gravity}\to\text{person}} + ma_y = mg + ma_y m(g + a_y)$$

$$= (66\text{ kg})\left(\left(9.8\frac{\text{m}}{\text{s}^2}\right) - \left(2.5\frac{\text{m}}{\text{s}^2}\right)\right) = \boxed{482\text{ N}}$$

REFLECT

We know that the force due to the parachute must be smaller than the force due to gravity since the person has a net downward acceleration.

4.75

SET UP

A 50-kg person is riding a bike up a 10-degree incline. The bike and the person have an acceleration of 1.0 m/s^2 up the incline. The forces acting on the person are the contact force due to the bike and the force of gravity. Although we don't know the magnitude *or* direction of the force due to the bike, we do know that gravity acts straight down. We can set up a coordinate system with axes that are parallel and perpendicular to the incline. We'll call up the plane and out of the plane positive. We can use Newton's second law to calculate the components of the bicycle's contact force and then use them to calculate the magnitude.

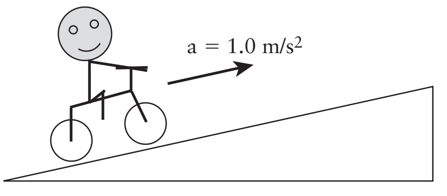


Figure 4-28 Problem 75

SOLVE

Free-body diagram of the rider:

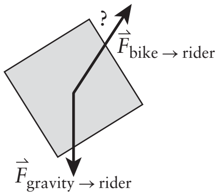


Figure 4-29 Problem 75

Newton's second law for the rider:

$$\sum F_\perp = F_{\text{bike}\to\text{rider},\,\perp} - F_{\text{gravity}\to\text{rider},\,\perp} = ma_\perp = 0$$

$$F_{\text{bike}\to\text{rider},\,\perp} = F_{\text{gravity}\to\text{rider},\,\perp} = mg\cos(10°) = (50\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)\cos(10°) = 483\text{ N}$$

$$\sum F_{\parallel} = F_{\text{bike}\to\text{rider},\,\parallel} - F_{\text{gravity}\to\text{rider},\,\parallel} = ma_{\parallel}$$

$$F_{\text{bike}\to\text{rider},\,\parallel} = F_{\text{gravity}\to\text{rider},\,\parallel} + ma_{\parallel} = mg\sin(10°) + ma_{\parallel} = m(g\sin(10°) + a_{\parallel})$$

$$= (50\text{ kg})\left(\left(9.8\frac{\text{m}}{\text{s}^2}\right)\sin(10°) + \left(1.0\frac{\text{m}}{\text{s}^2}\right)\right) = 135\text{ N}$$

$$F_{\text{bike}\to\text{rider}} = \sqrt{F^2_{\text{bike}\to\text{rider},\,\parallel} + F^2_{\text{bike}\to\text{rider},\,\perp}} = \sqrt{(135\text{ N})^2 + (483\text{ N})^2} = \boxed{502\text{ N}}$$

REFLECT

We could have used our "normal" coordinate system, where up and right point toward positive y and x, respectively. In this case the acceleration (not the force due to gravity) has components along both axes. The answer will be the same, though.

4.76

SET UP

A 65.0-kg person is sitting in a car that accelerates from rest to 100 km/hr in 4.5 s. The contact force of the accelerating car acts on the person. Assuming the acceleration is constant, we can relate this force to change in the vehicle's speed by Newton's second law.

SOLVE

$$\sum F = \frac{\Delta(mv)}{\Delta t} = m\frac{\Delta v}{\Delta t} + v\frac{\Delta m}{\Delta t} = m\frac{\Delta v}{\Delta t} + 0 = (65.0\text{ kg})\frac{\left(100\frac{\text{km}}{\text{hr}} \times \frac{1\text{ hr}}{3600\text{ s}} \times \frac{1000\text{ m}}{1\text{ km}}\right)}{4.5\text{ s}}$$

$$= \boxed{401\text{ N}}$$

REFLECT

This acceleration is approximately $0.63g$.

4.77

SET UP

A 60.0-kg person is riding in a car. While the car uniformly accelerates from 0 to 28 m/s, the person feels a horizontal force of 400 N; this is the only force acting on the person in the horizontal direction. We can use Newton's second law to calculate the acceleration and then use the definition of constant acceleration to calculate the time it takes the car to reach 28 m/s.

SOLVE

Newton's second law for the person:

$$\sum F_x = F_{\text{car}\to\text{person}} = ma_x$$

$$a_x = \frac{F_{\text{car}\to\text{person}}}{m}$$

Calculating the time:

$$a_x = \frac{\Delta v}{\Delta t}$$

$$\Delta t = \frac{\Delta v}{a_x} = \frac{\Delta v}{\left(\frac{F_{\text{car}\to\text{person}}}{m}\right)} = \frac{m\Delta v}{F_{\text{car}\to\text{person}}} = \frac{(60.0\text{ kg})\left(\left(28\frac{\text{m}}{\text{s}}\right) - 0\right)}{400\text{ N}} = \boxed{4.2\text{ s}}$$

REFLECT

The acceleration of the car is constant, so we could use the constant acceleration equations to calculate other quantities, for example, the distance the car travels.

4.78

SET UP

Adam and Ben are pulling on opposite ends of a rope while on a frozen pond. Since they are pulling on the same rope, the net force on each person is the same in magnitude and opposite in direction due to Newton's third law. We can relate the net force to the acceleration by Newton's second law.

SOLVE

$$\left(\sum F\right)_{\text{A}} = \left(\sum F\right)_{\text{B}}$$

$$m_{\text{A}}a_{\text{A}} = m_{\text{B}}a_{\text{B}}$$

$$a_{\text{B}} = \frac{m_{\text{A}}a_{\text{A}}}{m_{\text{B}}} = \frac{(75\text{ kg})\left(1\frac{\text{m}}{\text{s}^2}\right)}{50\text{ kg}} = \boxed{\frac{3\text{ m}}{2\text{ s}^2}}$$

Ben's acceleration points toward the $\boxed{\text{west}}$.

REFLECT

It makes sense that Ben's acceleration is larger than Adam's since Ben's mass is smaller than Adam's.

4.79

SET UP

We are given the position of a 2-kg object as a function of time. We can calculate the net force acting on the object by differentiating the position twice with respect to time to get the acceleration and multiplying this by the mass.

SOLVE

$$a(t) = \frac{d^2x}{dt^2} = \frac{d^2}{dt^2}(2t^2 + 3t - 5) = 8\frac{\text{m}}{\text{s}^2}$$

$$\sum F = ma = (2\text{ kg})\left(8\frac{\text{m}}{\text{s}^2}\right) = \boxed{16\text{ N}}$$

REFLECT

A constant acceleration is caused by a constant net force.

4.80

SET UP

Two blocks (M_1 and M_2) are connected by a string that passes over a pulley. Block $M_2 = 20$ kg and rests on a ramp at an incline of 30 degrees. We'll use two different but related coordinate systems for the two blocks. For block M_1, positive y will point upward. For block M_2, the axes will be parallel and perpendicular to the inclined plane, where down the ramp and out of the ramp are positive. We can draw free-body diagrams and apply Newton's second law in component form for both blocks. The tension acting on each block will be identical in magnitude. Since block M_2 does not leave the plane, the acceleration in the perpendicular direction is always zero. In part (a) both blocks are at rest, which means the acceleration of each block is zero and we can calculate M_1 in this case. In parts (b)–(d), $M_1 = 5$ kg, so we can plug this into Newton's second law and calculate the acceleration of the blocks. The magnitude of the acceleration will be the same for each block because they are tethered together with a string. If the acceleration is constant, we can use the constant acceleration equations to calculate how far M_2 travels starting from rest.

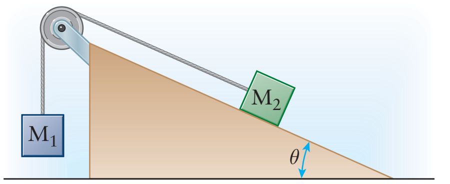


Figure 4-30 Problem 80

SOLVE

Part a)

Free-body diagram of M_2:

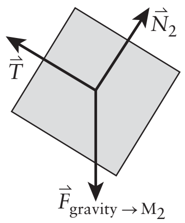


Figure 4-31 Problem 80

Newton's second law for M_2:

$$\sum F_{\parallel} = -T + F_{\text{gravity}\to M_2,\,\parallel} = M_2 a_{\parallel} = 0$$

$$T = F_{\text{gravity}\to M_2,\,\parallel} = M_2 g \sin(30°) = (20\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)\sin(30°) = 98\text{ N}$$

Free-body diagram of M_1:

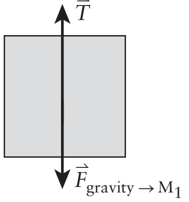


Figure 4-32 Problem 80

Newton's second law for M_1:

$$\sum F_y = T - F_{\text{gravity}\to M_1} = M_1 a_y = 0$$

$$F_{\text{gravity}\to M_1} = M_1 g = T$$

$$M_1 = \frac{T}{g} = \frac{98\ \text{N}}{\left(9.8\frac{\text{m}}{\text{s}^2}\right)} = \boxed{10\ \text{kg}}$$

Part b)
Newton's second law for M_1:

$$\sum F_y = T - F_{\text{gravity}\to M_1} = T - M_1 g = M_1 a_y = M_1 a$$

$$T = M_1(g + a)$$

Newton's second law for M_2:

$$\sum F_{\parallel} = -T + F_{\text{gravity}\to M_2,\parallel} = -(M_1(g+a)) + M_2 g\sin(30°) = M_2 a_{\parallel} = M_2 a$$

$$a = \frac{(M_2\sin(30°) - M_1)g}{M_1 + M_2} = \frac{((20\ \text{kg})\sin(30°) - (5\ \text{kg}))\left(9.8\frac{\text{m}}{\text{s}^2}\right)}{(20\ \text{kg}) + (5\ \text{kg})} = \boxed{1.96\frac{\text{m}}{\text{s}^2}}$$

Part c) M_2 moves $\boxed{\text{down the ramp}}$ since a is positive.

Part d)

$$\Delta x = v_0 t + \frac{1}{2}at^2 = 0 + \frac{1}{2}\left(1.96\frac{\text{m}}{\text{s}^2}\right)(2\ \text{s})^2 = \boxed{3.92\ \text{m}}$$

REFLECT

In part (a) we calculated that $M_1 = 10$ kg if the blocks are in equilibrium. For parts (b)–(d), we are told $M_1 = 5$ kg. Since this is smaller than 10 kg, we expect block M_1 to move up, which means M_2 moves down the ramp.

4.81

SET UP

Two blocks, resting on two different inclined planes, are attached by a string. The block on the left has a mass $M_1 = 6$ kg and rests on an incline at 60 degrees. The block on the right has a mass M_2 and rests on an incline at 25 degrees. We need to find the value of M_2 such that both blocks remain at rest. We'll use two different but related coordinate systems for the two blocks. For both blocks the axes will be parallel and perpendicular to the inclined plane. For block M_1 up the ramp and out of the ramp are positive, while down the ramp and out of the ramp are positive for M_2. We can draw free-body diagrams and apply Newton's second law in component form for both blocks. The tension acting on each block will be identical in magnitude. We are only interested in the forces and acceleration acting parallel to the planes

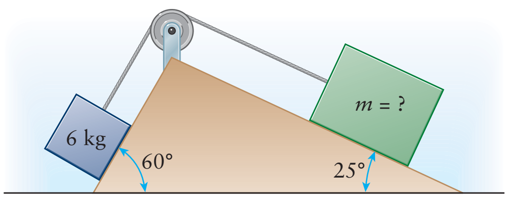


Figure 4-33 Problem 81

because the blocks do not leave the plane; the acceleration in the perpendicular direction is always zero. Both blocks are at rest, which means the acceleration of each block is zero and we can calculate M_2.

SOLVE

Free-body diagram of M_1:

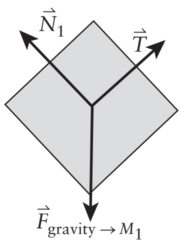


Figure 4-34 Problem 81

Newton's second law for M_1:

$$\sum F_{\parallel} = T - F_{\text{gravity}\to M_1,\parallel} = T - M_1 g\sin(60°) = M_1 a_{\parallel} = 0$$

$$T = M_1 g\sin(60°)$$

Free-body diagram for M_2:

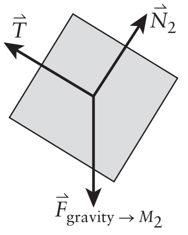


Figure 4-35 Problem 81

Newton's second law for M_2:

$$\sum F_{\parallel} = -T + F_{\text{gravity}\to M_2,\parallel} = -T + M_2 g\sin(25°) = M_2 a_{\parallel} = 0$$

$$-M_1 g\sin(60°) + M_2 g\sin(25°) = 0$$

$$M_2 = \frac{M_1\sin(60°)}{\sin(25°)} = \frac{(6\text{ kg})\sin(60°)}{\sin(25°)} = \boxed{12.3\text{ kg}}$$

REFLECT

The angle on the left is steeper than the angle on the right, so the mass on the left should be smaller than the mass on the right.

4.82

SET UP

A child on a sled (total mass M) starts from rest on a 20-degree incline. We are interested in the time it takes the child and sled to travel 210 m down the slope. We can calculate the acceleration of the sled from Newton's second law. We will use a coordinate system where the axes are

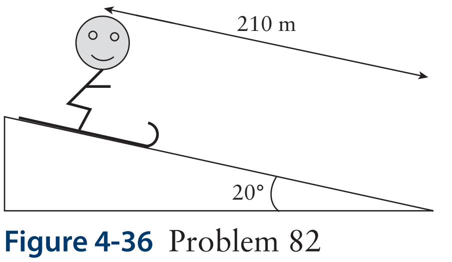


Figure 4-36 Problem 82

parallel and perpendicular to the inclined plane; down the plane and out of the plane will be positive. The forces acting on the sled are the normal force perpendicular to the plane and gravity straight down. We can calculate the acceleration in the direction parallel to the plane and use it to determine the time it takes the child and sled to travel 210 m.

SOLVE

Free-body diagram of the sled:

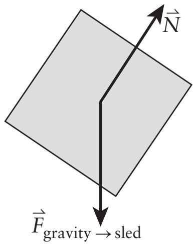


Figure 4-37 Problem 82

Newton's second law for the sled:

$$\sum F_{\parallel} = F_{\text{gravity}\to\text{sled},\parallel} = Mg\sin(20°) = Ma_{\parallel}$$

$$a_{\parallel} = g\sin(20°)$$

Time it takes to travel 210 m:

$$\Delta x = v_0 t + \frac{1}{2}a_{\parallel}t^2 = 0 + \frac{1}{2}a_{\parallel}t^2$$

$$t = \sqrt{\frac{2(\Delta x)}{a_{\parallel}}} = \sqrt{\frac{2(\Delta x)}{g\sin(20°)}} = \sqrt{\frac{2(210\text{ m})}{\left(9.8\frac{\text{m}}{\text{s}^2}\right)\sin(20°)}} = \boxed{11.2\text{ s}}$$

REFLECT

If the incline were steeper, the acceleration would increase and the time it takes for the sled to reach the bottom would decrease, which makes sense.

4.83

SET UP

A car (car 1) is traveling at a speed of 14 m/s when it crashes into a parked car (car 2). Car 1 moves a distance of 0.3 m before coming to a complete stop. The driver is wearing her seat belt, so she remains in her seat during the collision. Assuming the acceleration of the car (and, therefore, the driver) is constant, we can calculate its magnitude and direction from the constant acceleration equations. (We'll assume the car is initially traveling toward positive x.) The only force acting on the driver in the x direction is the contact force due to the seat belt. We can solve for the magnitude of the force of the belt using the acceleration and Newton's second law.

SOLVE

Acceleration of the driver:

$$v^2 - v_0^2 = 2a_x(\Delta x)$$

$$a_x = \frac{v^2 - v_0^2}{2(\Delta x)} = \frac{0 - \left(14\frac{\text{m}}{\text{s}^2}\right)^2}{2(0.3\text{ m})} = -326.7\frac{\text{m}}{\text{s}^2}$$

Free-body diagram of the driver:

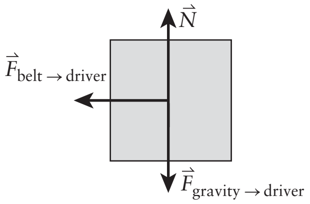


Figure 4-38 Problem 83

Newton's second law for the driver:

$$\sum F_x = -F_{\text{belt}\to\text{driver}} = ma_x$$

$$F_{\text{belt}\to\text{driver}} = -ma_x = -(52\text{ kg})\left(-326.7\frac{\text{m}}{\text{s}^2}\right) = \boxed{16{,}987\text{ N}}$$

REFLECT

This is a very large force acting on the driver, which makes sense since this is a car crash and the car, initially traveling at around 30 mph, comes to a stop in only 30 cm.

4.84

SET UP

You use your car to push your friend's car to the nearest gas station. Let's say that both cars are driving toward positive x. The cars accelerate from rest to 2.0 m/s in 60 s. For simplicity we will assume this acceleration to be constant and that the net force acting in the x direction is equal to the contact force between the two bumpers. Solving Newton's second law will give us the magnitude of this force.

SOLVE

Acceleration of your friend's car:

$$a_x = \frac{\Delta v}{\Delta t} = \frac{\left(2.0\frac{\text{m}}{\text{s}}\right)}{60\text{ s}} = 0.033\frac{\text{m}}{\text{s}^2}$$

Free-body diagram of your friend's car:

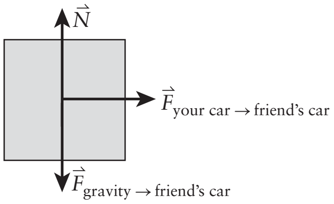


Figure 4-39 Problem 84

Newton's second law for your friend's car:

$$\sum F_x = F_{\text{your car}\rightarrow\text{friend's car}} = ma_x = (1200\text{ kg})\left(0.033\frac{\text{m}}{\text{s}^2}\right) = \boxed{40\text{ N}}$$

REFLECT

This may seem like a small force, but the acceleration of the cars is very small.

4.85

SET UP

A 30-kg dog stands on a scale, and both are placed in an elevator. The elevator begins to move and we are interested in calculating the reading on the scale for various accelerations of the elevator, which are equal to the accelerations of the scale and dog since they travel together as one object. The reading on the scale is equal in magnitude to the normal force acting on the dog. Gravity also acts on the dog. We can use Newton's second law to calculate N as a function of the acceleration. We'll define up to be positive y throughout our calculation.

SOLVE

Free-body diagram of the dog:

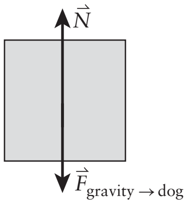


Figure 4-40 Problem 85

$$\sum F_y = N - F_{\text{gravity}\rightarrow\text{dog}} = ma_y$$

$$N = F_{\text{gravity}\rightarrow\text{dog}} + ma_y = mg + ma_y = m(g + a_y)$$

Part a)

$$N = m(g + a_y) = (30\text{ kg})\left(\left(9.8\frac{\text{m}}{\text{s}^2}\right) - \left(3.5\frac{\text{m}}{\text{s}^2}\right)\right) = \boxed{189\text{ N}}$$

Part b)

$$N = m(g + a_y) = (30\text{ kg})\left(\left(9.8\frac{\text{m}}{\text{s}^2}\right) + 0\right) = \boxed{294\text{ N}}$$

Part c)

$$N = m(g + a_y) = (30\text{ kg})\left(\left(9.8\frac{\text{m}}{\text{s}^2}\right) + \left(4.0\frac{\text{m}}{\text{s}^2}\right)\right) = \boxed{414\text{ N}}$$

REFLECT

When an elevator starts to move downward from rest, you feel lighter; when an elevator travels at a constant speed you feel "normal"; and, when an elevator moves upward from rest,

you feel heavier. All of these observations from everyday life correspond to our calculation with respect to the dog and the scale.

4.86

SET UP

The Inclinator is an elevator that travels up the side of the pyramid-shaped Luxor Hotel in Las Vegas. The Inclinator starts at the bottom from rest and accelerates up a slope of 39.0 degrees at rate of 1.25 m/s^2. A person is standing upright (relative to the ground) inside the Inclinator, which means he is at an angle relative to the surface of the pyramid. We will use a coordinate system where the y-axis is perpendicular to the ground and up is considered positive. When the person is standing on flat ground, his weight is 588 N. The effective weight of the person in the Inclinator is equal in magnitude to the normal force of the Inclinator on the person. We can use the y component of Newton's second law to calculate this magnitude. The only other force with a component along this axis is gravity.

SOLVE

Free-body diagram of the rider:

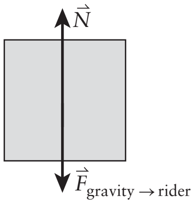


Figure 4-41 Problem 86

Newton's second law for the rider:

$$\sum F_y = N - F_{\text{gravity}\to\text{rider}} = N - mg = ma_y = ma\sin(39°)$$

$$N = mg + ma\sin(39°) = mg\left(1 + \left(\frac{a}{g}\right)\sin(39°)\right) = (588\ \text{N})\left(1 + \left(\frac{1.25\frac{\text{m}}{\text{s}^2}}{9.81\frac{\text{m}}{\text{s}^2}}\right)\sin(39°)\right) = \boxed{635\ \text{N}}$$

REFLECT

It makes sense that the person's effective weight is more than his actual weight since the lift is at an angle.

4.87

SET UP

We are asked to calculate effective weight of a rider in a moving elevator; this is equal to the magnitude of the normal force acting on the rider. We can use Newton's second law to calculate the magnitude of the normal force in terms of the weight of the rider and the acceleration of the elevator. In our coordinate system, positive y will point upward.

SOLVE

Free-body diagram of the rider:

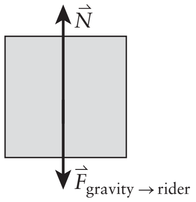


Figure 4-42 Problem 87

Newton's second law for the rider:

$$\sum F_y = N - F_{\text{gravity}\to\text{rider}} = ma_y$$

$$N = F_{\text{gravity}\to\text{rider}} + ma_y = mg + ma_y = m(g + a_y)$$

Starting at rest:

$$N = mg + ma_y = (700\text{ N}) + 0 = \boxed{700\text{ N}}$$

Accelerating upward at 3 m/s^2:

$$N = (71.4\text{ kg})\left(\left(9.8\frac{\text{m}}{\text{s}^2}\right) + \left(3.0\frac{\text{m}}{\text{s}^2}\right)\right) = \boxed{914\text{ N}}$$

Constant speed of 4 m/s:

$$N = mg + ma_y = (700\text{ N}) + 0 = \boxed{700\text{ N}}$$

Accelerating downward at -2 m/s^2:

$$N = (71.4\text{ kg})\left(\left(9.8\frac{\text{m}}{\text{s}^2}\right) - \left(2.0\frac{\text{m}}{\text{s}^2}\right)\right) = \boxed{557\text{ N}}$$

Free fall at $-g$:

$$N = mg + ma_y = mg + m(-g) = \boxed{0}$$

REFLECT

These calculations mesh with our qualitative observations when riding elevators—you feel heavier as the elevator accelerates up and lighter as it accelerates down.

4.88

SET UP

We are given the speed of an object of mass m as a function of its position. The net force acting on the object is equal to its mass multiplied by its acceleration. We can calculate its acceleration by differentiating the speed with respect to time. Since v is a function of x, we will need to use the chain rule to find the acceleration.

SOLVE

Part a)

$$a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = (2bx)(v) = (2bx)(bx^2) = 2b^2x^3$$

$$\sum F = ma = m(2b^2x^3) = \boxed{2mb^2x^3}$$

Part b)

$$\sum F = 2mb^2x^3 = 2(2\text{ kg})\left(\frac{8}{\text{s}\cdot\text{m}}\right)^2(10\text{ m})^3 = \boxed{2.56 \times 10^5\text{ N}}$$

REFLECT

Our expression in part (a) gives the correct SI units for force:

$$\text{kg}\cdot\left(\frac{1}{\text{s}\cdot\text{m}}\right)^2\cdot\text{m}^3 = \frac{\text{kg}\cdot\text{m}}{\text{s}^2} = \text{N}$$

4.89

SET UP

A froghopper has a mass of 12.3 mg and can flex its legs 2.0 mm in order to jump an additional 426 mm to a height of 428 mm above the ground. We are interested in the portion of the jump while the insect is still on the ground. We can assume that the acceleration of the insect is constant during this phase. In order to calculate the acceleration of the insect and the length of time during this phase of the jump we first need to calculate the speed with which the froghopper leaves the ground. While the insect is in the air, it is only under the influence of gravity, so we can use the constant acceleration equations and the height of the jump to calculate the takeoff speed. Once we have this value, we know the froghopper accelerated from rest through a distance of 2.0 mm to reach this takeoff speed. We can directly calculate the time because we are assuming the acceleration is constant. The forces acting on the insect while it is on the ground are the force of the ground on the froghopper pointing up (that is, the normal force) and the force of gravity pointing down. After defining a coordinate system where positive y points upward, we can calculate the magnitude of the normal force using Newton's second law.

SOLVE

Part a)

Takeoff speed:

$$v^2 - v_0^2 = 0^2 - v_0^2 = 2a_y(\Delta y)$$

$$v_0 = \sqrt{-2a_y(\Delta y)} = \sqrt{-2\left(-9.8\frac{\text{m}}{\text{s}^2}\right)(0.426\text{ m})} = 2.89\frac{\text{m}}{\text{s}}$$

Acceleration necessary to attain that speed from rest:

$$v^2 - v_0^2 = 2a_y(\Delta y)$$

$$a_y = \frac{v^2 - v_0^2}{2(\Delta y)} = \frac{\left(2.89\frac{\text{m}}{\text{s}}\right)^2 - 0}{2(0.0020\text{ m})} = \boxed{2087\frac{\text{m}}{\text{s}^2}}$$

Time:

$$a_y = \frac{\Delta v}{\Delta t}$$

$$\Delta t = \frac{\Delta v}{a_y} = \frac{2.89\frac{\text{m}}{\text{s}}}{2087\frac{\text{m}}{\text{s}^2}} = \boxed{0.00138 \text{ s} = 1.38 \text{ ms}}$$

Part b)

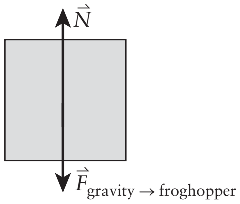


Figure 4-43 Problem 89

Part c)

$$\sum F_y = N - F_{\text{gravity}\to\text{froghopper}} = N - mg = ma_y$$

$$N = mg + ma_y = m(g + a_y) = (12.3 \times 10^{-6} \text{ kg})\left(\left(9.8\frac{\text{m}}{\text{s}^2}\right) + \left(2087\frac{\text{m}}{\text{s}^2}\right)\right)$$

$$= \boxed{0.0258 \text{ N} = 25.8 \text{ mN}}$$

This force is $\boxed{\text{214 times larger}}$ than the froghopper's weight.

REFLECT

A froghopper is around 5mm long, so a height of 426 mm is about 85 times its length.

4.90

SET UP

The maximum tension a fishing line can withstand is referred to as its "strength." A fishing line snaps when a fish is accelerated straight up at 10.2 m/s^2. The forces acting on the fish are the tension pulling upward and the force of gravity pointing downward. We can apply Newton's second law to the fish in order to calculate its mass. We are told the strength of the line is 260 N, which is the tension in the line at the instant it breaks. In our coordinate system, we will consider $+y$ to point up.

SOLVE

Free-body diagram of the fish:

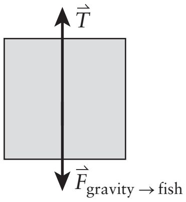


Figure 4-44 Problem 90

Newton's second law for the fish:

$$\sum F_y = T - F_{\text{gravity}\to\text{fish}} = T - mg = ma_y$$

$$m = \frac{T}{g + a_y} = \frac{260\text{ N}}{\left(9.8\frac{\text{m}}{\text{s}^2}\right) + \left(10.2\frac{\text{m}}{\text{s}^2}\right)} = \boxed{13\text{ kg}}$$

REFLECT

A mass of 13 kg is about 28.6 lb, which is reasonable for a decent-sized fish.

4.91

SET UP

A man pulls back a bowstring with a 22-g arrow in place. The tension in the string is 180 N, and the angle the string makes with the bow is 25 degrees. We will call the direction in which the arrow is pointing the positive x direction. We can use the x component of Newton's second law to calculate the acceleration of the arrow after the man lets go of the string. At that point the only forces with a component acting in the x direction are the tension in the string from the top and the tension in the string from the bottom.

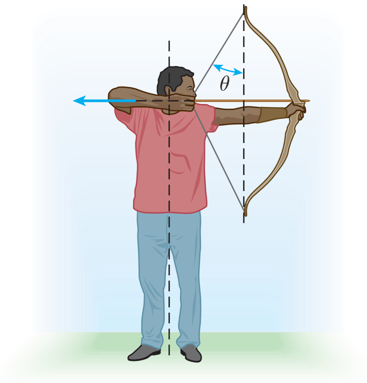


Figure 4-45 Problem 91

SOLVE

Free-body diagram of the arrow:

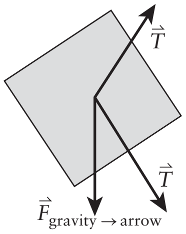


Figure 4-46 Problem 91

Newton's second law for the arrow:

$$\sum F_x = T\sin(25°) + T\sin(25°) = 2T\sin(25°) = ma_x$$

$$a_x = \frac{2T\sin(25°)}{m} = \frac{2(180\text{ N})\sin(25°)}{0.022\text{ kg}} = \boxed{6900\frac{\text{m}}{\text{s}^2}}$$

REFLECT

Although there is only one string, there is tension acting above and below the arrow.

4.92

SET UP

A fuzzy die of mass m is hanging from the ceiling of a car by a string. The car accelerates to the left at a rate of 2.7 m/s^2, and the string makes an angle θ with respect to the vertical. The forces acting on the die are the tension in the string and gravity. We will use a coordinate system where right and up are positive x and y, respectively. Since the string is at an angle, the tension has components along both axes. We can use the

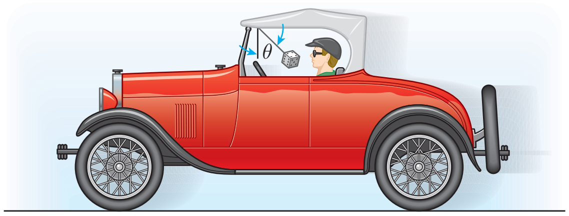


Figure 4-47 Problem 92

two component equations for Newton's second law to solve for the magnitude of the tension, divide the equations, and solve for the angle.

SOLVE

Free-body diagram of the die:

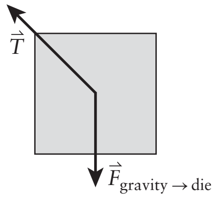


Figure 4-48 Problem 92

Newton's second law for the die:

$$\sum F_y = T\cos(\theta) - F_{\text{gravity}\to\text{die}} = ma_y = 0$$

$$T\cos(\theta) = F_{\text{gravity}\to\text{die}} = mg$$

$$\sum F_x = -T\sin(\theta) = ma_x$$

$$T\sin(\theta) = -ma_x$$

$$\frac{T\sin(\theta)}{T\cos(\theta)} = \frac{-ma_x}{mg}$$

$$\tan(\theta) = \frac{-a_x}{g}$$

$$\theta = \arctan\left(\frac{-a_x}{g}\right) = \arctan\left(\frac{-\left(-2.7\frac{\text{m}}{\text{s}^2}\right)}{\left(9.8\frac{\text{m}}{\text{s}^2}\right)}\right) = \boxed{15°}$$

REFLECT

This is a reasonable angle for the string to make with respect to the vertical. By dividing the equations, we don't need to use the weight, only the accelerations.

4.93

SET UP

Sue and Paul are attached by a rope while climbing a glacier with a 45-degree slope. Suddenly, Sue (m_{Sue} =66 kg) falls into a crevasse and falls 2 m in 10 s from rest. We'll use two different but related coordinate systems for the two people. For Sue, positive y will point upward. For Paul, the axes will be parallel and perpendicular to the inclined plane, where up the ramp and out of the ramp are positive. Tension from the rope pulling up and gravity pulling down are the only forces acting on Sue. Assuming her acceleration is constant, we can use the constant acceleration equations and Newton's second law

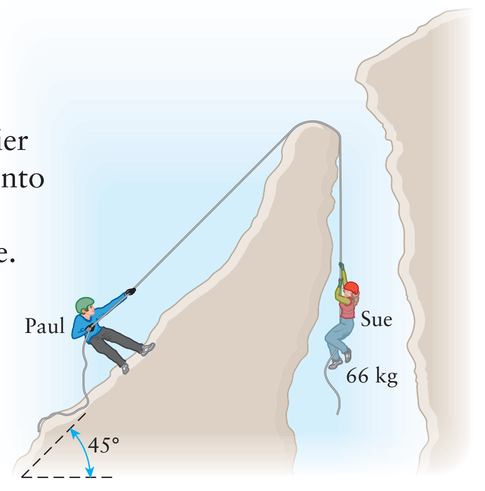


Figure 4-49 Problem 93

to calculate the magnitude of the tension. Since Paul and Sue are tethered to one another, the magnitudes of their accelerations are equal. The tension in the rope and gravity are the only forces acting on Paul that have components that are parallel to the face of the glacier. We can then solve the parallel component of Newton's second law for Paul's mass.

SOLVE

Part a)

Sue's acceleration:

$$\Delta y = v_0 t + \frac{1}{2} a_y t^2 = 0 + \frac{1}{2} a_y t^2$$

$$a_y = \frac{2(\Delta y)}{t^2} = \frac{2(-2\ \mathrm{m})}{(10\ \mathrm{s})^2} = -0.04 \frac{\mathrm{m}}{\mathrm{s}^2}$$

Free-body diagram of Sue:

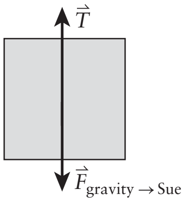


Figure 4-50 Problem 93

Newton's second law for Sue:

$$\sum F_y = T - F_{\text{gravity}\to\text{Sue}} = T - m_{\text{Sue}} g = m_{\text{Sue}} a_y$$

$$T = m_{\text{Sue}}(g + a_y) = (66\ \mathrm{kg})\left(\left(9.8\frac{\mathrm{m}}{\mathrm{s}^2}\right) + \left(-0.04\frac{\mathrm{m}}{\mathrm{s}^2}\right)\right) = \boxed{644\ \mathrm{N}}$$

Part b)

Free-body diagram of Paul:

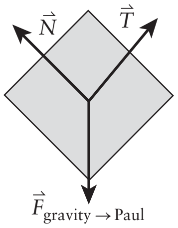


Figure 4-51 Problem 93

Newton's second law for Paul:

$$\sum F_{\parallel} = T - F_{\text{gravity}\to\text{Paul},\,\parallel} = T - m_{\text{Paul}} g \sin(45°) = m_{\text{Paul}} a_{\parallel}$$

$$m_{\text{Paul}} = \frac{T}{a_{\parallel} + g\sin(45°)} = \frac{(644\ \mathrm{N})}{\left(0.04\frac{\mathrm{m}}{\mathrm{s}^2}\right) + \left(9.8\frac{\mathrm{m}}{\mathrm{s}^2}\right)\sin(45°)} = \boxed{92.4\ \mathrm{kg}}$$

REFLECT

The tension in the rope (644 N) is less than Sue's weight (647 N), so there is a net force on Sue acting downward, which means she'll accelerate in that direction. Paul's mass of 92 kg is around 200 lb, which is a reasonable weight for a human being.

4.94

SET UP

A car is traveling at a speed of 28 m/s when it crashes into a bridge abutment. A 45-kg passenger in the car moves a distance of 55 cm while being brought to rest due to an inflated air bag. We'll assume the car is initially traveling toward positive x. The only force acting on the passenger in the x direction is the contact force due to the air bag. We are told to assume that the force stopping the passenger is constant, which means the acceleration is constant. We can calculate its magnitude and direction from the constant acceleration equations. We can solve for the magnitude of the force of the air bag using the acceleration and Newton's second law.

SOLVE

Acceleration of the passenger:

$$v^2 - v_0^2 = 2a_x(\Delta x)$$

$$a_x = \frac{v^2 - v_0^2}{2(\Delta x)} = \frac{0 - \left(28\frac{\text{m}}{\text{s}^2}\right)^2}{2(0.55\ \text{m})} = -712.7\frac{\text{m}}{\text{s}^2}$$

Free-body diagram of the passenger:

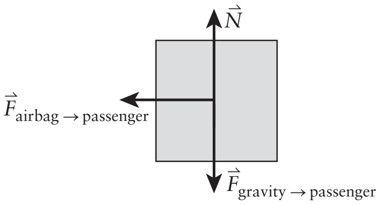


Figure 4-52 Problem 94

Newton's second law for the passenger:

$$\sum F_x = -F_{\text{airbag}\to\text{passenger}} = ma_x$$

$$F_{\text{airbag}\to\text{passenger}} = -ma_x = -(45\ \text{kg})\left(-712.7\frac{\text{m}}{\text{s}^2}\right) = \boxed{32{,}073\ \text{N}}$$

REFLECT

A speed of 28 m/s is around 63 mph and the car comes to a stop in about 0.04 s. Therefore, it's reasonable that there should be large forces involved in stopping the passenger.

4.95

SET UP

A car is traveling at 48 km/hr when it collides with another car. The airbags deploy and the person slows to rest at a rate of 60g. The acceleration is constant, so we can use the definition of average acceleration to calculate the time necessary for the person to come to rest. The forces acting on the passenger are the normal force from the seat, the force of gravity, and the contact force due to the airbag. Let's say the car is initially traveling toward negative x, which means the force due to the airbag on the passenger acts toward positive x. The other two forces act along the y-axis. We can solve Newton's second law in the x direction in order to calculate the magnitude of the airbag's contact force. Airbags are used as a safety device because they increase the length of time over which a force acts as well as increase the area over which that force acts.

SOLVE

Part a)

$$|a| = \frac{|\Delta v|}{\Delta t}$$

$$\Delta t = \frac{|\Delta v|}{|a|} = \frac{\left(48\frac{\text{km}}{\text{hr}} \times \frac{1\text{ hr}}{3600\text{ s}} \times \frac{1000\text{ m}}{1\text{ km}}\right)}{60\left(9.8\frac{\text{m}}{\text{s}^2}\right)} = \boxed{0.023\text{ s} = 23\text{ ms}}$$

Part b)

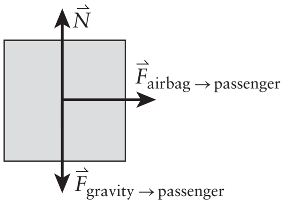


Figure 4-53 Problem 95

Part c)

$$\sum F_x = F_{\text{airbag}\to\text{passenger}} = ma_x = (72\text{ kg})(60)\left(9.8\frac{\text{m}}{\text{s}^2}\right) = \boxed{42{,}000\text{ N}}$$

Part d) The force doesn't injure the person because the airbag spreads out the application of the force and lengthens the time over which it acts (as compared to, say, hitting the steering wheel or windshield).

REFLECT

We did not need to use the fact that the torso comprises 43% of the body's weight.

4.96

SET UP

A 200-kg block is hoisted up using two massless pulleys (see Figure 4-54). Someone pulls downward on the free rope with a force of 1500 N. If the rope is pulled downward, both the small pulley and the block will rise. Because of this, we will consider the small pulley and block to be one object with a mass of $m_{(\text{Block + Pulley})} =$ 200 kg. (Remember, the pulleys are massless.) The forces acting on our system are two equal tension forces from the rope pulling straight up and the force of gravity acting downward. The tension in the rope is equal to 1500 N. Using a coordinate system where positive y points upward, we can solve Newton's second law for the (Block + Pulley) system and calculate its acceleration.

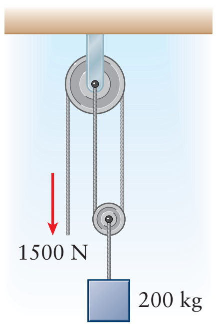


Figure 4-54 Problem 96

SOLVE

Free-body diagram of the block + small pulley:

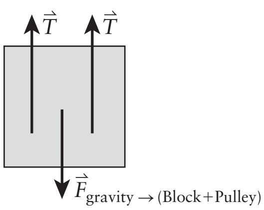


Figure 4-55 Problem 96

Newton's second law for the block + small pulley:

$$\sum F_y = 2T - F_{\text{gravity}\to(\text{Block}+\text{Pulley})} = 2T - m_{(\text{Block}+\text{Pulley})}g = m_{(\text{Block}+\text{Pulley})}a_y$$

$$a_y = \frac{2T - m_{(\text{Block}+\text{Pulley})}g}{m_{(\text{Block}+\text{Pulley})}} = \frac{2(1500\text{ N}) - (200\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)}{200\text{ kg}} = \boxed{5.2\frac{\text{m}}{\text{s}^2}}$$

REFLECT

A positive acceleration corresponds to the block moving upward, which is what we would expect if someone were pulling down on the rope. This problem also shows why pulleys are useful. By wrapping the rope around the pulley, the magnitude of the force pulling upward is double what it would be if the rope were connected directly to the block.

4.97

SET UP

A man sits in a bosun's chair (see figure). Since we are given the combined mass of the man, chair, and bucket ($m = 95.0$ kg), we will treat them together as one object. There are two tension forces upward on this object (one from each end of the rope) and the force of gravity acting downward. Using a coordinate system where positive y points upward, we can solve Newton's second law for the tension in terms of the acceleration. Recall that an object traveling at a constant velocity has an acceleration of zero.

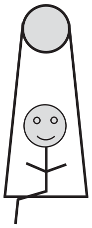


Figure 4-56 Problem 97

SOLVE

Free-body diagram of the (Man + Chair + Bucket):

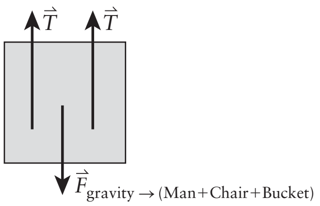


Figure 4-57 Problem 97

Newton's second law for the (Man + Chair + Bucket):

$$\sum F_y = 2T - F_{\text{gravity}\to(\text{Man}+\text{Chair}+\text{Bucket})} = 2T - mg = ma_y$$

$$T = \frac{m(g + a_y)}{2}$$

Part a) At constant speed:

$$T = \frac{(95.0\text{ kg})\left(\left(9.8\frac{\text{m}}{\text{s}^2}\right) + 0\right)}{2} = \boxed{466\text{ N}}$$

Part b) At a constant upward acceleration of 1.5 m/s^2:

$$T = \frac{(95.0\text{ kg})\left(\left(9.8\frac{\text{m}}{\text{s}^2}\right) + \left(1.5\frac{\text{m}}{\text{s}^2}\right)\right)}{2} = \boxed{537\text{ N}}$$

REFLECT

In order to remain stationary or travel at a constant speed, the man needs to pull with a force equal to the weight. The man should pull harder if he wants to accelerate upward, which makes sense.

4.98

SET UP

A 9800-kg cable car is attached to two cables—one below the car and one above the car—and accelerates at a rate of 0.78 m/s^2 up an incline of 30 degrees. Gravity also acts on the cable car. We will use a coordinate system with axes parallel and perpendicular to the slope, where up and out of the slope are considered positive. We are only interested in the difference between the magnitudes of the tensions of the two cables, which we can calculate from the parallel component of Newton's second law.

SOLVE

Free-body diagram of the cable car:

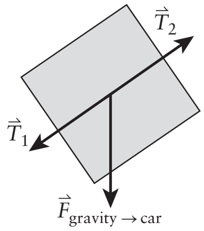


Figure 4-58 Problem 98

Newton's second law for the car:

$$\sum F_{\parallel} = T_2 - T_1 - F_{\text{gravity}\to\text{car},\,\parallel} = T_2 - T_1 - mg\sin(30°) = ma_y$$

$$T_2 - T_1 = m(g\sin(30°) + a_y) = (9800\text{ kg})\left(\left(9.8\frac{\text{m}}{\text{s}^2}\right)\sin(30°) + \left(0.78\frac{\text{m}}{\text{s}^2}\right)\right) = \boxed{55{,}664\text{ N}}$$

REFLECT

The tension in the cable above the car is larger than the tension in the cable below the car, which makes sense since the gravity is pulling the cable car down the ramp.

4.99

SET UP

Two blocks (m_1 = 1.0 kg, m_2 = 0.5 kg) are touching one another on a frictionless surface. An external force of magnitude 7.5 N is applied to block 1. The forces acting on block 1 in the x direction are the external force and the contact force of block 2 on block 1. The only force acting on block 2 in the x direction is the force of block 1 on block 2, which is related by Newton's third law to the force of block 2 on block 1. Because the blocks are touching, they will accelerate at the same rate, a_x. We can solve for a_x in terms of F_{ext} and then solve for $F_{1\to2}$.

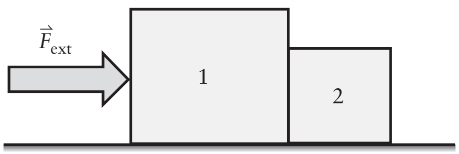


Figure 4-59 Problem 99

SOLVE

Free-body diagram of block 1:

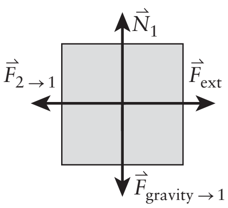


Figure 4-60 Problem 99

Free-body diagram of block 2:

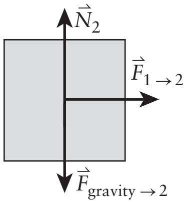


Figure 4-61 Problem 99

Newton's second law:

$$\sum F_{1,x} = F_{\text{ext}} - F_{2\to 1} = m_1 a_x$$

$$\sum F_{2,x} = F_{1\to 2} = m_2 a_x$$

Add the two equations together:

$$F_{\text{ext}} = (m_1 + m_2)a_x$$

$$a_x = \frac{F_{\text{ext}}}{m_1 + m_2}$$

Plugging back in:

$$\sum F_{2,x} = F_{1\to 2} = m_2 a_x = m_2\left(\frac{F_{\text{ext}}}{m_1 + m_2}\right) = (0.5\text{ kg})\left(\frac{7.5\text{ N}}{(1\text{ kg}) + (0.5\text{ kg})}\right) = \boxed{2.5\text{ N}}$$

REFLECT

Adding the two equations is the easiest way of solving a_x because the force of block 2 on block 1 and the force of block 1 on block 2 have the same magnitude. We could have also treated blocks 1 and 2 as a single object with a mass of $(m_1 + m_2)$ in order to calculate a_x. Finally, it makes sense that the force of block 1 on block 2 (2.5 N) should be less than the external force (7.5 N) and proportional to the mass of block 2.

4.100

SET UP

Three boxes ($m_A = 20$ kg, $m_B = 30$ kg, $m_C = 50$ kg) are sitting on a horizontal surface and touching one another (see Figure 4-62). An external force of magnitude F is pushing box A toward the right, which we will call positive x. We are told that the force of box B on box C has a magnitude of 200 N. This is the only force acting on box C in the x direction, so we can calculate the acceleration of box C. Since all of the boxes are touching, they will all have the same acceleration. Once we have the acceleration of all three boxes, we can treat boxes A, B, and C as a single object ABC of mass $m_{ABC} = 100$ kg with the same acceleration. The only force acting on ABC is the external force of magnitude F. We can solve for F using Newton's second law for the x component of box ABC.

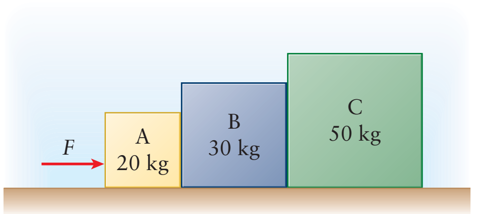


Figure 4-62 Problem 100

SOLVE

Free-body diagram of box C:

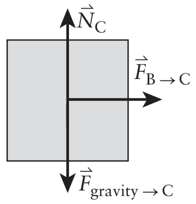


Figure 4-63 Problem 100

Acceleration of the boxes:

$$\sum F_{C,x} = F_{B\to C} = m_C a$$

$$a = \frac{F_{B\to C}}{m_C} = \frac{200 \text{ N}}{50 \text{ kg}} = \boxed{4\frac{\text{m}}{\text{s}^2}}$$

Free-body diagram of all three boxes as one object:

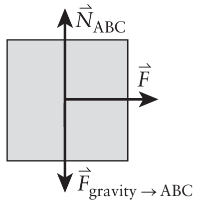


Figure 4-64 Problem 100

Magnitude of external force F:

$$\sum F_{ABC,x} = F = m_{ABC}a = (100 \text{ kg})\left(4\frac{\text{m}}{\text{s}^2}\right) = \boxed{400 \text{ N}}$$

REFLECT

All of the Newton's third law pairs between the boxes (for example, A → B, B → A) will cancel out if we treat all three boxes as one object because they are now internal to the system. Doing this allows us to easily calculate the magnitude of the external force F.

4.101

SET UP

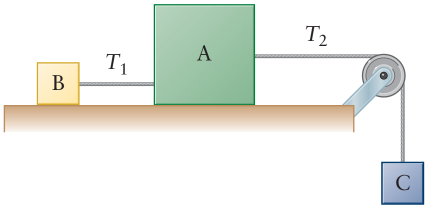


Figure 4-65 Problem 101

Blocks A and B are connected by a string with a tension T_1. Block C, which is attached to block A by a string with tension T_2, is hanging off of the table. Block A has a mass $m_A = 2$ kg, block B has a mass of $m_B = 1$ kg, and block C has a weight of 10 N (that is, $m_C = 1$ kg). We are told that blocks A and B accelerate to the right, which we'll call positive x. This means block C is falling (toward negative y). All three blocks have the same acceleration because they are tethered together. Using Newton's second law for each of the blocks, we can solve for the acceleration in terms of the known masses and weight. Once we have the acceleration, we can calculate the magnitudes of the tensions.

SOLVE

Free-body diagram of block A:

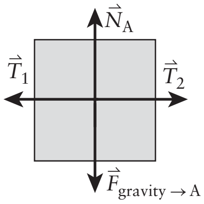


Figure 4-66 Problem 101

Free-body diagram of block B:

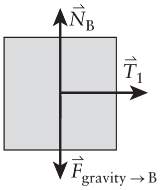


Figure 4-67 Problem 101

Free-body diagram of block C:

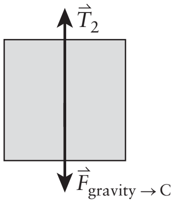


Figure 4-68 Problem 9101

Newton's second law:

$$\sum F_{A,x} = -T_1 + T_2 = m_A a$$

$$\sum F_{B,x} = T_1 = m_B a$$

$$\sum F_{C,y} = T_2 - F_{gravity \to C} = m_C(-a)$$

Solving for the acceleration:

$$T_2 = F_{gravity \to C} - m_C a = (m_A + m_B)a$$

$$a = \frac{F_{gravity \to C}}{m_A + m_B + m_C}$$

Solving for the tensions:

$$T_1 = m_B a = m_B\left(\frac{F_{gravity \to C}}{m_A + m_B + m_C}\right) = (1\ \text{kg})\left(\frac{10\ \text{N}}{(2\ \text{kg}) + (1\ \text{kg}) + (1\ \text{kg})}\right) = \boxed{2.5\ \text{N}}$$

$$T_2 = (m_A + m_B)a = (m_A + m_B)\left(\frac{F_{gravity \to C}}{m_A + m_B + m_C}\right)$$

$$= ((2\ \text{kg}) + (1\ \text{kg}))\left(\frac{10\ \text{N}}{(2\ \text{kg}) + (1\ \text{kg}) + (1\ \text{kg})}\right) = \boxed{7.5\ \text{N}}$$

REFLECT

It makes sense that $T_2 > T_1$ since the second string has to pull *both* blocks A and B, while the first string is only pulling block B. We could have also treated all three blocks as one object with a combined mass of 4 kg. The only external force acting on this system is $F_{\text{gravity}\rightarrow\text{C}} = 10\text{ N}$. This would have quickly given us the acceleration for the system.

4.102

SET UP

A 2-kg block resting on a 40-degree slope is connected to a hanging 3-kg block by a string. We'll use two different but related coordinate systems for the two blocks. For the 2-kg block, the axes will be parallel and perpendicular to the inclined plane, where up the ramp and out of the ramp are positive. For the 3-kg block, positive y will point downward. We can draw free-body diagrams and apply Newton's second law in component form for both blocks. The tension acting on each block will be identical in magnitude. The acceleration of the blocks will also be the same since they are tied together. We will first find the acceleration and then use that to calculate the tension.

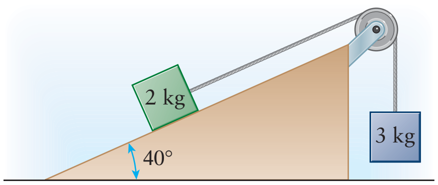


Figure 4-69 Problem 102

SOLVE

Free-body diagram of 2-kg mass:

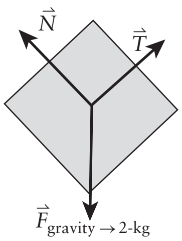


Figure 4-70 Problem 102

Free-body diagram of 3-kg mass:

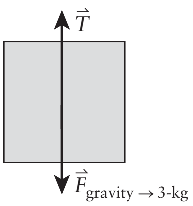


Figure 4-71 Problem 102

Newton's second law:

$$\sum F_{\text{2-kg},\,\parallel} = T - F_{\text{gravity}\rightarrow\text{2-kg},\,\parallel} = T - m_{\text{2-kg}}g\sin(40°) = m_{\text{2-kg}}a$$

$$\sum F_{\text{3-kg},\,y} = -T + F_{\text{gravity}\rightarrow\text{3-kg}} = -T + m_{\text{3-kg}}g = m_{\text{3-kg}}a$$

Solving for acceleration and tension:

$$m_{3\text{-kg}}g - m_{2\text{-kg}}g\sin(40°) = (m_{2\text{-kg}} + m_{3\text{-kg}})a$$

$$a = \frac{(m_{3\text{-kg}} - m_{2\text{-kg}}\sin(40°))g}{m_{2\text{-kg}} + m_{3\text{-kg}}} = \frac{((3\text{ kg}) - (2\text{ kg})\sin(40°))\left(9.8\frac{\text{m}}{\text{s}^2}\right)}{(2\text{ kg}) + (3\text{ kg})} = \boxed{3.4\frac{\text{m}}{\text{s}^2}}$$

$$T = m_{3\text{-kg}}(g - a) = (3\text{ kg})\left(\left(9.8\frac{\text{m}}{\text{s}^2}\right) - \left(3.4\frac{\text{m}}{\text{s}^2}\right)\right) = \boxed{19.2\text{ N}}$$

REFLECT

A positive acceleration means that the 3-kg block is falling and pulling the 2-kg block up the ramp, which is what we would expect to happen.

4.103

SET UP

A 1-kg block and a 2-kg block are connected by a string with a tension T_1. A 4-kg block, which is attached to the 2-kg block by a string with tension T_2, is hanging off of the table. We'll use a coordinate system where right and down are positive x and y, respectively. All three blocks have the same acceleration because they are tethered together. Using Newton's second law for each of the blocks, we can solve for the acceleration in terms of the known masses. Once we have the acceleration, we can calculate the magnitudes of the tensions.

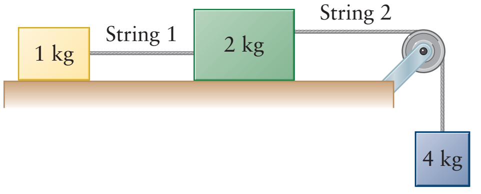


Figure 4-72 Problem 103

SOLVE

Free-body diagram of 1-kg block:

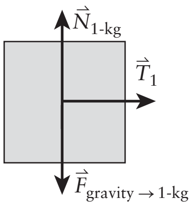


Figure 4-73 Problem 103

Free-body diagram of 2-kg block:

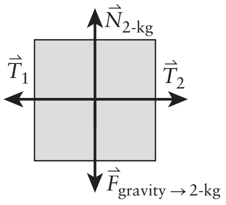


Figure 4-74 Problem 103

Free-body diagram of 4-kg block:

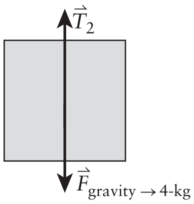


Figure 4-75 Problem 103

Newton's second law:

$$\sum F_{\text{1-kg}, x} = T_1 = m_{\text{1-kg}}a$$

$$\sum F_{\text{2-kg}, x} = -T_1 + T_2 = m_{\text{2-kg}}a$$

$$\sum F_{\text{4-kg}, y} = -T_2 + F_{\text{gravity}\to\text{4-kg}} = m_{\text{4-kg}}a$$

Solving for the acceleration:

$$T_2 = F_{\text{gravity}\to\text{4-kg}} - m_{\text{4-kg}}a = (m_{\text{1-kg}} + m_{\text{2-kg}})a$$

$$a = \frac{F_{\text{gravity}\to\text{4-kg}}}{m_{\text{1-kg}} + m_{\text{2-kg}} + m_{\text{4-kg}}} = \frac{(4\text{ kg})\left(9.8\dfrac{\text{m}}{\text{s}^2}\right)}{(1\text{ kg}) + (2\text{ kg}) + (4\text{ kg})} = \boxed{5.6\frac{\text{m}}{\text{s}^2}}$$

Solving for the tensions:

$$T_1 = m_{\text{1-kg}}a = (1\text{ kg})\left(5.6\frac{\text{m}}{\text{s}^2}\right) = \boxed{5.6\text{ N}}$$

$$T_2 = (m_{\text{1-kg}} + m_{\text{2-kg}})a = ((1\text{ kg}) + (2\text{ kg}))\left(5.6\frac{\text{m}}{\text{s}^2}\right) = \boxed{16.8\text{ N}}$$

REFLECT

A positive acceleration means that the 4-kg block is falling and pulling the other two blocks to the right, which is what we would expect to happen. It makes sense that $T_2 > T_1$ since the second string has to pull both the 1-kg and 2-kg blocks, while the first string is only pulling the 1-kg block. The solution to this problem is the same as Problem 4.101.

4.104

SET UP

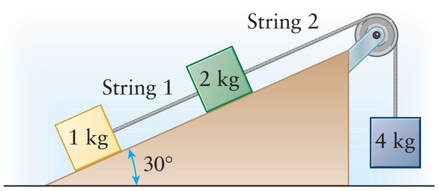


Figure 4-76 Problem 104

A 1-kg block and a 2-kg block are connected by a string with a tension T_1 and sitting on a 30-degree slope. A 4-kg block, which is attached to the 2-kg block by a string with tension T_2, is hanging off of the ramp. For the blocks on the ramp, we'll use a coordinate system where the axes are parallel and perpendicular to the face of the ramp and up the ramp is considered positive. For the 4-kg block, down will be positive y in order to stay consistent with the ramp's coordinate system. All three blocks have the same acceleration because they are tethered together. Using Newton's second law for each of the blocks, we can

solve for the acceleration in terms of the known masses. Once we have the acceleration, we can calculate the magnitudes of the tensions.

SOLVE

Free-body diagram of 1-kg block:

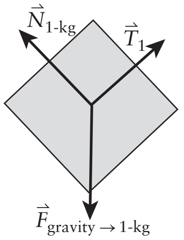


Figure 4-77 Problem 104

Free-body diagram of 2-kg block:

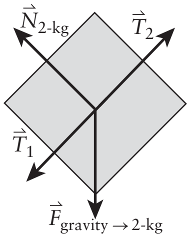


Figure 4-78 Problem 104

Free-body diagram of 4-kg block:

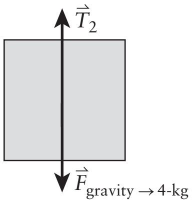


Figure 4-79 Problem 104

Newton's second law:

$$\sum F_{1\text{-kg},\,\parallel} = T_1 - F_{\text{gravity}\to 1\text{-kg},\,\parallel} = T_1 - m_{1\text{-kg}}g\sin(30°) = m_{1\text{-kg}}a$$

$$\sum F_{2\text{-kg},\,\parallel} = T_2 - T_1 - F_{\text{gravity}\to 2\text{-kg},\,\parallel} = T_2 - T_1 - m_{2\text{-kg}}g\sin(30°) = m_{2\text{-kg}}a$$

$$\sum F_{4\text{-kg},\,y} = -T_2 + F_{\text{gravity}\to 4\text{-kg}} = -T_2 + m_{4\text{-kg}}g = m_{4\text{-kg}}a$$

Solving for the acceleration:

$$T_2 = m_{4\text{-kg}}(g - a) = (m_{1\text{-kg}} + m_{2\text{-kg}})a + (m_{1\text{-kg}} + m_{2\text{-kg}})g\sin(30°)$$

$$a = \frac{m_{4\text{-kg}}g - (m_{1\text{-kg}} + m_{2\text{-kg}})g\sin(30°)}{m_{1\text{-kg}} + m_{2\text{-kg}} + m_{4\text{-kg}}} = \frac{(4\text{ kg})\left(9.8\dfrac{\text{m}}{\text{s}^2}\right) - ((1\text{ kg}) + (2\text{ kg}))\left(9.8\dfrac{\text{m}}{\text{s}^2}\right)\sin(30°)}{(1\text{ kg}) + (2\text{ kg}) + (4\text{ kg})}$$

$$= \boxed{3.5\frac{\text{m}}{\text{s}^2}}$$

Solving for the tensions:

$$T_1 = m_{1\text{-kg}}g\sin(30°) + m_{1\text{-kg}}a = (1\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)\sin(30°) + (1\text{ kg})\left(3.5\frac{\text{m}}{\text{s}^2}\right) = \boxed{8.4\text{ N}}$$

$$T_2 = m_{4\text{-kg}}(g - a) = (4\text{ kg})\left(\left(9.8\frac{\text{m}}{\text{s}^2}\right) - \left(3.5\frac{\text{m}}{\text{s}^2}\right)\right) = \boxed{25.2\text{ N}}$$

REFLECT

A positive acceleration means that the 4-kg block is falling and pulling the other two blocks to the right, which is what we would expect to happen. It makes sense that $T_2 > T_1$ since the second string has to pull both the 1-kg and 2-kg blocks, while the first string is only pulling the 1-kg block. It makes sense that these tensions are larger in this case than in Problem 4.103 because the strings are now counteracting a portion of the weights of the blocks.

4.105

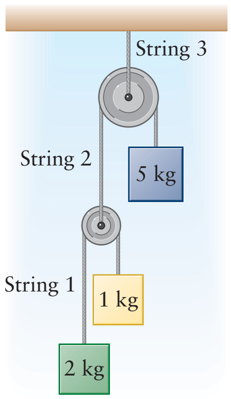


Figure 4-80 Problem 105

SET UP

A compound Atwood's machine is constructed out of a 1-kg block, a 2-kg block, and a 5-kg block. String 1 connects the 1-kg and 2-kg blocks, while string 2 connects the 5-kg block to the pulley holding the smaller blocks. We expect the 2-kg block to move down while the 1-kg block moves up in the smaller Atwood's machine. The smaller Atwood's machine, which we will treat as an object of 3-kg in mass, will be pulled up by the 5-kg block. The net acceleration of the 2-kg block is the acceleration due to the effects of each Atwood machine. In the larger Atwood machine's coordinate system we'll call the downward direction for the 5-kg block and the upward direction of the 3-kg object as positive y. For the smaller one, we'll call the downward direction for the 2-kg block and the upward direction of the 1-kg object as positive y.

SOLVE

Free-body diagram of the 5-kg block:

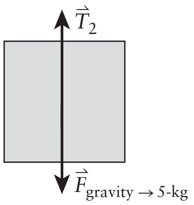


Figure 4-81 Problem 105

Free-body diagram of the (1 kg + 2 kg) object:

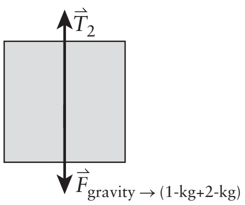


Figure 4-82 Problem 105

Newton's second law:

$$\sum F_{\text{5-kg},\, y} = F_{\text{gravity}\to\text{5-kg}} - T_2 = m_{\text{5-kg}}g - T_2 = m_{\text{5-kg}}a_{\text{5-kg}}$$

$$\sum F_{(\text{1-kg}+\text{2-kg}),\, y} = T_2 - F_{\text{gravity}\to(\text{1-kg}+\text{2-kg})} = T_2 - (m_{\text{1-kg}} + m_{\text{2-kg}})g = (m_{\text{1-kg}} + m_{\text{2-kg}})a_{\text{5-kg}}$$

$$m_{\text{5-kg}}g - (m_{\text{1-kg}} + m_{\text{2-kg}})g = (m_{\text{1-kg}} + m_{\text{2-kg}})a_{\text{5-kg}} + m_{\text{5-kg}}a$$

$$a_{\text{5-kg}} = \frac{(m_{\text{5-kg}} - m_{\text{1-kg}} - m_{\text{2-kg}})g}{m_{\text{1-kg}} + m_{\text{2-kg}} + m_{\text{5-kg}}} = \left(\frac{2}{8}\right)\left(9.8\frac{\text{m}}{\text{s}^2}\right) = 2.45\frac{\text{m}}{\text{s}^2}$$

The (1 kg + 2 kg) object is moving upward at an acceleration of 2.45 m/s^2.

Free-body diagram of the 2-kg block:

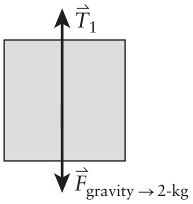


Figure 4-83 Problem 105

Free-body diagram of the 1-kg block:

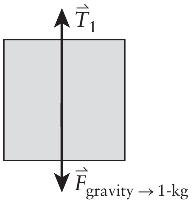


Figure 4-84 Problem 105

Newton's second law:

$$\sum F_{\text{2-kg},\, y} = F_{\text{gravity}\to\text{2-kg}} - T_1 = m_{\text{2-kg}}g - T_1 = m_{\text{2-kg}}a_{\text{2-kg}}$$

$$\sum F_{\text{1-kg},\, y} = T_1 - F_{\text{gravity}\to\text{1-kg}} = T_1 - m_{\text{1-kg}}g = m_{\text{1-kg}}a_{\text{2-kg}}$$

$$m_{\text{2-kg}}g - m_{\text{1-kg}}g = (m_{\text{2-kg}} + m_{\text{1-kg}})a_{\text{2-kg}}$$

$$a_{\text{2-kg}} = \frac{(m_{\text{2-kg}} - m_{\text{1-kg}})g}{m_{\text{2-kg}} + m_{\text{1-kg}}} = \left(\frac{1}{3}\right)\left(9.8\frac{\text{m}}{\text{s}^2}\right) = 3.27\frac{\text{m}}{\text{s}^2}$$

Net acceleration of block 2: (2.45 m/s^2) − (3.27 m/s^2) = $\boxed{-0.82\ \text{m/s}^2}$, where the negative sign implies $\boxed{\text{downward}}$ motion.

REFLECT

Be careful when defining coordinate systems in Atwood machines; it's usually easiest to use a curved axis that follows the string.

4.106

SET UP

An astronaut is sitting in a space shuttle that takes off vertically from rest at an acceleration of 29 m/s^2. Let's assume the space shuttle is accelerating toward positive y. The forces acting in the y direction are the contact force between the shuttle and the astronaut acting upward and the force of gravity acting downward. Since we know the acceleration of the shuttle (and, therefore, the astronaut), we can use Newton's second law to calculate the magnitude of the force of the shuttle on the astronaut.

SOLVE

Free-body diagram of the astronaut:

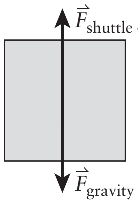


Figure 4-85 Problem 106

Newton's second law for the astronaut:

$$\sum F_y = F_{\text{shuttle}\to\text{astronaut}} - mg = ma_y$$

$$F_{\text{shuttle}\to\text{astronaut}} = m(g + a_y) = (75\text{ kg})\left(\left(29\frac{\text{m}}{\text{s}^2}\right) + \left(9.8\frac{\text{m}}{\text{s}^2}\right)\right) = \boxed{2910\text{ N}}$$

This is four times the astronaut's weight.

REFLECT

We should expect the force required to accelerate an astronaut into space to be large.

4.107

SET UP

A bulldozer travels at a constant speed of 4 m/s and pushes a mound of dirt with a constant force of 3500 N. We are asked to find the rate at which the dirt builds up in front of the blade of the bulldozer. We can relate the net force to the mass rate using the differential form of Newton's second law. Since the speed is constant, we can pull that out of the derivative, and the mass rate will be equal to the net force divided by the speed.

SOLVE

$$\sum F = \frac{d(mv)}{dt} = m\frac{dv}{dt} + v\frac{dm}{dt} = 0 + v\frac{dm}{dt}$$

$$\frac{dm}{dt} = \frac{\sum F}{v} = \frac{3500\text{ N}}{\left(4\frac{\text{m}}{\text{s}}\right)} = \boxed{875\frac{\text{kg}}{\text{s}}}$$

REFLECT

This seems like a reasonable mass for a bulldozer to push.

4.108

SET UP

A diver falls from a platform that is 10 m above the surface of a 5-m-deep pool. We need to calculate the force that the water needs to exert on the diver in order to stop her from hitting the bottom of the pool. First we need to calculate the speed with which she enters the water using the constant acceleration equations. We can then calculate the acceleration necessary for her to come to rest within 5 m, assuming it is constant. Once she is in the water there are two forces acting on her—the force of the water on her and the force of gravity. Because we're interested in the average force of the water on the diver, this will be a constant force and we can use Newton's second law directly to calculate its magnitude.

SOLVE

Speed entering the water:

$$v^2 = v_0^2 + 2a(\Delta y)$$

$$v = \sqrt{v_0^2 + 2a(\Delta y)} = \sqrt{0 + 2\left(-9.8\frac{\text{m}}{\text{s}^2}\right)(-10\text{ m})} = 14\frac{\text{m}}{\text{s}}$$

Acceleration necessary to stop the diver in the water:

$$v^2 = v_0^2 + 2a(\Delta y)$$

$$a = \frac{v^2 - v_0^2}{2(\Delta y)} = \frac{0 - \left(14\frac{\text{m}}{\text{s}}\right)^2}{2(-5\text{ m})} = 19.6\frac{\text{m}}{\text{s}^2}$$

Average force of the water on the diver:

$$\sum F = F_{\text{diver}} - F_{\text{gravity}\rightarrow\text{diver}} = F_{\text{diver}} - mg = ma$$

$$F_{\text{diver}} = m(g + a) = (62\text{ kg})\left(\left(9.8\frac{\text{m}}{\text{s}^2}\right) + \left(19.6\frac{\text{m}}{\text{s}^2}\right)\right) = \boxed{1823\text{ N}}$$

REFLECT

This is about three times the diver's weight. Don't forget that gravity is still acting on the diver while she is in the water.

4.109

SET UP

Three crates are connected to each other by strings that can withstand a maximum tension of 45.0 N before breaking. String A connects a 25.0-kg crate to an 18.0-kg crate; string B connects the 18.0-kg crate to a 15.0-kg crate; and a person pulls a third string attached to the 15.0-kg crate at an angle of 34.0 degrees above the horizontal. Let's use a coordinate system where the crates are being pulled toward positive x. We need to determine the largest force that can be exerted without breaking either string A or B, which

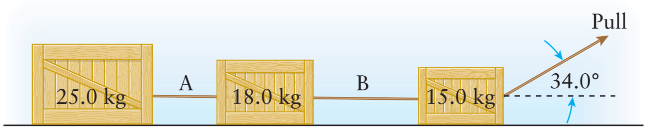


Figure 4-86 Problem 109

we can do through Newton's second law. We'll assume that string B is the first one to break since it is effectively pulling more mass than string A; therefore, we'll set $T_B = 45.0$ N. Once we find the acceleration of the system, we will treat all three crates as one object and apply Newton's second law in order to calculate the magnitude of the pull force. We can calculate the tension in string A directly from the acceleration of the system and the mass of the 25.0-kg crate.

SOLVE

Part a)

Free-body diagram for the 25-kg crate:

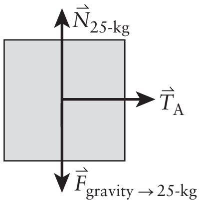


Figure 4-87 Problem 109

Free-body diagram for the 18-kg crate:

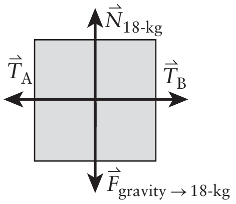


Figure 4-88 Problem 109

Free-body diagram for the (25 kg + 18 kg + 15 kg) object:

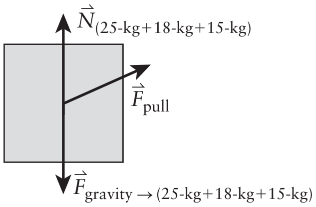


Figure 4-89 Problem 109

Newton's second law:

$$\sum F_{25\text{-kg},\,x} = T_A = m_{25\text{-kg}}a$$

$$\sum F_{18\text{-kg},\,x} = T_B - T_A = m_{18\text{-kg}}a$$

$$T_B = T_A + m_{18\text{-kg}}a = m_{25\text{-kg}}a + m_{18\text{-kg}}a$$

$$a = \frac{T_B}{m_{25\text{-kg}} + m_{18\text{-kg}}} = \frac{45.0\text{ N}}{(25.0\text{ kg}) + (18.0\text{ kg})} = 1.05\frac{\text{m}}{\text{s}^2}$$

$$\sum F_{(25\text{-kg}+18\text{-kg}+15\text{-kg}),\,x} = F_{\text{Pull}}\cos(34°) = m_{(25\text{-kg}+18\text{-kg}+15\text{-kg})}a$$

$$F_{\text{Pull}} = \frac{m_{(25\text{-kg}+18\text{-kg}+15\text{-kg})}a}{\cos(34°)} = \frac{((25.0\text{ kg}) + (18.0\text{ kg}) + (15.0\text{ kg}))}{\cos(34°)}\left(1.05\frac{\text{m}}{\text{s}^2}\right) = \boxed{73.5\text{ N}}$$

Part b)

$$\boxed{T_{\text{B}} = 45.0\text{ N}}$$

$$T_{\text{A}} = m_{25\text{-kg}}a = (25.0\text{ kg})\left(1.05\frac{\text{m}}{\text{s}^2}\right) = \boxed{26.3\text{ N}}$$

REFLECT

Our assumption that string B would break first is a good one. We can also show this through Newton's second law for the 18-kg crate. The net force acting on this box is $T_{\text{B}} - T_{\text{A}}$, which has to be positive since the crate is moving toward positive x. In order for this difference to be positive, $T_{\text{B}} > T_{\text{A}}$.

4.110

SET UP

A 275-g rubber ball falls from a height of 2.20 m, bounces off the ground, and reaches a height of 1.65 m (75% of its initial height). The ball is in contact with the ground for 18.4 ms. While the ball is in the air, gravity is the only force acting on it. Both gravity and the normal force are acting on the ball when it is in contact with the ground. We need to calculate the ball's change in velocity in order to determine the net force acting on the ball. We can use the constant acceleration equations to determine the speed of the ball right before and right after it hits the ground. The magnitude of the normal force is equal to the net force on the ball plus the ball's weight.

SOLVE

Part a)

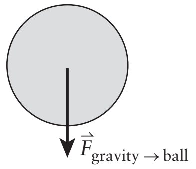


Figure 4-90 Problem 110

Part b)

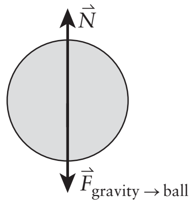


Figure 4-91 Problem 110

Part c)

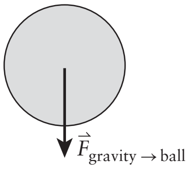


Figure 4-92 Problem 110

Part d)
Speed of the ball right before it hits the ground:

$$v^2 = v_0^2 + 2a_y(\Delta y)$$

$$v = \sqrt{v_0^2 + 2a_y(\Delta y)} = \sqrt{0 + 2\left(-9.8\frac{\text{m}}{\text{s}^2}\right)(-2.20 \text{ m})} = 6.57\frac{\text{m}}{\text{s}}$$

Speed with which the ball leaves the floor after bouncing:

$$v^2 - v_0^2 = 0 - v_0^2 = +2a_y(\Delta y)$$

$$v_0 = \sqrt{-2a_y(\Delta y)} = \sqrt{-2\left(-9.8\frac{\text{m}}{\text{s}^2}\right)(0.75)(2.20 \text{ m})} = 5.69\frac{\text{m}}{\text{s}}$$

Average magnitude of normal force:

$$\sum F = N - mg = \frac{m\Delta v}{\Delta t}$$

$$N = mg + \frac{m\Delta v}{\Delta t} = (0.275 \text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right) + \frac{(0.275 \text{ kg})\left(\left(5.69\frac{\text{m}}{\text{s}}\right) - \left(-6.57\frac{\text{m}}{\text{s}}\right)\right)}{0.0184 \text{ s}} = \boxed{186 \text{ N}}$$

The force of the floor on the ball (that is, the normal force) points $\boxed{\text{upward}}$ with a magnitude of $\boxed{186 \text{ N}}$.

REFLECT
The normal force is the force responsible for changing the ball's direction.

4.111

SET UP
The mass and velocity of an object both change with respect to time. The net force acting on the object is equal to the derivative of the product of the mass and speed with respect to time. Since both the mass and velocity are functions of time, we will need to use the product rule to find the derivative.

SOLVE

Part a)

$$\sum F = \frac{d(mv)}{dt} = m\frac{dv}{dt} + v\frac{dm}{dt} = (m_0e^{-bt})(a) + (at + v_0)(-bm_0e^{-bt})$$

$$= (m_0e^{-bt})(a - bat - bv_0) = \boxed{(m_0e^{-bt})(a - bv)}$$

Part b)

$$v(3\text{ s}) = \left(6\frac{\text{m}}{\text{s}^2}\right)(3\text{ s}) + \left(1\frac{\text{m}}{\text{s}}\right) = 19\frac{\text{m}}{\text{s}}$$

$$\sum F = ((2\text{ kg})e^{-(0.16\text{ s}^{-1})(3\text{ s})})\left(\left(6\frac{\text{m}}{\text{s}^2}\right) - (0.16\text{ s}^{-1})\left(19\frac{\text{m}}{\text{s}}\right)\right) = \boxed{3.66\text{ N}}$$

REFLECT

Our algebraic expression has dimensions of force, as expected. The mass will be the largest at $t = 0$ and exponentially decay as time goes on. As the mass decreases, the speed will increase as well, assuming that a and v_0 are positive.

Chapter 5
Applications of Newton's Laws

Conceptual Questions

5.1 The static frictional force between two surfaces is sometimes less than the normal force. If the coefficient of static friction is less than 1, the normal force will be larger than the static frictional force. If the coefficient of static friction is more than 1, the static frictional force can exceed the normal force or not depending on how much friction is required to prevent slipping.

5.2 It will be easier to keep pushing the box across the floor once you get it moving because $\mu_k < \mu_s$.

5.3 When a pickup truck accelerates forward, the contents will also accelerate forward due to the force of static friction between the box and the truck's bed.

5.4 The force of friction is independent of the area over which the force acts. The coefficient of friction depends only on the materials of the surfaces.

5.5 As you reduce the force you are applying to the book, the weight of the book remains the same; the normal force decreases; the frictional force is the same; and the maximum static frictional force decreases.

5.6 We will assume the object is traveling at a constant speed around the circle. (a) The displacement is zero because the object starts and ends at the same location. (b) The average velocity is zero because the displacement is zero. (c) The average acceleration is zero because the average velocity at the beginning and end of the rotation is the same. (d) The instantaneous velocity is constantly changing around the circle. (e) The instantaneous centripetal acceleration always points inward, so its direction changes depending on the location of the object.

5.7 If the water stays in the bucket, it is because the whirling is so fast that at the top of the circle, the bucket is accelerating downward faster than the acceleration of gravity. The bottom of the bucket is needed to pull the water down! The sides of the bucket prevent the water from sloshing ahead or behind with the rest of the water that is going in the same circle. The weight is always pointing straight down (whether at the top or at the bottom), but the normal force N changes direction, pointing down at the top of the swing and up at the lowest point. Thus, $\sum F_{\text{top}} = mg + N$ while $\sum F_{\text{bottom}} = mg - N$.

5.8 When a wheel skids on the road, it is being acted on by kinetic friction rather than static friction when the wheel is rolling. Because the maximum value of the coefficient of static friction is larger than the maximum value of the coefficient of kinetic friction, the force due to static friction will be larger than that due to kinetic friction and the car will stop in a shorter amount of time.

5.9 An object on a slope; an object in a stack of objects (the lower objects' normal forces are greater than their own weight; Figure 5-11a); an object pressed to a wall (Figure 5-14a); an object sitting in an elevator as the elevator starts or stops.

5.10 Although the coefficients of friction between synovial fluid and bone should be the same for everyone, the normal forces involved will be different from person to person. This means the magnitudes of the frictional forces will also be different from person to person.

5.11 His acceleration decreases because the net force on him decreases. Net force is equal to his weight minus his drag force. The faster an object is moving, the larger the drag force it experiences as a function of its speed. Because drag force increases with increasing speed, net force and acceleration decrease. As the drag force approaches his weight, the acceleration approaches zero.

5.12 The terminal speed of a raindrop is proportional to its weight. Larger drops should fall at a higher speed than smaller ones.

5.13 The frictional force between the tires and the road provides the centripetal force that keeps the car moving in a circle. If the frictional force is not strong enough to keep the car in a circle, the car continues in a straight line and starts to skid.

5.14 The forces acting on the car in either case are the force due to gravity and the normal force, but their relative magnitudes depend on the concavity of the road. Assuming the road is a portion of a circle and that the car is traveling at a constant speed, the car will undergo uniform circular motion while traveling over the hill or through the dip. The acceleration of the car (and, therefore, its net force) will point toward the center of the circle, which is down for the hill and up for the dip. The normal force will be smaller than the weight in the case of the hill and larger in the case of the dip. This corresponds with the sensations passengers feel in each situation—you feel like you're leaving your seat when zooming over a hill and squished when you're traveling through a dip.

5.15 If you know the car's instantaneous speed and the radius of curvature, you can use the formula $a_c = \frac{v^2}{r}$. A more direct alternative is to use an accelerometer—a circular glass tube with a ball in it. If the ball settles at angle α from the bottom, $\tan \alpha =$ (centripetal acceleration)/(acceleration of gravity).

5.16

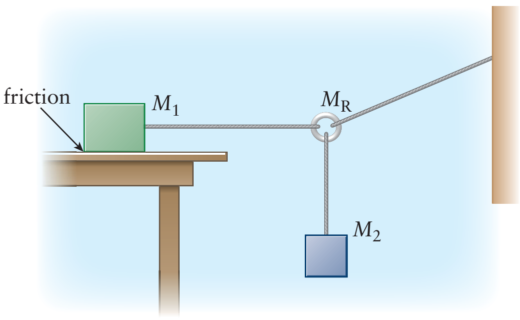


Figure 5-1 Problem 16

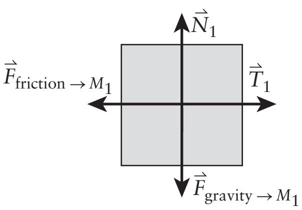


Figure 5-2 Problem 16

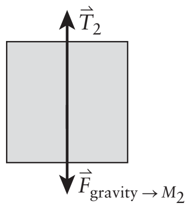


Figure 5-3 Problem 16

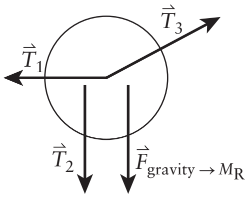


Figure 5-4 Problem 16

5.17 Banking the curve allows the normal force to help push the car into the turn. The additional normal force also increases the maximum frictional force.

5.18 Newton's second law relates the *net* force on an object to its acceleration.

5.19 Yes. If he lies flat in relation to the wind and she does not, she will have a smaller profile to the wind. Thus, she will experience less drag and be able to catch up.

Multiple-Choice Questions

5.20 **A** (1.0).
Free-body diagram of the SUV:

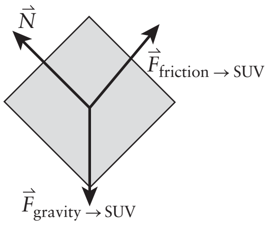


Figure 5-5 Problem 20

Newton's second law:

$$\sum F_{\perp} = N - mg\cos(45°) = ma_{\perp} = 0$$

$$N = mg\cos(45°)$$

$$\sum F_{\parallel} = \mu_s N - mg\sin(45°) = ma_{\parallel} = 0$$

$$\mu_s = \frac{mg\sin(45°)}{N} = \frac{mg\sin(45°)}{mg\cos(45°)} = \tan(45°) = 1.0$$

5.21 **A** (slide down at constant speed).
Free-body diagram of a mass sliding down a ramp:

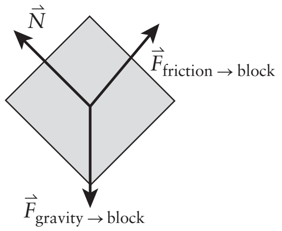


Figure 5-6 Problem 21

Newton's second law:

$$\sum F_\perp = N - mg\cos(45°) = ma_\perp = 0$$

$$N = mg\cos(45°)$$

$$\sum F_\parallel = \mu_k N - mg\sin(45°) = ma_\parallel$$

$$\mu_k(mg\cos(45°)) - mg\sin(45°) = ma_\parallel$$

$$a_\parallel = \mu_k g\cos(45°) - g\sin(45°)$$

The acceleration is independent of the mass, which means the blocks will have the same motion.

5.22 **B** (49 N). Because the crate doesn't slip relative to the conveyor belt, static friction is acting on it. The maximum magnitude of static friction is $\mu_s N = \mu_s mg = (0.5)(10\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right) = 49\text{ N}$ in this case.

5.23 **E** (cannot be determined without more information). We don't know anything about the drag force acting on or the acceleration of the bacterium.

5.24 **B** (The 5-kg lead ball has the larger acceleration). Assuming down is positive y:

$$\sum F_y = mg - F_{\text{drag}} = ma_y$$

$$a_y = g - \frac{F_{\text{drag}}}{m}$$

Since the balls have the same dimensions, their drag coefficients will be the same. The larger the mass, the smaller the contribution of the second term and the larger the overall acceleration.

5.25 **D** (the drag force is larger than the skydiver's weight). Once the skydiver opens his parachute, he will start to slow down because there is a net force acting upward. A net force upward means that the magnitude of the drag force is larger than the magnitude of his weight.

5.26 **D** ($T_1 = 2T_2$).

$$\frac{T_1}{T_2} = \frac{\left(\dfrac{mv^2}{R_1}\right)}{\left(\dfrac{mv^2}{R_2}\right)} = \frac{R_2}{R_1} = \frac{2R_1}{R_1} = 2$$

5.27 **C** ($m_1 = m_2$).

$$\frac{m_1}{m_2} = \frac{\left(\dfrac{F_{\text{tension, 1}}R_1}{\left(\dfrac{2\pi R_1}{T_1}\right)^2}\right)}{\left(\dfrac{F_{\text{tension, 2}}R_2}{\left(\dfrac{2\pi R_2}{T_2}\right)^2}\right)} = \frac{\left(\dfrac{T_1^2 F_{\text{tension, 1}}}{R_1}\right)}{\left(\dfrac{T_2^2 F_{\text{tension, 2}}}{R_2}\right)} = \frac{\left(\dfrac{T_1^2 F_{\text{tension, 1}}}{R_1}\right)}{\left(\dfrac{T_2^2 (2F_{\text{tension, 1}})}{(2R_1)}\right)} = 1$$

5.28 **A** (larger than its weight). At the top of the circle where down is positive y: $\sum F_y = T + mg$, where both terms are positive. Therefore, the net force causing the rotation is larger than the weight.

5.29 **B** ($N > mg$). There is a net force acting upward because there is an acceleration in that direction when you are at the bottom of a rotating Ferris wheel.

Estimation Questions

5.30 The coefficient of friction between the rubber tires and the road should be relatively large, so 0.7–0.8 is reasonable.

5.31 For a box of books that slides 2 m on a carpeted floor after being pushed at an initial speed of 5 m/s, the coefficient of friction is about 0.6.

5.32 Let's say the car travels around the ramp at a speed of about 15 m/s. The coefficient of friction between the rubber tires and the road is around 0.7.

$$F_{\text{friction}} = \mu N = \mu mg = ma = m\left(\frac{v^2}{R}\right)$$

$$R = \frac{v^2}{\mu g} = \frac{\left(15\dfrac{\text{m}}{\text{s}}\right)^2}{(0.7)\left(9.8\dfrac{\text{m}}{\text{s}^2}\right)} = \boxed{33\text{ m}}$$

5.33 Assume the initial speed of the runner to be 7 m/s and the slide covers 3 m. The acceleration would be $a = \dfrac{-\left(7\dfrac{\text{m}}{\text{s}}\right)^2}{2(3\text{ m})} = -8\dfrac{\text{m}}{\text{s}^2}$.

$$\sum F = \mu mg = ma$$

$$\mu = \frac{a}{g} = \frac{\left(8\dfrac{\text{m}}{\text{s}^2}\right)}{\left(9.8\dfrac{\text{m}}{\text{s}^2}\right)} = \boxed{0.8}$$

5.34 Assume the initial speed of the puck to be 8 m/s and the puck comes to rest in 30 m (half the rink). The acceleration would be $a = \dfrac{-\left(8\dfrac{\text{m}}{\text{s}}\right)^2}{2(30\text{ m})} = -1.1\dfrac{\text{m}}{\text{s}^2}$.

$$\sum F = \mu mg = ma$$

$$\mu = \frac{a}{g} = \frac{\left(1.1\dfrac{\text{m}}{\text{s}^2}\right)}{\left(9.8\dfrac{\text{m}}{\text{s}^2}\right)} = \boxed{0.1}$$

5.35 The coefficient of kinetic friction is about 0.6; the coefficient of static friction is about 0.8.

5.36 Brake pads are designed to press against the discs and stop the car in a reasonable amount of time. The amount of friction can't be too large, though, or the passengers would not experience a comfortable ride. The brakes and wheels would also get too hot from the heat generated. The coefficient of kinetic friction should be around 0.4.

5.37 It takes about 2 s to fall 2 m at a 30-degree angle from vertical (the length of the slide); with these data, the coefficient of kinetic friction is 0.34.

5.38 The coefficient of friction between the glass and the lacquered bar should be relatively small so that it's easy to sling a drink down the bar. A coefficient of friction around 0.2–0.3 is reasonable.

5.39 Assume your speed falls from 3 m/s to 1 m/s while sliding along a 4-m-long Slip 'N Slide. Then

$$t = \frac{v_\text{f} - v_\text{i}}{a} = \frac{\left(2\dfrac{\text{m}}{\text{s}}\right)}{a}$$

$$d = v_\text{i}t - \frac{1}{2}at^2$$

$$4\text{ m} = \left(3\frac{\text{m}}{\text{s}}\right)\left(\frac{\left(2\dfrac{\text{m}}{\text{s}}\right)}{a}\right) + \frac{1}{2}a\left(\frac{\left(2\dfrac{\text{m}}{\text{s}}\right)}{a}\right)^2$$

$$a = 2\frac{\text{m}}{\text{s}^2}$$

$$\sum F = \mu mg = ma$$

$$\mu = \frac{a}{g} = \frac{\left(2\frac{\text{m}}{\text{s}^2}\right)}{\left(9.8\frac{\text{m}}{\text{s}^2}\right)} = \boxed{0.2}$$

5.40

Time (s)	Force (N)
0.00	0.00
0.01	1.33
0.05	3.28
0.10	8.11
0.15	8.20
0.20	8.24
0.25	8.26
0.30	7.84
0.35	5.17
0.40	5.21
0.45	5.22
0.50	5.37

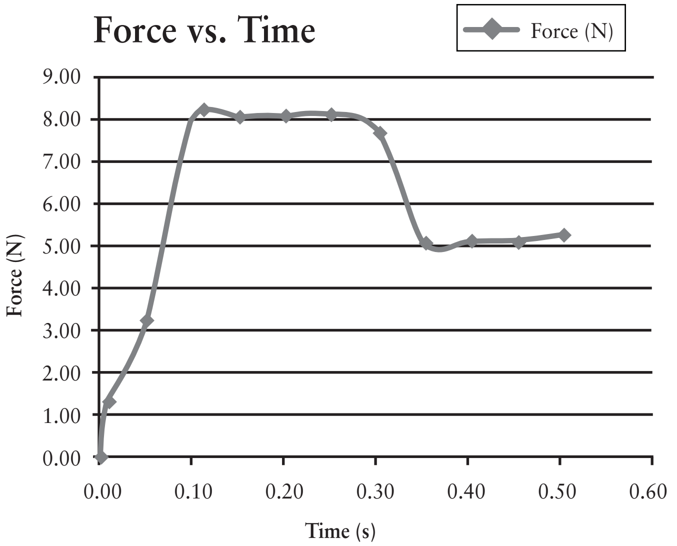


Figure 5-7 Problem 40

The block is stationary until the $t = 0.3$ s mark, at which time it starts to move and kinetic friction takes over.

At $t = 0.20$ s:

$$F_{\text{app}} = F_{\text{sf, max}} = \mu_s N = \mu_s mg$$

$$\mu_s = \frac{F_{\text{app}}}{mg} = \frac{8.24\text{ N}}{(1\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)} = \boxed{0.84}$$

At $t = 0.45$ s:

$$F_{\text{app}} = F_{\text{kf}} = \mu_k N = \mu_k mg$$

$$\mu_s = \frac{F_{\text{app}}}{mg} = \frac{5.22\text{ N}}{(1\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)} = \boxed{0.53}$$

Problems

5.41

SET UP

A 7.6-kg object rests on a level floor with a coefficient of static friction of 0.55. The minimum horizontal applied force that will cause the object to start sliding is equal to the maximum possible magnitude of the static frictional force. This will have a magnitude equal to the

coefficient of static friction multiplied by the normal force acting on the object. Because the object is stationary in the vertical direction, the normal force is equal in magnitude to the weight.

SOLVE

$$F_{\text{app}} = F_{\text{sf, max}} = \mu_s N = \mu_s mg = (0.55)(7.6\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right) = \boxed{41\text{ N}}$$

REFLECT

Usually the magnitude of static friction involves an inequality. We can use an equal sign here because we're looking for the maximum static frictional force.

5.42

SET UP

A 5-kg object rests on a level floor with a coefficient of static friction of 0.67. The minimum horizontal applied force that will cause the object to start sliding is equal to the maximum possible magnitude of the static frictional force. This will have a magnitude equal to the coefficient of static friction multiplied by the normal force acting on the object. Because the object is stationary in the vertical direction, the normal force is equal in magnitude to the weight.

SOLVE

$$F_{\text{app}} = F_{\text{sf, max}} = \mu_s N = \mu_s mg = (0.67)(5\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right) = \boxed{33\text{ N}}$$

REFLECT

Usually the magnitude of static friction involves an inequality. We can use an equal sign here because we're looking for the maximum static frictional force.

5.43

SET UP

A block is sitting at rest on a ramp and there is friction between the block and the ramp, which means we need to consider static friction. The forces acting on the block are the normal force, the force due to gravity, and static friction. The normal force acts perpendicular to the face of the plane; gravity acts straight down, and static friction acts parallel, pointing up the ramp.

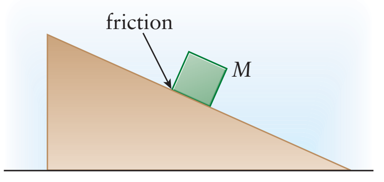


Figure 5-8 Problem 43

SOLVE

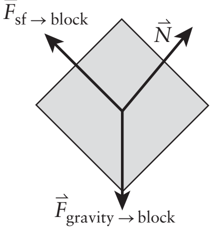


Figure 5-9 Problem 43

REFLECT

When deciding which way friction will act, think about what would happen if there were *no* friction. Friction will act in the opposite direction of the motion. In this case, the block would slide down the ramp, so static friction is acting up the ramp to oppose this motion.

5.44

SET UP

A horizontal force F is applied to a mass M_2, which rests on a frictionless table. Mass M_1 sits atop M_2 and there is friction between these two objects. We expect M_1 to remain stationary relative to M_2 as M_2 is being pushed, which means static friction is acting on it. The normal force, gravity, and force F act on M_2. The normal force, gravity, and static friction act on M_1.

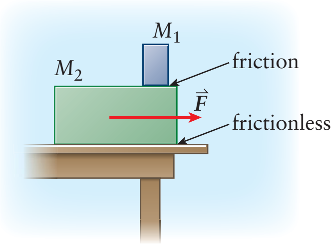


Figure 5-10 Problem 44

SOLVE

Free-body diagram of M_1:

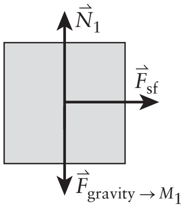


Figure 5-11 Problem 44

Free-body diagram of M_2:

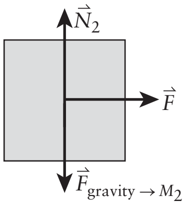


Figure 5-12 Problem 44

REFLECT

We expect M_1 to move with M_2, which means there must be a net force acting in the horizontal direction. It cannot be $\vec{F}$ since that is applied to M_2. It cannot be kinetic friction since M_1 does not slide relative to M_2. Therefore, it must be static friction.

5.45

SET UP

A horizontal force $F_{app} = 10$ N is applied to a stationary 2-kg block. The coefficient of static friction between the block and the floor is 0.75. In order to determine the motion of the block, we first need to determine which friction—static or kinetic—needs to be considered. Newton's second law in the vertical direction will give us the magnitude of the normal force, which is used in calculating the magnitudes of the frictional forces. We should first calculate the magnitude of the static frictional force. If it is larger than 10 N, the block won't move; if it's less than 10 N,

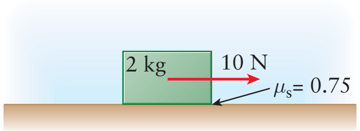


Figure 5-13 Problem 45

then we calculate the magnitude of kinetic friction and use that to determine the acceleration in the horizontal direction.

SOLVE

Free-body diagram of block:

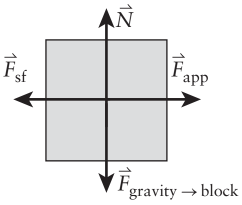


Figure 5-14 Problem 45

Newton's second law:

$$\sum F_y = N - F_{\text{gravity}\to\text{block}} = N - mg = ma_y = 0$$

$$N = mg$$

Magnitude of static friction:

$$F_{\text{sf}} \leq \mu_{\text{s}} N = \mu_{\text{s}} mg = (0.75)(2\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right) = 14.7\text{ N}$$

The applied force of 10 N is not enough to overcome static friction, which means the $\boxed{\text{block will remain stationary}}$.

REFLECT

The information regarding kinetic friction is irrelevant for this problem.

5.46

SET UP

A block of mass $m = 3.1$ kg is attached to a spring with spring constant $k = 250$ N/m and placed on a horizontal surface. The coefficient of static friction between the block and the horizontal surface is $\mu_{\text{s}} = 0.24$. We are interested in the maximum distance Δx that the block can be pulled and released from rest such that the block remains stationary. Since the block is at rest, the net force acting in the horizontal direction is equal to zero, which means the magnitude of the maximum static frictional force will be equal to the magnitude of the spring force. Solving Newton's second law in the y direction for the magnitude of the normal force and plugging this into the magnitude of the static frictional force in the x direction will give us the maximum distance Δx.

SOLVE

Free-body diagram of block:

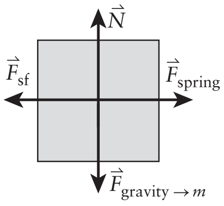


Figure 5-15 Problem 46

Newton's second law:

$$\sum F_y = N - F_{\text{gravity}\to\text{block}} = N - mg = ma_y = 0$$

$$N = mg$$

$$\sum F_x = F_{\text{spring}} - F_{\text{sf}} = k(\Delta x) - \mu_s N = ma_x = 0$$

$$k(\Delta x) = \mu_s N$$

$$\Delta x = \frac{\mu_s N}{k} = \frac{\mu_s mg}{k} = \frac{(0.24)(3.1\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)}{\left(250\frac{\text{N}}{\text{m}}\right)} = \boxed{0.029\text{ m} = 2.9\text{ cm}}$$

REFLECT

Since we used the maximum value of static friction, the block can be pulled (or pushed) a distance less than 2.9 cm and still remain at rest.

5.47

SET UP

A coin (mass $m = 25$ g) sits on a turntable that is rotating at 78 rev/min. The coin is located 13 cm from the center of the turntable, which means the coin travels $2\pi(0.13)$ m in one revolution. We'll draw a free-body diagram of the coin while it is traveling on the turntable and the center of the turntable is directly to the coin's right. This means the coin is undergoing centripetal acceleration in the positive x direction; static friction is the force causing this motion. (The coin is not slipping relative to the turntable.) The coin is not moving in the y direction, so $a_y = 0$. Solving Newton's second law in the y direction for the magnitude of the normal force and plugging this into the magnitude of the static frictional force in the x direction will give us the minimum value of μ_s.

SOLVE

Free-body diagram of the coin:

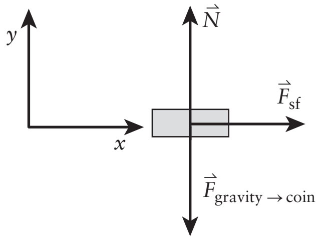


Figure 5-16 Problem 47

Newton's second law:

$$\sum F_y = N - F_{\text{gravity}\to\text{coin}} = N - mg = ma_y = 0$$

$$N = mg$$

$$\sum F_x = F_{\text{sf}} = \mu_s N = \mu_s mg = ma_x = m\left(\frac{v^2}{R}\right)$$

$$\mu_s g = \frac{v^2}{R}$$

$$\mu_s = \frac{v^2}{gR} = \frac{\left(\frac{78\ \text{rev}}{\text{min}} \times \frac{1\ \text{min}}{60\ \text{s}} \times \frac{2\pi(0.13\ \text{m})}{1\ \text{rev}}\right)^2}{\left(9.8\frac{\text{m}}{\text{s}^2}\right)(0.13\ \text{m})} = \boxed{0.885}$$

REFLECT

This is a reasonable value for a coefficient of static friction. Note that the mass of the coin did not factor into our calculation.

5.48

SET UP

An object experiences a horizontal kinetic frictional force equal to 12.7 N. The coefficient of kinetic friction in this case is 0.37. The magnitude of kinetic friction is equal in magnitude to the product of the coefficient of kinetic friction and the normal force. Assuming that the object is not accelerating in the vertical direction, the magnitude of the normal force is equal to the object's weight. Setting the forces equal we can solve for the mass of the object.

SOLVE

$$F_{\text{kf}} = \mu_k N = \mu_k mg$$

$$m = \frac{F_{\text{kf}}}{\mu_k g} = \frac{12.7\ \text{N}}{(0.37)\left(9.8\frac{\text{m}}{\text{s}^2}\right)} = \boxed{3.5\ \text{kg}}$$

REFLECT

The magnitude of the force of friction is almost one-third the weight of the object.

5.49

SET UP

A book is pushed across a horizontal table with a force equal to one-half the book's weight. The book is traveling at a constant speed, which means its acceleration in this direction is equal to zero. The only forces acting in the horizontal direction are the applied force and kinetic friction, and they act in opposite directions. The magnitude of the kinetic frictional force is constant and equal to the product of the coefficient of kinetic friction and the normal force. Assuming that the object is not accelerating in the vertical direction, the magnitude of the normal force is equal to the object's weight. Setting the forces equal we can solve for the coefficient of kinetic friction.

SOLVE

$$\sum F_{\text{horizontal}} = F_{\text{app}} - F_{\text{kf}} = ma_{\text{horizontal}} = 0$$

$$F_{\text{app}} = F_{\text{kf}} = \mu_k N = \mu_k F_{\text{gravity}\to\text{book}}$$

$$\mu_k = \frac{F_{app}}{F_{gravity \to book}} = \frac{\left(\frac{F_{gravity \to book}}{2}\right)}{F_{gravity \to book}} = \boxed{0.5}$$

REFLECT

This is a reasonable value for the coefficient of kinetic friction for a book sliding across a table.

5.50

SET UP

A 50-N force is applied to a 25-kg crate and the crate accelerates at a rate of 1 m/s^2. The forces acting in the vertical direction are the normal force pointing up and gravity pointing down. The net force in this direction is equal to zero because the crate does not accelerate in this direction. The forces acting in the horizontal direction are the applied force and kinetic friction. We'll use a coordinate system where right and up point toward positive x and y. We can use Newton's second law and the definition of the magnitude of kinetic friction to solve for the magnitudes of the normal force, kinetic friction, and the coefficient of kinetic friction.

SOLVE

Free-body diagram of the crate:

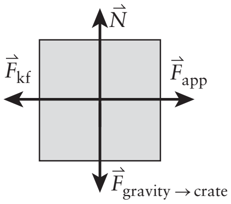


Figure 5-17 Problem 50

Part a)

$$\sum F_y = N - F_{gravity \to crate} = N - mg = ma_y = 0$$

$$N = mg = (25 \text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right) = \boxed{245 \text{ N}}$$

Part b)

$$\sum F_x = F_{app} - F_{kf} = ma_x$$

$$F_{kf} = F_{app} - ma_x = (50 \text{ N}) - (25 \text{ kg})\left(1\frac{\text{m}}{\text{s}^2}\right) = \boxed{25 \text{ N}}$$

Part c)

$$F_{kf} = \mu_k N$$

$$\mu_k = \frac{F_{kf}}{N} = \frac{25 \text{ N}}{245 \text{ N}} = \boxed{0.10}$$

REFLECT

This coefficient would imply that the crate is sliding on a well-lubricated surface. A value of 0.10 is relatively small.

5.51

SET UP

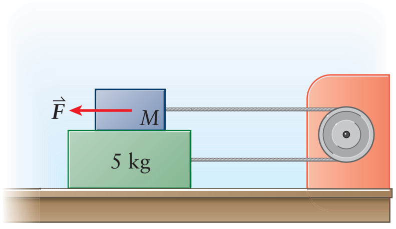


Figure 5-18 Problem 51

A block of mass M sits on top of a 5.00-kg block that is on a tabletop. A string with tension T connects the blocks around a frictionless peg. There is friction between both blocks and between the 5.00-kg block and the table; the coefficient of kinetic friction in both cases is $\mu_k = 0.330$. An external force of magnitude $F = 60.0$ N is pulling the top block to the left and the bottom block to the right at a constant speed, which means the acceleration of each block is zero. We'll consider up to be the positive y-axis. For the x-axis we'll use an axis that points along the string in the direction of the motion of the blocks. Therefore, right is positive for the bottom block and left is positive for the top block. The forces acting on the unknown mass are the normal force upward, gravity downward, the external force to the left, tension to the right, and kinetic friction to the right. The forces acting on the 5-kg block are the normal force upward, gravity downward, the contact force from the unknown mass downward, tension to the right, and two kinetic frictional forces to the right—one due to the interaction with the unknown mass and one due to the interaction with the tabletop. The friction acting on the 5-kg block due to the unknown mass is the third law partner to the kinetic friction in the free-body diagram of the unknown mass. The contact force acting on the 5-kg block due to the unknown mass is the third law partner to the normal force acting on the unknown mass. The friction acting on the 5-kg block due to the tabletop has a magnitude related to the normal force acting on the 5-kg block.

SOLVE

Free-body diagram of M:

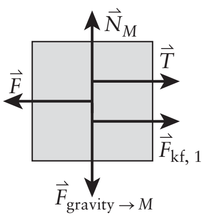


Figure 5-19 Problem 51

Free-body diagram of the 5-kg block:

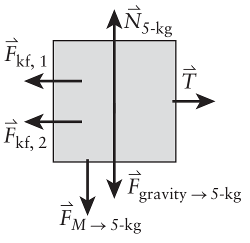


Figure 5-20 Problem 51

Newton's second law, y components:

$$\sum F_{M,\,y} = N_M - F_{\text{gravity}\to M} = N_M - Mg = Ma_y = 0$$

$$N_M = Mg$$

$$\sum F_{5\text{-kg},\,y} = N_{5\text{-kg}} - F_{\text{gravity}\to 5\text{-kg}} - F_{M\to 5\text{-kg}} = N_{5\text{-kg}} - m_{5\text{-kg}}g - N_M$$

$$= N_{5\text{-kg}} - m_{5\text{-kg}}g - Mg = m_{5\text{-kg}}a_y = 0$$

$$N_{5\text{-kg}} = (M + m_{5\text{-kg}})g$$

Newton's second law, x components:

$$\sum F_{M,\,x} = F - T - F_{\text{kf},\,1} = F - T - \mu_k N_M = F - T - \mu_k Mg = Ma_x = 0$$

$$F - T - \mu_k Mg = 0$$

$$\sum F_{5\text{-kg},\,x} = T - F_{\text{kf},1} - F_{\text{kf},\,2} = T - \mu_k N_M - \mu_k N_{5\text{-kg}}$$

$$= T - \mu_k Mg - \mu_k(M + m_{5\text{-kg}})g = m_{5\text{-kg}}a_x = 0$$

Adding the x component equations together:

$$F - 3\mu_k Mg - \mu_k m_{5\text{-kg}}g = 0$$

$$M = \frac{F - \mu_k m_{5\text{-kg}}g}{3\mu_k g} = \frac{(60.0\text{ N}) - (0.330)(5.00\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)}{3(0.330)\left(9.8\frac{\text{m}}{\text{s}^2}\right)} = \boxed{4.52\text{ kg}}$$

REFLECT

The magnitude of the normal force is not always equal to the magnitude of the force of gravity acting on the object. For example, two forces are acting downward on the 5-kg mass, so the normal force is larger than the weight of the 5-kg mass.

5.52

SET UP

A mop head (mass $m = 3.75$ kg) is being pushed around a floor with a force of $F = 50$ N at an angle of 50 degrees above the x-axis. The coefficient of kinetic friction between the mop head and the floor is $\mu_k = 0.330$. The mop head is only accelerating in the positive x direction, so $a_y = 0$. We can use Newton's second law to solve for the acceleration of the mop head in the x direction.

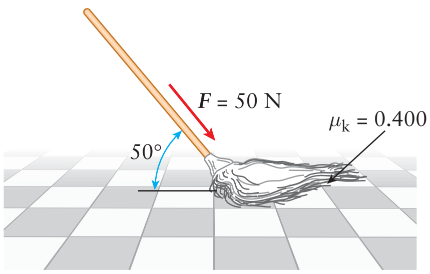


Figure 5-21 Problem 52

SOLVE

Free-body diagram of the mop head:

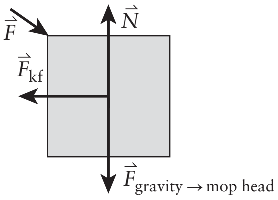


Figure 5-22 Problem 52

Newton's second law:

$$\sum F_y = N - F_{gravity \to mop} - F_{app,\,y} = N - mg - F\sin(50°) = ma_y = 0$$

$$N = mg + F\sin(50°)$$

$$\sum F_x = F_{app,\,x} - F_{kf} = F\cos(50°) - \mu_k N = F\cos(50°) - \mu_k(mg + F\sin(50°)) = ma_x$$

$$a_x = \frac{F\cos(50°) - \mu_k(mg + F\sin(50°))}{m}$$

$$= \frac{(50\ \text{N})\cos(50°) - (0.400)\left((3.75\ \text{kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right) + (50\ \text{N})\sin(50°)\right)}{3.75\ \text{kg}} = \boxed{0.57\frac{\text{m}}{\text{s}^2}}$$

REFLECT

Even though the mop head is not moving in the y direction, we still need to solve the y component of Newton's second law in order to determine the normal force acting on the mop. The normal force (that is, the contact force acting on the mop head due to the floor) is not equal to the weight of the mop head.

5.53

SET UP

A 12-kg block and a 5-kg block are connected by a taut string with tension T. The coefficient of static friction between the 12-kg block and the floor is $\mu_{s,\,12\text{-kg}} = 0.443$ and the coefficient of static friction between the 5-kg block and the floor is $\mu_{s,\,5\text{-kg}} = 0.573$. An external force F pulling to the right is applied to the 5-kg block. We need to determine, using Newton's second law, the minimum F necessary to overcome static friction and accelerate the blocks toward positive x. Since the mass is positive and the x component of the acceleration will be positive when the blocks move, the net force acting on each block also needs to be positive (that is, greater than zero).

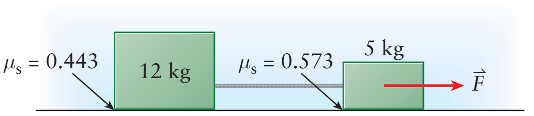


Figure 5-23 Problem 53

SOLVE

Free-body diagram of the 12-kg block:

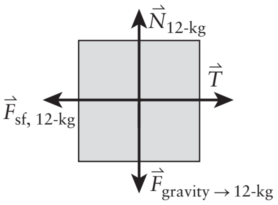


Figure 5-24 Problem 53

Free-body diagram of the 5-kg block:

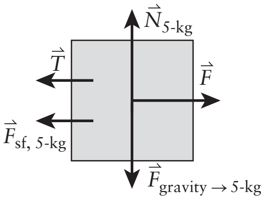


Figure 5-25 Problem 53

Newton's second law, y components:

$$\sum F_{\text{12-kg}, y} = N_{\text{12-kg}} - F_{\text{gravity}\rightarrow\text{12-kg}} = N_{\text{12-kg}} - m_{\text{12-kg}}g = m_{\text{12-kg}}a_y = 0$$

$$N_{\text{12-kg}} = m_{\text{12-kg}}g$$

$$\sum F_{\text{5-kg}, y} = N_{\text{5-kg}} - F_{\text{gravity}\rightarrow\text{5-kg}} = N_{\text{5-kg}} - m_{\text{5-kg}}g = m_{\text{5-kg}}a_y = 0$$

$$N_{\text{5-kg}} = m_{\text{5-kg}}g$$

Newton's second law, x components:

$$\sum F_{\text{12-kg}, x} = T - F_{\text{sf}\rightarrow\text{12-kg}} = T - \mu_{\text{s, 12-kg}}N_{\text{12-kg}} = T - \mu_{\text{s, 12-kg}}m_{\text{12-kg}}g = m_{\text{12-kg}}a_x$$

$$\sum F_{\text{5-kg}, x} = F - T - F_{\text{sf}\rightarrow\text{5-kg}} = F - T - \mu_{\text{s, 5-kg}}N_{\text{5-kg}} = F - T - \mu_{\text{s, 5-kg}}m_{\text{5-kg}}g = m_{\text{5-kg}}a_x$$

Adding the x-component equations together:

$$F - g(\mu_{\text{s, 12-kg}}m_{\text{12-kg}} + \mu_{\text{s, 5-kg}}m_{\text{5-kg}}) = (m_{\text{5-kg}} + m_{\text{12-kg}})a_x$$

Setting up and solving the inequality:

$$F - g(\mu_{\text{s, 12-kg}}m_{\text{12-kg}} + \mu_{\text{s, 5-kg}}m_{\text{5-kg}}) = (m_{\text{5-kg}} + m_{\text{12-kg}})a_x \geq 0$$

$$F \geq g(\mu_{\text{s, 12-kg}}m_{\text{12-kg}} + \mu_{\text{s, 5-kg}}m_{\text{5-kg}}) = \left(9.8\frac{\text{m}}{\text{s}^2}\right)((0.443)(12\text{ kg}) + (0.573)(5\text{ kg})) = \boxed{80.2\text{ N}}$$

REFLECT

The maximum value of static friction is when $F_{\text{sf}} = \mu_s N$. When the magnitude of the applied force is larger than this value, the system will start to accelerate.

5.54

SET UP

A 2-kg box is sitting on top of a 5-kg crate. The coefficient of static friction between the crate and the floor is $\mu_{s,1} = 0.400$ and the coefficient of static friction between the boxes is $\mu_{s,2} = 0.667$. A rope is attached to the left of the 2-kg box and is connected to the wall; the rope is initially slack. An external force F pulls the 5-kg crate to the right. While the rope is slack, the box and the crate will move as one 7-kg object to the right. The minimum force F necessary to accelerate the crate to the right is equal in magnitude to the static frictional force acting on this 7-kg object. As the two boxes move to the right, the rope will eventually become taut. The tension in the rope when it becomes taut will be equal to the magnitude of static friction acting on the 2-kg box. After this point, the 2-kg box will slide relative to the 5-kg crate.

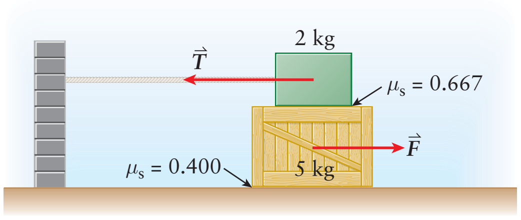


Figure 5-26 Problem 54

SOLVE

Free-body diagram for the 7-kg object:

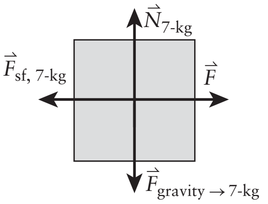


Figure 5-27 Problem 54

Newton's second law:

$$\sum F_{7\text{-kg}, y} = N_{7\text{-kg}} - F_{\text{gravity}\rightarrow 7\text{-kg}} = N_{7\text{-kg}} - m_{7\text{-kg}}g = m_{7\text{-kg}}a_y = 0$$

$$N_{7\text{-kg}} = m_{7\text{-kg}}g$$

$$\sum F_{7\text{-kg}, x} = F - F_{\text{sf, 7-kg}} = F - \mu_{s,1}N_{7\text{-kg}} = F - \mu_{s,1}m_{7\text{-kg}}g = m_{7\text{-kg}}a_x = 0$$

$$F = \mu_{s,1}m_{7\text{-kg}}g = (0.400)(7\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right) = \boxed{27.4\text{ N}}$$

Free-body diagram for the 2-kg box when the rope is taut:

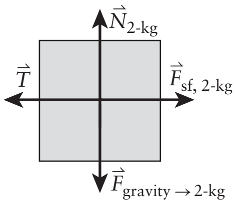


Figure 5-28 Problem 54

Newton's second law:

$$\sum F_{2\text{-kg}, y} = N_{2\text{-kg}} - F_{\text{gravity}\rightarrow 2\text{-kg}} = N_{2\text{-kg}} - m_{2\text{-kg}}g = m_{2\text{-kg}}a_y = 0$$

$$N_{2\text{-kg}} = m_{2\text{-kg}}g$$

$$\sum F_{2\text{-kg}, x} = -T + F_{\text{sf, 2-kg}} = -T + \mu_{\text{s, 2}}N_{2\text{-kg}} = -T + \mu_{\text{s, 2}}m_{2\text{-kg}}g = m_{2\text{-kg}}a_x = 0$$

$$T = \mu_{\text{s, 2}}m_{2\text{-kg}}g = (0.667)(2\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right) = \boxed{13.1\text{ N}}$$

REFLECT

Thinking about what would happen in real life usually simplifies the calculations. By realizing the box and crate will initially move as one we can define our system as a combined 7-kg object. The contact forces and frictional forces between the box and the crate are now internal to the system and will not factor into our free-body diagram.

5.55

SET UP

A 3-kg object is being pushed with a force F at an angle of 30 degrees below the positive x-axis. The coefficient of kinetic friction between the object and the floor is 0.400. We can use Newton's second law to calculate F such that the object accelerates at a rate of 2.5 m/s^2 in the x direction.

SOLVE

Free-body diagram of object:

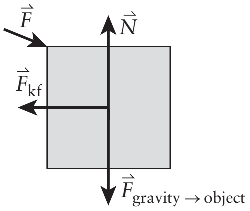


Figure 5-29 Problem 54

Newton's second law:

$$\sum F_y = N - F_{\text{app}, y} - F_{\text{gravity}\rightarrow\text{object}} = N - F\sin(30°) - mg = ma_y = 0$$

$$N = F\sin(30°) + mg$$

$$\sum F_x = F_{\text{app}, x} - F_{\text{kf}} = F_{\text{app}}\cos(30°) - \mu_{\text{k}}N = F_{\text{app}}\cos(30°) - \mu_{\text{k}}N = ma_x$$

$$F_{\text{app}}\cos(30°) - \mu_{\text{k}}(F\sin(30°) + mg) = ma_x$$

$$F_{\text{app}} = \frac{m(a_x + g)}{\cos(30°) - \mu_{\text{k}}\sin(30°)} = \frac{(3\text{ kg})\left(\left(2.5\frac{\text{m}}{\text{s}^2}\right) + \left(9.8\frac{\text{m}}{\text{s}^2}\right)\right)}{\cos(30°) - (0.4)\sin(30°)} = \boxed{28.9\text{ N}}$$

REFLECT

If we pushed the object straight on at an angle of 0 degrees, we would only need a force of 19.3 N.

5.56

SET UP

Two blocks are connected by a string. Block 1 ($m_1 = 2.85$ kg) is being held in place on a ramp at a 40-degree angle. The coefficient of static friction between block 1 and the ramp is $\mu_s = 0.552$. Block 2 ($m_2 = 4.75$ kg) is hanging off the edge of the ramp. The person then releases block 1 and both blocks begin to move. To determine the initial acceleration of the blocks, we can use the maximum static frictional force in our Newton's second law calculation. Once we have the acceleration of the system, we can calculate the tension in the string. We'll use an axis pointing along the string where up the ramp is considered positive. Out of the ramp will be positive for the perpendicular axis for block 1.

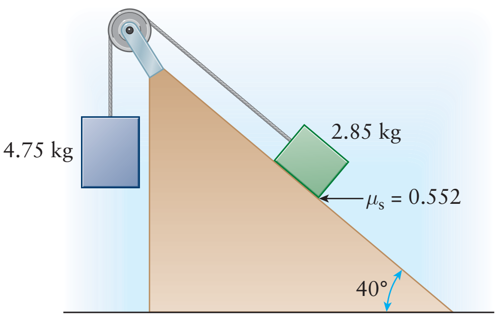


Figure 5-30 Problem 56

SOLVE

Free-body diagram of block 1 ($m_1 = 2.85$ kg):

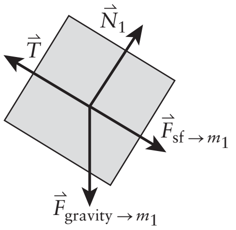


Figure 5-31 Problem 56

Free-body diagram of block 2 ($m_2 = 4.75$ kg):

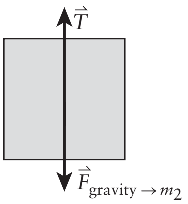


Figure 5-32 Problem 56

Newton's second law:

$$\sum F_{1,\perp} = N_1 - F_{gravity\to 1,\perp} = N_1 - m_1 g\cos(40°) = m_1 a_\perp = 0$$

$$N_1 = m_1 g\cos(40°)$$

$$\sum F_{1,\parallel} = T - F_{sf,1} - F_{gravity\to 1,\parallel} = T - \mu_s N_1 - m_1 g\sin(40°)$$

$$= T - \mu_s m_1 g\cos(40°) - m_1 g\sin(40°) = m_1 a_\parallel$$

$$\sum F_{2,y} = F_{gravity\to 2} - T = m_2 g - T = m_2 a_\parallel$$

Adding the equations together:

$$m_2 g - \mu_s m_1 g\cos(40°) - m_1 g\sin(40°) = (m_1 + m_2)a_\parallel$$

$$a_{\parallel} = \frac{m_2 g - \mu_s m_1 g\cos(40°) - m_1 g\sin(40°)}{m_1 + m_2}$$

$$= \frac{(4.75\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right) - (0.552)(2.85\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)\cos(40°) - (2.85\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)\sin(40°)}{(4.75\text{ kg}) + (2.85\text{ kg})}$$

$$= \boxed{6.92\frac{\text{m}}{\text{s}^2}}$$

$$T = m_2(g - a_{\parallel}) = (4.75\text{ kg})\left(\left(9.8\frac{\text{m}}{\text{s}^2}\right) - \left(6.92\frac{\text{m}}{\text{s}^2}\right)\right) = \boxed{13.7\text{ N}}$$

REFLECT

It makes sense that the block should be pulled up the ramp since the hanging block is heavier than the block on the ramp. After the blocks start moving, kinetic friction (rather than static friction) will act on block 1.

5.57

SET UP

Two blocks are connected by a string. Block 1 is hanging over the edge of a ramp. Block 2 ($m_2 = 8$ kg) is resting on a 28-degree slope. The coefficient of kinetic friction between block 2 and the ramp is 0.22. We are told block 2 is sliding down the ramp at a constant speed, which means its acceleration is constant in all directions. Because the two blocks are connected, block 1 is also traveling at a constant speed. We can use Newton's second law to calculate the mass of block 1. We will use an axis that points along the string, starting from block 1, and down the ramp will be positive.

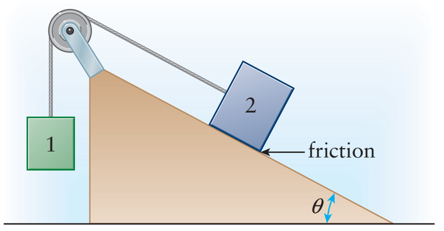


Figure 5-33 Problems 57 and 58

SOLVE

Free-body diagram of block 1:

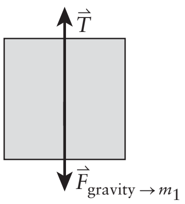


Figure 5-34 Problem 57

Free-body diagram of block 2:

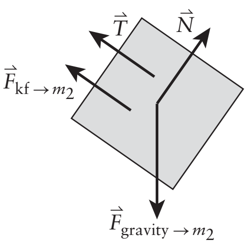


Figure 5-35 Problem 57

Newton's second law:

$$\sum F_{1,y} = T - F_{\text{gravity}\to 1} = T - m_1 g = m_1 a = 0$$

$$T = m_1 g$$

$$\sum F_{2,\perp} = N - F_{\text{gravity}\to 2,\perp} = N - m_2 g\cos(28°) = m_2 a_\perp = 0$$

$$N = m_2 g\cos(28°)$$

$$\sum F_{2,\parallel} = F_{\text{gravity}\to 2,\parallel} - T - F_{\text{kf}} = m_2 g\sin(28°) - T - \mu_k N$$

$$= m_2 g\sin(28°) - T - \mu_k(m_2 g\cos(28°))$$

$$= m_2 g\sin(28°) - m_1 g - \mu_k(m_2 g\cos(28°)) = 0$$

$$m_1 = m_2(\sin(28°) - \mu_k\cos(28°)) = (8\text{ kg})(\sin(28°) - (0.22)\cos(28°)) = \boxed{2.2\text{ kg}}$$

REFLECT

We are told block 2 is sliding down the ramp, so we should expect the mass of block 1 to be less than 8 kg; our answer makes sense.

5.58

SET UP

Two blocks are connected by a string. Block 1 is hanging over the edge of a ramp. Block 2 ($m_2 = 10$ kg) is resting on a 30-degree slope. The coefficient of kinetic friction between block 2 and the ramp is 0.20. We are told block 2 is sliding up the ramp at a constant speed, which means its acceleration is constant in all directions. Because the two blocks are connected, block 1 is also traveling at a constant speed. We can use Newton's second law to calculate the mass of block 1. We will use an axis that points along the string, starting from block 2, and up the ramp will be positive.

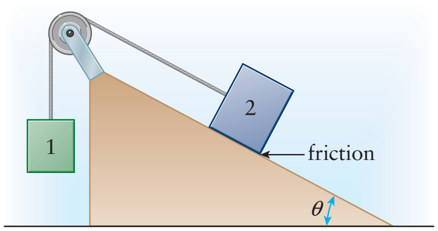


Figure 5-36 Problems 57 and 58

SOLVE

Free-body diagram of block 1:

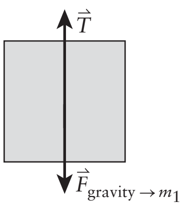


Figure 5-37 Problem 58

Free-body diagram of block 2:

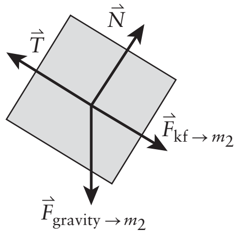


Figure 5-38 Problem 58

Newton's second law:

$$\sum F_{1,\,y} = -T + F_{\text{gravity}\to 1} = -T + m_1 g = m_1 a = 0$$

$$T = m_1 g$$

$$\sum F_{2,\,\perp} = N - F_{\text{gravity}\to 2,\,\perp} = N - m_2 g\cos(30°) = m_2 a_\perp = 0$$

$$N = m_2 g\cos(30°)$$

$$\sum F_{2,\,\parallel} = T - F_{\text{gravity}\to 2,\,\parallel} - F_{\text{kf}} = T - m_2 g\sin(30°) - \mu_{\text{k}} N$$

$$= T - m_2 g\sin(30°) - \mu_{\text{k}}(m_2 g\cos(30°))$$

$$= m_1 g - m_2 g\sin(30°) - \mu_{\text{k}} m_2 g\cos(30°) = m_2 a = 0$$

$$m_1 = m_2(\sin(30°) + \mu_{\text{k}}\cos(30°)) = (10\text{ kg})(\sin(30°) + (0.20)\cos(30°)) = \boxed{6.7\text{ kg}}$$

REFLECT

Although this question may seem the same as Problem 5.57, kinetic friction acts in different directions in the two problems. Remember that the direction in which kinetic friction acts is opposite to the motion of the object.

5.59

SET UP

A paramecium propels itself through the water by using its cilia, which generate a force $F_{\text{propulsion}}$. As the organism travels through the water, it experiences a drag force acting in a direction opposite to its motion with a magnitude of bv^2, where b is approximately 0.31 and v is the speed. The paramecium is traveling at a terminal speed of $0.15 \times 10^{-3}\frac{\text{m}}{\text{s}}$, which means the magnitude of the propulsion force equals the magnitude of the drag force. Setting these magnitudes equal, we can solve for the propulsion force. We'll assume the paramecium is traveling toward $+x$.

SOLVE

$$\sum F_x = F_{\text{propulsion}} - F_{\text{drag}} = F_{\text{propulsion}} - bv^2 = ma_x = 0$$

$$F_{\text{propulsion}} = bv^2 = (0.31)\left(0.15 \times 10^{-3}\frac{\text{m}}{\text{s}}\right)^2 = \boxed{7.0 \times 10^{-9}\text{ N}}$$

REFLECT

This seems like a reasonable force that a paramecium can generate.

5.60

SET UP

An *E.coli* bacterium propels itself with flagella. When the flagella exert a force of F_1 the bacterium travels at a constant speed of 20 microns/second. From Newton's second law we know the net force must equal zero if the bacterium is traveling at a constant speed. The only forces acting on the *E. coli* are the force from the flagella and the drag force. Since the speed of the bacterium is so slow, we expect the drag force to be proportional to its speed to the first power. Therefore, we can use proportional reasoning to determine the speed of the bacterium if the force due to the flagella is doubled.

SOLVE

$$F = bv$$

$$\frac{F_2}{F_1} = \frac{2F_1}{F_1} = 2 = \frac{bv_2}{bv_1} = \frac{v_2}{v_1}$$

$$v_2 = 2v_1 = 2\left(20\frac{\mu\text{m}}{\text{s}}\right) = \boxed{40\frac{\mu\text{m}}{\text{s}}}$$

REFLECT

The drag force is usually linear in the speed for very tiny objects, such as microorganisms.

5.61

SET UP

A 5-kg object is traveling at a terminal speed of 50 m/s. The magnitude of the drag force it experiences is equal to $F_{\text{drag}} = bv^{2.5}$. We can set the drag force equal to the weight of the object in order to calculate the value of b.

SOLVE

$$F_{\text{gravity}\to\text{object}} = F_{\text{drag}}$$

$$mg = bv^n$$

$$b = \frac{mg}{v^n} = \frac{(5\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)}{\left(50\frac{\text{m}}{\text{s}}\right)^{2.5}} = \boxed{0.00277}$$

REFLECT

The SI units for b in this case are $\text{kg}\cdot\left(\frac{\text{s}}{\text{m}^3}\right)^{0.5}$.

5.62

SET UP

The drag force of a person falling through the atmosphere has a magnitude of bv^2. The person will reach terminal velocity when the magnitude of the drag force equals the person's weight. Setting these equal, we can solve for either v or b.

SOLVE

Part a)

$$F_{\text{gravity}\rightarrow\text{person}} = F_{\text{drag}}$$

$$mg = bv^2$$

$$v = \sqrt{\frac{mg}{b}} = \sqrt{\frac{(70\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)}{\left(18\frac{\text{kg}}{\text{m}}\right)}} = \boxed{6.2\frac{\text{m}}{\text{s}}}$$

Part b)

$$mg = bv^2$$

$$b = \frac{mg}{v^2} = \frac{(70\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)}{\left(50\frac{\text{m}}{\text{s}}\right)^2} = \boxed{0.27\frac{\text{kg}}{\text{m}}}$$

REFLECT

It makes sense that the drag coefficient is much smaller for a person without a parachute than a person with a parachute.

5.63

SET UP

A girl is riding her scooter (combined mass $m = 50$ kg) down a 10-degree incline. If she coasts (that is, doesn't pedal) down the hill, she'll travel at a constant speed of 12 m/s while experiencing a drag force that is proportional to the square of her speed. Since her speed is constant in this case, the magnitude of the drag force is equal to the magnitude of her weight parallel to the plane. This allows us to solve for the drag coefficient b, which only depends on the object and the air, not the speed. In order to travel down the hill at a faster speed, an external applied force is required to accelerate her and overcome the drag force. Now there is another force acting in the same direction as the parallel component of her weight. We can use Newton's second law and the earlier expression for b to solve for the magnitude of this applied force F_{app}.

SOLVE

Free-body diagram of the girl at speed 1 ($v_1 = 12$ m/s):

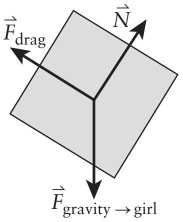


Figure 5-39 Problem 63

Newton's second law for speed 1 ($v_1 = 12$ m/s):

$$\sum F_{\parallel} = F_{\text{gravity}\to\text{girl},\,\parallel} - F_{\text{drag}} = mg\sin(10°) - bv_1^2 = ma_{\parallel} = 0$$

$$b = \frac{mg\sin(10°)}{v_1^2}$$

Free-body diagram of the girl at speed 1 ($v_1 = 20$ m/s):

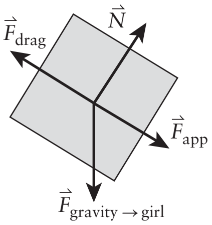


Figure 5-40 Problem 63

Newton's second law for speed 2 ($v_2 = 20$ m/s):

$$\sum F_{\parallel} = F_{\text{app}} + F_{\text{gravity}\to\text{girl},\,\parallel} - F_{\text{drag}} = F_{\text{app}} + mg\sin(10°) - bv_2^2 = ma_{\parallel} = 0$$

$$F_{\text{app}} = bv_2^2 - mg\sin(10°) = \left(\frac{mg\sin(10°)}{v_1^2}\right)v_2^2 - mg\sin(10°) = mg\sin(10°)\left(\frac{v_2^2}{v_1^2} - 1\right)$$

$$= (50\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)\sin(10°)\left(\frac{\left(20\frac{\text{m}}{\text{s}}\right)^2}{\left(12\frac{\text{m}}{\text{s}}\right)^2} - 1\right) = \boxed{151\text{ N}}$$

REFLECT

A force is required in order to overcome drag and increase the speed of the girl. It makes sense that the magnitude of this force should depend on the ratio of the old and new speeds.

5.64

SET UP

The net force acting on a free-falling object experiencing linear drag is $F = mg - bv$. The product bv must have dimensions of force. Acceleration is the first derivative of the velocity with respect to time, which means this is a differential equation. Specifically, since the velocity

is to the first power, this is a first-order linear ordinary differential equation. We can solve (that is, find $v(t)$) this type of differential equation by solving for the multiplicative factor μ, such that $\frac{d}{dt}(\mu v) = \mu g$.

SOLVE

Part a)

$$[F] = [b][v]$$

In SI units:

$$\frac{\mathrm{kg} \cdot \mathrm{m}}{\mathrm{s}^2} = [b]\left(\frac{\mathrm{m}}{\mathrm{s}}\right)$$

$$\boxed{[b] = \frac{\mathrm{kg}}{\mathrm{s}}}$$

Part b)

$$F = ma = m\left(\frac{dv}{dt}\right) = mg - bv$$

$$\frac{dv}{dt} + \frac{b}{m}v = g$$

Solving the first-order linear differential equation:

$$\mu = e^{\int \frac{b}{m} dt} = e^{\frac{bt}{m}}$$

$$\mu\left(\frac{dv}{dt} + \frac{b}{m}v = g\right)$$

$$e^{\frac{bt}{m}}\left(\frac{dv}{dt}\right) + \left(\frac{b}{m}\right)e^{\frac{bt}{m}}v = ge^{\frac{bt}{m}}$$

$$\frac{d}{dt}\left(e^{\frac{bt}{m}}v\right) = ge^{\frac{bt}{m}}$$

$$e^{\frac{bt}{m}}v = \int ge^{\frac{bt}{m}}dt = g\left[\frac{m}{b}e^{\frac{bt}{m}}\right] + C$$

$\boxed{v(t) = \frac{gm}{b} + Ce^{-\frac{bt}{m}}}$, where C is a constant of integration.

REFLECT

We can double-check that our function $v(t)$ satisfies the original differential equation:

$$v(t) = \frac{gm}{b} + Ce^{-\frac{bt}{m}}$$

$$\frac{dv}{dt} = \frac{d}{dt}\left(\frac{gm}{b} + Ce^{-\frac{bt}{m}}\right) = 0 + C\left(-\frac{b}{m}e^{-\frac{bt}{m}}\right) = \frac{-Cb}{m}e^{-\frac{bt}{m}}$$

$$a = \frac{dv}{dt} = \frac{-Cb}{m}e^{-\frac{bt}{m}} \stackrel{?}{=} g - \frac{b}{m}v = g - \frac{b}{m}\left(\frac{gm}{b} + Ce^{-\frac{bt}{m}}\right)$$

$$\frac{-Cb}{m}e^{-\frac{bt}{m}} \stackrel{?}{=} g - g - \frac{Cb}{m}e^{-\frac{bt}{m}}$$

$$\frac{-Cb}{m}e^{-\frac{bt}{m}} = \frac{-Cb}{m}e^{-\frac{bt}{m}}$$

5.65

SET UP

A 1500-kg truck is driving around an unbanked curve at a speed of 20 m/s. The maximum frictional force between the road and the tires is 8000 N. We can use Newton's second law to relate the maximum frictional force to the radius of the curvature by realizing the car is undergoing uniform circular motion and that static friction is the force responsible for the car traveling around the circle.

SOLVE

Free-body diagram of the truck:

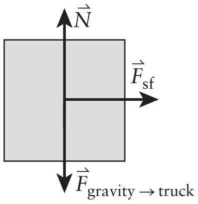


Figure 5-41 Problem 65

Newton's second law:

$$\sum F_y = N - F_{\text{gravity}\to\text{truck}} = N - mg = ma_y = 0$$

$$N = mg$$

$$\sum F_x = F_{sf} = ma_x = m\left(\frac{v^2}{R}\right)$$

$$R = \frac{mv^2}{F_{sf}} = \frac{(1500\text{ kg})\left(20\frac{\text{m}}{\text{s}}\right)^2}{8000\text{ N}} = \boxed{75\text{ m}}$$

REFLECT

Although it may seem large, this is a reasonable radius for a curve on a highway. The curves on a highway are usually gentle because cars are traveling at high speeds.

5.66

SET UP

A 0.170-kg hockey puck is tied to a string and spun in a circle of radius $R = 1.25$ m. The string can withstand a maximum tension of 5.00 N. We can use Newton's second law to calculate the angular speed that corresponds to this maximum tension.

SOLVE

$$T = \frac{mv^2}{R}$$

$$v = \sqrt{\frac{RT}{m}} = \sqrt{\frac{(1.25\text{ m})(5.00\text{ N})}{0.170\text{ kg}}} = 6.06\frac{\text{m}}{\text{s}} \times \frac{60\text{ s}}{1\text{ min}} \times \frac{1\text{ rev}}{2\pi(1.25\text{ m})} = \boxed{46.3\frac{\text{rev}}{\text{min}}}$$

REFLECT

This seems like a reasonable speed that someone can twirl a hockey puck on a string.

5.67

SET UP

A 0.025-kg washer is tied to a 0.60-m-long string and whirled in a vertical circle at a constant speed of 6 m/s. We are asked to find the tension in the string when the washer is at the bottom and top of its path, which we can do through Newton's second law. The two forces acting on the washer are tension and gravity. Gravity will always act downward, but the tension point will be either up or down, depending on the location of the washer. Although the acceleration always points toward the center, the location of the center will be either above or below the washer. Therefore, the sign of the centripetal acceleration will also change depending on the location of the washer. We will define our positive y as pointing up.

SOLVE

Part a)
Free-body diagram of the washer:

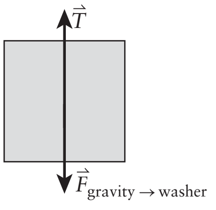


Figure 5-42 Problem 67

Newton's second law:

$$\sum F_y = T - F_{\text{gravity}\to\text{washer}} = T - mg = ma_y = m\frac{v^2}{R}$$

$$T = m\left(g + \frac{v^2}{R}\right) = (0.025\text{ kg})\left(\left(9.8\frac{\text{m}}{\text{s}^2}\right) + \frac{\left(6\frac{\text{m}}{\text{s}}\right)^2}{0.60\text{ m}}\right) = \boxed{1.75\text{ N}}$$

Part b)
Free-body diagram of the washer:

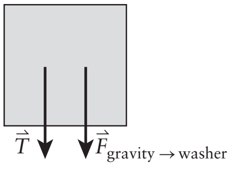


Figure 5-43 Problem 67

Newton's second law:

$$\sum F_y = -T - F_{\text{gravity}\to\text{washer}} = -T - mg = ma_y = m\left(-\frac{v^2}{R}\right)$$

$$T = m\left(\frac{v^2}{R} - g\right) = (0.025\text{ kg})\left(\frac{\left(6\frac{\text{m}}{\text{s}}\right)^2}{0.60\text{ m}} - \left(9.8\frac{\text{m}}{\text{s}^2}\right)\right) = \boxed{1.26\text{ N}}$$

REFLECT

Centripetal acceleration always points toward the center of the circle. The sign of the centripetal acceleration will depend on your coordinate system.

5.68

SET UP

The Moon ($m = 7.35 \times 10^{22}$ kg) rotates about the Earth once every 27.4 days. The distance between the Moon and Earth is 3.84×10^8 m. We can use Newton's second law to calculate the net force exerted on the Moon as it orbits Earth. We'll assume the orbit is circular in order to use the centripetal acceleration.

SOLVE

$$F = \frac{mv^2}{R} = \frac{m}{R}\left(\frac{2\pi R}{T}\right)^2 = \frac{4\pi^2 Rm}{T^2} = \frac{4\pi^2(3.84 \times 10^8\text{ m})(7.35 \times 10^{22}\text{ kg})}{\left(27.4\text{ days} \times \frac{24\text{ hr}}{1\text{ day}} \times \frac{3600\text{ s}}{1\text{ hr}}\right)^2} = \boxed{1.99 \times 10^{20}\text{ N}}$$

The rotation of the Moon is due to the $\boxed{\text{gravitational force of the Earth acting on the Moon}}$.

REFLECT

The magnitude of the gravitational force is equal to mg only near the surface of the Earth.

5.69

SET UP

A centrifuge of radius $R = 0.1$ m spins a 1-g sample at 1200 rev/min. The net force causing it to spin is equal to the mass of the sample multiplied by the centripetal acceleration of the sample.

SOLVE

$$F = ma = \frac{mv^2}{R} = \frac{(10^{-3}\text{ kg})\left(\frac{1200\text{ rev}}{\text{min}} \times \frac{1\text{ min}}{60\text{ s}} \times \frac{2\pi(0.1\text{ m})}{1\text{ rev}}\right)^2}{0.1\text{ m}} = \boxed{1.58\text{ N}}$$

REFLECT

This force is about 100 times larger than the weight of the sample.

5.70

SET UP

A centrifuge of radius $R = 0.1$ m spins a 0.0030-kg sample at 60,000 rev/min. The net force causing it to spin is equal to the mass of the sample multiplied by the centripetal acceleration of the sample.

SOLVE

$$F = ma = \frac{mv^2}{R} = \frac{(3.0 \times 10^{-3}\ \text{kg})\left(\frac{60{,}000\ \text{rev}}{\text{min}} \times \frac{1\ \text{min}}{60\ \text{s}} \times \frac{2\pi(0.1\ \text{m})}{1\ \text{rev}}\right)^2}{0.1\ \text{m}} = \boxed{12{,}000\ \text{m}}$$

REFLECT

This force is about 400,000 times larger than the weight of the sample!

5.71

SET UP

A proton travels in a circular orbit of 6300 m in circumference at a speed of $3 \times 10^8 \frac{\text{m}}{\text{s}}$. The radius of the orbit is (6300 m)/(2π). We can calculate the centripetal acceleration of a proton directly from its definition using these data.

SOLVE

$$a = \frac{v^2}{R} = \frac{\left(3 \times 10^8 \frac{\text{m}}{\text{s}}\right)^2}{\left(\frac{6300\ \text{m}}{2\pi}\right)} = \boxed{9.0 \times 10^{13} \frac{\text{m}}{\text{s}^2}}$$

REFLECT

Although this value seems very large, the mass of a proton is extremely small (around 10^{-27} kg), which means the net force acting on the proton will be on the order of 10^{-13} N.

5.72

SET UP

A 70-kg jet pilot is sitting in a plane that is at the bottom of a vertical loop. The radius of the loop is 500 m, and the plane is traveling at a speed of 200 m/s. The forces acting on the pilot when he is at the bottom of the loop are the normal force due to the seat pointing up and gravity pointing down. At this point, the center of the loop is above him. We can use Newton's second law to calculate the magnitude of the normal force acting on the pilot from the centripetal acceleration.

SOLVE

Free-body diagram of the pilot:

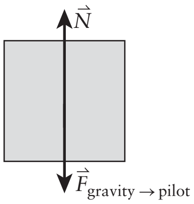


Figure 5-44 Problem 72

Newton's second law:

$$\sum F_y = N - F_{\text{gravity}\rightarrow\text{pilot}} = N - mg = ma_y = m\left(\frac{v^2}{R}\right)$$

$$N = m\left(g + \frac{v^2}{R}\right) = (70\text{ kg})\left(\left(9.8\frac{\text{m}}{\text{s}^2}\right) + \frac{\left(200\frac{\text{m}}{\text{s}}\right)^2}{500\text{ m}}\right) = \boxed{6286\text{ N}}$$

REFLECT
This force is approximately nine times the pilot's weight.

5.73

SET UP
A 0.75-kg tetherball is attached to a 1.25-m rope and is spun around a pole. The rope makes an angle of 35 degrees with the vertical. Although the ball is traveling in a circle in the horizontal plane, the ball is at rest with respect to the vertical direction. We can use Newton's second law to calculate the tension in the rope.

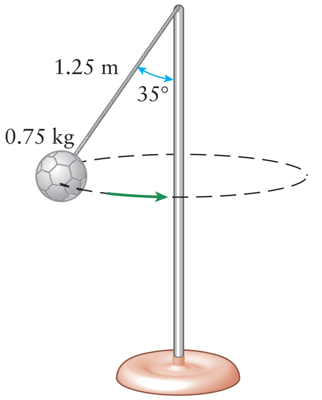


Figure 5-45 Problem 73

SOLVE
Free-body diagram of the ball:

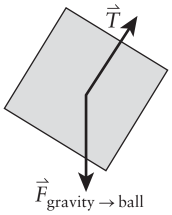


Figure 5-46 Problem 73

Newton's second law:

$$\sum F_y = T_y - F_{\text{gravity}} = T\cos(35°) - mg = ma_y = 0$$

$$T = \frac{mg}{\cos(35°)} = \frac{(0.75\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)}{\cos(35°)} = \boxed{8.97\text{ N}}$$

REFLECT
We didn't need to use the information regarding the ball's speed in order to find the tension.

5.74

SET UP
The radius of the Earth is $R_E = 6.38 \times 10^6$ m. An object located at the equator sweeps out a circle of radius R_E in one day. An object located at latitude 40 degrees north sweeps out a circle of radius $R_{40} = R_E\cos(40°)$ in one day. We can plug these radii into the definition in order to calculate the centripetal acceleration of the object.

SOLVE

Part a)

$$a_c = \frac{v^2}{R} = \frac{\left(\frac{2\pi R_E}{T}\right)^2}{R_E} = \frac{4\pi^2 R_E}{T^2} = \frac{4\pi^2(6.38 \times 10^6 \text{ m})}{\left(1 \text{ day} \times \frac{24 \text{ hr}}{1 \text{ day}} \times \frac{3600 \text{ s}}{1 \text{ hr}}\right)^2} = \boxed{0.0337 \frac{\text{m}}{\text{s}^2}}$$

Part b)
Radius of the circle at 40 degrees north:

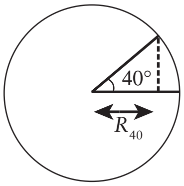


Figure 5-47 Problem 74

Centripetal acceleration:

$$a_c = \frac{v^2}{R_{40}} = \frac{\left(\frac{2\pi R_{40}}{T}\right)^2}{R_{40}} = \frac{4\pi^2 R_{40}}{T^2} = \frac{4\pi^2(6.38 \times 10^6 \text{ m})\cos(40°)}{\left(1 \text{ day} \times \frac{24 \text{ hr}}{1 \text{ day}} \times \frac{3600 \text{ s}}{1 \text{ hr}}\right)^2} = \boxed{0.0258 \frac{\text{m}}{\text{s}^2}}$$

REFLECT
It makes sense that the acceleration should decrease as the object moves closer to the North Pole and the distance from the rotation axis decreases.

5.75

SET UP
In a windmill pitch the pitcher makes a vertical circle or radius $R = 0.31$ m and releases the ball at the bottom of its arc. At the point of release, the force of her arm on the ball is pointing straight upward and is opposed by the force of gravity on the ball. We can use Newton's second law and the centripetal acceleration to calculate the magnitude of the force of the pitcher's arm on the ball. The ball has a mass of $m = 0.19$ kg and leaves her hand at a speed of $v = 24$ m/s.

SOLVE
Free-body diagram of the softball:

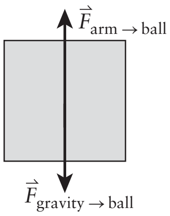


Figure 5-48 Problem 75

Newton's second law:

$$\sum F_y = F_{\text{arm}\to\text{ball}} - F_{\text{gravity}\to\text{ball}} = F_{\text{arm}\to\text{ball}} - mg = ma_y = m\left(\frac{v^2}{R}\right)$$

$$F_{\text{arm}\to\text{ball}} = m\left(g + \frac{v^2}{R}\right) = (0.19\text{ kg})\left(\left(9.8\frac{\text{m}}{\text{s}^2}\right) + \frac{\left(24\frac{\text{m}}{\text{s}}\right)^2}{0.31\text{ m}}\right) = \boxed{350\text{ N}}$$

REFLECT

Through Newton's third law, this is also the magnitude of the force acting on the pitcher's arm, which is relevant for sports medicine and injuries.

5.76

SET UP

A 150-kg crate is sitting in the bed of a pickup truck traveling at 50 km/hr. The driver applies the brakes and the truck comes to rest in 12 s. Since the crate is traveling with the truck, it experiences the same acceleration, which we'll assume is constant. Static friction (μ_s = 0.655) is the force responsible for bringing the crate to rest. We can calculate the maximum magnitude of static friction and the acceleration associated with it and compare it to the actual acceleration of the truck. If the truck decelerates at a rate faster than the maximum allowed by static friction, the crate will slide with respect to the truck bed. Because the maximum static friction is constant and the only force acting in the horizontal direction, the acceleration will be constant. We can use the constant acceleration equations to calculate the minimum stopping time for the truck in order to prevent the crate from slipping.

SOLVE

Part a)
Free-body diagram of the crate:

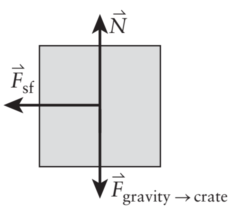


Figure 5-49 Problem 76

Newton's second law:

$$\sum F_y = N - F_{\text{gravity}\to\text{crate}} = N - mg = ma_y = 0$$

$$N = mg$$

$$\sum F_x = -F_{\text{sf}} = -\mu_s N = -\mu_s mg = ma_x$$

$$a_x = -\mu_s g = -(0.655)\left(9.8\frac{\text{m}}{\text{s}^2}\right) = -6.42\frac{\text{m}}{\text{s}^2}$$

Actual acceleration of the truck:

$$a_x = \frac{\Delta v}{\Delta t} = \frac{\left(0 - \left(50\frac{\text{km}}{\text{hr}} \times \frac{1\text{ hr}}{3600\text{ s}} \times \frac{1000\text{ m}}{1\text{ km}}\right)\right)}{12\text{ s}} = -1.16\frac{\text{m}}{\text{s}^2}$$

Since the truck is decelerating at a smaller rate than the maximum allowed by static friction, $\boxed{\text{the crate will not slide}}$ during the braking period.

Part b)

$$a_x = \frac{\Delta v}{\Delta t}$$

$$\Delta t = \frac{\Delta v}{a_x} = \frac{\left(0 - \left(50\frac{\text{km}}{\text{hr}} \times \frac{1\text{ hr}}{3600\text{ s}} \times \frac{1000\text{ m}}{1\text{ km}}\right)\right)}{\left(6.42\frac{\text{m}}{\text{s}^2}\right)} = \boxed{2.2\text{ s}}$$

REFLECT

The magnitude of static friction can range from around zero up to $\mu_s N$ and will take whatever value is necessary (within that range) in order to prevent relative sliding motion.

5.77

SET UP

The coefficient of static friction between a rubber tire and dry pavement is around 0.80. We can use Newton's second law in order to calculate the acceleration of the car and then the time required for the car to accelerate from rest to 26.8 m/s. We are told that the engine supplies power to only two of the four wheels, which means we need to divide the maximum force due to static friction by two. Friction opposes relative *slipping* motion between two objects but can still cause the car to accelerate forward.

SOLVE

Part a)
Free-body diagram of the car:

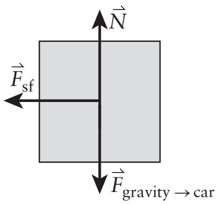


Figure 5-50 Problem 77

Newton's second law:

$$\sum F_y = N - F_{\text{gravity}\to\text{car}} = N - mg = ma_y = 0$$

$$N = mg$$

$$\sum F_x = F_{\text{sf}} = \mu_s\left(\frac{N}{2}\right) = \mu_s\left(\frac{mg}{2}\right) = ma_x$$

$$a_x = \frac{\mu_s g}{2} = \frac{(0.80)\left(9.8\frac{\text{m}}{\text{s}^2}\right)}{2} = 3.92\frac{\text{m}}{\text{s}^2}$$

Calculating the time:

$$a_x = \frac{\Delta v}{\Delta t}$$

$$\Delta t = \frac{\Delta v}{a_x} = \frac{\left(26.8\frac{\text{m}}{\text{s}}\right)}{\left(3.92\frac{\text{m}}{\text{s}^2}\right)} = \boxed{6.84 \text{ s}}$$

Part b) Static friction opposes the relative slipping motion between two surfaces. The bottom of the tire tends to slip backward relative to the wet pavement, so the frictional force opposes the backward motion and, hence, is forward.

REFLECT

On dry pavement, the car's tires do not slip relative to the surface of the road. An example of when the tires *do* slip would be on an icy road or on rain-soaked pavement.

5.78

SET UP

Two blocks are connected by a string over a pulley. Block m_1 ($m_1 = 1.0$ kg) is sitting on a 30-degree incline. The coefficient of static friction between block m_1 and the ramp is $\mu_s = 0.5$, and the coefficient of kinetic friction between block m_1 and the ramp is $\mu_s = 0.4$. Block m_2 ($m_2 = 0.4$ kg) is hanging over the edge of the ramp. We first need to determine whether or not static friction will keep the blocks stationary. In the case of static friction, both blocks are stationary and the tension is equal to the weight of block m_2. We'll assume that m_1 would move down the ramp in the frictionless case. Static friction needs to counteract the difference between the parallel component of gravity and the tension. If static friction is large enough to hold the blocks in place, then we've already calculated the tension. If the static friction is smaller, then the blocks will accelerate and kinetic friction will take over. Then we need to calculate the acceleration of the two blocks in order to determine the tension. We'll consider a coordinate system with an axis pointing along the string starting from m_2 and pointing down the ramp.

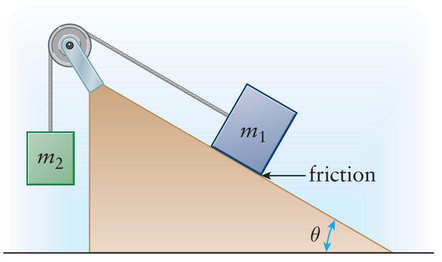


Figure 5-51 Problems 78 and 79

SOLVE

Free-body diagram of m_1:

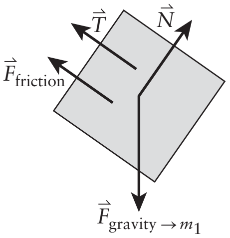


Figure 5-52 Problem 78

Free-body diagram of m_2:

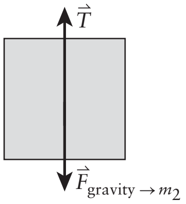


Figure 5-53 Problem 78

Newton's second law:

$$\sum F_{2,\,y} = T - F_{\text{gravity}\to m_2} = T - m_2 g = m_2 a$$

$$\sum F_{1,\,\perp} = N - F_{\text{gravity}\to m_2,\,\perp} = N = m_1 g\cos(30°) = m_1 a_\perp = 0$$

$$N = m_1 g\cos(30°)$$

$$\sum F_{1,\,\parallel} = F_{\text{gravity}\to m_1,\,\parallel} - T - F_{\text{friction}} = m_1 a$$

Testing static friction:

$$F_{\text{sf, max}} = \mu_s N = \mu_s m_1 g\cos(30°) = (0.50)(1.0\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)\cos(30°) = 4.24\text{ N}$$

$$F_{\text{gravity}\to m_2,\,\parallel} - T = m_1 g\sin(30°) - m_2 g = g(m_1\sin(30°) - m_2)$$

$$= \left(9.8\frac{\text{m}}{\text{s}^2}\right)((1.0\text{ kg})\sin(30°) - (0.4\text{ kg})) = 0.98\text{ N}$$

Because the maximum magnitude of static friction is larger than the forces opposing it, the blocks will remain at rest. Therefore, the tension in the string is equal to the weight of m_2: $T = m_2 g = \boxed{3.92\text{ N}}$.

REFLECT

If the blocks were accelerating, then the tension in the rope would not be equal to the weight of block m_2 since there would be a net force acting on it.

5.79

SET UP

Two blocks are connected by a string over a pulley. Block m_1 ($m_1 = 1.0$ kg) is sitting on a 30-degree incline. The coefficient of static friction between block m_1 and the ramp is $\mu_s = 0.5$, and the coefficient of kinetic friction between block m_1 and the ramp is $\mu_s = 0.4$. Block m_2 ($m_2 = 2.0$ kg) is hanging over the edge of the ramp. We first need to determine whether or not static friction will keep the blocks stationary. In the case of static friction, both blocks are stationary and the tension is equal to the weight of block m_2. Assuming that m_1 would move up the ramp due to m_2, the tension needs

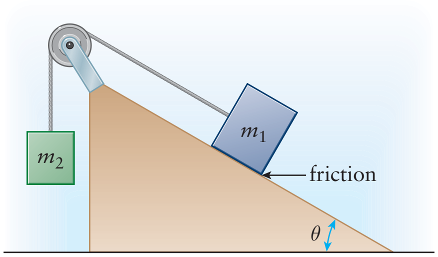


Figure 5-54 Problems 78 and 79

to be counteracted by both static friction and the parallel component of gravity. If the tension is larger than the other two forces, then the blocks will accelerate and kinetic friction will take over. This will let us calculate the acceleration of the two blocks. We'll consider a coordinate system with an axis pointing along the string starting from m_1 and pointing up the ramp.

SOLVE

Free-body diagram of m_1:

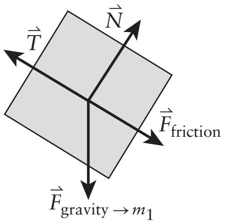


Figure 5-55 Problem 79

Free-body diagram of m_2:

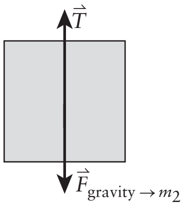


Figure 5-56 Problem 79

Newton's second law:

$$\sum F_{2,\,y} = F_{\text{gravity}\to m_2} - T = m_2 g - T = m_2 a$$

$$\sum F_{1,\,\perp} = N - F_{\text{gravity}\to m_1,\,\perp} = N - m_1 g\cos(30°) = m_1 a_\perp = 0$$

$$N = m_1 g\cos(30°)$$

$$\sum F_{1,\,\parallel} = T - F_{\text{gravity}\to m_1,\,\parallel} - F_{\text{friction}} = m_1 a$$

Testing static friction:

$$F_{\text{sf, max}} = \mu_s N = \mu_s m_1 g\cos(30°) = (0.50)(1.0\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)\cos(30°) = 4.24\text{ N}$$

$$T - F_{\text{gravity}\to m_1,\,\parallel} = m_2 g - m_1 g\sin(30°) = g(m_2 - m_1\sin(30°))$$

$$= \left(9.8\frac{\text{m}}{\text{s}^2}\right)((2.0\text{ kg}) - (1.0\text{ kg})\sin(30°)) = 14.7\text{ N}$$

Because the maximum magnitude of static friction is less than the forces opposing it, the block will start to accelerate. This means we must consider the kinetic friction acting on the block.

$$\sum F_{1,\,\parallel} = T - F_{\text{gravity}\to m_1,\,\parallel} - F_{\text{kf}} = T - m_1 g\sin(30°) - \mu_k N$$

$$= T - m_1 g\sin(30°) - \mu_k m_1 g\cos(30°) = m_1 a$$

Adding the equations for the two blocks together:

$$m_2g - m_1g\sin(30°) - \mu_k m_1 g\cos(30°) = (m_1 + m_2)a$$

$$a = \frac{m_2g - m_1g\sin(30°) - \mu_k m_1 g\cos(30°)}{m_1 + m_2}$$

$$= \frac{(2.0\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right) - (1.0\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)\sin(30°) - (0.40)(1.0\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)\cos(30°)}{(1.0\text{ kg}) + (2.0\text{ kg})} = \boxed{3.77\frac{\text{m}}{\text{s}^2}}$$

REFLECT

A positive acceleration means that block m_1 will travel up the ramp, as we would expect.

5.80

SET UP

A runaway ski slides down a 250-m-long slope that is inclined at 37 degrees. The ski has an initial speed of 10 m/s. The forces acting on the ski are the normal force, gravity, and kinetic friction. We can use Newton's second law to calculate the acceleration of the ski along the axis parallel to the slope. (We'll assume that down the ramp is positive.) If this acceleration is constant, we can use the constant acceleration equations to determine the time it takes the ski to travel 250 m for the different values of μ_k.

SOLVE

Free-body diagram of the ski:

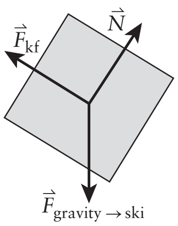


Figure 5-57 Problem 80

Newton's second law:

$$\sum F_\perp = N - F_{\text{gravity}\to\text{ski},\perp} = N - mg\cos(37°) = ma_\perp = 0$$

$$N = mg\cos(37°)$$

$$\sum F_\parallel = F_{\text{gravity}\to\text{ski},\parallel} - F_{kf} = mg\sin(37°) - \mu_k N = mg\sin(37°) - \mu_k mg\cos(37°) = ma_\parallel$$

$$a_\parallel = g(\sin(37°) - \mu_k\cos(37°))$$

Constant acceleration:

$$\Delta x = v_0 t + \frac{1}{2}a_\parallel t^2$$

$$t = \frac{-v_0 \pm \sqrt{v_0^2 - 4\left(\frac{1}{2}a_{\|}\right)(-\Delta x)}}{a_{\|}} = \frac{-v_0 \pm \sqrt{v_0^2 + 2a_{\|}(\Delta x)}}{a_{\|}}$$

$$= \frac{-v_0 \pm \sqrt{v_0^2 + 2g\,(\sin(37°) - \mu_k \cos(37°))(\Delta x)}}{g\,(\sin(37°) - \mu_k \cos(37°))}$$

Part a)

$$t = \frac{-\left(10\frac{\text{m}}{\text{s}}\right) \pm \sqrt{\left(10\frac{\text{m}}{\text{s}}\right)^2 + 2\left(9.8\frac{\text{m}}{\text{s}^2}\right)(\sin(37°) - (0.1)\cos(37°))(250\text{ m})}}{\left(9.8\frac{\text{m}}{\text{s}^2}\right)(\sin(37°) - (0.1)\cos(37°))}$$

Taking the positive root:

$$\boxed{t = 8.12\text{ s}}$$

Part b)

$$t = \frac{-\left(10\frac{\text{m}}{\text{s}}\right) \pm \sqrt{\left(10\frac{\text{m}}{\text{s}}\right)^2 + 2\left(9.8\frac{\text{m}}{\text{s}^2}\right)(\sin(37°) - (0.15)\cos(37°))(250\text{ m})}}{\left(9.8\frac{\text{m}}{\text{s}^2}\right)(\sin(37°) - (0.15)\cos(37°))}$$

Taking the positive root:

$$\boxed{t = 8.39\text{ s}}$$

REFLECT

It should take the ski longer to reach the bottom if the effect of friction is larger.

5.81

SET UP

A 2.5-kg package slides down a 20-degree incline with an initial speed of 2 m/s. The package slides a total distance of 12.0 m. We can calculate the constant acceleration of the package assuming it comes to rest at the bottom of the ramp. Using Newton's second law, we can relate this acceleration to the net force on the package and then calculate the coefficient of kinetic friction between the package and the ramp.

SOLVE

Finding the acceleration:

$$v^2 - v_0^2 = 2a_{\|}(\Delta x)$$

$$a_{\|} = \frac{v^2 - v_0^2}{2(\Delta x)} = \frac{0 - \left(2\frac{\text{m}}{\text{s}}\right)^2}{2(12.0\text{ m})} = -0.167\frac{\text{m}}{\text{s}^2}$$

Free-body diagram of the package:

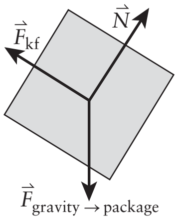


Figure 5-58 Problem 81

Newton's second law:

$$\sum F_{\perp} = N - F_{\text{gravity}\to\text{package},\,\perp} = N - mg\cos(20°) = ma_{\perp} = 0$$

$$N = mg\cos(20°)$$

$$\sum F_{\parallel} = F_{\text{gravity}\to\text{package},\,\parallel} - F_{kf} = mg\sin(20°) - \mu_k N = mg\sin(20°) - \mu_k mg\cos(20°) = ma_{\parallel}$$

$$\mu_k = \frac{g\sin(20°) - a_{\parallel}}{g\cos(20°)} = \frac{\left(9.8\frac{\text{m}}{\text{s}^2}\right)\sin(20°) - \left(-0.167\frac{\text{m}}{\text{s}^2}\right)}{\left(9.8\frac{\text{m}}{\text{s}^2}\right)\cos(20°)} = \boxed{0.382}$$

REFLECT

We can look at the limiting cases in order to double-check our algebraic solution. As the angle of the ramp approaches 0 degrees, the magnitude of kinetic friction should be smaller if the block comes to rest at the end of the ramp. As the angle of the ramp approaches 90 degrees, then the coefficient of kinetic friction will need to be extremely large in order to attain the same acceleration.

5.82

SET UP

Two blocks are connected to each other by a string over a pulley—block 1 ($m_1 = 6.00$ kg) is on the left and block 2 (mass m_2) is on the right. The coefficient of static friction between each block and the ramp is $\mu_s = 0.542$. We need to determine the range of values m_2 can take such that the system remains at rest (that is, all accelerations in all directions are zero). We can use Newton's second law to calculate the value of mass that gives the maximum magnitude of static friction in the system. We will need to consider two cases: (1) static friction prevents block 2 from sliding up the ramp and (2) static friction prevents block 2 from sliding down the ramp. Static friction will act in different directions on each block in each case, so we need to redraw the free-body diagram for each case. We'll consider a coordinate system with an axis pointing along the string starting from m_1 and pointing up and over the ramp.

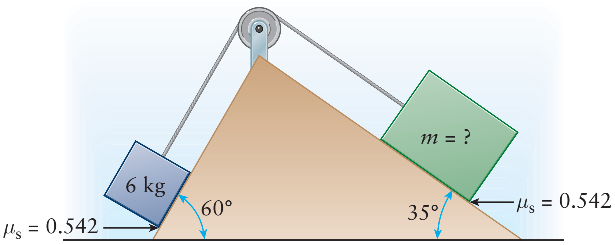


Figure 5-59 Problem 82

SOLVE

Assuming block 2 would slide up the ramp:

Free-body diagram of block 1:

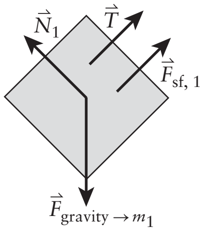


Figure 5-60 Problem 82

Free-body diagram of block 2:

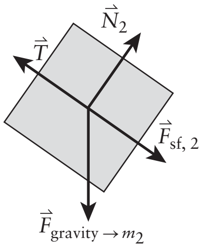


Figure 5-61 Problem 82

Newton's second law:

$$\sum F_{1,\perp} = N_1 - F_{\text{gravity}\to 1,\perp} = N_1 - m_1 g\cos(60°) = m_1 a_\perp = 0$$

$$N_1 = m_1 g\cos(60°)$$

$$\sum F_{1,\parallel} = T + F_{\text{sf, 1}} - F_{\text{gravity}\to 1,\parallel} = T + \mu_s N_1 - m_1 g\sin(60°)$$

$$= T + \mu_s m_1 g\cos(60°) - m_1 g\sin(60°) = m_1 a_\parallel = 0$$

$$T = m_1 g\sin(60°) - \mu_s m_1 g\cos(60°) = m_1 g\left(\sin(60°) - \mu_s\cos(60°)\right)$$

$$\sum F_{2,\perp} = N_2 - F_{\text{gravity}\to 2,\perp} = N_2 - m_2 g\cos(35°) = m_2 a_\perp = 0$$

$$N_2 = m_2 g\cos(35°)$$

$$\sum F_{2,\parallel} = F_{\text{sf, 2}} + F_{\text{gravity}\to 2,\parallel} - T = \mu_s N + m_2 g\sin(35°) - T$$

$$= \mu_s m_2 g\cos(35°) + m_2 g\sin(35°) - T$$

$$= \mu_s m_2 g\cos(35°) + m_2 g\sin(35°) - m_1 g\left(\sin(60°) - \mu_s\cos(60°)\right) = m_2 a_\parallel = 0$$

$$m_2 = \frac{m_1\left(\sin(60°) - \mu_s\cos(60°)\right)}{\mu_s\cos(35°) + \sin(35°)} = \frac{(6.00\text{ kg})\left(\sin(60°) - (0.542)\cos(60°)\right)}{(0.542)\cos(35°) + \sin(35°)} = 3.51\text{ kg}$$

Assuming block 2 would slide down the ramp:

Free-body diagram of block 1:

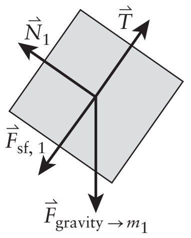


Figure 5-62 Problem 82

Free-body diagram of block 2:

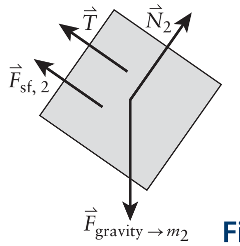


Figure 5-63 Problem 82

Newton's second law:

$$\sum F_{1,\perp} = N_1 - F_{\text{gravity}\to 1,\perp} = N_1 - m_1 g\cos(60°) = m_1 a_\perp = 0$$

$$N_1 = m_1 g\cos(60°)$$

$$\sum F_{1,\parallel} = T - F_{\text{sf, 1}} - F_{\text{gravity}\to 1,\parallel} = T - \mu_\text{s} N_1 - m_1 g\sin(60°)$$

$$= T - \mu_\text{s} m_1 g\cos(60°) - m_1 g\sin(60°) = m_1 a_\parallel = 0$$

$$T = m_1 g\sin(60°) + \mu_\text{s} m_1 g\cos(60°) = m_1 g(\sin(60°) + \mu_\text{s}\cos(60°))$$

$$\sum F_{2,\perp} = N_2 - F_{\text{gravity}\to 2,\perp} = N_2 - m_2 g\cos(35°) = m_2 a_\perp = 0$$

$$N_2 = m_2 g\cos(35°)$$

$$\sum F_{2,\parallel} = -F_{\text{sf, 2}} + F_{\text{gravity}\to 2,\parallel} - T = -\mu_\text{s} N_2 + m_2 g\sin(35°) - T$$

$$= -\mu_\text{s} m_2 g\cos(35°) + m_2 g\sin(35°) - T$$

$$= -\mu_\text{s} m_2 g\cos(35°) + m_2 g\sin(35°) - m_1 g(\sin(60°) + \mu_\text{s}\cos(60°)) = m_2 a_\parallel = 0$$

$$m_2 = \frac{m_1(\sin(60°) + \mu_\text{s}\cos(60°))}{-\mu_\text{s}\cos(35°) + \sin(35°)} = \frac{(6.00\text{ kg})(\sin(60°) + (0.542)\cos(60°))}{-(0.542)\cos(35°) + \sin(35°)} = \boxed{52.6\text{ kg}}$$

REFLECT

The system will remain at rest for values of m_2 between 3.51 kg and 52.6 kg.

5.83

SET UP

A 100-kg sailboat is moving at 10 m/s when its mast suddenly breaks. The boat starts to slow down due to a drag force that is proportional to the boat's speed; we'll assume that this is the only force acting in the direction of the boat's motion. We can use Newton's second law to calculate the speed of the boat as a function of time. Because the drag force is proportional to the speed, we will need to solve a separable differential equation. The initial condition that $v(t = 0\text{ s}) = 10$ m/s will allow us to solve for the constant of integration. Knowing that the boat's speed is 6 m/s after 5 s will allow us to find the drag coefficient b. Using all of this information, we can solve for the time it takes the boat to slow to a speed of 0.5 m/s.

SOLVE

Finding $v(t)$:

$$\sum F = ma = m\frac{dv}{dt} = F_{\text{drag}} = -bv$$

$$\frac{dv}{v} = -\frac{b}{m}dt$$

$$\int\frac{dv}{v} = \int -\frac{b}{m}dt$$

$$\ln(v) = -\frac{b}{m}t + C$$

$$v(t) = Ce^{-\frac{b}{m}t}$$

Applying the initial condition ($v(t = 0) = 10$ m/s):

$$v(0) = Ce^{-\frac{b}{m}(0)} = C = 10\frac{\text{m}}{\text{s}}$$

$$v(t) = \left(10\frac{\text{m}}{\text{s}}\right)e^{-\frac{b}{m}t}$$

Solving for b:

$$v(5\text{ s}) = \left(10\frac{\text{m}}{\text{s}}\right)e^{-\frac{b}{m}(5\text{ s})} = 6\frac{\text{m}}{\text{s}}$$

$$b = -\frac{m}{5\text{ s}}\ln(0.6) = -\frac{100\text{ kg}}{5\text{ s}}\ln(0.6) = 10.22\frac{\text{kg}}{\text{s}}$$

Solving for the time it takes to reach $v = 0.5$ m/s:

$$v(t) = \left(10\frac{\text{m}}{\text{s}}\right)e^{-\frac{b}{m}t} = 0.5\frac{\text{m}}{\text{s}}$$

$$t = -\frac{m}{b}\ln(0.05) = -\frac{100\text{ kg}}{\left(10.22\frac{\text{kg}}{\text{s}}\right)}\ln(0.05) = \boxed{29\text{ s}}$$

REFLECT

Because the force (and thus the acceleration) is proportional to the speed, we cannot use the constant acceleration equations to answer this question. This is why we needed to use calculus.

5.84

SET UP

A spherical raindrop with a diameter of 4.0 mm (radius $R = 2.0$ mm) falls at a terminal speed of 8.5 m/s. For an object traveling at terminal speed, the magnitude of the drag force acting on it is equal to its weight. We can relate the mass of the raindrop to its density and volume to calculate the mass as a function of R. This will give us a general answer for b when we set the magnitudes of the forces equal to one another. To determine the terminal speed for a larger drop, we can take the ratio of our algebraic expression for two different values of R, where $R_2 = 2R_1$.

SOLVE

Part a)
At terminal speed:

$$F_{\text{drag}} = F_{\text{gravity}\rightarrow\text{raindrop}}$$

$$bv_{\text{terminal}}^2 = mg = (\rho V)g$$

$$b = \frac{\rho V g}{v_{\text{terminal}}^2} = \frac{\rho\left(\frac{4}{3}\pi R^3\right)g}{v_{\text{terminal}}^2} = \frac{\left(1000\frac{\text{kg}}{\text{m}^3}\right)\left(\frac{4}{3}\pi(0.0020\text{ m})^3\right)\left(9.8\frac{\text{m}}{\text{s}^2}\right)}{\left(8.5\frac{\text{m}}{\text{s}}\right)^2} = \boxed{4.5 \times 10^{-6}\frac{\text{kg}}{\text{m}}}$$

Part b)

$$bv_{\text{terminal}}^2 = mg = (\rho V)g$$

$$v_{\text{terminal}} = \sqrt{\frac{\rho V g}{b}}$$

$$\frac{v_{\text{terminal, 2}}}{v_{\text{terminal, 1}}} = \frac{\sqrt{\frac{\rho V_2 g}{b}}}{\sqrt{\frac{\rho V_1 g}{b}}} = \sqrt{\frac{V_2}{V_1}} = \sqrt{\frac{\left(\frac{4}{3}\pi R_2^3\right)}{\left(\frac{4}{3}\pi R_1^3\right)}} = \sqrt{\frac{R_2^3}{R_1^3}} = \sqrt{\frac{(2R_1)^3}{R_1^3}} = \boxed{\sqrt{8}}$$

The terminal speed of the larger drop should be $\sqrt{8}$ times larger than the terminal speed of the smaller one.

REFLECT

We can also show the answer to part (b) through brute force:

$$bv_{\text{terminal}}^2 = mg = (\rho V)g$$

$$v_{\text{terminal, 2}} = \sqrt{\frac{\rho V_2 g}{b}} = \sqrt{\frac{\rho\left(\frac{4}{3}\pi R_2^3\right)g}{b}} = \sqrt{\frac{\left(1000\frac{\text{kg}}{\text{m}^3}\right)\left(\frac{4}{3}\pi(0.0040\text{ m})^3\right)\left(9.8\frac{\text{m}}{\text{s}^2}\right)}{\left(4.5\times10^{-6}\frac{\text{kg}}{\text{m}}\right)}} = 24\frac{\text{m}}{\text{s}}$$

$$\frac{v_{\text{terminal, 2}}}{v_{\text{terminal, 1}}} = \frac{24}{8.5} = 2.8 \approx \sqrt{8}$$

5.85

SET UP

In a weird parallel universe, an object moving in a liquid experiences a drag force inversely proportional to its speed: $F_{\text{drag}} = -bv^{-1}$. An object of mass m is initially moving with a speed of v_0. Assuming the drag force is the only force acting along the direction of the object's motion, we can use the differential form of Newton's second law to solve for the speed as a function of time. This is a separable differential equation with the initial condition of $v(0) = v_0$. We are told that a specific object enters the liquid with an initial speed of 10 m/s, and after 9 s its speed is 8 m/s. We can use our algebraic expression for $v(t)$ from part (a) along with these data to calculate the ratio b/m and then the time necessary for the object to come to a complete stop.

SOLVE

Part a)

$$\sum F = F_{\text{drag}} = -bv^{-1} = ma = m\frac{dv}{dt}$$

$$\frac{dv}{dt} = -\frac{b}{mv}$$

$$vdv = -\frac{b}{m}dt$$

$$\int vdv = \int -\frac{b}{m}dt$$

$$\frac{1}{2}v^2 = -\frac{b}{m}t + C$$

$$v(t) = \sqrt{-\frac{2b}{m}t + C}$$

Applying the initial condition:

$$v(t=0) = \sqrt{C} = v_0$$

$$C = v_0^2$$

$$\boxed{v(t) = \sqrt{-\frac{2b}{m}t + v_0^2}}$$

Part b)

$$v(9\text{ s}) = 8\frac{\text{m}}{\text{s}} = \sqrt{-\frac{2b}{m}(9\text{ s}) + \left(10\frac{\text{m}}{\text{s}}\right)^2}$$

$$\frac{b}{m} = 2\frac{\text{m}^2}{\text{s}^2}$$

$$v(t) = 0 = \sqrt{-\frac{2b}{m}t + v_0^2}$$

$$0 = -\frac{2b}{m}t + v_0^2$$

$$t = \frac{v_0^2}{2\left(\frac{b}{m}\right)} = \frac{\left(10\frac{\text{m}}{\text{s}}\right)^2}{2\left(2\frac{\text{m}^2}{\text{s}^3}\right)} = \boxed{25\text{ s}}$$

REFLECT

Because the drag force is inversely proportional to the speed, the largest force will be when the object is moving most slowly.

5.86

SET UP

An ultracentrifuge spins a 2.0-g sample at a rate of 100,000 rev/min. The sample is located a distance of $R = 0.1$ m from the rotation axis. We can calculate the net force acting on the sample using the centripetal acceleration of the sample and Newton's second law.

SOLVE

$$\sum F = ma = m\left(\frac{v^2}{R}\right) = (0.002\text{ kg})\frac{\left(100{,}000\frac{\text{rev}}{\text{min}} \times \frac{1\text{ min}}{60\text{ s}} \times \frac{2\pi(0.10\text{ m})}{1\text{ rev}}\right)^2}{0.10\text{ m}} = \boxed{21{,}932\text{ N}}$$

$\boxed{\text{No, the force would not be appreciably different}}$ if the sample were spun in a vertical or horizontal circle because the centripetal acceleration is much, much larger than the acceleration due to gravity.

REFLECT

The magnitude of the centripetal acceleration is approximately 10^7 m/s^2, so adding 9.8 to 10^7 will still give 10^7.

5.87

SET UP

A person is standing against the wall of a cylindrical amusement park ride that starts to rotate. Once the cylinder ($R = 3.5$ m) reaches a speed of 25 rev/min, the floor drops out and the passenger remains in place due to static friction acting upward. We can use Newton's second law and the centripetal acceleration to calculate the coefficient of static friction between the wall and the passenger.

SOLVE

Free-body diagram of the passenger:

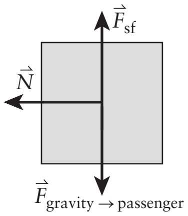


Figure 5-64 Problem 87

Newton's second law:

$$\sum F_x = N = ma_x = m\left(\frac{v^2}{R}\right)$$

$$\sum F_y = F_{sf} - F_{gravity \to passenger} = \mu_k N - mg = \mu_k\left(\frac{mv^2}{R}\right) - mg = ma_y = 0$$

$$\mu_k = \frac{gR}{v^2} = \frac{\left(9.8\frac{\text{m}}{\text{s}^2}\right)(3.5\text{ m})}{\left(25\frac{\text{rev}}{\text{min}} \times \frac{1\text{ min}}{60\text{ s}} \times \frac{2\pi(3.5\text{ m})}{1\text{ rev}}\right)^2} = \boxed{0.409}$$

REFLECT

The coefficient of static friction is less than 1, which makes sense. We were not given the mass of a passenger in the problem statement, which means we either need to estimate a value or it does not factor into our final answer.

5.88

SET UP

Block 1 ($m_1 = 0.125$ kg) undergoes uniform circular motion around a circle of radius $R = 1.00$ m on top of a frictionless table. Block 2 ($m_2 = 0.225$ kg) is connected to block 1 by a string and is hanging through a hole in the center of the table. We are told that block 2 is stationary, which allows us to determine the tension in the shared string through Newton's second law. We can use the centripetal acceleration and the tension to calculate the period Δt of block 1's motion.

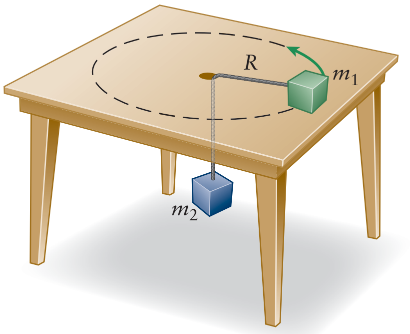


Figure 5-65 Problem 88

SOLVE

Free-body diagram of m_1:

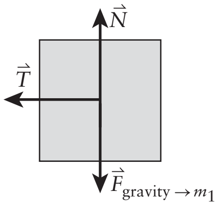


Figure 5-66 Problem 88

Free-body diagram of m_2:

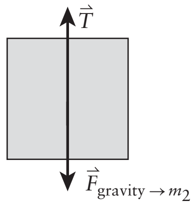


Figure 5-67 Problem 88

Newton's second law:

$$\sum F_{2,\,y} = T - F_{\text{gravity}\to 2} = T - m_2 g = m_2 a_y = 0$$

$$T = m_2 g$$

$$\sum F_{1,\,x} = T = m_1\left(\frac{v^2}{R}\right) = m_1\left(\frac{\left(\frac{2\pi R}{\Delta t}\right)^2}{R}\right) = m_1\left(\frac{4\pi^2 R}{(\Delta t)^2}\right)$$

$$\Delta t = \sqrt{m_1\left(\frac{4\pi^2 R}{T}\right)} = \sqrt{\frac{4\pi^2 R m_1}{m_2 g}} = \sqrt{\frac{4\pi^2(1.00\text{ m})(0.125\text{ kg})}{(0.225\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)}} = \boxed{1.50\text{ s}}$$

REFLECT

Block 1 is traveling at a speed of 0.67 rev/s or 4.2 m/s.

5.89

SET UP

A mass M is attached to a string of length L and is rotated. The angle the string makes with the vertical is θ. The speed of the mass is N revolutions per second. In one revolution the mass sweeps out a circle of radius $R = L\sin(\theta)$. We can use Newton's second law to solve for the observed value of θ. The only forces acting on the mass are tension from the string and gravity. The mass is stationary in the y direction and undergoing centripetal motion in the x direction. Keep in mind that we are only allowed to have M, L, N, and physical constants in our final answer.

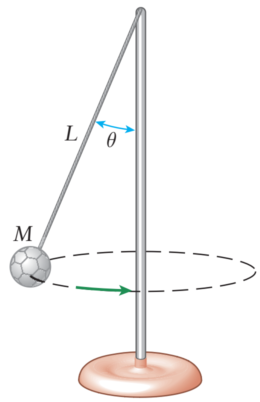


Figure 5-68 Problem 89

SOLVE

Free-body diagram of M:

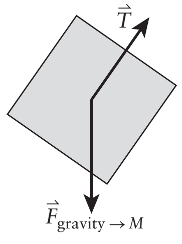


Figure 5-69 Problem 89

Newton's second law:

$$\sum F_x = T_x = T\sin(\theta) = M\left(\frac{v^2}{R}\right) = M\left(\frac{(N(2\pi R))^2}{R}\right) = M(4\pi^2 N^2 R) = 4\pi^2 N^2 M(L\sin(\theta))$$

$$T = 4\pi^2 N^2 ML$$

$$\sum F_y = T_y - F_{\text{gravity}\to M} = T\cos(\theta) - Mg = Ma_y = 0$$

$$T\cos(\theta) = 4\pi^2 N^2 ML\cos(\theta) = Mg$$

$$\cos(\theta) = \frac{g}{4\pi^2 N^2 L}$$

$$\boxed{\theta = \arccos\left(\frac{g}{4\pi^2 N^2 L}\right)}$$

REFLECT

If we observe that the mass makes a smaller angle with the vertical, we would assume the rotation speed has increased. It also makes sense that a smaller angle could result from a longer string (that is, increased L).

5.90

SET UP

In an accident, a person's head pivots about the base of his neck through an angle of 60 degrees; this motion lasts 250 ms. The head is about 20 cm from the base of the neck. Assuming it is constant, we can calculate the speed of the head from the distance it travels in 0.250 s. An angle of 60 degrees is 1/6 of a full circle, which means the head travels through 1/6 of the circumference of a circle of radius $R = 20$ cm. Using the speed, we can calculate both the centripetal acceleration and the force exerted on the head by the neck.

SOLVE

Part a)

$$v = \frac{\left(\frac{1}{6}\right)(2\pi)(0.20\text{ m})}{0.250\text{ s}} = 0.838\frac{\text{m}}{\text{s}}$$

$$a = \frac{v^2}{R} = \frac{\left(0.838\frac{\text{m}}{\text{s}}\right)^2}{0.20\text{ m}} = \boxed{3.5\frac{\text{m}}{\text{s}^2}}$$

Part b)

$$\sum F = ma = (0.06)(75\text{ kg})\left(3.5\frac{\text{m}}{\text{s}^2}\right) = \boxed{16\text{ N}}$$

Part c) Yes, headrests would help neck injuries since the head would pivot through an angle of less than 60 degrees before hitting the headrest. This would affect the strain in the neck muscles but not the contact force necessary in stopping the head from moving. Headrests prevent the head from snapping backward when the car comes to rest.

REFLECT

This force is approximately 36% of the weight of the head.

5.91

SET UP

A jet is flying in a horizontal circle at a speed of 680 km/hr. The wings of the plane are tilted at an angle of 60 degrees relative to the horizontal. The only forces acting on the plane are gravity acting straight down and lift acting perpendicular to the wings. We can use Newton's second law and the definition of centripetal acceleration in order to calculate the radius of the circle.

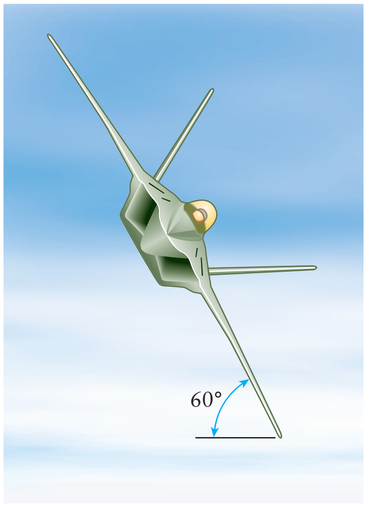


Figure 5-70 Problem 91

SOLVE

Free-body diagram of the plane:

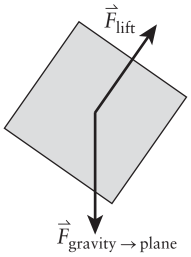


Figure 5-71 Problem 91

Newton's second law:

$$\sum F_y = F_{\text{lift},\,y} - F_{\text{gravity}\to\text{plane}} = F_{\text{lift}}\cos(60°) - mg = ma_y = 0$$

$$F_{\text{lift}} = \frac{mg}{\cos(60°)}$$

$$\sum F_x = F_{\text{lift},\,x} = F_{\text{lift}}\sin(60°) = \left(\frac{mg}{\cos(60°)}\right)\sin(60°) = m\left(\frac{v^2}{R}\right)$$

$$R = \frac{v^2}{g\tan(60^{\circ})} = \frac{\left(680\frac{\text{km}}{\text{hr}} \times \frac{1\ \text{hr}}{3600\ \text{s}} \times \frac{1000\ \text{m}}{1\ \text{km}}\right)^2}{\left(9.8\frac{\text{m}}{\text{s}^2}\right)\tan(60^{\circ})} = \boxed{2100\ \text{m} = 2.1\ \text{km}}$$

REFLECT

The radius of the turn is independent of the mass of the jet plane. A circle of radius $R = 2.1$ km may seem large, but the jet is traveling at around 425 mph!

5.92

SET UP

A 1000-kg car is driving around a curve ($R = 100$ m) at a speed of 65 km/hr (= 18.1 m/s). The curve is banked at an angle of 10 degrees. The forces acting on the car are gravity straight down, the normal force perpendicular to the road, and static friction acting down the surface of the road. We'll use a "standard" coordinate system where positive x points toward the center of the circle and positive y points straight up. Even though this looks like an inclined plane problem, the motion of the car is not parallel to the plane but in a horizontal circle. In this coordinate system, the acceleration in the y direction is zero. We can use Newton's second law and the centripetal acceleration to calculate the minimum coefficient of static friction between the pavement and the tires necessary such that the car does not skid around the curve.

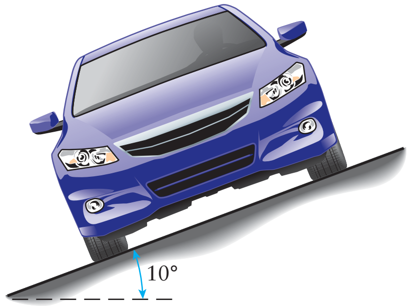


Figure 5-72 Problem 92

SOLVE

Free-body diagram of the car:

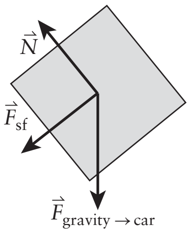


Figure 5-73 Problem 92

Newton's second law:

$$\sum F_y = N_y - F_{sf,\,y} - F_{gravity \to car} = N\cos(10°) - \mu_s N\sin(10°) - mg = ma_y = 0$$

$$N = \frac{mg}{\cos(10°) - \mu_s \sin(10°)}$$

$$\sum F_x = N_x + F_{sf,\,x} = N\sin(10°) + \mu_s N\cos(10°) = ma_x = m\frac{v^2}{R}$$

$$\left(\frac{mg}{\cos(10°) - \mu_s\sin(10°)}\right)\sin(10°) + \mu_s\left(\frac{mg}{\cos(10°) - \mu_s\sin(10°)}\right)\cos(10°) = m\frac{v^2}{R}$$

$$g\left(\frac{\tan(10°) + \mu_s}{1 - \mu_s\tan(10°)}\right) = \frac{v^2}{R}$$

$$Rg\tan(10°) + Rg\mu_s = v^2 - v^2\mu_s\tan(10°)$$

$$Rg\mu_s + v^2\mu_s\tan(10°) = v^2 - Rg\tan(10°)$$

$$\mu_s = \frac{v^2 - Rg\tan(10°)}{Rg + v^2\tan(10°)} = \frac{\left(18.1\frac{\text{m}}{\text{s}}\right)^2 - (100\text{ m})\left(9.8\frac{\text{m}}{\text{s}^2}\right)\tan(10°)}{(100\text{ m})\left(9.8\frac{\text{m}}{\text{s}^2}\right) + \left(18.1\frac{\text{m}}{\text{s}}\right)^2\tan(10°)} = \boxed{0.149}$$

REFLECT

When a curve is banked, the coefficient of static friction does not need to be as large as an unbanked curve because a component of the normal force contributes to the centripetal motion.

5.93

SET UP

A 75-kg skydiver is falling at terminal velocity, which means the magnitude of the drag force acting on her is equal to her weight. We are told the drag force has a magnitude of bv, where $b = 115$ kg/s. Setting this equal to her weight allows us to calculate v. Once she comes in contact with the ground, the forces acting on her are gravity and the normal force. We can calculate the average value of the normal force by first finding the skydiver's (constant) acceleration when coming to a stop and then plugging it into Newton's second law.

SOLVE

Part a)
At terminal velocity:

$$F_{\text{drag}} = F_{\text{gravity}\rightarrow\text{skydiver}}$$

$$bv_{\text{terminal}} = mg$$

$$v_{\text{terminal}} = \frac{mg}{b} = \frac{(75\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)}{\left(115\frac{\text{kg}}{\text{s}}\right)} = \boxed{6.39\frac{\text{m}}{\text{s}}}$$

Part b)

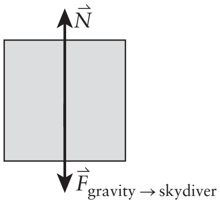


Figure 5-74 Problem 93

Part c)
Finding the acceleration of the skydiver:

$$v^2 - v_0^2 = 2a_y(\Delta y)$$

$$a_y = \frac{v^2 - v_0^2}{2(\Delta y)} = \frac{0 - \left(6.39\frac{\text{m}}{\text{s}}\right)^2}{2(-0.70\text{ m})} = 29\frac{\text{m}}{\text{s}^2}$$

$$\sum F_y = N - F_{\text{gravity}\rightarrow\text{skydiver}} = N - mg = ma_y$$

$$N = m(g + a_y) = (75\text{ kg})\left(\left(9.8\frac{\text{m}}{\text{s}^2}\right) + \left(29\frac{\text{m}}{\text{s}^2}\right)\right) = \boxed{2900\text{ N}}$$

This is over four times her weight; she is $\boxed{\text{most likely going to get hurt}}$.

REFLECT

She experiences an acceleration of almost $3g$, which is large, when she hits the ground.

5.94

SET UP

A small object moving through a vat of oil experiences a drag force of $F_{\text{drag}} = -bv^{1/2}$. Since we are ignoring the force of the gravity, the drag force is the only force acting on the object. Therefore, it is equal to its mass times its acceleration. The acceleration is the first derivative of velocity with respect to time, which means we have a differential equation. We can solve this separable differential equation for $v(t)$ by sequestering like terms and integrating. The initial condition that $v(t = 0) = v_0$ allows us to find the constant of integration.

SOLVE

$$\sum F = ma = m\frac{dv}{dt} = F_{\text{drag}} = -bv^{1/2}$$

$$\frac{dv}{v^{1/2}} = -\frac{b}{m}dt$$

$$\int \frac{dv}{v^{1/2}} = \int -\frac{b}{m}dt$$

$$2v^{1/2} = -\frac{b}{m}t + C$$

$$v^{1/2} = -\frac{b}{2m}t + C$$

Applying the initial conditions $(v(t = 0) = v_0)$:

$$v_0^{1/2} = C = \sqrt{v_0}$$

$$\boxed{v(t) = \left(-\frac{b}{2m}t + \sqrt{v_0}\right)^2}$$

REFLECT

The dimensions of our expression are valid since b has SI units of $\frac{\text{kg} \cdot \text{m}^{1/2}}{\text{s}^{3/2}}$. As time goes on, the speed of the object decreases as expected.

Chapter 6
Work and Energy

Conceptual Questions

6.1 Equation 6-2 is $W = F\cos(\theta)d$. The work done on an object can be equal to zero if (a) the force is zero, (b) the distance is zero, or (c) if the force and distance are perpendicular.

6.2 Most household appliances list the power they consume rather than the energy.

6.3 Stepping on the log would require raising the body, which would require doing work on it. Most of the work would not be recovered when coming down off the log.

6.4 (a) Conservation of energy. The ball should come back to exactly the same height at which it was released. (b) You might be hit by the ball if you push it ever so slightly rather than just releasing it.

6.5 (a) Yes. (b) No. She is dissipating energy inside herself in the futile attempt to move the boulder.

6.6 Positive. The force she applies to the cement block and the displacement of the block are in the same direction.

6.7 If the surface is moving, yes. Otherwise, the distance over which the force is applied is zero, so the work will be zero.

6.8 No, kinetic energy can never be negative. Kinetic energy is related to the mass of an object, which is always positive, and the square of the object's speed. If you square a real number, it will always be positive. Therefore, kinetic energy will always be positive.

6.9 Yes, every time kinetic energy decreases from one positive value to another smaller positive value, the change in kinetic energy is negative.

6.10 A nonconservative force changes the mechanical energy of the system. The work done by a nonconservative force depends on the path of the object. Three examples of nonconservative forces are kinetic friction, air drag, and a push.

6.11 The snowboarder walks to the top of the mountain, which requires converting stored chemical energy into heat and kinetic energy, which in turn is converted into gravitational potential energy and more heat (if the snowboarder rides a lift, the source will also include wherever the lift gets its energy). Then, riding down, the gravitational potential energy and some chemical energy will be converted into kinetic energy, which is in turn dissipated into heat.

6.12 The speed is the more significant factor when calculating the kinetic energy of an object, since it is squared, while the mass is to the first power. Therefore, swinging a smaller claw hammer at a faster speed requires more work.

6.13 The orbital path is along the circle, and the gravitational force points along the radii; the edges of circles are perpendicular to their radii.

6.14 (a) The kinetic energy of the heavier rocket is equal to the kinetic energy of the lighter one. They both start with the same amount of fuel (that is, chemical energy). (b) The launch speed of the heavier rocket will be less than the launch speed of the lighter one. Because their kinetic energies are equal, a larger mass will translate into a smaller speed.

6.15 (a) When it reaches the bottom. (b) When it reaches the bottom (but it's been close to that speed for a while).

6.16 This allows the person to change his or her gravitational potential energy (by doing work) gradually rather than quickly.

6.17 Yes. The reference value for height is set arbitrarily. If you're working from the ground level, going underground yields a negative gravitational potential energy.

6.18 You have more gravitational potential energy at the top of the hill since you are at a higher altitude.

6.19 In the special case of constant force, the integral is just a multiplication problem, so you do not need to be aware of calculus to be able to determine it. If the force is not constant and/or the path is complicated, you will need to calculate an integral.

6.20 The person does work on the toy gun by pulling the trigger in order to generate the energy to launch the dart over and over again.

Multiple-Choice Questions

6.21 C (3). Tension, gravity, kinetic friction.

6.22 (d) < (b) < (c) < (a). The work in case (d) is negative. The work in case (b) is zero. The work in cases (a) and (c) is positive in both cases, but the angles are different.

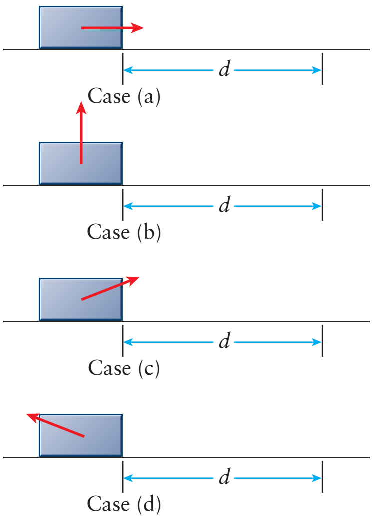


Figure 6-1 Problem 22

6.23 **C** (decreases). As θ increases to 90 degrees, $\cos(\theta)$ approaches zero.

6.24 **E** (first decreases, then increases). The car needs to slow down and stop before changing direction.

6.25 **E** (0). The tension is always perpendicular to the motion of the ball.

6.26 **D** ($2k$). It will be harder to stretch the two "half-springs."

6.27 **C** (They have the same kinetic energy.) Each twin starts with the same gravitational potential energy and falls the same distance.

6.28 **D** (All balls have the same speed). All three balls have the same initial kinetic energy and gravitational potential energy. They will all have the same speed if they fall the same distance.

Estimation Questions

6.29 An 8-kg suitcase being lifted into the trunk of a car that has an elevation of 0.75 m will gain 60 J, but that energy is not the minimum required to get the suitcase in the trunk. The lip of the trunk is 1.1 m off the ground, so the suitcase will, at one point, have 90 J of energy.

6.30 A 50-lb toolbox adds an extra 3% to the weight of a 1600-lb car. Most gasoline engines are only around 20% efficient, which means for every 1 gallon of gas, only 0.2 gallon is converted into useable energy. That means:

$$3\%\text{ useable energy} \times \frac{100\%\text{ gas}}{20\%\text{ useable energy}} = 15\%\text{ gas}$$

6.31 A standard estimate of the power of hard jogging is 7–8 kcal/kg/hr. For a runner who is 80 kg and who runs for 25 min, the energy is roughly 1 MJ.

6.32 A commercial jet has a mass on the order of 10^5 kg and touches down at a speed of around 70 m/s. The kinetic energy of the plane will be $K = \frac{1}{2}(10^5\text{ kg})\left(70\frac{\text{m}}{\text{s}}\right)^2 = 2.5 \times 10^8$ J. Most of the energy is converted into heat.

6.33 A cue ball has a mass of 0.17 kg and a speed of 1.5 m/s. The kinetic energy of the cue ball is around 0.5 J.

6.34 The work done in carrying the backpack up one flight of stairs is equal to the change in the backpack's gravitational potential energy. The mass of a backpack is around 10 kg and a flight of stairs is around 3 m. The work done is around 300 J.

6.35 Pumas have been seen to leap 5.4 m. If the puma's average mass is 62 kg, the maximum kinetic energy is about 300 J.

6.36 The 102nd floor tower of the Empire State Building is 373 m above the ground. Thor ($m \sim 70$ kg) probably rides to the top around 100 times in a day. If he works 5 days a week for 50 weeks for 40 years, he works for around 10^4 days in his life. Therefore, the total number of trips he makes is $N = 10^6$. Setting the total gravitational potential energy equal to kinetic energy and solving for the speed:

$$Nmgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2Ngh} = \sqrt{2(10^6)\left(10\frac{\text{m}}{\text{s}^2}\right)(373\text{ m})} \approx 86{,}000\frac{\text{m}}{\text{s}}$$

6.37

x(m)	F(N)
0.00	0.00
0.01	2.00
0.02	4.00
0.03	6.00
0.04	8.00
0.05	10.00
0.06	10.50
0.07	11.00
0.08	11.50
0.09	12.00
0.10	12.48
0.11	12.48
0.12	12.48
0.13	12.60
0.14	12.60
0.15	12.70
0.16	12.70
0.17	12.60
0.18	12.50
0.19	12.50
0.20	12.50
0.21	12.48
0.22	9.36
0.23	6.24
0.24	3.12
0.25	0.00

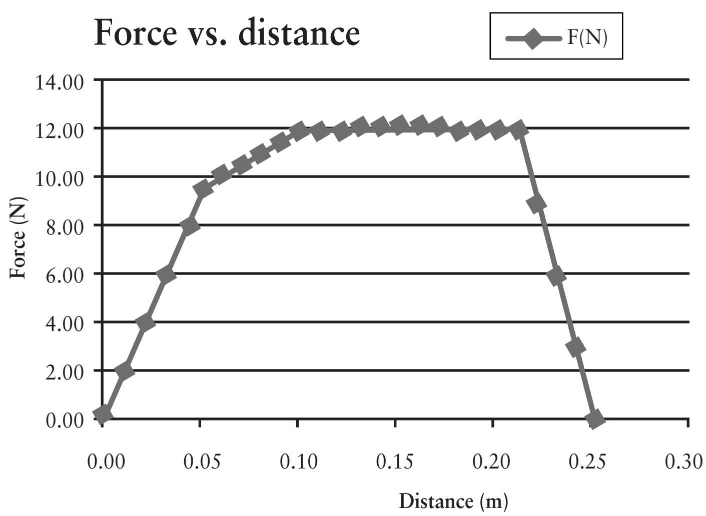


Figure 6-2 Problem 37

The work done by the force is equal to the area under the force versus distance curve. All answers are given to one to two significant figures.

Part a)

$$W_{0\to0.1} = \frac{1}{2}(0.0500\text{ m})(10.00\text{ N}) + \frac{1}{2}(0.0500\text{ m})((10.00\text{ N}) + (12.48\text{ N})) = \boxed{0.8\text{ J}}$$

Part b)

$$W_{0\to0.2} = W_{0\to0.1} + W_{0.1\to0.2} = (0.8\ \text{J}) + (0.100\ \text{m})(12.60\ \text{N}) = \boxed{2\ \text{J}}$$

Part c) We found this in part (b).

$$W_{0.1\to0.2} = (0.100\ \text{m})(12.60\ \text{N}) = \boxed{1.2\ \text{J}}$$

Part d)

$$W_{0\to0.25} = W_{0\to0.2} + W_{0.2\to0.25} = (2\ \text{J}) + \frac{1}{2}(0.050\ \text{m})(12.50\ \text{N}) = \boxed{2.3\ \text{J}}$$

Problems

6.38

SET UP

A crane slowly lifts a 200-kg crate 15 m directly up into the air. Because the crane is slowly lifting the crate, we can assume that the crate is moving at constant acceleration, which means the force of the crane on the crate is equal to the weight of the crate. Both of these forces are constant. The force of the crane on the crate is parallel to the displacement of the crate, while the force of gravity is antiparallel to the displacement.

SOLVE

Work done by the crane:

$$W_{\text{crane}} = F_{\text{crane}} d\cos(0°) = mgd = (200\ \text{kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)(15\ \text{m}) = \boxed{29{,}400\ \text{J}}$$

Work done by gravity:

$$W_{\text{gravity}} = F_{\text{gravity}} d\cos(180°) = -mgd = -(200\ \text{kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)(15\ \text{m}) = \boxed{-29{,}400\ \text{J}}$$

REFLECT

The forces are equal in magnitude but opposite in direction. Therefore, the work done by each force should have the same magnitude but opposite sign.

6.39

SET UP

A weightlifter lifts 446 kg a distance of 2.0 m. If the mass is moving at constant acceleration, then the magnitude of the force of the weightlifter on the mass is equal to the weight of the mass. Both of these forces are constant. The force of the weightlifter on the mass is parallel to the displacement of the mass.

SOLVE

$$W_{\text{weightlifter}} = F_{\text{weightlifter}} d\cos(0°) = mgd = (446\ \text{kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)(2.0\ \text{m}) = \boxed{8700\ \text{J}}$$

REFLECT

We are only allowed two significant figures in our final answer since d has two significant figures. The force and the displacement are in the same direction, so we expect W to be positive.

6.40

SET UP

A 350-kg box is pulled with an external force of 5000 N. The box moves 7 m up a 30-degree inclined plane with no friction. The three forces acting on the block are the external force, gravity, and the normal force. The external force is parallel to the displacement of the block. Because gravity acts straight down, the angle between the displacement and the force of gravity is $(30 + 90) = 120$ degrees; this means the work done by gravity should be negative because cos(120 degrees) is negative. The normal force acts perpendicular to the motion of the block for all time.

SOLVE

Part a)

$$W_{\text{ext}} = F_{\text{ext}} d \cos(0°) = (5000 \text{ N})(7 \text{ m}) = \boxed{35{,}000 \text{ J}}$$

Part b)

$$W_{\text{gravity}} = F_{\text{gravity}} d(120°) = mgd\cos(120°) = (350 \text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)(7 \text{ m})\cos(120°) = \boxed{-12{,}005 \text{ J}}$$

Part c)

$$W_N = Nd\cos(90°) = \boxed{0}$$

REFLECT

The potential energy of the box has increased so it makes sense that the net work on the box should be positive.

6.41

SET UP

Three clowns are pushing a 300-kg crate a distance of 12 m across a frictionless floor. Moe pushes to the right with a force of 500 N, Larry pushes to the left with a force of 300 N, and Curly pushes down with a force of 600 N. The net force on the block will push it to the right, which means Moe's force is parallel to the displacement, Larry's is antiparallel, and Curly's is perpendicular.

SOLVE

$$W_{\text{Moe}} = F_{\text{Moe}} d\cos(0°) = (500 \text{ N})(12 \text{ m}) = \boxed{6000 \text{ J}}$$

$$W_{\text{Larry}} = F_{\text{Larry}} d\cos(180°) = (300 \text{ N})(12 \text{ m})(-1) = \boxed{-3600 \text{ J}}$$

$$W_{\text{Curly}} = F_{\text{Curly}} d\cos(90°) = \boxed{0}$$

REFLECT

The work done by Curly's force is zero because not only is it acting perpendicular to the displacement, but the displacement in the vertical direction is zero as well. If there were friction involved, Curly's force would increase the normal force and, therefore, the frictional force.

6.42

SET UP

A person carries a 30-kg cooler down a distance of 20 m at a constant speed. Because it is moving at a constant speed, the cooler is moving at constant acceleration, which means the force of the person on the cooler is equal to the weight of the cooler. We'll assume that the person is pulling straight up on the cooler, which is antiparallel to the displacement. The force of gravity is parallel to the displacement of the cooler.

SOLVE

Part a)

$$W_{\text{person}} = F_{\text{person}} d\cos(180°) = -mgd = -(30\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)(20\text{ m}) = \boxed{-5880\text{ J}}$$

Part b)

$$W_{\text{gravity}} = F_{\text{gravity}} d\cos(0°) = mgd = (30\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)(20\text{ m}) = \boxed{5800\text{ J}}$$

REFLECT

The work done by gravity is not always negative because it points downward. It depends on the direction of the displacement of the object.

6.43

SET UP

A 150-kg crate is moving down a 40-degree incline. The coefficient of kinetic friction between the crate and the ramp is 0.54. A museum curator is pushing against the crate with enough force that the crate does not accelerate. We can use Newton's second law to determine the magnitude of the curator's force and the magnitude of kinetic friction. Once we know the magnitudes and directions of all of the forces acting on the crate, we need to determine the angle between each force and the displacement of the crate before calculating the work done by each force.

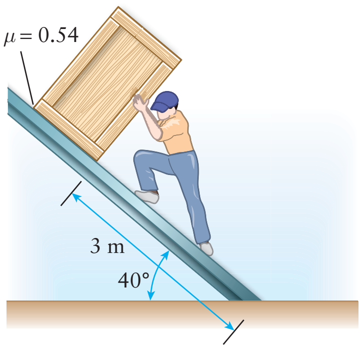


Figure 6-3 Problem 43

SOLVE

Free-body diagram of the crate:

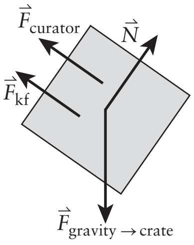


Figure 6-4 Problem 43

Part a)

$$W_{\text{gravity}} = F_{\text{gravity}} d\cos(50°) = mgd\cos(50°) = (150\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)(3\text{ m})\cos(50°) = \boxed{2.8 \times 10^3\text{ J}}$$

Part b)

Newton's second law:

$$\sum F_{\perp} = N - F_{\text{gravity},\,\perp} = N - mg\cos(40°) = ma_{\perp} = 0$$

$$N = mg\cos(40°)$$

$$\sum F_{\parallel} = F_{\text{gravity},\,\parallel} - F_{\text{curator}} - F_{\text{kf}} = mg\sin(40°) - F_{\text{curator}} - \mu_{\text{k}}N$$

$$= mg\sin(40°) - F_{\text{curator}} - \mu_{\text{k}}(mg\cos(40°)) = ma_{\parallel} = 0$$

$$F_{\text{curator}} = mg(\sin(40°) - \mu_{\text{k}}\cos(40°))$$

Calculating the work:

$$W_{\text{curator}} = F_{\text{curator}} d\cos(180°) = -mg(\sin(40°) - \mu_{\text{k}}\cos(40°))d$$

$$= -(150\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)(\sin(40°) - (0.54)\cos(40°))(3\text{ m}) = \boxed{-1.0 \times 10^3\text{ J}}$$

Part c)

$$W_{\text{kf}} = F_{\text{kf}} d\cos(180°) = -(\mu_{\text{k}}N)d = -\mu_{\text{k}}(mg\cos(40°))d$$

$$= -(0.54)(150\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)\cos(40°)(3\text{ m}) = \boxed{-1.8 \times 10^3\text{ J}}$$

Part d)

$$W_N = Nd\cos(90°) = \boxed{0}$$

REFLECT

Remember that the angle used when calculating work is the angle between the force vector and the displacement vector, not the angle of the inclined plane.

6.44

SET UP

We are given four vectors written in terms of unit vectors. The magnitude of each vector is equal to the square root of the dot product of the vector with itself. To determine the angle the vector makes with the $+x$-axis, we can find the dot product of the vector with the unit vector in the x direction; this will equal the magnitude of the vector multiplied by the cosine of the angle.

SOLVE

Part a)

$$A = \sqrt{\vec{A} \cdot \vec{A}} = \sqrt{3^2 + 6^2} = \boxed{\sqrt{45}}$$

$$B = \sqrt{\vec{B} \cdot \vec{B}} = \sqrt{(-6)^2 + 6^2} = \boxed{\sqrt{72}}$$

$$C = \sqrt{\vec{C} \cdot \vec{C}} = \sqrt{2^2 + (-3)^2} = \boxed{\sqrt{13}}$$

$$D = \sqrt{\vec{D} \cdot \vec{D}} = \sqrt{(-3)^2 + (-4)^2} = \boxed{5}$$

Part b)

$$\vec{A} \cdot \hat{x} = A_x = A\cos(\theta)$$

$$\theta = \arccos\left(\frac{A_x}{A}\right) = \arccos\left(\frac{3}{\sqrt{45}}\right) = \boxed{63°}$$

$$\vec{B} \cdot \hat{x} = B_x = B\cos(\theta)$$

$$\theta = \arccos\left(\frac{B_x}{B}\right) = \arccos\left(\frac{-6}{\sqrt{72}}\right) = \arccos\left(\frac{-1}{\sqrt{2}}\right) = \boxed{135°}$$

$$\vec{C} \cdot \hat{x} = C_x = C\cos(\theta)$$

$$\theta = \arccos\left(\frac{C_x}{C}\right) = \arccos\left(\frac{2}{\sqrt{13}}\right) = \boxed{304°}$$

$$\vec{D} \cdot \hat{x} = D_x = D\cos(\theta)$$

$$\theta = \arccos\left(\frac{D_x}{D}\right) = \arccos\left(\frac{-3}{5}\right) = \boxed{233°}$$

REFLECT

The dot product of a vector with a unit vector is equal to the component associated with that unit vector. It's a good idea to make sure you chose the correct value of the arccosine by comparing the angle to the components of the vector.

6.45

SET UP

We are given two vectors written in terms of unit vectors. We can calculate the angle between them by taking their dot product.

SOLVE

$$\vec{A} \cdot \vec{B} = AB\cos(\theta)$$

$$\vec{A} \cdot \vec{B} = (6)(-6) + (6)(6) = 0$$

Therefore, $\boxed{\theta = 90°}$.

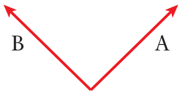


Figure 6-5 Problem 45

REFLECT

Taking the dot product between two vectors is the simplest way to determine if they are perpendicular to one another.

6.46

SET UP

We are asked to prove that the angle between two parallel vectors is zero. We can represent two arbitrary parallel vectors as differing by a scalar multiple a. We can take the dot product of the two arbitrary vectors and compare the result to the product of the magnitudes of the vectors in order to determine $\cos(\theta)$ and, therefore, θ.

SOLVE

$$\vec{A} \cdot \vec{B} = (x\hat{x} + y\hat{y}) \cdot (ax\hat{x} + ay\hat{y}) = ax^2 + ay^2 = a(x^2 + y^2)$$

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{x^2 + y^2}$$

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{a^2x^2 + a^2y^2} = a\sqrt{x^2 + y^2}$$

$$AB = (\sqrt{x^2 + y^2})(a\sqrt{x^2 + y^2}) = a(x^2 + y^2)$$

$\vec{A} \cdot \vec{B} = AB\cos(\theta)$, only if $\theta = 0°$, which means the vectors are parallel.

REFLECT

The dot product of two vectors is a maximum when the vectors are parallel, a minimum when they are antiparallel, and zero when they are perpendicular.

6.47

SET UP

A 0.25-g bumblebee is moving at a speed of 10 m/s. We can calculate its kinetic energy directly from these quantities.

SOLVE

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(2.5 \times 10^{-4}\ \text{kg})\left(10\frac{\text{m}}{\text{s}}\right)^2 = \boxed{1 \times 10^{-2}\ \text{J}}$$

REFLECT

The speed has one significant figure, so our answer has one significant figure.

6.48

SET UP

A 1250-kg car has an initial speed of 20 m/s. The engine does work in order to increase the car's speed to 30 m/s. Assuming this is the only force that does work, the net work required to increase the car's speed is equal to the change in the car's kinetic energy.

SOLVE

$$W_{\text{net}} = \Delta K = K_{\text{f}} - K_{\text{i}} = \frac{1}{2}mv_{\text{f}}^2 - \frac{1}{2}mv_{\text{i}}^2 = \frac{1}{2}m(v_{\text{f}}^2 - v_{\text{i}}^2)$$

$$= \frac{1}{2}(1250\ \text{kg})\left(\left(30\frac{\text{m}}{\text{s}}\right)^2 - \left(20\frac{\text{m}}{\text{s}}\right)^2\right) = \boxed{3.1 \times 10^5\ \text{J}}$$

REFLECT

Even though the car is only increasing its speed by 10 m/s, the work required depends on the mass (which is large) and the difference in the *squares* of the speeds.

6.49

SET UP

A 2100-kg truck has an initial speed of 22 m/s. We want to calculate the net work necessary to decrease the car's speed to 12 m/s, which is equal to the change in the car's kinetic energy.

SOLVE

$$W_{\text{net}} = \Delta K = K_{\text{f}} - K_{\text{i}} = \frac{1}{2}mv_{\text{f}}^2 - \frac{1}{2}mv_{\text{i}}^2 = \frac{1}{2}m(v_{\text{f}}^2 - v_{\text{i}}^2)$$

$$= \frac{1}{2}(2100\ \text{kg})\left(\left(12\frac{\text{m}}{\text{s}}\right)^2 - \left(22\frac{\text{m}}{\text{s}}\right)^2\right) = \boxed{-3.6 \times 10^5\ \text{J}}$$

A net work of $\boxed{3.6 \times 10^5\ \text{J}}$ is required to slow the truck from 22 m/s to 12 m/s.

REFLECT

The negative sign in our solution means that the net force must act opposite to the displacement of the truck.

6.50

SET UP

A 10-kg block starts at rest on level ground. A force of 200 N pushes the block and acts over a distance of 4 m. Kinetic friction also acts over this same distance; the coefficient of kinetic friction between the block and the floor is 0.44. Both of these forces are constant. The net work acting on the block is the sum of the work due to the push and the work due to kinetic friction. It is also equal to the change in the object's kinetic energy through the work–kinetic energy theorem.

SOLVE

Calculating the net work:

$$W_{\text{push}} = F_{\text{push}} d \cos(0°) = (200\ \text{N})(4\ \text{m}) = 800\ \text{J}$$

$$W_{\text{kf}} = F_{\text{kf}} d \cos(180°) = -\mu_{\text{k}} N d = -\mu_{\text{k}} m g d = -(0.44)(10\ \text{kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)(4\ \text{m}) = -173\ \text{J}$$

$$W_{\text{net}} = W_{\text{push}} + W_{\text{kf}} = (800\ \text{J}) + (-173\ \text{J}) = 627\ \text{J}$$

Calculating the final speed:

$$W_{\text{net}} = \Delta K = K_{\text{f}} - K_{\text{i}} = \frac{1}{2}mv_{\text{f}}^2 - \frac{1}{2}mv_{\text{i}}^2 = \frac{1}{2}mv_{\text{f}}^2 - 0$$

$$v_{\text{f}} = \sqrt{\frac{2W_{\text{net}}}{m}} = \sqrt{\frac{2(627\ \text{J})}{10\ \text{kg}}} = \boxed{11\frac{\text{m}}{\text{s}}}$$

REFLECT

The work done by kinetic friction is negative since kinetic friction opposes the motion of the object. The final speed if the surface were frictionless would be 12.6 m/s.

6.51

SET UP

A 2-kg block, which is initially at rest, is pushed by a force of 40 N for 22 m over a frictionless surface. This is the only force that does work on the block because the normal force and the force due to gravity are perpendicular to the displacement of the block. The net work on the block is equal to the change in its kinetic energy; this allows us to calculate the block's final speed.

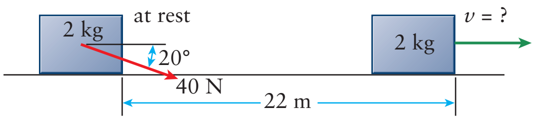


Figure 6-6 Problem 51

SOLVE

$$W_{\text{push}} = F_{\text{push}} d \cos(20°) = (40\ \text{N})(22\ \text{m})\cos(20°) = 830\ \text{J}$$

$$W_{\text{net}} = \Delta K = K_{\text{f}} - K_{\text{i}} = \frac{1}{2}mv_{\text{f}}^2 - \frac{1}{2}mv_{\text{i}}^2 = \frac{1}{2}m(v_{\text{f}}^2 - v_{\text{i}}^2) = \frac{1}{2}mv_{\text{f}}^2 - 0$$

$$v_{\text{f}} = \sqrt{\frac{2W_{\text{net}}}{m}} = \sqrt{\frac{2(830\ \text{J})}{2\ \text{kg}}} = \boxed{29\frac{\text{m}}{\text{s}}}$$

REFLECT

The final speed would be larger if the force were applied at an angle closer to 0 degrees.

6.52

SET UP

A bicycle starts at rest. An applied force of 1200 N then pushes a bicycle forward, while a force of 800 N due to air resistance pushes against it. These forces act on the bicycle over a distance of 20 m. The bicycle is traveling on level ground, which means the forces are parallel or antiparallel to the displacement. We can calculate the work done by each force, determine the net work done, set this equal to the change in the bicycle's kinetic energy, and solve for the final speed of the bicycle.

SOLVE

$$W_{\text{push}} = F_{\text{push}}d\cos(0°) = F_{\text{push}}d = (1200\ \text{N})(20\ \text{m}) = 24{,}000\ \text{J}$$

$$W_{\text{drag}} = F_{\text{drag}}d\cos(180°) = -F_{\text{drag}}d = -(800\ \text{N})(20\ \text{N}) = -16{,}000\ \text{J}$$

$$W_{\text{net}} = W_{\text{push}} + W_{\text{drag}} = (24{,}000\ \text{J}) + (-16{,}000\ \text{J}) = 8000\ \text{J}$$

$$W_{\text{net}} = \Delta K = K_{\text{f}} - K_{\text{i}} = \frac{1}{2}mv_{\text{f}}^2 - \frac{1}{2}mv_{\text{i}}^2 = \frac{1}{2}m(v_{\text{f}}^2 - v_{\text{i}}^2) = \frac{1}{2}mv_{\text{f}}^2 - 0$$

$$v_{\text{f}} = \sqrt{\frac{2W_{\text{net}}}{m}} = \sqrt{\frac{2(8000\ \text{J})}{90\ \text{kg}}} = \boxed{13\frac{\text{m}}{\text{s}}}$$

REFLECT

A speed of 13 m/s is around 30 mph, which seems like a reasonable speed for a bicycle. We could have first calculated the net force on the bicycle (=400 N) in order to calculate the net work:

$$W_{\text{net}} = F_{\text{net}}d\cos(0°) = (400\ \text{N})(20\ \text{m}) = 8000\ \text{J}$$

6.53

SET UP

A baseball of mass m is thrown at a wall with a speed v_{i}. The ball bounces off the wall with a speed of $v_{\text{i}}/3$. We can calculate the initial and final kinetic energies of the ball and then compare them to determine the change in the ball's kinetic energy.

SOLVE

$$K_{\text{i}} = \frac{1}{2}mv_{\text{i}}^2$$

$$K_f = \frac{1}{2}mv_f^2 = \frac{1}{2}m\left(\frac{1}{3}v_i\right)^2 = \frac{1}{9}\left(\frac{1}{2}mv_i^2\right) = \frac{1}{9}K_i$$

The final kinetic energy is one-ninth of the initial kinetic energy, which means the kinetic energy has $\boxed{\text{decreased by a factor of 8/9 (approximately 89\%)}}$. This energy is $\boxed{\text{converted into sound and heat}}$.

REFLECT

The exact mass of the baseball was not necessary to answer this question because it remained constant and we were just looking for percent change.

6.54

SET UP

A book of mass m slides across a carpet with an initial speed of 4 m/s. The book comes to rest after it travels a distance of 3.25 m. The only forces acting on the book are friction, gravity, and the normal force. Since the block is not accelerating in the vertical direction, the magnitude of the normal force is equal to the weight of the block. These forces are perpendicular to the displacement of the book and, therefore, do no work. The net work is equal to the work done by kinetic friction. This is also equal to the change in the book's kinetic energy through the work–kinetic energy theorem. This will allow us to calculate the coefficient of kinetic friction between the book and the carpet.

SOLVE

$$W_{\text{net}} = W_{\text{kf}} = F_{\text{kf}}d\cos(180°) = -(\mu_k N)d = -\mu_k mgd$$

$$\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = 0 - \frac{1}{2}mv_i^2$$

Using the work–kinetic energy theorem:

$$-\mu_k mgd = -\frac{1}{2}mv_i^2$$

$$\mu_k = \frac{v_i^2}{2gd} = \frac{\left(4\frac{\text{m}}{\text{s}}\right)^2}{2\left(9.8\frac{\text{m}}{\text{s}^2}\right)(3.25\text{ m})} = \boxed{0.25\text{ m}}$$

REFLECT

This is a reasonable value for the coefficient of kinetic friction. Remember that coefficients of friction are positive, dimensionless, and usually less than 1.

6.55

SET UP

A 0.15-kg baseball is initially traveling at a speed of 44 m/s when it is caught by a catcher's glove. The glove slows the ball to rest over a distance of 0.125 m. The net work on the ball is equal to the work done by the glove on the ball. The force of the glove on the ball points in

the opposite direction to the ball's displacement. We can use the work–kinetic energy theorem to calculate the magnitude of the average force of the glove during the catch.

SOLVE

$$\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = 0 - \frac{1}{2}mv_i^2$$

$$W_{net} = W_{glove \to ball} = F_{glove \to ball}d\cos(180°) = -F_{glove \to ball}d$$

Using the work–kinetic energy theorem:

$$-\frac{1}{2}mv_i^2 = -F_{glove \to ball}d$$

$$F_{glove \to ball} = \frac{mv_i^2}{2d} = \frac{(0.15\text{ kg})\left(44\frac{\text{m}}{\text{s}}\right)^2}{2(0.125\text{ m})} = \boxed{1200\text{ N}}$$

The force of the glove on the hand will be $\boxed{\text{slightly less}}$ than the force of the glove on the ball because the glove will compress somewhat and absorb energy.

REFLECT

The force of the glove on the ball may not necessarily be (and probably is not) constant. The average force, however, is constant.

6.56

SET UP

A 0.325-kg model boat, initially at rest, is floating on a pond. The wind provides a force on the boat of 1.85 N at an angle of 25 degrees north of east. The drag force of the water on the boat has a magnitude of 0.75 N and points to the west. These forces act continuously as the boat travels east for a distance of 3.55 m. We can calculate the work done by each force, determine the net work done, set this equal to the change in the boat's kinetic energy, and solve for the final speed of the boat.

SOLVE

$$W_{wind} = F_{wind}d\cos(25°) = (1.85\text{ N})(3.55\text{ m})\cos(25°) = 5.95\text{ J}$$

$$W_{drag} = F_{drag}d\cos(180°) = -F_{drag}d = -(0.75\text{ N})(3.55\text{ m}) = -2.66\text{ J}$$

$$W_{net} = W_{push} + W_{drag} = (5.95\text{ J}) + (-2.66\text{ J}) = 3.29\text{ J}$$

$$W_{net} = \Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}mv_f^2 - 0$$

$$v_f = \sqrt{\frac{2W_{net}}{m}} = \sqrt{\frac{2(3.29\text{ J})}{0.325\text{ kg}}} = \boxed{4.50\frac{\text{m}}{\text{s}}}$$

REFLECT

The final speed is 10 mph, which seems like a reasonable speed for a small model boat that weighs less than a pound.

6.57

SET UP

We are given a plot of force versus distance for a variable one-dimensional force. The work done by the force is equal to the area under this curve. We can split the plot up into simple geometric shapes, such as a trapezoid and a triangle.

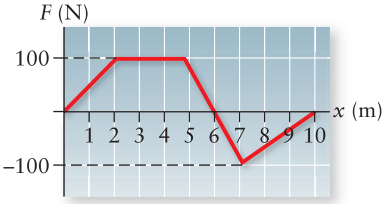


Figure 6-7 Problem 57

SOLVE

$$W_{0\to 10\text{ m}} = W_{0\to 6\text{ m}} + W_{6\to 10\text{ m}} = \frac{1}{2}(100\text{ N})((6\text{ m}) + (3\text{ m})) - \frac{1}{2}(4\text{ m})(100\text{ N}) = \boxed{250\text{ J}}$$

REFLECT

Recall that an area under the x-axis is considered to be negative.

6.58

SET UP

The equation of a force as a function of position $F(x)$ is given in SI units. The work done by this force over a certain displacement is equal to the area under the curve. Since we have the functional form of the force, we can integrate it directly over the desired region.

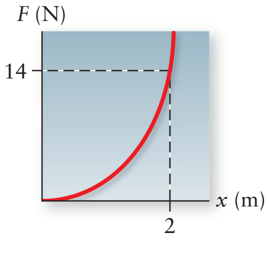


Figure 6-8 Problem 58

SOLVE

$$W = \int_0^2 F(x)dx = \int_0^2 (2x^2 + 3x)dx = \left[\frac{2}{3}x^3 + \frac{3}{2}x^2\right]_0^2 = \frac{2}{3}(2)^3 + \frac{3}{2}(2)^2 = \boxed{11.3\text{ J}}$$

REFLECT

The force and the displacement both point toward positive x, so the work done by the force should also be positive.

6.59

SET UP

An object is attached to a spring with a spring constant $k = 450$ N/m. The object starts at a position 0.12 m from equilibrium and is pulled to a position 0.18 m from equilibrium. The work done by the spring on the object is related to the force of the spring on the object and the displacement. Because the force of the spring is a function of position, we need to use the integral form to calculate the work done. There will also be a negative sign introduced because the force of the spring points in the opposite direction to the displacement.

SOLVE

$$W_{\text{spring}} = -\int_{0.12}^{0.18} F_{\text{spring}}dx = -\int_{0.12}^{0.18} (kx)dx = -\left[\frac{1}{2}kx^2\right]_{0.12}^{0.18} \text{ (SI units)}$$

$$= -\frac{1}{2}\left(450\frac{\text{N}}{\text{m}}\right)\left((0.18\text{ m})^2 - (0.12\text{ m})^2\right) = \boxed{-4.1\text{ J}}$$

REFLECT

The magnitude of the work done by the spring increases nonlinearly as the object is pulled farther and farther from equilibrium. For example, the magnitude of the work done in moving the object from $x = 12$ cm to $x = 18$ cm will be less than the magnitude of the work done in moving the object from $x = 18$ cm to $x = 24$ cm, even though the displacement is $+6$ cm in both cases.

6.60

SET UP

An object of mass m is located at the end of a spring of mass m_s and length L. We are asked to determine the kinetic energy of the entire system, which is equal to the kinetic energy of the object plus the kinetic energy of the spring. We will assume the object moves with a speed of v. In order to determine the kinetic energy of the spring, we need to split it up into infinitesimal elements of mass dm_s and integrate in order to add up the contributions of each small piece of the spring to its kinetic energy: $K_{\text{spring}} = \int \frac{1}{2}(dm_s)v_s^2$, where v_s is the speed of any small part dm_s of the spring. If we call the location of the fixed end of the spring $x = 0$ and the location of the object $x = L$, the mass of the spring will increase as a function of x: $dm_s = \frac{m_s}{L}dx$. Likewise, the speed v_s increases in a linear fashion starting from the fixed end: $v_s = v\frac{x}{L}$. Plugging these into the integral converts it from an integral over mass to an integral over x from $x = 0$ to $x = L$.

SOLVE

Kinetic energy of the mass m:

$$K_m = \frac{1}{2}mv^2$$

Kinetic energy of the spring:

$$K_{\text{spring}} = \int \frac{1}{2}(dm_{\text{s}})v_{\text{s}}^2 = \frac{1}{2}\int_0^L \left(\frac{m_{\text{s}}}{L}dx\right)\left(v\frac{x}{L}\right)^2 = \frac{m_{\text{s}}v^2}{2L^3}\int_0^L x^2 dx = \frac{m_{\text{s}}v^2}{2L^3}\left[\frac{1}{3}x^3\right]_0^L$$

$$= \frac{m_{\text{s}}v^2}{2L^3}\left[\frac{1}{3}L^3\right] = \frac{1}{6}m_{\text{s}}v^2$$

Total kinetic energy:

$$K_{\text{total}} = K_m + K_{\text{spring}} = \frac{1}{2}mv^2 + \frac{1}{6}m_{\text{s}}v^2 = \boxed{\frac{1}{2}\left(m + \frac{m_{\text{s}}}{3}\right)v^2}$$

REFLECT

Usually, we assume the spring is ideal, which means it is massless. This assumption allows us to ignore the contribution of the moving spring to the kinetic energy of the system.

6.61

SET UP

A 5.0-kg object is attached to a spring that is fixed to a wall. The spring constant $k = 250$ N/m. The spring is compressed 10 cm from its equilibrium position and then let go. The spring will push the object back toward equilibrium, which means the force of the spring on the block points in the same direction as the displacement of the block; this is the only force that does work on the object. We can use the work–kinetic energy theorem to relate the work done by the spring over the various displacements to the kinetic energy (and, therefore, the speed) of the object.

SOLVE

Part a)

$$\Delta K = W_{\text{net}}$$

$$\frac{1}{2}mv_{\text{f}}^2 - \frac{1}{2}mv_{\text{i}}^2 = \int_{0.08\text{ m}}^{0.10\text{ m}} F_{\text{spring}}dx$$

$$\frac{1}{2}mv_{\text{f}}^2 + 0 = \int_{0.08\text{ m}}^{0.10\text{ m}} (kx)dx = \left[\frac{1}{2}kx^2\right]_{0.08\text{ m}}^{0.10\text{ m}} = \frac{1}{2}k((0.10\text{ m})^2 - (0.08\text{ m})^2)$$

$$v_{\text{f}} = \sqrt{\frac{k}{m}((0.10\text{ m})^2 - (0.08\text{ m})^2)} = \sqrt{\frac{\left(250\frac{\text{N}}{\text{m}}\right)}{5.0\text{ kg}}((0.10\text{ m})^2 - (0.08\text{ m})^2)} = \boxed{0.4\frac{\text{m}}{\text{s}}}$$

Part b)

$$\frac{1}{2}mv_{\text{f}}^2 - \frac{1}{2}mv_{\text{i}}^2 = \int_{0.05\text{ m}}^{0.10\text{ m}} F_{\text{spring}}dx$$

$$\frac{1}{2}mv_{\text{f}}^2 + 0 = \int_{0.05\text{ m}}^{0.10\text{ m}} (kx)dx = \left[\frac{1}{2}kx^2\right]_{0.05\text{ m}}^{0.10\text{ m}} = \frac{1}{2}k((0.10\text{ m})^2 - (0.05\text{ m})^2)$$

$$v_f = \sqrt{\frac{k}{m}((0.10\text{ m})^2 - (0.05\text{ m})^2)} = \sqrt{\frac{\left(250\frac{\text{N}}{\text{m}}\right)}{5.0\text{ kg}}((0.10\text{ m})^2 - (0.05\text{ m})^2)} = \boxed{0.6\frac{\text{m}}{\text{s}}}$$

Part c)

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \int_{0\text{ m}}^{0.10\text{ m}} F_{\text{spring}}dx$$

$$\frac{1}{2}mv_f^2 + 0 = \int_{0\text{ m}}^{0.10\text{ m}} (kx)dx = \left[\frac{1}{2}kx^2\right]_{0\text{ m}}^{0.10\text{ m}} = \frac{1}{2}k((0.10\text{ m})^2 - (0\text{ m})^2)$$

$$v_f = \sqrt{\frac{k}{m}(0.10\text{ m})^2} = (0.10\text{ m})\sqrt{\frac{\left(250\frac{\text{N}}{\text{m}}\right)}{5.0\text{ kg}}} = \boxed{0.7\frac{\text{m}}{\text{s}}}$$

REFLECT

As the spring relaxes to its equilibrium length, we expect the stored potential energy to convert into kinetic energy, which means the object should move faster and faster. The maximum speed of the object will occur as the object passes through the equilibrium position.

6.62

SET UP

The equation of a force as a function of position $F(x)$ is given in SI units. The work done by this force over a certain displacement is equal to the area under the curve. Since we have the functional form of the force, we can integrate it directly over the desired region.

SOLVE

$$W = \int_0^1 F(x)dx = \int_0^1 (12x + 2x^2 - 0.25x^3)dx = \left[6x^2 + \frac{2}{3}x^3 - \frac{1}{16}x^4\right]_0^1$$

$$= 6(1)^2 + \frac{2}{3}(1)^3 - \frac{1}{16}(1)^4 = \boxed{6.6\text{ J}}$$

REFLECT

Even though the x^3 term is negative, its contribution is small over the region from $x = 0$ m to $x = 1$ m, so it makes sense that the work done is positive.

6.63

SET UP

We are given the variable force that acts on an object that moves from the origin to (3,4). We will need to use the integral form of the work because the force is a function of the object's position. We can separate the integral into a "dx" integral from $x = 0$ to $x = 3$ and a "dy" integral from $y = 0$ to $y = 4$. All of the values are in SI units, so our final answer will have units of joules.

SOLVE

$$\vec{F} = 3x\hat{x} + 4y\hat{y}$$

$$d\vec{r} = dx\hat{x} + dy\hat{y}$$

$$\int_{(0,0)}^{(3,4)} \vec{F} \cdot d\vec{r} = \int_{(0,0)}^{(3,4)} (3x\hat{x} + 4y\hat{y}) \cdot (dx\hat{x} + dy\hat{y}) = \int_0^3 3xdx + \int_0^4 4ydy$$

$$= \left[\frac{3}{2}x^2\right]_0^3 + [2y^2]_0^4 = \frac{3}{2}(3)^2 + 2(4)^2 + 0 + 0 = \boxed{46 \text{ J}}$$

REFLECT

Because x and y are positive, the force points in the same general direction as the displacement and the work should be positive.

6.64

SET UP

We are given the variable force that acts on an object that moves from the origin to (2,8,3). We will need to use the integral form of the work because the force is a function of the object's position. We can separate the integral into a "dx" integral from $x = 0$ to $x = 2$, a "dy" integral from $y = 0$ to $y = 8$, and a "dz" integral from $z = 0$ to $z = 3$. All of the values are in SI units, so our final answer will have units of joules.

SOLVE

$$\vec{F} = 2x\hat{x} + 3y\hat{y} + 0.2z^2\hat{z}$$

$$d\vec{r} = dx\hat{x} + dy\hat{y} + dz\hat{z}$$

$$\int_{(0,0,0)}^{(2,8,3)} \vec{F} \cdot d\vec{r} = \int_{(0,0,0)}^{(2,8,3)} (2x\hat{x} + 3y\hat{y} + 0.2z^2\hat{z}) \cdot (dx\hat{x} + dy\hat{y} + dz\hat{z})$$

$$= \int_0^2 2xdx + \int_0^8 3ydy + \int_0^8 0.2z^2dz = [x^2]_0^2 + \left[\frac{3}{2}y^2\right]_0^8 + \left[\frac{0.2}{3}z^3\right]_0^3$$

$$= (2)^2 + \frac{3}{2}(8)^2 + \left(\frac{0.2}{3}\right)(3)^3 = \boxed{101 \text{ J}}$$

REFLECT

Because x and y are positive, the force points in the same general direction as the displacement and the work should be positive.

6.65

SET UP

A 1-N apple is 2.5 m above the ground. The gravitational potential energy of the apple relative to the ground is equal to the product of its weight and the distance above the ground.

SOLVE

$$\Delta U_{\text{gravity}} = mg\Delta y = (1\ \text{N})(2.5\ \text{m}) = \boxed{2.5\ \text{J}}$$

REFLECT

Remember that only the difference in potential energy is physically meaningful.

6.66

SET UP

A 2000-kg object is dropped from rest from a height of 18 m above the ground. Its final position is 2 m above the ground. The change in the object's gravitational potential energy is equal to the final potential energy minus the initial potential energy.

SOLVE

$$\Delta U_{\text{gravity}} = mg\Delta y = (2000\ \text{kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)((2\ \text{m}) - (18\ \text{m})) = \boxed{-3.1 \times 10^5\ \text{J}}$$

REFLECT

The gravitational potential energy of the object should decrease since its final position is closer to the ground.

6.67

SET UP

A 40.0-kg boy glides down a hill on a skateboard. He ends up a vertical distance of 4.35 m below his initial location. The change in his gravitational potential energy is equal to this change in height multiplied by his weight.

SOLVE

$$\Delta U_{\text{gravity}} = mg\Delta y = (40.0\ \text{kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)(-4.35\ \text{m}) = \boxed{-1700\ \text{J} = -1.7\ \text{kJ}}$$

REFLECT

The boy's initial gravitational potential energy is converted into kinetic energy as he glides down the hill.

6.68

SET UP

A spring has a spring constant $k = 15.5$ N/m. We are asked to calculate the difference in potential energy stored in the spring stretched by two different amounts. The spring begins stretched 10 cm from its equilibrium position. It is then stretched to 15 cm from its equilibrium position. The potential energy of a spring is proportional to the square of the stretched distance.

SOLVE

$$\Delta U_{\text{spring}} = \frac{1}{2}kx_{\text{f}}^2 - \frac{1}{2}kx_{\text{i}}^2 = \frac{1}{2}k(x_{\text{f}}^2 - x_{\text{i}}^2) = \frac{1}{2}\left(15.5\frac{\text{N}}{\text{m}}\right)((0.15\ \text{m})^2 - (0.10\ \text{m})^2) = 0.097\ \text{J}$$

REFLECT

The spring is stretched more in the final state than the initial, so it makes sense that the difference between them should be positive.

6.69

SET UP

A spring with spring constant $k = 200$ N/m is resting vertically on the ground. A 2-kg object is placed on the upper end of the spring, which compresses the spring, and everything comes to rest. The net force on the object is zero at equilibrium, which means the force due to the spring acting on the object is equal in magnitude to the object's weight; this will tell us the amount by which the spring compresses, which we can then use to calculate the potential energy store in the spring.

SOLVE

Part a)

$$\sum F_y = F_{\text{spring}} - F_{\text{gravity}} = ky - mg = ma_y = 0$$

$$ky - mg = 0$$

$$y = \frac{mg}{k} = \frac{(2\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)}{\left(200\frac{\text{N}}{\text{m}}\right)} = \boxed{0.10\text{ m}}$$

Part b)

$$\Delta U_{\text{spring}} = \frac{1}{2}ky_{\text{f}}^2 - \frac{1}{2}ky_{\text{i}}^2 = \frac{1}{2}ky_{\text{f}}^2 - 0 = \frac{1}{2}\left(200\frac{\text{N}}{\text{m}}\right)(0.01\text{ m})^2 = \boxed{1\text{ J}}$$

REFLECT

The potential energy of the spring has increased because the object did nonconservative work on it.

6.70

SET UP

A 3.5-kg box is pushed 5 m up a frictionless ramp to an overall height of 3 m. This external force acts parallel to the surface of the ramp (that is, parallel to the displacement of the box). We are told the box travels at a constant speed, which means its acceleration parallel to the ramp is zero. We can use Newton's second law to calculate the magnitude of the external force. We can multiply this force by the distance it travels up the ramp to calculate the work done by the external force on the box. The change in the box's gravitational energy is equal to the product of its weight and its change in height.

SOLVE

Part a)
Newton's second law:

$$\sum F_{\parallel} = F_{\text{ext}} - F_{\text{gravity},\parallel} = F_{\text{ext}} - mg\sin(\theta) = ma_{\parallel} = 0$$

$$F_{\text{ext}} = mg\sin(\theta)$$

Work:

$$W_{\text{ext}} = F_{\text{ext}}d\cos(0°) = (mg\sin(\theta))d = mg\left(\frac{3}{5}\right)d = \left(\frac{3}{5}\right)(3.5\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)(5\text{ m}) = \boxed{103\text{ J}}$$

Part b)

$$\Delta U_{\text{gravity}} = mg\Delta y = (3.5\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)(3\text{ m}) = \boxed{103\text{ J}}$$

The work done by the external force on the box is equal to the box's change in gravitational potential energy.

REFLECT
The external force is nonconservative, which means the work done by it is equal to the change in the box's mechanical energy. Since the kinetic energy of the box remains constant, the work will be equal to the change in the box's gravitational potential energy.

6.71

SET UP
A 0.0335-kg coin is initially at the ground level of a building and then carried up to a height of 630 m. The change in the coin's gravitational potential energy is equal to its weight multiplied by the difference in its vertical position.

SOLVE

$$\Delta U_{\text{gravity}} = mg\Delta y = (0.0335\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)((630\text{ m}) - (0\text{ m})) = \boxed{210\text{ J}}$$

REFLECT
Technically, the answer will be a little less than this since g decreases with height.

6.72

SET UP
A 65-kg person walks along a walkway that is 356 m above the surface of the Earth. His potential energy relative to the surface of the Earth is equal to his weight multiplied by his height above the Earth.

SOLVE

$$\Delta U_{\text{gravity}} = mg\Delta y = (65\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)(356\text{ m}) = \boxed{227{,}000\text{ J} = 227\text{ kJ}}$$

REFLECT
The CN tower is the tallest structure in the Western Hemisphere.

6.73

SET UP
A spring is compressed a distance of 0.125 m from its equilibrium position. We are told that the potential energy stored by the spring in this configuration is 3.33 J. We can use the definition of the spring potential energy to calculate the spring constant of this spring.

SOLVE

$$U_{\text{spring}} = \frac{1}{2}kx^2$$

$$k = \frac{2U_{\text{spring}}}{x^2} = \frac{2(3.33\text{ J})}{(0.125\text{ m})^2} = \boxed{426\frac{\text{N}}{\text{m}}}$$

REFLECT
The larger the spring constant, the "stiffer" the spring.

6.74

SET UP
A ball is thrown straight up with an initial speed of 15 m/s. We can use conservation of energy to calculate the height of the ball when its speed is one-half of the initial speed. We'll define the initial position to have a potential energy of zero.

SOLVE

$$U_{\text{i}} + K_{\text{i}} = U_{\text{f}} + K_{\text{f}}$$

$$0 + \frac{1}{2}mv_{\text{i}}^2 = mgy_{\text{f}} + \frac{1}{2}mv_{\text{f}}^2$$

$$\frac{1}{2}v_{\text{i}}^2 = gy_{\text{f}} + \frac{1}{2}\left(\frac{1}{2}v_{\text{i}}\right)^2$$

$$y_{\text{f}} = \frac{3}{8}\frac{v_{\text{i}}^2}{g} = \frac{3}{8}\frac{\left(15\frac{\text{m}}{\text{s}}\right)^2}{\left(9.8\frac{\text{m}}{\text{s}^2}\right)} = \boxed{8.6\text{ m}}$$

REFLECT
This corresponds to a height of 28 feet, which seems reasonable because the ball was launched at a speed of around 34 mph.

6.75

SET UP

A water balloon is thrown straight down with an initial speed of 12 m/s from a height of 5.0 m above the ground. We can use conservation of energy to calculate the speed of the ball right before it hits the ground. We'll define the ground to have a potential energy of zero.

SOLVE

$$U_i + K_i = U_f + K_f$$

$$mgy_i + \frac{1}{2}mv_i^2 = 0 + \frac{1}{2}mv_f^2$$

$$v_f = \sqrt{2gy_i + v_i^2} = \sqrt{2\left(9.8\frac{\text{m}}{\text{s}^2}\right)(5.0\text{ m}) + \left(12\frac{\text{m}}{\text{s}}\right)^2} = \boxed{16\frac{\text{m}}{\text{s}}}$$

REFLECT

This is the same answer we would get if we used the constant acceleration equation $v^2 - v_0^2 = 2a_y(\Delta y)$.

6.76

SET UP

A gold coin of mass m is dropped from a height of 630 m above the ground. We can use conservation of energy to calculate the speed of the ball right before it hits the ground. We'll define the ground to have a potential energy of zero.

SOLVE

$$U_i + K_i = U_f + K_f$$

$$mgy_i + 0 = 0 + \frac{1}{2}mv_f^2$$

$$v_f = \sqrt{2gy_i} = \sqrt{2\left(9.8\frac{\text{m}}{\text{s}^2}\right)(630\text{ m})} = \boxed{111\frac{\text{m}}{\text{s}}}$$

REFLECT

The answer is independent of the mass of the coin.

6.77

SET UP

A child riding a sled (total mass m) slides down a frictionless 15-degree incline. At the bottom of the hill, her speed is 7 m/s. Although we are not given her initial height above the ground, we can calculate it using conservation of energy—all of her initial gravitational potential energy is converted to kinetic energy at the bottom of the hill. We can calculate the distance she slid down the incline using the sine of the angle.

SOLVE

$$U_i + K_i = U_f + K_f$$

$$mgy_i + 0 = 0 + \frac{1}{2}mv_f^2$$

$$y_i = \frac{v_f^2}{2g} = \frac{\left(7\frac{\text{m}}{\text{s}}\right)^2}{2\left(9.8\frac{\text{m}}{\text{s}^2}\right)} = 2.5\text{ m}$$

$$\sin(15°) = \frac{y_i}{d}$$

$$d = \frac{y_i}{\sin(15°)} = \frac{2.5\text{ m}}{\sin(15°)} = \boxed{9.7\text{ m}}$$

REFLECT

The mass of the child + sled was not necessary since it cancels out in the solution.

6.78

SET UP

Carl Lewis jumped to a maximum height of 1.2 m. At this height he was still moving with a speed of 6.6 m/s. We can use conservation of energy to determine Carl's initial speed. We'll define the initial gravitational potential energy (before he's jumped) to be zero.

SOLVE

$$U_i + K_i = U_f + K_f$$

$$0 + \frac{1}{2}mv_i^2 = mgy_f + \frac{1}{2}mv_f^2$$

$$v_f = \sqrt{2gy_f + v_i^2} = \sqrt{2\left(9.8\frac{\text{m}}{\text{s}^2}\right)(1.2\text{ m}) + \left(6.6\frac{\text{m}}{\text{s}}\right)^2} = \boxed{8.2\frac{\text{m}}{\text{s}}}$$

REFLECT

This corresponds to 18 mph, which is a reasonable running speed for an Olympic athlete.

6.79

SET UP

A pendulum is made of a ball of mass m and a string of length $L = 1.25$ m. The ball is lifted to a height such that the string makes an angle of 30 degrees with respect to the vertical. All of the ball's initial gravitational potential energy is converted to kinetic energy at the bottom of its swing. Applying conservation of mechanical energy, we can determine the speed of the ball at the bottom of its swing. We will measure the gravitational potential energy relative to the lowest part of the pendulum's swing. We can use trigonometry to relate the initial height y_i to the length L.

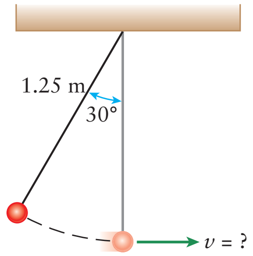


Figure 6-9 Problem 79

SOLVE

Trigonometry:

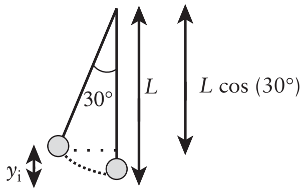


Figure 6-10 Problem 79

$$y_i = L - L\cos(30°) = L(1 - \cos(30°))$$

Conservation of energy:

$$U_i + K_i = U_f + K_f$$

$$mgy_i + 0 = 0 + \frac{1}{2}mv_f^2$$

$$gL(1 - \cos(30°)) = \frac{1}{2}v_f^2$$

$$v_f = \sqrt{2gL(1 - \cos(30°))} = \sqrt{2\left(9.8\frac{\text{m}}{\text{s}^2}\right)(1.25\text{ m})(1 - \cos(30°))} = \boxed{1.8\frac{\text{m}}{\text{s}}}$$

The mass impacts the speed only in that if the mass is too small, air resistance and the mass of the string will become significant.

REFLECT

The tension in the string is always perpendicular to the ball's motion; thus, the work done by the tension will be zero.

6.80

SET UP

An ice cube of mass m starts from rest at a height $y_A = 5$ m and slides down a track without friction. This initial gravitational potential energy is converted to kinetic energy (and vice versa) as the ice cube slides on the track. We can use conservation of mechanical energy to determine its speed at the marked locations.

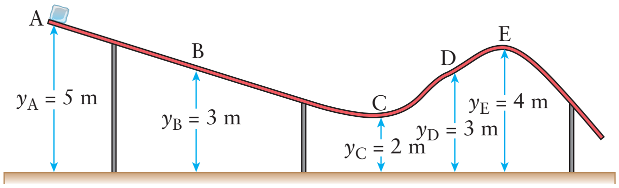


Figure 6-11 Problem 80

SOLVE

Point B:

$$U_A + K_A = U_B + K_B$$

$$mgy_A + 0 = mgy_B + \frac{1}{2}mv_B^2$$

$$v_B = \sqrt{2g(y_A - y_B)} = \sqrt{2\left(9.8\frac{\text{m}}{\text{s}^2}\right)((5\text{ m}) - (3\text{ m}))} = \boxed{6.26\frac{\text{m}}{\text{s}}}$$

Point C:

$$U_A + K_A = U_C + K_C$$

$$mgy_A + 0 = mgy_C + \frac{1}{2}mv_C^2$$

$$v_C = \sqrt{2g(y_A - y_C)} = \sqrt{2\left(9.8\frac{\text{m}}{\text{s}^2}\right)((5\text{ m}) - (2\text{ m}))} = \boxed{7.67\frac{\text{m}}{\text{s}}}$$

Point D:

$$U_A + K_A = U_D + K_D$$

$$mgy_A + 0 = mgy_D + \frac{1}{2}mv_D^2$$

$$v_D = \sqrt{2g(y_A - y_D)} = \sqrt{2\left(9.8\frac{\text{m}}{\text{s}^2}\right)((5\text{ m}) - (3\text{ m}))} = \boxed{6.26\frac{\text{m}}{\text{s}}}$$

Point E:

$$U_A + K_A = U_E + K_E$$

$$mgy_A + 0 = mgy_E + \frac{1}{2}mv_E^2$$

$$v_B = \sqrt{2g(y_A - y_E)} = \sqrt{2\left(9.8\frac{\text{m}}{\text{s}^2}\right)((5\text{ m}) - (4\text{ m}))} = \boxed{4.43\frac{\text{m}}{\text{s}}}$$

REFLECT

Without calculating anything, we can determine that the ice cube will be traveling the fastest at point C since it is the lowest point in the track. Points B and D will have the same speed because they are at the same height. Point E will have the smallest speed.

6.81

SET UP

A driver slams on her brakes and skids to a stop over a distance of 88 m. The coefficient of kinetic friction between the tires and the road is 0.48. We can use the work–kinetic energy theorem to determine the initial speed of the car. The force of kinetic friction, which is the only force that does work on the car, is constant and antiparallel to the displacement of the car. The car is not accelerating in the vertical direction, which means the magnitude of the normal force is equal to the weight of the car. We will call the mass of the car m.

SOLVE

$$\Delta K = W_{\text{net}}$$

$$\frac{1}{2}mv_{\text{f}}^2 - \frac{1}{2}mv_{\text{i}}^2 = W_{\text{kf}} = F_{\text{kf}}d\cos(180°) = -(\mu_{\text{k}}N)d = -\mu_{\text{k}}(mg)d$$

$$0 - \frac{1}{2}mv_{\text{i}}^2 = -\mu_{\text{k}}mgd$$

$$v_{\text{i}} = \sqrt{2\mu_{\text{k}}gd} = \sqrt{2(0.48)\left(9.8\frac{\text{m}}{\text{s}^2}\right)(88\text{ m})} = \boxed{29\frac{\text{m}}{\text{s}}}$$

REFLECT

Kinetic friction is a nonconservative force and the only one that does work on the car, so the work done by kinetic friction is equal to the change in the car's mechanical energy. Because the potential energy remains constant, the change in the mechanical energy is equal to the change in the kinetic energy. This is an equivalent statement to the work–kinetic energy theorem in this case.

6.82

SET UP

A 65-kg woman steps off a 10-m-high diving platform and drops straight into the water. The water exerts a force on her and she comes to rest at a depth of 4.5 m. The force of the water on the diver is the only nonconservative force that does work on her, which means we can set this equal to the change in her mechanical energy. She falls a total distance of $\Delta y = -14.5$ m and begins and ends at rest. The force of the water only acts over the distance $d = 4.5$ m that the diver is in the water.

SOLVE

$$W_{\text{nc}} = \Delta U + \Delta K = \Delta U_{\text{gravity}} + 0 = mg\Delta y$$

$$W_{\text{nc}} = W_{\text{water}} = F_{\text{water}}d\cos(180°) = -F_{\text{water}}d$$

$$-F_{\text{water}}d = mg\Delta y$$

$$F_{\text{water}} = -\frac{mg\Delta y}{d} = -\frac{(65\text{ kg})\left(9.8\dfrac{\text{m}}{\text{s}^2}\right)(-14.5\text{ m})}{4.5\text{ m}} = \boxed{2053\text{ N}}$$

REFLECT

The average magnitude of the force of the water on the diver must be positive. It should be reasonably large because she comes to rest in 4.5 m; this force is about three times her weight.

6.83

SET UP

A skier leaves the starting gate at an elevation of 4212 m with an initial speed of 4.00 m/s. The end of the ski slope is at an elevation of 4017 m. We are told that air resistance causes a 50% loss in the final kinetic energy (compared to the case with no air resistance). We can use conservation of mechanical energy to first determine the skier's final speed in the absence of air resistance (that is, the ideal case) and use this to calculate the skier's final speed taking air resistance into account (that is, the real case).

GPS elevation = 4212 m

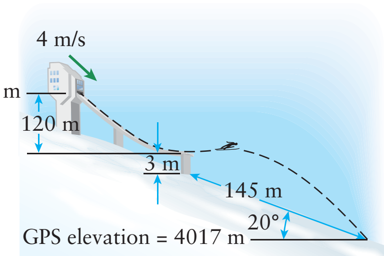


Figure 6-12 Problem 83

SOLVE

Conservation of mechanical energy:

$$U_{\text{i}} + K_{\text{i}} = U_{\text{f}} + K_{\text{f}}$$

$$\frac{1}{2}mv_{\text{i}}^2 + mgy_{\text{i}} = \frac{1}{2}mv_{\text{f, ideal}}^2 + mgy_{\text{f}}$$

$$v_{\text{f, ideal}}^2 = v_{\text{i}}^2 + 2g(y_{\text{i}} - y_{\text{f}})$$

Considering air resistance:

$$K_{\text{real}} = \frac{1}{2}K_{\text{ideal}}$$

$$\frac{1}{2}mv_{\text{f, real}}^2 = \frac{1}{2}\left(\frac{1}{2}mv_{\text{f, ideal}}^2\right)$$

$$v_{\text{f, real}} = \sqrt{\frac{v_{\text{f, ideal}}^2}{2}} = \sqrt{\frac{v_{\text{i}}^2 + 2g(y_{\text{i}} - y_{\text{f}})}{2}}$$

$$= \sqrt{\frac{\left(4.00\frac{\text{m}}{\text{s}}\right)^2 + 2\left(9.8\frac{\text{m}}{\text{s}^2}\right)((4212\text{ m}) - (4017\text{ m}))}{2}} = \boxed{43.8\frac{\text{m}}{\text{s}}}$$

REFLECT

The specifics regarding the middle of the skier's trip are not important since the surfaces are frictionless and we are told the change in energy caused by air resistance.

6.84

SET UP

An 18-kg suitcase was dropped from rest from a hot-air balloon floating at a height of y_i = 385 m. We are told the final speed of the suitcase right before it hits the ground is 30 m/s. Not all of the initial gravitational potential energy is converted into kinetic energy because air drag is doing work on the suitcase. We can use conservation of mechanical energy to determine the ideal final kinetic energy of the suitcase and take a ratio of the ideal to the actual final kinetic energies to find the percentage of energy that was "lost."

SOLVE

Ideal final kinetic energy:

$$U_i + K_i = U_f + K_f$$

$$mgy_i + 0 = 0 + K_{f,\text{ ideal}}$$

$$K_{f,\text{ ideal}} = (18\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)(385\text{ m}) = 67{,}900\text{ J}$$

Actual final kinetic energy:

$$K_{f,\text{ actual}} = \frac{1}{2}mv_{f,\text{ actual}}^2 = \frac{1}{2}(18\text{ kg})\left(30\frac{\text{m}}{\text{s}}\right)^2 = 8100\text{ J}$$

Percent energy "lost":

$$\frac{K_{f,\text{ actual}}}{K_{f,\text{ ideal}}} = \frac{8100\text{ J}}{67{,}900\text{ J}} = 0.12$$

The final kinetic energy is 12% of the initial kinetic energy, which means $\boxed{\text{88\% was "lost" to air resistance}}$.

REFLECT

If we didn't calculate the intermediate kinetic energies, we would see explicitly that the mass of the suitcase was not necessary:

$$\frac{K_{f,\text{ actual}}}{K_{f,\text{ ideal}}} = \frac{\left(\frac{1}{2}mv_{f,\text{ actual}}^2\right)}{mgy_i} = \frac{v_{f,\text{ actual}}^2}{2gy_i}$$

6.85

SET UP

A bicyclist travels $d = 20$ m up a 10-degree incline at a constant speed. The combined mass of the person and the bike is $m = 90$ kg. The work done by gravity is equal to the negative of the bicyclist's change in gravitational potential energy because gravity is a conservative force. We can use trigonometry to represent the final height of the rider in terms of d. The only other nonzero work is the work done by the person. (The normal force is perpendicular to the displacement for all time and, therefore, does no work on the system.) We can use the work–kinetic energy theorem to relate the net work to the change in the bicyclist's kinetic energy, which is zero since he remains at a constant speed.

SOLVE

Work done by gravity:

$$W_{\text{gravity}} = -\Delta U_{\text{gravity}} = -mg\Delta y = -mg(d\sin(10°))$$

$$= -(90 \text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)(20 \text{ m})\sin(10°) = \boxed{-3100 \text{ J} = -3.1 \text{ kJ}}$$

Work done by the person:

$$W_{\text{net}} = W_{\text{gravity}} + W_{\text{person}} + W_N = \Delta K = 0$$

$$W_{\text{gravity}} + W_{\text{person}} + 0 = 0$$

$$W_{\text{person}} = -W_{\text{gravity}} = \boxed{3.1 \text{ kJ}}$$

REFLECT

We could have also calculated the work done by the person using the definition of work. The bicyclist is traveling at a constant speed, which means his acceleration parallel to the ramp is zero. The magnitude of the force he applies is equal to the magnitude of the component of gravity parallel to the ramp: $F_{\text{person}} = mg\sin(10°)$. The force of the person on the bike and his displacement point in the same direction, so $W_{\text{person}} = F_{\text{person}}d = mgd\sin(10°)$, which is the same result.

6.86

SET UP

A child slides down a hill with an initial speed of 1 m/s. After traveling a distance of 25 m, her speed is 4 m/s. The net work done on the child + sled system is equal to the change in the kinetic energy of the system. The forces acting on the system are kinetic friction, gravity, and the normal force. The normal force is perpendicular to the displacement, so the work done by the normal force is zero. Kinetic friction has a magnitude that is one-fifth of the weight of the system and acts antiparallel to the displacement. The angle between the force of gravity and the displacement is equal to $(90° - \theta)$, where θ is the angle the hill makes with the horizontal.

SOLVE

$$\Delta K = W_{\text{net}} = W_{\text{kf}} + W_{\text{gravity}} + W_N$$

$$\frac{1}{2}m(v_f^2 - v_i^2) = F_{kf}d\cos(180°) + F_{gravity}d\cos(90° - \theta) + 0$$

$$\frac{1}{2}m(v_f^2 - v_i^2) = -\left(\frac{mg}{5}\right)d + (mg\sin(\theta))d$$

$$\sin(\theta) = \frac{1}{2gd}(v_f^2 - v_i^2) + \frac{1}{5}$$

$$\theta = \arcsin\left[\frac{1}{2gd}(v_f^2 - v_i^2) + \frac{1}{5}\right] = \arcsin\left[\frac{1}{2\left(9.8\frac{\text{m}}{\text{s}^2}\right)(25\text{ m})}\left(\left(4\frac{\text{m}}{\text{s}}\right)^2 - \left(1\frac{\text{m}}{\text{s}}\right)^2\right) + \frac{1}{5}\right] = \boxed{13°}$$

REFLECT

This is a reasonable angle for a hill. In finding the work done by gravity, we could have also used the component of the gravitational force that was parallel to the hill ($W_{gravity} = F_{gravity,\,\parallel}d\cos(0°) = (mg\sin(\theta))d\cos(0°) = mgd\sin(\theta)$) since the perpendicular component does not contribute to the work.

6.87

SET UP

The coefficient of restitution e of a ball is defined as the speed of the ball after impact divided by the speed of the ball just before impact. We can use conservation of mechanical energy to determine these speeds in terms of the heights h and H. The kinetic energy of the ball at the heights h and H is equal to zero since the ball is momentarily at rest.

SOLVE

Part a)
Before:

$$mgH = \frac{1}{2}mv_{before}^2$$

$$v_{before} = \sqrt{2gH}$$

After:

$$\frac{1}{2}mv_{after}^2 = mgh$$

$$v_{after} = \sqrt{2gh}$$

Finding the coefficient of restitution:

$$e = \frac{v_{after}}{v_{before}} = \frac{\sqrt{2gh}}{\sqrt{2gH}} = \boxed{\sqrt{\frac{h}{H}}}$$

Part b)

$$e = \sqrt{\frac{h}{H}} = \sqrt{\frac{60\text{ cm}}{80\text{ cm}}} = \boxed{\sqrt{\frac{3}{4}} = 0.87}$$

REFLECT

A coefficient of restitution of 1 refers to a completely elastic collision.

6.88

SET UP

We are given the vectors for the displacement of an object and the force acting on that object. The dot product of these two vectors is equal to the work done by the force on the object.

SOLVE

$$W = \vec{F} \cdot \vec{d} = (2.2\hat{x} + 4.5\hat{y}) \cdot (12\hat{x} + 20\hat{y}) = (2.2)(12) + (4.5)(20) = \boxed{116.4\text{ J}}$$

REFLECT

The angle between the two vectors is around 4 degrees.

6.89

SET UP

A force $\vec{F} = F_x\hat{x} + F_y\hat{y}$ has a magnitude 50 N and does 24 J of work on an object with a displacement of $\vec{d} = 2\hat{x} + 2\hat{y}$. We can find the components of the force from these two data. The work is equal to the force dotted into the displacement. The magnitude of the vector is $F = \sqrt{F_x^2 + F_y^2}$. Once we have the components of the force vector, we can divide both by the magnitude of the force (that is, 50 N) to get the direction vector of the force, $\hat{F}$.

SOLVE

$$F = \sqrt{F_x^2 + F_y^2} = 50 \text{ (SI units)}$$

$$F_x^2 + F_y^2 = 2500$$

$$W = \vec{F} \cdot \vec{d} = (F_x\hat{x} + F_y\hat{y}) \cdot (2\hat{x} + 2\hat{y}) = 2F_x + 2F_y = 24$$

$$F_y = 12 - F_x$$

$$F_x^2 + F_y^2 = F_x^2 + (12 - F_x)^2 = 2500$$

$$2F_x^2 - 24F_x - 2356 = 0$$

$$F_x = \frac{-(-24) \pm \sqrt{(-24)^2 - 4(2)(-2356)}}{2(2)} = \frac{24 \pm \sqrt{19{,}424}}{4}$$

Taking the positive root:

$$F_x = \frac{24 + \sqrt{19{,}424}}{4} = 40.8\text{ N}$$

$$F_y = 12 - 40.8 = -28.8 \text{ N}$$

$$\vec{F} = (40.8 \text{ N})\hat{x} - (28.8 \text{ N})\hat{y}$$

Direction vector for the positive root:

$$\hat{F} = \frac{\vec{F}}{|F|} = \frac{(40.8 \text{ N})\hat{x} - (28.8 \text{ N})\hat{y}}{50 \text{ N}} = \boxed{(0.817)\hat{x} - (0.577)\hat{y}}$$

Taking the negative root:

$$F_x = \frac{24 - \sqrt{19{,}424}}{4} = -28.8 \text{ N}$$

$$F_y = 12 - (-28.8) = 40.8 \text{ N}$$

$$\vec{F} = -(28.8 \text{ N})\hat{x} + (40.8 \text{ N})\hat{y}$$

Direction vector for the negative root:

$$\hat{F} = \frac{\vec{F}}{|F|} = \frac{-(28.8 \text{ N})\hat{x} + (40.8 \text{ N})\hat{y}}{50 \text{ N}} = \boxed{-(0.577)\hat{x} + (0.817)\hat{y}}$$

REFLECT

The "^" notation, as in $\hat{F}$, denotes the unit vector for force $\vec{F}$: the vector that points along $\vec{F}$ but has a magnitude of 1.

6.90

SET UP

You push a 20-kg crate at a constant velocity up a 33-degree ramp. The coefficient of kinetic friction between the crate and the ramp is 0.2. There are three nonconservative forces acting on the box—your push, kinetic friction, and the normal force. The work done by all of these forces is equal to the change in the crate's mechanical energy. (The change in the crate's kinetic energy is zero, so, in this case, the net nonconservative work is equal to the change in the crate's gravitational potential energy.) Although the normal force does no work on the crate, we still need to calculate the magnitude of this force in order to calculate the work done by kinetic friction. From Newton's second law and the fact that the crate is traveling at a constant velocity up the ramp (that is, its acceleration is zero in all directions), we see that the normal force is equal in magnitude to the component of the crate's weight that acts perpendicularly to the ramp.

SOLVE

$$W_{\text{nc}} = W_{\text{push}} + W_{\text{kf}} + W_N = \Delta U_{\text{gravity}} + \Delta K$$

$$W_{\text{push}} + F_{\text{kf}} d\cos(180°) + 0 = mg\Delta y + 0$$

$$W_{\text{push}} - (\mu_{\text{k}} N)d = mg(d\sin(33°))$$

$$W_{\text{push}} = \mu_{\text{k}}(mg\cos(33°))d + mgd\sin(33°) = mgd(\mu_{\text{k}}\cos(33°) + \sin(33°))$$

$$= (20\ \text{kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)(2\ \text{m})((0.2)\cos(33°) + \sin(33°)) = \boxed{279\ \text{J}}$$

REFLECT

We could have also solved for the magnitude of the push through Newton's second law and then found the work done by the push using the displacement.

6.91

SET UP

A 12-kg block is released from rest on a frictionless incline that makes an angle of 28 degrees with the horizontal. The block starts at some initial height h_1. The block slides down the ramp, comes into contact with a spring ($k = 13{,}500$ N/m), and comes to rest once the spring compresses a distance of 5.5 cm. Since there is no nonconservative work done on the block, we can use conservation of mechanical energy. The block starts and ends at rest, which means the kinetic energies are zero and the initial potential energy will equal the final potential energy. Initially, the block is not in contact with the spring, so the initial potential energy is all gravitational. If we define the height of the compressed spring to have a gravitational potential energy of zero, the final potential energy is equal to the potential energy stored in the spring. Setting these two equal, we can find the initial height of the block relative to its stopping point. Trigonometry will allow us to find the distance that the block traveled down the ramp.

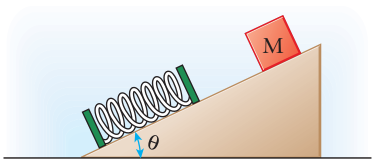


Figure 6-13 Problem 91

SOLVE

$$U_i + K_i = U_f + K_f$$

$$mgh_i + 0 = \frac{1}{2}kx^2 + 0$$

$$h_i = \frac{kx^2}{2mg}$$

$$d = \frac{h_i}{\sin(28°)} = \frac{kx^2}{2mg\sin(28°)} = \frac{\left(13{,}500\frac{\text{N}}{\text{m}}\right)(0.055\ \text{m})^2}{2(12\ \text{kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)\sin(28°)} = \boxed{0.37\ \text{m}}$$

REFLECT

The block starts out a distance of 31.5 cm from the front of the uncompressed spring.

6.92

SET UP

A man on his luge (total mass $m = 88$ kg) emerges onto a horizontal straight track with an initial speed of $v_i = 28$ m/s. The luge and rider slow at a rate of 2.8 m/s^2 before coming to a stop. In order to calculate the work done on the luge and rider by the force that slows them, we need to know the magnitude of the force and the distance over which this force acts. Assuming the slowing force is the only force acting on them in the horizontal direction, the magnitude of this force will equal *ma* from Newton's second law. We can calculate the distance they travel before coming to rest from the constant acceleration kinematic equations. The force will be antiparallel to the displacement of the luge and rider, which means the dot product will be negative.

SOLVE

Stopping distance:

$$v^2 - v_0^2 = 2a_x\Delta x$$

$$\Delta x = \frac{v^2 - v_0^2}{2a_x} = \frac{0 - \left(28\frac{\text{m}}{\text{s}}\right)^2}{2\left(-2.8\frac{\text{m}}{\text{s}^2}\right)} = 140 \text{ m}$$

Work done by force:

$$W = \vec{F}\cdot\vec{d} = (|ma_x|)(\Delta x)\cos(180°) = -m|a_x|\Delta x = -(88 \text{ kg})\left(2.8\frac{\text{m}}{\text{s}^2}\right)(140 \text{ m}) = \boxed{-3.4\times 10^4 \text{ J}}$$

REFLECT

From the work–kinetic energy theorem, we expect the net work done to be negative since the luge and rider slow down.

6.93

SET UP

A dolphin jumps out of the water and reaches a height of 2.5 m above the surface. We can use conservation of mechanical energy to calculate the dolphin's initial speed. We'll say the dolphin has a gravitational potential energy of zero at the surface of the water.

SOLVE

$$U_i + K_i = U_f + K_f$$

$$0 + \frac{1}{2}mv_i^2 = mgy_f + 0$$

$$v_i = \sqrt{2gy_f} = \sqrt{2\left(9.8\frac{\text{m}}{\text{s}^2}\right)(2.5 \text{ m})} = \boxed{7.0\frac{\text{m}}{\text{s}}}$$

REFLECT

Our answer is independent of the weight or length of the dolphin.

6.94

SET UP

A block is released from rest at the top of a track consisting of two frictionless quarter circles of radius $R = 2$ m connected by a 7-m-long, straight, horizontal, rough surface. The coefficient of kinetic friction between the block and the horizontal surface is $\mu_k = 0.1$. The work done by kinetic friction is equal to the change in the block's mechanical energy, since kinetic friction is the only nonconservative force doing work on the block. We'll define the horizontal surface to have a gravitational potential energy of zero, so that the block starts with an initial potential energy of mgR. When the block comes to rest, it will have a final *mechanical* energy of 0. We can calculate the total effective distance over which kinetic friction must act to bring the block to rest. If the distance required to stop the block is longer than the actual length of the horizontal surface, it means that the block will make multiple trips over the surface until it comes to rest.

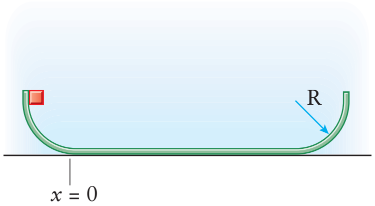


Figure 6-14 Problem 94

SOLVE

$$W_{nc} = \Delta U + \Delta K = U_f - U_i + 0 = 0 - mgh_i$$

$$W_{nc} = W_{kf} = (\mu_k N)d\cos(180°) = -\mu_k(mg)d = -mgR$$

$$d = \frac{R}{\mu_k} = \frac{2\text{ m}}{0.1} = 20\text{ m}$$

The total distance necessary to stop the block is 20 m, but the straight, rough patch is only 7 m. This means the block will pass over the rough patch once from left to right, back from right to left, and $\boxed{\text{stop a distance of 6 m from } x = 0}$.

REFLECT

Because the curved parts are frictionless, the block cannot come to rest on them and they don't affect our solution past the initial potential energy calculation. The block will rise to lower and lower heights on the curved portions after traveling over the rough surface due to the work done by friction.

6.95

SET UP

An object is released from rest on a frictionless ramp at a height of $H_1 = 12$ m. The bottom of this ramp merges smoothly with another frictionless ramp that makes an angle of $\theta_2 = 37$ degrees with the horizontal. Because both ramps are frictionless, the mechanical energy of the object is conserved. The block starts and ends at rest, which means the block will rise to the same

height on the second ramp regardless of the angle of the ramp. We can use trigonometry to determine the distance the object travels up the second ramp. In part (b), the block starts from rest at a height of $H_1 = 12$ m; we can use conservation of mechanical energy to calculate its speed at a height of $H_2 = 7$ m.

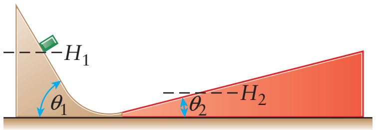


Figure 6-15 Problem 95

SOLVE

Part a)

$$d_2 = \frac{H_2}{\sin(37°)} = \frac{12\text{ m}}{\sin(37°)} = \boxed{20\text{ m}}$$

Part b)

$$U_\text{i} + K_\text{i} = U_\text{f} + K_\text{f}$$

$$mgh_\text{i} + 0 = mgh_\text{f} + \frac{1}{2}mv_\text{f}^2$$

$$v_\text{f} = \sqrt{2g(h_\text{i} - h_\text{f})} = \sqrt{2\left(9.8\frac{\text{m}}{\text{s}^2}\right)((12\text{ m}) - (7\text{ m}))} = \boxed{9.9\frac{\text{m}}{\text{s}}}$$

REFLECT

The angle of the first ramp is not used because the ramp is frictionless and mechanical energy is conserved.

6.96

SET UP

A game involves sliding a block of mass M down a frictionless ramp and then up and off the end of a second ramp of height $h_0 = 0.5$ m. The target is $d = 3.6$ m from the end of the second ramp. Player 1 releases block 1 from a height $h_1 = 2.6$ m and it lands 0.8 m short of the target (that is, $d_1 = 2.8$ m). Using the information from player 1's trial, player 2 wants to release her block from a height h_2 such that block 2 travels a horizontal distance of $d_2 = d = 3.6$ m. We can use conservation of mechanical energy to calculate the launch speed of each block and then use the range formula from kinematics to relate this launch speed to the horizontal distance covered by each block. Player 2 uses the same setup, which means the launch angle of blocks 1 and 2 will be the same.

SOLVE

Launch speed of block 1:

$$Mg(h_1 - h_0) = \frac{1}{2}Mv_{1,0}^2$$

$$v_{1,0}^2 = 2g(h_1 - h_0)$$

Horizontal distance traveled by block 1 after launch:

$$d_1 = \frac{2v_{1,0}^2\sin(\theta)\cos(\theta)}{g} = \frac{2(2g(h_1 - h_0))\sin(\theta)\cos(\theta)}{g} = 4(h_1 - h_0)\sin(\theta)\cos(\theta)$$

Launch speed of block 2:

$$Mg(h_2 - h_0) = \frac{1}{2}Mv_{2,0}^2$$

$$v_{2,0}^2 = 2g(h_2 - h_0)$$

Horizontal distance traveled by block 2 after launch:

$$d_2 = \frac{2v_{2,0}^2\sin(\theta)\cos(\theta)}{g} = \frac{2(2g(h_2 - h_0))\sin(\theta)\cos(\theta)}{g} = 4(h_2 - h_0)\sin(\theta)\cos(\theta)$$

Solving for h_2:

$$\frac{d_2}{d_1} = \frac{4(h_2 - h_0)\sin(\theta)\cos(\theta)}{4(h_1 - h_0)\sin(\theta)\cos(\theta)} = \frac{(h_2 - h_0)}{(h_1 - h_0)}$$

$$h_2 = \frac{d_2}{d_1}(h_1 - h_0) + h_0 = \frac{3.6\text{ m}}{2.8\text{ m}}((2.6\text{ m}) - (0.5\text{ m})) + (0.5\text{ m}) = \boxed{3.2\text{ m}}$$

REFLECT

It makes sense that we need to release block 2 from a larger height since block 1 was short of the target.

6.97

SET UP

A froghopper ($m = 12.3 \times 10^{-6}$ kg) can jump to a maximum height of 0.290 m when its takeoff angle is 58 degrees above the horizontal. We can use conservation of mechanical energy to calculate the initial speed v_{i} of the froghopper. From this initial speed, we can calculate the initial kinetic energy of the insect, which is equal to the energy stored in its legs that is used for the jump.

SOLVE

Part a)

$$U_{\mathrm{i}} + K_{\mathrm{i}} = U_{\mathrm{f}} + K_{\mathrm{f}}$$

$$0 + \frac{1}{2}mv_{\mathrm{i}}^2 = mgh_{\mathrm{f}} + \frac{1}{2}mv_{\mathrm{f}}^2$$

$$\frac{1}{2}v_{\mathrm{i}}^2 = gh_{\mathrm{f}} + \frac{1}{2}(v_{\mathrm{i}}\cos(58°))^2$$

$$v_{\mathrm{i}} = \sqrt{\frac{2gh}{1-\cos^2(58°)}} = \sqrt{\frac{2\left(9.8\frac{\mathrm{m}}{\mathrm{s}^2}\right)(0.290\ \mathrm{m})}{1-\cos^2(58°)}} = \boxed{2.8\frac{\mathrm{m}}{\mathrm{s}}}$$

Part b)

,

$$\frac{4.8 \times 10^{-5}\ \mathrm{J}}{12.3 \times 10^{-6}\ \mathrm{kg}} = \boxed{3.9\frac{\mathrm{J}}{\mathrm{kg}}}$$

REFLECT

Since the conversion from the potential energy stored in the froghopper's legs before the jump to the kinetic energy at launch is most likely not 100%, the froghopper will need to store more than 48 μJ in its legs for each jump.

6.98

SET UP

A 20-g object is placed against a spring ($k = 20$ N/m) that has been compressed by a distance $d = 0.10$ m. The block is released, slides for another 1.25 m, falls off the table, and lands 1.60 m from the edge of the table. In order to determine if the table is frictionless, we can first calculate the ideal distance the object would travel past the table in the absence of friction. First, we can use conservation of mechanical energy to calculate the speed of the object when it leaves the table. Then we can use kinematics to determine the horizontal distance that the object travels. If this ideal distance is larger than 1.60 m, then the table is not frictionless. We need to calculate the change in the mechanical energy of the system and set this equal to the work done by kinetic friction in order to calculate the coefficient of kinetic friction between the object and the tabletop.

SOLVE

Conservation of energy:

$$U_{\mathrm{i}} + K_{\mathrm{i}} = U_{\mathrm{f}} + K_{\mathrm{f}}$$

$$\frac{1}{2}kd^2 + 0 = 0 + \frac{1}{2}mv^2$$

$$v = d\sqrt{\frac{k}{m}}$$

Kinematics:

$$\Delta y = v_{0,y}t + \frac{1}{2}a_y t^2 = 0 - \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2(\Delta y)}{-g}}$$

$$\Delta x = v_{0,x}t + \frac{1}{2}a_x t^2 = v_{0,x}\sqrt{\frac{2(\Delta y)}{-g}} = d\sqrt{\frac{2k(\Delta y)}{-gm}}$$

$$= (0.1\text{ m})\sqrt{\frac{2\left(20\frac{\text{N}}{\text{m}}\right)(-1.00\text{ m})}{-\left(9.8\frac{\text{m}}{\text{s}^2}\right)(0.020\text{ kg})}} = 1.60\text{ m}$$

Therefore, there is $\boxed{\text{no friction}}$ between the object and the tabletop.

REFLECT

If there were friction, we would need to calculate the work done by kinetic friction over a distance of 1.35 m to find the coefficient of kinetic friction.

6.99

SET UP

Two objects ($m_1 = 1.0$ kg, $m_2 = 2.0$ kg) are attached by a thread, which is passed over a pulley. The objects start at the same height and are released from rest. We can use conservation of mechanical energy to calculate the speed of the objects when they are separated by a vertical distance of 1.0 m. The total mechanical energy of the system is equal to the mechanical energy of the 1.0-kg object plus the mechanical energy of the 2.0-kg object. The objects are released from rest, which means the initial kinetic energy of the system is zero. To make the math easier, we will define $y = 0$ to be the starting position of both objects, which will cause the initial gravitational potential energy of the system to be zero as well. Since the objects are attached and both start at the same location, they will move an equal distance above and below the starting point with the same speed v. Therefore, if the objects are separated by a vertical distance of 1.0 m, then the 1.0-kg object is 0.5 m above the starting position and the 2.0-kg object is 0.5 m below the starting position.

SOLVE

$$U_\text{i} + K_\text{i} = U_\text{f} + K_\text{f}$$

$$0 + 0 = U_\text{f} + K_\text{f}$$

$$K_\text{f} = -U_\text{f}$$

$$\frac{1}{2}m_1 v^2 + \frac{1}{2}m_2 v^2 = -[m_1 g(0.5\text{ m}) + m_2 g(-0.5\text{ m})]$$

$$v^2 = \frac{g(m_2 - m_1)}{(m_2 + m_1)}$$

$$v = \sqrt{\frac{g(m_2 - m_1)}{(m_2 + m_1)}} = \sqrt{\frac{\left(9.8\frac{\text{m}}{\text{s}^2}\right)((2.0\text{ kg}) - (1.0\text{ kg}))}{((2.0\text{ kg}) + (1.0\text{ kg}))}} = \boxed{1.8\frac{\text{m}}{\text{s}}}$$

REFLECT

The final gravitational potential energy of the 2.0-kg block is negative because its final location is below $y = 0$. A speed of 1.8 m/s seems reasonable; we could double-check our answer by performing a Newton's second law analysis on the system. Using energy (rather than forces and kinematics) to answer this question is much simpler.

6.100

SET UP

Runaway truck lanes are typically 35 m long and horizontal. We can use the work–kinetic energy theorem to calculate the coefficient of kinetic friction between the gravel in the truck lane and the truck's tires. The only nonzero work is the work due to kinetic friction, which is equal to the change in the truck's kinetic energy.

SOLVE

$$W_{\text{net}} = W_{\text{kf}} + W_{\text{gravity}} + W_N = \Delta K$$

$$F_{\text{kf}}d\cos(180°) + (-\Delta U_{\text{gravity}}) + 0 = \frac{1}{2}mv_{\text{f}}^2 - \frac{1}{2}mv_{\text{i}}^2$$

$$-(\mu_{\text{k}}N)d - (0) = 0 - \frac{1}{2}mv_{\text{i}}^2$$

$$-\mu_{\text{k}}(mg)d = -\frac{1}{2}mv_{\text{i}}^2$$

$$\mu_{\text{k}} = \frac{v_{\text{i}}^2}{2gd} = \frac{\left(24.6\frac{\text{m}}{\text{s}}\right)^2}{2\left(9.8\frac{\text{m}}{\text{s}^2}\right)(35\text{ m})} = \boxed{0.88}$$

REFLECT

We would expect the coefficient of kinetic friction to be large in order to quickly stop a careening tractor-trailer.

6.101

SET UP

In one year the United States produced 282×10^9 kWh of electrical energy from 4138 hydroelectric dams. On average each dam is 50.0 m high and is 90% efficient in converting mechanical energy into electrical energy. We can perform various unit conversions in order to answer the posed questions.

SOLVE

Part a)

$$P = \frac{(282 \times 10^9 \text{ kWh})}{(1 \text{ yr})(4138 \text{ dams})} \times \frac{3.60 \times 10^6 \text{ J}}{1.00 \text{ kWh}} \times \frac{1 \text{ yr}}{365.25 \text{ days}} \times \frac{1 \text{ day}}{24 \text{ hr}} \times \frac{1 \text{ hr}}{3600 \text{ s}}$$

$$= 7.77 \times 10^6 \frac{\text{W}}{\text{dam}} \times \frac{1 \text{ MW}}{10^6 \text{ W}} = \boxed{7.77 \frac{\text{MW}}{\text{dam}}}$$

Part b)

$$U_{\text{gravity}} = 282 \times 10^9 \text{ kWh} \times \frac{100\% \text{ mechanical energy}}{90\% \text{ electrical energy}} = 3.13 \times 10^{11} \text{ kWh}$$

$$m = \frac{U_{\text{gravity}}}{gh} = \frac{3.13 \times 10^{11} \text{ kWh}}{\left(9.8 \frac{\text{m}}{\text{s}^2}\right)(50.0 \text{ m})} \times \frac{3.60 \times 10^6 \text{ J}}{1.00 \text{ kWh}} = \boxed{2.30 \times 10^{15} \text{ kg}}$$

Part c)

$$\frac{2.3 \times 10^{15} \text{ kg}}{4138 \text{ dams}} = \boxed{5.56 \times 10^{11} \frac{\text{kg}}{\text{dam}}} \times \frac{1 \text{ m}^3}{10^3 \text{ kg}} = \boxed{5.56 \times 10^8 \frac{\text{m}^3}{\text{dam}}}$$

Part d)

$$282 \times 10^9 \text{ kWh} \times \frac{3.60 \times 10^6 \text{ J}}{1.00 \text{ kWh}} \times \frac{1 \text{ gal}}{45.0 \times 10^6 \text{ J}} = \boxed{22.6 \times 10^9 \text{ gal}}$$

REFLECT

An internal combustion engine is approximately 20% efficient.

6.102

SET UP

Gas jets on Mars, where $g_{\text{Mars}} = 3.7 \text{ m/s}^2$, throw sand and dust about 75 m above the surface. We can use conservation of mechanical energy to determine the speed of the material as it leaves the surface. We'll define the gravitational potential energy of the material at the surface to be zero. Before the gas leaves the jets, it is traveling underground at a speed of 160 km/hr. The energy lost per kilogram of material is equal to the work done by the nonconservative forces acting on it, which in turn is equal to the change in the material's mechanical energy. We'll assume that the gas is traveling just below the surface of the planet, so we can ignore the change in gravitational potential energy. Therefore, the energy lost is related to the difference in kinetic energy underground and just at the surface, which is related to our answer from part (a).

SOLVE

Part a)

$$U_\text{i} + K_\text{i} = U_\text{f} + K_\text{f}$$

$$0 + \frac{1}{2}mv_i^2 = mg_{\text{Mars}}h + 0$$

$$v_i = \sqrt{2g_{\text{Mars}}h} = \sqrt{2\left(3.7\frac{\text{m}}{\text{s}^2}\right)(75\text{ m})} = \boxed{24\frac{\text{m}}{\text{s}}}$$

Part b)
Converting km/hr to m/s:

$$160\frac{\text{km}}{\text{hr}} \times \frac{10^3\text{ m}}{1\text{ km}} \times \frac{1\text{ hr}}{3600\text{ s}} = 44.4\frac{\text{m}}{\text{s}}$$

Nonconservative work:

$$W_{\text{nc}} = \Delta K + \Delta U = \frac{1}{2}m(v_f^2 - v_i^2) + 0$$

$$\frac{W_{\text{nc}}}{m} = \frac{1}{2}(v_f^2 - v_i^2) = \frac{1}{2}\left(\left(24\frac{\text{m}}{\text{s}}\right)^2 - \left(44.4\frac{\text{m}}{\text{s}}\right)^2\right) = -700\frac{\text{J}}{\text{kg}}$$

REFLECT

A speed of 160 km/hr is nearly 100 mph. On Earth, the sand and dust would only rise to a height of 29 m since g is larger on Earth than on Mars.

6.103

SET UP

A block of mass m starts at the bottom of a ramp with an initial speed of 20 m/s. The ramp makes an angle of 37 degrees with the horizontal, and the coefficient of kinetic friction between the block and the ramp is 0.5. The block will eventually come to a stop a distance d up the ramp. The work done by kinetic friction is equal to the change in the mechanical energy of the system; from this we can calculate d. The force of kinetic friction, which acts opposite to the displacement of the block, has a magnitude of $F_{\text{kf}} = \mu_k N = \mu_k mg\cos(37°)$. We'll define the bottom of the ramp to have a gravitational potential energy of zero.

SOLVE

$$W_{\text{nc}} = \Delta K + \Delta U = \frac{1}{2}m(v_f^2 - v_i^2) + mg(\Delta y)$$

$$-(\mu_k mg\cos(37°))d = \frac{1}{2}m(0 - v_0^2) + mg(d\sin(37°) - 0)$$

$$\mu_k gd\cos(37°) - gd\sin(37°) = \frac{1}{2}v_0^2$$

$$d = \frac{v_0^2}{2g(\sin(37°) + \mu_k\cos(37°))} = \frac{\left(20\frac{\text{m}}{\text{s}}\right)^2}{2\left(9.8\frac{\text{m}}{\text{s}^2}\right)(\sin(37°) + (0.5)\cos(37°))} = \boxed{20\text{ m}}$$

REFLECT

We did not need to use the coefficient of static friction in this calculation since the block slides up the ramp until it reaches its maximum height. Static friction would tell us whether the block stays at the top of the ramp or starts to slide back down.

6.104

SET UP

A block of mass M is placed inside a frictionless loop of radius R. The block is released with some speed v_i from a height of R. We can use conservation of mechanical energy to calculate the initial speed such that the block just barely makes it around the circle. This means the block's initial mechanical energy needs to equal its mechanical energy at rest at the top of the ring (that is, a height of $2R$).

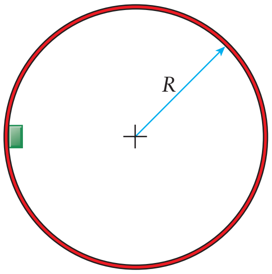


Figure 6-16 Problem 104

SOLVE

$$U_i + K_i = U_f + K_f$$

$$MgR + \frac{1}{2}Mv_i^2 = Mg(2R) + 0$$

$$\boxed{v_i = \sqrt{2gR}}$$

REFLECT

If the block started at rest, we would need to launch it (somehow) from a height of $2R$ in order to make it all the way around the loop.

6.105

SET UP

A 25-N object is placed on a horizontal, frictionless table in front of a spring with spring constant k. The spring is compressed a distance d and released, hitting the object off the table. When the object leaves the table, it becomes attached to a 3-m-long string and the object travels in a circular path. The string will break when the tension in it reaches 200 N; this occurs when the spring is compressed by a distance $d = 1$ m. We can use the information about the string breaking to calculate the value of the spring constant k. First we need to determine the speed of the object when it leaves the table in terms of k by applying conservation of mechanical energy. All of the potential energy stored in the spring is converted to the kinetic energy of the object. Assuming no air resistance, the speed of the object will

remain constant as it travels around the circular path, which means the object is undergoing centripetal motion with an acceleration equal to $\frac{v^2}{R}$, where R is the length of the string. The maximum tension in the string will occur when the object is at the bottom of its path. We can use Newton's second law to relate the net force on the object to its speed; the only forces acting on the object at the bottom of its swing is the tension acting upward and gravity acting downward.

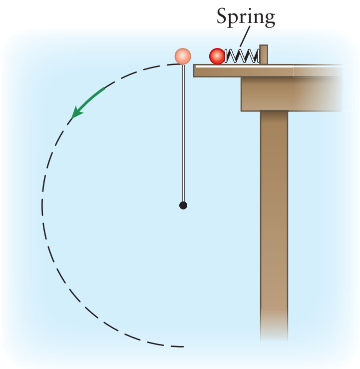


Figure 6-17 Problem 105

SOLVE

Conservation of energy to find the speed of the ball:

$$U_{\text{spring}} = K$$

$$\frac{1}{2}kd^2 = \frac{1}{2}mv^2$$

$$v^2 = \frac{kd^2}{m}$$

Newton's second law at the bottom of the circle:

$$\sum F_y = T - F_g = T - mg = ma_y = m\frac{v^2}{R}$$

$$T - mg = \frac{m}{R}\left(\frac{kd^2}{m}\right) = \frac{kd^2}{R}$$

$$k = \frac{(T - mg)R}{d^2} = \frac{((200\text{ N}) - (25\text{ N}))(3\text{ m})}{(1\text{ m})^2} = \boxed{525\frac{\text{N}}{\text{m}}}$$

REFLECT

To save time and a calculation, we solved for v^2 in the conservation of energy portion since we need v^{-2} (not v) in the Newton's second law portion. A spring constant of 525 N/m is reasonable for an everyday spring.

6.106

SET UP

The potential energy of an electron in an atom as a function of r is given. We can calculate the force exerted on the electron as a function of r by taking the derivative of $U(r)$ with respect to r and multiplying by -1. We can determine the SI units of a and b by dimensional analysis.

SOLVE

Part a)

$$U(r) = \frac{a}{r^6} - \frac{b}{r^{12}} = ar^{-6} - br^{-12}$$

$$F(r) = -\frac{dU}{dr} = -\frac{d}{dr}(ar^{-6} - br^{-12}) = \boxed{6ar^{-7} - 12br^{-13}}$$

Part b)

$$[U] = [a][r^{-6}]$$

$$[a] = \frac{[U]}{[r^{-6}]} = [U][r^6] = \boxed{\mathrm{J \cdot m^6}}$$

$$[U] = [b][r^{-12}]$$

$$[b] = \frac{[U]}{[r^{-12}]} = [U][r^{12}] = \boxed{\mathrm{J \cdot m^{12}}}$$

REFLECT

This potential energy function is referred to as the Lennard-Jones 6-12 potential and is very useful when modeling the potential energies involved in atoms and molecules.

6.107

SET UP

A 1-kg object has a potential energy function given by $U(x) = 3(x - 1) - (x - 3)^3$ (SI units). We are also given a plot of $U(x)$ versus x. We can see from the plot that $x = x_1$ and $x = x_3$ correspond to a local minimum and local maximum of $U(x)$. We can calculate their numerical values by taking the derivative of U with respect to x, setting this equal to zero, and solving for x. If the particle has a total energy E_2, it will be either confined to the neighborhood around $x = x_1$ or freely accelerating toward $+x$, depending on its starting location. We can calculate the speed of the particle at any given point by evaluating the potential energy function and subtracting this from the total energy to give the kinetic energy (and, therefore, the speed) of the particle. The force of the particle is equal to $-\frac{dU}{dx}$, which is related to the slope of the plot.

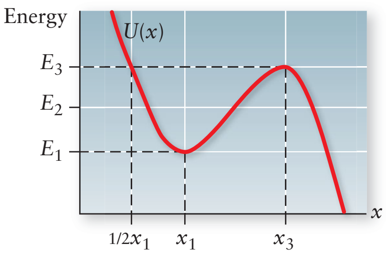


Figure 6-18 Problem 107

SOLVE

$$U(x) = 3(x-1) - (x-3)^3$$

Part a)

$$\frac{dU}{dx} = 0 = \frac{d}{dx}[3(x-1) - (x-3)^3] = 3 - 3(x-3)^2$$

$$x^2 - 6x + 8 = 0$$

$$(x-2)(x-4) = 0$$

$$\boxed{\begin{array}{l} x_1 = 2\text{ m} \\ x_3 = 4\text{ m} \end{array}}$$

Part b) Either it is sometimes at x_1, in which case it rocks back and forth in the trap, or it is off to the right, free and accelerating in the $+x$ direction.

Part c)

$$U(x=2) = 3(2-1) - (2-3)^3 = 3(1) - (-1)^3 = 4\text{ J}$$

$$U + K = E = 58\text{ J}$$

$$(4\text{ J}) + \frac{1}{2}mv^2 = 58\text{ J}$$

$$v = \sqrt{\frac{2(54\text{ J})}{m}} = \sqrt{\frac{2(54\text{ J})}{1\text{ kg}}} = \boxed{10\frac{\text{m}}{\text{s}}}$$

Part d)

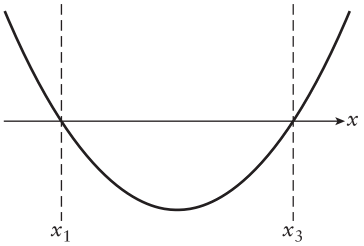


Figure 6-19 Problem 107

Part e)

$$U(x = 1) = 3(1 - 1) - (1 - 3)^3 = 0 - (-2)^3 = 8\text{ J}$$

$$U_\text{i} + K_\text{i} = U_\text{f} + K_\text{f}$$

$$(8\text{ J}) + 0 = (4\text{ J}) + \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2(4\text{ J})}{m}} = \sqrt{\frac{2(4\text{ J})}{1\text{ kg}}} = \boxed{2.8\frac{\text{m}}{\text{s}}}$$

REFLECT

If the particle has a total energy of E_2 and is in the neighborhood of $x = x_2$, it will be confined in that trap and oscillate back and forth as if it were attached to a spring. The effective spring constant in this case will be related to the second derivative of the potential energy function evaluated at $x = x_2$. This can be shown by taking a Taylor series expansion of $U(x)$ about $x = x_2$.

Chapter 7
Linear Momentum

Conceptual Questions

7.1 An impulse, commonly speaking, is a brief push or, psychologically, a spur-of-the-moment desire. This definition fits a common use of the physicist's notion of impulse, which is a collision. The forces in collisions will tend to be brief. Changes in momentum, of course, occur in other situations, and the term makes less sense there.

7.2 Newton's second law states that the net external force on a system is equal to the product of the system's mass and its acceleration. The net external force is also equal to the time derivative of the system's momentum. If the net external force is zero, then this means the momentum is a constant for all time (that is, momentum is conserved).

7.3 Yes, if the tennis ball moves 18 times faster.

7.4 Not necessarily. For example, consider the case of a 2-kg object traveling at a speed of 2 m/s and an 8-kg object traveling at a speed of 1 m/s. Their kinetic energies are equal (8 J), but their momenta are not (4 kg · m/s and 8 kg · m/s, respectively).

7.5 The force of the impact is less on carpet, which can be seen by considering the momentum (the impulse is spread over more time) or by considering energy (the work is done over more distance).

7.6 We'll consider the system to be the child and the board, both of which start at rest. (a) The board will move in the opposite direction that the child is running. (b) The center of mass of the system will remain stationary for all time. (c) The answers to parts (a) and (b) do not change if the child walks rather than runs.

7.7 Pushing off the boat to step onto the pier will push the boat away. You will make less progress than on solid ground and can easily fall in the gap.

7.8 The boat will appear to move in the direction opposite to that of the dog.

7.9 The momentum of Mars is around $1.5 \times 10^{28} \frac{\text{kg} \cdot \text{m}}{\text{s}}$; the momentum of the asteroid is around $7 \times 10^{12} \frac{\text{kg} \cdot \text{m}}{\text{s}}$. Because this is more than 15 orders of magnitude smaller, no noticeable change to the trajectory of Mars could result from the impact.

7.10 We would need to integrate the force with respect to time in order to calculate the impulse given to the object. This is also equal to the area under the force versus time graph.

7.11 Look for "loss" of energy. Did the collision make a sound? Did the object deform? If you can accurately gauge their velocities, do the objects have less total kinetic energy than before? Any of these conditions indicates an inelastic collision.

7.12 The air bag would increase the time over which the impulse is spread, thus decreasing the average net force on the cell phone.

7.13 Flopping down on a bed; catching a ball.

7.14 If only conservation of energy applied, it would be possible to drop two balls together and one single ball on the other side to twice the initial height of the first two balls. When we require that momentum as well as energy is conserved, the only solution when multiple balls are raised and released together is for the same number of balls to be raised to the same height on the other side.

7.15 (a) Choose three orthogonal directions, ideally so that at least one is aligned with a direction around which the sheet is symmetric. For any axis around which the sheet is symmetric, the center of mass in the perpendicular direction is along that axis. Otherwise, the center of mass in a given direction is found by adding up (that is, integrating) the product of the mass of infinitesimally small elements of the sheet and the distance each is from the axis and then dividing by the mass of the sheet. (b) The same process holds for a sheet with a hole, with the additional step that the hole is included in the process as a "negative" mass.

7.16 Momentum is conserved during the explosion but not before or after, since an external force (namely, gravity) is acting on the system. Mechanical energy is not conserved, since much of the energy will be "lost" to heat, light, and sound. The center of mass of the system will follow the parabolic path both before and after the explosion.

7.17 Part of the force opposing the arrow's advance is friction from the sides, which will depend linearly on how far the arrow has penetrated the target. This force is equivalent to a spring that resists compression but won't rebound. Doubling the speed will quadruple the kinetic energy. The kinetic energy is dissipated when the "spring" has absorbed all of the energy, which occurs at twice the depth. The arrow penetrates twice as far. We could alternately assume that the force is mainly at the front of the arrow and is constant. In this case, the penetration must quadruple for Fd to match the quadrupled kinetic energy. Other assumptions are possible, but they are more difficult to work with.

Multiple-Choice Questions

7.18 **E** (The truck has a much lower speed than the car.) The product mv is the same for each vehicle, but the mass of the truck is much larger than the mass of the car. Therefore, the truck's speed is much lower than the car's speed.

7.19 **D** ($2mv$). If the initial momentum of the ball points in the $-x$ direction, its initial momentum is $-mv$ and the final momentum is $+mv$. This is a total change in momentum of $2mv$.

7.20 **E** ($-mv$; $-2mv$). Assuming both objects are thrown toward $+x$, the initial momentum of each object is $+mv$. The final momentum of the clay is 0, while the final momentum of the ball is $-mv$. This means the overall change in momentum of the clay and the ball is $-mv$ and $-2mv$, respectively.

7.21 **A** (The velocities of both particles are zero.) In a completely inelastic collision, the two particles collide and stick together. The initial momentum of the system is $mv + (-mv) = 0$. Conservation of momentum dictates that the final momentum of the system following the collision must also be zero: $p_f = 0 = (2m)v$; therefore, $v = 0$.

7.22 **C** ($v/3$). The initial momentum of the system in the x direction is $p_{i,x} = mv + \frac{m}{2}(-v)$. The final momentum of the system in the x direction is $p_{f,x} = \left(m + \frac{m}{2}\right)v_f$. Setting these equal and solving for v_f in terms of v,

$$\frac{mv}{2} = \frac{3mv_f}{2}$$

$$v_f = \frac{v}{3}$$

7.23 **B** (The magnitudes of the velocities are the same but the directions are reversed). By rearranging the energy and momentum conservation equations for two objects undergoing a one-dimensional, elastic collision, we can write the final speeds of each object as:

$$v_{1,f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1,i} + \left(\frac{2m_2}{m_1 + m_2}\right)v_{2,i}$$

$$v_{2,f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1,i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_{2,i}$$

For the case of equal masses (that is, $m_1 = m_2$), we find that $v_{1,f} = v_{2,i}$ and $v_{2,f} = v_{1,i}$.

7.24 **C** (0; v). This is an example of the special case seen in Section 7-5 for an elastic collision where two objects are the same and one begins at rest.

7.25 **D** (Lilly's speed is twice John's speed.) The initial momentum of the system is zero, which means the final momentum of the system must also be zero. Since Lilly's mass is one-half John's mass, Lilly's speed must be twice John's speed.

7.26 **B** (You should deflect the ball back toward your friend at the same speed with which it hit your hand). Catching the ball would cause you and the ball to move together as one object, while deflecting the ball would cause it to move in the opposite direction. Applying momentum conservation will show that the second case maximizes your speed.

7.27 **A** (Block A rises to a height greater than H_A and block B rises to a height less than H_B).

Conservation of momentum:

$$m_A\sqrt{2gH_A} - m_B\sqrt{2gH_B} = m_A v_{A,f} + m_B v_{B,f}$$

Conservation of energy:

$$m_A(2gH_A) + m_B(2gH_B) = m_A v_{A,f}^2 + m_B v_{B,f}^2$$

Solving this system of equations for the final speeds and then applying conservation of energy to calculate the final heights, we'll find that the less massive block (block A) will rise to a greater height than the more massive block (block B).

We can also look at the extremes of the system. What if block B is way more massive than block A? If they are released from about the same heights, then certainly block A rises higher after the collision. If they are released so that block B is nearly at the bottom initially, then it's going slowly when they collide, perhaps nearly zero, but that, too, makes block A rise higher. What if block B is just slightly more massive than block A? If they are released from about the same heights, then they have about the same speeds when they collide, but that would result in block A having just a slightly higher speed afterward. So again, block A rises higher after the collision. And, finally, if block B is just slightly more massive than block A and block B is released from way down low, so that block B is going almost zero when they collide, since A is less massive than block B it rebounds at a slightly higher speed than its initial speed, which means it will rise higher.

Estimation Questions

7.28 A typical car has a mass of 1000 kg. A speed of 65 mph is around 30 m/s. Therefore, the magnitude of the car's momentum is 3×10^4 kg · m/s.

7.29 A baseball has a mass of about 145 g. A typical fastball's speed is 90–100 mph or 40–45 m/s. So the momentum is about 6 kg · m/s.

7.30 The mass of a tennis ball is 0.057 kg, and a professional tennis player can serve at around 125 mph (=55 m/s). The magnitude of the ball's momentum is around 3.1 kg · m/s.

7.31 Roughly speaking, it is around your navel. The ratio of the center of mass to height in humans is about 0.55.

7.32 A bumblebee has a mass of around 0.5 g and can fly at a speed of 4 m/s. The magnitude of its momentum is 0.002 kg · m/s.

7.33 The momentum of Earth as it orbits the Sun is

$$p = mv = (6 \times 10^{24}\ \text{kg})\left(30{,}000\frac{\text{m}}{\text{s}}\right) = 1.8 \times 10^{29}\frac{\text{kg}\cdot\text{m}}{\text{s}}.$$

7.34 A softball has a mass of around 0.18 kg and is pitched at around 33 m/s. A baseball has a mass of around 0.145 kg and is pitched at around 40 m/s.

7.35 A tennis ball has a mass of 0.057 kg and a coefficient of restitution of 0.94. If an incoming volley is 40 m/s, then we get an impulse of around 5 kg · m/s.

7.36 A car of mass M moving at 20 m/s hits a stationary car of mass M. After the collision, the two cars stick together and travel at a speed of 10 m/s (from momentum conservation). The work done by kinetic friction on the wreckage is equal to the change in the wreckage's mechanical energy:

$$W_{nc} = -\mu_k(2M)gd = \Delta K + \Delta U = -\frac{1}{2}(2M)v^2$$

The coefficient of kinetic friction between the road and the tires is around 0.7. Plugging all of this information in:

$$d = \frac{v^2}{2\mu_k g} = \frac{\left(10\frac{\text{m}}{\text{s}}\right)^2}{2(0.7)\left(9.8\frac{\text{m}}{\text{s}^2}\right)} = \boxed{7.3\text{ m}}$$

7.37

t (s)	0	1	2	3	4	5	6	7	8	9	10	11	12
F (N)	−20	−20	−10	0	10	15	18	20	25	25	25	25	25

t (s)	13	14	15	16	17	18	19	20	21	22	23	24	25
F (N)	25	25	25	25	25	25	25	25	20	15	10	5	0

Part a)

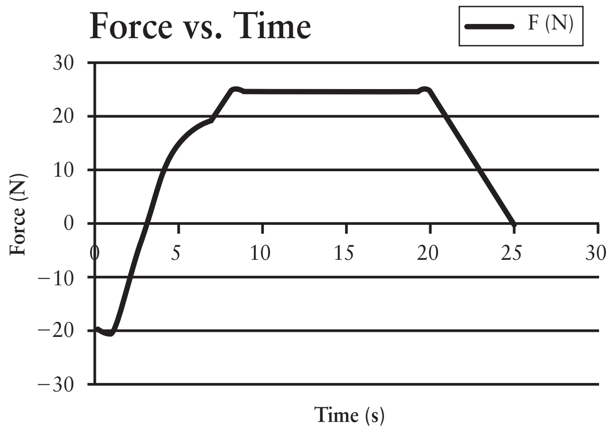


Figure 7-1 Problem 37

Part b)
The area under a force versus time curve is equal to the change in momentum of the object.

$$\text{Area} = \frac{1}{2}(3\text{ s})(-20\text{ N}) + \frac{1}{2}((22\text{ s}) + (12\text{ s}))(25\text{ N}) = 395\frac{\text{kg}\cdot\text{m}}{\text{s}} = \Delta p$$

$$\Delta p = mv_f - mv_i = mv_f - 0$$

$$v_f = \frac{\Delta p}{m} = \frac{\left(395\frac{\text{kg}\cdot\text{m}}{\text{s}}\right)}{2\text{ kg}} = 198\frac{\text{m}}{\text{s}}$$

Problems

7.38

SET UP

A 10,000-kg train car is traveling at a velocity of 15 m/s to the east. The momentum of the train car is the product of its mass and velocity.

SOLVE

$$p = mv = (10{,}000\text{ kg})\left(15\frac{\text{m}}{\text{s}}\right) = 150{,}000\frac{\text{kg}\cdot\text{m}}{\text{s}}$$

The momentum of the train car is $\boxed{1.5 \times 10^{5}\frac{\text{kg}\cdot\text{m}}{\text{s}}\text{ to the east}}$.

REFLECT

Remember that momentum is a vector quantity, which means we need to give its magnitude and its direction.

7.39

SET UP

The magnitude of the instantaneous momentum of a 0.057-kg tennis ball is 2.6 kg · m/s. We can use the definition of momentum to calculate the instantaneous speed of the ball.

SOLVE

$$p = mv$$

$$v = \frac{p}{m} = \frac{\left(2.6\frac{\text{kg}\cdot\text{m}}{\text{s}}\right)}{0.057\text{ kg}} = \boxed{46\frac{\text{m}}{\text{s}}}$$

REFLECT

We would expect a large speed since the mass of the ball is small. This is a reasonable speed for a tennis ball.

7.40

SET UP

A 1250-kg car is initially backing up at a speed of 5 m/s and then driving forward at 12 m/s. We can calculate the initial momentum and final momentum of the car by multiplying the mass by the velocity. We will assume the car is backing up in the negative direction. The change in the car's momentum is equal to its final momentum minus its initial momentum.

SOLVE

Initial momentum:

$$p_{\text{i}} = mv_{\text{i}} = (1250\text{ kg})\left(-5\frac{\text{m}}{\text{s}}\right) = \boxed{-6250\frac{\text{kg}\cdot\text{m}}{\text{s}}}$$

Final momentum:

$$p_{\mathrm{f}} = mv_{\mathrm{f}} = (1250\text{ kg})\left(14\frac{\mathrm{m}}{\mathrm{s}}\right) = \boxed{17{,}500\frac{\mathrm{kg}\cdot\mathrm{m}}{\mathrm{s}}}$$

Change in momentum:

$$\Delta p = p_{\mathrm{f}} - p_{\mathrm{i}} = \left(17{,}500\frac{\mathrm{kg}\cdot\mathrm{m}}{\mathrm{s}}\right) - \left(-6250\frac{\mathrm{kg}\cdot\mathrm{m}}{\mathrm{s}}\right) = \boxed{23{,}750\frac{\mathrm{kg}\cdot\mathrm{m}}{\mathrm{s}}}$$

REFLECT

Momentum is a vector, so we need to include the correct signs corresponding to the direction of the car's motion.

7.41

SET UP

A 135-kg football player is running at a speed of 7 m/s. We can use the definition of momentum to calculate the magnitude of his momentum.

SOLVE

$$p = mv = (135\text{ kg})\left(7\frac{\mathrm{m}}{\mathrm{s}}\right) = \boxed{945\frac{\mathrm{kg}\cdot\mathrm{m}}{\mathrm{s}}}$$

REFLECT

The momentum will point in the same direction as the football player's velocity.

7.42

SET UP

Ball 1 has a mass m_1 and is traveling at a speed of v_1. Ball 2 has four times the mass ($m_2 = 4m_1$) and twice the speed ($v_2 = 2v_1$) of ball 1. We can directly calculate the momentum and kinetic energy of each ball in order to compare these values.

SOLVE

Part a)

$$\frac{p_2}{p_1} = \frac{m_2 v_2}{m_1 v_1} = \frac{(4m_1)(2v_1)}{m_1 v_1} = \boxed{8}$$

Part b)

$$\frac{K_2}{K_1} = \frac{\left(\frac{1}{2}m_2 v_2^2\right)}{\left(\frac{1}{2}m_1 v_1^2\right)} = \frac{(4m_1)(2v_1)^2}{m_1 v_1^2} = \boxed{16}$$

REFLECT

Momentum is proportional to the speed, while kinetic energy is proportional to the speed squared.

7.43

SET UP

A 55-kg girl is riding a skateboard at a speed of 6 m/s. We can calculate the magnitude of her momentum directly from the definition of momentum. We are given the magnitude of the skateboard's momentum and we know that it must be traveling at the same speed as the girl. Therefore, we can find the mass of the skateboard through division.

SOLVE

Part a)

$$p = mv = (55\ \text{kg})\left(6\frac{\text{m}}{\text{s}}\right) = \boxed{330\frac{\text{kg}\cdot\text{m}}{\text{s}}}$$

Part b)

$$m = \frac{p}{v} = \frac{\left(30\frac{\text{kg}\cdot\text{m}}{\text{s}}\right)}{\left(6\frac{\text{m}}{\text{s}}\right)} = \boxed{5\ \text{kg}}$$

REFLECT

The momentum of the skateboard is one-eleventh of the momentum of the girl, which means the mass of the skateboard will be one-eleventh of the mass of the girl.

7.44

SET UP

An object ($m_1 = 2$ kg) is initially traveling east at a speed of $v_{1,\text{i}} = 4$ m/s. It collides with a second object ($m_2 = 6$ kg) that is initially at rest. After the collision, the 6-kg object is traveling east at a speed of $v_{2,\text{f}} = 1$ m/s. We can use conservation of momentum to find the final velocity of m_1. We will consider east to be the positive direction in our coordinate system.

SOLVE

$$m_1 v_{1,\text{i}} + m_2 v_{2,\text{i}} = m_1 v_{1,\text{f}} + m_2 v_{2,\text{f}}$$

$$v_{1,\text{f}} = \frac{m_1 v_{1,\text{i}} + m_2 v_{2,\text{i}} - m_2 v_{2,\text{f}}}{m_1} = \frac{(2\ \text{kg})\left(4\frac{\text{m}}{\text{s}}\right) + 0 - (6\ \text{kg})\left(1\frac{\text{m}}{\text{s}}\right)}{2\ \text{kg}} = 1\frac{\text{m}}{\text{s}}$$

The final velocity of the 2-kg object is $\boxed{\text{1 m/s to the east}}$.

REFLECT

The phrase "completely elastic collision" lets us know that the two objects do not stick together after the collision.

7.45

SET UP

An object ($m_1 = 3$ kg) is initially traveling to the right at a speed of $v_{1,\text{i}} = 6$ m/s. It collides with a second object ($m_2 = 5$ kg) that is initially traveling to the left at a speed of $v_{2,\text{i}} = 4$ m/s.

After the collision, the 3-kg object is traveling to the left at a speed of $v_{1,\,\mathrm{f}} = 2$ m/s. We can use conservation of momentum to find the final velocity of m_2. We will consider the positive direction in our coordinate system to point toward the right.

SOLVE

$$m_1 v_{1,\,\mathrm{i}} + m_2 v_{2,\,\mathrm{i}} = m_1 v_{1,\,\mathrm{f}} + m_2 v_{2,\,\mathrm{f}}$$

$$v_{2,\,\mathrm{f}} = \frac{m_1 v_{1,\,\mathrm{i}} + m_2 v_{2,\,\mathrm{i}} - m_1 v_{1,\,\mathrm{f}}}{m_1} = \frac{(3\text{ kg})\left(6\frac{\mathrm{m}}{\mathrm{s}}\right) + (5\text{ kg})\left(-4\frac{\mathrm{m}}{\mathrm{s}}\right) - (3\text{ kg})\left(-2\frac{\mathrm{m}}{\mathrm{s}}\right)}{5\text{ kg}} = 0.8\frac{\mathrm{m}}{\mathrm{s}}$$

The final velocity of the 5-kg object is $\boxed{\text{0.8 m/s to the right}}$.

REFLECT

Velocity is a vector, which means we need to report both its magnitude and its direction.

7.46

SET UP

Blythe ($m_\mathrm{B} = 50$ kg) and Geoff ($m_\mathrm{G} = 80$ kg) are standing together at rest on ice skates. Blythe pushes Geoff in the chest, and he moves off at a speed of 4 m/s relative to the ice. We can use conservation of momentum to calculate Blythe's velocity after the push. We'll assume that Geoff is traveling in the positive direction.

SOLVE

$$m_\mathrm{B} v_{\mathrm{B},\,\mathrm{i}} + m_\mathrm{G} v_{\mathrm{G},\,\mathrm{i}} = m_\mathrm{B} v_{\mathrm{B},\,\mathrm{f}} + m_\mathrm{G} v_{\mathrm{G},\,\mathrm{f}}$$

$$v_{\mathrm{B},\,\mathrm{f}} = \frac{m_\mathrm{B} v_{\mathrm{B},\,\mathrm{i}} + m_\mathrm{G} v_{\mathrm{G},\,\mathrm{i}} - m_\mathrm{G} v_{\mathrm{G},\,\mathrm{f}}}{m_\mathrm{B}} = \frac{0 + 0 - (80\text{ kg})\left(4\frac{\mathrm{m}}{\mathrm{s}}\right)}{50\text{ kg}} = -6.4\frac{\mathrm{m}}{\mathrm{s}}$$

Part a) Blythe's final speed is $\boxed{6.4\text{ m/s}}$.

Part b) Blythe moves in the $\boxed{\text{opposite direction as Geoff}}$.

REFLECT

Blythe is less massive than Geoff, so it makes sense that she moves at a faster speed than Geoff. The initial momentum of the system is zero; Blythe must move in the opposite direction as Geoff in order for momentum to be conserved.

7.47

SET UP

An object of mass $3M$ has an initial velocity of $\vec{v}_0 = v_0\hat{x}$. It breaks into two unequal pieces. Piece 1 has a mass of M and travels off at an angle of 45 degrees below the x-axis with a speed of v_1. Piece 2 has a mass of $2M$ and travels off at an angle of 30 degrees above the x-axis with a speed of v_2. We can use conservation of momentum in order to calculate the final velocities of the two pieces. Since this is a two-dimensional problem, we will need to split the momenta into components and solve the x and y component equations.

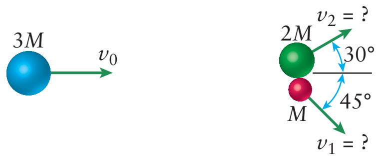


Figure 7-2 Problem 47

SOLVE

y component:

$$0 = (2M)v_2\sin(30°) - (M)v_1\sin(45°)$$

$$v_2 = \frac{v_1\sin(45°)}{2\sin(30°)} = \frac{v_1}{\sqrt{2}}$$

x component:

$$(3M)v_0 = (2M)v_2\cos(30°) + (M)v_1\cos(45°)$$

$$3v_0 = 2v_2\cos(30°) + v_1\cos(45°) = 2\left(\frac{v_1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \frac{v_1}{\sqrt{2}}$$

$$v_1 = \frac{3\sqrt{2}}{\sqrt{3}+1}v_0 = 1.55v_0$$

$$v_2 = \frac{v_1}{\sqrt{2}} = \frac{3}{\sqrt{3}+1}v_0 = 1.10v_0$$

Therefore,

$$\vec{v}_1 = (1.55v_0\cos(45°))\hat{x} - (1.55v_0\sin(45°))\hat{y} = \boxed{(1.1v_0)\hat{x} - (1.1v_0)\hat{y}}$$

$$\vec{v}_2 = (1.10v_0\cos(30°))\hat{x} + (1.10v_0\sin(30°))\hat{y} = \boxed{(0.95v_0)\hat{x} + (0.55v_0)\hat{y}}$$

REFLECT

Initially there is no momentum in the y direction, which means the final y momentum must also be zero. If we calculate it explicitly, we see that this is true: $(M)(-1.1v_0) + (2M)(0.55v_0) = 0$.

7.48

SET UP

A cue ball (mass $m_c = 0.170$ kg) is rolling at 2 m/s at a direction 30 degrees north of east. It collides elastically with the eight ball (mass $m_e = 0.156$ kg). The cue ball moves off at a speed of v_c at an angle of 10 degrees north of east, while the eight ball moves off at a speed of v_e due north. We can use conservation of momentum to calculate the final speeds of the two pool balls. Since this is a two-dimensional problem, we will need to split the momenta into components and solve for the x and y components of the final velocity. In our coordinate system east will point toward $+x$ and north will point toward $+y$.

SOLVE

x component:

$$m_c\left(2\frac{\mathrm{m}}{\mathrm{s}}\right)\cos(30°) = m_c v_c \cos(10°)$$

$$v_c = \frac{\left(2\frac{\mathrm{m}}{\mathrm{s}}\right)\cos(30°)}{\cos(10°)} = \boxed{1.8\frac{\mathrm{m}}{\mathrm{s}}}$$

y component:

$$m_c\left(2\frac{\mathrm{m}}{\mathrm{s}}\right)\sin(30°) = m_c v_c \sin(10°) + m_c v_c$$

$$v_c = \frac{m_c\left(2\frac{\mathrm{m}}{\mathrm{s}}\right)\sin(30°) - m_c v_c \sin(10°)}{m_c}$$

$$= \frac{(0.170\text{ kg}) - (0.170\text{ kg})\left(1.8\frac{\mathrm{m}}{\mathrm{s}}\right)\sin(10°)}{0.156\text{ kg}} = \boxed{0.76\frac{\mathrm{m}}{\mathrm{s}}}$$

REFLECT

Momentum is a vector, which means we always need to split it up into components before solving for the final velocities.

7.49

SET UP

A car ($m_C = 1000$ kg) is moving at an initial velocity of 30 m/s due north when it collides elastically with a truck ($m_T = 1250$ kg) that was heading 45 degrees north of east. After the collision, the car is moving due east and the truck is moving due north. In order to find the initial speed of the truck, we can use conservation of momentum to calculate the initial speed of the truck. Since this is a two-dimensional problem, we will need to split the momenta into components and solve for the x and y components of the final velocity. In our coordinate system east will point toward $+x$ and north will point toward $+y$.

SOLVE

Conservation of momentum, x component:

$$m_T v_{T,\,i} \cos(45°) = m_C v_{C,\,f}$$

$$v_{C,\,f} = \frac{m_T v_{T,\,i} \cos(45°)}{m_C}$$

Conservation of momentum, y component:

$$m_C v_{C,\,i} + m_T v_{T,\,i} \sin(45°) = m_T v_{T,\,f}$$

$$v_{T,\,f} = \frac{m_C v_{C,\,i} + m_T v_{T,\,i} \sin(45°)}{m_T}$$

Conservation of energy:

$$m_C v_{C,i}^2 + m_T v_{T,i}^2 = m_C v_{C,f}^2 + m_T v_{T,f}^2$$

$$m_C v_{C,i}^2 + m_T v_{T,i}^2 = m_C\left(\frac{m_T v_{T,i}\cos(45°)}{m_C}\right)^2 + m_T\left(\frac{m_C v_{C,i} + m_T v_{T,i}\sin(45°)}{m_T}\right)^2$$

$$m_C v_{C,i}^2 + m_T v_{T,i}^2 = \frac{m_T^2 v_{T,i}^2\cos^2(45°)}{m_C} + \frac{m_C^2 v_{C,i}^2 + m_T^2 v_{T,i}^2\sin^2(45°) + 2m_C m_T v_{C,i}v_{T,i}\sin(45°)}{m_T}$$

$$m_C v_{C,i}^2 + m_T v_{T,i}^2 = \frac{m_T^2 v_{T,i}^2}{2m_C} + \frac{m_C^2 v_{C,i}^2}{m_T} + \frac{m_T v_{T,i}^2}{2} + \sqrt{2}m_C v_{C,i}v_{T,i}$$

$$\left(m_T - \frac{m_T^2}{2m_C} - \frac{m_T}{2}\right)v_{T,i}^2 - (\sqrt{2}m_C v_{C,i})v_{T,i} + \left(m_C v_{C,i}^2 - \frac{m_C^2 v_{C,i}^2}{m_T}\right) = 0$$

Solving the quadratic equation (all numbers in SI units):

$$a = \left(m_T - \frac{m_T^2}{2m_C} - \frac{m_T}{2}\right) = \left(1250 - \frac{(1250)^2}{2(1000)} - \frac{1250}{2}\right) = -156.25$$

$$b = -(\sqrt{2}m_C v_{C,i}) = -\sqrt{2}(1000)(30) = -42{,}426.4$$

$$c = \left(m_C v_{C,i}^2 - \frac{m_C^2 v_{C,i}^2}{m_T}\right) = \left((1000)(30)^2 - \frac{(1000)^2(30)^2}{1250}\right) = 180{,}000$$

$$v_{T,i} = \frac{-(-42426.4) \pm \sqrt{(-42426.4)^2 - 4(-156.25)(180000)}}{2(-156.25)} = \frac{42426.4 \pm 43732.1}{-312.5}$$

Taking the negative root:

$$v_{T,i} = \frac{42426.4 - 43732.1}{-312.5} = \boxed{4.2\frac{\text{m}}{\text{s}}}$$

REFLECT

The final speeds of the car and truck are 3.7 m/s and 27 m/s, respectively.

7.50

SET UP

A firecracker, initially at rest, explodes into three pieces: a small piece ($m_S = 0.01$ kg), a medium-sized piece ($m_M = 0.03$ kg), and a large piece ($m_L = 0.06$ kg). We are given the final momentum vectors for each piece and the magnitudes of some of the components of the final velocities. We can use conservation of momentum to calculate the remaining components of the velocity of each piece. The sign of each component is given by the sign of the component in the final momentum vector.

SOLVE

x component:

$$0 = m_L v_{L,x} - m_S v_{S,x}$$

$$v_{L,x} = \frac{m_S v_{S,x}}{m_L} = \frac{(0.01\text{ kg})\left(4\frac{\text{m}}{\text{s}}\right)}{0.06\text{ kg}} = 0.67\frac{\text{m}}{\text{s}}$$

y component:

$$0 = m_M v_{M,y} - m_S v_{S,y}$$

$$v_{M,y} = \frac{m_S v_{S,y}}{m_M} = \frac{(0.01\text{ kg})\left(4\frac{\text{m}}{\text{s}}\right)}{0.03\text{ kg}} = 1.3\frac{\text{m}}{\text{s}}$$

z component:

$$0 = m_M v_{M,z} - m_L v_{L,z}$$

$$v_{M,z} = \frac{m_L v_{L,z}}{m_M} = \frac{(0.06\text{ kg})\left(1\frac{\text{m}}{\text{s}}\right)}{0.03\text{ kg}} = 2\frac{\text{m}}{\text{s}}$$

Velocities:

$$\boxed{\vec{v}_S = -\left(4\frac{\text{m}}{\text{s}}\right)\hat{x} - \left(4\frac{\text{m}}{\text{s}}\right)\hat{y}}$$

$$\boxed{\vec{v}_M = \left(1.3\frac{\text{m}}{\text{s}}\right)\hat{y} + \left(2\frac{\text{m}}{\text{s}}\right)\hat{z}}$$

$$\boxed{\vec{v}_L = \left(0.67\frac{\text{m}}{\text{s}}\right)\hat{x} - \left(1\frac{\text{m}}{\text{s}}\right)\hat{z}}$$

REFLECT

Because mass is a positive scalar, the velocity and momentum vectors will have the same direction and, therefore, the same signs of their components.

7.51

SET UP

Two bighorn sheep are in the midst of a head-butting contest. Sheep 1 ($m_1 = 95$ kg) moves at a speed of 10 m/s directly toward sheep 2 ($m_2 = 80$ kg) running at 12 m/s. To determine which sheep wins the contest, we need to compare the momenta of the two sheep; whichever sheep has the larger momentum will end up knocking the other sheep backward.

SOLVE

Sheep 1:

$$p_1 = m_1 v_1 = (95\text{ kg})\left(10\frac{\text{m}}{\text{s}}\right) = 950\frac{\text{kg}\cdot\text{m}}{\text{s}}$$

Sheep 2:

$$p_2 = m_2 v_2 = (80\text{ kg})\left(12\frac{\text{m}}{\text{s}}\right) = 960\frac{\text{kg}\cdot\text{m}}{\text{s}}$$

The lighter sheep has a larger momentum, so $\boxed{\text{sheep 2}}$ will win the head-butting contest.

REFLECT

If we knew how long the sheep were in contact, we could calculate the force of one sheep on the other. We could then relate this to the acceleration of the sheep via Newton's second law. If the net force on the sheep were constant, we could use the constant acceleration equations to calculate how far sheep 2 pushes sheep 1.

7.52

SET UP

We are told that a known particle of mass m and initial speed v_0 collides elastically with a stationary, unknown particle of mass m_{unknown}. The known particle then rebounds with a speed of v. We can invoke conservation of momentum and conservation of energy because it is an elastic collision. Using the expressions from Physicist's Toolbox 7-1, we can write the final speed of the unknown mass in terms of the known particle's initial speed and the two masses. This will allow us to find an expression of the unknown mass in terms of m, v, and v_0. For the case of a neutron and a proton, $m_n \approx m_p = 1.67 \times 10^{-27}$ kg; we can calculate the recoil speed of the proton and final speed of the proton directly from the expressions in Physicist's Toolbox 7-1.

SOLVE

Part a)

$$m_{\text{known}}v_{\text{known, i}} + m_{\text{unknown}}v_{\text{unknown, i}} = m_{\text{known}}v_{\text{known, f}} + m_{\text{unknown}}v_{\text{unknown, f}}$$

$$mv_0 + 0 = mv + m_{\text{unknown}}v_{\text{unknown, f}}$$

$$m_{\text{unknown}} = \frac{m(v_0 - v)}{v_{\text{unknown, f}}}$$

But

$$v_{\text{unknown, f}} = v_0\left(\frac{2m}{m + m_{\text{unknown}}}\right)$$

Plugging this in,

$$m_{\text{unknown}} = \frac{m(v_0 - v)}{v_0}\left(\frac{m + m_{\text{unknown}}}{2m}\right) = \frac{(v_0 - v)}{2v_0}(m + m_{\text{unknown}})$$

$$m_{\text{unknown}}\left(1 - \frac{v_0 - v}{2v_0}\right) = \left(\frac{v_0 - v}{2v_0}\right)m$$

$$\boxed{m_{\text{unknown}} = \left(\frac{v_0 - v}{v_0 + v}\right)m}$$

Part b)

$$v_{\text{p, f}} = v_0\left(\frac{m_p - m_n}{m_p + m_n}\right) = v_0\left(\frac{m_p - m_p}{m_p + m_p}\right) = \boxed{0}$$

$$v_{\mathrm{n,f}} = v_0\left(\frac{2m_\mathrm{p}}{m_\mathrm{p} + m_\mathrm{n}}\right) = v_0\left(\frac{2m_\mathrm{p}}{m_\mathrm{p} + m_\mathrm{p}}\right) = \boxed{2v_0}$$

REFLECT

The mass of a proton and the mass of a neutron are approximately equal. Therefore, it makes sense that, after a head-on collision, the proton should stop and the neutron should move off at the initial speed of the proton.

7.53

SET UP

A train car ($m_1 = 10{,}000$ kg) is moving at a speed of $v_{1,\mathrm{i}} = 20$ m/s to the east when it collides with a second train car ($m_2 = 20{,}000$ kg) that is at rest. The train cars stick together and travel with a final speed of v_f. We can calculate the final velocity of the two-car system by applying conservation of momentum.

SOLVE

$$m_1 v_{1,\mathrm{i}} + m_2 v_{2,\mathrm{i}} = (m_1 + m_2)v_\mathrm{f}$$

$$v_\mathrm{f} = \frac{m_1 v_{1,\mathrm{i}} + m_2 v_{2,\mathrm{i}}}{m_1 + m_2} = \frac{(10{,}000\ \mathrm{kg})\left(20\frac{\mathrm{m}}{\mathrm{s}}\right) + 0}{(10{,}000\ \mathrm{kg}) + (20{,}000\ \mathrm{kg})} = 6.7\frac{\mathrm{m}}{\mathrm{s}}$$

The final velocity of the train is $\boxed{6.7\ \text{m/s to the east}}$.

REFLECT

The speed of the two-car system should be smaller than the speed of the incoming car since the mass is larger. The final velocity will point in the same direction as the initial velocity because the second car started at rest.

7.54

SET UP

A large fish ($m_1 = 25$ kg) is moving at a speed of $v_{1,\mathrm{i}} = 1.0$ m/s toward a small fish ($m_2 = 1.0$ kg) that is at rest. The large fish eats the small fish, which means the mass of the large fish has increased by 1.0 kg. We can calculate the final speed v_f of the large fish by applying conservation of momentum.

SOLVE

$$m_1 v_{1,\mathrm{i}} + m_2 v_{2,\mathrm{i}} = (m_1 + m_2)v_\mathrm{f}$$

$$v_\mathrm{f} = \frac{m_1 v_{1,\mathrm{i}} + m_2 v_{2,\mathrm{i}}}{m_1 + m_2} = \frac{(25\ \mathrm{kg})\left(1.0\frac{\mathrm{m}}{\mathrm{s}}\right) + 0}{(25\ \mathrm{kg}) + (1.0\ \mathrm{kg})} = \boxed{0.96\frac{\mathrm{m}}{\mathrm{s}}}$$

REFLECT

The mass of the small fish is much less than the mass of the large fish, so the final speed should be very close to the initial speed.

7.55

SET UP

A howler monkey ($m_1 = 5.0$ kg) is swinging from a vine at a speed of $v_{1,\,i} = 12$ m/s due east toward a second monkey ($m_2 = 6.0$ kg) that is also moving east at a speed of $v_{2,\,i} = 8$ m/s. The monkeys grab onto one another and travel together on the same vine. We can calculate the final speed v_f of the two monkeys traveling together by applying conservation of momentum. In our coordinate system, east will point in the positive direction.

SOLVE

$$m_1 v_{1,\,i} + m_2 v_{2,\,i} = (m_1 + m_2) v_f$$

$$v_f = \frac{m_1 v_{1,\,i} + m_2 v_{2,\,i}}{m_1 + m_2} = \frac{(5.0\text{ kg})\left(12\frac{\text{m}}{\text{s}}\right) + (6.0\text{ kg})\left(8\frac{\text{m}}{\text{s}}\right)}{(5.0\text{ kg}) + (6.0\text{ kg})} = \boxed{9.8\frac{\text{m}}{\text{s}}}$$

REFLECT

It makes sense that the monkeys will continue to travel east at a speed in between their initial speeds.

7.56

SET UP

Car 1 (mass $m_1 = 1200$ kg) is traveling due north at 20 m/s. Car 2 (mass $m_2 = 1500$ kg) is traveling due east at 18 m/s. The two collide, stick together, and travel off at a speed of v_f and angle θ. We can use conservation of momentum in order to find the speed and direction of the wreckage following the collision. Since this is a two-dimensional problem, we will need to split the momenta into components and solve for the x and y components of the final velocity. In our coordinate system east will point toward $+x$ and north will point toward $+y$.

SOLVE

x component:

$$m_1 v_{1i,\,x} + m_2 v_{2i,\,x} = (m_1 + m_2) v_{f,\,x}$$

$$0 + m_2 v_{2i,\,x} = (m_1 + m_2) v_{f,\,x}$$

$$v_{f,\,x} = \frac{m_2 v_{2i,\,x}}{m_1 + m_2} = \frac{(1500\text{ kg})\left(18\frac{\text{m}}{\text{s}}\right)}{(1500\text{ kg}) + (1200\text{ kg})} = 10\frac{\text{m}}{\text{s}}$$

y component:

$$m_1 v_{1i,\,y} + m_2 v_{2i,\,y} = (m_1 + m_2) v_{f,\,y}$$

$$m_1 v_{1i,\,y} + 0 = (m_1 + m_2) v_{f,\,y}$$

$$v_{f,\,y} = \frac{m_1 v_{1i,\,y}}{m_1 + m_2} = \frac{(1200\text{ kg})\left(20\frac{\text{m}}{\text{s}}\right)}{(1500\text{ kg}) + (1200\text{ kg})} = 8.9\frac{\text{m}}{\text{s}}$$

Final speed:

$$v_\mathrm{f} = \sqrt{v_{\mathrm{f},x}^2 + v_{\mathrm{f},y}^2} = \sqrt{\left(10\frac{\mathrm{m}}{\mathrm{s}}\right)^2 + \left(8.9\frac{\mathrm{m}}{\mathrm{s}}\right)^2} = \boxed{13.4\frac{\mathrm{m}}{\mathrm{s}}}$$

Final direction:

$$\theta = \arctan\left(\frac{v_{\mathrm{f},y}}{v_{\mathrm{f},x}}\right) = \arctan\left(\frac{\left(8.9\frac{\mathrm{m}}{\mathrm{s}}\right)}{\left(10\frac{\mathrm{m}}{\mathrm{s}}\right)}\right) = \boxed{42^\circ \text{ north of east}}$$

REFLECT

The initial momentum of the more massive car is larger, so it makes sense that the momentum of the wreckage is more aligned to the east than the north.

7.57

SET UP

A linebacker ($m_1 = 85$ kg) is running toward the sidelines at a speed of 8 m/s. He tackles a running back ($m_2 = 75$ kg) that is running toward the goal line at 9 m/s. After the two players collide, they leave the ground and move off together with the same final velocity. We can use conservation of momentum to calculate the final speed and direction of the entangled players. We'll say that the direction toward the goal line is $+x$ and that the direction toward the sideline is $+y$.

SOLVE

x component:

$$m_1 v_{1\mathrm{i},x} + m_2 v_{2\mathrm{i},x} = (m_1 + m_2) v_{\mathrm{f},x}$$

$$v_{\mathrm{f},x} = \frac{m_1 v_{1\mathrm{i},x} + m_2 v_{2\mathrm{i},x}}{m_1 + m_2} = \frac{0 + (75\ \mathrm{kg})\left(9\frac{\mathrm{m}}{\mathrm{s}}\right)}{(85\ \mathrm{kg}) + (75\ \mathrm{kg})} = 4.22\frac{\mathrm{m}}{\mathrm{s}}$$

y component:

$$m_1 v_{1\mathrm{i},y} + m_2 v_{2\mathrm{i},y} = (m_1 + m_2) v_{\mathrm{f},y}$$

$$v_{\mathrm{f},y} = \frac{m_1 v_{1\mathrm{i},y} + m_2 v_{2\mathrm{i},y}}{m_1 + m_2} = \frac{(85\ \mathrm{kg})\left(8\frac{\mathrm{m}}{\mathrm{s}}\right) + 0}{(85\ \mathrm{kg}) + (75\ \mathrm{kg})} = 4.25\frac{\mathrm{m}}{\mathrm{s}}$$

Final speed and direction:

$$v_\mathrm{f} = \sqrt{v_{\mathrm{f},x}^2 + v_{\mathrm{f},y}^2} = \sqrt{\left(4.22\frac{\mathrm{m}}{\mathrm{s}}\right)^2 + \left(4.25\frac{\mathrm{m}}{\mathrm{s}}\right)^2} = \boxed{6.0\frac{\mathrm{m}}{\mathrm{s}}}$$

$$\theta = \arctan\left(\frac{v_{\mathrm{f},y}}{v_{\mathrm{f},x}}\right) = \arctan\left(\frac{\left(4.25\frac{\mathrm{m}}{\mathrm{s}}\right)}{\left(4.22\frac{\mathrm{m}}{\mathrm{s}}\right)}\right) = \boxed{45^\circ}$$

REFLECT

The components of the final velocity are approximately equal, which means the angle should be about 45 degrees since $\arctan(1) = 45$ degrees.

7.58

SET UP

A gust of wind exerts a force of 20 N on a bird for 1.2 s. The bird is initially traveling at a speed of 5 m/s and ends up moving in the opposite direction at 7 m/s. We'll assume that the force acts in the $+x$ direction, which means the bird is initially traveling in the $-x$ direction. We can calculate the mass of the bird from the contact time, the force, and the change in the bird's velocity.

SOLVE

$$\Delta p = m\Delta v = F\Delta t$$

$$m = \frac{F\Delta t}{\Delta v} = \frac{(20\text{ N})(1.2\text{ s})}{\left(7\frac{\text{m}}{\text{s}}\right) - \left(-5\frac{\text{m}}{\text{s}}\right)} = \boxed{2\text{ kg}}$$

REFLECT

If the bird were traveling in the $+x$ direction initially, then the force would act in the $-x$ direction. The answer would be the same.

7.59

SET UP

A 0.200-kg ball is initially traveling at 20 m/s toward your hand. You catch the ball and it comes to rest within 0.025 s. We can find the magnitude of the force your hand exerts on the ball from the change in the ball's momentum and the contact time.

SOLVE

$$F = \left|\frac{\Delta p}{\Delta t}\right| = \left|\frac{m(v_\text{f} - v_\text{i})}{\Delta t}\right| = \left|\frac{(0.200\text{ kg})\left(0 - \left(20\frac{\text{m}}{\text{s}}\right)\right)}{0.025\text{ s}}\right| = \boxed{160\text{ N}}$$

REFLECT

The force of your hand on the ball will act opposite to the ball's initial velocity.

7.60

SET UP

We are given a plot of force versus time. The impulse delivered to an object is equal to the area under this curve. We can split the total area into geometric shapes with simple areas to calculate, namely, a trapezoid and a triangle.

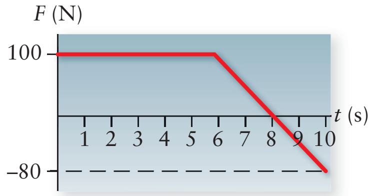


Figure 7-3 Problem 60

SOLVE

$$J_{0\to 10\text{ s}} = J_{0\to 8\text{ s}} + J_{8\to 10\text{ s}} = \frac{1}{2}(100\text{ N})((6\text{ s}) + (8\text{ s})) + \frac{1}{2}(-80\text{ N})(2\text{ s}) = \boxed{620\frac{\text{kg}\cdot\text{m}}{\text{s}}}$$

REFLECT

Remember that an area below the x-axis is considered to be negative.

7.61

SET UP

A 0.145-kg baseball is thrown toward a batter at a speed of 40 m/s. The batter hits the ball with a force of 25,000 N. The bat and ball are in contact for 0.5 ms ($= 5 \times 10^{-4}$ s). We can calculate the final speed of the ball from the force and the contact time. We'll assume that the ball is initially traveling in the $-x$ direction, which means the force acts in the $+x$ direction.

SOLVE

$$\Delta p = m\Delta v = m(v_f - v_i) = F\Delta t$$

$$v_f = \frac{F\Delta t}{m} + v_i = \frac{(25{,}000\text{ N})(5 \times 10^{-4}\text{ s})}{0.145\text{ kg}} + \left(-40\frac{\text{m}}{\text{s}}\right) = \boxed{46\frac{\text{m}}{\text{s}}}$$

REFLECT

A speed of 46 m/s is around 100 mph, which is a reasonable speed for a line drive.

7.62

SET UP

A 5-kg object moves along a straight line. Its initial speed is 12 m/s in one direction, and its final speed is 8 m/s in the opposite direction. We'll call the initial direction positive. We are given a plot of the net force acting on the object as a function of time with some values missing. The area under this curve is equal to the change in the object's momentum. We can determine numerical values for the missing quantities by explicitly calculating the area of the triangles. Areas underneath the t-axis are considered to be negative. We'll label the missing force values from the most negative to the most positive starting with F_1 up to F_3.

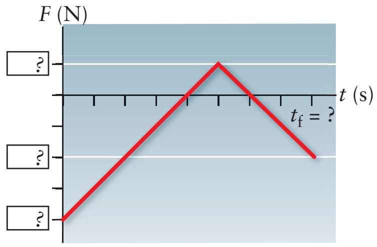


Figure 7-4 Problem 62

SOLVE

Let's say each tick mark on the time axis is 1 s. Therefore, $\boxed{t_f = 8\text{ s}}$.

Because we are given the tick marks on the force axis, we can enforce a relationship among F_1, F_2, and F_3:

$$F_1 = -4F_3$$

$$F_2 = -2F_3$$

Calculating the area:

$$\Delta p = m\Delta v = (5\text{ kg})\left(\left(-8\frac{\text{m}}{\text{s}}\right) - \left(12\frac{\text{m}}{\text{s}}\right)\right) = -100\frac{\text{kg}\cdot\text{m}}{\text{s}}$$

$$\Delta p = \frac{1}{2}(F_1)(4\text{ s}) + \frac{1}{2}(F_3)(2\text{ s}) + \frac{1}{2}(F_2)(2\text{ s}) = \frac{1}{2}(-4F_3)(4\text{ s}) + \frac{1}{2}(F_3)(2\text{ s}) + \frac{1}{2}(-2F_3)(2\text{ s})$$

$$-100 = -8F_3 + F_3 - 2F_3 \quad \text{(SI units)}$$

$$\boxed{F_3 = \frac{100}{9}\text{N}}$$

$$\boxed{F_1 = -4F_3 = -\frac{400}{9}\text{N}}$$

$$\boxed{F_2 = -2F_3 = -\frac{200}{9}\text{N}}$$

REFLECT

There are many correct answers to this problem. We can check to make sure our answers are consistent. The values of F_1 and F_2 should be negative because they are below the axis. The total area should equal $-100\text{ kg}\cdot\text{m/s}$:

$$\Delta p = \frac{1}{2}\left(-\frac{400}{9}\text{N}\right)(4\text{ s}) + \frac{1}{2}\left(\frac{100}{9}\text{N}\right)(2\text{ s}) + \frac{1}{2}\left(-\frac{200}{9}\text{N}\right)(2\text{ s})$$

$$= \left(-\frac{800}{9} + \frac{100}{9} - \frac{200}{9}\right)\frac{\text{kg}\cdot\text{m}}{\text{s}} = -100\frac{\text{kg}\cdot\text{m}}{\text{s}}$$

7.63

SET UP

We are given the functional form for a one-dimensional force as a function of time $F(t)$ in SI units. This force acts on an object that is initially at rest, which means its initial momentum is zero. We can calculate the momentum of the object after the force has acted for 1 s by integrating $F(t)$ with respect to time from $t = 0$ s to $t = 1$ s.

SOLVE

$$\Delta p = p_f - p_i = p_f - 0 = \int_0^1 F(t)dt$$

$$p_f = \int_0^1 F(t)dt = \int_0^1 (9t^2 + 8t^3)dt = [3t^3 + 2t^4]_0^1 = [3(1)^3 + 2(1)^4 - 0 - 0] = \boxed{5\frac{\text{kg}\cdot\text{m}}{\text{s}}}$$

REFLECT

The momentum of the object points in the same direction as the force.

7.64

SET UP

A baseball bat ($m_{\text{bat}} = 0.850$ kg) strikes a ball ($m_{\text{ball}} = 0.145$ kg) when both are moving at 31.3 m/s toward each other. (This speed is measured relative to the ground.) The bat and ball are in contact for 1.20 ms, after which the ball is traveling at a speed of 42.5 m/s in the opposite direction. We can use the definition of impulse to calculate the impulse given to the ball by the bat. Before we can calculate the impulse given to the bat by the ball, we need to apply conservation of momentum to find the bat's final velocity. The average force the bat exerts on the ball is equal to the ball's change in momentum divided by the contact time.

SOLVE

Part a)

$$J = m\Delta v = m(v_f - v_i) = (0.145\text{ kg})\left(\left(42.5\frac{\text{m}}{\text{s}}\right) - \left(-31.3\frac{\text{m}}{\text{s}}\right)\right) = 10.7\frac{\text{kg}\cdot\text{m}}{\text{s}}$$

The impulse given to the ball by the bat is $\boxed{10.7\frac{\text{kg}\cdot\text{m}}{\text{s}} \text{ in the positive } x \text{ direction}}$.

Part b)

Conservation of momentum:

$$m_{\text{bat}}v_{\text{bat, i}} + m_{\text{ball}}v_{\text{ball, i}} = m_{\text{bat}}v_{\text{bat, f}} + m_{\text{ball}}v_{\text{ball, f}}$$

$$v_{\text{bat, f}} = \frac{m_{\text{bat}}v_{\text{bat, i}} + m_{\text{ball}}v_{\text{ball, i}} - m_{\text{ball}}v_{\text{ball, f}}}{m_{\text{bat}}}$$

$$= \frac{(0.145\text{ kg})\left(-31.3\frac{\text{m}}{\text{s}}\right) + (0.850\text{ kg})\left(31.3\frac{\text{m}}{\text{s}}\right) - (0.145\text{ kg})\left(42.5\frac{\text{m}}{\text{s}}\right)}{0.850\text{ kg}} = 18.7\frac{\text{m}}{\text{s}}$$

Impulse:

$$J = m\Delta v = m(v_f - v_i) = (0.850\text{ kg})\left(\left(18.7\frac{\text{m}}{\text{s}}\right) - \left(31.3\frac{\text{m}}{\text{s}}\right)\right) = -10.7\frac{\text{kg}\cdot\text{m}}{\text{s}}$$

The impulse given to the ball by the bat is $\boxed{10.7\frac{\text{kg}\cdot\text{m}}{\text{s}} \text{ in the negative } x \text{ direction}}$.

Part c)

$$F_{\text{average}} = \frac{\Delta p}{\Delta t} = \frac{\left(10.7\frac{\text{kg}\cdot\text{m}}{\text{s}}\right)}{1.20 \times 10^{-3}\text{ s}} = 8920\text{ N}$$

The average force of the bat on the ball is $\boxed{8920 \text{ N in the positive } x \text{ direction}}$.

Part d) Although the force is large, the ball and bat are only in contact for 1.20 ms, so the bat doesn't shatter.

REFLECT

The impulse given to the ball by the bat and the impulse given to the bat by the ball must be equal in magnitude and opposite in direction due to conservation of momentum.

7.65

SET UP

A ball ($m_1 = 2$ kg) is moving at 3 m/s to the right when it collides elastically with a second ball ($m_2 = 4$ kg) that is initially at rest. We can use the equations from Physicist's Toolbox 7-1 to calculate the velocity of each ball after the collision.

SOLVE

$$v_{1,\text{f}} = v_{1,\text{i}}\left(\frac{m_1 - m_2}{m_1 + m_2}\right) = \left(3\frac{\text{m}}{\text{s}}\right)\left(\frac{(2\text{ kg}) - (4\text{ kg})}{(2\text{ kg}) + (4\text{ kg})}\right) = -1\frac{\text{m}}{\text{s}}$$

The final velocity of the 2-kg ball is $\boxed{1 \text{ m/s to the left}}$.

$$v_{2,\text{f}} = v_{2,\text{i}}\left(\frac{2m_1}{m_1 + m_2}\right) = \left(3\frac{\text{m}}{\text{s}}\right)\left(\frac{2(2\text{ kg})}{(2\text{ kg}) + (4\text{ kg})}\right) = 2\frac{\text{m}}{\text{s}}$$

The final velocity of the 4-kg ball is $\boxed{2 \text{ m/s to the right}}$.

REFLECT

Since the larger ball starts at rest, we would expect the smaller ball to bounce off of it and move in the opposite direction.

7.66

SET UP

A ball ($m_1 = 0.170$ kg) is moving at 4.00 m/s toward the right when it collides elastically with a second ball ($m_2 = 0.155$ kg) moving toward the left at 2.00 m/s. We can use conservation of momentum and conservation of mechanical energy to find the final velocities of the two balls. We can derive a useful relationship between the final velocities and the initial velocities of a one-dimensional elastic collision: $v_{1,\text{f}} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1,\text{i}} + \left(\frac{2m_2}{m_1 + m_2}\right)v_{2,\text{i}}$ and $v_{2,\text{f}} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1,\text{i}} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_{2,\text{i}}$. In our coordinate system, we'll consider $+x$ to point toward the right.

SOLVE

$$\begin{aligned} v_{1,\text{f}} &= \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1,\text{i}} + \left(\frac{2m_2}{m_1 + m_2}\right)v_{2,\text{i}} \\ &= \left(\frac{(0.170\text{ kg}) - (0.155\text{ kg})}{(0.170\text{ kg}) + (0.155\text{ kg})}\right)\left(4.00\frac{\text{m}}{\text{s}}\right) + \left(\frac{2(0.155\text{ kg})}{(0.170\text{ kg}) + (0.155\text{ kg})}\right)\left(-2.00\frac{\text{m}}{\text{s}}\right) \\ &= -1.72\frac{\text{m}}{\text{s}} \end{aligned}$$

Ball 1's final velocity is $\boxed{1.72 \text{ m/s to the left}}$.

$$v_{2,\,f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1,\,i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_{2,\,i}$$

$$= \left(\frac{2(0.170 \text{ kg})}{(0.170 \text{ kg}) + (0.155 \text{ kg})}\right)\left(4.00\frac{\text{m}}{\text{s}}\right) + \left(\frac{(0.155 \text{ kg}) - (0.170 \text{ kg})}{(0.170 \text{ kg}) + (0.155 \text{ kg})}\right)\left(-2.00\frac{\text{m}}{\text{s}}\right)$$

$$= 4.28\frac{\text{m}}{\text{s}}$$

Ball 2's final velocity is $\boxed{4.28 \text{ m/s to the right}}$.

REFLECT

In an elastic collision, we expect the balls to bounce off of one another and reverse direction after the collision.

7.67

SET UP

A block of ice ($m_1 = 10$ kg) is moving at 8 m/s toward the east when it collides elastically with a second block of ice ($m_2 = 6$ kg) moving toward the east at 4 m/s. We can use conservation of momentum and conservation of mechanical energy to find the final velocities of the two blocks. We can derive a useful relationship between the final velocities and the initial velocities of a one-dimensional elastic collision: $v_{1,\,f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1,\,i} + \left(\frac{2m_2}{m_1 + m_2}\right)v_{2,\,i}$ and $v_{2,\,f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1,\,i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_{2,\,i}$. In our coordinate system, we'll consider $+x$ to point toward the east.

SOLVE

$$v_{1,\,f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1,\,i} + \left(\frac{2m_2}{m_1 + m_2}\right)v_{2,\,i}$$

$$= \left(\frac{(10 \text{ kg}) - (6 \text{ kg})}{(10 \text{ kg}) + (6 \text{ kg})}\right)\left(8\frac{\text{m}}{\text{s}}\right) + \left(\frac{2(6 \text{ kg})}{(10 \text{ kg}) + (6 \text{ kg})}\right)\left(4\frac{\text{m}}{\text{s}}\right) = 5\frac{\text{m}}{\text{s}}$$

Block 1's final velocity is $\boxed{5 \text{ m/s to the right}}$.

$$v_{2,\,f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1,\,i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_{2,\,i}$$

$$= \left(\frac{2(10 \text{ kg})}{(10 \text{ kg}) + (6 \text{ kg})}\right)\left(8\frac{\text{m}}{\text{s}}\right) + \left(\frac{(6 \text{ kg}) - (10 \text{ kg})}{(10 \text{ kg}) + (6 \text{ kg})}\right)\left(4\frac{\text{m}}{\text{s}}\right) = 9\frac{\text{m}}{\text{s}}$$

Block 2's final velocity is $\boxed{9 \text{ m/s to the right}}$.

REFLECT

It makes sense that block 1 should slow down and block 2 should move faster after the collision.

7.68

SET UP

A neutron ($m_1 = 1.67 \times 10^{-27}$ kg) is moving at 2×10^5 m/s when it collides elastically with a deuteron ($m_2 = 3.34 \times 10^{-27}$ kg) that is initially at rest. We can use the equations from Physicist's Toolbox 7-1 to calculate the final speeds of each particle after the collision.

SOLVE

$$v_{1,\text{f}} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1,\text{i}} = \left(\frac{(1.67 \times 10^{-27}\text{ kg}) - (3.34 \times 10^{-27}\text{ kg})}{(1.67 \times 10^{-27}\text{ kg}) + (3.34 \times 10^{-27}\text{ kg})}\right)\left(2 \times 10^5 \frac{\text{m}}{\text{s}}\right) = -6.67 \times 10^4 \frac{\text{m}}{\text{s}}$$

The final speed of the neutron is $\boxed{6.67 \times 10^4\text{ m/s}}$.

$$v_{2,\text{f}} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1,\text{i}} = \left(\frac{2(1.67 \times 10^{-27}\text{ kg})}{(1.67 \times 10^{-27}\text{ kg}) + (3.34 \times 10^{-27}\text{ kg})}\right)\left(2 \times 10^5 \frac{\text{m}}{\text{s}}\right) = 1.33 \times 10^5 \frac{\text{m}}{\text{s}}$$

The final speed of the deuteron is $\boxed{1.33 \times 10^5\text{ m/s}}$.

REFLECT

The neutron, which is lighter than the deuteron, will bounce off the deuteron and travel in the negative x direction after the collision.

7.69

SET UP

Three objects are placed on a coordinate grid: object 1 ($m_1 = 4$ kg) is located at (-6 m, 2 m), object 2 ($m_2 = 2$ kg) is located at (2 m, 2 m), and object 3 ($m_3 = 3$ kg) is located at (-4 m, -4 m). We can calculate the x and y components of the center of mass by applying the definition of center of mass.

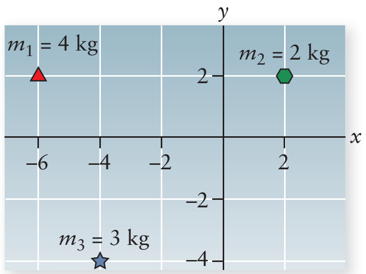


Figure 7-5 Problem 69

SOLVE

x component:

$$x_{\text{CM}} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{(4\text{ kg})(-6\text{ m}) + (2\text{ kg})(2\text{ m}) + (3\text{ kg})(-4\text{ m})}{(4\text{ kg}) + (2\text{ kg}) + (3\text{ kg})}$$

$$= \frac{-24 + 4 - 12}{9}\text{m} = \boxed{-3.6\text{ m}}$$

y component:

$$y_{CM} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{(4\text{ kg})(2\text{ m}) + (2\text{ kg})(2\text{ m}) + (3\text{ kg})(-4\text{ m})}{(4\text{ kg}) + (2\text{ kg}) + (3\text{ kg})}$$

$$= \frac{8 + 4 - 12}{9}\text{m} = \boxed{0\text{ m}}$$

REFLECT

The mass is symmetrically distributed about the *x*-axis, which means the *y* component of the center of mass is zero.

7.70

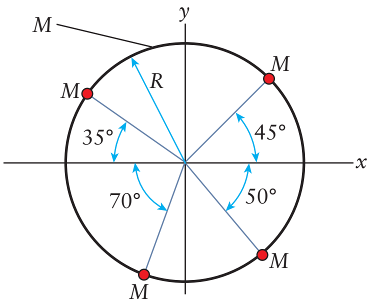


Figure 7-6 Problem 70

SET UP

Four beads of equal mass M are attached to a hoop of mass $m_5 = M$ and radius R. The center of mass of the system is the weighted average of the positions of the four beads and the hoop. The hoop is centered about the origin and the beads are located at various locations around the hoop. We are given the angles corresponding to the location of each bead, so we will need to do some trigonometry in order to determine their *x* and *y* components.

SOLVE

x component:

$$x_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4 + m_5x_5}{m_1 + m_2 + m_3 + m_4 + m_5}$$

$$= \frac{M(R\cos(45°)) + M(-R\cos(45°)) + M(-R\cos(70°)) + M(R\cos(50°)) + M(0)}{5M}$$

$$= \boxed{-0.038R}$$

y component:

$$y_{CM} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + m_4y_4 + m_5y_5}{m_1 + m_2 + m_3 + m_4 + m_5}$$

$$= \frac{M(R\sin(45°)) + M(R\sin(45°)) + M(-R\sin(70°)) + M(-R\sin(50°)) + M(0)}{5M}$$

$$= \boxed{-0.085R}$$

REFLECT
The center of mass of the hoop itself is at the origin, so it only contributes to the total mass of the system.

7.71

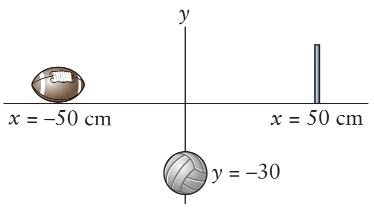


Figure 7-7 Problem 71

SET UP
The locations of three uniform objects (a rod, a football, and a volleyball) are given. Since the objects are uniform, the center of mass of each object is located at its geometrical center. The center of an ellipse is halfway along each axis. (Remember that the semimajor axis is the longer axis.)

SOLVE
Rod:

$$\boxed{\begin{aligned} x_{\text{CM}} &= 50\text{ cm} \\ y_{\text{CM}} &= 15\text{ cm} \end{aligned}}$$

Football:

$$\boxed{\begin{aligned} x_{\text{CM}} &= -50\text{ cm} \\ y_{\text{CM}} &= 4\text{ cm} \end{aligned}}$$

Volleyball:

$$\boxed{\begin{aligned} x_{\text{CM}} &= 0\text{ cm} \\ y_{\text{CM}} &= -30\text{ cm} \end{aligned}}$$

REFLECT
Because we knew the dimensions of each shape and that they were uniform, we did not need to know the mass of the objects. We would get the same answers if we explicitly calculated the center of mass in each case.

7.72

SET UP
A 0.145-kg baseball is dropped from rest from a height of 60 m. The ball will undergo free fall as it falls to the ground. We can use the constant acceleration equations to determine the speed of the ball when it hits the ground. The momentum of the ball at this point is equal to the product of the mass and the velocity of the ball.

SOLVE

Final speed of baseball:

$$v_f^2 = v_i^2 + 2a_y(\Delta y) = 0 + 2\left(-9.8\frac{m}{s^2}\right)(-60\text{ m}) = 1176\frac{m^2}{s^2}$$

$$v_f = 34.3\frac{m}{s}$$

Momentum:

$$p = mv_f = (0.145\text{ kg})\left(34.3\frac{m}{s}\right) = 4.97\frac{kg \cdot m}{s}$$

The momentum of the ball when it hits the ground is $\boxed{4.97\frac{kg \cdot m}{s}\text{ pointing straight down}}$.

REFLECT

The ball hits the ground with a speed of around 77 mph, which seems reasonable after falling from almost 200 feet.

7.73

SET UP

Two different bullets are shot with different speeds into a heavy block of unknown mass. We can use conservation of momentum and conservation of energy in order to determine which case will cause the block to swing higher. Once we know which bullet to use, we can apply the same procedure to calculate the mass of the wooden block. We will then use the same wooden block to calculate the speed a third bullet is traveling to cause the block to swing a certain height.

SOLVE

Part a)
Conservation of momentum:

$$m_{\text{bullet}}v_i = (m_{\text{bullet}} + m_{\text{block}})v_f$$

$$v_f = \frac{m_{\text{bullet}}}{m_{\text{bullet}} + m_{\text{block}}}v_i$$

Conservation of energy:

$$\frac{1}{2}(m_{\text{bullet}} + m_{\text{block}})v_f^2 = (m_{\text{bullet}} + m_{\text{block}})gh$$

$$h = \frac{v_f^2}{2g} = \frac{1}{2g}\left(\frac{m_{\text{bullet}}}{m_{\text{bullet}} + m_{\text{block}}}\right)^2 v_i^2$$

But we are told the block is heavy, which means

$$m_{\text{bullet}} << m_{\text{block}}$$

Therefore,

$$h \approx \frac{1}{2g}\left(\frac{m_{\text{bullet}}}{m_{\text{block}}}\right)^2 v_i^2$$

Comparing the two bullets:

$$\frac{h_{\text{Ruger}}}{h_{\text{Remington}}} \approx \frac{\frac{1}{2g}\left(\frac{m_{\text{Ruger}}}{m_{\text{block}}}\right)^2 v_{\text{i, Ruger}}^2}{\frac{1}{2g}\left(\frac{m_{\text{Remington}}}{m_{\text{block}}}\right)^2 v_{\text{i, Remington}}^2} = \frac{m_{\text{Ruger}}^2 v_{\text{i, Ruger}}^2}{m_{\text{Remington}}^2 v_{\text{i, Remington}}^2} = \frac{(2.14\text{ g})^2\left(1290\frac{\text{m}}{\text{s}}\right)^2}{(9.71\text{ g})^2\left(948\frac{\text{m}}{\text{s}}\right)^2} = 0.09$$

This is less than 1, which means the $\boxed{\text{Remington bullet}}$ will make the block swing higher.

Part b)
From trigonometry, the final height the block reaches is

$$h = L - L\cos(60°)$$

Conservation of energy:

$$v_{\text{f}} = \sqrt{2gh} = \sqrt{2g(L - L\cos(60°))}$$

Conservation of momentum:

$$m_{\text{bullet}}v_{\text{i}} = (m_{\text{bullet}} + m_{\text{block}})v_{\text{f}}$$

$$m_{\text{block}} = \frac{m_{\text{bullet}}v_{\text{i}}}{v_{\text{f}}} - m_{\text{bullet}} = \frac{m_{\text{bullet}}v_{\text{i}}}{\sqrt{2g(L - L\cos(60°))}} - m_{\text{bullet}} = \frac{m_{\text{bullet}}v_{\text{i}}}{\sqrt{gL}} - m_{\text{bullet}}$$

$$= \frac{(9.71\text{ g})\left(948\frac{\text{m}}{\text{s}}\right)}{\sqrt{\left(9.8\frac{\text{m}}{\text{s}^2}\right)(1.25\text{ m})}} - (9.71\text{ g}) = 2620\text{ g} = \boxed{2.62\text{ kg}}$$

Part c)
From trigonometry, the final height the block reaches is

$$h = L - L\cos(30°)$$

Conservation of energy:

$$v_{\text{f}} = \sqrt{2gh} = \sqrt{2g(L - L\cos(30°))}$$

Conservation of momentum:

$$m_{\text{bullet}}v_{\text{i}} = (m_{\text{bullet}} + m_{\text{block}})v_{\text{f}}$$

$$v_{\text{i}} = \left(\frac{m_{\text{bullet}} + m_{\text{block}}}{m_{\text{bullet}}}\right)v_{\text{f}} = \left(\frac{m_{\text{bullet}} + m_{\text{block}}}{m_{\text{bullet}}}\right)\sqrt{2gL(1 - \cos(30°))}$$

$$= \left(\frac{(8.41\text{ g}) + (2620\text{ g})}{8.41\text{ g}}\right)\sqrt{2\left(9.8\frac{\text{m}}{\text{s}^2}\right)(1.25)(1 - \cos(30°))} = \boxed{566\frac{\text{m}}{\text{s}}}$$

REFLECT
In order to save time, we left all of the masses in grams. Since we're taking a ratio of the mass, the exact unit does not matter as long as we remain consistent.

7.74

SET UP

Sally ($m_S = 60$ kg) is sitting at rest in the middle of a frozen pond with her boot ($m_B = 5$ kg). She throws her boot away from the shore with a speed of 30 m/s; we'll call this direction $-x$. We can use conservation of momentum to calculate Sally's speed immediately after she threw her boot. Because Sally and her boot started at rest, the initial momentum of the system is zero. Therefore, the final momentum of the system must also be zero, which means the final velocity of the center of mass is zero. If the velocity of the center of mass is zero, it will remain in the same place for all time. Sally's final speed will be constant on the frictionless ice, so we can calculate the time it takes her to reach the shore directly from the distance.

SOLVE

Part a)

$$m_S v_{S,\,i} + m_B v_{B,\,i} = m_S v_{S,\,f} + m_B v_{B,\,f}$$

$$v_{S,\,f} = \frac{m_S v_{S,\,i} + m_B v_{B,\,i} - m_B v_{B,\,f}}{m_S} = \frac{0 + 0 - (5\text{ kg})\left(-30\frac{\text{m}}{\text{s}}\right)}{60\text{ kg}} = \boxed{2.5\frac{\text{m}}{\text{s}}}$$

Part b) Since the system starts at rest, the center of mass will not move. Therefore, the center of mass is located at the point where she threw the boot.

Part c)

$$v_{S,\,f} = \frac{\Delta x}{\Delta t}$$

$$\Delta t = \frac{\Delta x}{v_{S,\,f}} = \frac{30\text{ m}}{\left(2.5\frac{\text{m}}{\text{s}}\right)} = 12\text{ s}$$

REFLECT

Our explanation for part (b) is the same as Newton's first law: "An object at rest will remain at rest."

7.75

SET UP

Three hundred million people fall from a height of 1 m onto the ground. We can use conservation of mechanical energy to calculate the speed with which the people land on Earth. From this speed, we can calculate the magnitude of the momentum of all 300 million people; this is the momentum that will be imparted to Earth. Dividing this quantity by the mass of the Earth will give us the change in speed of the Earth.

SOLVE

Part a)
Speed of a single person:

$$mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh} = \sqrt{2\left(9.8\frac{\text{m}}{\text{s}^2}\right)(1\text{ m})} = 4.4\frac{\text{m}}{\text{s}}$$

Momentum of 300 million people:

$$p = mv = (3.00 \times 10^8)(65\text{ kg})\left(4.4\frac{\text{m}}{\text{s}}\right) = \boxed{8.6 \times 10^{10}\frac{\text{kg}\cdot\text{m}}{\text{s}}}$$

Part b)

$$\Delta p_{\text{E}} = m_{\text{E}}\Delta v_{\text{E}}$$

$$\Delta v_{\text{E}} = \frac{\Delta p_{\text{E}}}{m_{\text{E}}} = \frac{\left(8.6 \times 10^{10}\frac{\text{kg}\cdot\text{m}}{\text{s}}\right)}{6.0 \times 10^{24}\text{ kg}} = \boxed{1.445 \times 10^{-14}\frac{\text{m}}{\text{s}}}$$

REFLECT

We could have also used kinematics to calculate the landing speed of the people. Technically Earth will recoil at a nonzero speed, but a speed on the order of 10^{-14} m/s is imperceptible, so the Earth effectively stays still.

7.76

SET UP

A 65-kg person jumps from a height of 2 m. We are asked to calculate the average force of the ground on the person (that is, the normal force) during his landing. We can use conservation of energy to calculate his speed when he lands. The net force acting on the person when he lands is constant, so we can use the constant acceleration equations to first find his acceleration and then use Newton's second law to find the average magnitude of the normal force in each case.

SOLVE

Landing speed:

$$mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh} = \sqrt{2\left(9.8\frac{\text{m}}{\text{s}^2}\right)(2\text{ m})} = 6.3\frac{\text{m}}{\text{s}}$$

Part a)

$$v^2 - v_0^2 = 2a_y(\Delta y)$$

$$a_y = \frac{v^2 - v_0^2}{2(\Delta y)} = \frac{0 - \left(6.3\frac{\text{m}}{\text{s}}\right)^2}{2(-0.010\text{ m})} = 1984.5\frac{\text{m}}{\text{s}^2}$$

$$\sum F_y = N - F_{\text{gravity}} = ma_y$$

$$N = F_{\text{gravity}} + ma_y = m(g + a_y) = (65\text{ kg})\left(\left(9.8\frac{\text{m}}{\text{s}^2}\right) + \left(1984.5\frac{\text{m}}{\text{s}^2}\right)\right) = \boxed{1.3 \times 10^5\text{ N}}$$

Part b)

$$v^2 - v_0^2 = 2a_y(\Delta y)$$

$$a_y = \frac{v^2 - v_0^2}{2(\Delta y)} = \frac{0 - \left(6.3\frac{\text{m}}{\text{s}}\right)^2}{2(-0.500\text{ m})} = 39.7\frac{\text{m}}{\text{s}^2}$$

$$\sum F_y = N - F_{\text{gravity}} = ma_y$$

$$N = F_{\text{gravity}} + ma_y = m(g + a_y) = (65\text{ kg})\left(\left(9.8\frac{\text{m}}{\text{s}^2}\right) + \left(39.7\frac{\text{m}}{\text{s}^2}\right)\right) = \boxed{3200\text{ N}}$$

REFLECT

People naturally bend their knees when landing in order to minimize the force of the impact and the likelihood of injury.

7.77

SET UP

Two cars are involved in a head-on collision. Car A weighs 1500 lb and was traveling eastward with an initial speed of v_A. Car B weighs 1100 lb and was traveling westward at an initial speed of $v_B = 45$ mph. The cars collide, stick together, and slide eastward for 19 ft before stopping. The coefficient of kinetic friction between the tires and the road is $\mu_k = 0.75$. We can use the work–kinetic energy theorem to calculate the speed of the wreckage just after the cars collide; the only force doing work on the cars is kinetic friction. Once we have the speed of the wreckage after the collision, we can use conservation of momentum to calculate what the speed of car A must have been.

SOLVE

Speed following collision:

$$\Delta K = W_{\text{net}} = W_{\text{kf}}$$

$$\frac{1}{2}(m_A + m_B)(v_f^2 - v_i^2) = -\mu_k(m_A + m_B)gd$$

$$0 - v_i^2 = -2\mu_k gd$$

$$v_i = \sqrt{2\mu_k gd} = \sqrt{2(0.75)\left(32\frac{\text{ft}}{\text{s}^2}\right)(19\text{ ft})} = 30.2\frac{\text{ft}}{\text{s}} \times \frac{3600\text{ s}}{1\text{ hr}} \times \frac{1\text{ mi}}{5280\text{ ft}} = 20.6\text{ mph}$$

Initial speed of car A:

$$m_A v_A - m_B v_B = (m_A + m_B)v_i$$

$$v_A = \frac{(m_A + m_B)v_i + m_B v_B}{m_A} = \frac{((1500\text{ lb}) + (1100\text{ lb}))(20.6\text{ mph}) + (1100\text{ lb})(45\text{ mph})}{1500\text{ lb}}$$

$$= \boxed{69\text{ mph}}$$

REFLECT

If the wreckage slides eastward, car A must have been traveling faster than car B when they collided.

7.78

SET UP

A train car ($m_t = 5000$ kg) is rolling at an initial speed of $v_i = 20.0$ m/s when, all of a sudden, rainwater ($m_w = 200$ kg) collects inside of it. We can calculate the final speed of the filled train car from conservation of momentum.

SOLVE

$$m_t v_i = (m_t + m_w)v_f$$

$$v_f = \frac{m_t}{m_t + m_w}v_i = \frac{5000 \text{ kg}}{(5000 \text{ kg}) + (200 \text{ kg})}\left(20.0\frac{\text{m}}{\text{s}}\right) = 19.2\frac{\text{m}}{\text{s}}$$

REFLECT

It makes sense that the train car should slow down once it fills up with water.

7.79

SET UP

A train car ($m_t = 8000$ kg) is rolling at an initial speed of $v_i = 20.0$ m/s when, all of a sudden, rainwater collects inside of it. The final speed of the filled train car is $v_f = 19$ m/s. We can calculate the mass of the rainwater from conservation of momentum.

SOLVE

$$m_t v_i = (m_t + m_w)v_f$$

$$m_w = \frac{m_t v_i}{v_f} - m_t = \frac{(8000 \text{ kg})\left(20.0\frac{\text{m}}{\text{s}}\right)}{\left(19\frac{\text{m}}{\text{s}}\right)} - (8000 \text{ kg}) = \boxed{421 \text{ kg}}$$

REFLECT

The speed of the train car decreased by about 5%, which means the mass must have increased by about 5%.

7.80

SET UP

An open rail car has an initial mass of 10,000 kg and an initial speed of 5 m/s. Rocks fall into the car at a rate of 500 kg/s for 3 s, which means the mass increases by 1500 kg. We can use conservation of momentum to find the final speed of the rail car filled with rocks.

SOLVE

$$m_i v_i = m_f v_f$$

$$v_f = \frac{m_i v_i}{m_f} = \frac{(10{,}000\text{ kg})\left(5\frac{\text{m}}{\text{s}}\right)}{(10{,}000\text{ kg}) + \left(500\frac{\text{kg}}{\text{s}}\right)(3\text{ s})} = \boxed{4.3\frac{\text{m}}{\text{s}}}$$

REFLECT

The final mass is larger, so we expect the rail car to slow down.

7.81

SET UP

A skier is initially traveling southward at a speed of $v_{1i} = 8$ m/s when he collides elastically with a snowboarder who is initially traveling westward at a speed of $v_{2i} = 12$ m/s. After they collide, the skier slides at an angle of 45 degrees south of west while the snowboarder slides at an angle of 45 degrees north of west. The skier and the snowboarder have the same mass. We can use conservation of momentum to calculate the final speed of each person. Since this is a two-dimensional problem, we will need to split the momenta into components and solve the x and y component equations. In our coordinate system east and north will point toward $+x$ and $+y$, respectively.

SOLVE

x component:

$$m_1 v_{1i,\,x} + m_2 v_{2i,\,x} = m_1 v_{1f,\,x} + m_2 v_{2f,\,x}$$

$$0 + \left(-12\frac{\text{m}}{\text{s}}\right) = -v_{1f}\cos(45°) - v_{2f}\cos(45°)$$

$$-12\frac{\text{m}}{\text{s}} = -\frac{v_{1f}}{\sqrt{2}} - \frac{v_{2f}}{\sqrt{2}}$$

y component:

$$m_1 v_{1i,\,y} + m_2 v_{2i,\,y} = m_1 v_{1f,\,y} + m_2 v_{2f,\,y}$$

$$\left(-8\frac{\text{m}}{\text{s}}\right) + 0 = -v_{1f}\sin(45°) + v_{2f}\sin(45°)$$

$$-8\frac{\text{m}}{\text{s}} = -\frac{v_{1f}}{\sqrt{2}} + \frac{v_{2f}}{\sqrt{2}}$$

Adding the two equations together:

$$-20\frac{\text{m}}{\text{s}} = -\frac{2v_{1f}}{\sqrt{2}}$$

$$v_{1f} = \left(10\frac{\text{m}}{\text{s}}\right)\sqrt{2} = \boxed{14\frac{\text{m}}{\text{s}}}$$

$$-8\frac{\text{m}}{\text{s}} = -\frac{\left(10\frac{\text{m}}{\text{s}}\right)\sqrt{2}}{\sqrt{2}} + \frac{v_{2\text{f}}}{\sqrt{2}}$$

$$v_{2\text{f}} = \boxed{3\frac{\text{m}}{\text{s}}}$$

REFLECT

The initial momentum of the system was fully aligned toward $-y$. There is a component of the final momentum of the system that points toward $+y$, which means the skier must slide faster than his initial speed.

7.82

SET UP

A curler slides a stone with an initial speed of 6.4 m/s toward a second, stationary stone of equal mass. We'll assume stone 1 is initially traveling toward $+x$ and that the two stones collide elastically. Stone 1 moves off at an angle of 120 degrees from its initial trajectory at a speed of $v_{1\text{f}}$, and stone 2 moves off at an angle θ relative to the horizontal and a speed of 5.6 m/s. We can use conservation of energy and conservation of momentum to calculate the final speed of stone 1 and the direction of stone 2.

SOLVE

Conservation of energy:

$$\frac{1}{2}m_1v_{1\text{i}}^2 = \frac{1}{2}m_1v_{1\text{f}}^2 + \frac{1}{2}m_1v_{2\text{f}}^2$$

$$v_{1\text{f}} = \sqrt{v_{1\text{i}}^2 - v_{2\text{f}}^2} = \sqrt{\left(6.4\frac{\text{m}}{\text{s}}\right)^2 - \left(5.6\frac{\text{m}}{\text{s}}\right)^2} = \boxed{3.1\frac{\text{m}}{\text{s}}}$$

Conservation of momentum, y component:

$$m_1v_{1\text{i},\,y} = m_1v_{1\text{f},\,y} + m_2v_{2\text{f},\,y}$$

$$0 = v_{1\text{f}}\sin(120°) + v_{2\text{f}}\sin(\theta)$$

$$\theta = \arcsin\left(\frac{-v_{1\text{f}}\sin(120°)}{v_{2\text{f}}}\right) = \arcsin\left(\frac{-\left(3.1\frac{\text{m}}{\text{s}}\right)\sin(120°)}{\left(5.6\frac{\text{m}}{\text{s}}\right)}\right) = -29°$$

$$\boxed{\theta = 29° \text{ below the horizontal}}$$

REFLECT

It makes sense that stone 2 is traveling toward $-y$ and at a faster speed than stone 1 after the collision.

7.83

SET UP

A lion ($m_L = 135$ kg) is running northward at 80 km/hr when it collides with and latches onto a gazelle ($m_G = 29$ kg) that is running eastward at 60 km/hr. In our coordinate system east will point toward $+x$ and north will point toward $+y$. We can use conservation of momentum to find the final speed and direction of the lion–gazelle system after the lion attacks. Since this is a two-dimensional problem, we will need to split the momenta into components and solve the x and y component equations.

SOLVE

x component:

$$m_L v_{L,x} + m_G v_{G,x} = (m_L + m_G)v_{f,x}$$

$$v_{f,x} = \frac{m_L v_{L,x} + m_G v_{G,x}}{(m_L + m_G)} = \frac{0 + (29\text{ kg})\left(60\frac{\text{km}}{\text{hr}}\right)}{(135\text{ kg}) + (29\text{ kg})} = 11\frac{\text{km}}{\text{hr}}$$

y component:

$$m_L v_{L,y} + m_G v_{G,y} = (m_L + m_G)v_{f,y}$$

$$v_{f,y} = \frac{m_L v_{L,y} + m_G v_{G,y}}{(m_L + m_G)} = \frac{(135\text{ kg})\left(80\frac{\text{km}}{\text{hr}}\right) + 0}{(135\text{ kg}) + (29\text{ kg})} = 66\frac{\text{km}}{\text{hr}}$$

Final speed:

$$v = \sqrt{v_{f,x}^2 + v_{f,y}^2} = \sqrt{\left(11\frac{\text{km}}{\text{hr}}\right)^2 + \left(66\frac{\text{km}}{\text{hr}}\right)^2} = \boxed{67\frac{\text{km}}{\text{hr}}}$$

Final direction:

$$\theta = \arctan\left(\frac{66}{11}\right) = \boxed{80.5^\circ\text{ north of east}}$$

REFLECT

The momentum of the lion is considerably larger than the momentum of the gazelle, so the final momentum of the system should point more north than east.

7.84

SET UP

A pigeon is flying northward at a speed of $v_P = 23$ m/s. A pigeon hawk is flying north at a speed of $v_H = 35$ m/s at an angle of 45 degrees below the horizontal when it latches onto the pigeon. The two birds then travel off together with the same velocity. We can use conservation of momentum to find the final velocity of the pigeon–hawk system after the hawk attacks. Since this is a two-dimensional problem, we will need to split the momenta into components and solve the x and y component equations separately. In our coordinate system north will point toward $+x$ and up will point toward $+y$.

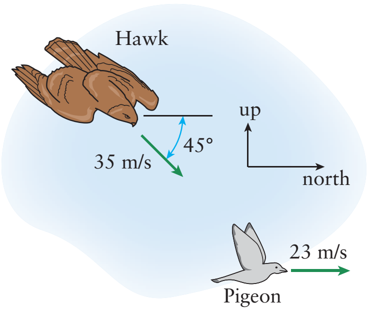


Figure 7-8 Problem 84

SOLVE

x component:

$$m_{\mathrm{H}}v_{\mathrm{H},x} + m_{\mathrm{P}}v_{\mathrm{P},x} = (m_{\mathrm{H}} + m_{\mathrm{P}})v_{\mathrm{f},x}$$

$$v_{\mathrm{f},x} = \frac{m_{\mathrm{H}}v_{\mathrm{H},x} + m_{\mathrm{P}}v_{\mathrm{P},x}}{(m_{\mathrm{H}} + m_{\mathrm{P}})} = \frac{(2m_{\mathrm{P}})v_{\mathrm{H},x} + m_{\mathrm{P}}v_{\mathrm{P},x}}{(2m_{\mathrm{P}} + m_{\mathrm{P}})} = \frac{2\left(35\frac{\mathrm{m}}{\mathrm{s}}\right)\cos(45°) + \left(23\frac{\mathrm{m}}{\mathrm{s}}\right)}{3} = 24.2\frac{\mathrm{m}}{\mathrm{s}}$$

y component:

$$-m_{\mathrm{H}}v_{\mathrm{H},y} + m_{\mathrm{P}}v_{\mathrm{P},y} = (m_{\mathrm{H}} + m_{\mathrm{P}})v_{\mathrm{f},y}$$

$$v_{\mathrm{f},y} = \frac{-m_{\mathrm{H}}v_{\mathrm{H},y} + m_{\mathrm{P}}v_{\mathrm{P},y}}{(m_{\mathrm{H}} + m_{\mathrm{P}})} = \frac{-(2m_{\mathrm{P}})v_{\mathrm{H},y} + 0}{(2m_{\mathrm{P}} + m_{\mathrm{P}})} = \frac{-2\left(35\frac{\mathrm{m}}{\mathrm{s}}\right)\sin(45°)}{3} = -16.5\frac{\mathrm{m}}{\mathrm{s}}$$

Final velocity:

$$\boxed{\vec{v}_{\mathrm{f}} = \left(24.2\frac{\mathrm{m}}{\mathrm{s}}\right)\hat{x} - \left(16.5\frac{\mathrm{m}}{\mathrm{s}}\right)\hat{y}}$$

REFLECT

We know the final velocity of the two birds must point down and north by considering the initial velocities.

7.85

SET UP

A bullet ($m_{\mathrm{B}} = 0.012$ kg) is shot into a block of wood of mass m_{W} at a speed of $v_{\mathrm{B\,i}} = 250$ m/s. The wood is attached to a spring with a spring constant $k = 200$ N/m. Once the bullet embeds itself into the wood, the spring compresses a distance of $x = 30$ cm before coming to a stop. We first need to use conservation of momentum in order to calculate the speed of the (bullet + wood) after the collision in terms of m_{W}. Then we can use conservation of mechanical energy to solve for the numerical value of m_{W}.

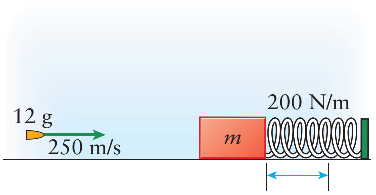


Figure 7-9 Problem 85

SOLVE

Conservation of momentum:

$$m_B v_{B,i} + m_W v_{W,i} = (m_B + m_W)v_f$$

$$v_f = \frac{m_B v_{B,i} + m_W v_{W,i}}{m_B + m_W} = \frac{m_B v_{B,i} + 0}{m_B + m_W} = \frac{m_B v_{B,i}}{m_B + m_W}$$

Conservation of mechanical energy:

$$U_i + K_i = U_f + K_f$$

$$0 + \frac{1}{2}(m_B + m_W)v_f^2 = \frac{1}{2}kx^2 + 0$$

$$(m_B + m_W)v_f^2 = kx^2$$

$$(m_B + m_W)\left(\frac{m_B v_{B,i}}{m_B + m_W}\right)^2 = kx^2$$

$$\frac{1}{(m_B + m_W)}(m_B v_{B,i})^2 = kx^2$$

$$m_W = \frac{(m_B v_{B,i})^2}{kx^2} - m_B = \frac{(0.012\text{ kg})^2\left(250\frac{\text{m}}{\text{s}}\right)^2}{\left(200\frac{\text{N}}{\text{m}}\right)(0.30\text{ m})^2} - (0.012\text{ kg}) = \boxed{0.49\text{ kg}}$$

REFLECT

The block of wood with the embedded bullet moves at a speed of 5.9 m/s.

7.86

SET UP

A marble ($m_M = 0.0075$ kg) is shot into a ballistic pendulum ($m_P = 0.250$ kg) at a speed of $v_{M,i} = 6$ m/s. The pendulum and the marble then swing together and rise to a final height y_f. We first need to use conservation of momentum in order to calculate the speed of the (pendulum + marble) after the collision. Then we can use conservation of mechanical energy to calculate the final height that the pendulum reaches.

SOLVE

Conservation of momentum:

$$m_M v_{M,i} + m_P v_{P,i} = (m_M + m_P)v_f$$

$$v_f = \frac{m_M v_{M,i} + m_P v_{P,i}}{m_M + m_P} = \frac{m_M v_{M,i} + 0}{m_M + m_P} = \frac{(0.0075\text{ kg})\left(6\frac{\text{m}}{\text{s}}\right)}{(0.0075\text{ kg}) + (0.250\text{ kg})} = 0.175\frac{\text{m}}{\text{s}}$$

Conservation of mechanical energy:

$$U_{\mathrm{i}} + K_{\mathrm{i}} = U_{\mathrm{f}} + K_{\mathrm{f}}$$

$$0 + \frac{1}{2}(m_{\mathrm{M}} + m_{\mathrm{P}})v_{\mathrm{f}}^2 = (m_{\mathrm{M}} + m_{\mathrm{P}})gy_{\mathrm{f}} + 0$$

$$y_{\mathrm{f}} = \frac{v_{\mathrm{f}}^2}{2g} = \frac{\left(0.175\frac{\mathrm{m}}{\mathrm{s}}\right)^2}{2\left(9.8\frac{\mathrm{m}}{\mathrm{s}^2}\right)} = \boxed{0.0016\ \mathrm{m} = 1.6\ \mathrm{mm}}$$

REFLECT

We would not expect the pendulum to swing very far since the mass of the pendulum is 33 times larger than the mass of the marble.

7.87

SET UP

We are given the equation for the z component of the force acting on an object. The integral of this force with respect to time from $t = 0$ to $t =$ infinity will give us the change in the object's momentum over that time interval. We are told the object starts from rest, which means its initial momentum is equal to zero.

SOLVE

$$\Delta p_z = p_{\mathrm{f},z} - p_{\mathrm{i},z} = p_{\mathrm{f},z} - 0 = \int_0^\infty F_z(t)dt$$

$$p_{\mathrm{f},z} = \int_0^\infty F_z(t)dt = \int_0^\infty t^2 e^{-t^2} dt = \boxed{\sqrt{\frac{\pi}{16}}\frac{\mathrm{kg}\cdot\mathrm{m}}{\mathrm{s}} = 0.44\frac{\mathrm{kg}\cdot\mathrm{m}}{\mathrm{s}}}$$

REFLECT

The indefinite integral of the function $t^2e^{-t^2}$ is not well-defined, but its definite integral from $t = 0$ to $t =$ infinity is.

7.88

SET UP

We are given the x component of the momentum of a particle as a function of time. The net force acting on the particle in the x direction is equal to the derivative of the x component of the momentum with respect to time. We will need to invoke the chain rule when calculating the derivative of the sine term. We can evaluate this derivative at $t = 2$ s to find the net force at that instant.

SOLVE

Arbitrary t:

$$\sum F_x = \frac{dp_x}{dt} = \frac{d}{dt}[3\sin(5t) + 2t^2] = \boxed{15\cos(5t) + 4t}$$

At $t = 2$ s:

$$\sum F_x\big|_{t=2\text{ s}} = 15\cos(5(2)) + 4(2) = 15\cos(10) + 8 = \boxed{-4.6\text{ N}}$$

REFLECT

For small t, the oscillatory nature of the motion (due to the sine function) will be more pronounced. As t becomes larger, the t^2 term becomes more significant.

7.89

SET UP

We are given the y component of the momentum of a particle as a function of time. The net force acting on the particle in the y direction is equal to the derivative of the y component of the momentum with respect to time. We will need to invoke both the product rule and the chain rule when calculating the derivative of the first term.

SOLVE

$$\sum F_y = \frac{dp_y}{dt} = \frac{d}{dt}[3e^{-t}\cos(5t) + 3t^{-1/2}] = 3\frac{d}{dt}[e^{-t}\cos(5t) + t^{-1/2}]$$

$$= 3\left[-e^{-t}\cos(5t) + e^{-t}(-5\sin(5t)) - \frac{1}{2}t^{-3/2}\right] = \boxed{-3\left[e^{-t}\cos(5t) + 5e^{-t}\sin(5t) + \frac{1}{2}t^{-3/2}\right]}$$

REFLECT

The net force takes the shape of a damped oscillatory function that goes to zero as t goes to infinity.

7.90

SET UP

An object ($m_1 = 2.5$ kg) is initially traveling at an unknown speed v_1 toward a second stationary object ($m_2 = 3.6$ kg). After they collide, m_1 has a speed of 4 m/s and travels off at an angle θ_1 above the horizontal and m_2 has a speed of 2.5 m/s and travels off at an angle θ_2 below the horizontal. Assuming that the collision is elastic, we can use conservation of energy to calculate v_1. We can calculate the two unknown angles by applying conservation of momentum.

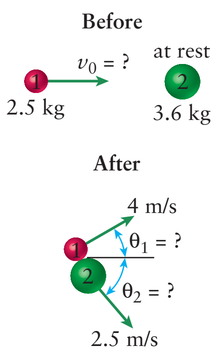


Figure 7-10 Problem 90

SOLVE

Conservation of energy:

$$m_1 v_0^2 = m_1 v_{1f}^2 + m_2 v_{2f}^2$$

$$v_0 = \sqrt{\frac{m_1 v_{1f}^2 + m_2 v_{2f}^2}{m_1}} = \sqrt{\frac{(2.5\ \text{kg})\left(4\frac{\text{m}}{\text{s}}\right)^2 + (3.6\ \text{kg})\left(2.5\frac{\text{m}}{\text{s}}\right)^2}{2.5\ \text{kg}}} = \boxed{5\frac{\text{m}}{\text{s}}}$$

Conservation of momentum, x component (numbers in SI units):

$$m_1 v_0 = m_1 v_1 \cos(\theta_1) + m_2 v_2 \cos(\theta_2)$$

$$(2.5)(5) = (2.5)(4)\cos(\theta_1) + (3.6)(2.5)\cos(\theta_2)$$

$$5 = 4\cos(\theta_1) + 3.6\cos(\theta_2)$$

$$4\cos(\theta_1) = 5 - 3.6\cos(\theta_2)$$

Conservation of momentum, y component (numbers in SI units):

$$0 = m_1 v_1 \sin(\theta_1) - m_2 v_2 \sin(\theta_2)$$

$$0 = (2.5)(4)\sin(\theta_1) - (3.6)(2.5)\sin(\theta_2)$$

$$4\sin(\theta_1) = 3.6\sin(\theta_2)$$

Solving for θ_2:

$$(4\sin(\theta_1))^2 + (4\cos(\theta_1))^2 = (3.6\sin(\theta_2))^2 + (5 - 3.6\cos(\theta_2))^2$$

$$16 = (3.6)^2\sin^2(\theta_2) + 25 + (3.6)^2\cos^2(\theta_2) - 36\cos(\theta_2)$$

$$16 = (3.6)^2 + 25 - 36\cos(\theta_2)$$

$$\cos(\theta_2) = 0.61$$

$$\theta_2 = \arccos(0.61) = \boxed{52.4°}$$

Solving for θ_1:

$$\sin(\theta_1) = \frac{3.6\sin(\theta_2)}{4}$$

$$\theta_1 = \arcsin\left(\frac{3.6\sin(\theta_2)}{4}\right) = \arcsin\left(\frac{3.6\sin(52.4°)}{4}\right) = \boxed{45.5°}$$

REFLECT

In an elastic collision between two objects of the same mass, the sum of their scattering angles must be 90 degrees.

7.91

SET UP

A 0.075-kg ball is thrown at 25 m/s toward a brick wall. We will call the direction toward the wall $+x$. The impulse that the wall imparts to the ball when it hits is equal to the change in the ball's momentum. The force the wall exerts on the ball is equal to the impulse imparted to the ball divided by the contact time.

SOLVE

Part a)

$$\Delta p_x = p_{\mathrm{f},x} - p_{\mathrm{i},x} = m(v_{\mathrm{f},x} - v_{\mathrm{i},x}) = (0.075\text{ kg})\left(\left(-25\frac{\text{m}}{\text{s}}\right) - \left(25\frac{\text{m}}{\text{s}}\right)\right) = \boxed{-3.75\frac{\text{kg}\cdot\text{m}}{\text{s}}}$$

Part b)

$$\Delta p_x = p_{\mathrm{f},x} - p_{\mathrm{i},x} = m(v_{\mathrm{f},x} - v_{\mathrm{i},x}) = m(-v_{\mathrm{f}}\cos(45°) - v_{\mathrm{i}}\cos(45°))$$

$$= (0.075\text{ kg})\left(-\left(25\frac{\text{m}}{\text{s}}\right)\cos(45°) - \left(25\frac{\text{m}}{\text{s}}\right)\cos(45°)\right) = -2.65\frac{\text{kg}\cdot\text{m}}{\text{s}}$$

$$\Delta p_y = p_{\mathrm{f},y} - p_{\mathrm{i},y} = m(v_{\mathrm{f},y} - v_{\mathrm{i},y}) = m(v_{\mathrm{f}}\sin(45°) - v_{\mathrm{i}}\sin(45°)) = 0$$

The total impulse the wall imparts to the ball is $\boxed{-2.65\frac{\text{kg}\cdot\text{m}}{\text{s}}}$.

Part c)

$$\sum F_x = \frac{\Delta p_x}{\Delta t} = \frac{-2.65\frac{\text{kg}\cdot\text{m}}{\text{s}}}{0.01\text{ s}} = -265\text{ N}$$

$$\boxed{\vec{F} = -(265\text{ N})\hat{x}}$$

REFLECT

A net force changes the momentum of an object. Since the momentum in the y direction did not change, there is no force acting in that direction.

7.92

SET UP

A uniform sheet of metal of total mass M is formed into a right triangle that is 12 m long in the x direction and 5 m long in the y direction. Since this is a solid, continuous object, we need to use calculus to find the center of mass coordinates of the triangle: $x_{\mathrm{CM}} = \dfrac{\int x\,dm}{M}$ and $y_{\mathrm{CM}} = \dfrac{\int y\,dm}{M}$. We split the triangle up into small rectangles of area dA. We can convert the integral from dm to dA through the density.

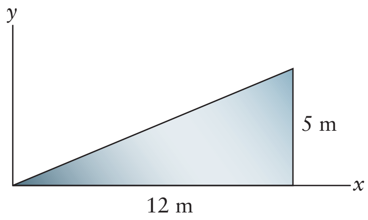


Figure 7-11 Problem 92

SOLVE

x component:

$$\frac{M}{A} = \frac{dm}{dA}$$

$$dm = \frac{M}{A}dA = \frac{M}{\left(\frac{1}{2}\right)(5)(12)}(ydx) = \frac{My}{30}dx$$

But, through similar triangles,

$$\frac{y}{x} = \frac{5}{12}$$

Therefore,

$$dm = \frac{My}{30}dx = \frac{M}{30}\left(\frac{5}{12}x\right)dx = \frac{M}{72}xdx$$

$$x_{\text{CM}} = \frac{1}{M}\int xdm = \frac{1}{M}\int_0^{12} x\left(\frac{M}{72}x\right)dx = \frac{1}{72}\int_0^{12} x^2dx = \frac{1}{72}\left[\frac{x^3}{3}\right]_0^{12} = \frac{12^3}{216} = \boxed{8\text{ m}}$$

y component:

$$\frac{M}{A} = \frac{dm}{dA}$$

$$dm = \frac{M}{A}dA = \frac{M}{\left(\frac{1}{2}\right)(5)(12)}(xdy) = \frac{Mx}{30}dy$$

But, through similar triangles,

$$\frac{5-y}{x} = \frac{5}{12}$$

Therefore,

$$dm = \frac{Mx}{30}dy = \frac{M}{30}\left(\frac{12}{5}\right)(5-y)dy = \frac{2M}{25}(5-y)dy$$

$$y_{\text{CM}} = \frac{1}{M}\int ydm = \frac{1}{M}\int_0^5 y\left(\frac{2M}{25}\right)(5-y)dy = \frac{2}{25}\int_0^5 (5y-y^2)dy = \frac{2}{25}\left[\frac{5y^2}{2} - \frac{y^3}{3}\right]_0^5$$

$$= \frac{2}{25}\left[\frac{5(5)^2}{2} - \frac{(5)^3}{3}\right] = \left[5 - \frac{10}{3}\right] = \boxed{1.67\text{ m}}$$

REFLECT

In general, the centroid of a right triangle is located one-third of the distance in each direction from the right angle.

7.93

SET UP

A uniform sheet of metal of total mass M is formed into an isosceles triangle that is 1 m long in the x direction and 2 m long in the y direction. Since this is a solid, continuous object, we need to use calculus to find the center of mass coordinates of the triangle: $x_{\text{CM}} = \dfrac{\int x\,dm}{M}$ and $y_{\text{CM}} = \dfrac{\int y\,dm}{M}$. We split the triangle up into small rectangles of area dA. We can convert the integral from dm to dA through the density.

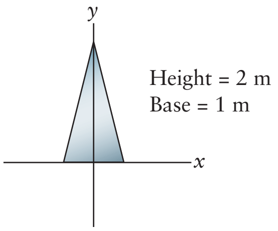


Figure 7-12 Problem 93

SOLVE

x component:

The triangle is symmetric about the y-axis; therefore,

$$\boxed{x_{\text{CM}} = 0}$$

y component:

$$\frac{M}{A} = \frac{dm}{dA}$$

$$dm = \frac{M}{A}dA = \frac{M}{\left(\frac{1}{2}\right)(1)(2)}(2x\,dy) = 2Mx\,dy$$

But, through similar triangles,

$$\frac{2-y}{x} = \frac{2}{0.5}$$

Therefore,

$$dm = 2Mx\,dy = 2M\left(\frac{2-y}{4}\right)dy = \frac{M}{2}(2-y)dy$$

$$y_{\text{CM}} = \frac{1}{M}\int y\,dm = \frac{1}{M}\int_0^2 My\frac{(2-y)}{2}dy = \frac{1}{2}\int_0^2 (2y-y^2)dy = \frac{1}{2}\left[y^2 - \frac{y^3}{3}\right]_0^2$$

$$= \frac{1}{2}\left[(2)^2 - \frac{(2)^3}{3}\right] = \left[2 - \frac{4}{3}\right] = \boxed{0.67\text{ m}}$$

REFLECT

More of the mass is located near the bottom of the triangle, so we would expect y_{CM} to be less than half the height.

7.94

SET UP

A thin trapezoidal plate has a mass M, a larger base length L, a shorter base length $L/2$, and a height H. Since this is a solid, continuous object, we need to use calculus to find the center of mass coordinates of the triangle: $x_{CM} = \frac{\int x dm}{M}$ and $y_{CM} = \frac{\int y dm}{M}$. We split the triangle up into small rectangles of area dA. We can convert the integral from dm to dA through the density. To make the math easier, we will flip the trapezoid upside-down, such that the smaller base is on the bottom.

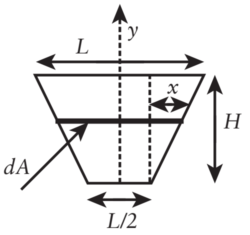


Figure 7-13 Problem 94

SOLVE

x component:

The trapezoid is symmetric about the y-axis; therefore,

$$\boxed{x_{CM} = 0}$$

y component:

$$y_{CM} = \frac{1}{M}\int y dm = \frac{1}{M}\int y(\sigma dA) = \frac{1}{M}\int_0^H y\sigma\left(2\left(\frac{L}{4} + x\right)dy\right) = \frac{2}{M}\int_0^H y\left(\frac{M}{\left(\frac{3}{4}HL\right)}\right)\left(\frac{L}{4} + x\right)dx$$

$$= 2\int_0^H y\left(\frac{4}{3HL}\right)\left(\frac{L}{4} + \left(\frac{Ly}{4H}\right)\right)dx = 2\left(\frac{4}{3HL}\right)\left(\frac{L}{4}\right)\int_0^H y\left(1 + \frac{y}{H}\right)dx$$

$$= 2\left(\frac{1}{3H}\right)\left[\frac{1}{2}y^2 + \frac{1}{3H}y^3\right]_0^H = \frac{2}{3H}\left[\frac{H^2}{2} + \frac{H^3}{3H}\right] = \frac{2}{3H}\left[\frac{5H^2}{6}\right] = \frac{5}{9}H$$

The center of mass of the trapezoid is located $5H/9$ from its top, which means it is located $\boxed{4H/9 \text{ from its bottom}}$.

REFLECT

There is more mass located in the lower portion of the trapezoid, so the center of mass should be located closer to the bottom than the top.

7.95

SET UP

We are given the vector describing the force acting on an object as a function of time. The integral of this force with respect to time will give us the change in the object's momentum. We are told the object starts from rest, which means its initial momentum is equal to zero.

SOLVE

$$\Delta\vec{p} = \vec{p}(t) - \vec{p}_{\mathrm{i}} = \vec{p}(t) - 0 = \int_0^t F(t)dt = \int_0^t \left[\left(\frac{14}{t^2+1}\right)\hat{x} + \left(\frac{12}{t^2-1}\right)\hat{y}\right]dt$$

$$= [14\arctan(t)]_0^t\hat{x} + [-12\ \mathrm{arctanh}(t)]_0^t\hat{y}$$

$$= \boxed{[14\arctan(t)]\hat{x} - [12\ \mathrm{arctanh}(t)]\hat{y}}$$

REFLECT

The antiderivative of $\dfrac{1}{t^2+1}$ is $\arctan(t)$, and the antiderivative of $\dfrac{1}{t^2-1}$ is $-\mathrm{arctanh}(t)$.

7.96

SET UP

The mass of an oxygen nucleus is 16 times that of a proton. A proton is initially traveling at a speed of 5×10^5 m/s toward an oxygen nucleus at rest. The proton collides with the nucleus elastically and leaves the interaction with a speed of 5×10^5 m/s. Since the initial and final kinetic energies of the proton are the same, none of the initial kinetic energy is transferred to the oxygen nucleus. Given the relative masses of the two objects, we would expect the proton to "bounce off" the oxygen nucleus and travel back the way it came after the collision.

SOLVE

Part a) Since the initial kinetic energy of the proton is equal to the final kinetic energy of the proton, $\boxed{\text{no energy is transferred}}$ to the oxygen nucleus.

Part b) The $\boxed{\text{oxygen nucleus remains in place}}$, while the $\boxed{\text{proton travels back the way it came}}$.

REFLECT

An oxygen nucleus is much more massive than a proton, so we expect the oxygen to remain at rest.

7.97

SET UP

A circular hole of radius $R/4$ is cut from a thin, uniform disk of mass M and radius R. The disk is centered about the origin, and the hole is centered at $x = R/2$ and $y = 0$. We can find the center of mass of this object by considering it as two parts—the disk and the hole—where the "mass" of the hole is negative. The object is uniform, which means its density is constant; from this we can relate the "mass" of the hole to the mass of the disk.

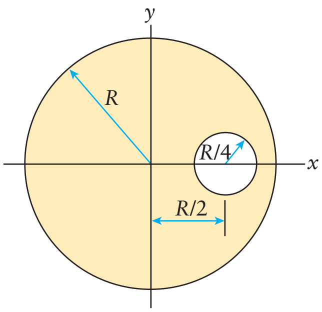


Figure 7-14 Problem 97

SOLVE

"Mass" of the hole:

$$\frac{m_{\text{disk}}}{A_{\text{disk}}} = \frac{m_{\text{hole}}}{A_{\text{hole}}}$$

$$m_{\text{hole}} = \frac{m_{\text{disk}} A_{\text{hole}}}{A_{\text{disk}}} = \frac{M\left(\pi\left(\frac{R}{4}\right)^2\right)}{\pi R^2} = \frac{M}{16}$$

x component:

$$x_{\text{CM}} = \frac{m_{\text{disk}} x_{\text{disk}} - m_{\text{hole}} x_{\text{hole}}}{m_{\text{total}}} = \frac{M(0) - \left(\frac{M}{16}\right)\left(\frac{R}{2}\right)}{\left(M - \frac{M}{16}\right)} = \frac{\left(-\frac{MR}{32}\right)}{\left(\frac{15M}{16}\right)} = \boxed{-\frac{R}{30}}$$

y component: The object is symmetric about the x-axis.

Therefore,

$$\boxed{y_{\text{CM}} = 0} \text{ from symmetry}$$

REFLECT

There is more mass on the left side of the object than the right side, so the x component of the center of mass should be negative.

7.98

SET UP

We are asked to find the center of mass of a solid hemisphere of mass M and radius R. We'll assume the hemisphere has a uniform density ρ and is centered about the z-axis. From symmetry, we know the center of mass must lie along the z-axis. Since the hemisphere is a continuous object, we will need to use the integral from the center of mass definition: $z_{\text{CM}} = \frac{\int z\,dm}{\int dm}$. We can convert the integral from dm to dV by means of the density. The denominator becomes the volume of a hemisphere, which is $\frac{2}{3}\pi R^3$. The numerator becomes a triple integral after we write the volume element in terms of spherical coordinates; the limits of integration are $r = 0$ to $r = R$, $\theta = 0$ to $\theta = \pi/2$, and $\phi = 0$ to $\phi = 2\pi$. As a reminder, in spherical coordinates, $z = r\cos(\theta)$ and $dV = r^2\sin(\theta)dr d\theta d\phi$.

SOLVE

$$z_{\text{CM}} = \frac{\int z\,dm}{\int dm} = \frac{\int z\rho\,dV}{\int \rho\,dV} = \frac{\int z\,dV}{\int dV} = \frac{\iiint (r\cos(\theta))r^2\sin(\theta)\,dr\,d\theta\,d\phi}{\left(\frac{2}{3}\pi R^3\right)}$$

$$= \frac{3}{2\pi R^3}\int_0^R r^3dr\int_0^{\pi/2}\sin(\theta)\cos(\theta)\,d\theta\int_0^{2\pi}d\phi$$

$$= \frac{3}{2\pi R^3}\left[\frac{1}{4}r^4\right]_0^R\left[-\frac{1}{2}\cos^2(\theta)\right]_0^{\pi/2}[\phi]_0^{2\pi} = \frac{3}{8\pi R^3}[R^4]\left(-\frac{1}{2}\right)\left[\cos^2\left(\frac{\pi}{2}\right) - \cos^2(0)\right][2\pi]$$

$$= -\frac{3R}{8}[0-1] = \boxed{\frac{3}{8}R}$$

REFLECT

More of the mass is located toward the base of the hemisphere, so it makes sense that the center of mass should be located less than $R/2$ from the base.

7.99

SET UP

A rocket accelerates by expelling stored gases from its tail section. In a time interval of Δt, the rocket ejects gas of mass Δm with a speed of v_g relative to the rocket. The rocket and the gas form an isolated system, which means the total momentum of the system will be constant. Starting with momentum conservation, we can differentiate both sides and solve for the net force acting on the rocket, which is equal to the time derivative of the rocket's momentum. We'll consider the forward direction to be positive.

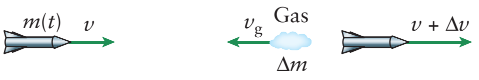


Figure 7-15 Problem 99

SOLVE

$$p_{\text{i}} = p_{\text{gas, f}} + p_{\text{rocket, f}}$$

$$\frac{dp_{\text{i}}}{dt} = 0 = \frac{dp_{\text{gas, f}}}{dt} + \frac{dp_{\text{rocket, f}}}{dt}$$

$$\frac{dp_{\text{rocket, f}}}{dt} = \sum F_{\text{rocket}} = -\frac{dp_{\text{gas, f}}}{dt} = -\frac{d}{dt}(-(\Delta m)v_g)$$

Over a finite time interval Δt, this becomes:

$$\sum F_{\text{rocket}} = \boxed{\frac{(\Delta m)v_g}{\Delta t}}$$

REFLECT

The net force acting on the rocket is positive, as expected. We also see that the change in the rocket's momentum with respect to time is equal to the negative of the change in the gases' momentum with respect to time; this is a statement of Newton's third law.

Chapter 8
Rotational Motion

Conceptual Questions

8.1 Rotation refers to an object spinning about its own axis. Revolution refers to one object rotating about another object. In terms of the solar system, the Earth spinning about its axis in one day is an example of rotational motion, whereas the Earth circling the Sun in a year is an example of orbital motion.

8.2 The SI units of angular velocity are radians per second. The factor of 2π arises from the conversion between revolutions and radians, namely, 1 revolution $= 2\pi$ radians.

8.3 The moment of inertia of a rotating object is calculated relative to the axis of rotation. If we don't explicitly define this axis at the outset of the problem, the calculated moment of inertia will be ambiguous at best and meaningless at worst. Even for axes crossing the center of mass, different axes will have different moments.

8.4 An object moving in a straight line can have a nonzero angular momentum as long as it does not travel through the rotation axis.

8.5 (a) The SI unit for rotational kinetic energy is the joule. (b) The SI unit for moment of inertia is (kilogram)(meter)^2. (c) The SI unit for angular momentum is (kilogram)(meter)^2/(second).

8.6 An ice skater is changing the way her mass is distributed and, therefore, changing her moment of inertia when she moves her arms in and out. Since there are no external torques acting on the skater once she is in the pirouette, angular momentum is conserved. This is why her angular velocity changes.

8.7 When riding a seesaw, the torque produced by each rider about the pivot should be approximately equal in magnitude. The riders are trying to move their centers of mass relative to the pivot of the seesaw by leaning forward or backward. This, in turn, will affect the magnitude of the torque about the pivot produced by each rider by changing the moment arms.We are told that the riders are sitting equidistant from the pivot point and the person at the top always leans backward, which means she has a smaller mass than the person at the bottom (who always leans forward).

8.8 We can use Table 8-1 to look up the moments of inertia for the various shapes listed. A cube is a rectangular parallelepiped in which all the sides are equal in length:

$$I_{\text{cube}} = \frac{1}{12}M((2R)^2 + (2R)^2) = \frac{1}{12}M(4R^2 + 4R^2) = \frac{2}{3}MR^2.$$

Ranking of moments of inertia from greatest to least:

$$(I_{\text{hoop}} = MR^2) > \left(I_{\text{cube}} = \frac{2}{3}MR^2\right) = \left(I_{\text{hollow sphere}} = \frac{2}{3}MR^2\right) > \left(I_{\text{solid cylinder}} = \frac{1}{2}MR^2\right) > \left(I_{\text{solid sphere}} = \frac{2}{5}MR^2\right).$$

8.9 Yes, the cat will fall on its feet, but it will have to undergo a more radical twisting motion of its torso to compensate for its missing tail. The tail, although small, has a large moment arm and that is why it is critical to the cat as it attempts to produce large torques and angular momenta during its fall.

8.10 The rotational kinetic energy, K_{rot}, of a point mass M rotating about an axis R away at an angular speed of ω is equal to $\frac{1}{2}I\omega^2 = \frac{1}{2}(MR^2)\omega^2$. Therefore, the first ball has a rotational kinetic energy of $K_{\text{rot, ball 1}} = \frac{1}{2}(mL^2)\omega^2$, and the second ball has a rotational kinetic energy of $K_{\text{rot, ball 2}} = \frac{1}{2}((2m)(2L)^2)(2\omega)^2 = 32\left(\frac{1}{2}(mL^2)\omega^2\right) = 32K_{\text{rot, ball 1}}$.

8.11 The term *dm* represents a tiny piece of mass from a large-scale object.

8.12 Technically, a newton-meter *is* a joule, but the standard is to call the SI units of torque newton-meter rather than joule to highlight the difference between torque and energy.

8.13 It depends *where* on the door the student is pushing. If she is pushing exactly on the axis of the door's hinges, she will be exerting zero torque on the door and it will not open.

8.14 An external force is doing work to pull the ball inward, which means the rotational kinetic energy will not remain constant. Since the mass is changing its location with respect to the axis of rotation, the moment of inertia will change as well. Accordingly, the angular speed should also change.

8.15 If we consider the system to be the cookie dough + turntable, there is **no net torque** acting on the system, and the **rotational kinetic energy remains constant** since there is no net work done on the system. This means the **angular momentum of the system is constant**. Dropping the cookie dough onto the edge of the turntable changes the way the mass in the system is distributed about the axis of rotation, thus **changing the moment of inertia.** Since the angular momentum is constant and the moment of inertia of the system changes, the **angular velocity of the system must change** after the cookie dough lands. (The moment of inertia increases. Therefore, the **magnitude of the angular velocity decreases**; that is, the turntable slows down.) A changing angular velocity means there is a **nonzero angular acceleration**.

8.16 A torque wrench is a tool used to tighten nuts onto bolts to a prescribed amount. A scale on the wrench shows the amount of torque being delivered to the nut as it is turned onto the bolt. A Canadian torque wrench is most likely expressed in units of

newton-meters, whereas an American torque wrench will be in the units of foot-pounds. The mechanic needs to make sure the wrench is calibrated in units he expects.

8.17 Translational motion ⇔ Rotational motion

$$x = x_0 + v_{0,x}t + \frac{1}{2}a_x t^2 \Leftrightarrow \theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$v = v_0 + at \Leftrightarrow \omega = \omega_0 + \alpha t$$

$$v^2 = v_0^2 + 2a(x - x_0) \Leftrightarrow \omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$\sum F = ma \Leftrightarrow \sum \tau = I\alpha$$

$$p = mv \Leftrightarrow L = I\omega$$

$$K = \frac{1}{2}mv^2 \Leftrightarrow K_{\text{rot}} = \frac{1}{2}I\omega^2$$

8.18 The moment of inertia of an object determines how that object responds to an applied torque.

8.19 We can use conservation of energy to answer this question. All three objects—a sphere, a cylinder, and a ring—start at rest from the same height and, thus, have the same initial potential energy. As the objects roll down the inclined plane, potential energy is converted into rotational *and* translational kinetic energy. The smaller the moment of inertia of an object, the more energy that will be converted into translational kinetic energy rather than rotational kinetic energy. The object that reaches the bottom of the ramp has the largest translational kinetic energy, so the sphere $\left(I_{\text{sphere}} = \frac{2}{5}MR^2\right)$ "wins," followed by the cylinder $\left(I_{\text{cylinder}} = \frac{1}{2}MR^2\right)$, and then the hoop $(I_{\text{hoop}} = MR^2)$.

8.20 The angular momentum is proportional to the moment of inertia, and the moment of inertia is proportional to the *square* of the distance to the rotation axis. The distance from the Earth to the Sun is much larger than the radius of the Earth, so the angular momentum of the Earth revolving about the Sun should be larger than the angular momentum of the Earth rotating about its axis.

8.21 A radian is the ratio of the arc length to the radius. Although we use "radian" as a unit, it is dimensionless. Whenever we are using angular quantities, we include "radian" to emphasize this fact. If we are using translational quantities, these have dimensions (for example, length) and we should use the correct units to show this.

8.22 There are three unique ways a vector cross product can equal zero: (1) the magnitude of $\vec{A}$ is zero, (2) the magnitude of $\vec{B}$ is zero, or (3) the angle between vectors $\vec{A}$ and $\vec{B}$ is equal to an integer multiple of π radians.

8.23 There are three main steps for the right-hand rule: (1) point the fingers on your right hand along the first vector, (2) fold your right fingers towards your palm until they point along the second vector, and (3) your right thumb will point in the direction of the cross product.

8.24 The positive direction is up the ramp.

a) Graph (a). The angular acceleration should behave the same as the linear acceleration, which is constant and negative.

b) Graph (e). The angular velocity should behave the same as the linear velocity. The cylinder initially rolls up the ramp, stops, and then rolls down the ramp.

8.25 Counterclockwise corresponds to the positive sense of rotation. We're told the merry-go-round undergoes constant acceleration.

a) Graph (c). The angular speed increases linearly as a function of time.

b) Graph (i). The angular displacement increases parabolically under constant angular acceleration.

c) Graph (a). The merry-go-round maintains a constant maximum angular speed.

d) Graph (ii). The angular displacement increases linearly for a constant angular speed.

e) Graph (d). The angular speed should decrease linearly from a maximum to zero.

f) Graph (iv). The angular position of the merry-go-round will eventually level out once it comes to a stop.

g) When the merry-go-round is speeding up in the counterclockwise direction, there is a positive angular acceleration caused by a positive net torque on the merry-go-round. Since we're told that the angular acceleration is constant, the net torque must also be constant. The net torque is equal to zero when the merry-go-round is spinning at a constant angular velocity. The merry-go-round slows down due to a constant, negative net torque being applied.

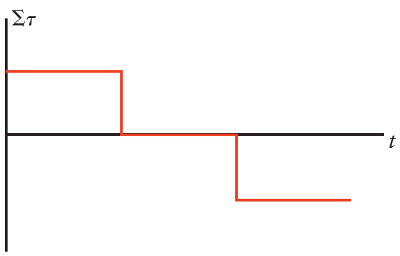


Figure 8-1 Problem 25

8.26 The magnitude of the torque exerted on the bolt is equal to $FR\sin(\theta)$, where F is the magnitude of the force, R is the distance from the rotation axis (the bolt, in this case) to the location of the force, and θ is the angle between these two vectors. Since F is the same in all four cases, the torque will be the largest when the force is applied to the end farthest from the bolt *and* perpendicular to the wrench (C). The torques in (B) and (D) are the next largest and are equal to one another. Finally, (A) is the smallest since R is much smaller in this case compared to the other three. To reiterate: (A) < (B) = (D) < (C).

Multiple-Choice Questions

8.27 **B** (the cylinder). A solid sphere of radius R, a solid cylinder of radius R, and a thin rod of length R have the same mass and are rotating with the same angular speed. The rotational kinetic energy of each object is proportional to its moment

of inertia. The cylinder has the largest moment of inertia of the three shapes—$I_{\text{sphere}} = \frac{2}{5}MR^2$, $I_{\text{cylinder}} = \frac{1}{2}MR^2$, $I_{\text{rod}} = \frac{1}{12}MR^2$—and will have the largest rotational kinetic energy.

8.28 **B** (0.2 times as large as before).

$$K_{\text{rot, i}} = \frac{1}{2}I_i\omega_i^2$$

$$K_{\text{rot, f}} = \frac{1}{2}I_f\omega_f^2 = \frac{1}{2}(5I_i)\left(\frac{\omega_i}{5}\right)^2 = \frac{1}{5}\left(\frac{1}{2}I_i\omega_i^2\right) = \boxed{\frac{1}{5}K_{\text{rot, i}}}$$

8.29 E (32).

$$\frac{I_2}{I_1} = \frac{\left(\frac{2}{5}M_2R_2^2\right)}{\left(\frac{2}{5}M_1R_1^2\right)} = \frac{(\rho V_2)R_2^2}{(\rho V_1)R_1^2} = \frac{\left(\frac{4}{3}\pi R_2^3\right)R_2^2}{\left(\frac{4}{3}\pi R_1^3\right)R_1^2} = \frac{R_2^5}{R_1^5} = \frac{(2R_1)^5}{R_1^5} = 2^5 = \boxed{32}$$

8.30 **E** (they all roll to the same height). If all of the objects start with the same energy, they will all end up with the same gravitational potential energy. Because the objects have the same mass, they will all rise to the same height.

8.31 **C** (the hoop).Three objects—a solid ball, a solid disk, and a hoop—that have the same mass and radius start rolling *and* translating up an incline from the same height, which we'll call $y = 0$. The objects all begin moving at the same linear speed. The moment of inertia for each of the objects is different, though: $I_{\text{solid sphere}} = \frac{2}{5}MR^2$, $I_{\text{solid disk}} = \frac{1}{2}MR^2$, $I_{\text{hoop}} = MR^2$.

We can use conservation of mechanical energy to solve this problem:

$$K_{\text{rot, i}} + K_{\text{tr, i}} + U_{\text{g, i}} = K_{\text{rot, f}} + K_{\text{tr, f}} + U_{\text{g, f}}$$

$$\frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 + 0 = 0 + 0 + mgh$$

$$\frac{1}{2}I\left(\frac{v}{R}\right)^2 + \frac{1}{2}mv^2 = mgh$$

Solving for the height h that each object makes it up the incline, $h = \frac{v^2}{2mg}\left(\frac{I}{R^2} + m\right)$.

Since the moment of inertia for the hoop has the largest coefficient (1 versus $\frac{2}{5}$ versus $\frac{1}{2}$), it will go up the incline the farthest.

8.32 **D** (the same as Lily's). Every point on the merry-go-round will rotate at the same constant angular speed regardless of its location.

8.33 **B** (larger than Lily's). The linear speed of a point on the merry-go-round is proportional to the distance from the rotation axis. Since Bob is farther from the rotation axis, his linear speed will be larger than Lily's.

8.34 **E** (angular momentum about her center of mass). Gravity, an external force, is acting on the gymnast, which will change her position, velocity, and momentum. Her angular velocity may change (for example, spinning in the air), but her angular momentum cannot change since there is no net torque acting on her.

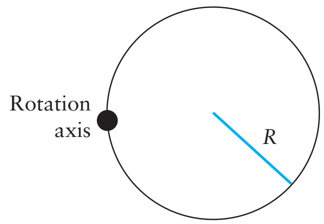


Figure 8-2 Problem 35

8.35 **B** ($2MR^2$). We need to find the moment of inertia of a thin ring of radius R rotating about a point on its rim. The parallel-axis theorem will give us the moment of inertia of an object rotating about an axis *not* passing through its center of mass: $I = I_{CM} + Mh^2$, where I_{CM} is the moment of inertia of the object rotating about an axis through its center of mass and h is the distance between that axis and the actual rotation axis. For a thin ring, $I_{CM} = MR^2$. The distance from the center of mass to a point on the edge of the ring is R. Therefore, $I = I_{CM} + Mh^2 = MR^2 + MR^2 = 2MR^2$.

8.36 **D** (remain constant). After the push has ended, there is no net torque acting on the system, which means the angular momentum will remain constant. The moment of inertia is not changing, which means the angular velocity will not change either.

Estimation Questions

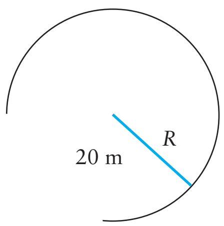


Figure 8-3 Problem 37

8.37 We're told that the cloverleaf is approximately three-quarters of a circle, which corresponds to a total angular displacement of $\Delta\theta = \frac{3}{4}(2\pi \text{ rad}) = \frac{3\pi}{2}\text{rad}$. We need to estimate the amount of time the car is driving on the cloverleaf. Let's say that the cloverleaf is three-quarters of a circle of 25 m in radius, and the car is driving a little over 30 mph (15 m/s). The total distance the car travels is the arc length, or $\left(\frac{3\pi}{2}\text{rad}\right)(25 \text{ m}) = \frac{75\pi}{2}\text{m}$. It takes the car $\frac{\left(\frac{75\pi}{2}\text{m}\right)}{\left(15\frac{\text{m}}{\text{s}}\right)} = \frac{5\pi}{2}\text{s}$ to drive around the cloverleaf. The angular speed is the angular displacement over the time it took, so $\omega = \frac{\Delta\theta}{\Delta t} = \frac{\left(\frac{3\pi}{2}\text{rad}\right)}{\left(\frac{5\pi}{2}\text{s}\right)} = \boxed{\frac{3}{5}\frac{\text{rad}}{\text{s}} = 0.6\frac{\text{rad}}{\text{s}}}$.

8.38 A fan is designed to operate up to 1 billion rotations. In order to estimate the lifetime of the fan, we can first estimate how many rotations it makes in a single day. It takes the fan 2 s to reach its operating speed of 750 rpm from rest and 10 s for it to come to

a stop. Chances are the fan will be in operation each day for much longer than 12 s. Assuming that is the case, then almost all the rotations occur while the fan is rotating at its constant operating speed of 750 rpm. If the fan is in operation for 10 hr a day, it will complete approximately 4.5×10^5 revolutions/day. At this rate it will take about 2200 days (or about 6 years) to reach 1 billion revolutions. (We could get a better estimate by explicitly considering the constant acceleration and deceleration of the fan, but this will only affect our estimate slightly.)

8.39 A door is about a meter wide. If you push 1 m from the rotation axis through the hinges with a force of about 10 N, the maximum torque you can provide will be 10 N · m.

8.40 We can model the pencil as a thin rod rotating about its center. A pencil has a mass of about 2 g and a length of 17.5 cm. This gives a moment of inertia of

$$I_{\text{pencil}} = \frac{1}{12}ML^2 = \frac{1}{12}(2 \times 10^{-3}\ \text{kg})(0.175\ \text{m})^2 = \boxed{5 \times 10^{-6}\ \text{kg} \cdot \text{m}^2}.$$

8.41 Let's assume the skater has a total mass of 50 kg and is made up of cylinders of various sizes and masses: Her body is about 30 cm in diameter and 40 kg, and each arm is 50 cm long and 5 kg. When her arms are outstretched, we will treat them as thin rods being rotated about an axis a distance h from the center of mass of the arm. Here h is the radius of her body plus the distance to the center of her arm, or $h = (0.15\ \text{m}) + (0.5)\cdot(0.50\ \text{m}) = 0.40\ \text{m}$. Her moment of inertia with her arms stretched out is

$$I_{\text{outstretched}} = I_{\text{torso}} + I_{\text{arms}} = \frac{1}{2}M_{\text{body}}R^2_{\text{body}} + 2\left(\frac{1}{12}M_{\text{arm}}L^2_{\text{arm}} + M_{\text{arm}}h^2_{\text{arm}}\right)$$

$$= \frac{1}{2}(40\ \text{kg})(0.15\ \text{m})^2 + 2\left(\frac{1}{12}(5\ \text{kg})(0.50\ \text{m})^2 + (5\ \text{kg})(0.40\ \text{m})^2\right) = \boxed{2.3\ \text{kg} \cdot \text{m}^2}$$

When she is spinning with her arms drawn in, her arms no longer affect the moment of inertia and her body acts as if it were the entire 50 kg:

$$I_{\text{tucked in}} = I_{\text{body}} = \frac{1}{2}(50\ \text{kg})(0.15\ \text{m})^2 = \boxed{0.56\ \text{kg} \cdot \text{m}^2}$$

8.42 The Earth requires 365.25 days to complete one revolution about the Sun. Assuming a circular orbit, in one day the Earth will travel

$$1\ \text{day} \times \frac{1\ \text{rev}}{365.25\ \text{day}} \times \frac{2\pi\ \text{rad}}{1\ \text{rev}} \approx \boxed{0.2\ \text{rad}}\ \text{or}\ 1\ \text{day} \times \frac{1\ \text{rev}}{365.25\ \text{day}} \times \frac{360°}{1\ \text{rev}} \approx \boxed{1°}.$$

8.43 We will assume the Sun sweeps out a half-circle in a 12-hr period (from sunrise to sunset). This is an angular displacement of π radians.

$$\omega = \frac{\pi\ \text{rad}}{12\ \text{hr}} \times \frac{1\ \text{hr}}{3600\ \text{s}} \approx \boxed{7 \times 10^{-5}\frac{\text{rad}}{\text{s}}}$$

8.44 A washing machine has an operating speed of about 1500 rpm. It reaches this speed in approximately 5 s, corresponding to an angular acceleration of

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\left(1500\frac{\text{rev}}{\text{min}} \times \frac{1\text{ min}}{60\text{ s}} \times \frac{2\pi\text{ rad}}{1\text{ rev}}\right)}{5\text{ s}} \approx \boxed{30\frac{\text{rad}}{\text{s}^2}}.$$

8.45 The magnitude of the "skip-it" ball's angular momentum is equal to $L = I\omega = MR^2\omega$, where we plugged in the moment of inertia for a point mass. Since the toy is made of plastic and is attached to a child's leg, the mass of the ball should be around 0.25 kg. The toy is approximately a meter long. The ball makes about a revolution per second when the child is swinging it around her leg; this corresponds to an angular speed of $\omega = 1\frac{\text{rev}}{\text{s}} \times \frac{2\pi\text{ rad}}{1\text{ rev}} = 2\pi\frac{\text{rad}}{\text{s}}$. Plugging all of this in yields $L = (0.25\text{ kg})(1\text{ m})^2 \times \left(2\pi\frac{\text{rad}}{\text{s}}\right) = \frac{\pi}{2}\frac{\text{kg}\cdot\text{m}^2}{\text{s}} \approx \boxed{1.6\frac{\text{kg}\cdot\text{m}^2}{\text{s}}}$.

8.46 We can model the Earth as a uniform, solid sphere. The mass of the Earth is about 6×10^{24} kg and the radius of the Earth is about 6.4×10^6 m. The moment of inertia of the Earth is $I_{\text{Earth}} = \frac{2}{5}m_{\text{E}}R_{\text{E}}^2 = \frac{2}{5}(6 \times 10^{24}\text{ kg})(6.4 \times 10^6\text{ m})^2 \approx \boxed{1 \times 10^{38}\text{ kg}\cdot\text{m}^2}$.

8.47 Below is a spreadsheet containing the given time and angle data:

Time (s)	θ (rad)	ω (rad/s)	α (rad/s^2)	τ (N · m)	I (kg · m^2)
0	0				0.25
1	0.349	0.349			
2	0.700	0.351	0.002	0.0005	
3	1.05	0.350	−0.001	−0.00025	
4	1.40	0.350	0.000	0	
5	1.75	0.350	0.000	0	
6	2.10	0.350	0.000	0	
7	2.44	0.340	−0.010	−0.0025	
8	2.80	0.360	0.020	0.005	
9	3.14	0.340	−0.020	−0.005	
10	3.50	0.360	0.020	0.005	
11	3.50	0.000	−0.360	−0.09	
12	3.49	−0.010	−0.010	−0.0025	
13	3.50	0.010	0.020	0.005	
14	3.51	0.010	0.000	0	
15	3.51	0.000	−0.010	−0.0025	
16	3.98	0.470	0.470	0.1175	
17	5.01	1.030	0.560	0.14	

18	6.48	1.470	0.440	0.11	
19	8.53	2.050	0.580	0.145	
20	11.0	2.470	0.420	0.105	
21	14.1	3.100	0.630	0.1575	
22	17.6	3.500	0.400	0.1	
23	21.6	4.000	0.500	0.125	
24	26.2	4.600	0.600	0.15	
25	31.0	4.800	0.200	0.05	

The third column is the angular speed for each interval—the speed listed for $t = 1$ s is $\omega_{t=1} = \frac{\theta_{t=1} - \theta_{t=0}}{1\text{ s}}$. The angular speed over the first 10 s is approximately constant with an average of $\boxed{0.35\text{ rad/s}}$. The fourth column is the angular acceleration for each angular speed interval—the acceleration listed for $t = 2$ s is $a_{t=2} = \frac{\omega_{t=2} - \omega_{t=1}}{1\text{ s}}$. The angular acceleration from 15−25 s is not constant, but the average is $\boxed{0.48\text{ rad/s}^2}$. The fifth column is the net torque exerted in each interval. It is calculated using Newton's second law for rotation, $\sum\tau = I\alpha$, where the moment of inertia is given in the sixth column. The torque during the time interval from 0–10 s $\boxed{\text{should be zero}}$ since the angular speed is constant. During 10–15 s, the torque $\boxed{\text{should also be zero}}$ because the angular position is constant. Due to errors introduced in the calculation process and the fact that these are real data, the calculated entries do not come out to exactly zero in these cases. The average net torque during 15–25 s is $\sum\tau = (0.25\text{ kg}\cdot\text{m}^2)\left(0.48\frac{\text{rad}}{\text{s}^2}\right) = \boxed{0.12\text{ N}\cdot\text{m}}$. By interpreting the data rather than just blindly calculating, we can come up with an accurate model of the object's motion.

Problems

8.48

SET UP

The angular speed is angular displacement divided by the time interval. We are told that the object completes two full revolutions in 12 s. One revolution corresponds to an angular displacement of 2π radians.

SOLVE

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\text{ rev}}{12\text{ s}} \times \frac{2\pi\text{ rad}}{1\text{ rev}} = \frac{\pi}{3}\frac{\text{rad}}{\text{s}} = \boxed{1.05\frac{\text{rad}}{\text{s}}}$$

REFLECT

The object moves approximately 1 radian, or ~57 degrees, every second.

8.49

SET UP

We are asked to find the angular speed of a car rounding a corner and are given its translational speed and the radius of the turn. We can use the equation that relates these quantities, $v = \omega r$, to solve for the angular speed.

SOLVE

$$\omega = \frac{v}{r} = \frac{\left(12\frac{\text{m}}{\text{s}}\right)}{7\ \text{m}} = \boxed{1.71\frac{\text{rad}}{\text{s}}}$$

REFLECT

In 1 s the car travels through an angle of 1.71 radians, or 97 degrees. This is a little more than a quarter of a circle, which seems reasonable.

8.50

SET UP

These are conversions between common units of angular speed. Keep in mind that rpm is short for "revolutions per minute" and that one revolution corresponds to an angular displacement of 2π radians.

SOLVE

$$45\frac{\text{rev}}{\text{min}} \times \frac{2\pi\ \text{rad}}{1\ \text{rev}} \times \frac{1\ \text{min}}{60\ \text{s}} = \frac{3\pi}{2}\frac{\text{rad}}{\text{s}} = \boxed{4.71\frac{\text{rad}}{\text{s}}}$$

$$33\frac{1}{3}\frac{\text{rev}}{\text{min}} \times \frac{2\pi\ \text{rad}}{1\ \text{rev}} \times \frac{1\ \text{min}}{60\ \text{s}} = \boxed{3.49\frac{\text{rad}}{\text{s}}}$$

$$2\pi\frac{\text{rev}}{\text{s}} \times \frac{2\pi\ \text{rad}}{1\ \text{rev}} = (2\pi)^2\frac{\text{rad}}{\text{s}} = \boxed{39.5\frac{\text{rad}}{\text{s}}}$$

REFLECT

Recall that the SI units for angular speed are rad/s and that radians are dimensionless.

8.51

SET UP

It is reasonable to assume that the angular speed of the Moon is roughly constant. In this case, the angular speed is given by the angular displacement divided by the time it took. We're told that the Moon completes one orbit (assuming a circular orbit) in 27.4 days. One circular orbit is an angular displacement of 2π radians.

SOLVE

$$1\ \text{orbit} = 27.4\ \text{day} \times \frac{24\ \text{hr}}{1\ \text{day}} \times \frac{3600\ \text{s}}{1\ \text{hr}} = 2.367 \times 10^6\ \text{s}$$

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi\ \text{rad}}{2.367 \times 10^6\ \text{s}} = \boxed{2.65 \times 10^{-6}\frac{\text{rad}}{\text{s}}}$$

REFLECT
Since it takes such a long time for the Moon to complete one orbit, we expect its angular speed to be a small number. The Earth–Moon distance is not necessary for finding the angular speed.

8.52

SET UP
A point mass (mass $m = 0.25$ kg) is rotating at an angular speed of 3 rev/s about an axis 0.5 m away. In order to calculate the rotational kinetic energy of the object we first need to find its moment of inertia.

SOLVE

$$I_{\text{point mass}} = mR^2 = (0.25\text{ kg})(0.5\text{ m})^2 = 0.0625\text{ kg}\cdot\text{m}^2$$

$$K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}(0.0625\text{ kg}\cdot\text{m}^2)\left(3\frac{\text{rev}}{\text{s}} \times \frac{2\pi\text{ rad}}{1\text{ rev}}\right)^2 = \boxed{11.1\text{ J}}$$

REFLECT
Remember to convert revolutions into radians.

8.53

SET UP
We are asked to find the rotational kinetic energy of any object and are given the object's moment of inertia and angular speed. We can use the definition of rotational kinetic energy to accomplish this.

SOLVE

$$K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}(0.28\text{ kg}\cdot\text{m}^2)\left(4\frac{\text{rad}}{\text{s}}\right)^2 = \boxed{2.24\text{ J}}$$

REFLECT
Since we were given the moment of inertia of the object, the exact axis of rotation was not necessary to solve the problem.

8.54

SET UP
We are asked to find the moment of inertia of any object about an unspecified axis and are given the object's rotational kinetic energy and angular speed. First we need to convert the angular speed into rad/s before we can use the definition of rotational kinetic energy.

SOLVE

$$\omega = 13\frac{\text{rev}}{\text{min}} \times \frac{2\pi\text{ rad}}{1\text{ rev}} \times \frac{1\text{ min}}{60\text{ s}} = \frac{13\pi}{30}\frac{\text{rad}}{\text{s}}$$

$$K_{\text{rot}} = \frac{1}{2}I\omega^2$$

$$I = \frac{2K_{\text{rot}}}{\omega^2} = \frac{2(18\text{ J})}{\left(\frac{13\pi}{30}\frac{\text{rad}}{\text{s}}\right)^2} = \boxed{19.4\text{ kg}\cdot\text{m}^2}$$

REFLECT
Remember to convert revolutions into radians.

8.55

SET UP
We are given the moment of inertia I and rotational kinetic energy K_{rot} of an object and asked to find its angular speed. We can use the definition of rotational kinetic energy, $K_{\text{rot}} = \frac{1}{2}I\omega^2$, and solve for ω. To convert from radians per second, we need to realize that one revolution corresponds to an angular displacement of 2π radians.

SOLVE
Solving for ω and plugging in the given values, we find that

$$\omega = \sqrt{\frac{2K_{\text{rot}}}{I}} = \sqrt{\frac{2(2.75\ \text{J})}{0.33\ \text{kg}\cdot\text{m}^2}}$$

$$= 4.08\frac{\text{rad}}{\text{s}} \times \frac{60\ \text{s}}{1\ \text{min}} \times \frac{1\ \text{rev}}{2\pi\ \text{rad}} = \boxed{39.0\frac{\text{rev}}{\text{min}}}$$

REFLECT
Since we are given the moment of inertia explicitly, we do not need to use the fact that it is a rotating wheel.

8.56

SET UP
Our system consists of three point objects (with masses $m_1 = 2.4$ kg, $m_2 = 1.8$ kg, $m_3 = 3.0$ kg) located on a single line. We need to find the moment of inertia of the system rotating about an axis through the middle object (m_2). We can use the definition of moment of inertia because these are all point objects.

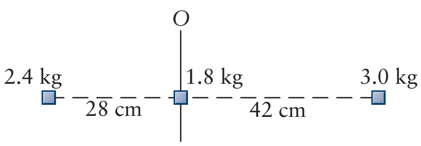


Figure 8-4 Problem 56

SOLVE

$$I = \sum_i m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$$

$$= (2.4\ \text{kg})(0.28\ \text{m})^2 + (1.8\ \text{kg})(0\ \text{m})^2 + (3.0\ \text{kg})(0.42\ \text{m})^2 = \boxed{0.717\ \text{kg}\cdot\text{m}^2}$$

REFLECT
Since the axis of rotation passes through the middle object, m_2 does not contribute to the moment of inertia (that is, $r_2 = 0$).

8.57

SET UP

Our system consists of three point objects (with masses $m_1 = 1.0$ kg, $m_2 = 1.5$ kg, $m_3 = 2.0$ kg) located on a single line. We need to find the moment of inertia of the system rotating about an axis 1 m to the left of m_1. We can use the definition of moment of inertia because these are all point objects. The distances between neighboring masses are given in the figure, but we need the distance of each mass from the axis of rotation for our calculation.

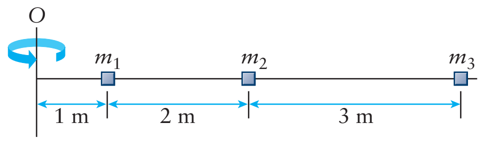


Figure 8-5 Problem 57

SOLVE

$$I = \sum_i m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$$

$$= (1.0\text{ kg})(1\text{ m})^2 + (1.5\text{ kg})(3\text{ m})^2 + (2.0\text{ kg})(6\text{ m})^2 = \boxed{86.5\text{ kg}\cdot\text{m}^2}$$

REFLECT

Remember that r in the moment of inertia equation is the distance the mass is from the axis of rotation, which is not necessarily what is always given in the problem statement.

8.58

SET UP

A thin, uniform washer has an inner radius of r_1, an outer radius of r_0, and is centered at $r = 0$.We need to determine the moment of inertia of the washer about an axis through the center of the washer using integration: $I = \int r^2 dm$. Since we are told the washer is uniform, the density σ throughout the object is constant. This allows us to convert our integration from dm to dr and $d\theta$.

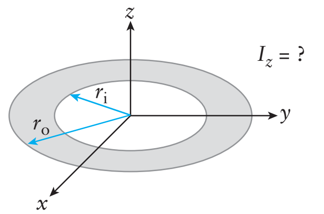


Figure 8-6 Problem 58

SOLVE

$$I_{\text{washer}} = \int r^2 dm$$

We can find dm by remembering that

$$\frac{\text{area of piece}}{\text{area of disk}} = \frac{\text{mass of piece}}{\text{mass of disk}}$$

Therefore,

$$dm = \sigma dA = \left(\frac{M}{\pi(r_0^2 - r_1^2)}\right)dA = \left(\frac{M}{\pi(r_0^2 - r_1^2)}\right)r dr d\theta$$

Plugging this into the integral,

$$I = \int r^2 dm = \int_0^{2\pi}\int_{r_1}^{r_0} r^2\left(\frac{M}{\pi(r_0^2 - r_1^2)}\right) r\,dr\,d\theta = \frac{M}{\pi(r_0^2 - r_1^2)}\int_0^{2\pi} r^3 dr \int_0^{2\pi} d\theta$$

$$= \frac{M}{\pi(r_0^2 - r_1^2)}\left[\frac{1}{4}r^4\right]_{r_1}^{r_0}[\theta]_0^{2\pi} = \frac{M}{\pi(r_0^2 - r_1^2)}\left[\frac{1}{4}(r_0^4 - r_1^4)\right][2\pi]$$

$$= \frac{M}{2(r_0^2 - r_1^2)}[(r_0^2 - r_1^2)(r_0^2 + r_1^2)] = \boxed{\frac{M}{2}(r_0^2 + r_1^2)}$$

REFLECT

The moment of inertia for a hoop of mass M rotating about an axis through its center is MR^2. The moment of inertia of the washer looks like the moment of inertia for the hoop with $R = R_{\text{average}} = \dfrac{r_0^2 + r_1^2}{2}$.

8.59

SET UP

A uniform, hollow cylinder of radius R, length L, and mass M is rotating about an axis—say, the x-axis—through its central axis. We can show that the moment of inertia is MR^2 through integration. Because it is a cylinder, all of the mass is located a distance R from the rotation axis. We are assuming that the mass density is constant, which allows us to relate dm to dx.

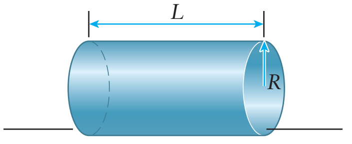


Figure 8-7 Problem 59

SOLVE

$$I = \int r^2 dm$$

$$dm = \sigma dA = \left(\frac{M}{2\pi RL}\right)(2\pi R dx) = \frac{M}{L}dx$$

$$I = \int r^2 dm = \int_0^L R^2\left(\frac{M}{L}dx\right) = \frac{MR^2}{L}\int_0^L dx = \frac{MR^2}{L}[x]_0^L = \frac{MR^2}{L}[L] = \boxed{MR^2}$$

REFLECT

This answer makes sense because each infinitesimal mass dm is located a constant distance R from the rotation axis.

8.60

SET UP

A baton is made up of two objects, each of mass m_{end} = 350 g, attached to each end of a rod of mass m_{rod} and length L = 60 cm. The total mass of the baton is between 940–950 g, which means m_{rod} is between 240–250 g. The manufacturer of the baton says that the moment of inertia about an axis through the center of the baton should be

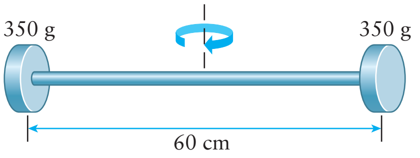


Figure 8-8 Problem 60

between $0.075 - 0.080\ \text{kg}\cdot\text{m}^2$, but the twirler believes this is a mistake. Since we know the mass and length of the baton, we can explicitly calculate the minimum and maximum moments of inertia and compare them to the manufacturer's specifications. By modeling the baton as a uniform rod and two point masses, we can use the moments of inertia for those objects.

SOLVE

$$I_{\text{baton}} = I_{\text{rod}} + 2I_{\text{rod}} = \frac{1}{12}m_{\text{rod}}L^2 + 2m_{\text{rod}}\left(\frac{L}{2}\right)^2$$

Minimum moment of inertia:

$$I_{\text{baton, minimum}} = \frac{1}{12}(0.940\ \text{kg})(0.60\ \text{m})^2 + 2(0.350\ \text{kg})(0.30\ \text{m})^2 = 0.0702\ \text{kg}\cdot\text{m}^2$$

Maximum moment of inertia:

$$I_{\text{baton, maximum}} = \frac{1}{12}(0.950\ \text{kg})(0.60\ \text{m})^2 + 2(0.350\ \text{kg})(0.30\ \text{m})^2 = 0.0705\ \text{kg}\cdot\text{m}^2$$

REFLECT

The actual moment of inertia lies between $0.0702\ \text{kg}\cdot\text{m}^2$ and $0.0705\ \text{kg}\cdot\text{m}^2$, which is outside the stated range from the manufacturer; the $\boxed{\text{twirler is correct}}$.

8.61

SET UP

The steering wheel is rotating about an axis that passes through its center and is made up of a rim of radius R and mass M and five thin rods of length R and mass $M/2$. The overall moment of inertia of the steering wheel is equal to the moment of inertia of the rim plus the moments of inertia for each of the thin rods. We can model the rim as a uniform ring of radius R rotating about an axis through its center, and each of the thin rods is rotating about a perpendicular line through one end.

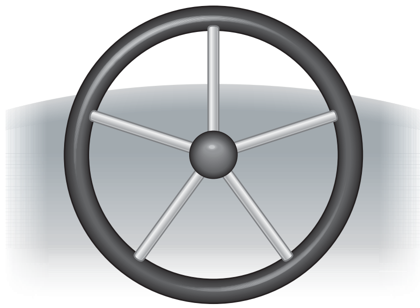


Figure 8-9 Problem 61

SOLVE

$$I_{\text{rim}} = I_{\text{ring}} = MR^2$$

$$I_{\text{rod}} = \frac{1}{3}\left(\frac{M}{2}\right)R^2 = \frac{1}{6}MR^2$$

$$I_{\text{total}} = I_{\text{rim}} + 5I_{\text{rod}} = MR^2 + 5\left(\frac{1}{6}MR^2\right) = MR^2 + \frac{5}{6}MR^2 = \boxed{\frac{11}{6}MR^2}$$

REFLECT

Since the steering wheel is made up of simple geometric objects, we can use the results in Table 8-1 of the textbook. We then determine the axis of rotation for the overall object. This allows us to determine which moments of inertia to use (for example, a thin rod rotating about an axis through its center versus a thin rod rotating about an axis through one end).

8.62

SET UP

A solid, uniform sphere is rotating about an axis tangent to its surface. We can use the parallel-axis theorem to determine the moment of inertia in this case. The center of mass of the sphere is a distance R from the axis of rotation.

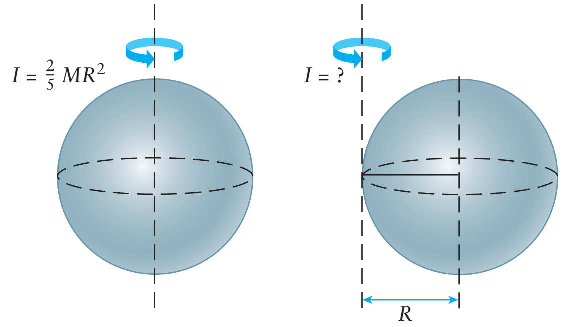


Figure 8-10 Problem 62

SOLVE

$$I = I_{\text{sphere}} + I_{\text{from axis}} = \frac{2}{5}MR^2 + MR^2 = \boxed{\frac{7}{5}MR^2}$$

REFLECT

Since the rotation axis is a distance R from the center of mass, it makes sense that the moment of inertia in this case is larger than the moment of inertia of the sphere about an axis through its center of mass.

8.63

SET UP

A solid, uniform cylinder is rotating about an axis tangent to its length. We can use the parallel-axis theorem to determine the moment of inertia in this case. The center of mass of the cylinder is a distance R from the axis of rotation.

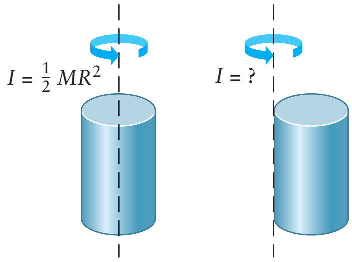


Figure 8-11 Problem 63

SOLVE

$$I = I_{\text{cylinder}} + I_{\text{from axis}} = \frac{1}{2}MR^2 + MR^2 = \boxed{\frac{3}{2}MR^2}$$

REFLECT

Since the rotation axis is a distance R from the center of mass, it makes sense that the moment of inertia in this case is larger than the moment of inertia of the cylinder about an axis through its center of mass and parallel to its length.

8.64

SET UP

A thin rod of mass $M = 2.25$ kg and length $L = 1.25$ m is rotating about an axis one-third of the way from the left end of the rod. We will assume that the rod is uniform. The center of mass of the rod is located a distance $\frac{1}{2}L - \frac{1}{3}L = \frac{1}{6}L$ from the rotation axis.

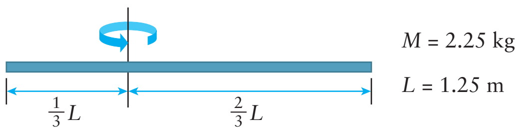


Figure 8-12 Problem 64

SOLVE

$$I = I_{\text{thin rod}} + I_{\text{from axis}} = \frac{1}{12}ML^2 + M\left(\frac{L}{6}\right)^2 = \frac{1}{12}ML^2 + \frac{1}{36}ML^2$$

$$= \frac{1}{9}ML^2 = \frac{1}{9}(2.25\ \text{kg})(1.25\ \text{m})^2 = \boxed{0.39\ \text{kg} \cdot \text{m}^2}$$

REFLECT

Rather than using the parallel-axis theorem, we could have modeled the system as two separate rods of lengths $\frac{L}{3}$ and $\frac{2L}{3}$ rotating about an axis through the end of each rod. Assuming the rods are uniform allows us to determine the mass of each rod given the initial mass. This analysis yields the same answer:

$$I = I_{\text{left rod}} + I_{\text{right rod}} = \frac{1}{3}M_{\text{left}}\left(\frac{L}{3}\right)^2 + \frac{1}{3}M_{\text{right}}\left(\frac{2L}{3}\right)^2$$

$$= \frac{1}{3}\left(\frac{1}{3}M\right)\left(\frac{L}{3}\right)^2 + \frac{1}{3}\left(\frac{2}{3}M\right)\left(\frac{2L}{3}\right)^2 = 0.39\ \text{kg}\cdot\text{m}^2$$

8.65

SET UP

We are given the dimensions (5 cm × 7 cm) and the mass density (1.5 g/cm^2) for a thin plate rotating about an axis through its left side. According to Table 8-1 in the text, the moment of inertia for a solid rectangular parallelepiped is $\frac{1}{12}M(a^2 + b^2)$. In the case of the rectangle, $a = 0$ and $b = 7$ cm, which gives a moment of inertia of $\frac{1}{12}Mb^2$. We can use this result and the parallel-axis theorem to determine the moment of inertia of a rectangle rotating about an axis a distance $b/2$ from its center of mass.

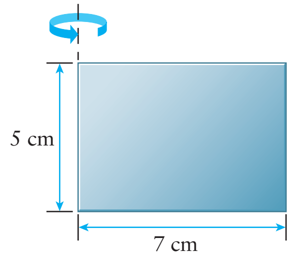


Figure 8-13 Problem 65

SOLVE

$$I = I_{\text{rectangle}} + I_{\text{from axis}} = \frac{1}{12}Mb^2 + M\left(\frac{b}{2}\right)^2 = \frac{1}{3}Mb^2 = \frac{1}{3}(\sigma A)b^2$$

$$= \frac{1}{3}\left(1.5\frac{\text{g}}{\text{cm}^2}\right)(35\ \text{cm}^2)(7\ \text{cm}) = \boxed{860\ \text{g}\cdot\text{cm}^2}$$

REFLECT

Even though the moment of inertia for a two-dimensional rectangle is not listed in Table 8-1 of the text, we can derive it from the three-dimensional version.

8.66

SET UP

A sphere of mass M and unknown radius r is rotating about an axis through its center. A second sphere of the same mass M and known radius R is rotating about an axis that is tangent to its surface. We are asked to find r such that the moments of inertia for the two cases are equal. We need to use the parallel-axis theorem to find the moment of inertia for the sphere of radius R.

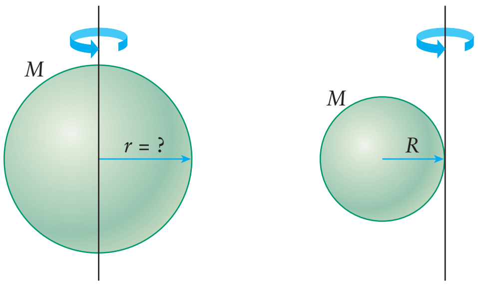


Figure 8-14 Problem 66

SOLVE

$$I_{\text{sphere, radius } r} = I_{\text{sphere, radius } R}$$

$$\frac{2}{5}Mr^2 = \frac{2}{5}MR^2 + MR^2$$

$$\frac{2}{5}Mr^2 = \frac{7}{5}MR^2$$

$$\boxed{r = \sqrt{\frac{7}{2}}R}$$

REFLECT

Since the masses are the same in both cases, we expect $r > R$ if the moments of inertia are the same.

8.67

SET UP

The system is made up of three separate objects: a solid sphere of radius R and mass M, another solid sphere of radius $2R$ and mass M, and a thin, uniform rod of length $3R$ and mass M. We need to find the moment of inertia about the axis through the center of the rod by considering the moments of inertia for each of the component objects in order to find the overall moment of inertia of the system. Since the two spheres are not located at the axis of rotation, we need to determine the distance their centers of mass are from the axis of rotation and invoke the parallel-axis theorem. The center of mass for the large sphere is $3.5R$ from the axis of rotation, and the small sphere is $2.5R$ from the axis.

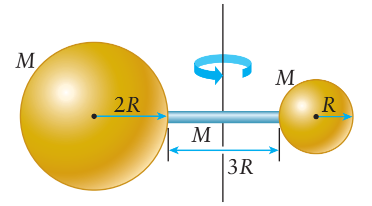


Figure 8-15 Problem 67

SOLVE

$$I_{\text{total}} = I_{\text{rod}} + I_{\text{left sphere}} + I_{\text{right sphere}}$$

$$I_{\text{rod}} = \frac{1}{12}M(3R)^2 = \frac{9}{12}MR^2$$

$$I_{\text{left sphere}} = I_{\text{sphere}} + I_{\text{from axis}} = \frac{2}{5}M(2R)^2 + M\left(2R + \frac{3}{2}R\right)^2 = \frac{8}{5}MR^2 + \frac{49}{4}MR^2 = \frac{277}{20}MR^2$$

$$I_{\text{right sphere}} = I_{\text{sphere}} + I_{\text{from axis}} = \frac{2}{5}MR^2 + M\left(R + \frac{3}{2}R\right)^2 = \frac{2}{5}MR^2 + \frac{25}{4}MR^2 = \frac{133}{20}MR^2$$

$$I_{\text{total}} = I_{\text{rod}} + I_{\text{left sphere}} + I_{\text{right sphere}} = \frac{9}{12}MR^2 + \frac{277}{20}MR^2 + \frac{133}{20}MR^2 = \boxed{\frac{255}{12}MR^2}$$

REFLECT

The system is made up of simple geometric objects, so we can use the results in Table 8-1 of the textbook. We need to be careful to add the distance the center of mass of each sphere is from the rotation axis in determining the overall moment of inertia. Since the solid spheres are not located along the axis of rotation, it makes sense that the overall moment of inertia of the system is larger than just the sum of

$$I_{\text{rod}} + I_{\text{sphere of }2R} + I_{\text{sphere of R}} = \frac{9}{12}MR^2 + \frac{8}{5}MR^2 + \frac{2}{5}MR^2 = \frac{33}{12}MR^2$$

8.68

SET UP

A sphere of radius R and mass M is attached to the right end of a massless rod of length L. The sphere–rod system is rotated about an axis through the left end of the rod. We can use the parallel-axis theorem to find the moment of inertia of the sphere–rod system. Because the rod is massless, it does not contribute to the moment of inertia of the object.

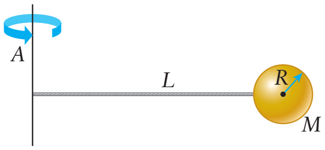


Figure 8-16 Problem 68

SOLVE

$$I = I_{\text{sphere}} + I_{\text{from axis}} = \boxed{\frac{2}{5}MR^2 + M(L+R)^2}$$

REFLECT

The sphere is attached to the end of the rod, which means the center of mass of the sphere is a distance $(L + R)$ from the axis of rotation.

8.69

SET UP

A door of mass $M = 9.0$ kg, width $W = 0.81$ m, and length $L = 1.78$ m is rotating about its hinges. We can model this situation as a uniform rectangle rotating about an axis that is $W/2$ from its center of mass. According to Table 8-1 in the text, the moment of inertia for a solid rectangular parallelepiped rotating about an axis through its center of mass is $\frac{1}{12}M(a^2 + b^2)$. In the case of the rectangle, $a = 0$ and $b = W$, which gives a moment of inertia of $\frac{1}{12}MW^2$. The parallel-axis theorem will allow us to find the moment of inertia of the door about the hinges.

SOLVE

$$I = I_{\text{rectangle}} + I_{\text{from axis}} = \frac{1}{12}MW^2 + M\left(\frac{W}{2}\right)^2 = \frac{1}{3}MW^2 = \frac{1}{3}(9\text{ kg})(0.81\text{ m})^2 = \boxed{1.97\text{ kg}\cdot\text{m}^2}$$

REFLECT

Note that the length of the door does not factor into the moment of inertia of the door.

8.70

SET UP

A thin, spherical shell of mass M and radius R sits atop a solid cylinder of mass M, radius $R/2$, and length $2R$. This object is rotating about an axis through the center of both the spherical shell and the cylinder. Since the rotation axis passes through the center of mass of both objects, we don't need to use the parallel-axis theorem. The overall moment of inertia of the object is the sum of the moments of inertia for a spherical shell and a solid, uniform cylinder rotating about an axis through the center of mass.

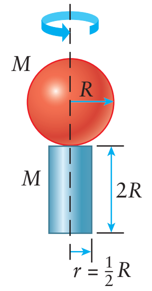


Figure 8-17 Problem 70

SOLVE

$$I = I_{\text{spherical shell}} + I_{\text{cylinder}} = \frac{2}{3}MR^2 + \frac{1}{2}M\left(\frac{R}{2}\right)^2 = \boxed{\frac{19}{24}MR^2}$$

REFLECT

The parallel-axis theorem is used for rotation axes that do *not* pass through the center of mass.

8.71

SET UP

A pencil of length L is made up of a piece of wood of mass M and length $7L/8$ and an eraser of mass M and length $L/8$. We need to determine the moment of inertia of the pencil about an axis through the end of the eraser using integration: $I = \int r^2 dm$. The overall moment of inertia of the pencil is the moment of inertia due to the eraser plus the moment of inertia due to the wood. We can model the pencil as a uniform, thin rod with the end of the eraser located at $y = 0$ and the top of the wooden part at $y = L$ to make the calculation easier because we only need to consider one dimension. In this coordinate system the eraser starts at $y = 0$ and ends at $y = L/8$ and the wood starts at $y = L/8$ and ends at $y = L$. Since we are assuming the pencil is uniform, the density throughout each part (eraser, wood) is constant. This allows us to convert our integration from dm to dy.

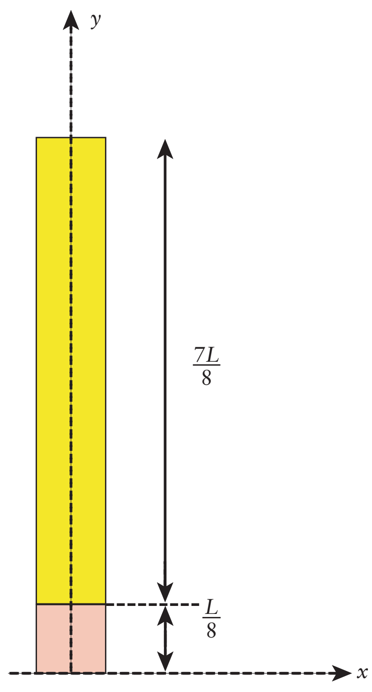


Figure 8-18 Problem 71

SOLVE

$$I_{\text{pencil}} = \int_0^L y^2 dm = \int_0^{L/8} y^2 dm_{\text{eraser}} + \int_{L/8}^{L} y m_{\text{wood}}$$

We can find dm_{eraser} and dm_{wood} by remembering that

$$\frac{\text{length of slice}}{\text{length of eraser}} = \frac{\text{mass of slice}}{\text{mass of eraser}} \text{ and } \frac{\text{length of slice}}{\text{length of wood}} = \frac{\text{mass of slice}}{\text{mass of wood}}.$$

Therefore,

$$dm_{\text{eraser}} = \frac{M}{\left(\frac{L}{8}\right)}dy = \frac{8M}{L}dy \text{ and } dm_{\text{wood}} = \frac{M}{\left(\frac{7L}{8}\right)}dy = \frac{8M}{7L}dy$$

Plugging these into the integral,

$$I_{\text{pencil}} = \int_0^{L/8} y^2 dm_{\text{eraser}} + \int_{L/8}^{L} y^2 dm_{\text{wood}} = \int_0^{L/8} y^2\left(\frac{8M}{L}\right)dy + \int_{L/8}^{L} y^2\left(\frac{8M}{7L}\right)dy$$

$$= \left(\frac{8M}{L}\right)\int_0^{L/8} y^2 dy + \left(\frac{8M}{7L}\right)\int_{L/8}^{L} y^2 dy = \left(\frac{8M}{L}\right)\left[\frac{y^3}{3}\right]_0^{L/8} + \left(\frac{8M}{7L}\right)\left[\frac{y^3}{3}\right]_{L/8}^{L}$$

$$= \left(\frac{8M}{3L}\right)\left[\left(\frac{L}{8}\right)^3\right] + \left(\frac{8M}{21L}\right)\left[L^3 - \left(\frac{L}{8}\right)^3\right] = \frac{ML^2}{192} + \left(\frac{8M}{21L}\right)\left[\frac{511L^3}{512}\right]$$

$$= \frac{ML^2}{192} + \left(\frac{M}{3}\right)\left[\frac{73L^2}{64}\right] = \frac{74ML^2}{192} = \boxed{0.385\ ML^2}$$

REFLECT

At first, it's difficult to see if this is a reasonable value or not. Let's consider a few similar cases for comparison. The moment of inertia for a uniform pencil of mass $2M$ and length L rotating about an axis through one end would be $I = \frac{2ML^2}{3} = 0.667ML^2$. It makes sense that our answer should be *smaller* than this since the $2M$ is not distributed uniformly. The moment of inertia of an object made up of two rods—one of length $L/8$ and mass M, one of length $7L/8$ and mass M—about an axis through each of their ends would be $I = \frac{1}{3}M\left(\frac{L}{8}\right)^2 + \frac{1}{3}M\left(\frac{7L}{8}\right)^2 = \frac{ML^2}{192} + \frac{49ML^2}{192} = \frac{50ML^2}{192} = 0.26\ ML^2$. In this case, the mass is distributed closer to the rotation axis than in our pencil example. Therefore, we would expect our answer to be *larger* than this moment of inertia. Comparing similar cases is a quick and useful way of determining whether an answer, especially an algebraic one, is reasonable or not.

8.72

SET UP

A weighted rod of length L is rotated about an axis through its left end. The left half of the rod has a mass of M, while the right half has a mass of $4M$. We can use integration to determine the moment of inertia of the rod in this case.

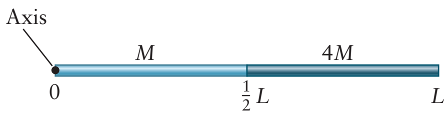


Figure 8-19 Problem 72

SOLVE

$$I_{rod} = \int_0^L x^2 dm = \int_0^{L/2} x^2 dm_{left} + \int_{L/2}^L x^2 dm_{right}$$

We can find dm_{left} and dm_{right} by remembering that

$$\frac{\text{length of slice}}{\text{length of rod}} = \frac{\text{mass of slice}}{\text{mass of rod}}$$

Therefore,

$$dm_{left} = \frac{M}{\left(\frac{L}{2}\right)} dx \text{ and } dm_{right} = \frac{4M}{\left(\frac{L}{2}\right)} dx$$

Plugging these into the integral, we find that

$$I_{rod} = \int_0^{L/2} x^2 dm_{left} + \int_{L/2}^L x^2 dm_{right} = \int_0^{L/2} x^2 \left(\frac{2M}{L}\right) dx + \int_{L/2}^L x^2 \left(\frac{8M}{L}\right) dx$$

$$= \left(\frac{2M}{L}\right)\left[\frac{x^3}{3}\right]_0^{L/2} + \left(\frac{8M}{L}\right)\left[\frac{x^3}{3}\right]_{L/2}^{L} = \left(\frac{ML^2}{12}\right) + \left(\frac{7ML^2}{3}\right) = \boxed{\frac{29ML^2}{12}}$$

REFLECT

We need to use integration to find the moment of inertia because the mass of the rod is not uniform throughout its entire length.

8.73

SET UP

A uniform, solid cylinder of radius $R = 5$ cm and mass $M = 3$ kg starts from rest at the top of an inclined plane of length $L = 2$ m tilted at an angle of 25° with the horizontal. The cylinder rolls without slipping down the ramp. We can calculate the final speed of the cylinder through conservation of energy. All of the initial gravitational potential energy is converted into both rotational kinetic energy and translational kinetic energy.

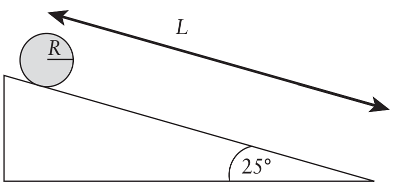


Figure 8-20 Problem 73

SOLVE

$$U_{g,\,i} = K_{rot,\,f} + K_{tr,\,f}$$

$$Mgh_i = \frac{1}{2} I \omega_f^2 + \frac{1}{2} M v_f^2$$

$$MgL \sin(25°) = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_f}{R}\right)^2 + \frac{1}{2} M v_f^2$$

$$gL \sin(25°) = \frac{1}{4} v_f^2 + \frac{1}{2} v_f^2 = \frac{3}{4} v_f^2$$

$$v_f = \sqrt{\frac{4}{3} gL \sin(25°)} = \sqrt{\frac{4}{3}\left(9.8 \frac{\text{m}}{\text{s}^2}\right)(2 \text{ m}) \sin(25°)} = \boxed{3.32 \frac{\text{m}}{\text{s}}}$$

REFLECT

The cylinder is both translating and rolling down the ramp, so we need to consider both types of kinetic energy. Also, the exact radius of the cylinder is not necessary in our calculation since it cancels out.

8.74

SET UP

A uniform, solid sphere of radius $R = 5$ cm and mass $M = 3$ kg starts with a translational speed of $v_{\text{i}} = 2$ m/s at the top of an inclined plane of length $L = 2$ m tilted at an angle of 25° with the horizontal. The sphere rolls without slipping down the ramp. We can calculate the final speed of the cylinder through conservation of energy. The initial gravitational potential energy, translational kinetic energy, and rotational kinetic energy are converted into both rotational kinetic energy and translational kinetic energy.

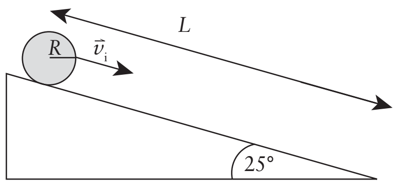


Figure 8-21 Problem 74

SOLVE

$$U_{\text{g, i}} + K_{\text{rot, i}} + K_{\text{tr, i}} = K_{\text{rot, f}} + K_{\text{tr, f}}$$

$$Mgh_{\text{i}} + \frac{1}{2}I\omega_{\text{i}}^2 + \frac{1}{2}Mv_{\text{i}}^2 = \frac{1}{2}I\omega_{\text{f}}^2 + \frac{1}{2}Mv_{\text{f}}^2$$

$$MgL\sin(25°) + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v_{\text{i}}}{R}\right)^2 + \frac{1}{2}Mv_{\text{i}}^2 = \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v_{\text{f}}}{R}\right)^2 + \frac{1}{2}Mv_{\text{f}}^2$$

$$gL\sin(25°) + \frac{1}{5}v_{\text{i}}^2 + \frac{1}{2}v_{\text{i}}^2 = \frac{1}{5}v_{\text{f}}^2 + \frac{1}{2}v_{\text{f}}^2$$

$$gL\sin(25°) + \frac{7}{10}v_{\text{i}}^2 = \frac{7}{10}v_{\text{f}}^2$$

$$v_{\text{f}} = \sqrt{\frac{10}{7}gL\sin(25°) + v_{\text{i}}^2} = \sqrt{\frac{10}{7}\left(9.81\frac{\text{m}}{\text{s}^2}\right)(2\text{ m})\sin(25°) + (2\text{ m})^2} = \boxed{3.98\frac{\text{m}}{\text{s}}}$$

REFLECT

At the beginning, the sphere is both translating and rolling down the ramp, so we need to consider both types of kinetic energy. Also, the exact radius of the sphere is not necessary in our calculation since it cancels out.

8.75

SET UP

A spherical marble (mass $M = 50$ g, radius $R_{\text{marble}} = 0.5$ cm, moment of inertia $I = \frac{2}{5}MR_{\text{marble}}^2$) starts from rest at the top of a ramp. The marble then begins to roll (without slipping) down the ramp toward a loop of radius $R_{\text{loop}} = 20$ cm. We are told the marble *just barely* makes it around the loop; this means the

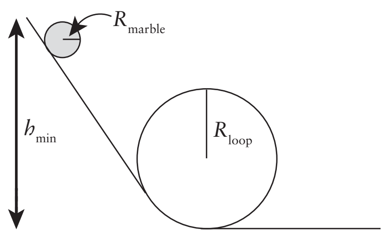


Figure 8-22 Problem 75

normal force from the loop on the marble is approximately zero. We also know the marble is undergoing centripetal motion when in the loop. We can analyze the forces on the marble at the top of the loop using Newton's second law to give us the (translational) speed of the ball at that point. With this in hand, we can use conservation of energy to determine the minimum height from which the marble must start—some of the initial gravitational potential energy is converted into rotational and translational kinetic energy.

SOLVE

Free-body diagram of the marble and Newton's second law (where up is $+y$):

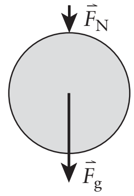


Figure 8-23 Problem 75

$$\sum F_y = -F_{\mathrm{N}} - F_{\mathrm{g}} = M\left(-\frac{v^2}{R_{\mathrm{loop}}}\right)$$

$$0 + Mg = M\left(\frac{v_{\mathrm{min}}^2}{R_{\mathrm{loop}}}\right)$$

$$v_{\mathrm{min}} = \sqrt{gR_{\mathrm{loop}}}$$

Conservation of energy:

$$Mgh_{\mathrm{min}} = Mg(2R_{\mathrm{loop}}) + \frac{1}{2}Mv_{\mathrm{min}}^2 + \frac{1}{2}I\omega_{\mathrm{min}}^2$$

$$Mgh_{\mathrm{min}} = Mg(2R_{\mathrm{loop}}) + \frac{1}{2}Mv_{\mathrm{min}}^2 + \frac{1}{2}\left(\frac{2}{5}MR_{\mathrm{marble}}^2\right)\left(\frac{v_{\mathrm{min}}^2}{R_{\mathrm{marble}}^2}\right)$$

$$gh_{\mathrm{min}} = g(2R_{\mathrm{loop}}) + \frac{1}{2}(gR_{\mathrm{loop}}) + \frac{1}{5}(gR_{\mathrm{loop}})$$

$$h_{\mathrm{min}} = \frac{27}{10}R_{\mathrm{loop}} = \frac{27}{10}(20 \times 10^{-2}\ \mathrm{m}) = 54 \times 10^{-2}\ \mathrm{m} = \boxed{54\ \mathrm{cm}}$$

REFLECT

We should place the marble 54 cm off the ground, or 14 cm above the top of the loop. If we replace the marble with a block that doesn't roll, the minimum height necessary would be smaller (50 cm) since the rotational kinetic energy of the object would be zero.

8.76

SET UP

A billiard ball of mass $M = 160$ g, radius $R_{\mathrm{ball}} = 2.5$ cm, and moment of inertia $I = \frac{2}{5}MR_{\mathrm{ball}}^2$ is rolling with a translational speed of $v_{\mathrm{i}} = 2$ m/s at point A. The ball continues rolling along the track toward point B, which is on the top of a hill with a radius of curvature $R_{\mathrm{hill}} = 60$ cm and is located 10 cm below point A. Conservation

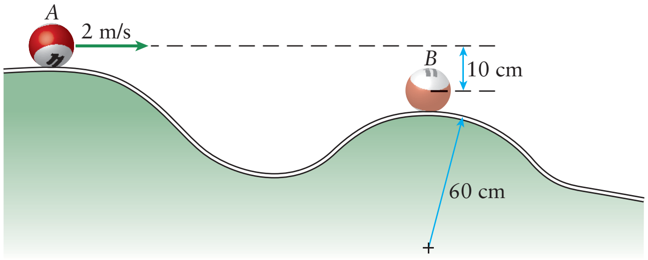


Figure 8-24 Problem 76

of energy will give us the speed of the ball at point B. For our potential energy calculation, we'll say point B is at a height of zero. The ball undergoes centripetal motion when traveling over the hill. We can use Newton's second law to determine the magnitude of the normal force on the ball at point B.

SOLVE

Conservation of energy:

$$U_{g,\,i} + K_{tr,\,i} + K_{rot,\,i} = U_{g,\,f} + K_{tr,\,f} + K_{rot,\,f}$$

$$Mgh_i + \frac{1}{2}Mv_i^2 + \frac{1}{2}I\omega_i^2 = Mgh_f + \frac{1}{2}Mv_f^2 + \frac{1}{2}I\omega_f^2$$

$$Mgh_i + \frac{1}{2}Mv_i^2 + \frac{1}{2}\left(\frac{2}{5}MR_{ball}^2\right)\left(\frac{v_i^2}{R_{ball}^2}\right) = 0 + \frac{1}{2}Mv_f^2 + \frac{1}{2}\left(\frac{2}{5}MR_{ball}^2\right)\left(\frac{v_f^2}{R_{ball}^2}\right)$$

$$gh_i + \frac{1}{2}v_i^2 + \frac{1}{5}v_i^2 = \frac{1}{2}v_f^2 + \frac{1}{5}v_f^2$$

$$v_f^2 = \frac{10}{7}gh_i + v_i^2 = \frac{10}{7}\left(9.81\frac{\text{m}}{\text{s}^2}\right)(0.1\text{ m}) + \left(2\frac{\text{m}}{\text{s}}\right)^2 = 5.40\frac{\text{m}^2}{\text{s}^2}$$

Free-body diagram of the marble and Newton's second law (where up is $+y$):

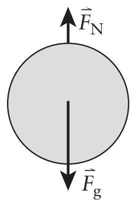


Figure 8-25 Problem 76

$$\sum F_y = F_N - F_g = M\left(-\frac{v_f^2}{R_{ball}}\right)$$

$$F_N = F_g - M\left(\frac{v_f^2}{R_{ball}}\right) = Mg - M\left(\frac{v_f^2}{R_{ball}}\right) = M\left(g - \frac{v_f^2}{R_{ball}}\right)$$

$$= (0.160\text{ kg})\left(\left(9.81\frac{\text{m}}{\text{s}^2}\right) - \frac{\left(5.40\frac{\text{m}}{\text{s}^2}\right)}{(0.60\text{ m})}\right) = \boxed{0.129\text{ N}}$$

REFLECT

Because we only need to know v_f^2 in the Newton's second law portion, we save ourselves a step by not explicitly calculating v_f.

8.77

SET UP

A bowling ball of known mass $m = 5$ kg and radius $R = 11$ cm is rolling without slipping down a lane at an angular speed of $\omega = 2\frac{\text{rad}}{\text{s}}$. We need to determine the ratio of the ball's translational kinetic energy $\left(K_{tr} = \frac{1}{2}mv_{CM}^2\right)$ to its rotational kinetic energy $\left(K_{rot} = \frac{1}{2}I\omega^2\right)$. The translational speed of the ball is related to the angular speed by $v_{CM} = R\omega$. The ball is a sphere, so its moment of inertia is $I = \frac{2}{5}mR^2$.

SOLVE

$$\frac{K_{tr}}{K_{rot}} = \frac{\left(\frac{1}{2}mv_{CM}^2\right)}{\left(\frac{1}{2}I\omega^2\right)} = \frac{m(R\omega)^2}{\left(\frac{2}{5}mR^2\right)\omega^2} = \boxed{\frac{5}{2}}$$

REFLECT

The solution is *independent* of the mass of the ball, the size of the ball, or the speed the ball is rolling!

8.78

SET UP

The Earth has a mass of $m_E = 5.98 \times 10^{24}$ kg and a radius of $R_E = 6.38 \times 10^6$ m. It completes one full rotation (2π radians) about its own axis once in a day. The Earth can be modeled as a solid sphere rotating about an axis through its center of mass, which has a moment of inertia of $I = \frac{2}{5}m_E R_E^2$. We can then calculate the rotational kinetic energy of the Earth from all of this information.

SOLVE

$$\omega = \frac{2\pi \text{ rad}}{1 \text{ day}} \times \frac{1 \text{ day}}{24 \text{ hr}} \times \frac{1 \text{ hr}}{3600 \text{ s}} = 7.27 \times 10^{-5}\frac{\text{rad}}{\text{s}}$$

$$K_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{2}{5}m_E R_E^2\right)\omega^2 = \frac{1}{5}(5.98 \times 10^{24} \text{ kg})(6.38 \times 10^6 \text{ m})^2\left(7.27 \times 10^{-5}\frac{\text{rad}}{\text{s}}\right)^2$$

$$= \boxed{2.57 \times 10^{29} \text{ J}}$$

REFLECT

Although the angular speed of the Earth's rotation is small, its moment of inertia is huge (as one would expect).

8.79

SET UP

The Earth has a mass of $m_E = 5.98 \times 10^{24}$ kg and is, on average, $R_{ES} = 1.5 \times 10^{11}$ m from the Sun. It completes one full revolution around the Sun in a year. If we assume the orbit is circular, the distance the Earth travels in one year is $2\pi R_{ES}$. Assuming the translational speed of the Earth around the Sun is constant, the speed of the Earth is $(2\pi R_{ES})/(1 \text{ year})$. We can then calculate the translational kinetic energy of the Earth and compare it to the rotational kinetic energy we calculated in Problem 8.78.

SOLVE

$$v = \frac{2\pi(1.5 \times 10^{11} \text{ m})}{1 \text{ yr}} \times \frac{1 \text{ yr}}{365.25 \text{ days}} \times \frac{1 \text{ day}}{24 \text{ hr}} \times \frac{1 \text{ hr}}{3600 \text{ s}} = 2.99 \times 10^4\frac{\text{m}}{\text{s}}$$

$$K_{\text{tr}} = \frac{1}{2}m_{\text{E}}v^2 = \frac{1}{5}(5.98 \times 10^{24}\text{ kg})\left(2.99 \times 10^4\frac{\text{m}}{\text{s}}\right)^2 = \boxed{2.67 \times 10^{33}\text{ J}}$$

$$\frac{K_{\text{tr}}}{K_{\text{rot}}} = \frac{2.67 \times 10^{33}\text{ J}}{2.57 \times 10^{29}\text{ J}} = \boxed{10{,}400}$$

REFLECT

The translational kinetic energy of the Earth around the Sun is four orders of magnitude larger than the rotational kinetic energy of the Earth about its own axis.

8.80

SET UP

A potter's flywheel is a 5-cm-thick, round piece of concrete with a mass of $M = 60$ kg and a diameter of $d = 35$ cm. Since the disk rotates about an axis passing through its center, perpendicular to its round area, we can model it as a cylinder with a moment of inertia of $I = \frac{1}{2}MR^2$, where $R = d/2$. We are given the rotational kinetic energy of the flywheel and need to find its angular speed.

SOLVE

$$K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}M\left(\frac{d}{2}\right)^2\right)\omega^2 = \frac{1}{16}Md^2\omega^2$$

$$\omega = \sqrt{\frac{16K_{\text{rot}}}{Md^2}} = \sqrt{\frac{16(15\text{ J})}{(60\text{ kg})(0.35\text{ m})^2}} = \boxed{5.71\frac{\text{rad}}{\text{s}}} \times \frac{60\text{ s}}{1\text{ min}} \times \frac{1\text{ rev}}{2\pi\text{ rad}} = \boxed{54.6\text{ rpm}}$$

REFLECT

The thickness of the disk is not necessary for this calculation.

8.81

SET UP

A Frisbee of mass $m = 0.160$ kg and diameter $d = 0.25$ m is spinning at an angular speed of 300 rpm about an axis through its center. First we are asked to find the rotational kinetic energy of the Frisbee. We are given the angular speed of the Frisbee but not the moment of inertia. The mass is *not* uniformly distributed in this case—70% of the mass can be modeled as a thin ring of diameter d, while the remaining 30% of the mass can be modeled as a flat disk of diameter d. The moment of inertia of the Frisbee is the sum of the moments of inertia for the ring and the disk. Knowing that only 60% of the rotational kinetic energy is converted into gravitational potential energy, we can solve for the maximum height the Frisbee attains above the release point.

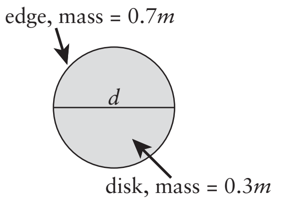


Figure 8-26 Problem 81

SOLVE

Kinetic energy of the Frisbee:

$$I_{\text{Frisbee}} = I_{\text{ring}} + I_{\text{disk}} = m_{\text{ring}}r_{\text{ring}}^2 + \frac{1}{2}m_{\text{disk}}r_{\text{disk}}^2 = (0.7m)\left(\frac{d}{2}\right)^2 + \frac{1}{2}(0.3m)\left(\frac{d}{2}\right)^2$$

$$= (0.7)(0.160\text{ kg})\left(\frac{0.25\text{ m}}{2}\right)^2 + \frac{1}{2}(0.3)(0.160\text{ kg})\left(\frac{0.25\text{ m}}{2}\right)^2 = 0.002125\text{ kg}\cdot\text{m}^2$$

$$K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}(0.002125\text{ kg}\cdot\text{m}^2)\left(300\frac{\text{rev}}{\text{min}} \times \frac{1\text{ min}}{60\text{ s}} \times \frac{2\pi\text{ rad}}{1\text{ rev}}\right)^2 = 1.05\text{ J}$$

Maximum height:

$$0.6K_{\text{rot}} = mgh$$

$$h = \frac{0.6K_{\text{rot}}}{mg} = \frac{0.6(1.05\text{ J})}{(0.16\text{ kg})\left(9.81\frac{\text{m}}{\text{s}^2}\right)} = \boxed{0.401\text{ m}}$$

REFLECT

This works out to be about 1.3 feet, which is reasonable. If the energy conversion were 100% efficient, the maximum height would have been 0.669 m or 2.2 ft.

8.82

SET UP

We are given the angular acceleration as a function of time of a rotating disk. The disk starts from rest, and we are asked to find the angular displacement of the disk from $t = 0$ s to $t = 18$ s. Angular acceleration is the second derivative of the angle with respect to time, which means we need to integrate twice with respect to time: first to find the angular speed, then to find the angular displacement.

SOLVE

$$\alpha(t) = \frac{d\omega}{dt} = 0.2t^2 - 1.25t + 12$$

$$\omega(t) = \int d\omega = \int(0.2t^2 - 1.25t + 12)dt = \frac{0.2}{3}t^3 - \frac{1.25}{2}t^2 + 12t + C$$

$$\omega(t = 0) = 0\text{, so } C = 0.$$

Therefore,

$$\omega(t) = \frac{d\theta}{dt} = \frac{0.2}{3}t^3 - \frac{1.25}{2}t^2 + 12t$$

$$\Delta\theta = \int d\theta = \int_0^{18}\left(\frac{0.2}{3}t^3 - \frac{1.25}{2}t^2 + 12t\right)dt = \left[\frac{0.2}{12}t^4 - \frac{1.25}{6}t^3 + 6t^2\right]_0^{18}$$

$$= \left[\frac{0.2}{12}(18)^4 - \frac{1.25}{6}(18)^3 + 6(18)^2\right] = \boxed{2479\text{ rad}}$$

REFLECT

This is approximately 395 revolutions, which corresponds to an average angular speed of about 22 rad/s.

8.83

SET UP

A spinning top completes 6000 revolutions before it starts to topple over. The average angular speed of the top is 800 rpm. We can directly solve for the time interval from the definition of average angular speed.

SOLVE

$$\omega_{\text{average}} = \frac{\Delta\theta}{\Delta t}$$

$$\Delta t = \frac{\Delta\theta}{\omega_{\text{average}}} = \frac{6000 \text{ rev}}{\left(800\frac{\text{rev}}{\text{min}}\right)} = \boxed{7.5 \text{ min}}$$

REFLECT

The given units for the angular displacement and average angular speed are both in terms of revolutions, so we do not need to convert them before our calculation.

8.84

SET UP

A merry-go-round with a diameter of $d = 4$ m starts from rest and is spun up to a final angular speed of 18 rpm in 43 s. From this information we can directly calculate the tangential speed of a child on the rim, $v = \omega R$, and the average angular acceleration of the merry-go-round, $\alpha_{\text{average}} = \frac{\Delta\omega}{\Delta t}$. Once we have the average angular acceleration, we can then use the kinematics equations for constant angular acceleration to find the angular displacement.

SOLVE

$$18\frac{\text{rev}}{\text{min}} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} = 1.88\frac{\text{rad}}{\text{s}}$$

$$v = \omega R = \omega\left(\frac{d}{2}\right) = \left(1.88\frac{\text{rad}}{\text{s}}\right)\left(\frac{4 \text{ m}}{2}\right) = \boxed{3.77\frac{\text{m}}{\text{s}}}$$

$$\alpha_{\text{average}} = \frac{\Delta\omega}{\Delta t} = \frac{\left(1.88\frac{\text{rad}}{\text{s}}\right) - 0}{43 \text{ s}} = \boxed{0.0438\frac{\text{rad}}{\text{s}^2}}$$

$$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2}\left(0.0438\frac{\text{rad}}{\text{s}^2}\right)(43 \text{ s})^2 = \boxed{40.5 \text{ rad}}$$

REFLECT

We don't know the exact angular acceleration as a function of time, but we could easily calculate the average angular acceleration. The average value is a constant, which allows us to use the constant angular acceleration kinematics equations to solve this problem.

8.85

SET UP

An open umbrella (radius $R = 55$ cm) starts at rest and is then twirled around its central axis. It completes 24 full revolutions in 30 s. Assuming the angular acceleration is constant, we can calculate it from the angular kinematics equations. Once we have the angular acceleration, we can use it to find the final angular speed and the maximum tangential speed.

SOLVE

$$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2}\alpha t^2$$

$$\alpha = \frac{2(\Delta\theta)}{t^2} = \frac{2\left(24\text{ rev} \times \dfrac{2\pi\text{ rad}}{1\text{ rev}}\right)}{(30\text{ s})^2} = \boxed{0.335\frac{\text{rad}}{\text{s}^2}}$$

$$v = \omega_f R = (\omega_0 + \alpha t)R = (0 + \alpha t)R = \left(0.335\frac{\text{rad}}{\text{s}^2}\right)(30\text{ s})(0.55\text{ m}) = \boxed{5.53\frac{\text{m}}{\text{s}}}$$

REFLECT

One rotation is the same as one revolution; both are equal to an angular displacement of 2π radians.

8.86

SET UP

It takes a turntable 4.5 s to reach a final angular speed of (100/3) rpm when starting from rest. We can use the average angular speed to calculate the total number of rotations the turntable undergoes when starting. We can follow the same procedure to determine the number of rotations the turntable undergoes when slowing down. In between, the turntable spins at a constant angular speed for 2827 s (47 min, 7 s); we can easily determine the number of rotations from the definition of angular speed. The sum of these three values—speeding up, spinning at constant speed, slowing down—will be the total number of rotations of the album.

SOLVE

Starting up:

$$\Delta\theta = \omega_{\text{average}} t = \left(\frac{\omega_f + \omega_i}{2}\right)t = \left(\frac{\left(\dfrac{100\text{ rev}}{3\text{ min}} \times \dfrac{1\text{ min}}{60\text{ s}}\right) + 0}{2}\right)(4.5\text{ s}) = \boxed{1.25\text{ rev}}$$

Spinning at constant speed:

$$\Delta\theta = \omega t = \left(\frac{100\text{ rev}}{3\text{ min}} \times \frac{1\text{ min}}{60\text{ s}}\right)(2827\text{ s}) = \boxed{1570.56\text{ rev}}$$

Slowing down:

$$\Delta\theta = \omega_{\text{average}} t = \left(\frac{\omega_f + \omega_i}{2}\right)t = \left(\frac{0 + \left(\dfrac{100\text{ rev}}{3\text{ min}} \times \dfrac{1\text{ min}}{60\text{ s}}\right)}{2}\right)(8\text{ s}) = \boxed{2.22\text{ rev}}$$

Total number of rotations:

$$\Delta\theta_{\text{total}} = (1.25 \text{ rev}) + (1570.56 \text{ rev}) + (2.22 \text{ rev}) = \boxed{1574.03 \text{ rev}}$$

REFLECT

As a quick mental check, we can estimate the angular speed by dividing the total number of rotations (approximately 1574) by the total time of the album (approximately 47 min). This comes out to about 33.5 rpm, which is reasonable.

8.87

SET UP

A CD player initially rotates at 300 rpm. It speeds up to 450 rpm within 0.75 s. We will assume that the CD player undergoes constant acceleration from 300 rpm to 450 rpm and we will use rotational kinematics. We will need to convert from rpm to rad/s.

SOLVE

$$450\frac{\text{rev}}{\text{min}} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} = 15\pi\frac{\text{rad}}{\text{s}}$$

$$300\frac{\text{rev}}{\text{min}} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} = 10\pi\frac{\text{rad}}{\text{s}}$$

$$\omega_{\text{f}} = \omega_{\text{i}} + \alpha t$$

$$\alpha = \frac{\omega_{\text{f}} - \omega_{\text{i}}}{t} = \frac{\left(15\pi\frac{\text{rad}}{\text{s}}\right) - \left(10\pi\frac{\text{rad}}{\text{s}}\right)}{0.75 \text{ s}} = \frac{20\pi}{3}\frac{\text{rad}}{\text{s}^2} = \boxed{21\frac{\text{rad}}{\text{s}^2}}$$

REFLECT

Although this seems large, it is reasonable given that the increase of 150 rpm happens in under a second. Remember to convert rpm into rad/s using 1 minute = 60 seconds and 1 revolution = 2π radians.

8.88

SET UP

A satellite circles the Earth in a geosynchronous orbit, which means it takes the satellite one day to make a full rotation (just like the Earth). Knowing this, we can set up a ratio to determine the angular displacement of the satellite in one hour. The fact that the satellite's orbit is geosynchronous also lets us calculate its angular speed directly from the definition, assuming it is constant.

SOLVE

$$\frac{2\pi \text{ rad}}{24 \text{ hr}} = \frac{\Delta\theta}{1 \text{ hr}}$$

$$\Delta\theta = \left(\frac{2\pi \text{ rad}}{24 \text{ hr}}\right)(1 \text{ hr}) = \frac{\pi}{12}\text{rad} = \boxed{0.26 \text{ rad}}$$

$$\omega = \frac{1\text{ rev}}{24\text{ hr}} \times \frac{1\text{ hr}}{60\text{ min}} = \boxed{6.94 \times 10^{-4}\text{ rpm}}$$

$$\omega = \frac{1\text{ rev}}{24\text{ hr}} \times \frac{1\text{ hr}}{3600\text{ s}} \times \frac{2\pi\text{ rad}}{1\text{ rev}} = \boxed{7.27 \times 10^{-5}\frac{\text{rad}}{\text{s}}}$$

REFLECT

Since the satellite's orbit is geosynchronous, it should have the same angular speed as the Earth rotating about its axis.

8.89

SET UP

A roulette wheel is initially spinning at an angular speed of $\omega_i = 1$ rev/s and experiences a constant angular acceleration of -0.02 rad/s^2. We can determine the time it takes for the wheel to come to rest (that is, a final angular speed of zero) and the number of rotations it completes by angular kinematics.

SOLVE

$$\omega_f = \omega_i + \alpha t$$

$$t = \frac{\omega_f - \omega_i}{\alpha} = \frac{0 - \left(1\frac{\text{rev}}{\text{s}} \times \frac{2\pi\text{ rad}}{1\text{ rev}}\right)}{\left(-0.02\frac{\text{rad}}{\text{s}^2}\right)} = \boxed{314.2\text{ s}}$$

$$\omega_f^2 - \omega_i^2 = 2\alpha(\Delta\theta)$$

$$\Delta\theta = \frac{\omega_f^2 - \omega_i^2}{2\alpha} = \frac{0 - \left(2\pi\frac{\text{rad}}{\text{s}}\right)^2}{2\left(-0.02\frac{\text{rad}}{\text{s}^2}\right)} = 987\text{ rad} \times \frac{1\text{ rev}}{2\pi\text{ rad}} = \boxed{157\text{ rev}}$$

REFLECT

An angular acceleration of -0.02 rad/s^2 is approximately -0.003 rev/s^2, so it is reasonable that it takes about 333 s (= 1000/3) for the wheel to come to rest.

8.90

SET UP

A driver applies a force of 20 N to the right to the top of a steering wheel of radius $R = 18$ cm and moment of inertia $0.097\text{ kg}\cdot\text{m}^2$. We can use the definition of torque to calculate the magnitude of the torque exerted about the center of the steering wheel by the driver. The angle θ between R and the force is 90 degrees. The torque is related to the angular acceleration of the steering wheel by Newton's second law for rotation.

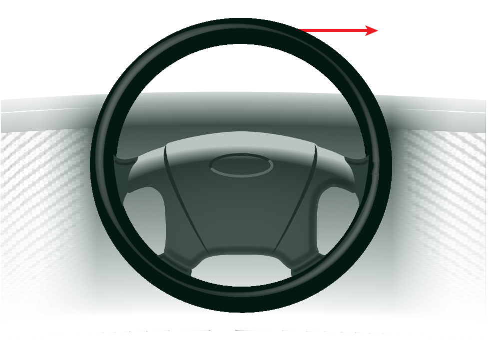


Figure 8-27 Problem 90

SOLVE

$$\sum \tau = \tau_{\text{driver}} = RF\sin(\theta) = I\alpha$$

$$\alpha = \frac{RF\sin(\theta)}{I} = \frac{(0.18 \text{ m})(20 \text{ N})\sin(90°)}{(0.097 \text{ kg} \cdot \text{m}^2)} = \boxed{37.1\frac{\text{rad}}{\text{s}^2}}$$

REFLECT

Remember that R is the distance from the rotation axis to the location where the force is applied, which is the radius of the steering wheel in this case.

8.91

SET UP

The palmaris longus muscle, which causes the wrist to move back and forth, can exert a force of 45 N on the wrist with an effective lever arm of 22 cm. The lever arm is the perpendicular distance between the axis of rotation and the line of force, which means the angle is 90 degrees when applying the definition of torque.

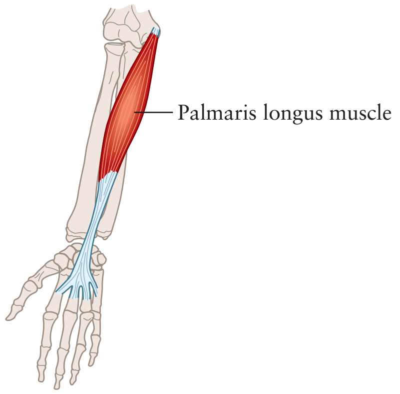


Figure 8-28 Problem 91

SOLVE

$$\tau = RF\sin(\theta) = (0.22 \text{ m})(45 \text{ N})\sin(90°) = \boxed{9.9 \text{ N} \cdot \text{m}}$$

REFLECT

The term *lever arm* refers to the shortest distance between the rotation axis and the line of force and corresponds to the $R\sin(\theta)$ portion of the torque.

8.92

SET UP

A force of 120 N is applied to a torque wrench, 25 cm from the nut and bolt. The angle between the $\vec{r}$ vector and the force is 110 degrees. (Recall that the $\vec{r}$ vector starts from the axis of rotation and ends at the location where the force is applied.) The definition of torque can be used to calculate torque about the axis that passes through the bolt.

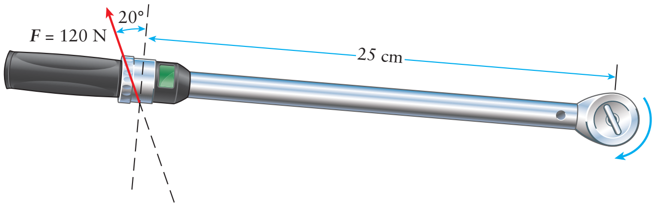


Figure 8-29 Problem 92

SOLVE

$$\tau = RF\sin(\theta) = (0.25 \text{ m})(120 \text{ N})\sin(110°) = \boxed{28.2 \text{ N} \cdot \text{m}}$$

REFLECT

We could also use the $\tau = r_\perp F$ form of torque, where $r_\perp$ is the moment arm. Recall that the moment arm is the perpendicular distance from the rotation axis to the line of force. From the geometry in this example, $r_\perp = (0.25 \text{ m})\cos(20°) = 0.235$ m. Multiplying this by 120 N gives the same answer as above.

8.93

SET UP

A 75-N force acts on an 85-cm-wide door at various angles in parts (a)–(d). We are asked to calculate the torque that this force exerts about an axis through the hinges in each case. Because the force is exerted in the same spot on the door, the magnitude of $\vec{r}$ is the same in each case, namely, 85 cm. The angle θ between $\vec{r}$ and $\vec{F}$ changes, though.

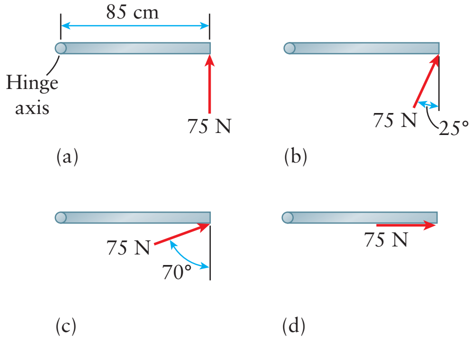


Figure 8-30 Problem 93

SOLVE

Part a) $\theta = 90°$, so

$\tau = RF\sin(\theta) = (0.85\text{ m})(75\text{ N})\sin(90°) = \boxed{63.8\text{ N}\cdot\text{m}}$.

Part b) $\theta = 115°$, so $\tau = RF\sin(\theta) = (0.85\text{ m})(75\text{ N})\sin(115°) = \boxed{57.8\text{ N}\cdot\text{m}}$.

Part c) $\theta = 160°$, so $\tau = RF\sin(\theta) = (0.85\text{ m})(75\text{ N})\sin(160°) = \boxed{21.8\text{ N}\cdot\text{m}}$.

Part d) $\theta = 0°$, so $\tau = RF\sin(\theta) = (0.85\text{ m})(75\text{ N})\sin(0°) = \boxed{0}$.

REFLECT

The maximum torque should occur when the force is applied perpendicularly to the door (part a), and the minimum torque should occur when the force is applied parallel to the door (part d). For angles in between, we expect the torque to be an intermediate value.

8.94

SET UP

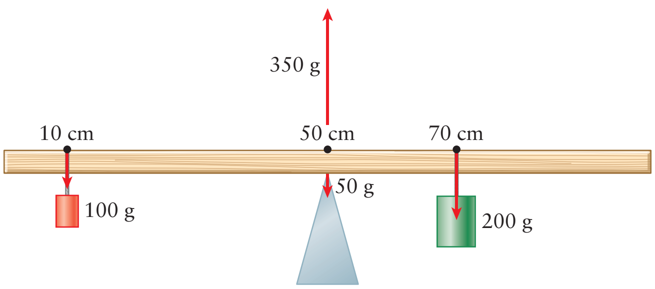


Figure 8-31 Problem 94

A 50-g meter stick has two masses hanging from it—a 100-g mass is 10 cm from the left end and a 200-g mass is 70 cm from the right end. The meter stick and masses are balanced on a fulcrum at the 50-cm point. The entire system is at rest. We need to calculate the clockwise and counterclockwise torques for all of the forces acting on the board about three different pivot points. First we need to draw a free-body diagram of the board to show the four forces—the contact force of the 100-g mass on the board, the contact force of the 200-g mass on the board, the weight of the meter stick, and the normal force from the fulcrum. Because the individual hanging masses are each at rest, the two contact forces on the board have magnitudes equal to the respective weights of the masses. Using this information and Newton's second law, we can find the magnitude of the normal force. Once we have the magnitudes of all of the forces we can calculate the torque due to each force about each axis. Remember that R in the definition of torque is the distance from the *rotation axis* to the application of the force. All of the forces acting on the board act perpendicularly to the board, which means $\theta = 90$ degrees for each torque.

SOLVE

Free-body diagram of the board:

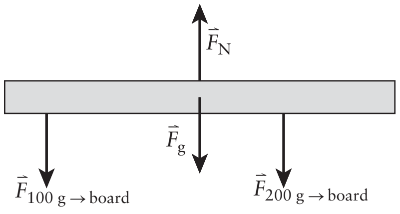


Figure 8-32 Problem 94

Finding the magnitude of the normal force:

$$\sum F_y = F_N - F_{100\text{ g}\to\text{board}} - F_g - F_{200\text{ g}\to\text{board}} = 0$$

$$F_N = F_{100\text{ g}\to\text{board}} + F_g + F_{200\text{ g}\to\text{board}} = (0.1\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right) + (0.05\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right) + (0.2\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)$$
$$= 3.43\text{ N}$$

Part a) Summing torques about an axis at 0 cm:

$$\tau_{CW} = \tau_{100\text{ g}} + \tau_g + \tau_{200\text{ g}} = (0.1\text{ m})(0.1\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)\sin(90°)$$
$$+(0.5\text{ m})(0.05\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)\sin(90°) + (0.7\text{ m})(0.2\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)\sin(90°) = \boxed{1.715\text{ N}\cdot\text{m}}$$

$$\tau_{CCW} = \tau_N = (0.5\text{ m})(0.35\text{ N})\sin(90°) = \boxed{1.715\text{ N}\cdot\text{m}}$$

Part b) Summing torques about an axis at 50 cm:

$\tau_N = 0 = \tau_g$ because the force acts at the chosen pivot and, therefore, $R = 0$ in both cases.

$$\tau_{CW} = \tau_{200\text{ g}} = (0.2\text{ m})(0.2\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)\sin(90°) = \boxed{0.392\text{ N}\cdot\text{m}}.$$

$$\tau_{CCW} = \tau_{100\text{ g}} = (0.4\text{ m})(0.1\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)\sin(90°) = \boxed{0.392\text{ N}\cdot\text{m}}.$$

Part c) Summing torques about an axis at 100 cm:

$$\tau_{CW} = \tau_N = (0.5\text{ m})(0.35\text{ N})\sin(90°) = \boxed{1.715\text{ N}\cdot\text{m}}.$$

$$\tau_{CCW} = \tau_{100\text{ g}} + \tau_g + \tau_{200\text{ g}} = (0.9\text{ m})(0.1\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)\sin(90°)$$
$$+ (0.5\text{ m})(0.05\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)\sin(90°) + (0.3\text{ m})(0.2\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)\sin(90°) = \boxed{1.715\text{ N}\cdot\text{m}}.$$

REFLECT

When the system is in static equilibrium, the torques must sum to zero about *any* axis. The system must remain at rest regardless of the pivot chosen.

8.95

SET UP

A robotic arm can withstand a maximum torque of $\tau_{\text{max}} = 3000\ \text{N} \cdot \text{m}$ about the axis O. The distance of the arm from O to the claw is 3 m. The maximum mass the arm can lift occurs when the contact force of the barrel on the arm, which has a magnitude of the barrel's weight, acts 3 m from O at an angle of 90 degrees to the arm.

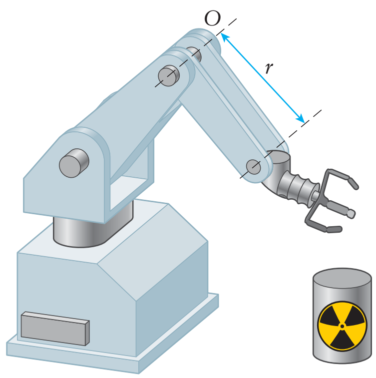


Figure 8-33 Problem 95

SOLVE

$$\tau_{\text{max}} = RF\sin(90°) = R(Mg)$$

$$M = \frac{\tau_{\text{max}}}{Rg} = \frac{3000\ \text{N} \cdot \text{m}}{(3\ \text{m})\left(9.81\frac{\text{m}}{\text{s}^2}\right)} = \boxed{102\ \text{kg}}.$$

REFLECT

This works out to a weight of approximately 220 lb, which seems reasonable for a canister that would require a robot to lift.

8.96

SET UP

An average adult can exert $10\ \text{N} \cdot \text{m}$ of torque when unscrewing a bottle cap. Assuming the force is applied tangent to the cap (and, thus, perpendicular to the radius), the maximum force exerted can be calculated directly.

SOLVE

$$\tau = RF\sin(90°) = RF$$

$$F = \frac{\tau}{R} = \frac{10\ \text{N} \cdot \text{m}}{0.01\ \text{m}} = \boxed{1000\ \text{N}}$$

REFLECT

This value seems a bit high (approximately 220 lb of force) but is probably on the high end of "reasonable."

8.97

SET UP

A person is holding a barbell of mass $m = 10$ kg in a completely outstretched hand of length $L = 0.75$ m. We're asked to calculate the magnitude of the torque due to the barbell about

the shoulder, $\tau = RF\sin(\theta)$. The force of interest is the force of the barbell on the hand, which acts downward. We need to draw a free-body diagram of the barbell at rest and use Newton's second law in order to determine the magnitude of this force. The force of the arm on the barbell is related to the force of the barbell on the arm through Newton's third law. The distance from the rotation axis to the point where the force is applied is the length of the arm, L. Since the force of the barbell on the arm acts straight down and the arm is completely outstretched, the angle between the force vector and the $\vec{r}$ vector is 90°.

SOLVE

Free-body diagram of the barbell and Newton's second law:

$$\sum F_y = F_{\text{arm}\to\text{barbell}} - F_g = ma_y = 0$$

$$F_{\text{arm}\to\text{barbell}} = F_g = mg$$

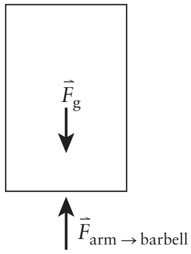


Figure 8-34 Problem 97

Newton's third law:

$$F_{\text{arm}\to\text{barbell}} = F_{\text{barbell}\to\text{arm}} = mg$$

Solving for the magnitude of the torque,

$$\tau = RF\sin(\theta) = Lmg\sin(90°) = (0.75\text{ m})(10\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right) = \boxed{73.5\text{ N}\cdot\text{m}}$$

REFLECT

Since we're interested in the forces acting on the arm, we use the contact force of the barbell on the arm, not the *weight* of the barbell, in calculating the torque. The weight of the barbell is the force on the barbell due to the Earth, which would not appear on a free-body diagram of the arm and is *not* the third law partner of the contact force. The two forces are related, though; we just need to draw a free-body diagram for the barbell at rest and solve Newton's second law.

8.98

SET UP

A thin disk of mass $M = 2.5$ kg and diameter $d = 18$ cm is spinning at a constant angular speed of $\omega = 1.25$ rad/s about its central axis. We can use the moment of inertia of a thin disk, $I = \frac{1}{2}MR^2$, to calculate the magnitude of its angular momentum using $L = I\omega$.

SOLVE

$$L = I\omega = \left(\frac{1}{2}MR^2\right)\omega = \frac{1}{2}M\left(\frac{d}{2}\right)^2\omega$$

$$= \frac{1}{8}Md^2\omega = \frac{1}{8}(2.5\text{ kg})(0.18\text{ m})^2\left(1.25\frac{\text{rad}}{\text{s}}\right) = \boxed{0.0127\frac{\text{kg}\cdot\text{m}^2}{\text{s}}}.$$

REFLECT

Remember that the radius, not the diameter, is used in the moment of inertia of the thin disk.

8.99

SET UP

A 300-g tetherball is rotating a distance $R = 125$ cm from the central pole at an angular speed of 60 rpm. We can treat the ball as a point mass with a moment of inertia of $I = MR^2$ and then calculate its angular momentum.

SOLVE

$$\omega = 60\frac{\text{rev}}{\text{min}} \times \frac{1\text{ min}}{60\text{ s}} \times \frac{2\pi\text{ rad}}{1\text{ rev}} = 2\pi\frac{\text{rad}}{\text{s}}$$

$$L = I\omega = (MR^2)\omega = (0.3\text{ kg})(1.25\text{ m})^2\left(2\pi\frac{\text{rad}}{\text{s}}\right) = \boxed{2.95\frac{\text{kg}\cdot\text{m}^2}{\text{s}}}$$

REFLECT

Remember to convert from rpm to rad/s when using the angular speed.

8.100

SET UP

The Earth orbits the Sun once in 365.3 days (= 1 year). It is easiest to use $L = MvR$, where M is the mass of the Earth, v is the linear speed of the Earth, and R is the distance between the Earth and the Sun, to calculate the angular momentum of the Earth in this case because we are given the period of the Earth's motion and R. Assuming the Earth's orbit is circular, the Earth traverses a total distance of $2\pi R$ in 365.3 days. The linear speed v is this distance divided by the time interval.

SOLVE

$$T = 365.3\text{ days} \times \frac{24\text{ hr}}{1\text{ day}} \times \frac{3600\text{ s}}{1\text{ hr}} = 3.16 \times 10^7\text{ s}$$

$$L = MvR = M\left(\frac{2\pi R}{T}\right)R = \frac{2\pi MR^2}{T} = \frac{2\pi(5.98 \times 10^{24}\text{ kg})(1.50 \times 10^{11}\text{ m})^2}{3.16 \times 10^7\text{ s}}$$

$$= \boxed{2.68 \times 10^{40}\frac{\text{kg}\cdot\text{m}^2}{\text{s}}}$$

REFLECT

We should expect the angular momentum of the Earth orbiting the Sun to be very large due to the size of the numbers involved.

8.101

SET UP

The Earth completes a full rotation of 2π radians in 1 day. The moment of inertia of the Earth about its central axis is $I = \frac{2}{5}MR^2$, assuming that it is a uniform, solid sphere. Converting the angular speed of the Earth from rad/day to rad/s and multiplying it by the moment of inertia will give the magnitude of the Earth's angular momentum about its axis.

SOLVE

$$L = I\omega = \left(\frac{2}{5}MR^2\right)\omega = \frac{2}{5}(5.98 \times 10^{24}\text{ kg})(6.38 \times 10^6\text{ m})^2\left(\frac{2\pi\text{ rad}}{1\text{ day}} \times \frac{1\text{ day}}{24\text{ hr}} \times \frac{1\text{ hr}}{3600\text{ s}}\right)$$

$$= \boxed{7.08 \times 10^{33}\frac{\text{kg}\cdot\text{m}^2}{\text{s}}}$$

REFLECT

It makes sense that the magnitude of the angular momentum of the Earth spinning about its own axis is much smaller than the angular momentum of the Earth orbiting about the Sun (Problem 8.100) because the radius of the Earth is much smaller than the Earth–Sun distance.

8.102

SET UP

An electron (mass $m_e = 9.11 \times 10^{-31}$ kg) is in the lowest energy orbital (radius $R = 5.29 \times 10^{-11}$ m) of hydrogen. Since the angular momentum L of this electron is given, we can easily calculate the speed of the electron using $L = m_e vR$.

SOLVE

$$L = m_e vR$$

$$v = \frac{L}{m_e R} = \frac{1.055 \times 10^{-34}\text{ J}\cdot\text{s}}{(9.11 \times 10^{-31}\text{ kg})(5.29 \times 10^{-11}\text{ m})} = \boxed{2.19 \times 10^6\frac{\text{m}}{\text{s}}}$$

REFLECT

It is reasonable that the speed is large given the small radius of the orbital.

8.103

SET UP

We are asked to find the magnitude of the angular momentum ($L = I\omega$) of a 70-kg person riding in a Ferris wheel of diameter $d = 35$ m that makes a complete revolution in 25 s. The person can be treated as a point mass rotating at a distance $d/2$ (that is, the radius) about the center of the Ferris wheel. We can calculate the angular speed of the person by knowing that 1 revolution is 2π radians and assuming the angular speed is constant.

SOLVE

$$I_{\text{point mass}} = mr^2 = m\left(\frac{d}{2}\right)^2 = (70\text{ kg})\left(\frac{35\text{ m}}{2}\right)^2 = 21{,}438\text{ kg}\cdot\text{m}^2$$

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi\text{ rad}}{25\text{ s}} = 0.251\frac{\text{rad}}{\text{s}}$$

$$L = I\omega = (21{,}438\text{ kg}\cdot\text{m}^2)\left(0.251\frac{\text{rad}}{\text{s}}\right) = \boxed{5388\frac{\text{kg}\cdot\text{m}^2}{\text{s}}}$$

REFLECT

Recall that r for the moment of inertia of a point mass is the distance the mass is located from the axis of rotation. In this case it is equal to the radius of the Ferris wheel since the passenger cars are located along the rim.

8.104

SET UP

A professor spins at $\omega_i = 10$ rpm while holding a 1-kg mass in each hand. Each mass is a distance $R_i = 0.75$ m from the rotation axis. We can treat them as point masses when calculating their moment of inertia. She pulls the masses in toward her body, which changes her moment of inertia and increases her angular speed to $\omega_f = 20$ rpm. We can use conservation of angular momentum to calculate her final moment of inertia and then the final distance the masses are from the rotation axis.

SOLVE

$$L_i = L_f$$

$$I_i\omega_i = I_f\omega_f$$

$$(MR_i^2 + MR_i^2)\omega_i = (MR_f^2 + MR_f^2)\omega_f$$

$$2MR_i^2\omega_i = 2MR_f^2\omega_f$$

$$R_i^2\omega_i = R_f^2\omega_f$$

$$R_f = R_i\sqrt{\frac{\omega_i}{\omega_f}} = (0.75\text{ m})\sqrt{\frac{10\text{ rpm}}{20\text{ rpm}}} = \boxed{0.53\text{ m}}$$

REFLECT

The distance we calculated is from the axis of rotation to each mass. We did *not* take into account the mass of her arms or the size of her torso. Also, since we only needed the ratio of the angular speeds, we saved time by leaving them in rpm.

8.105

SET UP

We are given a position vector for the location where a specific force vector acts. The torque resulting from this force is equal to the cross product, $\vec{\tau} = \vec{r} \times \vec{F}$. We can use the determinant method of calculating cross products to find the torque vector.

SOLVE

$$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 3 & 2 & 1 \\ 10 & -20 & 5 \end{vmatrix} = \hat{x}\begin{vmatrix} 2 & 1 \\ -20 & 5 \end{vmatrix} - \hat{y}\begin{vmatrix} 3 & 1 \\ 10 & 5 \end{vmatrix} + \hat{z}\begin{vmatrix} 3 & 2 \\ 10 & -20 \end{vmatrix}$$

$$= (10 + 20)\hat{x} - (15 - 10)\hat{y} + (-60-20)\hat{z} = \boxed{30\hat{x} - 5\hat{y} - 80\hat{z}}, \text{ in units of N} \cdot \text{m}$$

REFLECT

The cross product is NOT commutative, which means $\vec{r} \times \vec{F} \neq \vec{F} \times \vec{r}$, so the order of the elements in the determinant is important.

8.106

SET UP

We are given a position vector and velocity vector for a 2-kg mass at a specific moment in time. The angular momentum of the object is equal to the cross product, $\vec{L} = \vec{r} \times \vec{p}$, where the momentum vector is $\vec{p} = m\vec{v}$. We can use the determinant method of calculating cross products to find the angular momentum vector, assuming that the vectors are all given in terms of SI units.

SOLVE

$$\vec{p} = m\vec{v} = (2)(\hat{x} - 3\hat{y} - 5\hat{z}) = 2\hat{x} - 6\hat{y} - 10\hat{z}$$

$$\vec{L} = \vec{r} \times \vec{p} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -0.5 & 2 & 0.75 \\ 2 & -6 & -10 \end{vmatrix} = \hat{x}\begin{vmatrix} 2 & 0.75 \\ -6 & -10 \end{vmatrix} - \hat{y}\begin{vmatrix} -0.5 & 0.75 \\ 2 & -10 \end{vmatrix} + \hat{z}\begin{vmatrix} -0.5 & 2 \\ 2 & -6 \end{vmatrix}$$

$$= (-20 + 4.5)\hat{x} - (5 - 15)\hat{y} + (3 - 4)\hat{z} = \boxed{-15.5\hat{x} - 3.5\hat{y} - \hat{z}}, \text{ in units of } \frac{\text{kg} \cdot \text{m}^2}{\text{s}}.$$

REFLECT

The cross product is NOT commutative, which means $\vec{r} \times \vec{p} \neq \vec{p} \times \vec{r}$, so the order of the elements in the determinant is important.

8.107

SET UP

There are two mathematical representations for the cross product—the determinant representation, which preserves the vector nature of the cross product, and the angle representation, which gives only the magnitude of the cross product. By explicitly calculating $\vec{A} \times \vec{B}$ and finding this vector's magnitude, we can set it equal to $AB\sin(\theta)$ from the angle representation to find θ.

SOLVE

$$|\vec{A} \times \vec{B}| = AB\sin(\theta)$$

$$\sin(\theta) = \frac{|\vec{A} \times \vec{B}|}{AB}$$

$$\theta = \text{arc sin}\left(\frac{|\vec{A} \times \vec{B}|}{AB}\right)$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 3 & 4 & 2 \\ 5 & -2 & -3 \end{vmatrix} = \hat{x}\begin{vmatrix} 4 & 2 \\ -2 & -3 \end{vmatrix} - \hat{y}\begin{vmatrix} 3 & 2 \\ 5 & -3 \end{vmatrix} + \hat{z}\begin{vmatrix} 3 & 4 \\ 5 & -2 \end{vmatrix}$$

$$= (-12 + 4)\hat{x} - (-9 - 10)\hat{y} + (-6 - 20)\hat{z} = -8\hat{x} + 19\hat{y} - 26\hat{z}$$

$$|\vec{A} \times \vec{B}| = \sqrt{(-8)^2 + (19)^2 + (-26)^2} = \sqrt{1101}$$

$$A = \sqrt{(3)^2 + (4)^2 + (2)^2} = \sqrt{29}$$

$$B = \sqrt{(5)^2 + (-2)^2 + (-3)^2} = \sqrt{38}$$

$$\theta = \arcsin\left(\frac{|\vec{A} \times \vec{B}|}{AB}\right) = \arcsin\left(\frac{\sqrt{1101}}{\sqrt{29}\sqrt{38}}\right) = \arcsin\left(\frac{\sqrt{1101}}{\sqrt{1102}}\right) = \boxed{1.54 \text{ rad}}$$

REFLECT

This is approximately 88.3 degrees. We find that $|\vec{A} \times \vec{B}| \approx AB$, which is reasonable since $\sin(88.3°) \approx 1$.

8.108

SET UP

We are given an angular momentum vector as a function of time and asked to find the torque as a function of time associated with the rotational motion. The first derivative of the angular momentum with respect to time will give the net torque on the object. Once we have the torque vector, we can evaluate it at $t = 2$ s and then find its magnitude.

SOLVE

$$\vec{\tau}(t) = \frac{d\vec{L}(t)}{dt} = \frac{d}{dt}(3t\hat{x} + 4t^2\hat{y} + 0.5t^3\hat{z}) = \boxed{3\hat{x} + 8t\hat{y} + 1.5t^2\hat{z}}$$

$$\vec{\tau}(t = 2 \text{ s}) = 3\hat{x} + 8(2)\hat{y} + 1.5(2)^2\hat{z} = 3\hat{x} + 16\hat{y} + 6\hat{z}$$

$$|\vec{\tau}(t = 2 \text{ s})| = \sqrt{(3)^2 + (16)^2 + (6)^2} = \sqrt{301} = \boxed{17.3 \text{ N} \cdot \text{m}}$$

REFLECT

We assume that the angular momentum vector is given in terms of SI units.

8.109

SET UP

The first derivative of the angular momentum vector with respect to time is the torque vector. Since we are given the torque vector as a function of time, we need to integrate it with respect to time to get the angular momentum vector. Assuming the magnitude of the angular momentum vector is zero at $t = 0$ allows us to evaluate a definite integral from $t = 0$ up to some arbitrary t. This will give us the angular momentum vector as a function of time. Once we solve for the angular momentum as a function of time, we can plug in $t = 0.5$ s into the expression to get the angular momentum vector at that time and determine its magnitude. For ease of calculation, we'll assume that the torque vector is given in terms of SI units.

SOLVE

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

$$\vec{L}(t) = \int_{L(0)}^{L(t)} d\vec{L} = \int_0^t \vec{\tau}\,dt = \int_0^t (3\cos(\pi t)\hat{x} + 4\cos(\pi t)\hat{y})dt$$

$$= \left[\frac{3}{\pi}\sin(\pi t)\hat{x} + \frac{4}{\pi}\sin(\pi t)\hat{y}\right]_0^t = \frac{3}{\pi}\sin(\pi t)\hat{x} + \frac{4}{\pi}\sin(\pi t)\hat{y}$$

$$\vec{L}(t = 0.5\text{ s}) = \frac{3}{\pi}\sin\left(\frac{\pi}{2}\right)\hat{x} + \frac{4}{\pi}\sin\left(\frac{\pi}{2}\right)\hat{y} = \frac{3}{\pi}\hat{x} + \frac{4}{\pi}\hat{y}$$

$$L = \sqrt{L_x^2 + L_y^2} = \sqrt{\left(\frac{3}{\pi}\right)^2 + \left(\frac{4}{\pi}\right)^2} = \frac{1}{\pi}\sqrt{(3)^2 + (4)^2} = \boxed{\frac{5}{\pi}\frac{\text{kg}\cdot\text{m}^2}{\text{s}} = 1.59\frac{\text{kg}\cdot\text{m}^2}{\text{s}}}$$

REFLECT

The torque vector is only a function of one variable (t) even though it has two components (x and y); $\hat{x}$ and $\hat{y}$ are unit vectors, not variables. This allows for a straightforward integration of each component.

8.110

SET UP

The outer diameter of a Blu-ray disk is 11.75 cm and the inner diameter is 4.5 cm. The disk spins such that the laser maintains a constant linear speed relative to the disc of 7.5 m/s as it moves across the disc from the inner edge to the outer edge. From this linear speed and the measurements of the disc, we can calculate the maximum and minimum angular speeds. The disc should spin fastest when the laser is at the inner edge of the disc, where the radius is smaller, in order to keep a constant linear speed. Finally, knowing that the laser starts at the inner edge and moves to the outer edge, we can calculate the average angular acceleration over an 8.0-hr period.

SOLVE

Part a)

$$\omega = \frac{v}{R} = \frac{v}{\left(\frac{d}{2}\right)} = \frac{2v}{d}$$

$$\omega_{\text{outer}} = \frac{2\left(7.5\frac{\text{m}}{\text{s}}\right)}{0.1175\text{ m}} = \boxed{127.7\frac{\text{rad}}{\text{s}}} \times \frac{60\text{ s}}{1\text{ min}} \times \frac{1\text{ rev}}{2\pi\text{ rad}} = \boxed{1219\text{ rpm}}$$

$$\omega_{\text{inner}} = \frac{2\left(7.5\frac{\text{m}}{\text{s}}\right)}{0.045\text{ m}} = \boxed{333.3\frac{\text{rad}}{\text{s}}} \times \frac{60\text{ s}}{1\text{ min}} \times \frac{1\text{ rev}}{2\pi\text{ rad}} = \boxed{3183\text{ rpm}}$$

Part b) The Blu-ray disk spins the slowest when the laser is at the outer edge of the playing area and fastest when the laser is at the inner edge of the playing area.

Part c)

$$\alpha_{\text{average}} = \frac{\Delta\omega}{\Delta t} = \frac{\left(127.7\frac{\text{rad}}{\text{s}}\right) - \left(333.3\frac{\text{rad}}{\text{s}}\right)}{8\text{ hr}} \times \frac{1\text{ hr}}{3600\text{ s}} = \boxed{-7.1 \times 10^{-3}\frac{\text{rad}}{\text{s}^2}}$$

REFLECT

The disc should slow down as the laser moves from the inner edge of the disk to the outer edge of the disc, so a negative angular acceleration makes sense.

8.111

SET UP

A table saw is equipped with a safety mechanism that brings a saw blade, initially spinning at $\omega_i = 7000$ rpm, to rest in 5 ms. We can calculate the average angular acceleration of the blade within this time interval from the change in angular speed. Once we have the average angular acceleration we can calculate the number of rotations the blade completes in this 5-ms period.

SOLVE

Part a)

$$\omega_i = 7000\frac{\text{rev}}{\text{min}} \times \frac{1\text{ min}}{60\text{ s}} \times \frac{2\pi\text{ rad}}{1\text{ rev}} = 733\frac{\text{rad}}{\text{s}}$$

$$\alpha_{\text{average}} = \frac{\Delta\omega}{\Delta t} = \frac{0 - \left(733\frac{\text{rad}}{\text{s}}\right)}{5 \times 10^{-3}\text{ s}} = \boxed{-1.47 \times 10^{5}\frac{\text{rad}}{\text{s}^2}}$$

Part b)

$$\omega_f^2 - \omega_i^2 = 2\alpha(\Delta\theta)$$

$$\Delta\theta = \frac{\omega_f^2 - \omega_i^2}{2\alpha} = \frac{0 - \left(733\frac{\text{rad}}{\text{s}}\right)^2}{2\left(-1.47 \times 10^{5}\frac{\text{rad}}{\text{s}^2}\right)} = 1.83\text{ rad} \times \frac{1\text{ rev}}{2\pi\text{ rad}} = \boxed{0.29\text{ rev}}$$

REFLECT

The size of the saw blade does not factor into our calculations. It makes sense that the angular acceleration should be a large, negative number because the saw blade comes to a stop in a very short amount of time. Accordingly, we expect the number of rotations to be small.

8.112

SET UP

A Dremel tool starts from rest and reaches an operating speed of 35,000 rpm in 1.2 s. We can calculate the average angular acceleration over this time interval and use it to find the number of rotations the tool makes while spinning up. The tool spins at a constant angular speed of 35,000 rpm for 45 s; multiplying these two figures together gives the number of rotations

the tool makes while spinning at a constant speed. Putting all of this together gives the total number of rotations the tool has made thus far.

We are also told the tool takes 8.5 s to come to a complete stop after spinning at 35,000 rpm. We can follow the same process as above to find the average angular acceleration for the slowdown period and the total number of rotations the tool makes while being used.

SOLVE

Part a)

Angular displacement while reaching the operating speed:

$$\omega_f = 35{,}000\frac{\text{rev}}{\text{min}} \times \frac{1\text{ min}}{60\text{ s}} \times \frac{2\pi\text{ rad}}{1\text{ rev}} = 3665.2\frac{\text{rad}}{\text{s}}$$

$$\alpha_{\text{average}} = \frac{\Delta\omega}{\Delta t} = \frac{\left(3665.2\frac{\text{rad}}{\text{s}}\right) - 0}{1.2\text{ s}} = 3054.3\frac{\text{rad}}{\text{s}^2}$$

$$\Delta\theta_{\text{speeding up}} = \omega_i t + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2}\left(3665.2\frac{\text{rad}}{\text{s}^2}\right)(1.2\text{ s})^2 = 2199\text{ rad}$$

Angular displacement while spinning at the operating speed:

$$\Delta\theta_{\text{constant speed}} = \omega t = \left(3665.2\frac{\text{rad}}{\text{s}}\right)(45\text{ s}) = 164{,}934\text{ rad}$$

Total number of rotations:

$$\Delta\theta_{\text{total}} = \Delta\theta_{\text{speeding up}} + \Delta\theta_{\text{constant speed}} = ((2199\text{ rad}) + (164{,}934\text{ rad})) \times \frac{1\text{ rev}}{2\pi\text{ rad}} = \boxed{26{,}600\text{ rev}}$$

Part b)

We calculated the angular acceleration for the start-up period earlier:

$$\alpha_{\text{start-up}} = \boxed{3054.3\frac{\text{rad}}{\text{s}^2}}$$

Angular acceleration for slowdown period:

$$\alpha_{\text{slowdown, average}} = \frac{\Delta\omega}{\Delta t} = \frac{0 - \left(3665.2\frac{\text{rad}}{\text{s}}\right)}{8.5\text{ s}} = \boxed{-431.2\frac{\text{rad}}{\text{s}^2}}$$

Part c)

Number of rotations while slowing down:

$$\omega_f^2 - \omega_i^2 = 2\alpha(\Delta\theta)$$

$$\Delta\theta = \frac{\omega_f^2 - \omega_i^2}{2\alpha} = \frac{0 - \left(3665.2\frac{\text{rad}}{\text{s}}\right)^2}{2\left(-431.2\frac{\text{rad}}{\text{s}^2}\right)} = 14{,}659.2\text{ rad} \times \frac{1\text{ rev}}{2\pi\text{ rad}} = 2479\text{ rev}$$

Total number of rotations from start to finish:

$$\Delta\theta_{\text{total}} = (\Delta\theta_{\text{speeding up}} + \Delta\theta_{\text{constant speed}}) + \Delta\theta_{\text{slowing down}} = (26{,}600 \text{ rev}) + (2479 \text{ rev}) = \boxed{29{,}079 \text{ rev}}$$

REFLECT

The angular acceleration during the slowdown period is negative, as expected. The Dremel tool spins very quickly, so it is reasonable that the number of rotations it completes should be large.

8.113

SET UP

A baton is made of a thin rod of length L and uniform mass M with a point mass M attached to each end. The total moment of inertia of the baton is the sum of the moments of inertia due to the rod and the two point masses. The baton is being twirled about a point that is (3/8) L from one end. This means we need to use the parallel-axis theorem to find the moment of inertia for the rod (only) in this case since it is the object that is being rotated about an axis parallel to its center of mass; we can still treat the two masses on the ends as point masses located at a distance $(3/8)L$ and $(5/8)L$, respectively.

SOLVE

$$I_{\text{rod}} = I_{\text{CM}} + I_{\text{from axis}} = \frac{1}{12}ML^2 + M\left(\frac{L}{8}\right)^2 = \frac{19}{192}ML^2$$

$$I_{\text{total}} = I_{\text{rod}} + I_{\text{left mass}} + I_{\text{right mass}} = \frac{19}{192}ML^2 + M\left(\frac{3}{8}L\right)^2 + M\left(\frac{5}{8}L\right)^2$$

$$= \frac{19}{192}ML^2 + \frac{9}{64}ML^2 + \frac{25}{64}ML^2 = \boxed{\frac{121}{192}ML^2}$$

REFLECT

This works out to be about $(0.63)ML^2$. The moment of inertia when twirling the baton about its center of mass is less than this value: $I_{\text{total}} = \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 + M\left(\frac{L}{2}\right)^2 = \frac{7}{12}ML^2$, or $(0.58)ML^2$, which makes sense.

8.114

SET UP

We are asked to derive the moment of inertia of a uniform, solid sphere of mass M and radius R about an axis passing through its center of mass. In order to use the integration method, we need to determine both r and dm. The r in the moment of inertia integral is the perpendicular distance from the axis of rotation to the location of the infinitesimal mass element dm. In the case of a solid sphere in spherical coordinates, $r_\perp = r\sin(\theta)$. The mass element dm is equal to density ρ multiplied by the infinitesimal volume element dV given in spherical coordinates. Working through the triple integral will give the moment of inertia for a solid sphere.

SOLVE

$$dm = \rho dV = \frac{M}{\left(\frac{4}{3}\pi R^3\right)} r^2 dr \sin(\theta) d\theta d\phi$$

$$I = \int r_\perp^2 dm = \int (r\sin(\theta))^2 \left(\frac{M}{\left(\frac{4}{3}\pi R^3\right)}\right) r^2 dr \sin(\theta) d\theta d\phi$$

$$= \frac{3M}{4\pi R^3}\int_0^R r^4 dr \int_0^\pi \sin^3(\theta) d\theta \int_0^{2\pi} d\phi = \frac{3M}{4\pi R^3}\left[\frac{1}{5}r^5\right]_0^R \left[-\frac{1}{3}\cos(\theta)(\sin^2(\theta)+2)\right]_0^\pi [\phi]_0^{2\pi}$$

$$= \frac{3M}{4\pi R^3}\left[\frac{1}{5}R^5\right]\left[-\frac{1}{3}\cos(\pi)(\sin^2(\pi)+2) + \frac{1}{3}\cos(0)(\sin^2(0)+2)\right][2\pi]$$

$$= \frac{MR^2}{10}[2+2] = \boxed{\frac{2}{5}MR^2}$$

REFLECT

This is the same moment of inertia stated in Table 8-1 of the text. Remember that "r" in the definition of the moment of inertia is the distance from the rotation axis to the mass element dm, which is not the same as the variable r in spherical coordinates.

8.115

SET UP

We can model a spinning ice skater of mass M as cylinders put together—the head + trunk + legs as a vertical cylinder with mass $M_{\text{torso}} = 0.87M$ and $d_{\text{torso}} = 35$ cm and each arm + hand as a thin rod with mass $M_{\text{arm}} = 0.065M$ and length $L = 65$ cm. The skater starts spinning at an angular speed of $\omega_{\text{i}} = 70$ rpm with his arms outstretched. The moment of inertia in this case is the sum of the moment of inertia for the torso cylinder and the two moments of inertia for the arms rotated about an axis that is $\left(\frac{L_{\text{arm}}}{2} + R_{\text{torso}}\right)$ from their centers; this means we need to use the parallel-axis theorem for the arms. The final moment of inertia is the sum of the moment of inertia for the torso cylinder, which is the same as before, and the two arm rods by the skater's side. Because we are modeling the arms as thin rods, each arm contributes $M_{\text{arm}}R_{\text{torso}}^2$ to the final moment of inertia. Finally, since angular momentum is conserved, we can put all of this information together and find the skater's final angular speed.

SOLVE

Initial moment of inertia:

$$I_{\text{i}} = I_{\text{torso}} + I_{\text{arms, i}} = \frac{1}{2}M_{\text{torso}}R_{\text{torso}}^2 + 2(I_{\text{CM}} + I_{\text{from axis}})$$

$$= \frac{1}{2}M_{\text{torso}}R_{\text{torso}}^2 + 2\left(\frac{1}{12}M_{\text{arm}}L_{\text{arm}}^2 + M_{\text{arm}}\left(\frac{L_{\text{arm}}}{2} + R_{\text{torso}}\right)^2\right)$$

$$= \frac{1}{2}(0.87M)R_{\text{torso}}^2 + 2\left(\frac{1}{12}(0.065M)L_{\text{arm}}^2 + (0.065M)\left(\frac{L_{\text{arm}}}{2} + R_{\text{torso}}\right)^2\right)$$

$$= \frac{1}{2}(0.87)(62\text{ kg})(0.175\text{ m})^2 + \frac{1}{6}(0.065)(62\text{ kg})(0.65\text{ m})^2$$

$$+ 2(0.065)(62\text{ kg})\left(\left(\frac{0.65\text{ m}}{2}\right) + (0.175\text{ m})\right)^2$$

$$= \boxed{3.125\text{ kg}\cdot\text{m}^2}$$

Final moment of inertia:

$$I_{\mathrm{f}} = I_{\mathrm{torso}} + I_{\mathrm{arms,\,f}} = \frac{1}{2}M_{\mathrm{torso}}R_{\mathrm{torso}}^2 + 2(M_{\mathrm{arm}}R_{\mathrm{torso}}^2)$$

$$= \frac{1}{2}(0.87)(62\text{ kg})(0.175\text{ m})^2 + 2(0.065)(62\text{ kg})(0.175\text{ m})^2 = \boxed{1.073\text{ kg}\cdot\text{m}^2}$$

Conservation of angular momentum:

$$L_{\mathrm{i}} = L_{\mathrm{f}}$$

$$I_{\mathrm{i}}\omega_{\mathrm{i}} = I_{\mathrm{f}}\omega_{\mathrm{f}}$$

$$\omega_{\mathrm{f}} = \frac{I_{\mathrm{i}}\omega_{\mathrm{i}}}{I_{\mathrm{f}}} = \frac{(3.125\text{ kg}\cdot\text{m}^2)(70\text{ rpm})}{1.073\text{ kg}\cdot\text{m}^2} = \boxed{204\text{ rpm}}$$

REFLECT

The skater spins about three times faster when his arms are pulled in to his sides, which seems reasonable.

8.116

SET UP

A spherical shell has a mass M and radius R. We can use the parallel-axis theorem to find its moment of inertia about an axis tangent to its surface, which is a distance R from its center of mass.

SOLVE

$$I = I_{\text{spherical shell}} + I_{\text{from axis}} = \frac{2}{3}MR^2 + MR^2 = \boxed{\frac{5}{3}MR^2}$$

REFLECT

Be sure to use the moment of inertia for a spherical *shell*, not a *solid* sphere.

8.117

SET UP

The bike rim is to be made up of a hoop of mass $m_{\mathrm{hoop}} = 1$ kg and radius $R = 0.5$ m and an unknown number of spokes, each of mass $m_{\mathrm{spoke}} = 0.01$ kg. Since the spokes connect the center of a typical bicycle wheel to the hoop, the length of each one is equal to the radius of the hoop. The overall moment of inertia of the bike rim rotating about an axis through its center is equal to the moment of inertia of the hoop plus the moment of inertia for *each* spoke. We can model each spoke as a thin rod of length R rotating about an axis through one end. From this we can calculate the number of spokes necessary to give the stated overall moment of inertia. Once we know the number of spokes, we can determine the mass of all of the spokes and add that to the mass of the hoop to get the overall mass of the rim.

SOLVE

Part a)

$$I_{\text{rim}} = I_{\text{hoop}} + NI_{\text{spoke}} = m_{\text{hoop}}R^2 + N\left(\frac{1}{3}m_{\text{spoke}}L^2\right) = m_{\text{hoop}}R^2 + N\left(\frac{1}{3}m_{\text{spoke}}R^2\right)$$

$$N = (I_{\text{rim}} - \mathrm{m}_{\text{hoop}}R^2)\left(\frac{3}{m_{\text{spoke}}R^2}\right)$$

$$= ((0.280\ \text{kg}\cdot\text{m}^2) - (1\ \text{kg})(0.5\ \text{m})^2)\left(\frac{3}{(0.01\ \text{kg})(0.5\ \text{m})^2}\right) = \boxed{36}$$

Part b)

$$m_{\text{rim}} = m_{\text{hoop}} + Nm_{\text{spoke}} = (1\ \text{kg}) + 36(0.01\ \text{kg}) = \boxed{1.36\ \text{kg}}$$

REFLECT

This is a reasonable mass for a bike rim. We aren't explicitly given the length of the spokes, but from our everyday knowledge about bicycle wheels we are able to get started on the problem. Picturing how a typical bike wheel is constructed and how it rotates helps determine which moment of inertia formulas are appropriate to use. Using everyday knowledge to make reasonable assumptions is an invaluable skill in solving physics problems.

8.118

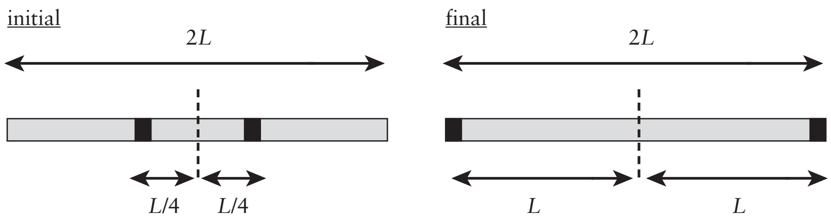


Figure 8-35 Problem 118

SET UP

Two beads, which we can treat as a point mass M each, are attached to a thin rod of mass $M/8$ and length $2L$. Initially the beads are a distance $L/4$ on either side of the center of the rod. The initial moment of inertia of the system is the moment of inertia of a thin rod rotating about an axis through its center plus the moment of inertia for each point mass. The initial angular speed of the rod is 20π rad/s. The beads then slide to the ends of the rod, which correspond to a distance L from the center. This changes the moment of inertia of the system and, through conservation of angular momentum, changes the angular speed of the system.

SOLVE

Moments of inertia:

$$I_{\text{i}} = I_{\text{rod}} + 2I_{\text{point mass}} = \frac{1}{12}\left(\frac{M}{8}\right)(2L)^2 + 2M\left(\frac{L}{4}\right)^2 = \frac{1}{6}ML^2$$

$$I_{\text{f}} = I_{\text{rod}} + 2I_{\text{point mass}} = \frac{1}{12}\left(\frac{M}{8}\right)(2L)^2 + 2ML^2 = \frac{49}{24}ML^2$$

Conservation of angular momentum:

$$L_{\mathrm{i}} = L_{\mathrm{f}}$$

$$I_{\mathrm{i}}\omega_{\mathrm{i}} = I_{\mathrm{f}}\omega_{\mathrm{f}}$$

$$\omega_{\mathrm{f}} = \frac{I_{\mathrm{i}}\omega_{\mathrm{i}}}{I_{\mathrm{f}}} = \frac{\left(\frac{1}{6}ML^2\right)\left(20\pi\frac{\mathrm{rad}}{\mathrm{s}}\right)}{\left(\frac{49}{24}ML^2\right)} = \frac{80\pi}{49}\ \frac{\mathrm{rad}}{\mathrm{s}} = \boxed{5.13\frac{\mathrm{rad}}{\mathrm{s}}}$$

REFLECT

The final moment of inertia is about 12 times larger than the initial moment of inertia, which means the final angular speed should be 12 times smaller than the initial angular speed. We calculate a final angular speed of about 5 rad/s, which is around one-twelfth of 20π rad/s (20π is close to 60).

8.119

SET UP

A uniform, thin disk has a mass $M = 0.3$ kg and a radius $R = 0.27$ m and is traveling with an initial translational speed of 4.8 m/s. The disk then rolls, without slipping, up a ramp that is angled $\theta = 55$ degrees from the horizontal and reaches some final height h_{f} off the ground. We can use conservation of mechanical energy and geometry to determine the total distance the disk travels on the surface of the ramp, D. Taking the ground to be the zero for gravitational potential energy, the disk only has translational and rotational kinetic energy initially. At its final location, all of the kinetic energy of the disk has converted to gravitational potential energy and the disk comes to rest. The distance D is related to h_{f} by the sine of the angle θ.

SOLVE

$$K_{\mathrm{tr,\,i}} + K_{\mathrm{rot,\,i}} = U_{\mathrm{g,\,f}}$$

$$\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = Mgh_{\mathrm{f}}$$

$$\frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v}{R}\right)^2 = MgD\sin(\theta)$$

$$\frac{1}{2}v^2 + \frac{1}{4}v^2 = gD\sin(\theta)$$

$$\frac{3v^2}{4gD} = \sin(\theta)$$

$$D = \frac{3v^2}{4g\sin(\theta)} = \frac{3\left(4.8\frac{\mathrm{m}}{\mathrm{s}}\right)^2}{4\left(9.81\frac{\mathrm{m}}{\mathrm{s}^2}\right)\sin(55°)} = \boxed{2.15\ \mathrm{m}}$$

REFLECT
Even though only the initial linear speed of the disk is explicitly given, remember to include its rotational kinetic energy in your calculation.

8.120

SET UP
A sphere of mass M and radius R is spinning at an angular speed ω. The sphere is replaced with a ring with the same mass M and a radius R_{ring}. The ring spins at the same angular speed ω and has the same rotational kinetic energy of the sphere. By setting the rotational kinetic energies equal, we find that the moments of inertia should be equal. This lets us solve for R_{ring} in terms of R. The magnitude of the angular momenta of an object is equal to the product of the moment of inertia and the angular speed. Since the moment of inertia and the angular speed are equal for the two objects, their angular momenta should also be equal.

SOLVE
Part a)

$$K_{\text{rot, ring}} = K_{\text{rot, sphere}}$$

$$\frac{1}{2}I_{\text{ring}}\omega^2 = \frac{1}{2}I_{\text{sphere}}\omega^2$$

$$I_{\text{ring}} = I_{\text{sphere}}$$

$$MR_{\text{ring}}^2 = \frac{2}{5}MR^2$$

$$\boxed{R_{\text{ring}} = R\sqrt{\frac{2}{5}}}$$

Part b) Since both parts have the same angular speed and same moment of inertia, they will have the $\boxed{\text{same angular momentum}}$.

REFLECT
It makes sense that the radius of the ring should be smaller than the radius of the sphere if the two moments of inertia are equal.

8.121

SET UP
A computer disk that has a diameter $d = 6.35$ cm is initially at rest, $\omega_f = 0$. The disk drive applies a torque to it, and the disk ends up spinning at a final angular speed of $\omega_f = 7200$ rpm. We can calculate the average torque the drive exerts on the disk through Newton's second law for rotation. The average acceleration can be calculated from the angular speeds and the time interval. We can model the computer disk as a thin disk of radius $(d/2)$ spinning about an axis through its center.

SOLVE

$$\omega_f = 7200\frac{\text{rev}}{\text{min}} \times \frac{1\text{ min}}{60\text{ s}} \times \frac{2\pi\text{ rad}}{1\text{ rev}} = 754\frac{\text{rad}}{\text{s}}$$

$$\alpha_{\text{average}} = \frac{\omega_f - \omega_i}{\Delta t} = \frac{\left(754\frac{\text{rad}}{\text{s}}\right) - 0}{2.5\text{ s}} = 301.6\frac{\text{rad}}{\text{s}^2}$$

$$\tau_{\text{average}} = I\alpha_{\text{average}} = \left(\frac{1}{2}MR^2\right)\alpha_{\text{average}} = \frac{1}{2}M\left(\frac{d}{2}\right)^2\alpha_{\text{average}}$$

$$= \frac{1}{2}(0.0075\text{ kg})\left(\frac{0.0635\text{ m}}{2}\right)^2\left(301.6\frac{\text{rad}}{\text{s}^2}\right) = \boxed{0.0011\text{ N}\cdot\text{m}}$$

REFLECT

Given that the disk has a small diameter and a small mass, it is reasonable that a small average torque is necessary to spin it.

8.122

SET UP

In his famous dive, Greg Louganis completed 3.5 revolutions while falling from an overall height of 12.0 m. First, we need to determine how long it took him to fall those 12.0 m using the kinematic equations for constant acceleration. Initially he starts at rest, $v_{i,y} = 0$, and gravity is the only external force acting on him. We can find the average angular speed by dividing 3.5 revolutions by the time his dive takes. If we assume there are no external torques, then angular momentum will be conserved throughout his dive. Initially, while he is tucked in, we can model his moment of inertia as a uniform, solid cylinder of diameter $d = 0.75$ m. When he stretches out, his moment of inertia can be modeled as a uniform, thin rod. We can determine Louganis's angular speed when he is stretched out by applying conservation of angular momentum. Once we have the final angular speed, we can calculate the change in his rotational kinetic energy while he extends his body.

SOLVE

Part a)

Time it takes Louganis to reach the water from his highest point:

$$\Delta y = v_{i,y}t + \frac{1}{2}a_y t^2 = 0 - \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2(\Delta y)}{-g}} = \sqrt{\frac{2(-12\text{ m})}{-\left(9.81\frac{\text{m}}{\text{s}^2}\right)}} = 1.57\text{ s}$$

Average angular velocity:

$$\omega_{\text{average}} = \frac{3.5\text{ rev}}{1.56\text{ s}} \times \frac{2\pi\text{ rad}}{1\text{ rev}} = \boxed{14\frac{\text{rad}}{\text{s}}}$$

Part b)

$$L_i = L_f$$

$$I_i\omega_i = I_f\omega_f$$

$$\left(\frac{1}{2}MR^2\right)\omega_i = \left(\frac{1}{12}ML^2\right)\omega_f$$

$$\frac{1}{2}\left(\frac{d}{2}\right)^2\omega_i = \frac{1}{12}L^2\omega_f$$

$$\omega_f = \frac{3d^2}{8L^2}\omega_i = \frac{3(0.75\text{ m})^2}{8(2.0\text{ m})^2}\left(14\frac{\text{rad}}{\text{s}}\right) = \boxed{3.0\frac{\text{rad}}{\text{s}}}$$

Part c)

$$\Delta K_{\text{rot}} = K_{\text{rot, f}} - K_{\text{rot, i}} = \frac{1}{2}I_f\omega_f^2 - \frac{1}{2}I_i\omega_i^2 = \frac{1}{2}\left(\frac{1}{12}ML^2\right)\omega_f^2 - \frac{1}{2}\left(\frac{1}{2}M\left(\frac{d}{2}\right)^2\right)\omega_i^2$$

$$= \frac{1}{24}ML^2\omega_f^2 - \frac{1}{16}Md^2\omega_i^2$$

$$= \frac{1}{24}(75\text{ kg})(2.0\text{ m})^2\left(3.0\frac{\text{rad}}{\text{s}}\right)^2 - \frac{1}{16}(75\text{ kg})(0.75\text{ m})^2\left(14\frac{\text{rad}}{\text{s}}\right)^2 = \boxed{-404\text{ J}}$$

REFLECT

We expect Louganis to spin faster while he is tucked rather than outstretched, so our answers to parts (b) and (c) are reasonable.

8.123

SET UP

A person holds an object of mass M in her hand while her forearm is horizontal and upper arm (made up of her humerus and biceps muscle) is vertical. Her biceps muscle connects to her forearm $R_{\text{biceps}} = 2$ cm from her elbow joint. The object is $R_{\text{object}} = 40$ cm from her elbow. We can find the magnitude of the force exerted by the biceps by considering the sum of the torques acting on the forearm about an axis through the person's elbow. The net torque must equal zero since everything is at rest. Because everything is at rest, the magnitude of the force of the object on her hand is equal to Mg due to Newton's second and third laws. All of the forces act perpendicularly to the forearm, so all of the angles when calculating the torques are equal to 90 degrees.

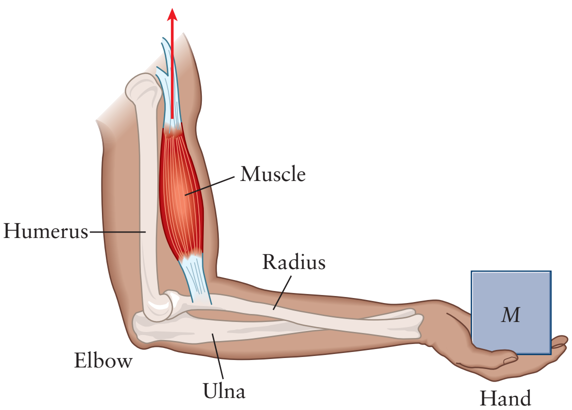


Figure 8-36 Problem 123

SOLVE

Free-body diagram of the forearm:

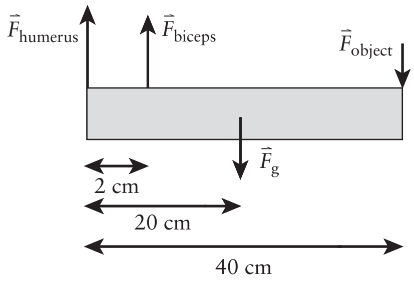


Figure 8-37 Problem 123

Sum of the torques acting on the forearm about an axis through the elbow (counterclockwise rotation is positive):

$$\sum \tau = \tau_{\text{humerus}} + \tau_{\text{biceps}} - \tau_{\text{g}} - \tau_{\text{object}} = 0$$

$$= F_{\text{humerus}} R_{\text{humerus}} \sin(\theta_{\text{humerus}}) + F_{\text{biceps}} R_{\text{biceps}} \sin(\theta_{\text{biceps}}) - F_{\text{g}} R_{\text{g}} \sin(\theta_{\text{g}}) - F_{\text{object}} R_{\text{object}} \sin(\theta_{\text{object}})$$

$$= F_{\text{humerus}}(0) \sin(90°) + F_{\text{biceps}}(0.02 \text{ m}) \sin(90°)$$

$$- (m_{\text{arm}} g)(0.20 \text{ m})\sin(90°) - (Mg)(0.40 \text{ m})\sin(90°)$$

$$F_{\text{biceps}} = \frac{(m_{\text{arm}} g)(0.20 \text{ m}) + (Mg)(0.40 \text{ m})}{0.02 \text{ m}} = \frac{(2 \text{ kg})\left(9.8 \frac{\text{m}}{\text{s}^2}\right)(0.20 \text{ m}) + M\left(9.8 \frac{\text{m}}{\text{s}^2}\right)(0.40 \text{ m})}{0.02 \text{ m}}$$

$$= \boxed{(196.2 \text{ N}) + M\left(196.2 \frac{\text{m}}{\text{s}^2}\right)}$$

REFLECT

The magnitude of the biceps force has a constant term and a term that depends on the mass of the object. The constant term corresponds to the force necessary to support the forearm itself.

8.124

SET UP

A patient's femur has a mass m_{f} and length L. Its center of gravity is located a distance $(L/3)$ from the pelvis. The femur is in traction and is supported by three ropes—one attached to the patient's hip $(\vec{F}_1)$ and two attached to her foot ($\vec{F}_2$ and $\vec{F}_3$)—as well as the force from the body acting at the hip joint, $\vec{F}_{\text{body}}$, pulling to the left. The rope attached to her hip pulls straight up with a magnitude of $m_1 g$; rope 2 is attached to her foot and pulls straight up with a magnitude

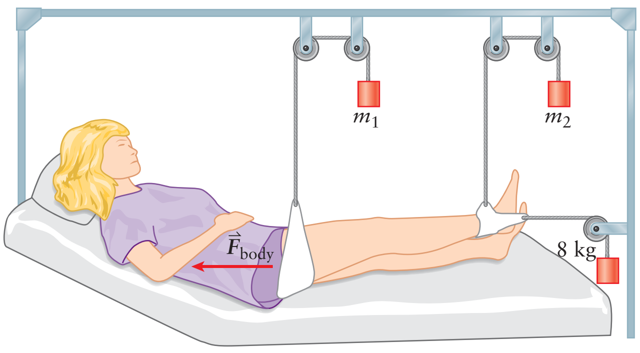


Figure 8-38 Problem 124

of m_2g; and rope 3 is attached to her foot and pulls to the right with a magnitude of $(8\text{ kg})\left(9.8\dfrac{\text{m}}{\text{s}^2}\right) = 78\text{ N}$. The femur is in static equilibrium, which means the sum of the forces must equal zero and the sum of the torques about any axis must equal zero. By summing the torques about the hip joint, the torques due to $\vec{F}_{\text{body}}$ and $\vec{F}_1$ are equal to zero because they act at the hip. The torque due to $\vec{F}_3$ is zero because the angle between $\vec{F}_3$ and $\vec{R}_3$ is 0 degrees. The sum of the forces in the y direction, coupled with the torque information, allows us to find a mathematical relationship between m_1 and m_2. Note that the sum of the forces in the x direction is not necessary. In part (b), the location of $\vec{F}_1$ has moved from the hip to the center of mass. This will affect the torque calculation, but not the force calculation, for part (b).

SOLVE

Part a)

Free-body diagram of the femur:

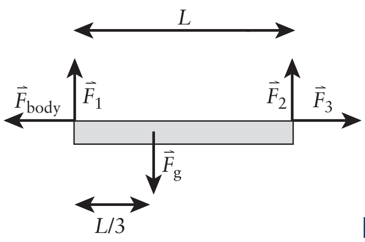


Figure 8-39 Problem 124

Sum of the forces in the y direction:

$$\sum F_y = F_1 + F_2 - F_g = m_1g + m_2g - m_fg = 0$$

$$m_f = m_1 + m_2$$

Sum of the torques acting on the femur about an axis through the hip joint (counterclockwise rotation is positive):

$$\sum \tau = \tau_{\text{body}} + \tau_1 - \tau_g + \tau_2 + \tau_3 = 0$$

$$= F_{\text{body}}(0)\sin(\theta) + F_1(0)\sin(\theta) - (m_fg)\left(\frac{L}{3}\right)\sin(90°) + (m_2g)L\sin(90°) + F_3L\sin(0°)$$

$$m_fg\left(\frac{L}{3}\right) = m_2gL$$

$$m_2 = \frac{m_f}{3}$$

Solving for m_1 in terms of m_f:

$$m_1 = m_f - m_2 = m_f - \left(\frac{m_f}{3}\right) = \frac{2m_f}{3}$$

Looking at the ratio of m_1 to m_2:

$$\frac{m_1}{m_2} = \frac{\left(\dfrac{2m_f}{3}\right)}{\left(\dfrac{m_f}{3}\right)} = 2$$

No, this relationship is $\boxed{\text{not unique}}$; it will hold as long as $\boxed{m_1 = 2m_2}$.

Part b)

Sum of the forces in the y direction is the same as before, so $m_f = m_1 + m_2$.

Sum of the torques acting on the femur about an axis through the hip (counterclockwise rotation is positive):

$$\sum\tau = \tau_{body} + \tau_1 - \tau_g + \tau_2 + \tau_3 = 0$$

$$= F_{body}(0)\sin(\theta) + F_1\left(\frac{L}{3}\right)\sin(90°) - (m_f g)\left(\frac{L}{3}\right)\sin(90°) + (m_2 g)L\sin(90°) + F_3 L\sin(0°)$$

$$m_1 g\left(\frac{L}{3}\right) - m_f g\left(\frac{L}{3}\right) + m_2 gL = 0$$

$$\frac{m_f}{3} = \frac{m_1}{3} + m_2$$

$$m_f = m_1 + 3m_2$$

Comparing the results from the force balance and the torque balance, we see that $m_1 = m_f$ and $m_2 = 0$. This relationship $\boxed{\text{is unique}}$.

REFLECT

The result in part (b) may seem odd—that m_2 is not necessary—but you can double-check the torque calculation by calculating the net torque about a *different* pivot (for example, the center of mass). Since the system is in static equilibrium, the sum of the torques must equal zero *regardless* of the pivot we choose. If we sum the torques about the center of mass, the only torque that is nonzero is the torque due to the force of m_2. This torque must equal zero, which can only occur if $m_2 = 0$.

8.125

SET UP

When the space debris accumulates on the surface of the Earth, it changes both the mass and moment of inertia of the Earth, which is why we expect the Earth's angular speed to decrease. We can use conservation of angular momentum if we treat the space debris and the Earth as an isolated system. Our initial state will be the Earth spinning at its "normal" rate (1 revolution in 24 hr) as the space debris falls radially inward from space. We assume the debris falls radially inward until it hits the Earth's surface because this simplifies the calculation since the initial angular momentum of the space debris will be zero, $\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times (-\hat{r}) = 0$. The final state will be the Earth and space debris spinning together. We will assume the debris is uniformly distributed on the surface of the Earth in a thin layer. This allows us to model the final moment of inertia as a thin, spherical shell with the radius of the Earth plus the moment of inertia of the Earth itself, $I_f = I_{Earth} + I_{debris} = I_{solid\ sphere} + I_{spherical\ shell}$. Part (a) of the question asks for the *change* in the angular speed per year, so we'll be solving for $\Delta\omega = \omega_f - \omega_i$. Once we know $\Delta\omega$ for one year we can use the definition of the period to determine the total

change in angular speed if the Earth's rotation period changed by 1 s. Comparing these two values of $\Delta\omega$ will tell us how many years of accumulating debris it would take for that change to occur.

SOLVE

Part a)

$$L_i = L_f$$

$$I_i\omega_i = I_f\omega_f$$

$$\omega_f = \frac{I_i\omega_i}{I_f}$$

$$\Delta\omega = \omega_f - \omega_i = \left(\frac{I_i\omega_i}{I_f}\right) - \omega_i = \omega_i\left(\frac{I_i}{I_f} - 1\right) = \omega_i\left(\frac{\left(\frac{2}{5}m_E R_E^2\right)}{\left(\frac{2}{5}m_E R_E^2\right) + \left(\frac{2}{3}m_{debris}R_E^2\right)} - 1\right)$$

$$= \omega_i\left(\frac{m_E}{m_E + \left(\frac{5}{3}m_{debris}\right)} - 1\right) = \omega_i\left(\frac{m_E}{m_E + \left(\frac{5}{3}m_{debris}\right)} - \frac{m_E + \left(\frac{5}{3}m_{debris}\right)}{m_E + \left(\frac{5}{3}m_{debris}\right)}\right)$$

$$= -\omega_i\left(\frac{\left(\frac{5}{3}m_{debris}\right)}{m_E + \left(\frac{5}{3}m_{debris}\right)}\right)$$

Converting the angular speed and mass of the debris:

$$\omega_i = \frac{2\pi \text{ rad}}{24 \text{ hr}} \times \frac{1 \text{ hr}}{3600 \text{ s}} = 7.3 \times 10^{-5}\frac{\text{rad}}{\text{s}}$$

$$m_{debris} = 60{,}000 \text{ tons} \times \frac{2000 \text{ lb}}{1 \text{ ton}} \times \frac{1 \text{ kg}}{2.2 \text{ lb}} = 5.5 \times 10^7 \text{ kg}$$

Plugging in:

$$\Delta\omega = -\left(7.3 \times 10^{-5}\frac{\text{rad}}{\text{s}}\right)\left(\frac{\frac{5}{3}(5.5 \times 10^7 \text{ kg})}{(5.97 \times 10^{24} \text{ kg}) + \frac{5}{3}(5.5 \times 10^7 \text{ kg})}\right) = \boxed{-1.1 \times 10^{-21}\frac{\text{rad}}{\text{s}}}$$

This is the change in angular speed in one year.

Part b)

$$T = \frac{2\pi}{\omega}$$

$$\Delta T = \frac{2\pi}{\omega_f} - \frac{2\pi}{\omega_i} = \frac{2\pi}{\omega_i + \Delta\omega} - \frac{2\pi}{\omega_i} = 2\pi\left(\frac{1}{\omega_i + \Delta\omega} - \frac{1}{\omega_i}\right)$$

$$\frac{\Delta T}{2\pi} + \frac{1}{\omega_i} = \frac{1}{\omega_i + \Delta\omega}$$

$$\omega_i + \Delta\omega = \frac{1}{\left(\frac{\Delta T}{2\pi} + \frac{1}{\omega_i}\right)} = \frac{1}{\left(\frac{\omega_i(\Delta T) + 2\pi}{2\pi\omega_i}\right)} = \frac{2\pi\omega_i}{\omega_i(\Delta T) + 2\pi}$$

$$\Delta\omega = \frac{2\pi\omega_i}{\omega_i(\Delta T) + 2\pi} - \omega_i = \omega_i\left(\frac{2\pi}{\omega_i(\Delta T) + 2\pi} - 1\right)$$

$$= \left(7.3 \times 10^{-5}\frac{\text{rad}}{\text{s}}\right)\left(\frac{2\pi}{\left(7.3 \times 10^{-5}\frac{\text{rad}}{\text{s}}\right)(1\text{ s}) + 2\pi} - 1\right)$$

$$= -8.5 \times 10^{-10}\frac{\text{rad}}{\text{s}}$$

This is the overall change in angular speed. To find how long it takes to reach this overall change in angular speed, we need to divide it by the change in angular speed *per year*:

$$\text{time} = \frac{\left(-8.5 \times 10^{-10}\frac{\text{rad}}{\text{s}}\right)}{\left(-1 \times 10^{-21}\frac{\text{rad}}{\text{s}\cdot\text{yr}}\right)} = \boxed{7.7 \times 10^{11}\text{ yr}}$$

REFLECT

Although 60,000 tons of debris sounds like a lot, it is negligible compared to the mass of the Earth. The amount of time we calculated for the Earth's rotation to change by a full second is longer than the age of the Earth (approximately 4 × 109 years), so we don't have to worry about space debris changing the Earth's rotation significantly…which is good.

8.126

SET UP

We can model Earth as a uniform, solid sphere of mass $m_E = 5.98 \times 10^{24}$ kg and radius $R_E = 6.38 \times 10^6$ m spinning about an axis through its center of mass. Earth makes one complete rotation in 1 day. Converting this period into an angular speed, we can calculate Earth's rotational kinetic energy. This is the maximum amount of energy that could be obtained, assuming the conversion is 100% efficient. To determine how long it would take to convert all of Earth's rotational kinetic energy, we can divide the energy we calculated by the energy

consumption rate per year. We can also use this rate to find how long it takes for an Earth day to become 48 hr, rather than 24, by looking at the difference in Earth's rotational kinetic energy for these two periods.

SOLVE

Part a)

$$K_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{2}{5}m_E R_E^2\right)\left(\frac{2\pi}{T}\right)^2 = \frac{4\pi^2 m_E R_E^2}{5T^2}$$

$$= \frac{4\pi^2(5.97 \times 10^{24}\ \text{kg})(6.38 \times 10^6\ \text{m})^2}{5\left(1\ \text{day} \times \frac{24\ \text{hr}}{1\ \text{day}} \times \frac{3600\ \text{s}}{1\ \text{hr}}\right)^2} = \boxed{2.57 \times 10^{29}\ \text{J}}$$

Part b)

$$2.57 \times 10^{29}\ \text{J} \times \frac{1\ \text{yr}}{6.4 \times 10^{20}\ \text{J}} = \boxed{4.0 \times 10^8\ \text{yr}}$$

This is 400 million years, so it might be worth the effort and expense.

Part c)

$$K_{rot,\,f} = \frac{4\pi^2 m_E R_E^2}{5T_f^2} = \frac{4\pi^2(5.97 \times 10^{24}\ \text{kg})(6.38 \times 10^6\ \text{m})^2}{5\left(48\ \text{hr} \times \frac{3600\ \text{s}}{1\ \text{hr}}\right)^2} = 6.4 \times 10^{28}\ \text{J}$$

$$\Delta K = (2.57 \times 10^{29}\ \text{J}) - (6.4 \times 10^{28}\ \text{J}) = 1.9 \times 10^{29}\ \text{J}$$

$$1.9 \times 10^{29}\ \text{J} \times \frac{1\ \text{yr}}{6.4 \times 10^{20}\ \text{J}} = \boxed{3 \times 10^8\ \text{yr}}$$

REFLECT

Although the math seems to work out, this is not really a viable method of energy production. Not only will the conversion of the Earth's rotational energy into "usable" energy not be 100% efficient, but also this method would cause enormous environmental changes!

8.127

SET UP

The Sun has a mass m_S and radius $R_{S,\,i} = 6.96 \times 10^8$ m. In 5 billion years the Sun's mass will be $0.8m_S$, and it will have a radius of $R_{S,\,f} = 8.0 \times 10^6$ m. The Sun is an isolated object, so its angular momentum will remain the same after its collapse. Because the moment of inertia of the Sun decreases, we expect its rotation rate to increase. We can calculate the Sun's final rotation rate in terms of its initial rotation rate from the conservation of angular momentum, assuming the Sun is a uniform, solid sphere. We can then explicitly calculate and compare the Sun's initial and final rotational kinetic energies.

SOLVE

Part a)

$$L_i = L_f$$

$$I_i\omega_i = I_f\omega_f$$

$$\left(\frac{2}{5}m_{S,\,i}R_{S,\,i}^2\right)\omega_i = \left(\frac{2}{5}m_{S,\,f}R_{S,\,f}^2\right)\omega_f$$

$$m_{S,\,i}R_{S,\,i}^2\omega_i = (0.8m_{S,\,i})R_{S,\,f}^2\omega_f$$

$$\omega_f = \frac{R_{S,\,i}^2}{0.8R_{S,\,f}^2}\omega_i = \frac{(6.96 \times 10^8 \text{ m})^2}{(0.8)(8.0 \times 10^6 \text{ m})^2}\omega_i = \boxed{9500\omega_i}$$

Part b)

$$\frac{K_f}{K_i} = \frac{\left(\frac{1}{2}I_f\omega_f^2\right)}{\left(\frac{1}{2}I_i\omega_i^2\right)} = \frac{(I_f\omega_f)\omega_f}{(I_i\omega_i)\omega_i} = \frac{\omega_f}{\omega_i} = \frac{9500\omega_i}{\omega_i} = \boxed{9500}$$

The final kinetic energy is 9500 times larger than the initial kinetic energy.

REFLECT

In part (b), we invoke conservation of angular momentum to make our calculation easier ($I_i\omega_i = I_f\omega_f$). We could also use the "angular momentum" form of rotational kinetic energy $\left(K_{rot} = \frac{L^2}{2I}\right)$ to arrive at the same answer.

8.128

SET UP

We can treat the Earth (mass m_E, radius R_E) and all of its inhabitants as an isolated system, which means angular momentum will be conserved. Initially we will assume that the people are uniformly distributed around the Earth, that the mass of the Earth includes the mass of the 7 billion people, and that the spinning Earth is a uniform, solid sphere. We can then find the moment of inertia for the case when everyone is located on the equator by treating the people as a hoop of mass m_{people} and radius R_E and using an effective mass of the Earth of $(m_E - m_{people})$. This, coupled with the fact that the Earth completes a full rotation in 1 day, will let us calculate the final angular speed of the Earth. If all of the people on Earth were to disappear, the effective mass of the Earth would be $(m_E - m_{people})$, which would correspond to a decreased moment of inertia and result in an increased angular speed.

SOLVE

Part a)

$$L_i = L_f$$

$$I_i\omega_i = I_f\omega_f$$

$$\left(\frac{2}{5}m_E R_E^2\right)\omega_i = \left(\frac{2}{5}(m_E - m_{people})R_E^2 + m_{people}R_E^2\right)\omega_f$$

$$\frac{2}{5}m_E R_E^2 \omega_i = \left(\frac{2}{5}m_E R_E^2 + \frac{3}{5}m_{people}R_E^2\right)\omega_f$$

$$\omega_f = \left(\frac{\frac{2}{5}m_E R_E^2}{\frac{2}{5}m_E R_E^2 + \frac{3}{5}m_{people}R_E^2}\right)\omega_i = \left(\frac{1}{1 + \frac{3}{2}\frac{m_{people}}{m_E}}\right)\omega_i$$

$$= \left(\frac{1}{1 + \frac{3}{2}\frac{(7 \times 10^9)(70 \text{ kg})}{(5.97 \times 10^{24} \text{ kg})}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ day}} \times \frac{1 \text{ day}}{24 \text{ hr}} \times \frac{1 \text{ hr}}{3600 \text{ s}}\right) < 7.3 \times 10^{-5}\frac{\text{rad}}{\text{s}}$$

Although very close to the initial angular speed of the Earth, this number will be smaller, so the Earth's rotation rate will $\boxed{\text{decrease}}$.

Part b) If there were no people on Earth, the effective mass of the Earth will decrease, which will decrease its moment of inertia. Angular momentum is conserved, so this change in mass will cause an $\boxed{\text{increase}}$ in the angular speed of the Earth.

REFLECT

Thinking generally, our answers make sense. Distributing mass further from the rotation axis (as in part a) should cause a decrease in the angular speed, while decreasing the mass altogether should increase the angular speed.

8.129

SET UP

Ten children, each of mass $M_{child} = 50$ kg, are standing in the center of a merry-go-round spinning at an angular speed of ω_i. We are told that the merry-go-round is a flat, solid cylinder with a mass of $M_{MGR} = 1000$ kg. We need to redistribute the children around the merry-go-round such that the final angular speed is $\omega_i/2$. For ease of calculation, we will assume that the children are point masses and that each one is the same distance, R_{child}, from the rotation axis. The merry-go-round and the 10 children make up an isolated system, which means angular momentum is conserved. We are told that the initial angular momentum of the children is zero. Although we are not given the exact radius of the merry-go-round, we can find R_{child} in terms of R_{MGR} to determine how the children should redistribute themselves in order to halve the angular speed of the merry-go-round.

SOLVE

$$L_{MGR,\,i} + L_{children,\,i} = L_{MGR,\,f} + L_{children,\,f}$$

$$I_{MGR}\omega_i = (I_{MGR} + I_{children,\,f})\omega_f$$

$$I_{MGR}\omega_i = (I_{MGR} + I_{children,\,f})\frac{\omega_i}{2}$$

$$I_{\text{MGR}} = I_{\text{children, f}}$$

$$\frac{1}{2}M_{\text{MGR}}R_{\text{MGR}}^2 = 10M_{\text{child}}R_{\text{child}}^2$$

$$R_{\text{child}} = \sqrt{\frac{M_{\text{MGR}}R_{\text{MGR}}^2}{20M_{\text{child}}}} = R_{\text{MGR}}\sqrt{\frac{(1000\text{ kg})}{20(50\text{ kg})}} = \boxed{R_{\text{MGR}}}$$

All 10 children should move to the edge of the merry-go-round.

REFLECT

Since all 10 children have to move to the edge of the merry-go-round to halve the angular speed, assuming they were all equidistant from the rotation axis turned out to be a reasonable assumption.

8.130

SET UP

A pilot is sitting in a human centrifuge of radius $R = 8.9$ m. The device starts at rest and then accelerates at a constant rate such that it makes 30 revolutions in 2 min. We can calculate the angular acceleration and final angular speed from kinematics. After this 2-min period, the centrifuge spins at a constant speed. The "g-force" the pilot experiences is related to the centripetal acceleration of the centrifuge, which we can calculate from the radius of the device and the angular speed from part (b). Finally, the centrifuge continues to accelerate at the same rate up to a state where the pilot experiences a g-force of $12g$. We can find the time it takes for the centrifuge to accelerate from this state to the $12g$ state from the definition of angular acceleration.

SOLVE

Part a)

$$\Delta\theta = \omega_{\text{i}}t + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2}\alpha t^2$$

$$\alpha = \frac{2(\Delta\theta)}{t^2} = \frac{2\left(30\text{ rev} \times \dfrac{2\pi\text{ rad}}{1\text{ rev}}\right)}{\left(2\text{ min} \times \dfrac{60\text{ s}}{1\text{ min}}\right)^2} = \frac{\pi}{120}\frac{\text{rad}}{\text{s}^2} = \boxed{0.026\frac{\text{rad}}{\text{s}^2}}$$

Part b)

$$\omega_{\text{f}} = \omega_{\text{i}} + \alpha t = 0 + \left(\frac{\pi}{120}\frac{\text{rad}}{\text{s}^2}\right)(120\text{ s}) = \pi\frac{\text{rad}}{\text{s}} = \boxed{3.1\frac{\text{rad}}{\text{s}}}$$

Part c)

$$\text{“}g\text{”} = \frac{v^2}{R} = R\omega^2 = \left(\frac{17.8\text{ m}}{2}\right)\left(3.1\frac{\text{rad}}{\text{s}}\right)^2 = 87.8\frac{\text{m}}{\text{s}^2} = \boxed{9g}$$

Part d)

$$12g = R\omega_f^2$$

$$\omega_f = \sqrt{\frac{12g}{R}} = \sqrt{\frac{12\left(9.81\frac{\text{m}}{\text{s}^2}\right)}{8.9\text{ m}}} = 3.6\frac{\text{rad}}{\text{s}}$$

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

$$\Delta t = \frac{\Delta\omega}{\alpha} = \frac{\omega_f - \omega_i}{\alpha} = \frac{\left(3.6\frac{\text{rad}}{\text{s}}\right) - \left(3.1\frac{\text{rad}}{\text{s}}\right)}{0.026\frac{\text{rad}}{\text{s}^2}} = \boxed{19.2\text{ s}}$$

REFLECT

For comparison, a force of $1g$ is equal in magnitude to the person's weight. A force of $12g$ for a typical 70-kg person would be approximately 8400 N or a little less than 1900 pounds.

8.131

SET UP

A marble of mass M, radius R, and moment of inertia $I = \frac{2}{5}MR^2$ is placed in front of a spring with spring constant k that has been compressed an amount x_c. The spring is released and, when the spring reaches its equilibrium length, the marble comes off the spring and begins to roll without slipping. The statement that static friction does no work suggests that we use conservation of mechanical energy to determine the time t it takes for the marble to travel a distance D. The initial elastic potential energy stored in the spring is converted into both translational and rotational kinetic energy of the marble. By solving for the linear speed of the marble as it leaves the spring, we can then use the definition of speed to determine the time it takes to travel a distance D. In part (b) we can use dimensional analysis to ensure our algebraic answer from part (a) has the correct dimension of time.

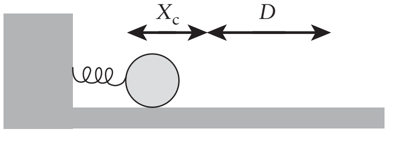


Figure 8-40 Problem 131

SOLVE

Part a)

Conservation of mechanical energy:

$$U_{\text{elastic}} = K_{\text{rot}} + K_{\text{tr}}$$

$$\frac{1}{2}kx_c^2 = \frac{1}{2}I\omega^2 + \frac{1}{2}Mv^2$$

$$\frac{1}{2}kx_c^2 = \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v}{R}\right)^2 + \frac{1}{2}Mv^2$$

$$\frac{1}{2}kx_c^2 = \frac{1}{5}Mv^2 + \frac{1}{2}Mv^2 = \frac{7}{10}Mv^2$$

$$v = \sqrt{\frac{5kx_c^2}{7M}} = \frac{D}{t}$$

$$t = \frac{D}{v} = \frac{D}{\sqrt{\frac{5kx_c^2}{7M}}} = D\sqrt{\frac{7M}{5kx_c^2}} = \boxed{\frac{D}{x_c}\sqrt{\frac{7M}{5k}}}$$

Part b)

$$[t] = [\mathrm{T}]$$

$$[M] = [\mathrm{M}]$$

$$[D] = [\mathrm{L}]$$

$$[x_c] = [\mathrm{L}]$$

$$[k] = [\mathrm{M}][\mathrm{T}]^{-2}$$

$$[t] \stackrel{?}{=} \frac{[D]}{[x_c]}\sqrt{\frac{[M]}{[k]}}$$

$$[\mathrm{T}] = \frac{[\mathrm{L}]}{[\mathrm{L}]}\sqrt{\frac{[\mathrm{M}]}{[\mathrm{M}][\mathrm{T}]^{-2}}} = \sqrt{[\mathrm{T}]^2} = [\mathrm{T}]$$

REFLECT

Because the work done by all the nonconservative forces on the marble is equal to zero, we could invoke the conservation of mechanical energy because $W_{\text{nonconservative}} = \Delta E_{\text{mechanical}}$. We also explicitly use dimensional analysis in this problem. It's always a good idea to check the dimensions of your final answer whether or not it is required to do so.

8.132

SET UP

Two triangles, each of base B and height H, are cut from a thin sheet of metal. Each triangle has a total mass M. They are then attached to a massless circular rim of radius R. We can relate R to H and B by the Pythagorean theorem: $R = \sqrt{H^2 + \frac{1}{4}B^2}$.

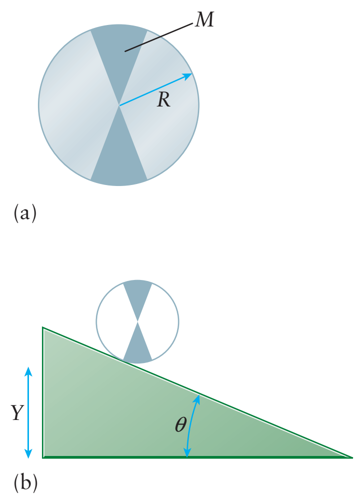


Figure 8-41 Problem 132

The moment of inertia of this object is equal to the sum of the moments of inertia of each slice, which we can calculate via integration. We will first find the moment of inertia for the right half of one triangle, multiply it by two to get the moment of inertia for one entire triangle, and multiply it by two again to find the moment of inertia for the entire object. Once we have the moment of inertia of the object, we can apply conservation

of mechanical energy to determine the linear speed of the wheel as it leaves the ramp. All of the initial gravitational potential energy is converted into both rotational and translational kinetic energy.

SOLVE

Part a)

$$I = \int r_{\perp}^2 dm$$

Finding dm:

$$dm = \sigma dA = \frac{M}{\left(\frac{1}{2}BH\right)}(xdy) = \frac{2M}{BH}xdy$$

We can eliminate the variable x by looking at the ratios from similar triangles:

$$\frac{\left(\frac{B}{2}\right)}{H} = \frac{x}{y}$$

$$x = \frac{B}{2H}y$$

Finding the moment of inertia of one-half of one triangle:

$$I_{\text{half}-\text{triangle}} = \int r_{\perp}^2 dm = \int_0^H y^2\left(\frac{2M}{BH}xdy\right) = \frac{2M}{BH}\int_0^H y^2\left(\frac{B}{2H}y\right)dy = \frac{M}{H^2}\int_0^H y^3 dy$$

$$= \frac{M}{H^2}\left[\frac{1}{4}y^4\right]_0^H = \frac{M}{H^2}\left[\frac{1}{4}H^4\right] = \frac{MH^2}{4}$$

Moment of inertia for one triangle:

$$I_{\text{triangle}} = 2I_{\text{half-triangle}} = 2\left(\frac{MH^2}{4}\right) = \frac{MH^2}{2}$$

Moment of inertia for the object:

$$I_{\text{object}} = 2I_{\text{triangle}} = 2\left(\frac{MH^2}{2}\right) = \boxed{MH^2}.$$

Part b)

$$U_{\text{g, i}} = K_{\text{tr, f}} + K_{\text{rot, f}}$$

$$(2M)gY = \frac{1}{2}(2M)v_{\text{f}}^2 + \frac{1}{2}I\omega_{\text{f}}^2$$

$$2MgY = Mv_{\text{f}}^2 + \frac{1}{2}(MH^2)\left(\frac{v_{\text{f}}^2}{R^2}\right)$$

$$2gY = v_f^2 + \frac{1}{2}H^2\left(\frac{v_f^2}{H^2 + \frac{1}{4}B^2}\right) = v_f^2\left(1 + \frac{H^2}{2\left(H^2 + \frac{1}{4}B^2\right)}\right)$$

$$\boxed{v_f = \sqrt{\frac{2gY}{\left(1 + \frac{H^2}{2\left(H^2 + \frac{1}{4}B^2\right)}\right)}}}$$

REFLECT

We can check the dimensions of our algebraic expression in part (b) to make sure it is a speed. The denominator term under the square root is dimensionless. The numerator term has dimensions of $\frac{[\mathrm{L}]}{[\mathrm{T}]^2} \cdot [\mathrm{L}] = \frac{[\mathrm{L}]^2}{[\mathrm{T}]^2}$. If we take the square root of this, we get dimensions of length per time, which are indeed the dimensions of speed.

Chapter 9
Elasticity and Fracture

Conceptual Questions

9.1 (a) Young's modulus accounts for the stretching or compressing of an object in one dimension. The bulk modulus describes the expansion or compression of an entire volume. (b) Both of the variables have the same units (N/m^2).

9.2 Yield stress is the stress at which the material becomes permanently deformed.

9.3 Yes. Real cables have weight, and that weight can be sufficient to break the cable.

9.4 Tensile strength is the maximum tensile stress a material can withstand before it begins to irreversibly deform. Ultimate strength is the maximum tensile stress the material can withstand. Breaking strength is the tensile stress that results in a material breaking apart.

9.5 Young's modulus describes the material's response to linear strain (like pulling on the ends of a wire); the bulk modulus describes the material's response to uniform pressure (acting over the entire volume); and the shear modulus describes the material's response to shearing strains. Determining the appropriate modulus requires determining which modulus (Young's, bulk, or shear) applies.

9.6 There is more than one answer to this question. For example, the stiffer a spring is, the larger the force necessary to stretch or compress it. The size dependence (cross-sectional area, length) of the material is not necessarily intuitive.

9.7 The strain is the more fundamental physical property. The strain determines the force the material is applying or its stress. There is no guarantee that each stress occurs at exactly one strain. In particular, very stretched metal begins to lose strength, and so the stress decreases.

9.8 The disparity may exist due to the different sizes of the ACL in men and women. For example, a larger cross-sectional area will result in a larger ultimate stress.

9.9 When an object experiences a stress below the yield strength, it should return to its original shape unchanged.

9.10 (a) The bottom side of the wood is under compression. (b) The top side of the wood is under tension. (c) The exact middle of the wood is where the transition from compression to tension occurs.

9.11 $\kappa = -\frac{1}{V}\frac{dV}{dP}$; $\kappa = -\frac{1}{V}\frac{\Delta V}{\Delta P}$.

9.12 (a) Strain is dimensionless and is a measure of how much an object's shape changes relative to its initial shape. (b) Tensile strain is a measure of how much an object stretches relative to its initial length. Volume strain is a measure of how much the volume of an object changes relative to its initial volume. Shear strain is a measure of how much one face of an object moves or slides relative to the opposite face.

9.13 As the metal approaches the breaking point, it is being damaged, which causes it to lose strength. For ductile metals in particular, when the metal becomes thin, its strength decreases.

9.14 The compressive stress due to the top of a cone is smaller than the compressive stress due to a cylinder.

9.15 The shear modulus is a measure of how much an object will deform under a given stress. The larger the shear modulus, the less it will deform; it is more rigid. So, the term *rigidity* applies because a material will deform less if it has a larger shear modulus.

9.16 Yes; this means you've applied a shear stress that was larger than the shear strength of the bolt.

9.17 If the strings are hit by the ball, they stretch; if they are hit hard enough, the tensile stress exceeds the breaking strength, and the strings break.

9.18 The skin loses some of its elasticity as people age and the tension in the skin decreases. Since the skin is not held as tightly, it will wrinkle.

9.19 Leg bones need to hold form in order to fulfill their role in holding the body together; they are cushioned by hooves and cartilage, so making them have a large breaking strain would not give much benefit but would require sacrificing other needs such as housing bone marrow. Antlers, on the other hand, need to be able to bend a great deal in order to survive the strong impacts to which they are subjected.

Multiple-Choice Questions

9.20 **E** (None of the above.) Strain is dimensionless.

9.21 **B** (N/m^2). Stress has units of newtons/meter^2.

9.22 **B** ($2\Delta L$). The stress and Young's modulus remain constant in each case:

$$\left(\frac{\Delta L}{L_0}\right)_1 = \left(\frac{\Delta L}{L_0}\right)_2$$

$$\frac{\Delta L}{L_0} = \frac{(\Delta L)_2}{2L_0}$$

$$(\Delta L)_2 = 2\Delta L$$

9.23 **B** ($\sqrt{2}$). The applied force, Young's modulus, and original length of the cable remain constant:

$$A_1(\Delta L)_1 = A_2(\Delta L)_2$$

$$\pi\left(\frac{D_1}{2}\right)^2(\Delta L) = \pi\left(\frac{D_2}{2}\right)^2\left(\frac{\Delta L}{2}\right)$$

$$D_2 = D_1\sqrt{2}$$

9.24 **C** (equal to F). The ultimate strength is the same in both cases because the wires are made of the same material and have the same cross-sectional area.

9.25 **A** (1/4). The applied force, Young's modulus, and original length of the cable remain constant:

$$A_1(\Delta L)_1 = A_2(\Delta L)_2$$

$$(\pi R^2)(\Delta L)_1 = \pi\left(\frac{R}{2}\right)^2(\Delta L)_2$$

$$\frac{(\Delta L)_1}{(\Delta L)_2} = \frac{1}{4}$$

9.26 **E** (A, B, and C). The wall mount is undergoing bending (compression and tension) and shear from the wall and the television.

9.27 **A** (materials with a relatively large bulk modulus). Usually, you would not want the volume of the building to change much when a force acts on it.

9.28 **C** (shear modulus). The force is being applied tangential to the face of the book, causing one face to move relative to the opposite face.

9.29 **A** (Young's modulus). The actor is stretching the steel cable, so the cable is under tension.

Estimation Questions

9.30 (a) 5×10^6 N/m^2. (b) 10×10^9 N/m^2

9.31 10×10^9 N/m^2 (approximately the same as for oak wood).

9.32 The shear modulus of cold butter is probably on the order of 10^3 N/m^2. The shear modulus will decrease as the butter warms up.

9.33 Assuming the breaking strength is about 10×10^6 N/m^2 and the diameter of your bone is about 1 cm, this leads to about 800 N; a value of 1000 N or more is probably more accurate.

9.34 The Young's modulus of iron is 190×10^9 N/m^2. The force required to bend the 0.5-inch-diameter rebar will be around 10^6 N.

9.35 17,000 N.

9.36 The sole of an athletic shoe is made of rubber. The sole probably shears a few percent (~5%) during a stop.

9.37

Strain (%)	Stress (10^9 N/m^2)
0.0	0
0.1	125
0.2	250
0.3	230
0.4	230
0.5	235
0.6	240
0.7	250
0.8	260
0.9	270
1.0	300
1.5	325
2.0	350
2.5	375
3.0	400
3.5	375
4.0	350
4.5	325
5.0	300

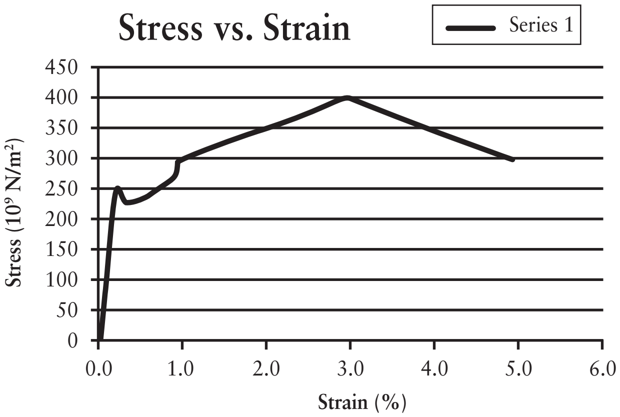


Figure 9-1 Problem 37

Part a) The yield strength is the tensile stress at which the material becomes permanently deformed: 250×10^9 N/m^2.

Part b) The ultimate strength is the maximum tensile stress the material can withstand: 400×10^9 N/m^2.

Part c) The Young's modulus is the slope of the initial linear region: $125{,}000 \times 10^9$ N/m^2.

Part d) Point of rupture occurs when the stress of an object is so large that the material starts to lose its structural integrity: 300×10^9 N/m^2. (This is where the graph stops.)

9.38

Strain (%)	Stress (10^6 N/m^2)
0.2	35
0.4	70
0.6	105
1.0	140
1.5	175

Part a) The elastic region is the initial linear region: up to a stress of 105×10^6 N/m^2.

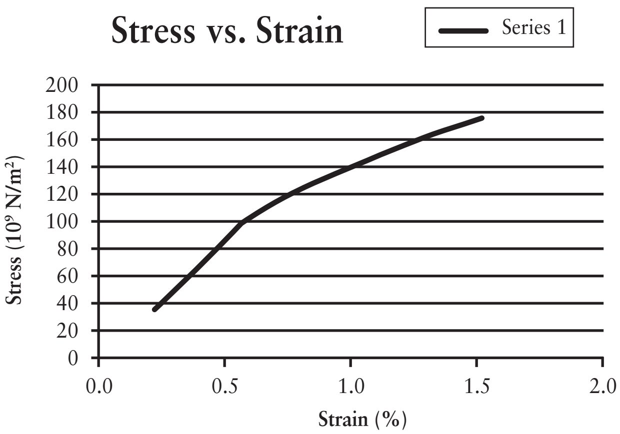


Figure 9-2 Problem 38

Part b) The yield strength is the tensile stress at which the material becomes permanently deformed: $105 \times 10^6\ \mathrm{N/m^2}$.

Part c) The Young's modulus is the slope of the initial linear region: $17{,}500 \times 10^6\ \mathrm{N/m^2}$.

Problems

9.39

SET UP

A cylindrical steel rod has an initial length of 250 cm and a diameter of 0.254 cm. A force stretches the rod 0.85 cm. We can calculate the magnitude of this force from the Young's modulus. The Young's modulus of steel from Table 9-1 is $200 \times 10^9\ \mathrm{N/m^2}$.

SOLVE

$$\frac{F}{A} = Y\frac{\Delta L}{L_0}$$

$$F = YA\frac{\Delta L}{L_0} = (200 \times 10^9\ \mathrm{Pa})(\pi)\left(\frac{0.00254\ \mathrm{m}}{2}\right)^2\left(\frac{0.85\ \mathrm{cm}}{250\ \mathrm{cm}}\right) = \boxed{3450\ \mathrm{N}}$$

REFLECT

A change of 0.85 cm is a change in length of approximately 0.3%.

9.40

SET UP

A rubber band with a circular cross section of radius 0.25 cm is stretched a distance of 3 cm by a force of 87 N. We can calculate the original length of the band from the Young's modulus. The Young's modulus of rubber from Table 9-1 is $0.005 \times 10^9\ \mathrm{N/m^2}$.

SOLVE

$$\frac{F}{A} = Y\frac{\Delta L}{L_0}$$

$$L_0 = YA\frac{\Delta L}{F} = (0.005 \times 10^9\ \mathrm{Pa})(\pi)(0.0025\ \mathrm{m})^2\left(\frac{3\ \mathrm{cm}}{87\ \mathrm{N}}\right) = \boxed{3.4\ \mathrm{cm}}$$

REFLECT

It's reasonable that a small force can stretch a rubber band almost twice its length.

9.41

SET UP

An aluminum bar has a cross section of $1\ \mathrm{cm^2}$ and an initial length of 88 cm. An applied force stretches the bar to a new length of 100 cm. We can calculate the magnitude of the force from the Young's modulus of aluminum and the tensile strain in the bar from the definition. The Young's modulus of aluminum from Table 9-1 is $70 \times 10^9\ \mathrm{N/m^2}$.

SOLVE

$$\frac{F}{A} = Y\frac{\Delta L}{L_0}$$

$$F = YA\frac{\Delta L}{L_0} = \left(70 \times 10^9 \frac{\text{N}}{\text{m}^2}\right)(10^{-4}\ \text{m}^2)\left(\frac{(100\ \text{cm}) - (88\ \text{cm})}{88\ \text{cm}}\right) = \boxed{9.55 \times 10^5\ \text{N} = 955\ \text{kN}}$$

$$\frac{\Delta L}{L_0} = \frac{(100\ \text{cm}) - (88\ \text{cm})}{88\ \text{cm}} = \boxed{0.136 = 13.6\%}$$

REFLECT
We would expect the force required to stretch an aluminum bar to be large.

9.42

SET UP
The tensile stress on a concrete block is 0.52×10^9 N/m^2. The Young's modulus of concrete from Table 9-1 is 30×10^9 N/m^2. We can calculate the tensile strain by dividing the stress by the Young's modulus.

SOLVE

$$\frac{\Delta L}{L_0} = \frac{\left(\frac{F}{A}\right)}{Y} = \frac{\left(0.52 \times 10^9 \frac{\text{N}}{\text{m}^2}\right)}{\left(30 \times 10^9 \frac{\text{N}}{\text{m}^2}\right)} = \boxed{0.017 = 1.7\%}$$

REFLECT
Strain is usually reported as a percentage.

9.43

SET UP
A physicist measures the ratio of an unknown material's tensile stress to its tensile strain to be 95×10^9 N/m^2. The ratio of tensile stress to tensile strain is the Young's modulus of the material. We can look in Table 9-1 to find a material with a Young's modulus of approximately 95×10^9 N/m^2.

SOLVE
Brass has a Young's modulus of 100×10^9 N/m^2, so the metal sample is most likely $\boxed{\text{brass}}$.

REFLECT
The values aren't exactly the same because there is some uncertainty due to the measurement in the physicist's experiment.

9.44

SET UP
A copper wire has an initial length of 10 m. A 1200-N force is applied to it and stretches the wire by 0.10 m. The Young's modulus of copper is 110×10^9 N/m^2. We will assume the wire

has a circular cross section. We can calculate the radius of the wire from the force, the strain, and the Young's modulus.

SOLVE

$$\frac{F}{A} = Y\frac{\Delta L}{L_0}$$

$$A = \pi R^2 = \frac{FL_0}{Y\Delta L}$$

$$R = \sqrt{\frac{FL_0}{\pi Y\Delta L}} = \sqrt{\frac{(1200\ \text{N})(10\ \text{m})}{\pi\left(110 \times 10^9 \frac{\text{N}}{\text{m}^2}\right)(0.10\ \text{m})}} = \boxed{5.9 \times 10^{-4}\ \text{m} = 590\ \mu\text{m}}$$

REFLECT

We can quickly double-check our answer by looking at the orders of magnitude of the quantities:

$$R \sim \sqrt{\frac{(10^3)(10)}{\pi(10^{11})(10^{-1})}} \approx \sqrt{3 \times 10^{-7}} \approx 5 \times 10^{-4}$$

9.45

SET UP

We can calculate the ratio of the tensile strains of an aluminum bar and a steel bar from their respective Young's moduli because the force and cross-sectional area are constant in both cases. The Young's modulus of steel is $200 \times 10^9\ \text{N/m}^2$; the Young's modulus of aluminum is $70 \times 10^9\ \text{N/m}^2$. The original length of each bar does not affect the strain since the strain is essentially the percent change in the object's length.

SOLVE

Part a)

$$\frac{\left(\frac{\Delta L}{L_0}\right)_{\text{Al}}}{\left(\frac{\Delta L}{L_0}\right)_{\text{Steel}}} = \frac{\left(\frac{F}{AY}\right)_{\text{Al}}}{\left(\frac{F}{AY}\right)_{\text{Steel}}} = \frac{Y_{\text{Steel}}}{Y_{\text{Al}}} = \frac{\left(200\frac{\text{N}}{\text{m}^2}\right)}{\left(70\frac{\text{N}}{\text{m}^2}\right)} = \frac{20}{7} = \boxed{2.86}$$

Part b) The strain does not depend on the relative lengths of the bars (but the relative amount of compression does).

REFLECT

It makes sense that aluminum, with a smaller Young's modulus, should experience a larger strain than steel for a given force.

9.46

SET UP

The elastic limit of an alloy is $0.6 \times 10^9\ \text{N/m}^2$. A 4-m-long wire made out of this alloy is designed to hang a sign that weighs 8000 N. The maximum distance the wire can stretch

is 5 cm. Assuming that the wire has a circular cross section, we can calculate the minimum radius of the wire from the weight of the sign, the elastic limit, and the length of the wire.

SOLVE

$$\frac{F}{A} = Y\frac{\Delta L}{L_0}$$

$$A = \pi R^2 = \frac{FL_0}{Y\Delta L}$$

$$R = \sqrt{\frac{FL_0}{\pi Y\Delta L}} = \sqrt{\frac{(8000\ \text{N})(4\ \text{m})}{\pi\left(0.6 \times 10^9 \frac{\text{N}}{\text{m}^2}\right)(0.05\ \text{m})}} = \boxed{0.018\ \text{m} = 1.8\ \text{cm}}$$

REFLECT

A stretch of 5 cm corresponds to a percent strain of 1.25%.

9.47

SET UP

A steel piano wire has a diameter of 0.2 cm. The tensile strength, which is the maximum tensile stress the material can withstand, of steel is 5.0×10^8 N/m^2. We can calculate the magnitude of the tension required to break the cylindrical wire directly from the strength of steel.

SOLVE

$$5.0 \times 10^8 \frac{\text{N}}{\text{m}^2} = \left(\frac{F}{A}\right)_{\text{max}} = \frac{F_{\text{max}}}{\pi R^2}$$

$$F_{\text{max}} = \left(5.0 \times 10^8 \frac{\text{N}}{\text{m}^2}\right)(\pi R^2) = \left(5.0 \times 10^8 \frac{\text{N}}{\text{m}^2}\right)(\pi)\left(\frac{0.20 \times 10^{-2}\ \text{m}}{2}\right)^2 = \boxed{1600\ \text{N} = 1.6\ \text{kN}}$$

REFLECT

The original length of the wire is irrelevant in this problem.

9.48

SET UP

Your weight exerts a force evenly over both of the soles of your shoes. Your mass is 55 kg, and each sole has an area of 200 cm^2. Dividing the magnitude of your weight by the total area of your shoes will give the compressive stress on your feet.

SOLVE

$$\frac{F}{A} = \frac{(55\ \text{kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)}{2\left(200\ \text{cm}^2 \times \frac{1\ \text{m}^2}{10^4\ \text{cm}^2}\right)} = \boxed{13{,}500\frac{\text{N}}{\text{m}^2}}$$

REFLECT

Standing on one foot would spread your weight out over half as much area and the stress on that foot would double.

9.49

SET UP

Your weight exerts a force evenly over both of the soles of your shoes. Your mass is 55 kg, and each sole has an area of 10 cm^2. Dividing the magnitude of your weight by the total area of your shoes will give the compressive stress on your feet.

SOLVE

$$\frac{F}{A} = \frac{(55\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)}{2\left(10\text{ cm}^2 \times \frac{1\text{ m}^2}{10^4\text{ cm}^2}\right)} = \boxed{27{,}000\frac{\text{N}}{\text{m}^2}}$$

REFLECT

This stress is 20 times larger than the stress in Problem 9.48.

9.50

SET UP

A woman's ACL experiences a tension of 3000 N. We can calculate the distance the ligament stretches from the original length of the ligament, the cross-sectional area of the ligament, and the Young's modulus. The ACL is 2.5 cm long and has a cross-sectional area of 0.54 cm^2. The Young's modulus of the ACL is 0.1×10^9 N/m^2.

SOLVE

$$\Delta L = \frac{FL_0}{AY} = \frac{(3000\text{ N})(2.5\text{ cm})}{\left(0.54\text{ cm}^2 \times \frac{1\text{ m}^2}{10^4\text{ cm}^2}\right)\left(0.1 \times 10^9\frac{\text{N}}{\text{m}^2}\right)} = \boxed{1.4\text{ cm}}$$

REFLECT

This is a strain of 56%.

9.51

SET UP

A model for the length of a person's ACL (in mm) relates it to the height of the person (in cm). We can plug in the height of the basketball player to calculate the approximate length of his ACL. We can calculate the distance the ligament stretches from the original length of the ligament, the tensile stress applied to the ligament, and the Young's modulus. The Young's modulus of the ACL is 0.1×10^9 N/m^2.

SOLVE

Part a)

$$L_{\text{ACL}} = 0.4606h - 41.29 = 0.4606(229) - 41.29 = \boxed{64.2\text{ mm} = 6.42\text{ cm}}$$

Part b)

$$\Delta L = \frac{\left(\frac{F}{A}\right)L_0}{Y} = \frac{\left(10 \times 10^6 \frac{\text{N}}{\text{m}^2}\right)(6.42 \text{ cm})}{\left(0.1 \times 10^9 \frac{\text{N}}{\text{m}^2}\right)} = \boxed{0.642 \text{ cm}}$$

REFLECT

A height of 229 cm is a little over 7 feet, 6 inches; his ACL is around 2.5 inches long.

9.52

SET UP

An artery has a Young's modulus of 16.8×10^3 N/m^2. Multiplying this by the tensile strain of the artery will give us the pressure acting on the artery. We can convert the pressure from newtons per square meter to mm Hg using the conversion factor 101,300 N/m^2 = 760 mm Hg. Blood pressures are routinely given in the units of mm Hg, as in "120 over 80." The first number is the systolic pressure, and the second number is the diastolic pressure.

SOLVE

Part a)

$$\frac{F}{A} = Y\frac{\Delta L}{L_0} = \left(16.8 \times 10^3 \frac{\text{N}}{\text{m}^2}\right)(0.952) = \boxed{16{,}000 \frac{\text{N}}{\text{m}^2}}$$

Part b)

$$16{,}000 \frac{\text{N}}{\text{m}^2} \times \frac{760 \text{ mm Hg}}{\left(101{,}300 \frac{\text{N}}{\text{m}^2}\right)} = 120 \text{ mm Hg}$$

This most likely corresponds to a $\boxed{\text{systolic pressure}}$.

REFLECT

The systolic pressure is always larger than the diastolic pressure. The systole is the contraction of the heart. The diastole is when the heart fills up with blood after the systole.

9.53

SET UP

A rigid cube of volume V_0 is filled with water and then frozen solid. The water expands by 9%, which means the volume changes by $+0.09V_0$. We can calculate the pressure exerted on the sides of the cube from the bulk modulus. The bulk modulus of water is 2.0×10^9 N/m^2.

SOLVE

$$\frac{F}{A} = B\frac{\Delta V}{V_0} = B\frac{0.09V_0}{V_0} = \left(2.0 \times 10^9 \frac{\text{N}}{\text{m}^2}\right)(0.09) = \boxed{1.8 \times 10^8 \frac{\text{N}}{\text{m}^2}}$$

REFLECT

We ignored the negative sign in the bulk modulus equation because we were just interested in the magnitude of the pressure. If we included the negative sign, we would need to determine the pressure required to squeeze the ice back into the original cube.

9.54

SET UP

A lead cube is pressed equally on all sides by forces of 100,000 N. Each side of the cube is 5 cm. The bulk modulus of lead is $46 \times 10^9\ \text{N/m}^2$. We can calculate the new dimensions of the lead cube from the bulk modulus. Since we are interested in the new size of the cube, we can write the area and the volumes in terms of the length.

SOLVE

$$\frac{F}{A} = -B\frac{\Delta V}{V_0}$$

$$\frac{F}{L_0^2} = -B\frac{(L_{\text{new}}^3 - L_0^3)}{L_0^3}$$

$$L_{\text{new}} = \sqrt[3]{-\frac{FL_0}{B} + L_0^3} = \sqrt[3]{-\frac{(100{,}000\ \text{N})(0.05\ \text{m})}{46 \times 10^9\ \text{Pa}} + (0.05\ \text{m})^3} = \boxed{0.04999\ \text{m} = 4.999\ \text{cm}}$$

REFLECT

It makes sense that a lead block would not change its shape much.

9.55

SET UP

A spherical air bubble has a radius of $R_0 = 4$ cm when it is 8 m below the surface, where the pressure is $0.78 \times 10^5\ \text{N/m}^2$ above the pressure at the surface. We can relate the change in pressure to the change in volume through the bulk modulus, $\Delta P = -B\frac{\Delta V}{V_0}$.

SOLVE

$$\Delta P = -B\frac{\Delta V}{V_0} = -B\frac{\left(\frac{4}{3}\pi R_f^3\right) - \left(\frac{4}{3}\pi R_0^3\right)}{\left(\frac{4}{3}\pi R_0^3\right)} = -B\left(\frac{R_f^3}{R_0^3} - 1\right)$$

$$R_f = R_0\sqrt[3]{1 - \frac{\Delta P}{B}} = (4\ \text{cm})\sqrt[3]{1 - \frac{\left(-0.78 \times 10^5 \frac{\text{N}}{\text{m}^2}\right)}{\left(1.01 \times 10^5 \frac{\text{N}}{\text{m}^2}\right)}} = \boxed{4.84\ \text{cm}}$$

REFLECT

The pressure is larger under the surface of the water, so we expect the size of the bubble to increase as it makes its way toward the surface.

9.56

SET UP

A sphere of copper is pressed uniformly by a force of 2×10^8 N. The initial radius of the sphere was 5 cm. The bulk modulus of copper is $140 \times 10^9\ \mathrm{N/m^2}$. We can calculate the change in volume and the new radius of the sphere from the bulk modulus.

SOLVE

$$\Delta P = -B\frac{\Delta V}{V_0}$$

$$\Delta V = -\frac{V_0 F}{BA} = -\frac{\left(\frac{4}{3}\pi\right)(0.05\ \mathrm{m})^2(2 \times 10^8\ \mathrm{N})}{\left(140 \times 10^9 \frac{\mathrm{N}}{\mathrm{m}^2}\right)(4\pi(0.05\ \mathrm{m})^2)} = \boxed{-2.4 \times 10^{-5}\ \mathrm{m}^3}$$

$$\Delta V = V_\mathrm{f} - V_0 = \frac{4}{3}\pi(R_\mathrm{f}^3 - R_0^3)$$

$$R_\mathrm{f} = \sqrt[3]{\frac{3\Delta V}{4\pi} + R_0^3} = \sqrt[3]{\frac{3(-2.4 \times 10^{-5}\ \mathrm{m}^3)}{4\pi} + (0.05\ \mathrm{m})^3} = \boxed{0.0492\ \mathrm{m} = 4.92\ \mathrm{cm}}$$

REFLECT

We wouldn't expect a metal sphere to change its shape by very much.

9.57

SET UP

A force of 200,000 N is acting tangentially to the top face of a brass plate. The top face is 2 cm by 20 cm. We can calculate the shear strain from the shear stress and the shear modulus. The shear modulus of brass from Table 9-1 is $40 \times 10^9\ \mathrm{N/m^2}$. The shear strain is equal to the tangent of φ.

Figure 9-3 Problem 57

SOLVE

$$\frac{F}{A} = S\frac{x}{h}$$

$$\frac{x}{h} = \frac{F}{AS} = \frac{200{,}000\ \text{N}}{(0.02\ \text{m})(0.20\ \text{m})\left(40 \times 10^9 \dfrac{\text{N}}{\text{m}^2}\right)} = \boxed{0.00125}$$

$$\tan(\varphi) = \frac{x}{h}$$

$$\varphi = \arctan\left(\frac{x}{h}\right) = \arctan(0.0025) = \boxed{0.072°}$$

REFLECT

The top face will hang over the bottom face by 125 microns.

9.58

SET UP

Shear forces act along the top and bottom sides of a door during an earthquake. The dimensions of the top and bottom of the door are 0.044 m by 0.81 m. We are told the shear strain of the door is 0.005. We can calculate the magnitude of the shear force from the shear stress, shear modulus, and cross-sectional area. The shear modulus of steel from Table 9-1 is $78 \times 10^9\ \text{N/m}^2$.

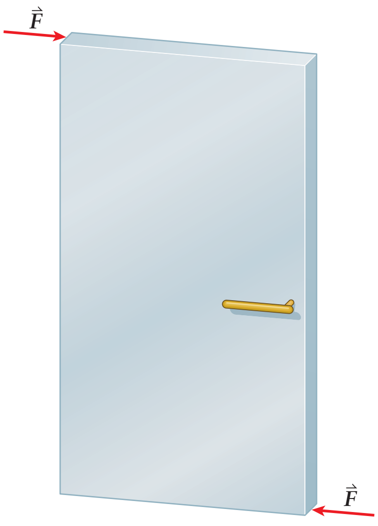


Figure 9-4 Problem 58

SOLVE

$$\frac{F}{A} = S\frac{x}{h}$$

$$F = AS\frac{x}{h} = (0.044\ \text{m})(0.81\ \text{m})\left(78 \times 10^9 \frac{\text{N}}{\text{m}^2}\right)(0.005) = \boxed{1.4 \times 10^7\ \text{N}}$$

REFLECT

The forces experienced during an earthquake should be large, even though the door does not change its shape by much.

9.59

SET UP

A force of 5×10^6 N is applied tangentially to one face of a brass cube. The resulting angle of shear $\varphi = 0.65$ degrees. The shear strain is equal to the tangent of φ. We can calculate the cross-sectional area of the cube from the force, the shear strain, and the shear modulus. The volume of the cube is the area raised to (3/2). The shear modulus of brass from Table 9-1 is 40×10^9 N/m^2.

SOLVE

$$\frac{F}{A} = S\frac{x}{h} = S\tan(\varphi)$$

$$A = L^2 = \frac{F}{S\tan(\varphi)}$$

$$V = L^3 = (L^2)^{\frac{3}{2}} = \left(\frac{F}{S\tan(\varphi)}\right)^{\frac{3}{2}} = \left(\frac{5 \times 10^6\text{ N}}{\left(40 \times 10^9 \frac{\text{N}}{\text{m}^2}\right)\tan(0.65°)}\right)^{\frac{3}{2}}$$

$$= 0.00116\text{ m}^3 \times \frac{1000\text{ L}}{1\text{ m}^3} = \boxed{1.16\text{ L}}$$

REFLECT

The top face hangs over the bottom face by about 1 mm, which seems reasonable for a metal block.

9.60

SET UP

The shear modulus of an asthmatic airway is approximately 50% of the shear modulus of a healthy airway. We'll assume that the same constant force is being applied to the same cross-sectional area in both cases. The ratio of the resulting shear strains will then be related to the ratio of the shear moduli.

SOLVE

$$\frac{F}{A} = S\frac{x}{h}$$

$$\frac{\left(\frac{x}{h}\right)_{\text{asthmatic}}}{\left(\frac{x}{h}\right)_{\text{healthy}}} = \frac{\left(\frac{F}{AS_{\text{asthmatic}}}\right)}{\left(\frac{F}{AS_{\text{healthy}}}\right)} = \frac{S_{\text{healthy}}}{S_{\text{asthmatic}}} = \frac{S_{\text{healthy}}}{\left(\frac{S_{\text{healthy}}}{2}\right)} = \boxed{2}$$

REFLECT

The asthmatic airway with the smaller shear modulus should experience a larger shear strain than the healthy airway when subject to the same force.

9.61

SET UP

In parts of the cardiovascular system, the cells lining the walls of the blood vessels experience a shear stress of 20 dyne/cm^2. We are told the shear strain is 0.008. We can find the shear modulus directly from these data. We will need the conversion factor from dynes to newtons: 1 N = 10^5 dyne.

SOLVE

$$S = \frac{\left(\frac{F}{A}\right)}{\left(\frac{x}{h}\right)} = \frac{\left(20\frac{\text{dyne}}{\text{cm}^2}\right)}{0.008} \times \frac{1\text{ N}}{10^5\text{ dyne}} \times \frac{10^4\text{ cm}^2}{1\text{ m}^2} = \boxed{250\frac{\text{N}}{\text{m}^2}}$$

REFLECT

We expect cells to have a much smaller shear modulus than, say, metal.

9.62

SET UP

A piece of granite is 200 m thick and has a shear modulus of 50 GPa. An earthquake exerts a shear force of 275×10^9 N to a square area of L^2. This results in a shear strain of 0.125. We can calculate the area over which the force is applied from the force, the shear modulus, and the shear strain. Because the area is a square, the value of L is the square root of the area.

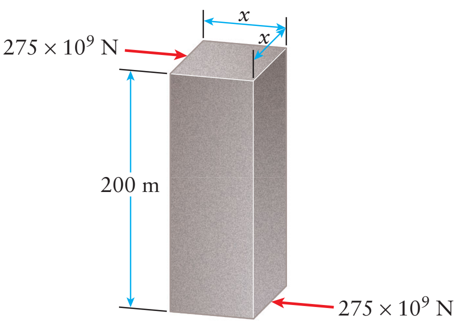


Figure 9-5 Problem 62

SOLVE

$$\frac{F}{A} = S(\text{shear strain})$$

$$A = \frac{F}{S(\text{shear strain})} = \frac{275 \times 10^9\text{ N}}{\left(50 \times 10^9\frac{\text{N}}{\text{m}^2}\right)(0.125)} = 44\text{ m}^2$$

$$L = \sqrt{A} = \sqrt{44\ \text{m}^2} = \boxed{6.6\ \text{m}}$$

REFLECT

The thickness of the granite is irrelevant because we were given the shear strain.

9.63

SET UP

A steel bolt has a diameter of 0.50 cm. The shear strength, which is the maximum shear stress the material can withstand, of steel is $400 \times 10^6\ \text{N/m}^2$. We can calculate the magnitude of the shear required to break the steel bolt directly from the shear strength of steel.

SOLVE

$$400 \times 10^6 \frac{\text{N}}{\text{m}^2} = \left(\frac{F}{A}\right)_{\text{max}} = \frac{F_{\text{max}}}{\pi R^2}$$

$$F_{\text{max}} = \left(400 \times 10^6 \frac{\text{N}}{\text{m}^2}\right)(\pi R^2) = \left(400 \times 10^6 \frac{\text{N}}{\text{m}^2}\right)(\pi)\left(\frac{0.50 \times 10^{-2}\ \text{m}}{2}\right)^2 = \boxed{7850\ \text{N} = 7.85\ \text{kN}}$$

REFLECT

The strength of a material depends on the mode of deformation; that is, shear strength is not necessarily equal to the tensile strength.

9.64

SET UP

A stress–strain curve for a steel spring is given. The initial linear region is the elastic region, followed by the plastic region. The yield strength (point A) is the tensile stress at which the spring enters the plastic region. The maximum value of the stress (point B) is the ultimate strength. The maximum stress the spring can withstand (point C) is the breaking strength. We can use Hooke's law and the cross-sectional area of the spring to calculate the value of the stress at point D. We are told point E corresponds to a strain of 100%.

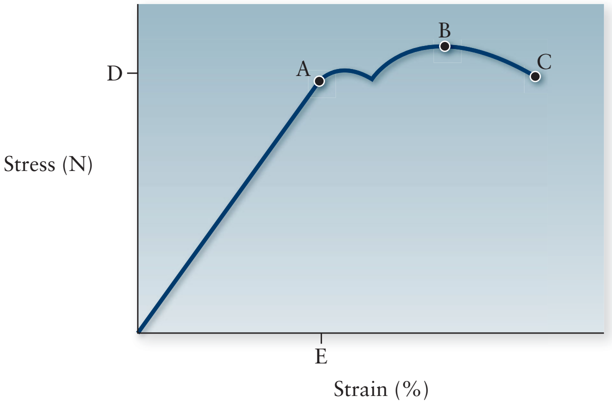


Figure 9-6 Problem 64

SOLVE
Point A is the yield strength.

Point B is the ultimate strength.

Point C is the breaking strength.

Point D)

$$F = k\Delta L = \left(100\frac{\text{N}}{\text{m}}\right)(0.12\text{ m}) = 12\text{ N}$$

$$\frac{F}{A} = \frac{12\text{ N}}{0.1\text{ cm}^2} \times \frac{10^4\text{ cm}^2}{1\text{ m}^2} = \boxed{1.2 \times 10^6\frac{\text{N}}{\text{m}^2}}$$

Point E is a strain of 100%.

REFLECT
The slope of the stress–strain curve in the elastic region is equal to the Young's modulus.

9.65

SET UP
A given rope can support 10,000 N/m^2 before breaking. A knot is tied in it, which decreases the ultimate breaking strength by a factor of 2. A worker wants to use this knotted rope to hang a load of 1000 N/m^2 but needs to make sure it meets the safety standards set by his company. The company uses a safety factor of 10 for all of its ropes, which means the ultimate breaking strength of the rope must be at least 10 times larger than the load it is supporting. To determine the safety factor for using this rope to hold this load, we need to compare the maximum force the rope can support to the force of the load. If this is more than 10, then the worker can safely use the rope; if not, he'll need to find a new, stronger rope.

SOLVE
Part a)
The overestimated ultimate breaking strength of the knotted rope is

$$\frac{\left(10{,}000\frac{\text{N}}{\text{m}^2}\right)}{2} = 5000\frac{\text{N}}{\text{m}^2}$$

The safety factor in this case is

$$\frac{\left(5000\frac{\text{N}}{\text{m}^2}\right)}{\left(1000\frac{\text{N}}{\text{m}^2}\right)} = \boxed{5}$$

Part b) $\boxed{\text{No}}$, the rigging company cannot use this rope because the safety factor is less than 10.

REFLECT

Since the cross section of the rope remains constant, the ratio between the ultimate breaking strength and the applied stress is equal to the ratio between the maximum force and the applied force.

9.66

SET UP

A 50-kg air conditioner slips and the cord gets caught in the mount. The power cord stretches from 3.0 m to 4.5 m. The cord has a diameter of 0.5 cm. We can calculate the Young's modulus of the cord by dividing the tensile stress by the tensile strain. We'll assume that the acceleration of the air conditioner throughout this process is negligible, which means the tension in the cord is equal in magnitude to the weight of the air conditioner.

SOLVE

$$\frac{F}{A} = Y\frac{\Delta L}{L_0}$$

$$Y = \frac{FL_0}{A\Delta L} = \frac{(50\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)(3.0\text{ m})}{\pi\left(\frac{0.005\text{ m}}{2}\right)^2((4.5\text{ m}) - (3.0\text{ m}))} = \boxed{50 \times 10^6\frac{\text{N}}{\text{m}^2}}$$

REFLECT

Although 50 MPa seems low for a Young's modulus, a force of ~500 N applied over the narrow cross section of the cord causes a strain of 50%.

9.67

SET UP

A typical Achilles tendon has an initial length of 25.0 cm and a diameter of 5.0 mm. A force stretches the tendon to a final length of 26.1 cm. We can calculate the magnitude of this force from the Young's modulus. The Young's modulus of the tendon is 1.47×10^9 N/m^2. We can convert from newtons to pounds using the conversion that 1 lb = 4.45 N.

SOLVE

$$\frac{F}{A} = Y\frac{\Delta L}{L_0}$$

$$F = YA\frac{\Delta L}{L_0} = \left(1.47 \times 10^9\frac{\text{N}}{\text{m}^2}\right)(\pi)\left(\frac{5.0 \times 10^{-3}\text{ m}}{2}\right)^2\left(\frac{(26.1\text{ cm}) - (25.0\text{ cm})}{25.0\text{ cm}}\right)$$

$$= \boxed{1300\text{ N} = 1.3\text{ kN}}$$

$$1300\text{ N} \times \frac{1\text{ lb}}{4.45\text{ N}} = \boxed{292\text{ lb}}$$

REFLECT

An increase of 1.1 cm corresponds to a strain of around 4%.

9.68

SET UP

Two rods have the same diameter but different Young's moduli and are welded together to form a longer rod of length L_0 (see figure). The left rod has a Young's modulus of Y_1 and the right rod has a Young's modulus of Y_2. We can calculate the combined Young's modulus for the new rod by comparing it to a single rod with the same dimensions and the same effective Young's modulus. We'll apply the same force F to each rod. This new rod will stretch a distance ΔL_{total}. The original rod will stretch the same amount, ΔL_{total}, which is equal to the distance the left rod stretches (ΔL_1) plus the distance the right rod stretches (ΔL_2). We can compare the two expressions to determine Y_{eff} in terms of Y_1 and Y_2.

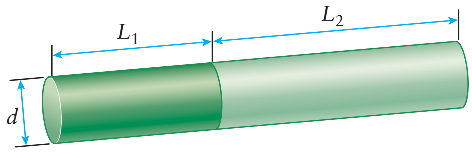


Figure 9-7 Problem 68

SOLVE

$$\frac{F}{A} = Y_{\text{eff}}\frac{\Delta L_{\text{total}}}{L_0}$$

$$\Delta L_{\text{total}} = \frac{FL_0}{AY_{\text{eff}}}$$

But $\Delta L_{\text{total}} = \Delta L_1 + \Delta L_2$:

$$\Delta L_1 + \Delta L_2 = \frac{FL_0}{AY_1} + \frac{FL_0}{AY_2} = \frac{FL_0}{A}\left(\frac{1}{Y_1} + \frac{1}{Y_2}\right)$$

By comparison,

$$\boxed{\frac{1}{Y_{\text{eff}}} = \frac{1}{Y_1} + \frac{1}{Y_2}} \text{ or } \boxed{Y_{\text{eff}} = \frac{Y_1Y_2}{Y_1 + Y_2}}$$

REFLECT

This is the same relationship for the effective spring constant of two springs in series, which makes sense since we can represent the spring constant in terms of the Young's modulus.

9.69

SET UP

Strands of caterpillar silk are typically 2.0 μm in diameter and have a Young's modulus of 4.0×10^9 N/m^2. We are asked to consider a climbing rope made of silk strands that is initially 9.0 m long and only stretches a distance of 1.00 cm when supporting the weight of two 85-kg climbers. We can calculate the cross-sectional area of this rope from the weight, the Young's modulus, and the strain. The ratio of this cross-sectional area to the cross-sectional area of one strand will give us the number of strands in the rope. Assuming the strands are distributed in a cylinder, we can use the area of a circle to calculate the diameter of the rope.

SOLVE

Part a)

$$\frac{F}{A} = Y\frac{\Delta L}{L_0}$$

$$A = \frac{FL_0}{Y\Delta L} = \frac{2(85\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)(9.0\text{ m})}{\left(4 \times 10^9\frac{\text{N}}{\text{m}^2}\right)(0.0100\text{ m})} = 0.000375\text{ m}^2$$

$$N_{\text{strands}} = \frac{A_{\text{total}}}{A_{1\text{ strand}}} = \frac{0.000375\text{ m}^2}{\pi\left(\dfrac{2.0 \times 10^{-6}\text{ m}}{2}\right)^2} = \boxed{1.2 \times 10^8\text{ strands}}$$

Part b)

$$A_{\text{total}} = \pi\left(\frac{D}{2}\right)^2$$

$$D = \sqrt{\frac{4A_{\text{total}}}{\pi}} = \sqrt{\frac{4(0.000375\text{ m}^2)}{\pi}} = \boxed{0.022\text{ m} = 2.2\text{ cm}}$$

Yes, this a reasonably-sized rope for mountain climbers to carry.

REFLECT

Climbing ropes made out of nylon have a comparable Young's modulus, $3.0 \times 10^9\text{ N/m}^2$.

9.70

SET UP

A brass sphere that has a radius of $R_0 = 0.06$ m sinks 2000 m to the bottom of the ocean. The pressure at this location is $199 \times 10^5\text{ N/m}^2$ larger than the pressure at the surface. We can relate the change in pressure to the change in volume through the bulk modulus, $\Delta P = -B\frac{\Delta V}{V_0}$. The bulk modulus of brass is $80 \times 10^9\text{ N/m}^2$.

SOLVE

$$\Delta P = -B\frac{\Delta V}{V_0}$$

$$\Delta V = -\frac{(\Delta P)V_0}{B} = -\frac{(\Delta P)\left(\frac{4}{3}\pi R_0^3\right)}{B}$$

$$= -\frac{\left(199 \times 10^5\frac{\text{N}}{\text{m}^2}\right)\left(\frac{4}{3}\pi\right)(0.06\text{ m})^3}{\left(80 \times 10^9\frac{\text{N}}{\text{m}}\right)} = \boxed{-2.25 \times 10^{-7}\text{ m}^3}$$

REFLECT
This difference in volume is only 0.225 mL.

9.71

SET UP
A 2.8-carat spherical diamond is grown under a pressure of 58×10^9 N/m^2. The diamond will expand once it is removed from the high-pressure chamber. The change in volume of the diamond is related to the change in pressure, the volume of the diamond, and the bulk modulus of the diamond. We can calculate the volume of the diamond from its density. One carat is equal to 0.2 g, and the density of diamond is 3.52 g/cm^3. The bulk modulus of diamond is 200×10^9 N/m^2. Since the diamond is spherical, we can calculate the change in its radius from its change in volume.

SOLVE
Part a)

$$\Delta P = -B\frac{\Delta V}{V_0}$$

$$\Delta V = -(\Delta P)\frac{V_0}{B} = \left(-58 \times 10^9 \frac{\text{N}}{\text{m}^2}\right)\frac{\left(2.8\text{ carat} \times \dfrac{0.200\text{ g}}{1\text{ carat}} \times \dfrac{1\text{ cm}^3}{3.52\text{ g}}\right)}{\left(200 \times 10^9 \dfrac{\text{N}}{\text{m}^2}\right)} = \boxed{0.046\text{ cm}^3}$$

Part b)

Initial volume:

$$V_0 = 2.8\text{ carat} \times \frac{0.200\text{ g}}{1\text{ carat}} \times \frac{1\text{ cm}^3}{3.52\text{ g}} = 0.159\text{ cm}^3$$

Initial radius:

$$R_0 = \left(\frac{3V_0}{4\pi}\right)^{\frac{1}{3}} = \left(\frac{3(0.159\text{ cm}^3)}{4\pi}\right)^{\frac{1}{3}} = 0.336\text{ cm}$$

Final volume:

$$V_f = V_0 + \Delta V = (0.159\text{ cm}^3) + (0.046\text{ cm}^3) = 0.205\text{ cm}^3$$

Final radius:

$$R_f = \left(\frac{3V_f}{4\pi}\right)^{\frac{1}{3}} = \left(\frac{3(0.205\text{ cm}^3)}{4\pi}\right)^{\frac{1}{3}} = 0.366\text{ cm}$$

Change in radius:

$$\Delta R = R_f - R_0 = (0.366\text{ cm}) - (0.336\text{ cm}) = \boxed{0.030\text{ cm}}$$

REFLECT
Atmospheric pressure is only 1.0×10^5 N/m^2, so the magnitude of the change in pressure is effectively the magnitude of the initial high pressure.

9.72

SET UP

A glass marble with a diameter of 1 cm is dropped into a 20-cm column of mercury. The change in volume of the marble is related to the increase in pressure, the volume of the marble, and the bulk modulus of glass. The pressure exerted by the column of mercury is 26.7×10^3 N/m^2. We can calculate the volume of the diamond from its diameter. The bulk modulus of glass is 50×10^6 N/m^2. Since the marble is spherical, we can calculate the change in its radius from its change in volume.

SOLVE

Part a)

$$\frac{F}{A} = -B\frac{\Delta V}{V_0}$$

$$\Delta V = -\left(\frac{F}{A}\right)\frac{V_0}{B} = -\left(26.7 \times 10^3 \frac{\text{N}}{\text{m}^2}\right)\frac{\left(\frac{4}{3}\pi\right)(1\text{ cm})^3}{\left(50 \times 10^6 \frac{\text{N}}{\text{m}^2}\right)} = \boxed{-0.0022\text{ cm}^3}$$

Part b)

$$\Delta V = \frac{4}{3}\pi(\Delta R)^3$$

$$\Delta R = \sqrt[3]{\frac{3\Delta V}{4\pi}} = \sqrt[3]{\frac{3(0.0022\text{ cm}^3)}{4\pi}} = \boxed{0.081\text{ cm}}$$

REFLECT

Even though mercury is very dense (13.6 times more dense than water), a pressure of 26.7×10^3 N/m^2 will not drastically change the size of the marble. For comparison, the pressure of the atmosphere is 1.01×10^5 N/m^2.

9.73

SET UP

A weightlifter lifts a mass of 263.5 kg above his head. All of this weight is supported by the lifter's two tibias. The average length of a tibia is 385 mm and its diameter is 3.0 cm. The Young's modulus for bone is 2.0×10^{10} N/m^2. We can use these data to calculate the distance his tibia is compressed upon lifting the weight. In order to determine if that distance is significant, we need to compare it to the initial length of the tibia. Throughout the calculation, we will assume that the given length of the tibia (385 mm) includes compression due to his weight.

SOLVE

Part a)

$$\frac{F}{A} = Y\frac{\Delta L}{L_0}$$

$$\Delta L = \frac{FL_0}{AY} = \frac{(263.5\ \text{kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)(385\ \text{mm})}{2(\pi)\left(\frac{0.03\ \text{m}}{2}\right)^2\left(2.0 \times 10^{10}\frac{\text{N}}{\text{m}^2}\right)} = \boxed{0.035\ \text{mm}}$$

Part b) No, a change of 0.035 mm is not a significant compression.

Part c) No, we do not need to include his weight because we are interested in the compression due to extra weight. His tibia is initially compressed slightly due to his weight; we've assumed the value of 385 mm includes that initial compression.

REFLECT
If the value of 385 mm did *not* include the initial compression of his weight, his weight would compress the tibia by 0.02 mm, which is also negligible.

9.74

SET UP
A 54,000-kg house rests atop six stacks of wooden blocks. The wood has a Young's modulus of $13 \times 10^9\ \text{N/m}^2$, and each block has dimensions of 25 cm by 75 cm. The wooden blocks are stacked 1.5 m high. We can calculate the compression of each stack of wood directly from these data.

SOLVE
Part a)

$$\frac{F}{A} = Y\frac{\Delta L}{L_0}$$

$$\Delta L = \frac{FL_0}{AY} = \frac{(54{,}000\ \text{kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)(1.5\ \text{m})}{6(0.25\ \text{m})(0.75\ \text{m})\left(13 \times 10^9\frac{\text{N}}{\text{m}^2}\right)} = \boxed{5.43 \times 10^{-5}\ \text{m} = 54.3\ \mu\text{m}}$$

Part b) The blocks should be able to withstand the compression since they are only compressed by 0.0036%.

REFLECT
Remember to multiply the cross-sectional area by 6 because the weight is spread out over six stacks of wood.

9.75

SET UP
The ultimate strength of steel is $400 \times 10^6\ \text{N/m}^2$. We can use this to find the minimum diameter necessary for a steel cable to hold a 4000-kg engine. Using our answer from part (a), we can calculate how far a 10-m-long cable would stretch once the engine is lifted off the ground. The Young's modulus of steel is $200 \times 10^9\ \text{N/m}^2$.

SOLVE

Part a)

$$400 \times 10^6 \text{ Pa} = \frac{(4000 \text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)}{\pi\left(\frac{D}{2}\right)^2} = \frac{4(4000 \text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)}{\pi D^2}$$

$$D = \sqrt{\frac{4(4000 \text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)}{\pi(400 \times 10^6 \text{ Pa})}} = \boxed{0.0112 \text{ m} = 1.12 \text{ cm}}$$

Part b)

$$\frac{F}{A} = Y\frac{\Delta L}{L_0}$$

$$\Delta L = \frac{FL_0}{AY} = \frac{(4000 \text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)(10 \text{ m})}{\pi\left(\frac{0.0112 \text{ m}}{2}\right)^2\left(200 \times 10^9\frac{\text{N}}{\text{m}^2}\right)} = \boxed{0.020 \text{ m} = 2.0 \text{ cm}}$$

REFLECT

The minimum diameter of the cable is 0.44 inch. In practice, a much thicker cable would be used to ensure it did not snap.

9.76

SET UP

A runner's foot applies a shearing force of 25 N to the sole of her shoe. The force is distributed over an area of 15 cm^2. The shear angle is 5.0 degrees. We can use all of this information to calculate the shear modulus of the rubber sole. The shear strain is equal to the tangent of the shear angle.

Figure 9-8 Problem 76

SOLVE

$$\frac{F}{A} = S\frac{x}{h} = S\tan(\theta)$$

$$S = \frac{F}{A\tan(\theta)} = \frac{25 \text{ N}}{\left(15 \text{ cm}^2 \times \frac{1 \text{ m}^2}{10^4 \text{ cm}^2}\right)\tan(5.0°)} = \boxed{1.91 \times 10^5\frac{\text{N}}{\text{m}^2}}$$

REFLECT

This is a reasonable shear modulus for a rubber sole. (In fact, looking ahead, this is equal to the modulus given in Problem 9.77.) Since we were given the shear angle, we did not need to use the thickness of the sole.

9.77

SET UP

A runner's foot applies a shearing force of 28 N to the sole of her shoe. The force is distributed over an area of 20 cm^2. The shear modulus of the rubber sole is $1.9 \times 10^5\ N/m^2$. We can use all of this information to calculate the shear strain and, therefore, the shear angle. The shear strain is equal to the tangent of the shear angle.

Figure 9-9 Problem 77

SOLVE

$$\frac{F}{A} = S\frac{x}{h} = S\tan(\varphi)$$

$$\varphi = \arctan\left(\frac{F}{AS}\right) = \arctan\left(\frac{28\ \text{N}}{\left(20\ \text{cm}^2 \times \frac{1\ \text{m}^2}{10^4\ \text{cm}^2}\right)\left(1.9 \times 10^5 \frac{\text{N}}{\text{m}^2}\right)}\right) = \boxed{4.2°}$$

REFLECT

This is a comparable answer to the shear angle given in Problem 9.76 with similar data.

9.78

SET UP

A strand of human hair has a Young's modulus of $4.0 \times 10^9\ N/m^2$ and a diameter of 150 microns. A 250-g object is hung from a 20.0-cm-long hair. We can calculate the amount the hair stretches from the weight of the object, the cross-sectional area of the hair strand, the original length of the strand, and the Young's modulus. When comparing this value to the amount an identical aluminum wire would stretch, we can take the ratio of the two distances because the force applied and the dimensions of the hair/wire are the same in both cases rather than redoing the calculation from scratch. We can also treat the hair strand as a spring and relate the Young's modulus of hair to the spring constant of the strand.

SOLVE

Part a)

$$\frac{F}{A} = Y\frac{\Delta L}{L_0}$$

$$\Delta L = \frac{FL_0}{AY} = \frac{(0.250\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)(20.0\text{ cm})}{\pi\left(\frac{150\times10^{-6}\text{ m}}{2}\right)^2\left(4.0\times10^9\frac{\text{N}}{\text{m}^2}\right)} = \boxed{0.693\text{ cm}}$$

Part b)

$$(\Delta L)_{\text{Al}} = \frac{Y_{\text{hair}}}{Y_{\text{Al}}}(\Delta L)_{\text{hair}} = \left(\frac{4.0\times10^9\text{ Pa}}{70\times10^9\text{ Pa}}\right)(0.693\text{ cm}) = \boxed{0.0396\text{ cm}}$$

$$(\Delta L)_{\text{Al}} = \frac{Y_{\text{hair}}}{Y_{\text{Al}}}(\Delta L)_{\text{hair}} = \frac{\left(4.0\times10^9\frac{\text{N}}{\text{m}^2}\right)}{\left(70\times10^9\frac{\text{N}}{\text{m}^2}\right)}(0.693\text{ cm}) = \boxed{0.0396\text{ cm}}$$

Part c)

$$k = \frac{AY}{L_0} = \frac{\pi\left(\frac{150\times10^{-6}\text{ m}}{2}\right)^2\left(4.0\times10^9\frac{\text{N}}{\text{m}^2}\right)}{0.200\text{ m}} = \boxed{353\frac{\text{N}}{\text{m}}}$$

Part d) A spring constant of 353 N/m is comparable to everyday springs in the lab.

REFLECT

Common springs in the laboratory have a spring constant of a few hundred N/m.

9.79

SET UP

On Earth, the spherical bubbles near the surface of a glass of water have a diameter $D_0 =$ 2.5 mm. This glass of water is taken to Mars. The atmospheric pressure on Earth is 1.0×10^5 N/m^2, and the atmospheric pressure on Mars is 650 N/m^2. We can relate the change in the diameter and, thus, the volume to the change in atmospheric pressure through the bulk modulus.

SOLVE

$$\Delta P = -B\frac{\Delta V}{V_0}$$

$$\Delta P = -B\frac{\left(\frac{4}{3}\pi R_{\text{f}}^3\right) - \left(\frac{4}{3}\pi R_0^3\right)}{\left(\frac{4}{3}\pi R_0^3\right)} = -B\left(\frac{R_{\text{f}}^3}{R_0^3} - 1\right)$$

$$R_f = R_0 \sqrt[3]{1 - \frac{\Delta P}{B}}$$

$$D_f = D_0 \sqrt[3]{1 - \frac{\Delta P}{B}} = (2.5\text{ mm}) \sqrt[3]{1 - \frac{\left(\left(650\frac{\text{N}}{\text{m}^2}\right) - \left(1.0 \times 10^5\frac{\text{N}}{\text{m}^2}\right)\right)}{\left(1.01 \times 10^5\frac{\text{N}}{\text{m}^2}\right)}} = \boxed{3.14\text{ mm}}$$

REFLECT

The change in pressure is approximately 10^5 as is the bulk modulus of air. Therefore, the percent change in volume is about 100%, which means the volume on Mars is about $2^{1/3}$ that on Earth, or $(2^{1/3})(2.5\text{ mm}) = 3.15\text{ mm}$.

9.80

SET UP

The femur bone has an effective cross section of 3.0 cm^2. The ultimate strength of the bone is $1.7 \times 10^8\text{ N/m}^2$. We can find the maximum compressive force the femur can withstand by multiplying the ultimate strength by the cross-sectional area.

SOLVE

$$\text{strength} = \left(\frac{F}{A}\right)_{\text{max}}$$

$$F_{\text{max}} = \left(1.7 \times 10^9\frac{\text{N}}{\text{m}^2}\right)\left(3.0\text{ cm}^2 \times \frac{1\text{ m}^2}{10^4\text{ cm}^2}\right) = \boxed{51{,}000\text{ N}}$$

REFLECT

A force of 51,000 N is about 70 times larger than an average person's weight. It's very difficult to break a femur under compression; more often, breaks are caused by twisting or bending.

9.81

SET UP

The femur bone resembles a hollow cylinder with an outer radius of 1.1 cm and an inner radius of 0.48 cm. The ultimate strength of the bone is $1.7 \times 10^8\text{ N/m}^2$. We can find the maximum compressive force the femur can withstand by multiplying the ultimate strength by the cross-sectional area.

SOLVE

$$\text{strength} = \left(\frac{F}{A}\right)_{\text{max}}$$

$$F_{\text{max}} = \left(1.7 \times 10^8\frac{\text{N}}{\text{m}^2}\right)(\pi)((1.1\text{ cm})^2 - (0.48\text{ cm})^2)\left(\frac{1\text{ m}^2}{10^4\text{ cm}^2}\right) = \boxed{52{,}000\text{ N} = 52\text{ kN}}$$

REFLECT

By comparing our answers to Problems 9.80 and 9.81, we see that a hollow cylinder is a stronger shape than a solid cylinder.

9.82

SET UP

A beam is attached to a vertical wall with a hinge and is stabilized by a steel support wire tied to its other end. The beam is 1000 kg and has a length of $L = 4$ m. The support wire makes a 30-degree angle with the beam. We can calculate the magnitude of the tension in the support wire by summing the torques acting on the beam—the torque due to gravity and the torque due to the tension—about the hinge. The sum of the torques must equal zero because the beam is in static equilibrium. To determine the minimum cross-sectional area of the steel wire so that it's not permanently stretched we need to use the yield strength of steel. The yield strength is the stress at which the material becomes permanently deformed.

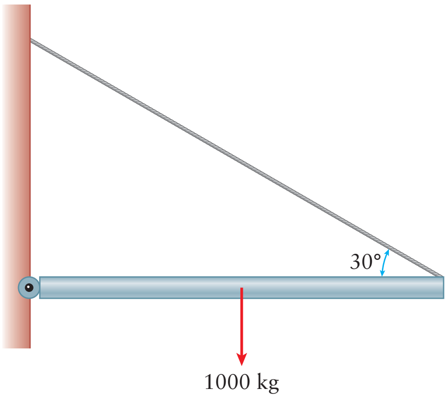


Figure 9-10 Problem 82

SOLVE

Part a)

$$\sum \tau = \tau_{\text{gravity}} + \tau_{\text{tension}} = -F_g R_g \sin(90°) + TR_T \sin(30°) = 0$$

$$(mg)\left(\frac{L}{2}\right) = T(L)\sin(30°)$$

$$T = \frac{mg}{2\sin(30°)} = \frac{(1000\text{ kg})\left(9.8\dfrac{\text{m}}{\text{s}^2}\right)}{2\sin(30°)} = \boxed{9800\text{ N}}$$

Part b)

$$\text{yield stress} = \frac{F}{A_{\text{min}}}$$

$$A_{\text{min}} = \frac{9800\text{ N}}{\left(290 \times 10^6 \dfrac{\text{N}}{\text{m}^2}\right)} = \boxed{3.38 \times 10^{-5}\text{ m}^2 = 0.338\text{ cm}^2}$$

REFLECT

The wire has a radius of 3.3 mm and a length of 4.6 m. The wire will stretch a distance of 6.7 mm.

9.83

SET UP

Table 9-2 contains values for various bone properties for humans and cows. We can compare the values between the two species in order to answer the questions posed. The ultimate stress is the maximum stress a material can withstand before breaking, and a large ultimate stress means a large force is required to break the bone. By looking over the values of the stresses in the table, we notice that they are all approximately the same, regardless of the species. Finally, we can calculate the amount a bone compresses from the force, longitudinal Young's modulus, cross-sectional area, and uncompressed length.

SOLVE

Part a) The compressive transverse ultimate stress is larger for cows ($178 \times 10^6\ \text{N/m}^2$) than for humans ($133 \times 10^6\ \text{N/m}^2$), which suggests that cows are more resistant to such breaks.

Part b) The longitudinal compressive ultimate stress is much larger in cows than humans. Also, cows have thicker bones.

Part c) Certainly. Most of the figures are quite similar, yet cows are much more massive than humans. Moreover, maximum running speeds in humans tend to be muscle-limited rather than bone-limited.

Part d)

$$\text{strength} = \left(\frac{F}{A}\right)_{\text{max}}$$

$$F_{\text{max}} = \left(95 \times 10^6 \frac{\text{N}}{\text{m}^2}\right)\left(3\ \text{cm}^2 \times \frac{1\ \text{m}^2}{10^4\ \text{cm}^2}\right) = \boxed{28{,}500\ \text{N} = 28.5\ \text{kN}}$$

$$\frac{F}{A} = Y\frac{\Delta L}{L_0}$$

$$\Delta L = \frac{FL_0}{AY} = \frac{(2850\ \text{N})(35\ \text{cm})}{\left(3\ \text{cm}^2 \times \frac{1\ \text{m}^2}{10^4\ \text{cm}^2}\right)\left(9.6 \times 10^9 \frac{\text{N}}{\text{m}}\right)} = \boxed{0.035\ \text{cm}}$$

REFLECT

Because cow bones are thicker than human bones, the effective cross-sectional area is larger for a cow. This means cow bone can withstand a (slightly) larger force than a human bone since the ultimate stresses are approximately equal in both cases.

9.84

SET UP

A bar under a constant tensile stress has a square cross-sectional area A. We'll call the side of the square L. Within the bar is a thin slab that makes an angle θ with the horizontal. We can relate the area of the angled slab to the cross-sectional area A through trigonometry; one side of the slab is still L, while the other is the hypotenuse of a right triangle related to θ and L. We

can decompose the force $\vec{F}$ into components along axes that are parallel and perpendicular to the slab. The tensile stress is the perpendicular component of the force divided by the area of the slab. The shear stress uses the component of the force parallel to the slab. Each of these expressions will depend on θ so we can determine the value of θ that gives a maximum value of the sine or cosine and, thus, the maximum value of the stress.

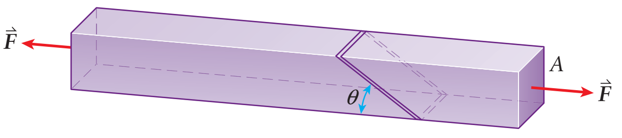


Figure 9-11 Problem 84

SOLVE

Part a)

The component of the force acting parallel to the slab is $\boxed{F\cos(\theta)}$.

The component of the force acting perpendicular to the slab is $\boxed{F\sin(\theta)}$.

Part b)

Area of the slab:

$$A_{\text{slab}} = (L)\left(\frac{L}{\sin(\theta)}\right) = \frac{L^2}{\sin(\theta)} = \frac{A}{\sin(\theta)}$$

Tensile stress:

$$\left(\frac{F}{A}\right)_{\text{tensile}} = \frac{F\sin(\theta)}{\left(\dfrac{A}{\sin(\theta)}\right)} = \boxed{\frac{F\sin^2(\theta)}{A}}$$

Shear stress:

$$\left(\frac{F}{A}\right)_{\text{shear}} = \frac{F\cos(\theta)}{\left(\dfrac{A}{\sin(\theta)}\right)} = \boxed{\frac{F\sin(\theta)\cos(\theta)}{A}}$$

Part c) The tensile stress will be a maximum at $\boxed{\theta = 90 \text{ degrees}}$.

Part d) The shear stress will be a maximum at $\boxed{\theta = 45 \text{ degrees}}$.

REFLECT

The cross-sectional area of the slab should be equal to A for $\theta = 0$ degrees and larger than A for $\theta > 0$ degrees. For $\theta = 90$ degrees, the shear stress is zero and the slab only experiences a tensile stress.

9.85

SET UP

A rectangular object made from a material that has a Young's modulus of Y and a bulk modulus of B has dimensions of ℓ, w, and t. A stress acts along ℓ, which changes the

dimensions of the object to $(\ell + \Delta\ell)$, $(w + \Delta w)$, and $(t + \Delta t)$, where $\Delta\ell < 0$, $\Delta w > 0$, and $\Delta t > 0$. We can find an expression for the fractional change in volume $\frac{\Delta V}{V_0}$ in terms of these dimensions and relate it to the bulk modulus through $\Delta P = -B\frac{\Delta V}{V_0}$. We will need to invoke the definition of Poisson's ratio ν, which is the induced strain divided by the primary strain: $\nu = -\frac{\Delta w/w}{\Delta\ell/\ell} = -\frac{\Delta t/t}{\Delta\ell/\ell}$. The fact that the fluid exerts a pressure, which is the primary applied stress, allows us to connect the bulk modulus to the Young's modulus.

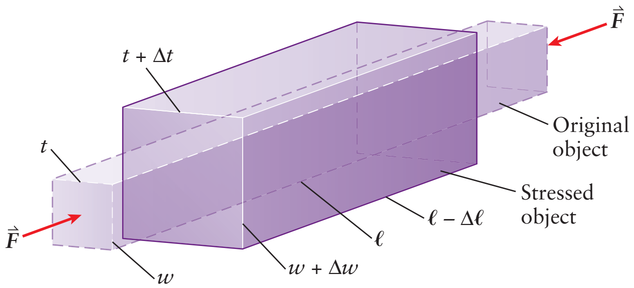


Figure 9-12 Problem 85

SOLVE

$$V_0 = \ell w t$$

$$V_f = (\ell + \Delta\ell)(w + \Delta w)(t + \Delta t)$$

$$\Delta V = V_f - V_0 = (\ell + \Delta\ell)(w + \Delta w)(t + \Delta t) - \ell w t$$

$$\frac{\Delta V}{V_0} = \frac{(\ell + \Delta\ell)(w + \Delta w)(t + \Delta t) - \ell w t}{\ell w t} = \left(\frac{\ell + \Delta\ell}{\ell}\right)\left(\frac{w + \Delta w}{w}\right)\left(\frac{t + \Delta t}{t}\right) - 1$$

$$= \left(1 + \frac{\Delta\ell}{\ell}\right)\left(1 + \frac{\Delta w}{w}\right)\left(1 + \frac{\Delta t}{t}\right) - 1$$

But $\nu = -\frac{\Delta w/w}{\Delta\ell/\ell} = -\frac{\Delta t/t}{\Delta\ell/\ell}$, which means

$$\frac{\Delta w}{w} = -\nu\left(\frac{\Delta\ell}{\ell}\right) = -\nu\varepsilon \quad \text{and} \quad \frac{\Delta t}{t} = -\nu\left(\frac{\Delta\ell}{\ell}\right) = -\nu\varepsilon$$

Plugging this into our expression for $\frac{\Delta V}{V_0}$:

$$\frac{\Delta V}{V_0} = \left(1 + \frac{\Delta\ell}{\ell}\right)\left(1 + \frac{\Delta w}{w}\right)\left(1 + \frac{\Delta t}{t}\right) - 1 = (1 + \varepsilon)(1 - \nu\varepsilon)(1 - \nu\varepsilon) - 1$$

$$= (1 + \varepsilon)(1 - 2\nu\varepsilon + \nu^2\varepsilon^2) - 1 = 1 + \varepsilon - 2\nu\varepsilon - 2\nu\varepsilon^2 + \nu^2\varepsilon^2 + \nu^2\varepsilon^3 - 1$$

We can neglect all terms with ε^2 or higher since $\varepsilon << 1$:

$$\frac{\Delta V}{V_0} \approx \varepsilon - 2\nu\varepsilon = \varepsilon(1 - 2\nu)$$

When this object is immersed in a fluid and symmetrically stressed, this strain will need to be multiplied by 3 for the three dimensions:

$$\frac{\Delta V}{V_0} \approx 3\varepsilon(1 - 2\nu)$$

Setting the pressure change equal to the stress applied to the object:

$$\Delta P = \left(0 - \frac{F}{A}\right) = -\frac{F}{A}$$

$$-B\frac{\Delta V}{V_0} = -Y\frac{\Delta \ell}{\ell}$$

$$B(3\varepsilon(1 - 2\nu)) = Y\varepsilon$$

$$\boxed{B = \frac{Y}{3(1 - 2\nu)}}$$

REFLECT

For an object under compression, the strain will always be less than 1 since the final length is smaller than the initial length. We are assuming the fluid does not change the overall length by very much.

Chapter 10
Gravitation

Conceptual Questions

10.1 Newton reasoned that the Moon, as it orbits, falls back toward the center of Earth due to the pull of gravity. Incrementally, it will move tangent to the orbital path, and then it will fall back to Earth. The combined motion leads to the familiar circular path. An apple falls straight toward Earth without the tangential motion. The vertical motion is the same for both the apple and the Moon.

10.2 The magnitude of the acceleration due to gravity near Earth's surface would increase more if Earth's radius were halved. The acceleration due to gravity is proportional to the mass but (inversely) proportional to the *square* of the radius.

10.3 When the mass of one object is doubled, the force between two objects doubles. If both masses are halved, the force is one-fourth the original value.

10.4 The magnitude of the gravitational force between everyday objects is negligible. Recall that G is on the order of 10^{-11} in SI units.

10.5 The mass cancels when applying Newton's second law to get the acceleration.

10.6 The magnitude of the gravitational force on an object increases as the distance between the objects decreases.

10.7 Orbiting the Sun is an incomplete description of the Moon's motion, but the Moon is definitely orbiting the Sun as a part of the Earth–lunar system.

10.8 The square of the period would now be proportional to the distance between two objects to the fourth power. Simplifying, the period would be proportional to the square of the distance between the two objects.

10.9 The most noticeable effects would be the comparison between the dynamics at relatively small distances (such as the Earth–Moon system) with the dynamics at extremely large distances (such as galaxies and galaxy clusters). The deviation in the inverse square law results in a difference in G or M. Also noncircular orbits would not be elliptical, although if the power is close enough to 2 the difference in orbit could be too small to notice without watching for a very long time indeed.

10.10 Even though it's inside of a hollow sphere, the marble still experiences the gravitational force due to the Earth.

10.11 The gravitational potential energy is negative because we choose to place the zero point of potential energy at $r = \infty$. With that choice, the gravitational potential energy is

negative because the gravitational force is attractive. The two objects will gain kinetic energy as they approach one another.

10.12 If we take ground level to be the zero potential energy, anything below it has a negative potential energy. That negative potential energy is equal in magnitude to the amount of positive energy needed to raise the frog up to ground level. Therefore, if the frog jumps with a larger positive amount of kinetic energy, the frog can escape the well.

10.13 By Kepler's law, the faster an object is moving as it orbits the Sun, the closer it is to the Sun. Thus, Earth is closer to the Sun when the Northern Hemisphere is in winter.

10.14 From Kepler's law, the larger the orbit, the longer the period.

10.15 When a satellite is in a circular equatorial orbit with a period equal to that of Earth's rotation, it will appear to be stationary relative to a single spot on the ground. The altitude of the required orbit, which can be found using Kepler's law of periods, is about 3.59×10^7 m.

10.16 Since the effect of gravity is so much less, it is easier to fabricate a perfectly spherical object.

10.17 Cells are probably the least affected by gravity (owing to strong electromagnetic forces at that level), but tissues and organs certainly depend on the constant supply of blood. This supply is intrinsically tied to the gravimetric forces on fluids in our body. Our vertebrae, for example, are continually compressed when we live on Earth. Our feet become flatter due to our continuous weight pushing down on them; our muscles sag over time, and so on.

Multiple-Choice Questions

10.18 **A** (four times as much as the original value). The magnitude of the gravitational force is proportional to the product of the two masses.

10.19 **E** (one-fourth of the original value). The magnitude of the gravitational force is inversely proportional to the square of the distance between the two masses.

10.20 **C** (they pull on each other equally). Newton's third law states that the gravitational force of the Earth on the Sun has the same magnitude as the gravitational force of the Sun on the Earth. You can also see this by explicitly applying Newton's universal law of gravitation.

10.21 **A** (she weighs more at the bottom). The value of g decreases with increasing altitude because the distance between the Earth and the mountain climber is increasing.

10.22 **B** ($2F$). From the shell theorem, the magnitude of the gravitational force of m will be proportional to the distance r. If r is doubled, the force will also double.

10.23 **B** ($\sqrt{2}v$). We can calculate the speed of the satellite in the initial orbit using Newton's second law: $v = \sqrt{\frac{GM}{R}}$. Comparing this to the escape speed formula, we see that the escape speed must be $\sqrt{2}v$.

10.24 **B** ($\sqrt{2}v$). The escape speed is proportional to the square root of the mass of the planet.

10.25 **A** (Satellite A has more kinetic energy, less potential energy, and less mechanical energy (potential energy plus kinetic energy) than satellite B). The kinetic energy of an orbiting satellite is inversely proportional to the distance. Less potential energy in this case means "more negative."

10.26 **A** (1/4).

$$\frac{F_{\text{c, satellite}}}{F_{\text{c, ISS}}} = \frac{\left(\dfrac{GMm}{R_{\text{satellite}}^2}\right)}{\left(\dfrac{GMm}{R_{\text{ISS}}^2}\right)} = \frac{R_{\text{ISS}}^2}{R_{\text{satellite}}^2} = \frac{R_{\text{ISS}}^2}{(2R_{\text{ISS}})^2} = \boxed{\frac{1}{4}}$$

10.27 **B** ($8T$).

$$\frac{T_{\text{B}}}{T_{\text{A}}} = \frac{\sqrt{\dfrac{4\pi^2 a_{\text{B}}^3}{GM}}}{\sqrt{\dfrac{4\pi^2 a_{\text{A}}^3}{GM}}} = \sqrt{\frac{(4a_{\text{A}})^3}{a_{\text{A}}^3}} = \boxed{8}$$

The M in the denominator of the square root refers to the mass of the *star*, not the *planet*.

Estimation Questions

10.28 The distance from the center of the Moon to the closest surface of the Earth is:

$$3.84 \times 10^8 \text{ m} - 6.38 \times 10^6 \text{ m} = 3.78 \times 10^8 \text{ m}$$

The distance from the center of the Moon to the farthest surface of the Earth is:

$$3.84 \times 10^8 \text{ m} + 6.38 \times 10^6 \text{ m} = 3.90 \times 10^8 \text{ m}$$

The force on a 1-kg mass at each of these locations will be:

$$F_{\text{close}} = \frac{\left(6.67 \times 10^{-11} \dfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(7.35 \times 10^{22} \text{ kg})(1 \text{ kg})}{(3.78 \times 10^8 \text{ m})^2} = 3.4 \times 10^{-5} \text{ N}$$

$$F_{\text{far}} = \frac{\left(6.67 \times 10^{-11} \dfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(7.35 \times 10^{22} \text{ kg})(1 \text{ kg})}{(3.90 \times 10^8 \text{ m})^2} = 3.2 \times 10^{-5} \text{ N}$$

Ratio:

$$\frac{3.4 \times 10^{-5}\ \text{N}}{3.2 \times 10^{-5}\ \text{N}} = \boxed{1.1}$$

10.29 Assuming it is possible to measure a force of about 10^{-7} N, the weights should have about 1 mm center-to-center separation.

10.30

$$g_{\text{Everest}} = \frac{GM}{r^2} = \frac{\left(6.67 \times 10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24}\ \text{kg})}{((6.38 \times 10^6\ \text{m}) + (8848\ \text{m}))^2} = \boxed{9.771\frac{\text{m}}{\text{s}^2}}$$

$$g_{\text{Dead Sea}} = \frac{GM}{r^2} = \frac{\left(6.67 \times 10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24}\ \text{kg})}{((6.38 \times 10^6\ \text{m}) - (394\ \text{m}))^2} = \boxed{9.800\frac{\text{m}}{\text{s}^2}}$$

10.31 The potential energy difference between the top of Mt. Everest and sea level is only 0.14% when using a constant g model as opposed to using the full law of gravitation.

10.32 The difference in gravitational fields between the peak of Mt. Everest and sea level is about $(9.80 - 9.77)\ \text{m/s}^2 = 0.03\ \text{m/s}^2$. The sherpa would feel a great compression in his vertebra of about $0.03/9.8 = 0.31\%$. If we assume that his height is directly related to this force, then the sherpa would be about 0.5 cm shorter at sea level.

10.33 The force between Earth and Jupiter is 1/18,000 of the force between Earth and the Sun. The impact would be small.

10.34

	T (days)	a (AU)	T^2 (days^2)	a^3 (AU^3)
Mercury	87.97	0.3871	7738.721	0.058
Venus	244.7	0.7233	50490.09	0.3784
Earth	365.2	1.000	133371	1
Mars	687.0	1.5234	471969	3.5354
Jupiter	4332	5.204	18766224	140.93
Saturn	10832	9.582	1.17E+08	879.77
Uranus	30799	19.23	9.49E+08	7111.1
Neptune	60190	30.10	3.62E+09	27271
Pluto	90613	39.48	8.21E+09	61536
Halley	27507	?	7.57E+08	?

(1 day = 86,400 s; 1 AU = 1.49598×10^{11} m)

Part a)

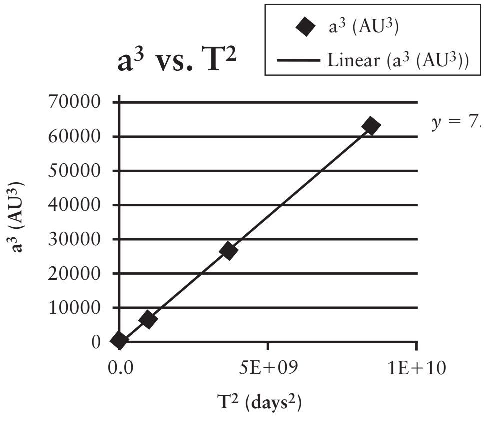


Figure 10-1 Problem 34

Best-fit constant: $\boxed{7.50 \times 10^{-6}\ (\text{AU}^3/\text{days}^2)}$.

Part b)

$$a_{\text{Halley}} = \sqrt[3]{\left(7.50 \times 10^{-6}\frac{\text{AU}^3}{\text{day}^2}\right)(27{,}507\ \text{days})^2} + (7.61\ \text{AU}) = \boxed{17.8\ \text{AU}}$$

Problems

10.35

SET UP

A tree stump ($m_s = 500$ kg) is 1000 m from a boulder ($m_b = 12{,}000$ kg). The gravitational force between the two is attractive. We can calculate its magnitude from Newton's universal law of gravitation.

SOLVE

$$F_g = \frac{Gm_sm_b}{r^2} = \frac{\left(6.67 \times 10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(500\ \text{kg})(12{,}000\ \text{kg})}{(1000\ \text{m})^2} = \boxed{4 \times 10^{-10}\ \text{N}}$$

The gravitational force exerted by the stump on the boulder points $\boxed{\text{toward the stump}}$.

REFLECT

This force is 15 orders of magnitude smaller than the force of gravity due to the Earth acting on the boulder.

10.36

SET UP

A bowling ball ($m_b = 7.0$ kg) is 0.2 m from a bowling pin ($m_p = 1.5$ kg). We can calculate the magnitude of the gravitational force between the two from Newton's universal law of gravitation.

SOLVE

$$F_g = \frac{Gm_b m_p}{r^2} = \frac{\left(6.67 \times 10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(7.0\text{ kg})(1.5\text{ kg})}{(0.2\text{ m})^2} = \boxed{1.8 \times 10^{-8}\text{ N}}$$

This force is $\boxed{\text{not large enough to affect the motion}}$ of the ball.

REFLECT

If this is the only force acting in the plane of the ball's motion, the acceleration of the ball (from Newton's second law) will be on the order of 10^{-9} m/s^2.

10.37

SET UP

A baseball ($m_{ball} = 0.150$ kg) is 100 m from a bat ($m_{bat} = 0.935$ kg). We can calculate the magnitude of the gravitational force between the two from Newton's universal law of gravitation.

SOLVE

$$F_g = \frac{Gm_{ball}m_{bat}}{r^2} = \frac{\left(6.67 \times 10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(0.150\text{ kg})(0.935\text{ kg})}{(1000\text{ m})^2} = \boxed{9.35 \times 10^{-16}\text{ N}}$$

REFLECT

As expected, this force is negligible.

10.38

SET UP

A throng of 3 million people ($m_t = (3 \times 10^6)(80\text{ kg})$) is 300 m from an eagle ($m_e = 4.5$ kg). We can calculate the magnitude of the gravitational force between the two from Newton's universal law of gravitation. Since the eagle is relatively close to the Earth's surface, we can use $g = 9.8$ m/s^2 to calculate the force between the Earth and the eagle.

SOLVE

Part a)

$$F_g = \frac{Gm_t m_e}{r^2} = \frac{\left(6.67 \times 10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(3 \times 10^6)(80\text{ kg})(4.5\text{ kg})}{(300\text{ m})^2} = \boxed{8.0 \times 10^{-7}\text{ N}}$$

Part b)

$$F_g = mg = (4.5\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right) = \boxed{44.1\text{ N}}$$

REFLECT

Using Newton's universal law of gravitation, the force between the Earth and the eagle has a magnitude of 44.0 N, so our assumption to use $g = 9.8$ m/s^2 is correct.

10.39

SET UP

The magnitude of the gravitational force between the Earth and an apple ($m_a = 1$ kg) on the Earth's surface is equal to $m_a g$. We can use Newton's universal law of gravitation to calculate the magnitude of the gravitational force between the same apple and the Moon.

SOLVE

Earth:

$$F_g = mg = (1\ \text{kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right) = \boxed{9.8\ \text{N}}$$

Moon:

$$F_g = \frac{Gm_a m_M}{r^2} = \frac{\left(6.67 \times 10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(1\ \text{kg})(7.35 \times 10^{22}\ \text{kg})}{(3.844 \times 10^{8}\ \text{m})^2} = \boxed{3.32 \times 10^{-5}\ \text{N}}$$

REFLECT

We would get the same answer for the force of the Earth on the apple if we used the universal law of gravitation.

10.40

SET UP

We can use Newton's universal law of gravitation to find the magnitude of the gravitational force between the Earth and the Sun. The mass of the Earth is $m_E = 5.98 \times 10^{24}$ kg, the mass of the Sun is $m_S = 1.99 \times 10^{30}$ kg, and the distance between the two is 1.50×10^{11} m.

SOLVE

$$F_g = \frac{Gm_E m_S}{r^2} = \frac{\left(6.67 \times 10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24}\ \text{kg})(1.99 \times 10^{30}\ \text{kg})}{(1.50 \times 10^{11}\ \text{m})^2} = \boxed{3.53 \times 10^{22}\ \text{N}}$$

REFLECT

The mean Earth–Sun distance (1.50×10^{11} m) is referred to as an "astronomical unit" or AU.

10.41

SET UP

We can use Newton's universal law of gravitation to calculate the net force acting on the Moon during an eclipse. The net force is equal to the vector sum of the gravitational force due to the Sun and the gravitational force due to the Earth. In our coordinate system, we'll assume that all three satellites lie along the x-axis and that the Sun is to the left of the Moon (toward $-x$).

SOLVE

$$\sum F_x = F_{S\to M,\,x} + F_{E\to M,\,x} = -\frac{Gm_S m_M}{r_{SM}^2} + \frac{Gm_E m_M}{r_{EM}^2} = GM_M\left(\frac{m_E}{r_{EM}^2} - \frac{m_S}{(r_{ES} - r_{EM})^2}\right)$$

$$= \left(6.67 \times 10^{-11}\frac{\mathrm{N\cdot m^2}}{\mathrm{kg^2}}\right)(7.35 \times 10^{22}\ \mathrm{kg})$$

$$\left(\frac{5.98 \times 10^{24}\ \mathrm{kg}}{3.84 \times 10^{8}\ \mathrm{m}^2} - \frac{1.99 \times 10^{30}\ \mathrm{kg}}{((1.50 \times 10^{11}\ \mathrm{m}) - (3.84 \times 10^{8}\ \mathrm{m}))^2}\right)$$

$$= -2.37 \times 10^{20}\ \mathrm{N}$$

The net force acting on the Moon has a magnitude of $\boxed{2.37 \times 10^{20}\ \text{N and points toward the Sun}}$.

REFLECT

The gravitational force on the Moon due to the Sun is still larger than the force due to the Earth even though the Earth is much closer.

10.42

SET UP

Star 1 has a mass equal to the mass of the Sun, while star 2 has a mass equal to one-half the mass of the Sun. The two stars are 50 AU from one another. We are interested in the point where the net gravitational force due to the two stars on a space probe (mass m_p) is equal to zero; the point where the net force is equal to zero must be between the two stars. We will let x be the distance from star 1 to the probe and y be the distance from star 2 to the probe. We'll assume that star 1 is to the left of the probe and star 2 is to the right.

SOLVE

$$\frac{GM_S M_p}{x^2} = \frac{G\left(\frac{M_S}{2}\right)M_p}{y^2}$$

$$2y^2 = x^2$$

$$x + y = 50\ \mathrm{AU}$$

$$y\sqrt{2} + y = 50\ \mathrm{AU}$$

$$y = \frac{50\ \mathrm{AU}}{1 + \sqrt{2}} = 20.7\ \mathrm{AU}$$

$$x = (50\ \mathrm{AU}) - y = 29.3\ \mathrm{AU}$$

The point where the net gravitational force due to the two stars is equal to zero is $\boxed{29.3\ \text{AU to the right of star 1}}$.

REFLECT

The exact masses of the stars or the space probe are not necessary to solve this problem. It makes sense that the point where the net force is zero should be closer to the less massive star.

10.43

SET UP

We are interested in the point where the net gravitational force due to the Earth ($m_E = 5.98 \times 10^{24}$ kg) and the Moon ($m_M = 7.35 \times 10^{22}$ kg) on a space probe (mass m_p) is equal to zero; the point where the net force is equal to zero must be between the Earth and the Moon. The Moon is 3.84×10^8 m from the Earth. We will let x be the distance from the Earth to the probe and y be the distance from the Moon to the probe. We'll assume that the Earth is to the left of the probe and the Moon is to the right.

SOLVE

$$\frac{GM_EM_p}{x^2} = \frac{GM_MM_p}{y^2}$$

$$\frac{x^2}{y^2} = \frac{M_E}{M_M}$$

$$x = y\sqrt{\frac{M_E}{M_M}}$$

$$x + y = 3.84 \times 10^8 \text{ m}$$

$$y\sqrt{\frac{M_E}{M_M}} + y = 3.84 \times 10^8 \text{ m}$$

$$y = \frac{3.84 \times 10^8 \text{ m}}{1 + \sqrt{\frac{M_E}{M_M}}} = \frac{3.84 \times 10^8 \text{ m}}{1 + \sqrt{\frac{5.98 \times 10^{24} \text{ kg}}{7.35 \times 10^{22} \text{ kg}}}} = 3.83 \times 10^7 \text{ m}$$

$$x = (3.84 \times 10^8 \text{ m}) - y = 3.46 \times 10^8 \text{ m}$$

The point where the net gravitational force due to the Earth and Moon is equal to zero is $\boxed{3.46 \times 10^8 \text{ m to the right of the Earth}}$ or $\boxed{3.83 \times 10^7 \text{ m to the left of the Moon}}$.

REFLECT

The exact mass of the space probe is not necessary to solve this problem. It makes sense that the point where the net force is zero should be closer to the Moon than the Earth since it is less massive.

10.44

SET UP

Eight stars are located on the vertices of a cube with sides of $d = 100$ AU. Each star has a mass of $m_S = 1.99 \times 10^{30}$ kg. We can use Newton's universal law of gravitation to calculate the net gravitational force acting on one of the stars, which we'll call star 1, due to the other seven stars. We'll set star 1 at the origin. We can first calculate the magnitude of each gravitational force and then the unit vector along which each force points. The net force is the vector sum of the seven individual forces.

SOLVE

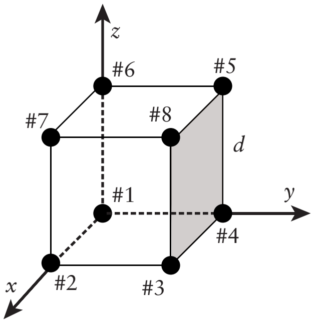


Figure 10-2 Problem 44

Magnitudes of nearest neighbors:

$$F_2 = F_4 = F_6 = \frac{Gm_S^2}{d^2}$$

Magnitudes of next nearest neighbors:

$$F_3 = F_5 = F_7 = \frac{Gm_S^2}{(d\sqrt{2})^2} = \frac{Gm_S^2}{2d^2}$$

Magnitudes of farthest neighbor:

$$F_8 = \frac{Gm_S^2}{(d\sqrt{3})^2} = \frac{Gm_S^2}{3d^2}$$

Force vectors:

$$\vec{F}_2 = F_2\hat{x} = \frac{Gm_S^2}{d^2}\hat{x}$$

$$\vec{F}_3 = F_3\left(\frac{\hat{x}+\hat{y}}{\sqrt{2}}\right) = \frac{Gm_S^2}{2\sqrt{2}d^2}(\hat{x}+\hat{y})$$

$$\vec{F}_4 = F_4\hat{y} = \frac{Gm_S^2}{d^2}\hat{y}$$

$$\vec{F}_5 = F_5\left(\frac{\hat{y}+\hat{z}}{\sqrt{2}}\right) = \frac{Gm_S^2}{2\sqrt{2}d^2}(\hat{y}+\hat{z})$$

$$\vec{F}_6 = F_6\hat{z} = \frac{Gm_S^2}{d^2}\hat{z}$$

$$\vec{F}_7 = F_7\left(\frac{\hat{x}+\hat{z}}{\sqrt{2}}\right) = \frac{Gm_S^2}{2\sqrt{2}d^2}(\hat{x}+\hat{z})$$

$$\vec{F}_8 = F_8\left(\frac{\hat{x}+\hat{y}+\hat{z}}{\sqrt{3}}\right) = \frac{Gm_S^2}{3\sqrt{3}d^2}(\hat{x}+\hat{y}+\hat{z})$$

Net force, x component:

$$\sum F_x = \left(\frac{Gm_S^2}{d^2}\right) + 2\left(\frac{Gm_S^2}{2\sqrt{2}d^2}\right) + \left(\frac{Gm_S^2}{3\sqrt{3}d^2}\right) = \left(1 + \frac{1}{\sqrt{2}} + \frac{1}{3\sqrt{3}}\right)\left(\frac{Gm_S^2}{d^2}\right)$$

$$= \left(1 + \frac{1}{\sqrt{2}} + \frac{1}{3\sqrt{3}}\right)\frac{\left(6.67 \times 10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(1.99 \times 10^{30}\text{ kg})^2}{\left(100\text{ AU} \times \frac{1.496 \times 10^{11}\text{ m}}{1\text{ AU}}\right)^2} = 2.24 \times 10^{24}\text{ N}$$

Net force, y component:

$$\sum F_y = \left(\frac{Gm_S^2}{d^2}\right) + 2\left(\frac{Gm_S^2}{2\sqrt{2}d^2}\right) + \left(\frac{Gm_S^2}{3\sqrt{3}d^2}\right) = \left(1 + \frac{1}{\sqrt{2}} + \frac{1}{3\sqrt{3}}\right)\left(\frac{Gm_S^2}{d^2}\right)$$

$$= \left(1 + \frac{1}{\sqrt{2}} + \frac{1}{3\sqrt{3}}\right)\frac{\left(6.67 \times 10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(1.99 \times 10^{30}\text{ kg})^2}{\left(100\text{ AU} \times \frac{1.496 \times 10^{11}\text{ m}}{1\text{ AU}}\right)^2} = 2.24 \times 10^{24}\text{ N}$$

Net force, z component:

$$\sum F_z = \left(\frac{Gm_S^2}{d^2}\right) + 2\left(\frac{Gm_S^2}{2\sqrt{2}d^2}\right) + \left(\frac{Gm_S^2}{3\sqrt{3}d^2}\right) = \left(1 + \frac{1}{\sqrt{2}} + \frac{1}{3\sqrt{3}}\right)\left(\frac{Gm_S^2}{d^2}\right)$$

$$= \left(1 + \frac{1}{\sqrt{2}} + \frac{1}{3\sqrt{3}}\right)\frac{\left(6.67 \times 10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(1.99 \times 10^{30}\text{ kg})^2}{\left(100\text{ AU} \times \frac{1.496 \times 10^{11}\text{ m}}{1\text{ AU}}\right)^2} = 2.24 \times 10^{24}\text{ N}$$

Net force:

$$\boxed{\sum \vec{F} = (2.24 \times 10^{24}\text{ N})(\hat{x} + \hat{y} + \hat{z})}$$

REFLECT

The symmetry of the system dictates that the net force must point along the long diagonal of the cube.

10.45

SET UP

The weight of a 5-kg object on the Earth's surface is equal to mg. We can use Newton's universal law of gravitation to calculate the magnitude of the gravitational force between the 5-kg object and a second point mass with a mass equal to $m_E = 5.98 \times 10^{24}$ kg that is a distance $R_E = 6.38 \times 10^6$ m away.

SOLVE

Weight on Earth's surface:

$$F_g = mg = (5\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right) = \boxed{49.0\text{ N}}$$

Gravitational force an Earth radius away:

$$F_g = \frac{Gm_1m_2}{r^2} = \frac{\left(6.67 \times 10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5\text{ kg})(5.98 \times 10^{24}\text{ kg})}{(6.38 \times 10^6\text{ m})^2} = \boxed{49.0\text{ N}}$$

The forces are the $\boxed{\text{same}}$.

REFLECT

The acceleration due to gravity near the Earth's surface g is equal to $\frac{GM_E}{R_E^2}$, so it is no surprise that the two forces we calculated are equal.

10.46

SET UP

The gravitational field due to a planet or star is the gravitational force per unit mass. It is found by dividing the gravitational force experienced by a small object by the mass of the small object. We can calculate the magnitude of the gravitational field due to Earth at the location of the Moon; the direction of the field will point toward Earth. The centripetal acceleration of the Moon as it orbits the Earth is related to the distance between the Earth and the Moon and the period of its rotation ($T = 27.32$ days). As a reminder, the mass of the Earth is $m_E = 5.98 \times 10^{24}$ kg and the distance between the Earth and the Moon is 3.84×10^8 m.

SOLVE

Part a)

$$\vec{g} = \frac{GM_E}{R_{EM}^2}(-\hat{r}) = \frac{\left(6.67 \times 10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24}\text{ kg})}{(3.84 \times 10^8\text{ m})^2}(-\hat{r}) = \left(-0.00270\frac{\text{m}}{\text{s}^2}\right)\hat{r}$$

The gravitational field due to Earth at the location of the Moon has a magnitude of $\boxed{0.00270\text{ m/s}^2\text{ and points toward Earth}}$.

Part b)

$$a_c = \frac{v^2}{R_{EM}} = \frac{\left(\frac{2\pi R_{EM}}{T}\right)^2}{R_{EM}} = \frac{4\pi^2 R_{EM}}{T^2} = \frac{4\pi^2(3.84 \times 10^8\text{ m})}{\left(27.32\text{ days} \times \frac{24\text{ hr}}{1\text{ day}} \times \frac{3600\text{ s}}{1\text{ hr}}\right)^2} = \boxed{0.00272\frac{\text{m}}{\text{s}^2}}$$

Part c) The values are $\boxed{\text{essentially the same}}$.

REFLECT

The Moon orbits the Earth because of the gravitational force of the Earth acting on the Moon.

10.47

SET UP

A hollow sphere has a mass m_1 and radius r_1. A second mass m_2 is located a distance r_2 from the surface of the sphere. We can use Newton's universal law of gravitation to calculate the magnitude of the gravitational force between the two objects. Mass m_2 is located outside of the hollow sphere, which means the sphere acts as a point mass located a distance $(r_1 + r_2)$ from m_2 due to the shell theorem.

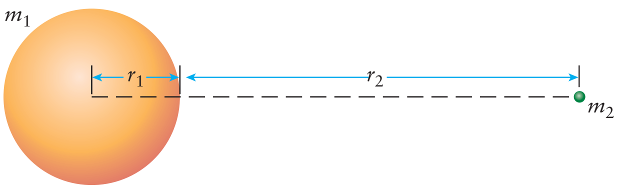


Figure 10-3 Problem 47

SOLVE

$$\boxed{F_g = \frac{Gm_1m_2}{(r_1 + r_2)^2}}$$

REFLECT

The gravitational force between the two objects is attractive and points along the line connecting the two.

10.48

SET UP

We are asked to find the value of g at a point 86 m below sea level. We can use the shell theorem to determine the fraction of the mass of the Earth that contributes to the value of g at this point; the shell has a radius of $(R_E - 86\text{ m})$. We will assume that the density of the Earth is constant in order to calculate the mass of the shell.

SOLVE

$$g = \frac{GM}{R^2} = \frac{G\left(\dfrac{M_E}{\left(\frac{4}{3}\pi R_E^3\right)}\left(\frac{4}{3}\pi(R_E - (86\text{ m}))^3\right)\right)}{(R_E - (86\text{ m}))^2} = \frac{GM_E(R_E - (86\text{ m}))}{R_E^3}$$

$$= \frac{\left(6.67 \times 10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24}\text{ kg})((6.38 \times 10^6\text{ m}) - (86\text{ m}))}{(6.38 \times 10^6\text{ m})^3} = \boxed{9.80\frac{\text{m}}{\text{s}^2}}$$

REFLECT

The value of g at an altitude of 86 m below sea level should not change much from the value of g at sea level.

10.49

SET UP

The center of a ping-pong ball ($m_p = 0.0027$ kg) is located 100 cm from the center of a basketball ($m_b = 0.600$ kg). The magnitude of the gravitational force between the two balls can be calculated directly from Newton's universal law of gravitation. We can treat each ball as a point mass located at its center due to the shell theorem.

SOLVE

$$F_g = \frac{Gm_p m_b}{r^2} = \frac{\left(6.67 \times 10^{-11}\frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}\right)(0.0027\ \mathrm{kg})(0.600\ \mathrm{kg})}{(1.00\ \mathrm{m})^2} = \boxed{1.08 \times 10^{-13}\ \mathrm{N}}$$

REFLECT

The dimensions of each ball are irrelevant since we are outside of each object.

10.50

SET UP

The acceleration due to gravity g at the top of Mt. Everest is proportional to the mass of the Earth and inversely proportional to the square of the distance between the center of the Earth and the top of Mt. Everest. In order to calculate g at the Dead Sea, which is located below sea level, we need to invoke the shell theorem to calculate the mass of the Earth that contributes to g at this location; the mass will be proportional to the ratio of the volume of the sphere of radius $(R_E - 400\ \mathrm{m})$ and the volume of the sphere of radius R_E.

SOLVE

Part a)

$$g_{\text{Everest}} = \frac{GM_E}{R^2} = \frac{\left(6.67 \times 10^{-11}\frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}\right)(5.98 \times 10^{24}\ \mathrm{kg})}{((6.38 \times 10^6\ \mathrm{m}) + (8850\ \mathrm{m}))^2} = \boxed{9.77\frac{\mathrm{m}}{\mathrm{s^2}}}$$

Part b)

$$M' = M_E \frac{\left(\frac{4}{3}\pi(R_E - (400\ \mathrm{m}))^3\right)}{\left(\frac{4}{3}\pi R_E^3\right)} = M_E \frac{(R_E - (400\ \mathrm{m}))^3}{R_E^3}$$

$$= (5.98 \times 10^{24}\ \mathrm{kg})\frac{((6.38 \times 10^6\ \mathrm{m}) - (400\ \mathrm{m}))^3}{(6.38 \times 10^6\ \mathrm{m})^3} = 5.9789 \times 10^{24}\ \mathrm{kg}$$

$$g_{\text{Everest}} = \frac{GM'}{R^2} = \frac{\left(6.67 \times 10^{-11}\frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}\right)(5.9789 \times 10^{24}\ \mathrm{kg})}{((6.38 \times 10^6\ \mathrm{m}) - (400\ \mathrm{m}))^2} = \boxed{9.80\frac{\mathrm{m}}{\mathrm{s^2}}}$$

$$\frac{g_{\text{Dead Sea}}}{g_{\text{Everest}}} = \frac{\left(9.80\frac{\mathrm{m}}{\mathrm{s^2}}\right)}{\left(9.77\frac{\mathrm{m}}{\mathrm{s^2}}\right)} = 1.003$$

The fractional change in g between Mt. Everest and the Dead Sea is $\boxed{0.003 \text{ or } 0.3\%}$.

REFLECT

The value of g does not change much over distances less than the Earth's radius.

10.51

SET UP

A 1-kg point mass is positioned within a 100,000-kg hollow sphere of radius $R = 10{,}000$ m. The mass per unit area of the sphere is equal to the total mass of the sphere divided by the surface area of the sphere. Because the 1-kg mass is located inside the hollow sphere, the net gravitational force acting on it is zero from the shell theorem.

SOLVE

Part a)

$$\frac{M}{A} = \frac{100{,}000 \text{ kg}}{4\pi(10{,}000 \text{ m})^2} = \boxed{7.96 \times 10^{-5} \frac{\text{kg}}{\text{m}^2}}$$

Part b) The net force acting on the 1-kg object is $\boxed{0}$.

REFLECT

Since all of the mass is confined to the surface of the sphere, the net gravitational force on an object inside the shell is zero.

10.52

SET UP

A uniform sphere ($m_1 = 100{,}000$ kg) has an inner radius $R_{\text{inner}} = 9500$ m and an outer radius $R_{\text{outer}} = 10{,}000$ m. A point mass ($m_2 = 1$ kg) is located 50,000 m from the surface of the sphere. Since the point mass is located outside of the sphere, we can treat the sphere as a point mass located at its center when calculating the gravitational force acting on the 1-kg mass. The volume of the spherical shell is equal to the volume of the outer sphere (radius R_{outer}) minus the volume of the inner sphere (radius R_{inner}).

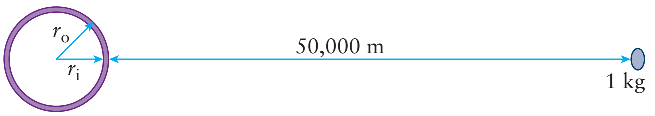


Figure 10-4 Problem 52

SOLVE

Part a)

$$F_g = \frac{Gm_1m_2}{r^2} = \frac{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)(100{,}000 \text{ kg})(1 \text{ kg})}{((50{,}000 \text{ m}) + (10{,}000 \text{ m}))^2} = 1.85 \times 10^{-15} \text{ N}$$

The net force on the 1-kg object is $\boxed{1.85 \times 10^{-15} \text{ N toward the sphere}}$.

Part b)

$$V = \frac{4}{3}\pi(R_{\text{outer}}^3 - R_{\text{inner}}^3) = \frac{4}{3}\pi((10{,}000 \text{ m})^3 - (9500 \text{ m})^3) = 5.97 \times 10^{11} \text{ m}^3$$

$$\frac{M}{V} = \frac{100{,}000 \text{ kg}}{5.97 \times 10^{11} \text{ m}^3} = \boxed{1.67 \times 10^{-7} \frac{\text{kg}}{\text{m}^3}}$$

REFLECT

We are told the distance that m_2 is from the *surface* of the large sphere. We need to add in the outer radius when calculating the gravitational force because we are treating the sphere as a point mass located at its center.

10.53

SET UP

Five spherical storage tanks are separated by 75 m and filled with natural gas ($\rho_{gas} = 0.8\ kg/m^3$). Each storage tank stores a volume of 5000 m^3; we can use the inner volume of the tank to find the radius of the inside of each tank. The tanks are constructed from a 10-cm-thick wall of steel, which has a density of $\rho_{steel} = 8000\ kg/m^3$. The distance d between the centers of neighboring tanks is equal to twice the radius of the tank plus twice the thickness of the steel wall plus the 75 m separating the tanks. The total mass of each tank is the mass of the steel plus the mass of the gas. We can use Newton's universal law of gravitation to calculate the gravitational force on the end tank, which we'll call tank 1, by the other tanks. The net gravitational force will point toward the remaining tanks.

SOLVE

Figure 10-5 Problem 53

Radius of each tank:

$$\frac{4}{3}\pi R^3 = 5000\ m^3$$

$$R = \sqrt[3]{\frac{3(5000\ m^3)}{4\pi}} = \sqrt[3]{\frac{3(5000\ m^3)}{4\pi}} = 10.61\ m$$

Distance between the center of mass of the tanks:

$$d = 2(10.61\ m) + 2(0.1\ m) + (75\ m) = 96.42\ m$$

Mass of each tank with gas:

$$M = M_{steel} + M_{gas} = \left[\frac{4}{3}\pi(R + (0.1\ m))^3 - \frac{4}{3}\pi R^3\right]\rho_{steel} + (5000\ m^3)\rho_{gas}$$

$$= \frac{4}{3}\pi[((10.61\ m) + (0.1\ m))^3 - (10.61\ m)^3]\left(8000\frac{kg}{m^3}\right) + (5000\ m^3)\left(0.8\frac{kg}{m^3}\right)$$

$$= (1.142 \times 10^6\ kg) + (4000\ kg) = 1.146 \times 10^6\ kg$$

Force of tank 2 on tank 1:

$$F_{2\to 1} = \frac{Gm_2m_1}{d_{21}^2} = \frac{\left(6.67 \times 10^{-11}\frac{N \cdot m^2}{kg^2}\right)(1.146 \times 10^6\ kg)^2}{(96.42\ m)^2} = 0.00942\ N$$

Force of tank 3 on tank 1:

$$F_{3\to1} = \frac{Gm_3m_1}{d_{31}^2} = \frac{\left(6.67\times10^{-11}\dfrac{\mathrm{N\cdot m^2}}{\mathrm{kg^2}}\right)(1.146\times10^6\ \mathrm{kg})^2}{[2(96.42\ \mathrm{m})]^2} = 0.00236\ \mathrm{N}$$

Force of tank 4 on tank 1:

$$F_{4\to1} = \frac{Gm_4m_1}{d_{41}^2} = \frac{\left(6.67\times10^{-11}\dfrac{\mathrm{N\cdot m^2}}{\mathrm{kg^2}}\right)(1.146\times10^6\ \mathrm{kg})^2}{[3(96.42\ \mathrm{m})]^2} = 0.00105\ \mathrm{N}$$

Force of tank 5 on tank 1:

$$F_{5\to1} = \frac{Gm_5m_1}{d_{51}^2} = \frac{\left(6.67\times10^{-11}\dfrac{\mathrm{N\cdot m^2}}{\mathrm{kg^2}}\right)(1.146\times10^6\ \mathrm{kg})^2}{[4(96.42\ \mathrm{m})]^2} = 0.000589\ \mathrm{N}$$

Net force on tank 1:

$$\sum F = (0.00942\ \mathrm{N}) + (0.00236\ \mathrm{N}) + (0.00105\ \mathrm{N}) + (0.000589\ \mathrm{N}) = 0.0134\ \mathrm{N}$$

The net gravitational force acting on tank 1 has a magnitude of $\boxed{\text{0.0134 N and points toward the other tanks}}$.

REFLECT

The weight of tank 1 is approximately 1×10^7 N, which is nine orders of magnitude larger than the net gravitational force due to the other four tanks.

10.54

SET UP

A planet is made up of material of density ρ and has a radius R. A space probe of mass m is traveling at a speed of v at a distance r from the center of the planet. We can use conservation of energy in order to calculate the speed of the probe at a distance of $2r$ from the center of the planet. We can also use conservation of energy to calculate the escape velocity of the space probe from its new orbit; in this calculation, the final potential energy will be equal to zero.

SOLVE

Part a)

$$U_\mathrm{i} + K_\mathrm{i} = U_\mathrm{f} + K_\mathrm{f}$$

$$-\frac{G\left(\frac{4}{3}\pi R^3\rho\right)m}{r} + \frac{1}{2}mv^2 = -\frac{G\left(\frac{4}{3}\pi R^3\rho\right)m}{2r} + \frac{1}{2}mv_\mathrm{f}^2$$

$$v_\mathrm{f}^2 = v^2 + 2G\left(\frac{4}{3}\pi R^3\rho\right)\left(\frac{1}{2r} - \frac{1}{r}\right) = v^2 - \frac{G}{r}\left(\frac{4}{3}\pi R^3\rho\right)$$

$$\boxed{v_\mathrm{f} = \sqrt{v^2 - \frac{4G\pi R^3\rho}{3r}}}$$

Part b)

$$\frac{1}{2}mv_f^2 - \frac{G\left(\frac{4}{3}\pi R^3\rho\right)m}{2r} = \frac{1}{2}mv_{esc}^2 + 0$$

$$v_f^2 - \frac{G\left(\frac{4}{3}\pi R^3\rho\right)}{r} = v_{esc}^2$$

$$v_{esc}^2 = \left(v^2 - \frac{4G\pi R^3\rho}{3r}\right) - \frac{4G\pi R^3\rho}{3r} = v^2 - \frac{8G\pi R^3\rho}{3r}$$

$$\boxed{v_{esc} = \sqrt{v^2 - \frac{8G\pi R^3\rho}{3r}}}$$

REFLECT

Escape velocity refers to the speed required of an object to be infinitely far from a planet. Recall that the gravitational potential energy of an object is equal to zero when $r = \infty$.

10.55

SET UP

A 10-kg object moves from a point 6000 m above sea level to a point 1000 m above sea level. The work done by gravity is equal to the negative of the change in the gravitational potential energy of the object. We need to include the radius of the Earth when determining the initial and final positions due to the shell theorem.

SOLVE

$$W = -\Delta U = -\left(\frac{-Gm_Em}{R_f} - \left(\frac{-Gm_Em}{R_i}\right)\right) = Gm_Em\left(\frac{1}{R_E + (1000\text{ m})} - \frac{1}{R_E + (6000\text{ m})}\right)$$

$$= \left(6.67 \times 10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24}\text{ kg})(10\text{ kg})$$

$$\left(\frac{1}{(6.38 \times 10^6\text{ m}) + (1000\text{ m})} - \frac{1}{(6.38 \times 10^6\text{ m}) + (6000\text{ m})}\right)$$

$$= \boxed{489{,}400\text{ J} = 489.4\text{ kJ}}$$

REFLECT

The force of gravity and the displacement are in the same direction, so we expect the work done by gravity to be positive. If we used $U = mgy$ instead, the work done by gravity would be 490 kJ.

10.56

SET UP

We can use conservation of energy to calculate the escape velocity of a space probe of mass m initially located on the Earth's surface. The initial position is at $r = R_E$ and the final energy of

the probe is equal to zero. If the probe were launched from the top of a mountain of height H, the initial position would be $r = R_E + H$.

SOLVE

Part a)

$$U_i + K_i = U_f + K_f$$

$$-\frac{Gm_E m}{R_E} + \frac{1}{2}mv_{esc}^2 = 0 + 0$$

$$v_{esc} = \sqrt{\frac{2Gm_E}{R_E}} = \sqrt{\frac{2\left(6.67 \times 10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24}\text{ kg})}{6.38 \times 10^6\text{ m}}} = \boxed{11{,}200\frac{\text{m}}{\text{s}}}$$

Part b) If $r_i > R_E$, then $\boxed{v_{esc}\text{ would be smaller}}$ than our answer from part (a).

REFLECT

As the initial position of the probe (relative to the Earth's surface) becomes larger, the initial potential energy increases (becomes less negative).

10.57

SET UP

The Schwarzschild radius is defined as the distance from a black hole where the escape velocity is equal to speed of light, $c = 3 \times 10^8$ m/s. We can use conservation of energy to calculate the initial position. The mass of the black hole is equal to the mass of the Sun, $m_S = 1.99 \times 10^{30}$ kg.

SOLVE

$$U_i + K_i = U_f + K_f$$

$$-\frac{Gm_S m}{r} + \frac{1}{2}mc^2 = 0 + 0$$

$$r = \frac{2Gm_S}{c^2} = \frac{2\left(6.67 \times 10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(1.99 \times 10^{30}\text{ kg})}{\left(3.00 \times 10^8\frac{\text{m}}{\text{s}}\right)^2} = \boxed{2950\text{ m}}$$

REFLECT

A radius of 2950 m is equal to 1.8 miles!

10.58

SET UP

We can determine the approximate diameter of the visible universe, D_{univ}, by finding the Schwarzschild radius of a sphere with a uniform density equal to $\rho_c = \frac{3H^2}{8\pi G}$, where

$H = 70$ (km/s)/Mpc. One pc (parsec) is equal to 30.857×10^{12} km. Recall that the Schwarzschild radius is defined as the distance from a black hole where the escape velocity is equal to speed of light, $c = 3 \times 10^8$ m/s. We can use conservation of energy to calculate this radius.

SOLVE

$$U_i + K_i = U_f + K_f$$

$$-\frac{Gm_{univ}m}{R_{univ}} + \frac{1}{2}mc^2 = 0 + 0$$

$$R_{univ} = \frac{2Gm_{univ}}{c^2} = \frac{2G\left(\rho_c\left(\frac{4}{3}\pi R_{univ}^3\right)\right)}{c^2} = \frac{8G\pi R_{univ}^3}{c^2}\left(\frac{3H^2}{8\pi G}\right) = \frac{R_{univ}^3 H^2}{c^2}$$

$$R_{univ} = \frac{c}{H} = \frac{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)}{\left(70\frac{\text{km}}{\text{s} \cdot \text{Mpc}} \times \frac{1\text{ Mpc}}{10^6\text{ pc}} \times \frac{1\text{ pc}}{30.857 \times 10^{12}\text{ km}}\right)} = 1.3 \times 10^{26}\text{ m}$$

$$D_{univ} = 2R_{univ} = 2(1.3 \times 10^{26}\text{ m}) = \boxed{2.6 \times 10^{26}\text{ m}}$$

REFLECT

This is about 19 orders of magnitude larger than the diameter of the Earth. For comparison, the ratio of the size of an atom to the size of the Earth is only about 16 orders of magnitude.

10.59

SET UP

The volume of water in the Pacific Ocean is 7×10^8 km^3. The density of seawater is 1030 kg/m^3. The mass of the Moon is 7.35×10^{22} kg, and the distance from the center of the Moon to the center of the Earth is 3.84×10^8 m. We can use these data to calculate the gravitational potential energy between the Pacific Ocean and the Moon when the Pacific is facing away and facing toward the Moon. Although it is a very rough approximation, we will treat the ocean as a point mass. Therefore, we will need to add and subtract, respectively, the radius of the Earth in each calculation. The maximum speed of the water due to the tidal influence of the Moon can be calculated through conservation of energy—the change in the potential energies calculated in parts (a) and (b) is equal to the change in kinetic energy.

SOLVE

$$m_w = \rho_w V = \left(1030\frac{\text{kg}}{\text{m}^3}\right)\left(7 \times 10^8\text{ km}^3 \times \left(\frac{10^3\text{ m}}{1\text{ km}}\right)^3\right) = 7.21 \times 10^{20}\text{ kg}$$

Part a)

$$U = -\frac{Gm_w m_M}{r_{away}} = -\frac{\left(6.67 \times 10^{-11}\frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)(7.21 \times 10^{20}\text{ kg})(7.35 \times 10^{22}\text{ kg})}{(3.84 \times 10^8\text{ m}) + (6.38 \times 10^6\text{ m})}$$

$$= \boxed{-9.054 \times 10^{24}\text{ J}}$$

Part b)

$$U = -\frac{Gm_{\mathrm{w}}m_{\mathrm{M}}}{r_{\mathrm{toward}}} = -\frac{\left(6.67 \times 10^{-11}\frac{\mathrm{N}\cdot\mathrm{m}^2}{\mathrm{kg}^2}\right)(7.21 \times 10^{20}\ \mathrm{kg})(7.35 \times 10^{22}\ \mathrm{kg})}{(3.84 \times 10^{8}\ \mathrm{m}) - (6.38 \times 10^{6}\ \mathrm{m})} = \boxed{-9.360 \times 10^{24}\ \mathrm{J}}.$$

Part c)

$$\Delta K = -\Delta U$$

$$\frac{1}{2}m_{\mathrm{w}}v^2 = -((-9.360 \times 10^{24}\ \mathrm{J}) - (-9.054 \times 10^{24}\ \mathrm{J})) = 3.06 \times 10^{23}\ \mathrm{J}$$

$$v = \sqrt{\frac{2(3.06 \times 10^{23}\ \mathrm{J})}{m_{\mathrm{w}}}} = \sqrt{\frac{2(3.06 \times 10^{23}\ \mathrm{J})}{7.21 \times 10^{20}\ \mathrm{kg}}} = \boxed{29\frac{\mathrm{m}}{\mathrm{s}}}$$

REFLECT

This is a very rough approximation and overestimation, as it treats the Pacific Ocean as a point mass and doesn't take into account drag.

10.60

SET UP

The nonconservative work required to move the Moon from its present orbit to a location twice as far away is equal to the change in the Moon's mechanical energy. We need to determine the initial and final speeds of the Moon in order to calculate the change in the kinetic energy. We can relate the initial speed of the Moon to its orbital period ($T_0 = 27.3$ days). The final orbital period can be found in terms of the original orbital period through Newton's second law. The net force acting on the Moon is the gravitational force due to the Earth; this net force causes the Moon to undergo centripetal motion. The mass of the Earth is 5.98×10^{24} kg, the mass of the Moon is 7.35×10^{22} kg, and the initial distance from the center of the Moon to the center of the Earth is 3.84×10^{8} m.

SOLVE

New period from Newton's second law:

$$\frac{Gm_{\mathrm{E}}m_{\mathrm{M}}}{r_0^2} = \frac{v_0^2}{r_0}$$

$$\frac{Gm_{\mathrm{E}}m_{\mathrm{M}}}{(2r_0)^2} = \frac{v_1^2}{2r_0}$$

$$v_0^2 = 2v_1^2$$

$$\left(\frac{2\pi r_0}{T_0}\right)^2 = 2\left(\frac{2\pi(2r_0)}{T_1}\right)^2$$

$$T_1 = \sqrt{2^3}T_0$$

Initial and final speeds:

$$v_0 = \frac{2\pi r_0}{T_0} = \frac{2\pi(3.84 \times 10^8\text{ m})}{\left(27.3\text{ days} \times \frac{24\text{ hr}}{1\text{ day}} \times \frac{3600\text{ s}}{1\text{ hr}}\right)} = 1023\frac{\text{m}}{\text{s}}$$

$$v_0 = \frac{2\pi(2r_0)}{T_1} = \frac{4\pi r_0}{\sqrt{2^3}T_0} = \frac{4\pi(3.84 \times 10^8\text{ m})}{\sqrt{2^3}\left(27.3\text{ days} \times \frac{24\text{ hr}}{1\text{ day}} \times \frac{3600\text{ s}}{1\text{ hr}}\right)} = 723.3\frac{\text{m}}{\text{s}}$$

Work required:

$$W_{\text{nc}} = \Delta K + \Delta U = \frac{1}{2}m_{\text{M}}(v_{\text{f}}^2 - v_{\text{i}}^2) + \left(-\frac{Gm_{\text{M}}m_{\text{E}}}{2r_0} - \left(-\frac{Gm_{\text{M}}m_{\text{E}}}{r_0}\right)\right) = \frac{1}{2}m_{\text{M}}(v_{\text{f}}^2 - v_{\text{i}}^2) + \frac{Gm_{\text{M}}m_{\text{E}}}{2r_0}$$

$$= \frac{1}{2}(7.35 \times 10^{22}\text{ kg})\left(\left(723.3\frac{\text{m}}{\text{s}}\right)^2 - \left(1023\frac{\text{m}}{\text{s}}\right)^2\right)$$

$$+ \frac{\left(6.67 \times 10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(7.35 \times 10^{22}\text{ kg})(5.98 \times 10^{24}\text{ kg})}{2(3.84 \times 10^8\text{ m})}$$

$$= (-1.923 \times 10^{28}\text{ J}) + (3.817 \times 10^{28}\text{ J}) = \boxed{1.89 \times 10^{28}\text{ J}}$$

REFLECT

The final speed should be slower than the initial speed because the radius of the final orbit is larger. It makes sense that the work required to move the Moon to a larger orbit should be positive.

10.61

SET UP

We can use conservation of energy to calculate the impact speed of a 100-kg asteroid that is initially moving at a speed of 200 m/s at a distance of 1000 km from the Moon's surface. When calculating the initial and final gravitational potential energies of the asteroid, we need to include the radius of the Moon ($r_{\text{M}} = 1.737 \times 10^6$ m). The force of the Moon on the asteroid is nonconservative and the only force doing work to stop the asteroid. Therefore, the work done by the Moon on the asteroid is equal to the change in the asteroid's mechanical energy. We will assume that the asteroid does not move very deeply into the surface during the impact, so $\Delta U \approx 0$.

SOLVE

Part a)

$$U_{\text{i}} + K_{\text{i}} = U_{\text{f}} + K_{\text{f}}$$

$$-\frac{Gm_{\text{M}}m}{r_{\text{i}}} + \frac{1}{2}mv_{\text{i}}^2 = -\frac{Gm_{\text{M}}m}{r_{\text{f}}} + \frac{1}{2}mv_{\text{f}}^2$$

$$v_{\text{f}} = \sqrt{-\frac{2Gm_{\text{M}}}{r_{\text{i}}} + \frac{2Gm_{\text{M}}}{r_{\text{f}}} + v_{\text{i}}^2} = \sqrt{-2Gm_{\text{M}}\left(\frac{1}{(r_{\text{M}} + (10^6\text{ m}))} - \frac{1}{r_{\text{M}}}\right) + v_{\text{i}}^2}$$

$$= \sqrt{-2\left(6.67 \times 10^{-11}\frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}\right)(7.35 \times 10^{22}\ \mathrm{kg})\left(\frac{1}{((1.737 \times 10^6\ \mathrm{m}) + (10^6\ \mathrm{m}))} - \frac{1}{(1.737 \times 10^6\ \mathrm{m})}\right) + \left(200\frac{\mathrm{m}}{\mathrm{s}}\right)^2}$$

$$= \boxed{1450\frac{\mathrm{m}}{\mathrm{s}}}$$

Part b)

$$W_{\mathrm{nc}} = \Delta K + \Delta U = \frac{1}{2}m(v_{\mathrm{f}}^2 - v_{\mathrm{i}}^2) + 0 = \frac{1}{2}m(0 - v_{\mathrm{i}}^2) = \frac{1}{2}(100\ \mathrm{kg})\left(1450\frac{\mathrm{m}}{\mathrm{s}}\right)^2 = -1.05 \times 10^8\ \mathrm{J}$$

The Moon does $\boxed{1.05 \times 10^8\ \mathrm{J}}$ of work.

REFLECT

We would expect the asteroid to heat up and/or break apart upon impact as well as embed itself into the Moon's surface, so our calculation gives an upper bound to the work done by the Moon.

10.62

SET UP

The space shuttle usually orbited Earth at altitudes of around 300 km. We can use Kepler's law of periods to calculate the time it takes the shuttle to make one orbit around Earth; the radius of the orbit is equal to the radius of the Earth plus 300 km. To determine the number of sunrises the astronauts saw per day, we can divide 24 hours by the period of the orbit.

SOLVE

Part a)

$$T^2 = \frac{4\pi^2 R^3}{GM}$$

$$T = \sqrt{\frac{4\pi^2 R^3}{GM}} = \sqrt{\frac{4\pi^2((6.38 \times 10^6\ \mathrm{m}) + (3.00 \times 10^5\ \mathrm{m}))^3}{\left(6.67 \times 10^{-11}\frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}\right)(5.98 \times 10^{24}\ \mathrm{kg})}}$$

$$= 5432\ \mathrm{s} \times \frac{1\ \mathrm{hr}}{3600\ \mathrm{s}} = \boxed{1.51\ \mathrm{hr}}$$

Part b)

$$N_{\mathrm{sunrises}} = \frac{24\ \mathrm{hr}}{1.51\ \mathrm{hr}} = \boxed{15.9\frac{\mathrm{sunrises}}{\mathrm{day}}}$$

REFLECT

In one day, you can't see "0.9" of a sunrise, but on average you can.

10.63

SET UP

A space shuttle orbited the Earth at an altitude of 300 km. We can rearrange Kepler's law of periods to solve for the shuttle's speed directly. The space shuttle travels $2\pi R$ meters (one

complete orbit) within T seconds (one period). The distance traveled divided by the time it took is equal to the speed.

SOLVE

$$T^2 = \frac{4\pi^2 R^3}{GM}$$

$$\frac{GM}{R} = \frac{4\pi^2 R^2}{T^2} = v^2$$

$$v = \sqrt{\frac{GM}{R}} = \sqrt{\frac{\left(6.67 \times 10^{-11} \frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}\right)(5.98 \times 10^{24}\ \mathrm{kg})}{(6.38 \times 10^6\ \mathrm{m}) + (3.00 \times 10^5\ \mathrm{m})}} = \boxed{7730 \frac{\mathrm{m}}{\mathrm{s}}}$$

REFLECT

The radius of the orbit is equal to the radius of the Earth plus 300 km.

10.64

SET UP

A satellite orbits Earth at an altitude of around 80,000 km. We can use Kepler's law of periods to calculate the time it takes the shuttle to make one orbit around Earth; the radius of the orbit is equal to the radius of the Earth plus 80,000 km.

SOLVE

$$T^2 = \frac{4\pi^2 R^3}{GM}$$

$$T = \sqrt{\frac{4\pi^2 R^3}{GM}} = \sqrt{\frac{4\pi^2((6.38 \times 10^6\ \mathrm{m}) + (8.00 \times 10^7\ \mathrm{m}))^3}{\left(6.67 \times 10^{-11} \frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}\right)(5.98 \times 10^{24}\ \mathrm{kg})}}$$

$$= 2.53 \times 10^5\ \mathrm{s} \times \frac{1\ \mathrm{hr}}{3600\ \mathrm{s}} = \boxed{70.2\ \mathrm{hr}}$$

REFLECT

We can quickly estimate the order of magnitude of the period to double-check our answer:

$$T \sim \sqrt{\frac{(10)(8 \times 10^7)^3}{(10)(10^{-11})(10^{24})}} \sim \sqrt{10^{10}} = 10^5$$

10.65

SET UP

A satellite orbits the Earth once every 86.5 min. Assuming its orbit is circular, we can use Kepler's law of periods to calculate the radius of the orbit, and then the circumference of the satellite's orbit, from the period.

SOLVE

$$T^2 = \frac{4\pi^2 R^3}{GM}$$

$$R = \sqrt[3]{\frac{GMT^2}{4\pi^2}} = \sqrt[3]{\frac{\left(6.67 \times 10^{-11}\frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}\right)(5.98 \times 10^{24}\ \mathrm{kg})\left(86.5\ \mathrm{min} \times \frac{60\ \mathrm{s}}{1\ \mathrm{min}}\right)^2}{4\pi^2}} = 6.48 \times 10^6\ \mathrm{m}$$

$$C = 2\pi R = 2\pi(6.48 \times 10^6\ \mathrm{m}) = \boxed{4.07 \times 10^7\ \mathrm{m}}$$

REFLECT

Be sure to use a consistent set of units when performing your calculations.

10.66

SET UP

Saturn makes one rotation about the Sun every 29.46 (Earth) years. We can use Kepler's law of periods to calculate the radius of the orbit, which is the distance between the Sun and Saturn. As a reminder, the mass of the Sun is 1.99×10^{30} kg.

SOLVE

Converting the period into seconds:

$$T = 29.46\ \mathrm{yr} \times \frac{365.25\ \mathrm{day}}{1\ \mathrm{yr}} \times \frac{24\ \mathrm{hr}}{1\ \mathrm{day}} \times \frac{3600\ \mathrm{s}}{1\ \mathrm{hr}} = 9.297 \times 10^8\ \mathrm{s}$$

Calculating the orbital radius:

$$T^2 = \frac{4\pi^2 R^3}{GM}$$

$$R = \sqrt[3]{\frac{GMT^2}{4\pi^2}} = \sqrt[3]{\frac{\left(6.67 \times 10^{-11}\frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}\right)(1.99 \times 10^{30}\ \mathrm{kg})(9.297 \times 10^8\ \mathrm{s})^2}{4\pi^2}} = \boxed{1.427 \times 10^{12}\ \mathrm{m}}$$

REFLECT

We could have also set up a proportionality of the Earth's information with respect to the Sun:

$$\frac{R^3_{\mathrm{Earth-Sun}}}{T^2_{\mathrm{Earth}}} = \frac{R^3_{\mathrm{Saturn-Sun}}}{T^2_{\mathrm{Saturn}}}$$

10.67

SET UP

The orbit of Mars around the Sun has a radius that is 1.524 times larger than the radius of the Earth's orbit around the Sun. We can set up a proportionality from Kepler's law of periods to calculate the period of Mars's orbit. The Earth completes one orbit around the Sun in one year.

SOLVE

$$\frac{R^3_{\mathrm{Earth-Sun}}}{T^2_{\mathrm{Earth}}} = \frac{R^3_{\mathrm{Mars-Sun}}}{T^2_{\mathrm{Mars}}}$$

$$T_{\text{Mars}} = \sqrt{\frac{R^3_{\text{Mars}-\text{Sun}} T^2_{\text{Earth}}}{R^3_{\text{Earth}-\text{Sun}}}} = \sqrt{\frac{(1.524 R_{\text{Earth}-\text{Sun}})^3 T^2_{\text{Earth}}}{R^3_{\text{Earth}-\text{Sun}}}} = \sqrt{(1.524)^3 T^2_{\text{Earth}}}$$

$$= \sqrt{(1.524)^3 (1\text{ yr})^2} = \boxed{1.881\text{ yr}}$$

REFLECT

Setting up a proportionality is the easiest way to solve this problem since we do not know the exact distance between Mars and the Sun.

10.68

SET UP

The eccentricity and semimajor axis of Venus's orbit are $e = 0.0068$ and $a = 1.082 \times 10^{11}$ m, respectively. We can represent the eccentricity, as well as the semimajor axis, in terms of the perigee distance r_{p} and apogee distance r_{a}, which are the closest and farthest distances from the Sun to Venus. One Venusian year is equal to the time required for Venus to complete one orbit around the Sun, which we can calculate using Kepler's law of periods. As a reminder, the mass of the Sun is $m_{\text{S}} = 1.99 \times 10^{30}$ kg.

SOLVE

Apogee and perigee distances:

$$e = \frac{r_{\text{a}} - r_{\text{p}}}{r_{\text{a}} + r_{\text{p}}}$$

$$r_{\text{a}} = r_{\text{p}}\left(\frac{1+e}{1-e}\right)$$

$$2a = r_{\text{a}} + r_{\text{p}} = r_{\text{p}}\left(\frac{1+e}{1-e}\right) + r_{\text{p}}$$

$$r_{\text{p}} = a(1-e) = (1.082 \times 10^{11}\text{ m})(1 - 0.0068) = \boxed{1.075 \times 10^{11}\text{ m}}$$

$$r_{\text{a}} = r_{\text{p}}\left(\frac{1+e}{1-e}\right) = (1.075 \times 10^{11}\text{ m})\left(\frac{1+0.0068}{1-0.0068}\right) = \boxed{1.090 \times 10^{11}\text{ m}}$$

Venusian year:

$$T^2 = \frac{4\pi^2 a^3}{GM_{\text{S}}}$$

$$T = \sqrt{\frac{4\pi^2 a^3}{GM_{\text{S}}}} = \sqrt{\frac{4\pi^2 (1.082 \times 10^{11}\text{ m})^3}{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)(1.99 \times 10^{30}\text{ kg})}}$$

$$= 1.941 \times 10^7\text{ s} \times \frac{1\text{ hr}}{3600\text{ s}} \times \frac{1\text{ day}}{24\text{ hr}} = \boxed{224.7\text{ days}}$$

REFLECT

A circle has an eccentricity equal to 0 since its "semimajor axis" and "semiminor axis" are both equal to the radius. The closer the eccentricity is to zero for an ellipse, the more circular it looks.

10.69

SET UP

A geosynchronous satellite has the same orbit period as the Earth, which means that its period is also 1 day, or 86,400 s. We can use Kepler's law of periods to calculate the altitude H of a geosynchronous satellite. We need to include the Earth's radius in the radius of the orbit.

SOLVE

$$T^2 = \frac{4\pi^2 R^3}{GM} = \frac{4\pi^2 (R_{\text{Earth}} + H)^3}{GM}$$

$$H = \sqrt[3]{\frac{GMT^2}{4\pi^2}} - R_{\text{Earth}} = \sqrt[3]{\frac{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24}\text{ kg})(86{,}400\text{ s})^2}{4\pi^2}} - (6.38 \times 10^6\text{ m})$$

$$= \boxed{3.59 \times 10^7\text{ m}}$$

REFLECT

Since a geosynchronous satellite rotates at the same speed as the Earth, it will appear fixed in space to an observer on the Earth's surface.

10.70

SET UP

The Earth orbits the Sun with a semimajor axis of 1.000 AU and orbital period of 365.24 days. From these data we can use Kepler's law of periods to solve for the mass of the Sun. An astronomical unit (AU) is the mean distance between the Earth and the Sun, which is around 1.496×10^{11} m.

SOLVE

$$T^2 = \frac{4\pi^2 R^3}{GM}$$

$$M = \frac{4\pi^2 R^3}{GT^2} = \frac{4\pi^2 \left(1.000\text{ AU} \times \frac{1.496 \times 10^{11}\text{ m}}{1\text{ AU}}\right)^3}{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)\left(365.24\text{ days} \times \frac{24\text{ hr}}{1\text{ day}} \times \frac{3600\text{ s}}{1\text{ hr}}\right)^2} = \boxed{1.99 \times 10^{30}\text{ kg}}$$

REFLECT

This is how they can "measure" the mass of the Sun.

10.71

SET UP

The Moon orbits the Earth in a nearly circular orbit once every 27.32 days. We can use Kepler's law of periods to help calculate the distance d from the surface of the Moon to the surface of Earth. The radius in Kepler's law is the center-to-center distance between the Moon and the Earth, so we will need to subtract out the radius of the Earth and the radius of the Moon in order to find d.

SOLVE

Converting the period into seconds:

$$27.32 \text{ days} \times \frac{24 \text{ hr}}{1 \text{ day}} \times \frac{3600 \text{ s}}{1 \text{ hr}} = 2.360 \times 10^{6} \text{ s}$$

Calculating the distance:

$$T^2 = \frac{4\pi^2 R^3}{GM} = \frac{4\pi^2 (R_{\text{Earth}} + R_{\text{Moon}} + d)^3}{GM}$$

$$d = \sqrt[3]{\frac{GMT^2}{4\pi^2}} - R_{\text{Earth}} - R_{\text{Moon}}$$

$$= \sqrt[3]{\frac{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24} \text{ kg})(2.360 \times 10^{6} \text{ s})^2}{4\pi^2}}$$

$$- (6.38 \times 10^{6} \text{ m}) - (1.73 \times 10^{6} \text{ m})$$

$$= \boxed{3.751 \times 10^{8} \text{ m}}$$

REFLECT

The mean distance from the center of the Earth to the center of the Moon is 3.84×10^8 m. This is larger than the distance between their surfaces, which makes sense.

10.72

SET UP

A planet orbits a star with an orbital radius of 1 AU around a star with a mass that is 1.75 times the mass of our Sun. We can use Kepler's law of periods to calculate the period of this planet's rotation. We will need the conversion between AU and meters ($1 \text{ AU} = 1.496 \times 10^{11}$ m) and the mass of the Sun (1.99×10^{30} kg).

SOLVE

$$T^2 = \frac{4\pi^2 R^3}{GM}$$

$$T = \sqrt{\frac{4\pi^2 R^3}{GM}} = \sqrt{\frac{4\pi^2 (1.496 \times 10^{11} \text{ m})^3}{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)(1.75)(1.99 \times 10^{30} \text{ kg})}}$$

$$= 2.385 \times 10^{7} \text{ s} \times \frac{1 \text{ hr}}{3600 \text{ s}} \times \frac{1 \text{ day}}{24 \text{ hr}} \times \frac{1 \text{ yr}}{365.25 \text{ days}} = \boxed{0.756 \text{ yr}}$$

REFLECT

It makes sense that a planet orbiting a more massive star at a distance equal to the Earth–Sun distance should have a shorter period.

10.73

SET UP

A thin, uniform rod with a mass M and a length L is located on the x-axis. A small object of mass m is located a distance D away along the x-axis at point P. We can calculate the gravitational force of the rod on the small object by integrating the infinitesimal gravitational field due to each portion of the rod and then multiplying by the mass m. Through symmetry, we see that the y component of the field (and force) is equal to zero, so we only need to calculate the x component. The x component of the infinitesimal field dg_x is equal to the value of the field $dg = \frac{Gdm}{r^2}$, where $r = (D + x)$ is the distance from the point P to the location of the mass dm. We are told that the rod is uniform, which means we can use the linear mass density to convert from an integral with respect to mass to an integral with respect to x.

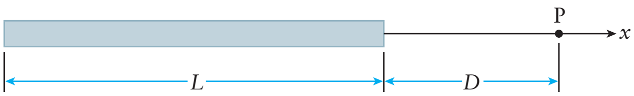


Figure 10-6 Problem 73

SOLVE

$$g_x = \int -dg_x = -\int \frac{Gdm}{r^2}$$

But $dm = \frac{M}{L}dx$:

$$g_x = -\int \frac{Gdm}{r^2} = -\int_0^L \frac{G}{(D+x)^2}\frac{M}{L}dx = \frac{GM}{L}\int_L^0 \frac{1}{(D+x)^2}dx = \frac{GM}{L}\left[-\frac{1}{D+x}\right]_L^0$$

$$= -\frac{GM}{L}\left[\frac{1}{D} - \frac{1}{D+L}\right]$$

Therefore, the force exerted on a small object at point P is:

$$\vec{F} = -\frac{GMm}{L}\left[\frac{1}{D} - \frac{1}{D+L}\right]\hat{x} = -\frac{GMm}{L}\left[\frac{(D+L)-D}{D(D+L)}\right]\hat{x} = \boxed{-\frac{GMm}{D(D+L)}\hat{x}}$$

REFLECT

Thinking about the situation beforehand and invoking symmetry will usually save you some time and effort when solving a problem. The gravitational force is always attractive, so the force on the small object must point toward $-x$. If $D \gg L$, the expression for the force reduces to $\vec{F} = -\frac{GMm}{D^2}\hat{x}$; in other words, from far away, the rod looks like a point mass.

10.74

SET UP

A thin, uniform rod with a mass M and a length L is located on the y-axis. A small object of mass m is located a distance D away along the x-axis at point P. We can calculate the gravitational force of the rod on the small object by integrating the infinitesimal gravitational field due to each portion of the rod and then multiplying by the mass m. Through symmetry, we see that the y component of the field (and force) cancel out, so we only need to calculate the x component. The x component of the infinitesimal field dg_x is equal to the value of the field $dg = \frac{Gdm}{r^2}$, where r is the distance from the point P to the location of the mass dm, multiplied by $\cos(\theta) = D/r$. (The distance r is the hypotenuse of the right triangle formed by D and y.) We are told that the rod is uniform, which means we can use the linear mass density to convert from an integral with respect to mass to an integral with respect to y.

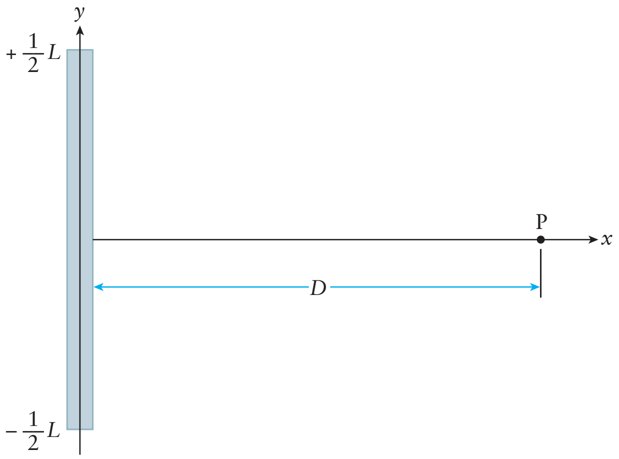


Figure 10-7 Problem 74

SOLVE

$$g_x = \int -dg_x = -\int dg\cos(\theta) = -\int \frac{Gdm}{r^2}\frac{D}{r}$$

But $dm = \frac{M}{L}dy$:

$$g_x = -\int \frac{Gdm}{r^2}\frac{D}{r} = -\int_{-L/2}^{L/2} \frac{GD}{r^3}\frac{M}{L}dy = -\frac{GDM}{L}\int_{-L/2}^{L/2} \frac{1}{(y^2+D^2)^{3/2}}dy = -\frac{GDM}{L}\left[\frac{y}{D^2\sqrt{y^2+D^2}}\right]_{-L/2}^{L/2}$$

$$= -\frac{GM}{DL}\left[\frac{\left(\frac{L}{2}\right)}{\sqrt{\left(\frac{L}{2}\right)^2+D^2}} - \frac{\left(-\frac{L}{2}\right)}{\sqrt{\left(-\frac{L}{2}\right)^2+D^2}}\right] = -\frac{GM}{DL}\left[\frac{L}{\sqrt{\left(\frac{L}{2}\right)^2+D^2}}\right] = -\frac{GM}{D\sqrt{\left(\frac{L}{2}\right)^2+D^2}}$$

Therefore, the force exerted on a small object at point P is:

$$\boxed{\vec{F} = -\frac{GMm}{D\sqrt{\left(\frac{L}{2}\right)^2+D^2}}\hat{x}}$$

REFLECT

The gravitational force is always attractive, so the force on the small object must point toward $-x$.

10.75

SET UP

A 1900-kg satellite is launched into an elliptical orbit that has a period of 702 min. The apogee of the orbit (farthest distance from the Earth) is $r_a = 39{,}200$ km and the perigee (closest distance to the Earth) is $r_p = 560$ km. We can use the definition of gravitational potential energy and the radius of the Earth to find the value of the potential energy at the apogee and perigee. The nonconservative work required to place the satellite in orbit is equal to the change in the satellite's mechanical energy. We will assume it starts from rest on the surface of the Earth. Its final speed is equal to the circumference of the orbit divided by the period of the orbit. The circumference of an ellipse can be approximated by $C \approx \pi[3(a+b) - \sqrt{(3a+b)(a+3b)}]$. We can calculate the values of a and b from the distances r_a and r_p.

SOLVE

Part a)

Apogee:

$$U_a = -\frac{Gm_E m}{r_E + r_a} = -\frac{\left(6.67 \times 10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24}\text{ kg})(1900\text{ kg})}{(6.38 \times 10^6\text{ m}) + (3.92 \times 10^7\text{ m})} = \boxed{-1.66 \times 10^{10}\text{ J}}$$

Perigee:

$$U_p = -\frac{Gm_E m}{r_E + r_p} = -\frac{\left(6.67 \times 10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24}\text{ kg})(1900\text{ kg})}{(6.38 \times 10^6\text{ m}) + (5.60 \times 10^5\text{ m})} = \boxed{-1.09 \times 10^{11}\text{ J}}$$

Part b)

Semimajor and semiminor axes:

$$a = \frac{r_a + r_p}{2} = \frac{(39{,}200\text{ km}) + (560\text{ km})}{2} = 19{,}880\text{ km}$$

$$e = \frac{r_a - r_p}{r_a + r_p} = \frac{(39{,}200\text{ km}) - (560\text{ km})}{(39{,}200\text{ km}) + (560\text{ km})} = 0.972$$

$$b = a\sqrt{1 - e^2} = (19{,}880\text{ km})\sqrt{1 - (0.972)^2} = 1104.2\text{ km}$$

Circumference of ellipse:

$$C \approx \pi[3(a+b) - \sqrt{(3a+b)(a+3b)}] = \pi[3((19{,}880\text{ km}) + (1104.2\text{ km}))$$

$$- \sqrt{(3(19{,}880\text{ km}) + (1104.2\text{ km}))((19{,}880\text{ km}) + 3(1104.2\text{ km}))}]$$

$$= \pi[(62{,}953\text{ km}) - \sqrt{(60{,}744\text{ km})(23{,}193\text{ km})}] = 79{,}855\text{ km}$$

Speed of satellite in orbit:

$$v_f = \frac{C}{T} = \frac{\left(79{,}855\text{ km} \times \dfrac{1000\text{ m}}{1\text{ km}}\right)}{\left(702\text{ min} \times \dfrac{60\text{ s}}{1\text{ min}}\right)} = 1896\frac{\text{m}}{\text{s}}$$

Work:

$$\begin{aligned}
W_{nc} = \Delta K + \Delta U &= \frac{1}{2}m(v_f^2 - v_i^2) + \left(\frac{-Gm_Em}{a} - \frac{-Gm_Em}{r_E}\right) \\
&= \frac{1}{2}m(v_f^2 - 0) - Gm_Em\left(\frac{1}{a} - \frac{1}{r_E}\right) \\
&= \frac{1}{2}(1900\text{ kg})\left(1896\frac{\text{m}}{\text{s}}\right)^2 - \left(6.67 \times 10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24}\text{ kg}) \\
&\quad \times (1900\text{ kg})\left(\frac{1}{1.988 \times 10^7\text{ m}} - \frac{1}{6.38 \times 10^6\text{ m}}\right) \\
&= \boxed{8.41 \times 10^{10}\text{ J}}
\end{aligned}$$

REFLECT

A stick of dynamite has approximately 2×10^6 J of energy. The energy required to launch the satellite is 40,000 times larger than this.

10.76

SET UP

We are asked to draw an ellipse with an eccentricity of $e = 0.3$ and a semimajor axis of $a = 5$ cm. We can calculate the length of the semiminor axis from the definition of the eccentricity, $e = \sqrt{1 - \frac{b^2}{a^2}}$. The center of the ellipse C is located at the intersection of the semimajor and semiminor axes. The distance between the two foci is equal to $2ea$.

SOLVE

Semiminor axis:

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$b = a\sqrt{1 - e^2} = (5\text{ cm})\sqrt{1 - (0.3)^2} = \boxed{4.77\text{ cm}}$$

Distance between foci:

$$2ea = 2(0.3)(5\text{ cm}) = \boxed{3\text{ cm}}$$

Ellipse:

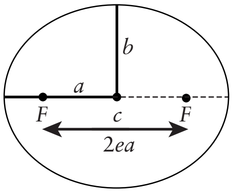


Figure 10-8 Problem 76

REFLECT

If a star were located at the right focus, the perihelion of this orbit would be 2.6 cm and the aphelion would be 7.4 cm.

10.77

SET UP

A shuttle is launched into a low-Earth orbit ($R \approx R_E$) near the equator. We can use conservation of energy to calculate the necessary launch speed of the shuttle v_L. A shuttle launched due east is launched with the rotation of the Earth, while a shuttle launched due west is launched against the rotation of the Earth, so the rotation speed of the Earth will either be added or subtracted to the launch speed v_L of the rocket. U.S. space shuttles were launched from the east coast of Florida because it is close to the equator and an eastward launch would fly directly over the ocean.

SOLVE

Tangential speed of the Earth's surface:

$$v_E = \frac{2\pi R_E}{T} = \frac{2\pi(6.38 \times 10^6\text{ m})}{\left(24\text{ hr} \times \frac{3600\text{ s}}{1\text{ hr}}\right)} = 464\frac{\text{m}}{\text{s}}$$

Part a)

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}m\left(v_L + \left(464\frac{\text{m}}{\text{s}}\right)\right)^2 - \frac{GM_Em}{R_E} = 0$$

$$v_L = \sqrt{\frac{2GM_E}{R_E}} - \left(464\frac{\text{m}}{\text{s}}\right) = \sqrt{\frac{2\left(6.67 \times 10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24}\text{ kg})}{(6.38 \times 10^6\text{ m})}} - \left(464\frac{\text{m}}{\text{s}}\right)$$

$$= \boxed{10{,}720\frac{\text{m}}{\text{s}} = 10.72\frac{\text{km}}{\text{s}}}$$

Part b)

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}m\left(v_L - \left(464\frac{\text{m}}{\text{s}}\right)\right)^2 - \frac{GM_Em}{R_E} = 0$$

$$v_L = \sqrt{\frac{2GM_E}{R_E}} + \left(464\frac{\text{m}}{\text{s}}\right) = \sqrt{\frac{2\left(6.67 \times 10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24}\text{ kg})}{(6.38 \times 10^6\text{ m})}} + \left(464\frac{\text{m}}{\text{s}}\right)$$

$$= \boxed{11{,}650\frac{\text{m}}{\text{s}} = 11.65\frac{\text{km}}{\text{s}}}$$

Part c) U.S. space missions are launched from the east coast of Florida so that the launch site will be as close to the equator as possible while still being on the U.S. mainland. Also, an eastward launch from its location won't pass over inhabited areas.

REFLECT

It makes sense that the space shuttle needs to launch with a smaller speed if it launches with the rotation of the Earth.

10.78

SET UP

Sputnik ($m = 84$ kg) made one orbit around Earth every 96 min. We can use Kepler's law of periods to calculate the altitude of *Sputnik*'s orbit around the Earth; the radius of the orbit is the radius of the Earth plus the altitude h. *Sputnik*'s weight (that is, the magnitude of the force due to gravity) is equal to its mass multiplied by the acceleration due to gravity at that point. For the weight in orbit, we can use Newton's universal law of gravitation at the distance we found in part (a); for the weight at the Earth's surface, we can use 9.8 m/s^2.

SOLVE

Part a)

$$T^2 = \frac{4\pi^2 R^3}{GM} = \frac{4\pi^2(R_E + h)^3}{GM}$$

$$h = \sqrt[3]{\frac{GMT^2}{4\pi^2}} - R_E$$

$$= \sqrt[3]{\frac{\left(6.67 \times 10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24}\text{ kg})\left(96\text{ min} \times \frac{60\text{ s}}{1\text{ min}}\right)^2}{4\pi^2}} - (6.38 \times 10^6\text{ m})$$

$$= \boxed{5.67 \times 10^5\text{ m} = 567\text{ km}}$$

Part b)

Weight in orbit:

$$F_g = m\frac{GM}{R^2} = (84\text{ kg})\frac{\left(6.67 \times 10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24}\text{ kg})}{((6.38 \times 10^6\text{ m}) + (5.67 \times 10^5\text{ m}))^2} = \boxed{694\text{ N}}$$

Weight on Earth's surface:

$$F_g = mg = (84\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right) = \boxed{823\text{ N}}$$

REFLECT

The weight of the satellite should decrease as its altitude increases.

10.79

SET UP

The Mars rovers have a vertical velocity of zero when they are 12 m above the surface of the planet. The rovers then undergo free-fall as they fall to the surface. In order to calculate the time required for the probes to reach the surface and the vertical speed when they land, we first need to calculate the acceleration due to gravity on Mars. This will be the y acceleration we will use in the constant acceleration equations.

SOLVE

$$g_{\text{Mars}} = \frac{GM_{\text{Mars}}}{R^2} = \frac{\left(6.67\times10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(6.419\times10^{23}\text{ kg})}{(3.397\times10^{6}\text{ m})^2} = 3.71\frac{\text{m}}{\text{s}^2}$$

Part a)

$$\Delta y = v_{0,y}t + \frac{1}{2}a_y t^2 = 0 - \frac{1}{2}g_{\text{Mars}}t^2$$

$$t = \sqrt{-\frac{2\Delta y}{g_{\text{Mars}}}} = \sqrt{-\frac{2(-12\text{ m})}{3.71\frac{\text{m}}{\text{s}^2}}} = \boxed{2.54\text{ s}}$$

Part b)

$$v_{f,y}^2 - v_{0,y}^2 = 2a_y\Delta y$$

$$v_{f,y} = \sqrt{v_{0,y}^2 + 2a_y\Delta y} = \sqrt{0 - 2g_{\text{Mars}}\Delta y} = \sqrt{0 - 2\left(3.71\frac{\text{m}}{\text{s}^2}\right)(-12\text{ m})} = \boxed{9.44\frac{\text{m}}{\text{s}}}$$

REFLECT

An object on Earth would take 1.56 s to fall 12 m and would land with an impact speed of 15.3 m/s. This is consistent with the reduced gravity on Mars.

10.80

Moon	Semimajor Axis (km)	Orbital Period (days)
Io	421,700	1.769
Europa	671,034	?
Ganymede	?	7.155
Callisto	?	16.689

SET UP

We are given a table of the semimajor axes and orbital periods of Jupiter's four largest moons. Since all of the moons are orbiting the same planet, we can set up ratios between the semimajor axis and period of Io and the semimajor axis and period of the moon we are interested in to solve for the missing information. Using the information regarding Io and Kepler's law of periods will let us calculate the mass of Jupiter.

SOLVE

Part a)

Europa:

$$\frac{T_{\text{Io}}^2}{a_{\text{Io}}^3} = \frac{T_{\text{Europa}}^2}{a_{\text{Europa}}^3}$$

$$T_{\text{Europa}} = \sqrt{\left(\frac{T_{\text{Io}}^2}{a_{\text{Io}}^3}\right)a_{\text{Europa}}^3} = \sqrt{\left(\frac{(1.769\text{ days})^2}{(4.21700 \times 10^8\text{ m})^3}\right)(6.71034 \times 10^8\text{ m})^3} = \boxed{3.551\text{ days}}$$

Ganymede:

$$\frac{T_{\text{Io}}^2}{a_{\text{Io}}^3} = \frac{T_{\text{Ganymede}}^2}{a_{\text{Ganymede}}^3}$$

$$a_{\text{Ganymede}} = \sqrt[3]{\left(\frac{a_{\text{Io}}^3}{T_{\text{Io}}^2}\right)T_{\text{Ganymede}}^2} = \sqrt[3]{\left(\frac{(4.21700 \times 10^8\text{ m})^3}{(1.769\text{ days})^2}\right)(7.155\text{ days})^2}$$

$$= \boxed{1.07 \times 10^9\text{ m} = 1.07 \times 10^6\text{ km}}.$$

Callisto:

$$\frac{T_{\text{Io}}^2}{a_{\text{Io}}^3} = \frac{T_{\text{Callisto}}^2}{a_{\text{Callisto}}^3}$$

$$a_{\text{Callisto}} = \sqrt[3]{\left(\frac{a_{\text{Io}}^3}{T_{\text{Io}}^2}\right)T_{\text{Callisto}}^2} = \sqrt[3]{\left(\frac{(4.21700 \times 10^8\text{ m})^3}{(1.769\text{ days})^2}\right)(16.689\text{ days})^2}$$

$$= \boxed{1.88 \times 10^9\text{ m} = 1.88 \times 10^6\text{ km}}.$$

Part b)

$$T_{\text{Io}}^2 = \frac{4\pi^2 a_{\text{Io}}^3}{GM_{\text{Jupiter}}}$$

$$M_{\text{Jupiter}} = \frac{4\pi^2 a_{\text{Io}}^3}{GT_{\text{Io}}^2} = \frac{4\pi^2(4.21700 \times 10^8\text{ m})^3}{\left(6.67 \times 10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\left(1.769\text{ days} \times \frac{24\text{ hr}}{1\text{ day}} \times \frac{3600\text{ s}}{1\text{ hr}}\right)^2}$$

$$= \boxed{1.90 \times 10^{27}\text{ kg}}$$

REFLECT

The larger the semimajor axis, the longer the orbital period and vice versa. The mass of Jupiter is about three orders of magnitude larger than the mass of Earth and about three orders of magnitude smaller than the mass of the Sun, which seems reasonable.

10.81

SET UP

The semimajor axis of an elliptical orbit of a planet around a star is $a = 2.25$ AU and the semiminor axis is $b = 1.75$ AU. The eccentricity e of an ellipse is defined as $e = \sqrt{1 - \frac{b^2}{a^2}}$. The perihelion r_p and aphelion r_a are the closest and farthest distances from the star to the planet; we can also represent the eccentricity, as well as the semimajor axis, in terms of these distances. Finally, the area of an ellipse is $A = \pi ab$.

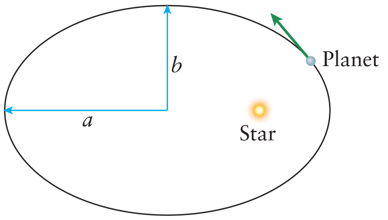


Figure 10-9 Problem 81

SOLVE

Eccentricity:

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{(1.75\ \text{AU})^2}{(2.25\ \text{AU})^2}} = \boxed{0.63}$$

Perihelion and aphelion:

$$e = \frac{r_a - r_p}{r_a + r_p}$$

$$er_a + er_p = r_a - r_p$$

$$r_a = \left(\frac{1+e}{1-e}\right) r_p$$

$$2a = r_a + r_p = \left(\frac{1+e}{1-e}\right) r_p + r_p = \left(\frac{1+e+1-e}{1-e}\right) r_p = \left(\frac{2}{1-e}\right) r_p$$

$$r_p = (1-e)a = (1-(0.63))(2.25\ \text{AU}) = \boxed{0.833\ \text{AU}}$$

$$r_a = 2a - r_p = 2(2.25\ \text{AU}) - (0.833\ \text{AU}) = \boxed{3.67\ \text{AU}}$$

Area:

$$A = \pi ab = \pi(2.25\ \text{AU})(1.75\ \text{AU}) = \boxed{12.4\ \text{AU}^2} \times \frac{(1.496 \times 10^{11}\ \text{m})^2}{1\ \text{AU}^2} = \boxed{2.78 \times 10^{23}\ \text{m}^2}$$

REFLECT

The planet moves the fastest at the perihelion and the slowest at the aphelion.

10.82

SET UP

A froghopper can jump with a speed of 2.8 m/s. You bring a bunch of froghoppers to an asteroid. If the asteroid is small enough, the froghoppers can jump straight up, free themselves from the gravity of the asteroid, and escape into space. We can use the expression for escape speed to calculate the diameter of the largest asteroid from which the froghoppers are able to escape. Newton's second law lets us calculate the diameter of the asteroid necessary for the froghoppers to jump horizontally and orbit the asteroid just above the surface; the gravitational force exerted on the froghoppers by the asteroid will cause the insects' centripetal motion.

SOLVE

Part a)

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G\rho V}{R}} = \sqrt{\frac{2G\rho\left(\frac{4}{3}\pi R^3\right)}{R}} = \sqrt{\frac{8G\rho\pi R^2}{3}} = \sqrt{\frac{8G\rho\pi\left(\frac{D}{2}\right)^2}{3}} = \sqrt{\frac{2G\rho\pi D^2}{3}}$$

$$D = \sqrt{\frac{3v_e^2}{2G\rho\pi}} = \sqrt{\frac{3\left(2.8\frac{\text{m}}{\text{s}}\right)^2}{2\left(6.67\times10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\left(2.0\frac{\text{g}}{\text{cm}^3}\times\frac{1\text{ kg}}{1000\text{ g}}\times\frac{10^6\text{ cm}^3}{1\text{ m}^3}\right)\pi}} = \boxed{5300\text{ m}}$$

Part b)

$$\frac{GMm}{R^2} = \frac{mv^2}{R}$$

$$\frac{G\rho\left(\frac{4}{3}\pi R^3\right)}{R^2} = \frac{v^2}{R}$$

$$R = \sqrt{\frac{3v^2}{4\pi G\rho}}$$

$$2R = D = 2\sqrt{\frac{3\left(2.8\frac{\text{m}}{\text{s}}\right)^2}{4\pi\left(6.67\times10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\left(2.0\frac{\text{g}}{\text{cm}^3}\times\frac{1\text{ kg}}{1000\text{ g}}\times\frac{10^6\text{ cm}^3}{1\text{ m}^3}\right)}} = \boxed{7500\text{ m}}$$

REFLECT

It makes sense that the asteroid should be smaller if the froghoppers escape from it rather than orbit around it.

10.83

SET UP

The International Space Station (ISS) orbits Earth in a nearly circular orbit at an altitude of 3.45×10^5 m. We can use Kepler's law of periods to calculate the time it takes the ISS to orbit

the Earth. The radius of the orbit is the radius of the Earth plus the altitude of the ISS. We can use Newton's universal law of gravitation to calculate the magnitude of the gravitational force that Earth exerts on a 10.0-kg object in the ISS. The magnitude of the gravitational force that Earth would exert on the object on the Earth's surface is equal to mg. Weightlessness implies that the force of gravity is zero; this is not the case for the object located in the ISS. Since both the object and the ISS are falling at the same rate, it appears as if the object is not attracted to the ISS.

SOLVE

Part a)

$$T^2 = \frac{4\pi^2 R^3}{GM}$$

$$T = \sqrt{\frac{4\pi^2 R^3}{GM}} = \sqrt{\frac{4\pi^2((6.38 \times 10^6 \text{ m}) + (3.45 \times 10^5 \text{ m}))^3}{\left(6.67 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24} \text{ kg})}}$$

$$= 5487 \text{ s} \times \frac{1 \text{ hr}}{3600 \text{ s}} = \boxed{1.52 \text{ hr}}$$

Part b)

$$F_\text{g} = \frac{Gm_\text{E}m}{R^2} = \frac{\left(6.67 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24} \text{ kg})(10.0 \text{ kg})}{((6.38 \times 10^6 \text{ m}) + (3.45 \times 10^5 \text{ m}))^2} = \boxed{88.2 \text{ N}}$$

$$\frac{F_\text{g, space}}{F_\text{g, surface}} = \frac{88.2 \text{ N}}{(10.0 \text{ kg})\left(9.8 \frac{\text{m}}{\text{s}^2}\right)} = \frac{88.2 \text{ N}}{98.0 \text{ N}} = \boxed{90\%}$$

Part c) The rest of the space station is falling with the object. The only weight relative to the station is from very weak tidal effects.

REFLECT

If an elevator's cables were to suddenly snap and the elevator car were to free-fall, the person inside would also be considered "weightless."

10.84

SET UP

The aphelion (farthest distance from the Sun) for the asteroid Apophis is $d_\text{a} = 1.099$ AU and the perihelion is 0.746 AU. The distance between these two points is equal to the major axis of its elliptical orbit. Therefore, the semimajor axis a is equal to half of the sum of those distances. We can use the semimajor axis and Kepler's law of periods to calculate the number of days it takes Apophis to orbit the Sun. Kepler's law of areas, which is just a restatement of the conservation of angular momentum, states that the speed of the asteroid will be the largest when it is closest to the Sun. Conservation of angular momentum will also allow us to find the ratio of the maximum speed to the minimum speed.

SOLVE

Part a)

$$d_a + d_p = 2a$$

$$a = \frac{d_a + d_p}{2} = \frac{(1.099\text{ AU}) + (0.746\text{ AU})}{2} = \boxed{0.923\text{ AU}} \times \frac{1.496 \times 10^{11}\text{ m}}{1\text{ AU}} = \boxed{1.38 \times 10^{11}\text{ m}}$$

Part b)

$$T^2 = \frac{4\pi^2 a^3}{GM}$$

$$T = \sqrt{\frac{4\pi^2 a^3}{GM}} = \sqrt{\frac{4\pi^2(1.38 \times 10^{11}\text{ m})^3}{\left(6.67 \times 10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(1.99 \times 10^{30}\text{ kg})}}$$

$$= 2.796 \times 10^7\text{ s} \times \frac{1\text{ hr}}{3600\text{ s}} \times \frac{1\text{ day}}{24\text{ hr}} = \boxed{323.6\text{ days}}$$

Part c) Kepler's law of areas, which is just the conservation of angular momentum, says that mvr = constant, so the speed is the greatest when the distance is the shortest and vice versa. Therefore, Apophis is traveling the $\boxed{\text{fastest at the perihelion}}$ and the $\boxed{\text{slowest at the aphelion}}$.

Part d)

$$mv_p r_p = mv_a r_a$$

$$\frac{v_{max}}{v_{min}} = \frac{v_p}{v_a} = \frac{r_a}{r_p} = \frac{1.099\text{ AU}}{0.746\text{ AU}} = \boxed{1.47}$$

REFLECT

The mass of the asteroid is not necessary to solve this problem. Since the asteroid is orbiting the Sun, the mass of the Sun is used in Kepler's law of periods.

10.85

SET UP

Seven spherical bowling balls (each of mass m_b = 8 kg with a radius of 11 cm) are lined up in a row and positioned 100 cm from a ping-pong ball (m_p = 2.7 g) located at point P (see figure). (For simplicity and clarity, we'll number the balls from left to right starting with ball 1.) We are asked to find the net gravitational force on the ping-pong ball due to the seven bowling balls. The net force on the ping-pong ball is the vector sum of the force due to each bowling ball. Through symmetry, we see that the x component of the force is equal to zero, so we only need to calculate the y component. Furthermore, we see that the mass distribution is symmetric about the line connecting point P with the center bowling ball. This means the y component of the force due to ball 1 is equal to the y component of the force due to ball 7,

the y component of ball 2 is equal to the y component of ball 6, and the y component of ball 3 is equal to the y component of ball 5. The shell theorem allows us to treat each bowling ball as a point mass located at its center of mass. The y component of each force is related to the cosine of the angle θ; the angle θ is equal to the arctangent of the distance each bowling ball is from the center divided by 100 cm. (The second figure shows the angle θ_1 and r_1 for bowling ball 1.)

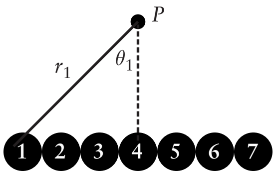


Figure 10-10 Problem 85

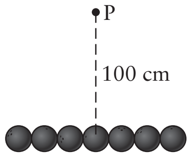


Figure 10-11 Problem 85

SOLVE

Balls 1, 7:

$$F_{1,y} = 2\left(-\frac{Gm_\mathrm{b}m_\mathrm{p}}{r_1^2}\right)\left(\cos\left(\arctan\left(\frac{66\text{ cm}}{100\text{ cm}}\right)\right)\right)$$

$$= 2\left(-\frac{\left(6.67\times10^{-11}\dfrac{\mathrm{N\cdot m^2}}{\mathrm{kg^2}}\right)(8\text{ kg})(2.7\times10^{-3}\text{ kg})}{(0.66\text{ m})^2+(1.00\text{ m})^2}\right)(0.834) = -1.674\times10^{-12}\text{ N}$$

Balls 2, 6:

$$F_{2,y} = 2\left(-\frac{Gm_\mathrm{b}m_\mathrm{p}}{r_2^2}\right)\left(\cos\left(\arctan\left(\frac{44\text{ cm}}{100\text{ cm}}\right)\right)\right)$$

$$= 2\left(-\frac{\left(6.67\times10^{-11}\dfrac{\mathrm{N\cdot m^2}}{\mathrm{kg^2}}\right)(8\text{ kg})(2.7\times10^{-3}\text{ kg})}{(0.44\text{ m})^2+(1.00\text{ m})^2}\right)(0.915) = -2.209\times10^{-12}\text{ N}$$

Balls 3, 5:

$$F_{3,y} = 2\left(-\frac{Gm_\mathrm{b}m_\mathrm{p}}{r_3^2}\right)\left(\cos\left(\arctan\left(\frac{22\text{ cm}}{100\text{ cm}}\right)\right)\right)$$

$$= 2\left(-\frac{\left(6.67\times10^{-11}\dfrac{\mathrm{N\cdot m^2}}{\mathrm{kg^2}}\right)(8\text{ kg})(2.7\times10^{-3}\text{ kg})}{(0.22\text{ m})^2+(1.00\text{ m})^2}\right)(0.977) = -2.685\times10^{-12}\text{ N}$$

Center ball (4):

$$F_{4,y} = -\frac{Gm_\mathrm{b}m_\mathrm{p}}{r_4^2} = -\frac{\left(6.67\times10^{-11}\dfrac{\mathrm{N\cdot m^2}}{\mathrm{kg^2}}\right)(8\text{ kg})(2.7\times10^{-3}\text{ kg})}{(1.00\text{ m})^2} = -1.441\times10^{-12}\text{ N}$$

Net force:

$$\sum F_y = (-1.674 \times 10^{-12}\ \mathrm{N}) - (2.209 \times 10^{-12}\ \mathrm{N}) - (2.685 \times 10^{-12}\ \mathrm{N}) - (1.44 \times 10^{-12}\ \mathrm{N})$$

$$= -8.01 \times 10^{-12}\ \mathrm{N}$$

$$\boxed{\sum \vec{F} = (-8.01 \times 10^{-12}\ \mathrm{N})\hat{y}}$$

REFLECT

The force due to ball 4 should be the largest since it is the closest, followed by ball 3 (and ball 5), then ball 2 (and ball 6), and finally ball 1 (and ball 7).

10.86

SET UP

The Sun and solar system actually orbit the center of the Milky Way galaxy once every 2.25×10^8 years at a distance of 52,000 light-years. We can use Kepler's law of periods to calculate the mass of the Milky Way, assuming that it is concentrated at the center of the galaxy. We will need the following conversion factor for light-years: 1 light year $= 9.47 \times 10^{15}$ m.

SOLVE

Converting from light-years to meters:

$$52{,}000 \text{ light-years} \times \frac{9.47 \times 10^{15}\ \mathrm{m}}{1 \text{ light-year}} = 4.92 \times 10^{20}\ \mathrm{m}$$

Converting from years to seconds:

$$2.25 \times 10^8\ \mathrm{yr} \times \frac{365.25\ \mathrm{days}}{1\ \mathrm{yr}} \times \frac{24\ \mathrm{hr}}{1\ \mathrm{day}} \times \frac{3600\ \mathrm{s}}{1\ \mathrm{hr}} = 7.10 \times 10^{15}\ \mathrm{s}$$

Kepler's law of periods:

$$T^2 = \frac{4\pi^2 R^3}{GM}$$

$$M = \frac{4\pi^2 R^3}{GT^2} = \frac{4\pi^2 (4.92 \times 10^{20}\ \mathrm{m})^3}{\left(6.67 \times 10^{-11} \frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}\right)(7.10 \times 10^{15}\ \mathrm{s})^2} = \boxed{1.40 \times 10^{42}\ \mathrm{kg}}$$

REFLECT

A light-year is the distance light travels in a vacuum during one year. The speed of light in a vacuum is 3×10^8 m/s.

10.87

SET UP

We are asked to derive Kepler's law of periods from Kepler's law of areas. Starting with the derivative of the area with respect to time, we can integrate both sides to relate the area A to the period T. We can then isolate T and square both sides and start to solve for T^2 in terms of the semimajor axis a. The eccentricity e of an ellipse is defined as $e = \sqrt{1 - \frac{b^2}{a^2}}$.

SOLVE

$$\frac{dA}{dt} = \frac{L}{2m}$$

$$\int dA = \int_0^T \frac{L}{2m} dt$$

$$A = \pi ab = \frac{LT}{2m}$$

$$T = \frac{2m\pi ab}{L}$$

$$T^2 = \left(\frac{2m\pi ab}{L}\right)^2 = \frac{4m^2\pi^2a^2b^2}{L^2}$$

But $L^2 = aGMm^2(1 - e^2)$ and $e = \sqrt{1 - \frac{b^2}{a^2}}$ or $b^2 = a^2(1 - e^2)$:

$$T^2 = \frac{4m^2\pi^2a^2b^2}{L^2} = \frac{4m^2\pi^2a^2(a^2(1 - e^2))}{aGMm^2(1 - e^2)}$$

$$\boxed{T^2 = \frac{4\pi^2a^3}{GM}}$$

REFLECT

The eccentricity of an ellipse is between 0 and 1. If the eccentricity is 0, then $a = b$, which means the shape is a circle.

10.88

SET UP

Three objects ($m_1 = 2.00 \times 10^6$ kg, $m_2 = 6.00 \times 10^6$ kg, $m_3 = 8.00 \times 10^6$ kg) are located on the x, y, and z axes, respectively, as shown in the figure. A 1-kg object is located at point P. We can calculate the net gravitational force that the mass distribution exerts on the 1-kg object by first finding the gravitational force that each individual mass exerts on the object. We will calculate the magnitude of each force and then the unit vector along which that force points.

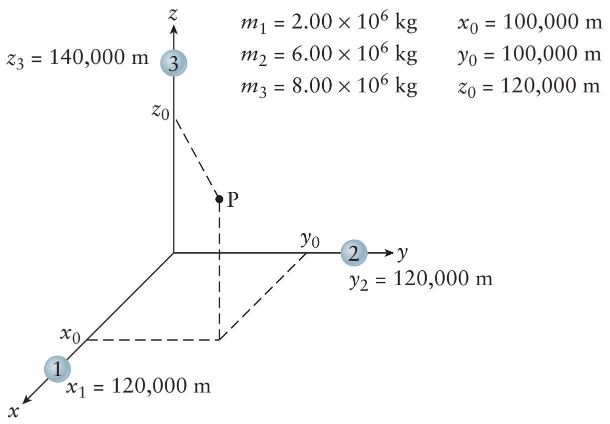


Figure 10-12 Problem 88

SOLVE

Force due to m_1:

Magnitude:

$$F_1 = \frac{Gm_1m}{r_1^2} = \frac{\left(6.67 \times 10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(2.00 \times 10^6\ \text{kg})(1\ \text{kg})}{((2.00 \times 10^4\ \text{m})^2 + (1.00 \times 10^5\ \text{m})^2 + (1.20 \times 10^5\ \text{m})^2)} = 5.38 \times 10^{-15}\ \text{N}$$

Direction:

$$\hat{r}_1 = \frac{(0.2\hat{x} - \hat{y} - 1.2\hat{z})}{\sqrt{(0.2)^2 + (-1)^2 + (-1.2)^2}} = \frac{(0.2\hat{x} - \hat{y} - 1.2\hat{z})}{1.575}$$

Force:

$$\begin{aligned}\vec{F}_1 &= (5.38 \times 10^{-15}\ \text{N})\frac{(0.2\hat{x} - \hat{y} - 1.2\hat{z})}{1.575}\\ &= (6.83 \times 10^{-16}\ \text{N})\hat{x} - (3.42 \times 10^{-15}\ \text{N})\hat{y} - (4.10 \times 10^{-15}\ \text{N})\hat{z}.\end{aligned}$$

Force due to m_2:

Magnitude:

$$F_2 = \frac{Gm_2m}{r_2^2} = \frac{\left(6.67 \times 10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(6.00 \times 10^6\ \text{kg})(1\ \text{kg})}{((1.00 \times 10^5\ \text{m})^2 + (2.00 \times 10^4\ \text{m})^2 + (1.20 \times 10^5\ \text{m})^2)} = 1.61 \times 10^{-14}\ \text{N}$$

Direction:

$$\hat{r}_2 = \frac{(-\hat{x} + 0.2\hat{y} - 1.2\hat{z})}{\sqrt{(-1)^2 + (0.2)^2 + (-1.2)^2}} = \frac{(-\hat{x} + 0.2\hat{y} - 1.2\hat{z})}{1.575}$$

Force:

$$\begin{aligned}\vec{F}_2 &= (1.61 \times 10^{-14}\ \text{N})\frac{(-\hat{x} + 0.2\hat{y} - 1.2\hat{z})}{1.575}\\ &= (-1.02 \times 10^{-14}\ \text{N})\hat{x} + (2.04 \times 10^{-15}\ \text{N})\hat{y} - (1.23 \times 10^{-14}\ \text{N})\hat{z}\end{aligned}$$

Force due to m_3:

Magnitude:

$$F_3 = \frac{Gm_3m}{r_3^2} = \frac{\left(6.67 \times 10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(8.00 \times 10^6\ \text{kg})(1\ \text{kg})}{((1.00 \times 10^5\ \text{m})^2 + (1.00 \times 10^5\ \text{m})^2 + (2.00 \times 10^4\ \text{m})^2)} = 2.62 \times 10^{-14}\ \text{N}$$

Direction:

$$\hat{r}_3 = \frac{(-\hat{x} - \hat{y} + 0.2\hat{z})}{\sqrt{(-1)^2 + (-1)^2 + (0.2)^2}} = \frac{(-\hat{x} - \hat{y} + 0.2\hat{z})}{1.428}$$

Force:

$$\vec{F}_1 = (2.62 \times 10^{-14}\ \mathrm{N})\frac{(-\hat{x} - \hat{y} + 0.2\hat{z})}{1.428}$$

$$= (-1.83 \times 10^{-14}\ \mathrm{N})\hat{x} - (1.83 \times 10^{-14}\ \mathrm{N})\hat{y} + (3.67 \times 10^{-15}\ \mathrm{N})\hat{z}$$

Net force:

$$\sum F_x = (6.83 \times 10^{-16}\ \mathrm{N}) - (1.02 \times 10^{-14}\ \mathrm{N}) - (1.83 \times 10^{-14}\ \mathrm{N}) = -2.78 \times 10^{-14}\ \mathrm{N}$$

$$\sum F_y = (-3.42 \times 10^{-15}\ \mathrm{N}) + (2.04 \times 10^{-15}\ \mathrm{N}) - (1.83 \times 10^{-14}\ \mathrm{N}) = -1.97 \times 10^{-14}\ \mathrm{N}$$

$$\sum F_x = (-4.10 \times 10^{-15}\ \mathrm{N}) - (1.23 \times 10^{-14}\ \mathrm{N}) + (3.67 \times 10^{-15}\ \mathrm{N}) = -1.27 \times 10^{-14}\ \mathrm{N}$$

$$\boxed{\sum \vec{F} = (-2.78 \times 10^{-14}\ \mathrm{N})\hat{x} - (1.97 \times 10^{-14}\ \mathrm{N})\hat{y} - (1.27 \times 10^{-14}\ \mathrm{N})\hat{z}}$$

REFLECT

The gravitational force is always attractive, so the force vectors will point from point P toward each of the masses.

10.89

SET UP

Two objects ($m_{10} = 10$ kg and $m_3 = 3$ kg) are separated by 40 cm. A third object ($m_1 = 1$ kg) is placed at a location along the line connecting them such that the net force acting on m_1 is zero. By considering the force vectors, this location must be between the two original objects. We will define x as the distance between m_{10} and m_1 and y as the distance between m_3 and m_1. Setting the magnitudes of the gravitational forces equal and using the fact that $x + y = 40$ cm, we can solve for x.

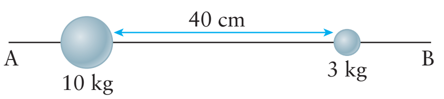


Figure 10-13 Problem 89

SOLVE

$$\frac{Gm_{10}m_1}{x^2} = \frac{Gm_3m_1}{y^2}$$

$$x^2 = \frac{m_{10}y^2}{m_3}$$

$$x = y\sqrt{\frac{m_{10}}{m_3}}$$

But $x + y = 40$ cm:

$$x + y = y\sqrt{\frac{m_{10}}{m_3}} + y = y\sqrt{\frac{10\text{ kg}}{3\text{ kg}}} + y = 2.826y = 40\text{ cm}$$

$$y = \frac{40\text{ cm}}{2.826} = 14.2\text{ cm}$$

$$x = (40\text{ cm}) - y = (40\text{ cm}) - (14.2\text{ cm}) = \boxed{25.8\text{ cm}}$$

REFLECT

It makes sense that the 1-kg object needs to be closer to the less massive object if the net force acting on it is zero.

10.90

SET UP

Within a uniform, solid, spherical object, the gravitational force increases as you move radially away from the center. Since gravity is a conservative force, the magnitude of the force is equal to derivative of the potential energy with respect to space. We can solve for the potential energy by integrating the force with respect to r.

SOLVE

$$\vec{F}(r) = -\Gamma m\vec{r} = -\frac{dU}{dr}$$

$$\int dU = U(r) - U(0) = -\int_0^r -\Gamma mr\,dr = \Gamma m\left[\frac{r^2}{2}\right]$$

REFLECT

The force has the same algebraic form as Hooke's law for springs, so the potential energy should also have the same algebraic form as Hooke's law.

Chapter 11
Fluids

Conceptual Questions

11.1 No, the pieces could be different sizes.

11.2 The pressure in a fluid depends only on the depth. Since the height of the water is the same in both vessels, they will have the same pressure at the bottom. The total weight of the fluid is not directly related to the pressure.

11.3 No. The pressure on the water is determined by its depth, gravity, and the pressure pushing down on the surface (air pressure). The ability of the dam to withstand that pressure does, of course, depend on its shape.

11.4 While jumping up, the person will have very little blood pumped to the brain, as most of the blood will be in the lower trunk and in the legs due to gravity. This may result in a fainting spell. Once the person is horizontal, the blood will evenly distribute in the body and will reach the brain and consciousness will be regained.

11.5 The pressure would be higher because of the additional pressure from fluid depth.

11.6 Raising your hand will decrease the blood pressure and, thus, slow the bleeding.

11.7 The collection bag should be held below the body so that the blood can flow down into it.

11.8 When the ice floats with the water line at the rim of the glass, some of the ice is above the rim, so not all of the ice cube is displacing water. However, the volume of water displaced is equal in weight to the weight of the entire ice cube. When the entire ice cube melts the resulting water is more dense than the ice, so it takes up less volume—and that volume is equal to the volume of water displaced by the submerged part of the floating ice cube.

11.9 It is easier to float in the Great Salt Lake because salt water is denser than freshwater.

11.10 The water level will fall. The boat displaces a volume of water equal to the weight of the boat. If the boat is removed from the water, the water will no longer be displaced and the water level will fall.

11.11 The same. Density is intrinsic to the material, and weight only depends on mass and gravity, not on the rest of the environment.

11.12 The equation of continuity applies since there is no water "lost" along the way. The speed will increase when the cross-sectional area decreases. Therefore, the speed

of the water in the wide valley will be slower than the speed of the water in the narrow channel.

11.13 The dam will be the same thickness as before. Pressure depends on depth, not length.

11.14 The landing planes should approach from the east (into the wind). This allows pilots to use the wind to provide lift so that they can cut the plane's engines and drift down in a controlled manner. If the wind is at their back, pilots will have to maintain airspeed with the engines to stay airborne. In addition, the speed of the plane at impact will be much greater and require significantly more braking to come to a stop.

11.15 The wind flow over the roof is very fast, while the airflow inside the house is not; this lowers the exterior pressure. If the roof is not made to hold down against a large part of an atmosphere of pressure pushing it up, it will fly off.

11.16 The stream of water is accelerated by gravity as it leaves the faucet. According to the equation of continuity, the cross-sectional area must decrease as the speed of the water flow increases.

11.17 The water will flow out horizontally from the hole.

Multiple-Choice Questions

11.18 **A** ($\rho_2 = 3\rho_1$).

$$\rho_1 = \frac{M}{V}$$

$$\rho_2 = \frac{3M}{V} = 3\rho_1$$

11.19 **D** (be increased, but not necessarily doubled). Because the gauge pressure is the pressure above 1 atm, doubling the gauge pressure will not necessarily double the absolute pressure.

11.20 **C** (the toy under water). The magnitude of the buoyant force is proportional to the volume displaced by the toy.

11.21 **B** (5/8).

$$F_b = F_g$$

$$\rho_{water} V_{displaced} g = \rho_{obj} V_{obj} g$$

$$\frac{\rho_{obj}}{\rho_{water}} = \frac{V_{displaced}}{V_{obj}} = \frac{\left(\frac{5}{8}V_{obj}\right)}{V_{obj}} = \boxed{\frac{5}{8}}$$

11.22 **B** (It remains the same). The volume of ice that displaces the water will be exactly the same once the ice melts and becomes water.

11.23 **A** (one-quarter the speed in the 0.5-cm pipe).

$$v_1A_1 = v_2A_2$$

$$v_2 = v_1\frac{A_1}{A_2} = v_1\left(\frac{\pi(0.5\text{ cm})^2}{\pi(1\text{ cm})^2}\right) = \boxed{\frac{1}{4}v_1}$$

11.24 **A** (increases). The equation of continuity applies since there is no blood "lost" along the way. The speed will increase when the cross-sectional area decreases.

11.25 **A** (the conservation of energy for a moving fluid). Bernoulli's equation is a statement of the work–energy theorem for a piece of moving fluid.

11.26 **C** (total energy). Conservation of energy states that the total energy of the system must remain constant. Bernoulli's equation shows that the kinetic energy, potential energy, velocity, or pressure can change but the total energy must remain constant.

11.27 **A** (Box A).

$$\sum F_y = F_b - F_g = \rho_{\text{water}}V_{\text{object}}g - \rho_{\text{object}}V_{\text{object}}g = ma_y = \rho_{\text{object}}V_{\text{object}}a_y$$

$$a_y = \frac{\rho_{\text{water}}V_{\text{object}}g - \rho_{\text{object}}V_{\text{object}}g}{\rho_{\text{object}}V_{\text{object}}} = g\left(\frac{\rho_{\text{water}}}{\rho_{\text{object}}} - 1\right)$$

The object with the smaller density will experience a larger acceleration upward.

Estimation Questions

11.28 Toronto (the largest city in Canada) has about 5.5×10^6 people. If a single person uses around 10 gallons a day, the city of Toronto would use about 5.5×10^7 gallons.

11.29 A pint (of mercury) is a stone the world around. Yes, that's the British unit of weight (1 stone equals about 14 lb).

11.30 There are about 1.23×10^{21} liters of water in the oceans, which represent about 1.23×10^{21} kg of the Earth's total mass (of 6×10^{24} kg). If all of this water were replaced with mercury, then the mass would be 1.68×10^{22} kg. This is an increase of 1.56×10^{22} kg, so the new mass of the Earth would be 6.02×10^{24} kg, which is an increase of 0.33%.

11.31 Mercury is, of course, 760 mm; water is 13.6 times greater, or 10.4 m; and seawater is 10.1 m.

11.32 Fresh water has a density of around 1000 kg/m^3 and salt water is about 1030 kg/m^3, the buoyant force in freshwater will be about 1000/1030 times (=97%) less than in salt water.

11.33 Using a 6:1 hydraulic jack operated by hand, a 300-kg object could be easily lifted.

11.34 The pressure in a typical garden hose is 50–100 psi (or 3.4–6.8 atm).

11.35 A wind that is 33 m/s (barely hurricane level) passing over the top of a 1-m^2 prone body resting on top of still air results in a lift of approximately 0.67 kN.

11.36 The systolic pressure (the first number reported in a blood pressure) corresponds to the pressure generated by the contraction of the heart. A typical systolic pressure is 120 mmHg gauge pressure, which corresponds to 880 mmHg ($\sim 1.2 \times 10^5$ Pa).

Problems

11.37

SET UP

An iron cube is 2 cm along each side. The density of iron is given as 7800 kg/m^3. We can multiply the volume of the cube by the density of iron to get the mass of the cube.

SOLVE

$$\rho = \frac{M}{V}$$

$$M = \rho V = \left(7800 \frac{\text{kg}}{\text{m}^3}\right)\left(2\text{ cm} \times \frac{1\text{ m}}{100\text{ cm}}\right)^3 = \boxed{0.062\text{ kg} = 62\text{ g}}$$

REFLECT

A cube that is 2 cm on each side is about ¾ in. on each side. A mass of 62 g seems reasonable for a small cube of iron.

11.38

SET UP

A sphere of aluminum has a mass of 24 kg. Aluminum has a density of 2700 kg/m^3. We can solve for the volume, and then the radius, of the sphere using the definition of density.

SOLVE

$$\rho = \frac{M}{V} = \frac{M}{\left(\frac{4}{3}\pi R^3\right)}$$

$$R = \sqrt[3]{\frac{3M}{4\pi\rho}} = \sqrt[3]{\frac{3(24\text{ kg})}{4\pi\left(2700\frac{\text{kg}}{\text{m}^3}\right)}} = \boxed{0.13\text{ m}}$$

REFLECT
A sphere with a radius of 13 cm is about 10 in. across in diameter, which seems reasonable for a 24-kg sphere.

11.39

SET UP
A cylinder made of an unknown material is 20 cm long, 1 cm in radius, and has a mass of 490 g. We can calculate the average density of the material by dividing the cylinder's mass by its volume. The volume of a cylinder is the cross-sectional area multiplied by the length. We can compare our answer to part (a) to the densities in Table 11-1 in order to figure out the material from which the cylinder is made. The specific gravity of a material is equal to its density divided by the density of water, $\rho_{\text{water}} = 1000 \text{ kg/m}^3$.

SOLVE

$$\rho = \frac{M}{V} = \frac{M}{\pi R^2 L} = \frac{(0.490 \text{ kg})}{\pi(0.01 \text{ m})^2(0.20 \text{ m})} = \boxed{7800\frac{\text{kg}}{\text{m}^3}}$$

Part b)
The cylinder is most likely made of iron.

Part c)

$$\text{SG} = \frac{\rho}{\rho_{\text{water}}} = \frac{7800\frac{\text{kg}}{\text{m}^3}}{1000\frac{\text{kg}}{\text{m}^3}} = \boxed{7.80}$$

REFLECT
A "uniform" object has a constant density throughout it. If an object is not uniform, we can calculate the average density of the object as we did above.

11.40

SET UP
An ice cube has a mass of 0.35 kg. The density of ice is 917 kg/m^3. Assuming the ice cube is actually cubic, the side of the ice cube is equal to the cube root of the mass divided by the density.

SOLVE

$$\rho = \frac{M}{V} = \frac{M}{L^3}$$

$$L = \sqrt[3]{\frac{M}{\rho}} = \sqrt[3]{\frac{0.35 \text{ kg}}{917\frac{\text{kg}}{\text{m}^3}}} = \boxed{0.073 \text{ m} = 7.3 \text{ cm}}$$

REFLECT
The density of ice is less than that of liquid water, which is why ice cubes float.

11.41

SET UP

The human body is approximately 65% water. We can approximate the volume of water in a 65-kg man by first assuming the total 65-kg mass is due to water. Dividing the mass of 65 kg by the density of water will give us the corresponding volume of water. We then need to multiply this volume by 0.65 to approximate the volume of water in the man.

SOLVE

$$\rho = \frac{M}{V}$$

$$V = \frac{M}{\rho} = \frac{65\text{ kg}}{1000\frac{\text{kg}}{\text{m}^3}} = (0.065\text{ m}^3)(0.65) = 0.042\text{ m}^3 \times \frac{1000\text{ L}}{1\text{ m}^3} = \boxed{42\text{ L}}$$

REFLECT

A volume of 42 L is over 10 gal.

11.42

SET UP

Density rods are cylinders of equal diameters and mass but varying lengths that are designed to illustrate the concept of density—the longer the cylinder, the smaller the density of that material. Density is equal to mass divided by volume. Since the mass and diameter are constant for each rod, we can rearrange the expression for the density and the length of each rod in order to find the ratio between the lengths. The specific gravity is equal to the density divided by the density of water, so the ratio of the densities will equal the ratio of the specific gravities. Once we have an algebraic expression for the lengths in terms of the specific gravities, we can plug in the given data ($SG_{\text{alum}} = 2.7$, $SG_{\text{iron}} = 7.8$, $SG_{\text{copper}} = 8.9$, $SG_{\text{brass}} = 8.5$, $SG_{\text{lead}} = 11.3$).

SOLVE

Part a)

$$\rho = \frac{M}{V} = \frac{M}{\pi R^2 L}$$

$$\rho L = \frac{M}{\pi R^2} = \text{constant}$$

Therefore,

$$\rho_1 L_1 = \rho_2 L_2$$

$$\boxed{\frac{L_1}{L_2} = \frac{\rho_2}{\rho_1} = \frac{\left(\frac{\rho_2}{\rho_{\text{water}}}\right)}{\left(\frac{\rho_1}{\rho_{\text{water}}}\right)} = \frac{SG_2}{SG_1}}$$

Part b)

Aluminum:

$$\frac{L_{\text{alum}}}{L_{\text{lead}}} = \frac{\text{SG}_{\text{lead}}}{\text{SG}_{\text{alum}}} = \frac{11.3}{2.7} = \boxed{4.2}$$

Iron:

$$\frac{L_{\text{iron}}}{L_{\text{lead}}} = \frac{\text{SG}_{\text{lead}}}{\text{SG}_{\text{iron}}} = \frac{11.3}{7.8} = \boxed{1.4}$$

Copper:

$$\frac{L_{\text{copper}}}{L_{\text{lead}}} = \frac{\text{SG}_{\text{lead}}}{\text{SG}_{\text{copper}}} = \frac{11.3}{8.9} = \boxed{1.3}$$

Brass:

$$\frac{L_{\text{brass}}}{L_{\text{lead}}} = \frac{\text{SG}_{\text{lead}}}{\text{SG}_{\text{brass}}} = \frac{11.3}{8.5} = \boxed{1.3}$$

REFLECT

Since the product of the density and the length of the cylinder is constant, the density of the material is inversely proportional to the length of the cylinder.

11.43

SET UP

We can calculate the density of Earth by dividing the mass of the Earth ($m_{\text{E}} = 5.98 \times 10^{24}$ kg) by the volume of the Earth. We'll treat the Earth as a sphere of radius $R_{\text{E}} = 6.380 \times 10^{6}$ m.

SOLVE

$$\rho = \frac{M}{V} = \frac{m_{\text{E}}}{\left(\frac{4}{3}\pi R_{\text{E}}^3\right)} = \frac{5.98 \times 10^{24}\text{ kg}}{\left(\frac{4\pi}{3}\right)(6.38 \times 10^{6}\text{ m})^3} = \boxed{5500\frac{\text{kg}}{\text{m}^3}}$$

REFLECT

This is consistent with the average density of Earth given in Table 11-1.

11.44

SET UP

The density of a sphere of radius R varies as a function of the distance from the center r: $\rho(r) = \rho_0\left(1 - \frac{r}{R}\right)$. We can find the mass of the sphere by integrating the density over the volume of the sphere: $m = \int \rho dV$. Since ρ is a function of r, we need to write the differential volume dV in terms of r. The easiest way of doing this is by differentiating the volume of a sphere $\left(V = \frac{4}{3}\pi r^3\right)$ with respect to r and then solving for dV. Plugging in this expression for dV will convert the integral from an integral over volume to an integral over the radius from $r = 0$ to $r = R$.

SOLVE

Finding dV:

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dr} = 4\pi r^2$$

$$dV = 4\pi r^2 dr$$

Finding m:

$$m = \int \rho dV = \int_0^R \rho_0 \left(1 - \frac{r}{R}\right)(4\pi r^2 dr) = 4\pi\rho_0 \int_0^R \left(r^2 - \frac{r^3}{R}\right) dr = 4\pi\rho_0 \left[\frac{r^3}{3} - \frac{r^4}{4R}\right]_0^R$$

$$= 4\pi\rho_0 \left[\frac{R^3}{3} - \frac{R^4}{4R}\right] = 4\pi\rho_0 \left[\frac{R^3}{12}\right] = \boxed{\frac{\pi}{3}\rho_0 R^3}$$

REFLECT

Our answer has dimensions of mass, as expected. A uniform sphere of density ρ_0 would have a mass of $\frac{4\pi}{3}\rho_0 R^3$. Because the density of this sphere decreases with the radius, we would expect the mass to be less than a uniform sphere.

11.45

SET UP

A neutron star has the same density as a neutron. We can calculate the radius of a neutron star that has the same mass as our Sun ($m_{\text{star}} = 1.99 \times 10^{30}$ kg) by setting the ratio of its mass to volume equal to the ratio of a neutron's mass to volume. We will assume both the star and the neutron are spheres. The mass of a neutron is $m_{\text{n}} = 1.7 \times 10^{-27}$ kg, and its approximate radius is 1.2×10^{-15} m.

SOLVE

$$\rho_{\text{n}} = \rho_{\text{star}}$$

$$\frac{m_{\text{n}}}{\left(\frac{4}{3}\pi R_{\text{n}}^3\right)} = \frac{m_{\text{star}}}{\left(\frac{4}{3}\pi R_{\text{star}}^3\right)}$$

$$R_{\text{star}} = R_{\text{n}}\sqrt[3]{\frac{m_{\text{star}}}{m_{\text{n}}}} = (1.2 \times 10^{-15}\text{ m})\sqrt[3]{\frac{1.99 \times 10^{30}\text{ kg}}{1.7 \times 10^{-27}\text{ kg}}} = \boxed{13\text{ km}}$$

REFLECT

A radius of 13 km seems reasonable for a "very small" star. Most astronomical distances are orders of magnitude larger. For instance, the radius of the Earth is approximately 6400 km and the distance between the Earth and the Moon is about 384,000 km.

11.46

SET UP

An object of mass m_1 is supported by a smaller square platform with sides of length s_1. The pressure that the platform exerts on the object is equal to the magnitude of the force the platform exerts on the object divided by the area over which this force acts. Assuming that the objects are at rest, the normal force of the platform acting on the object will have the same magnitude as the weight of the object. We can set the pressure that platform 1 exerts on object 1 equal to the pressure that platform 2 exerts on object 2 and solve for m_2. Once we have an expression for m_2 in terms of m_1, s_1, and s_2, we can plug in $s_2 = 5s_1$ to find the ratio between the two masses.

SOLVE

Part a)

$$P_1 = P_2$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$\frac{m_1 g}{s_1^2} = \frac{m_2 g}{s_2^2}$$

$$\boxed{m_2 = m_1 \frac{s_2^2}{s_1^2}}$$

Part b)

$$\frac{m_2}{m_1} = \frac{s_2^2}{s_1^2} = \frac{(5s_1)^2}{s_1^2} = \frac{25s_1^2}{s_1^2} = \boxed{25}$$

REFLECT

Pressure is inversely proportional to the cross-sectional area, so a larger mass is required to exert the same pressure on a larger platform.

11.47

SET UP

The pressure at the surface of the Earth is 1 atm, or 1.01×10^5 Pa. The weight of the atmosphere is equal to the atmospheric pressure multiplied by the surface area of the Earth. We can divide the weight of the atmosphere by g to get the mass of the atmosphere. The radius of the Earth is $R_E = 6.38 \times 10^6$ m.

SOLVE

$$P = \frac{F}{A} = \frac{Mg}{4\pi R_E^2}$$

$$M = \frac{4\pi P_{atm} R_E^2}{g} = \frac{4\pi(1.01 \times 10^5 \text{ Pa})(6.38 \times 10^6 \text{ m})^2}{9.8\frac{\text{m}}{\text{s}^2}} = \boxed{5.27 \times 10^{18} \text{ kg}}$$

REFLECT

This is a maximum estimate of the mass because the atmospheric pressure varies with altitude.

11.48

SET UP

An elephant has a mass of 6000 kg. Each foot is circular with a diameter of 50 cm. Since the mass is evenly distributed over the elephant's four feet, each foot supports one-quarter of the elephant's weight. For a human, each foot supports one-half of the weight. An average male has a mass of 70 kg. We can treat the bottom of a foot as a rectangle; a foot is about 25 cm long by about 10 cm across, so the area of one foot is 0.025 m^2. For both the elephant and the person the pressure on each foot is equal to the force exerted on the foot divided by the cross-sectional area of the foot.

SOLVE

Part a)

$$P_{1\text{ foot}} = \frac{F_{1\text{ foot}}}{A} = \frac{\left(\frac{mg}{4}\right)}{\pi\left(\frac{d}{2}\right)^2} = \frac{mg}{\pi d^2} = \frac{(6000\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)}{\pi(0.50\text{ m})^2} = \boxed{7.5 \times 10^4\text{ Pa}}$$

Part b)

$$P_{1\text{ foot}} = \frac{F_{1\text{ foot}}}{A} = \frac{\left(\frac{mg}{2}\right)}{A} = \frac{mg}{2A} = \frac{(70\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)}{2(0.025\text{ m}^2)} = \boxed{1.4 \times 10^4\text{ Pa}}$$

REFLECT

Even though the weight of an elephant is about 100 times larger than that of a human, the pressure per foot is only different by a factor of 5.

11.49

SET UP

A force of 25 N is applied to the head of a nail that is 0.32 cm in diameter. The pointed end of the nail is 0.032 cm in diameter. The pressure on the head of the nail or the pointed end of the nail is equal to the applied force divided by the cross-sectional area at that location.

SOLVE

Part a)

$$P = \frac{F}{A} = \frac{F}{\pi\left(\frac{d}{2}\right)^2} = \frac{4F}{\pi d^2} = \frac{4(25\text{ N})}{\pi(0.32 \times 10^{-2}\text{ m})^2} = \boxed{3.11 \times 10^6\text{ Pa} = 3.11\text{ MPa}}$$

Part b)

$$P = \frac{F}{A} = \frac{F}{\pi\left(\frac{d}{2}\right)^2} = \frac{4F}{\pi d^2} = \frac{4(25\text{ N})}{\pi(0.032 \times 10^{-2}\text{ m})^2} = \boxed{3.11 \times 10^8\text{ Pa} = 311\text{ MPa}}$$

REFLECT

Since the pressure is inversely proportional to the area, a decrease of a factor of 10 in the radius corresponds to an increase by a factor of 100 in the pressure!

11.50

SET UP

A 0.25-m-tall graduated cylinder is filled halfway with mercury and halfway with water. The pressure at the bottom of the cylinder is equal to the atmospheric pressure plus the additional pressures due to the two liquids. Each column of liquid has a height of $d = 0.125$ m. The density of mercury is $\rho_{\mathrm{Hg}} = 13{,}600\frac{\mathrm{kg}}{\mathrm{m}^3}$.

SOLVE

$$P = P_{\mathrm{atm}} + \rho_{\mathrm{Hg}} g d_{\mathrm{Hg}} + \rho_{\mathrm{water}} g d_{\mathrm{water}}$$

$$P = (1.01 \times 10^5\ \mathrm{Pa}) + \left(13600\frac{\mathrm{kg}}{\mathrm{m}^3}\right)\left(9.8\frac{\mathrm{m}}{\mathrm{s}^2}\right)(0.125\ \mathrm{m}) + \left(1000\frac{\mathrm{kg}}{\mathrm{m}^3}\right)\left(9.8\frac{\mathrm{m}}{\mathrm{s}^2}\right)(0.125\ \mathrm{m})$$

$$= \boxed{1.19 \times 10^5\ \mathrm{Pa}}$$

REFLECT

This is equivalent to a pressure of 1.17 atm.

11.51

SET UP

A swimming pool is filled to a depth of $d = 10$ m. The gauge pressure at the bottom of the pool is equal to the product of the density of water, g, and the depth d.

SOLVE

$$P_{\mathrm{gauge}} = P - P_{\mathrm{atm}} = \rho g d = \left(1000\frac{\mathrm{kg}}{\mathrm{m}^3}\right)\left(9.8\frac{\mathrm{m}}{\mathrm{s}^2}\right)(10\ \mathrm{m}) = \boxed{9.8 \times 10^4\ \mathrm{Pa}}$$

REFLECT

The length and width of the pool are not necessary to solve this problem.

11.52

SET UP

A diver is 10.0 m below the surface of the ocean. The pressure at the surface is 1 atm, or 1.01×10^5 Pa. The diver will experience an additional pressure due to the seawater, which has a density of 1025 kg/m^3. The gauge pressure is equal to the product of the density of seawater, g, and the depth. We can find the absolute pressure by adding the atmospheric pressure to the gauge pressure.

SOLVE

Gauge pressure:

$$P_{\text{gauge}} = P - P_{\text{atm}} = \rho g d = \left(1025\frac{\text{kg}}{\text{m}^3}\right)\left(9.8\frac{\text{m}}{\text{s}^2}\right)(10.0\text{ m}) = \boxed{1.005 \times 10^5\text{ Pa}}$$

Absolute pressure:

$$P = P_{\text{gauge}} + P_{\text{atm}} = (1.005 \times 10^5\text{ Pa}) + (1.01 \times 10^5\text{ Pa}) = \boxed{2.015 \times 10^5\text{ Pa}}$$

REFLECT

The diver will experience an additional 1 atm of pressure (due to the seawater) in the ocean at a depth of 10.1 m.

11.53

SET UP

The difference in blood pressure between the top of the head and the bottom of the feet of a 1.75-m-tall person is equal to product of the density of blood ($\rho_{\text{blood}} = 1.06 \times 10^3$ kg/m^3), g, and the height. The conversion between pascals and mmHg is 1.01×10^5 Pa = 760 mmHg.

SOLVE

$$\Delta P = \rho_{\text{blood}} g h = \left(1.06 \times 10^3\frac{\text{kg}}{\text{m}^3}\right)\left(9.8\frac{\text{m}}{\text{s}^2}\right)(1.75\text{ m}) = 18179\text{ Pa} \times \frac{760\text{ mmHg}}{1.01 \times 10^5\text{ Pa}} = \boxed{137\text{ mmHg}}.$$

REFLECT

Since we are just looking for the difference in blood pressure between the top of the head and the bottom of the feet, we do not need to explicitly include the pressure of the atmosphere in our calculation.

11.54

SET UP

Three manometers are filled with air, water, and mercury, respectively. Point O is located at the top of each manometer and is open to the atmosphere. The pressure at a point P that is at a depth of 0.37 m in each manometer is equal to the atmospheric pressure plus the increase in pressure due to the column of fluid. The densities of the three fluids are $\rho_{\text{air}} = 1.3$ kg/m^3, $\rho_{\text{water}} = 1000$ kg/m^3, and $\rho_{\text{Hg}} = 13{,}600$ kg/m^3.

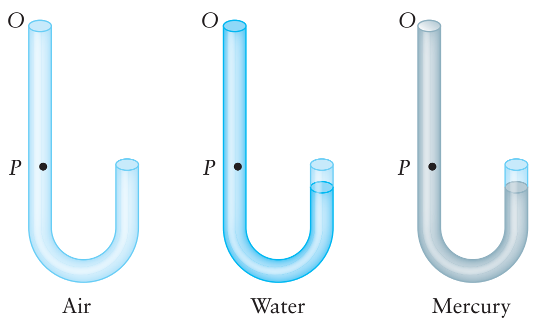


Figure 11-1 Problem 54

SOLVE

Air:

$$P = P_{\text{atm}} + \rho_{\text{air}} g d = (1.01 \times 10^5\ \text{Pa}) + \left(1.3\frac{\text{kg}}{\text{m}^3}\right)\left(9.8\frac{\text{m}}{\text{s}^2}\right)(0.37\ \text{m}) = \boxed{1.01 \times 10^5\ \text{Pa}}$$

Water:

$$P = P_{\text{atm}} + \rho_{\text{water}} g d = (1.01 \times 10^5\ \text{Pa}) + \left(1000\frac{\text{kg}}{\text{m}^3}\right)\left(9.8\frac{\text{m}}{\text{s}^2}\right)(0.37\ \text{m}) = \boxed{1.05 \times 10^5\ \text{Pa}}$$

Mercury:

$$P = P_{\text{atm}} + \rho_{\text{Hg}} g d = (1.01 \times 10^5\ \text{Pa}) + \left(13{,}600\frac{\text{kg}}{\text{m}^3}\right)\left(9.8\frac{\text{m}}{\text{s}^2}\right)(0.37\ \text{m}) = \boxed{1.50 \times 10^5\ \text{Pa}}$$

REFLECT

The pressure at P should be the largest in the manometer holding the densest fluid, which is mercury in this case.

11.55

SET UP

A cylinder is filled with iodine ($\rho_{\text{iodine}} = 4930\ \text{kg/m}^3$) to a depth of $d_{\text{iodine}} = 1.5\ \text{m}$. We need to fill a second cylinder with ether ($\rho_{\text{ether}} = 72.7\ \text{kg/m}^3$) such that the pressure at the bottom of each cylinder is the same. Since the pressure differences over the cylinders are equal, we can set up a proportionality between the density and depth of the iodine column and the ether column to solve for the depth of the ether column, d_{ether}.

SOLVE

$$\Delta P_{\text{iodine}} = \Delta P_{\text{ether}}$$

$$\rho_{\text{iodine}} g d_{\text{iodine}} = \rho_{\text{ether}} g d_{\text{ether}}$$

$$d_{\text{ether}} = \frac{\rho_{\text{iodine}}}{\rho_{\text{ether}}} d_{\text{iodine}} = \left(\frac{4930\frac{\text{kg}}{\text{m}^3}}{72.7\frac{\text{kg}}{\text{m}^3}}\right)(1.5\ \text{m}) = \boxed{102\ \text{m}}$$

REFLECT

Because the pressure difference is constant in both cases, the height of the column of fluid will be inversely proportional to the density of the fluid.

11.56

SET UP

We are asked to convert from various common units of pressure into the SI unit pascal. The conversion factors we'll need are 1 atm = 14.7 psi = 1.01×10^5 Pa = 760 torr.

SOLVE

Part a)

$$1500 \text{ kPa} \times \frac{1000 \text{ Pa}}{1 \text{ kPa}} = \boxed{1.5 \times 10^6 \text{ Pa}}$$

Part b)

$$35 \text{ psi} \times \frac{1 \text{ atm}}{14.7 \text{ psi}} \times \frac{1.01 \times 10^5 \text{ Pa}}{1 \text{ atm}} = \boxed{2.4 \times 10^5 \text{ Pa}}$$

Part c)

$$2.85 \text{ atm} \times \frac{1.01 \times 10^5 \text{ Pa}}{1 \text{ atm}} = \boxed{2.88 \times 10^5 \text{ Pa}}$$

Part d)

$$883 \text{ torr} \times \frac{1.01 \times 10^5 \text{ Pa}}{760 \text{ torr}} = \boxed{1.17 \times 10^5 \text{ Pa}}$$

REFLECT

A pressure of 1 Pa is relatively small, which is part of the reason why there are so many units for pressure used in everyday life.

11.57

SET UP

Blood pressure is usually measured as a gauge pressure, which is the pressure above the atmospheric pressure. To convert a gauge pressure into an absolute pressure, we need to add the pressure of the atmosphere. In mmHg, the atmospheric pressure is 760 mmHg.

SOLVE

$$P = P_{\text{gauge}} + P_{\text{atm}} = (120 \text{ mmHg}) + (760 \text{ mmHg}) = \boxed{880 \text{ mmHg}}$$

REFLECT

Two common examples in which gauge pressures are used are for blood pressures and for car tires. Be sure to convert to absolute pressure by adding the atmospheric pressure before using the pressures in an equation.

11.58

SET UP

The atmospheric pressure at sea level is 1 atm. Badwater is at an elevation of 85 m below sea level. The atmospheric pressure will increase by an amount equal to the product of the density of air ($\rho_{\text{air}} = 1.3 \text{ kg/m}^3$), the acceleration due to gravity, and the depth. In order to calculate the absolute atmospheric pressure at Badwater, we first need to convert from atmospheres to pascals: $1 \text{ atm} = 1.01 \times 10^5 \text{ Pa}$.

SOLVE

$$P = P_0 + \rho g d = (1.01 \times 10^5\ \text{Pa}) + \left(1.30\frac{\text{kg}}{\text{m}^3}\right)\left(9.8\frac{\text{m}}{\text{s}^2}\right)(85\ \text{m}) = \boxed{1.02 \times 10^5\ \text{Pa}}$$

REFLECT

Badwater is below sea level, so the atmospheric pressure should be larger. Also, when working with SI units, be sure to convert to pascals.

11.59

SET UP

A woman is wearing a wide-brimmed hat with a diameter of 0.45 m. The atmospheric pressure at her location is exactly 1 atm, or 1.01×10^5 Pa. This pressure is equal to the weight of the column of air above her hat divided by the cross-sectional area of her hat. Rearranging this relationship will give us the weight of the column of air.

SOLVE

$$P = \frac{F}{A}$$

$$F = PA = P_{\text{atm}}\left(\pi\left(\frac{d}{2}\right)^2\right) = \frac{\pi}{4}P_{\text{atm}}d^2 = \frac{\pi}{4}(1.01 \times 10^5\ \text{Pa})(0.45\ \text{m})^2 = \boxed{16{,}000\ \text{N} = 16\ \text{kN}}$$

REFLECT

This force is about 23 times larger than the weight of a 70-kg person. We are under this pressure constantly, so our body is accustomed to it and we only notice changes in this pressure, not the absolute value.

11.60

SET UP

A bicycle tire is inflated to a gauge pressure of 65 psi. In order to calculate the absolute pressure to which this corresponds, we first need to convert from psi to pascals (14.7 psi = 1 atm = 1.01×10^5 Pa) and then add the atmospheric pressure ($P_{\text{atm}} = 1.01 \times 10^5$ Pa).

SOLVE

Converting to pascals:

$$65\ \text{psi} \times \frac{1\ \text{atm}}{14.7\ \text{psi}} \times \frac{1.01 \times 10^5\ \text{Pa}}{1\ \text{atm}} = 4.47 \times 10^5\ \text{Pa}$$

Converting to absolute pressure:

$$P = P_{\text{gauge}} + P_{\text{atm}} = (4.47 \times 10^5\ \text{Pa}) + (1.01 \times 10^5\ \text{Pa}) = \boxed{5.48 \times 10^5\ \text{Pa}}$$

REFLECT

Most passenger car tires are inflated to about 30 psi, whereas bicycle tires are inflated to 40–60 psi.

11.61

SET UP

An airplane window has an area of 1000 cm^2. The pressure inside the airplane is 0.95 atm, while the pressure outside the airplane is 0.85 atm. The net force due to this pressure difference will point from the location of high pressure (inside the plane, in this case) toward the location of low pressure and have a magnitude equal to the pressure difference multiplied by the cross-sectional area of the window.

SOLVE

Pressure difference:

$$(0.95 \text{ atm}) - (0.85 \text{ atm}) = 0.10 \text{ atm} \times \frac{1.01 \times 10^5 \text{ Pa}}{1 \text{ atm}} = 1.01 \times 10^4 \text{ Pa}$$

Net force:

$$P = \frac{F}{A}$$

$$F = PA = (1.01 \times 10^4 \text{ Pa})\left(1000 \text{ cm}^2 \times \frac{1 \text{ m}^2}{10^4 \text{ cm}^2}\right) = 1.0 \times 10^3 \text{ N} = 1.0 \text{ kN}$$

The net force on the airplane window has a magnitude of $\boxed{\text{1.0 kN and points outward}}$.

REFLECT

The net force due to a pressure difference always points from high pressure to low pressure.

11.62

SET UP

A rectangular swimming pool has a cross-sectional area of 8 m × 35 m. The depth varies linearly from 1 m in the shallow end to 2 m in the deep end. The pressure at the shallow and deep ends is equal to the atmospheric pressure plus the pressure due to the water at that depth ($=\rho g d$). The net force on the bottom of the pool due to the water has the same magnitude as the weight of the water since the water is at rest.

SOLVE

Part a)

Deep end:

$$P = P_{\text{atm}} + \rho_{\text{water}} g d = (1.01 \times 10^5 \text{ Pa}) + \left(1000 \frac{\text{kg}}{\text{m}^3}\right)\left(9.8 \frac{\text{m}}{\text{s}^2}\right)(2 \text{ m}) = \boxed{1.21 \times 10^5 \text{ Pa}}$$

Shallow end:

$$P = P_{\text{atm}} + \rho_{\text{water}} g d = (1.01 \times 10^5 \text{ Pa}) + \left(1000 \frac{\text{kg}}{\text{m}^3}\right)\left(9.8 \frac{\text{m}}{\text{s}^2}\right)(1 \text{ m}) = \boxed{1.11 \times 10^5 \text{ Pa}}$$

Part b)

$$\sum F = F_{\text{water}\to\text{pool}} = \rho_{\text{water}} V_{\text{water}} g = \left(1000 \frac{\text{kg}}{\text{m}^3}\right)\left(\left(\frac{(2 \text{ m}) + (1 \text{ m})}{2}\right)(35 \text{ m})(8 \text{ m})\right)\left(9.8 \frac{\text{m}}{\text{s}^2}\right)$$

$$= \boxed{4.1 \times 10^6 \text{ N}}$$

REFLECT

The swimming pool is shaped like a trapezoidal prism, which has a volume of
$V = \frac{(b_1 + b_2)}{2} lw = \frac{(d_{\text{deep}} + d_{\text{shallow}})}{2} A$.

11.63

SET UP

A basketball is inflated to a gauge pressure of 8.5 psi. The ball is 0.23 m in diameter. The force that the walls of the basketball must withstand is equal to this pressure difference multiplied by the surface area of the spherical ball. The conversion between psi and pascals is 14.7 psi = 1.01×10^5 Pa.

SOLVE

$$\Delta P = \frac{F}{A}$$

$$F = (\Delta P)A = \left(8.5\text{ psi} \times \frac{1.01 \times 10^5\text{ Pa}}{14.7\text{ psi}}\right)(4\pi)\left(\frac{0.23\text{ m}}{2}\right)^2 = \boxed{9.7 \times 10^3\text{ N} = 9.7\text{ kN}}$$

REFLECT

A pressure of 8.5 psi is about 0.6 atm.

11.64

SET UP

A 55-gal drum is in the ocean at a depth of $d = 250$ m. The cylindrical drum has a diameter of 21.625 in. and a height of 34.5 in. The pressure inside the drum is 1 atm, or 1.01×10^5 Pa. The pressure outside the drum is equal to atmospheric pressure plus the pressure due to the seawater, which we can calculate from density of seawater ($\rho_{\text{seawater}} = 1025\text{ kg/m}^3$) and d. The net force acting on the walls of the drum is equal to the force due to the difference between the outside pressure the inside pressure.

SOLVE

Pressure at a depth of 250 m:

$$P = P_{\text{atm}} + \rho_{\text{seawater}} g d = (1.01 \times 10^5\text{ Pa}) + \left(1025\frac{\text{kg}}{\text{m}^3}\right)\left(9.8\frac{\text{m}}{\text{s}^2}\right)(250\text{ m}) = 2.61 \times 10^6\text{ Pa}$$

Surface area of the walls of the drum:

$$A = 2\pi Rh = 2\pi\left(\frac{21.625\text{ in}}{2} \times \frac{2.54\text{ cm}}{1\text{ in}} \times \frac{1\text{ m}}{100\text{ cm}}\right)\left(34.5\text{ in} \times \frac{2.54\text{ cm}}{1\text{ in}} \times \frac{1\text{ m}}{100\text{ cm}}\right) = 1.51\text{ m}^2$$

Net force on the walls of the drum:

$$\Delta P = \frac{F}{A}$$

$$F = (\Delta P)A = ((2.61 \times 10^6\text{ Pa}) - (1.01 \times 10^5\text{ Pa}))(1.51\text{ m}^2) = 3.79 \times 10^6\text{ N} = 3.79\text{ MN}$$

The net force on the walls of the drum is $\boxed{\text{3.79 MN toward the inside of the drum}}$.

REFLECT
This is a very large force. For comparison, the weight of a 2000-kg passenger car is "only" 0.02 MN.

11.65

SET UP
The hatch on the Mars lander is built and tested on Earth such that the internal pressure of the lander exactly balances the external pressure (that is, atmospheric pressure). The lander is then brought to Mars, where the external pressure is 650 Pa. The net force acting on the hatch is equal to the difference in the internal and external pressures multiplied by the cross-sectional area of the hatch. The hatch is round with a diameter of 0.500 m. Since the internal pressure of the lander is larger than the external pressure on Mars, the net force will point outward.

SOLVE

$$\Delta P = \frac{F}{A}$$

$$F = (\Delta P)A = ((1.01 \times 10^5 \text{ Pa}) - (650 \text{ Pa}))(\pi)\left(\frac{0.500 \text{ m}}{2}\right)^2$$

$$= \boxed{1.97 \times 10^4 \text{ N} \times \frac{0.22 \text{ lb}}{1 \text{ N}} = 4.33 \times 10^3 \text{ lb}}$$

This force points $\boxed{\text{outward}}$.

REFLECT
We can check our answer by looking at the orders of magnitude of each quantity:

$$F = (\Delta P)A \approx (10^5)\frac{(5 \times 10^{-1})^2}{4} = (10^5)\frac{(25)}{4}(10^{-2}) = (10^5)(10^1)(10^{-2}) = 10^4 \text{ N}$$

11.66

SET UP
A force of $F_1 = 150$ N is applied over an area $A_1 = 8 \text{ cm}^2$ of a hydraulic lift. We can use Pascal's principle to calculate the maximum weight that can be raised over an area $A_2 = 750$ cm^2. Setting the pressure at points 1 and 2 equal allows us to solve for F_2.

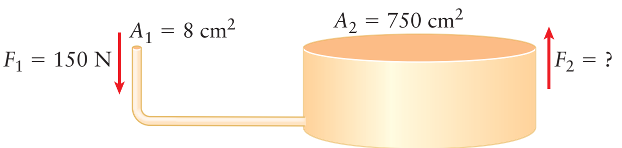


Figure 11-2 Problem 66

SOLVE

$$P_1 = P_2$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$F_2 = F_1\frac{A_2}{A_1} = (150\text{ N})\frac{(750\text{ cm}^2)}{(8\text{ cm}^2)} = \boxed{14{,}100\text{ N}}$$

REFLECT

Area 2 is larger than area 1 by a factor of 100; therefore, force 2 should be larger than force 1 by a factor of 100.

11.67

SET UP

A hydraulic lift is designed to lift a 900-kg car. The radius of the large piston is $r_{\text{large}} = 35$ cm, and the radius of the small piston is $r_{\text{small}} = 2$ cm. We can use Pascal's principle to calculate the minimum force exerted on the small piston F_{small} that will lift the weight of the car. Setting the pressure at the small piston and the large piston equal allows us to solve for F_{small}.

SOLVE

$$P_{\text{small}} = P_{\text{large}}$$

$$\frac{F_{\text{small}}}{A_{\text{small}}} = \frac{F_{\text{large}}}{A_{\text{large}}}$$

$$F_{\text{small}} = F_{\text{large}}\frac{A_{\text{small}}}{A_{\text{large}}} = (900\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)\frac{\pi(2\text{ cm})^2}{\pi(35\text{ cm})^2} = \boxed{28.8\text{ N}}$$

REFLECT

The force applied to the small piston should be much less than the force acting on the large piston, so an applied force of 28.8 N seems reasonable.

11.68

SET UP

A leaky hydraulic lift is only 75% efficient, which means that pressure at the large piston is equal to 75% of the pressure at the small piston. A force $F_{\text{small}} = 15$ N is applied to the small piston, which exerts a force of $F_{\text{large}} = 150$ N on the large piston. The small piston has a radius of $R_{\text{small}} = 0.05$ m. We can use Pascal's principle to calculate the radius of the large piston R_{large}.

SOLVE

$$0.75(P_{\text{small}}) = P_{\text{large}}$$

$$0.75\left(\frac{F_{\text{small}}}{A_{\text{small}}}\right) = \frac{F_{\text{large}}}{A_{\text{large}}}$$

$$0.75\left(\frac{F_{\text{small}}}{\pi R_{\text{small}}^2}\right) = \left(\frac{F_{\text{large}}}{\pi R_{\text{large}}^2}\right)$$

$$R_{\text{large}} = \sqrt{\frac{1}{0.75}\left(\frac{F_{\text{large}}}{F_{\text{small}}}\right)R_{\text{small}}^2} = \sqrt{\frac{1}{0.75}\left(\frac{150\text{ N}}{15\text{ N}}\right)(0.05\text{ m})^2} = \boxed{0.183\text{ m}}$$

REFLECT

If the lift were 100% efficient, the radius would be 0.158 m.

11.69

SET UP

A hydraulic lift is made up of a small piston of area $A_1 = 0.033\text{ m}^2$ and a large piston of area $A_2 = 4\text{ m}^2$. An applied force of $F_1 = 16\text{ N}$ causes the small piston to move downward a distance Δy_1. Pascal's principle lets us calculate the force that the large piston provides. The volume of the fluid moved on the left side must equal the volume of the fluid moved on the right side. We can find Δy_2 in terms of Δy_1 by setting these volumes equal to one another. The work done in moving each piston is equal to the magnitude of the force applied to the piston multiplied by the distance through which the piston travels.

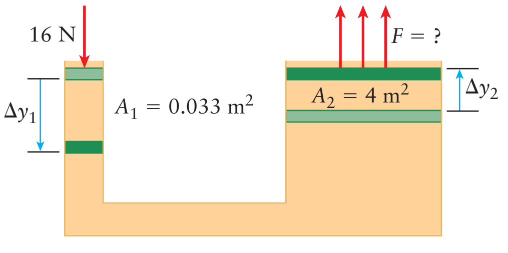


Figure 11-3 Problem 69

SOLVE

Part a)

$$P_1 = P_2$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$F_2 = F_1\frac{A_2}{A_1} = (16\text{ N})\frac{(4\text{ m}^2)}{(0.033\text{ m}^2)} = \boxed{1940\text{ N}}$$

Part b)

$$A_1\Delta y_1 = A_2\Delta y_2$$

$$\Delta y_2 = \frac{A_1}{A_2}\Delta y_1 = \left(\frac{0.033\text{ m}^2}{4\text{ m}^2}\right)\Delta y_1 = \boxed{0.00825\Delta y_1}$$

Part c)

$$W_1 = F_1\Delta y_1 = (16\text{ N})(0.20\text{ m}) = \boxed{3.2\text{ J}}$$

$$W_2 = F_2\Delta y_2 = F_2(0.00825\Delta y_1) = (1940\text{ N})(0.00825)(0.20\text{ m}) = \boxed{3.2\text{ J}}$$

The work done in slowly pushing the small piston is $\boxed{\text{equal}}$ to the work done in raising the large piston.

REFLECT

Since the density and mass of the liquid inside the lift remain constant, the volume must also remain constant.

11.70

SET UP

A rectangular block of wood is 10 cm × 15 cm × 40 cm and has a specific gravity of 0.6. The block is placed into a pool of water and allowed to come to rest. At that point, the only forces acting on the block are the buoyant force pointing up and the force due to gravity pointing down. We can calculate the magnitude of the buoyant force through Newton's second law. The magnitude of the buoyant force is also equal to the density of the fluid displaced (water, in this case) multiplied by the submerged volume of the block $V_{\text{submerged}}$ and by g. We can solve for $V_{\text{submerged}}$ and compare it to the entire volume of the block to determine the fraction of the block that is submerged. Finally, the volume of the water displaced is equal to $V_{\text{submerged}}$; this will let us find the weight of the water displaced by the block.

SOLVE

Part a)

Free-body diagram of the block:

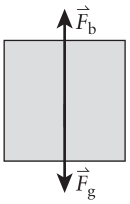


Figure 11-4 Problem 70

Newton's second law:

$$\sum F_y = F_{\text{b}} - F_{\text{g}} = ma_y = 0$$

$$F_{\text{b}} = F_{\text{g}} = mg = \rho_{\text{wood}}Vg = (\text{SG}_{\text{wood}})\rho_{\text{water}}Vg$$

$$= (0.6)\left(1000\frac{\text{kg}}{\text{m}^3}\right)(0.10\text{ m})(0.15\text{ m})(0.40\text{ m})\left(9.8\frac{\text{m}}{\text{s}^2}\right)$$

$$= \boxed{35.3\text{ N}}$$

Part b)

$$F_{\text{b}} = \rho_{\text{water}}V_{\text{submerged}}g$$

$$V_{\text{submerged}} = \frac{F_{\text{b}}}{\rho_{\text{water}}g} = \frac{35.3\text{ N}}{\left(1000\frac{\text{kg}}{\text{m}^3}\right)\left(9.8\frac{\text{m}}{\text{s}^2}\right)} = 0.0036\text{ m}^3$$

$$\frac{V_{\text{submerged}}}{V} = \frac{0.0036 \text{ m}^3}{(0.10 \text{ m})(0.15 \text{ m})(0.40 \text{ m})} = \boxed{0.6}$$

Part c)

$$F_{\text{g, water}} = \rho_{\text{water}} V_{\text{submerged}} g = \left(1000\frac{\text{kg}}{\text{m}^3}\right)(0.0036 \text{ m}^3)\left(9.8\frac{\text{m}}{\text{s}^2}\right) = \boxed{35.3 \text{ N}}$$

REFLECT

In general, for a partially submerged object, the weight of the fluid displaced is equal to the weight of the object.

11.71

SET UP

A block of cross-sectional area A is placed in a pool of freshwater ($SG_{\text{fresh}} = 1.025$). When it comes to rest, the block is submerged a distance $y_{\text{fresh}} = 10$ cm in the water. The same block is then placed into seawater ($SG_{\text{sea}} = 1.025$) and allowed to come to rest. In this case, the block is submerged y_{sea} in the water. In both situations the only forces acting on the block when it comes to rest are the buoyant force pointing up and the force due to gravity pointing down. Since we are using the same block, the force of gravity remains the same. Therefore, the buoyant force in the freshwater is equal to the buoyant force in the seawater. We can then relate the magnitude of each buoyant force to the density of the fluid and the volume displaced by the block.The volume displaced by the block is equal to its cross-sectional area A multiplied by the depth the block is submerged (y_{fresh} or y_{sea}), so we can solve for y_{sea} in terms of y_{fresh} and the specific gravities.

SOLVE

Free-body diagram of the block:

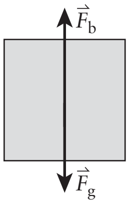


Figure 11-5 Problem 71

Newton's second law:

$$\sum F_y = F_b - F_g = ma_y = 0$$

$$F_b = F_g = mg$$

This means the buoyant force when the block is in freshwater is equal to the buoyant force in seawater:

$$F_{\text{b, fresh}} = F_{\text{b, sea}}$$

$$\rho_{\text{fresh}} V_{\text{fresh}} g = \rho_{\text{sea}} V_{\text{sea}} g$$

$$\rho_{\text{fresh}} A y_{\text{fresh}} = \rho_{\text{sea}} A y_{\text{sea}}$$

$$y_{\text{sea}} = \frac{\rho_{\text{fresh}} y_{\text{fresh}}}{\rho_{\text{sea}}} = \frac{(\text{SG}_{\text{fresh}}) y_{\text{fresh}}}{\text{SG}_{\text{sea}}} = \frac{(1)(10\text{ cm})}{1.025} = \boxed{9.756\text{ cm}}$$

REFLECT

It is often said that seawater is "more buoyant" than freshwater. This means an object will float higher in seawater than freshwater, as we see in this problem.

11.72

SET UP

A cube of side s is completely submerged in water of density ρ_{fluid}. The bottom of the cube is located at a depth of d_{b} and the top of the cube is located at a depth of d_{t}. The difference in pressure between the bottom and top of the cube is proportional to the distance $d_{\text{b}} - d_{\text{t}}$, which is equal to s. When the block is at rest and fully submerged in the fluid, the only forces acting on it are the buoyant force up and the force due to gravity down. The magnitude of the buoyant force is equal to the difference in pressure between the top and bottom face of the cube multiplied by the cross-sectional area of the cube. The weight of the displaced water is equal to the volume of the cube multiplied by $\rho_{\text{fluid}} g$.

SOLVE

Part a)

$$P_{\text{b}} - P_{\text{t}} = (P_{\text{atm}} + \rho_{\text{fluid}} g d_{\text{b}}) - (P_{\text{atm}} + \rho_{\text{fluid}} g d_{\text{t}}) = \rho_{\text{fluid}} g (d_{\text{b}} - d_{\text{t}}) = \boxed{\rho_{\text{fluid}} g s}$$

Part b)

Free-body diagram of the block:

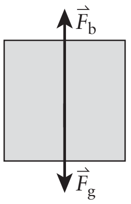


Figure 11-6 Problem 72

Newton's second law:

$$\sum F_y = F_{\text{b}} - F_{\text{g}} = (\Delta P)A - m_{\text{cube}} g = (\rho_{\text{fluid}} g s)(s^2) - m_{\text{cube}} g = \boxed{\rho_{\text{fluid}} g s^3 - m_{\text{cube}} g}$$

Part c)

$$F_{\text{g, water}} = m_{\text{water}} g = \rho_{\text{fluid}} V_{\text{cube}} g = \boxed{\rho_{\text{fluid}} s^3 g}$$

REFLECT

This derivation illustrates Archimedes' principle: The magnitude of the buoyant force (due to the difference in pressure as a function of depth in a fluid) is equal to the weight of fluid displaced by the object.

11.73

SET UP

A crown is weighed in air and in water. In air, the weight of the crown is measured as 5.15 N; we can calculate the mass of the crown from this measurement. In water, the apparent weight is 4.88 N. The apparent weight is equal to the weight of the crown in air minus the magnitude of the buoyant force, which is related to the volume of the crown. From both of these measurements, we can calculate the average density of the crown and compare it to the density of gold, which is 19.3 times the density of water. If the average density of the crown is close to the density of gold, then the crown is most likely made of gold.

SOLVE

$$F_{\mathrm{g}} = m_{\mathrm{crown}} g$$

$$m_{\mathrm{crown}} = \frac{F_{\mathrm{g}}}{g} = \frac{5.15\ \mathrm{N}}{9.8\frac{\mathrm{m}}{\mathrm{s}^2}} = 0.526\ \mathrm{kg}$$

$$W_{\mathrm{app}} = F_{\mathrm{g}} - F_{\mathrm{b}} = F_{\mathrm{g}} - \rho_{\mathrm{water}} g V_{\mathrm{crown}}$$

$$V_{\mathrm{crown}} = \frac{F_{\mathrm{g}} - W_{\mathrm{app}}}{\rho_{\mathrm{water}} g} = \frac{(5.15\ \mathrm{N}) - (4.88\ \mathrm{N})}{\left(1000\frac{\mathrm{kg}}{\mathrm{m}^3}\right)\left(9.8\frac{\mathrm{m}}{\mathrm{s}^2}\right)} = 2.76 \times 10^{-5}\ \mathrm{m}^3$$

$$\rho_{\mathrm{crown}} = \frac{m_{\mathrm{crown}}}{V_{\mathrm{crown}}} = \frac{0.526\ \mathrm{kg}}{2.76 \times 10^{-5}\ \mathrm{m}^3} = 1.91 \times 10^4 \frac{\mathrm{kg}}{\mathrm{m}^3}$$

$$\mathrm{SG}_{\mathrm{crown}} = \frac{\rho_{\mathrm{crown}}}{\rho_{\mathrm{water}}} = \frac{1.91 \times 10^4 \frac{\mathrm{kg}}{\mathrm{m}^3}}{1000\frac{\mathrm{kg}}{\mathrm{m}^3}} = 19.1$$

This is close to the specific gravity of gold, so the $\boxed{\text{crown is most likely made of gold}}$.

REFLECT

The crown could still be fake, though. The density we calculated is the average density of the crown, which means it could be made up of a mixture of less dense and more dense materials.

11.74

SET UP

A raft is 3 m × 4 m × 0.15 m and has an average density of 700 kg/m^3. We want to know the number N of 70-kg people that can stand on the raft before the raft is fully submerged in the water. The forces acting on the raft are the buoyant force pointing up, the force of gravity on the raft pointing down, and the force due to the people on the raft. The force due to the people is equal in magnitude to the weight of one person multiplied by N. We are interested in

the moment when the buoyant force exactly balances out the other two forces. We can set up Newton's second law for the raft and solve for N.

SOLVE

Free-body diagram of the raft:

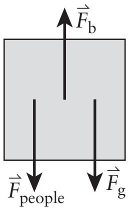


Figure 11-7 Problem 74

Newton's second law:

$$\sum F_y = F_b - F_g - F_{people} = ma_y = 0$$

$$F_b = F_g + F_{people}$$

$$\rho_{water} V_{raft} g = \rho_{raft} V_{raft} g + N m_{person} g$$

$$N = \frac{(\rho_{water} - \rho_{raft}) V_{raft}}{m_{person}} = \frac{\left(\left(1000\frac{kg}{m^3}\right) - \left(700\frac{kg}{m^3}\right)\right)(3\ m)(4\ m)(0.15\ m)}{70\ kg} = 7.7$$

The raft can hold $\boxed{\text{7 people}}$ before it is fully submerged.

REFLECT

You can't have 0.7 of a person, so the maximum number of "whole" people who can stand on the raft is 7. The eighth person will cause the raft to sink.

11.75

SET UP

A wooden buoy ($SG_{wood} = 0.6$) is released from rest at the bottom of a freshwater lake. We can use Newton's second law to calculate the acceleration of the buoy once it is released. As soon as the buoy is let go, the only forces acting on the buoy are the buoyant force pointing up and the force due to gravity pointing down. We will consider up to be the positive y axis. In part (b), we are told that the buoy begins from rest at a depth of 10 m below the surface of the water. Once we know the acceleration of the buoy, we can use the constant acceleration equations to first determine the speed of the buoy as it exits the water and then conservation of mechanical energy to determine the height the buoy reaches above the surface of the water. Since we are ignoring drag (both in the water and in the air), the height we calculate will be the maximum possible height the buoy could attain.

SOLVE

Part a)

Free-body diagram of the buoy:

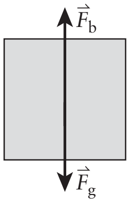


Figure 11-8 Problem 75

Newton's second law:

$$\sum F_y = F_b - F_g = ma_y$$

$$\rho_{water} V_{buoy} g - \rho_{wood} V_{buoy} g = \rho_{wood} V_{buoy} a_y$$

$$V_{buoy} g - \frac{\rho_{wood} V_{buoy}}{\rho_{water}} g = \frac{\rho_{wood}}{\rho_{water}} V_{buoy} a_y$$

$$g - (SG_{wood})g = (SG_{wood})a_y$$

$$a_y = \frac{1-(SG_{wood})}{SG_{wood}} g = \frac{1-0.6}{0.6}\left(9.8\frac{m}{s^2}\right) = \boxed{6.53\frac{m}{s^2}}$$

Part b)

Speed of the buoy when it exits the water:

$$v^2 - v_0^2 = 2a_y \Delta y$$

$$v = \sqrt{v_0^2 + 2a_y\Delta y} = \sqrt{0 + 2\left(6.53\frac{m}{s^2}\right)(10\ m)} = 11.43\frac{m}{s}$$

Maximum height the buoy reaches above the water:

$$K_i = U_f$$

$$\frac{1}{2}mv_2^2 = mgh$$

$$h = \frac{v_2^2}{2g} = \frac{\left(11.43\frac{m}{s}\right)^2}{2\left(9.8\frac{m}{s^2}\right)} = \boxed{6.7\ m}$$

REFLECT

We know that wood floats in water, so we would expect the acceleration of the buoy to point in the positive y direction. Since we've ignored the drag force due to the water, the height the buoy will reach will be less than 6.6 m above the surface of the water.

11.76

SET UP

A woman ($\rho_{\text{woman}} = 985\ \text{kg/m}^3$) is floating in the Great Salt Lake ($\rho_{\text{lake}} = 1130\ \text{kg/m}^3$). We can use Newton's second law to calculate the percentage of her volume that is above the waterline of the lake. Since she is at rest in the vertical direction, which we'll call y, the net force acting on her in that direction is zero. If we write both the magnitude of the buoyant force and the magnitude of the force due to gravity in terms of the density and volume, we can solve for the percentage of the woman's volume that is below the waterline. The volume *above* the water line is equal to 100% minus the percentage below.

SOLVE

Free-body diagram of the woman:

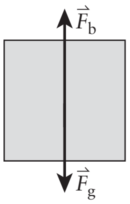


Figure 11-9 Problem 76

Newton's second law:

$$\sum F_y = F_b - F_g = ma_y = 0$$

$$F_b = F_g$$

$$\rho_{\text{lake}} V_{\text{displaced}}\, g = \rho_{\text{woman}} V_{\text{woman}}\, g$$

$$\frac{V_{\text{displaced}}}{V_{\text{woman}}} = \frac{\rho_{\text{woman}}}{\rho_{\text{lake}}} = \frac{985\dfrac{\text{kg}}{\text{m}^3}}{1130\dfrac{\text{kg}}{\text{m}^3}} = 0.87$$

$$\text{Percent above water} = 1 - \left(\frac{V_{\text{displaced}}}{V_{\text{woman}}}\right) = 1 - 0.87 = \boxed{0.13 \text{ or } 13\%}$$

REFLECT

We did not need to use the woman's mass in solving this problem, only her density. In a freshwater lake, only 1.5% of her volume would be above the water line.

11.77

SET UP

A hose with a radius of 1 cm is connected to a faucet and used to fill a 5-L container in 45 s. The volume flow rate is equal to the total volume filled divided by the time it took to do so. The volume flow rate is also equal to the cross-sectional area of the hose multiplied by the speed of the water in the hose.

SOLVE

Part a)

$$Q = \frac{V}{t} = \frac{5\text{ L}}{45\text{ s}} \times \frac{1\text{ m}^3}{1000\text{ L}} = \boxed{1.11 \times 10^{-4}\frac{\text{m}^3}{\text{s}}}$$

Part b)

$$Q = Av = \pi R^2 v$$

$$v = \frac{Q}{\pi R^2} = \frac{1.111 \times 10^{-4}\frac{\text{m}^3}{\text{s}}}{\pi(1 \times 10^{-2}\text{ m}^2)^2} = \boxed{0.354\frac{\text{m}}{\text{s}}}$$

REFLECT

The volume flow rate will be constant since the hose is a closed system.

11.78

SET UP

The volume flow rate of water through a hose is 0.45 m^3/s. The nozzle of the hose has a diameter of 7.5×10^{-3} m. The volume flow rate is equal to the cross-sectional area of the hose multiplied by the speed of the water in the hose.

SOLVE

$$Q = Av = \pi R^2 v = \pi\left(\frac{D}{2}\right)^2 v$$

$$v = \frac{4Q}{\pi D^2} = \frac{4\left(0.45\frac{\text{m}^3}{\text{s}}\right)}{\pi(7.5 \times 10^{-3}\text{ m})^2} = \boxed{1.0 \times 10^4\frac{\text{m}}{\text{s}}}$$

REFLECT

The assumption that there are no leaks means we can apply the equation of continuity.

11.79

SET UP

A 50-L container is filled with water using a hose that has a cross-sectional area of 3 cm^2. The water is flowing at a speed of 0.25 m/s. The volume flow rate is equal to the total volume filled divided by the time it took to do so. The volume flow rate is also equal to the cross-sectional area of the hose multiplied by the speed of the water in the hose. We can solve for the time necessary to fill the container by setting these two definitions equal to one another.

SOLVE

$$Q = \frac{V}{t} = Av$$

$$t = \frac{V}{Av} = \frac{50 \text{ L} \times \dfrac{1 \text{ m}^3}{1000 \text{ L}}}{\left(3 \text{ cm}^2 \times \dfrac{1 \text{ m}^2}{10^4 \text{ cm}^2}\right)\left(0.25 \dfrac{\text{m}}{\text{s}}\right)} = \boxed{670 \text{ s}}$$

REFLECT

A time of 11 minutes seems reasonable to fill a 50-L container with a small hose.

11.80

SET UP

A cylindrical blood vessel is partially blocked by the buildup of plaque. In the unblocked region, the diameter is D_0 and the blood is traveling at a speed of v_0. At one point, the plaque decreases the diameter to $D_1 = 0.4D_0$. Because the blood vessel is a closed system, we can apply the equation of continuity to solve for v_1 in terms of v_0.

SOLVE

$$A_0 v_0 = A_1 v_1$$

$$\pi R_0^2 v_0 = \pi R_1^2 v_1$$

$$\left(\frac{D_0}{2}\right)^2 v_0 = \left(\frac{D_1}{2}\right)^2 v_1$$

$$D_0^2 v_0 = D_1^2 v_1$$

$$v_1 = \frac{D_0^2}{D_1^2} v_0 = \frac{D_0^2}{(0.4D_0)^2} v_0 = \frac{v_0}{0.16} = \boxed{6.25 v_0}$$

REFLECT

We expect the speed of the blood to increase through the blocked region.

11.81

SET UP

The inner diameter of a needle used for an injection is 0.114×10^{-3} m. An injection of 2.5×10^{-3} L was given in 0.65 s. We can set the two expressions for the volume flow rate—the total volume divided by the time and the cross-sectional area of the needle multiplied by the speed of the fluid—equal and solve for the speed of the fluid as it leaves the needle.

SOLVE

$$Q = \frac{V}{t} = Av$$

$$v = \frac{V}{At} = \frac{\left(2.5 \times 10^{-3} \text{ L} \times \dfrac{1 \text{ m}^3}{1000 \text{ L}}\right)}{\pi\left(\dfrac{0.114 \times 10^{-3} \text{ m}}{2}\right)^2 (0.65 \text{ s})} = \boxed{380 \frac{\text{m}}{\text{s}}}$$

REFLECT
This large speed is a reason why most injections are given very slowly.

11.82

SET UP
The gate on a dam broke and a total of 1.35×10^9 gallons of water was lost before the gate was fixed. The maximum volume flow rate through the gate was 40,000 ft^3/s. The time that the gate was open can be estimated by dividing the volume of water lost by the maximum flow rate. The conversion between gallons and cubic feet is 7.48 gal = 1 ft^3.

SOLVE

$$Q = \frac{V}{t}$$

$$t = \frac{V}{Q} = \frac{\left(1.35 \times 10^9 \text{ gal} \times \dfrac{1 \text{ ft}^3}{7.48 \text{ gal}}\right)}{4 \times 10^4 \dfrac{\text{ft}^3}{\text{s}}} = 4512 \text{ s} \times \frac{1 \text{ hr}}{3600 \text{ s}} = \boxed{1.25 \text{ hr}}$$

REFLECT
Since we are given the maximum volume flow rate, the time we calculated is the shortest possible duration that the gate was open.

11.83

SET UP
A ventilation duct has a cross-sectional area of 900 cm^2. This ventilation system recirculates all of the air inside of a 7 m × 10 m × 2.4 m room within 30 min. We can set the two expressions for the volume flow rate—the total volume divided by the time and the cross-sectional area of the duct multiplied by the speed of the air—equal and solve for the speed of the air in the duct.

SOLVE

$$Q = \frac{V}{t} = Av$$

$$v = \frac{V}{At} = \frac{(7 \text{ m})(10 \text{ m})(2.4 \text{ m})}{\left(900 \text{ cm}^2 \times \dfrac{1 \text{ m}^2}{10^4 \text{ cm}^2}\right)\left(30 \text{ min} \times \dfrac{60 \text{ s}}{1 \text{ min}}\right)} = \boxed{1.04 \frac{\text{m}}{\text{s}}}$$

REFLECT
This is around 2.2 mph, which seems like a reasonable speed for a ventilation system.

11.84

SET UP
One end of a fire hose that is 11.7 cm in diameter is attached to a pressurized water tank. A group of firefighters is 5 m above the tank holding the other end, which is attached to a nozzle

that is 2 cm in diameter. We are asked to find the gauge pressure in the tank. We can do this by using the Bernoulli equation—one location will be where the hose connects to the tank and the other location will be the nozzle. The pressure at the nozzle is equal to P_{atm}, so the difference in the pressure at the tank and the nozzle will be equal to the gauge pressure. We are told that the speed of the water at the nozzle is $v_{\text{nozzle}} = 20$ m/s. We can use the continuity equation to calculate the speed of the water as it enters the hose.

SOLVE

Continuity equation:

$$A_{\text{nozzle}}v_{\text{nozzle}} = A_{\text{hose}}v_{\text{hose}}$$

$$v_{\text{hose}} = \frac{A_{\text{nozzle}}}{A_{\text{hose}}}v_{\text{nozzle}} = \frac{\pi\left(\frac{D_{\text{nozzle}}}{2}\right)^2}{\pi\left(\frac{D_{\text{hose}}}{2}\right)^2}v_{\text{nozzle}} = \frac{D^2_{\text{nozzle}}}{D^2_{\text{hose}}}v_{\text{nozzle}} = \frac{(2\text{ cm})^2}{(11.7\text{ cm})^2}\left(20\frac{\text{m}}{\text{s}}\right) = 0.584\frac{\text{m}}{\text{s}}$$

Bernoulli equation:

$$\frac{1}{2}\rho v^2_{\text{hose}} + P_{\text{hose}} + \rho g y_{\text{hose}} = \frac{1}{2}\rho v^2_{\text{nozzle}} + P_{\text{nozzle}} + \rho g y_{\text{nozzle}}$$

$$P_{\text{hose}} - P_{\text{nozzle}} = P_{\text{hose}} - P_{\text{atm}} = P_{\text{gauge}} = \frac{1}{2}\rho(v^2_{\text{nozzle}} - v^2_{\text{hose}}) + \rho g(y_{\text{nozzle}} - y_{\text{hose}})$$

$$= \frac{1}{2}\left(1000\frac{\text{kg}}{\text{m}^3}\right)\left(\left(20\frac{\text{m}}{\text{s}}\right)^2 - \left(0.584\frac{\text{m}}{\text{s}}\right)^2\right) + \left(1000\frac{\text{kg}}{\text{m}^3}\right)\left(9.8\frac{\text{m}}{\text{s}^2}\right)((5\text{ m}) - 0)$$

$$= \boxed{248{,}800\text{ Pa}}$$

REFLECT

This pressure corresponds to a gauge pressure of 2.5 atm, which sounds reasonable for the large tank of a fire truck.

11.85

SET UP

At one point, the maximum wind speed during Hurricane Katrina was 240 km/hr and the pressure in the eye was 0.877 atm. We can use the Bernoulli equation and the measured wind speed to calculate the theoretical pressure inside the eye. We will assume the pressure is 1 atm and that the air is still outside of the eye.

SOLVE

$$\frac{1}{2}\rho v_1^2 + P_1 + \rho g y_1 = \frac{1}{2}\rho v_2^2 + P_2 + \rho g y_2$$

$$0 + P_{\text{atm}} + 0 = \frac{1}{2}\rho v^2_{\text{eye}} + P_{\text{eye}} + 0$$

$$P_{\text{eye}} = P_{\text{atm}} - \frac{1}{2}\rho v_{\text{eye}}^2 = (1.01 \times 10^5 \text{ Pa}) - \frac{1}{2}\left(1.3\frac{\text{kg}}{\text{m}^3}\right)\left(240\frac{\text{km}}{\text{hr}} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ hr}}{3600 \text{ s}}\right)$$

$$= 98{,}111 \text{ Pa} \times \frac{1 \text{ atm}}{1.01 \times 10^5 \text{ Pa}} = \boxed{0.971 \text{ atm}}$$

This is larger than the measured value of 0.877 atm. The pressure outside of the eye is most likely not 1 atm.

REFLECT

The wind is made to go in a circle by the low pressure in the still center, so most of the pressure drop is due to a lower baseline pressure (that is, less than 1 atm), not the Bernoulli effect.

11.86

SET UP

The end of a 1-cm pipe attached to a pressurized tank has a nozzle point straight up attached to it. When the atmospheric pressure is 1 atm, the water from the nozzle rises to a height of 5 m. We can use conservation of mechanical energy to calculate the speed of the water as it exits the nozzle. We can then use Bernoulli's equation to compare the energy of the water at the nozzle to the energy of the water in the tank at the same height. Since the tank is so large, we can say that the speed of the water in the tank (far from the pipe) is approximately zero. This allows us to find the pressure inside the tank. The same tank is used in the middle of a hurricane when the atmospheric pressure is 0.877 atm. Reapplying Bernoulli's equation and the conservation of mechanical energy, we can calculate the new speed of the water out of the nozzle and its new height.

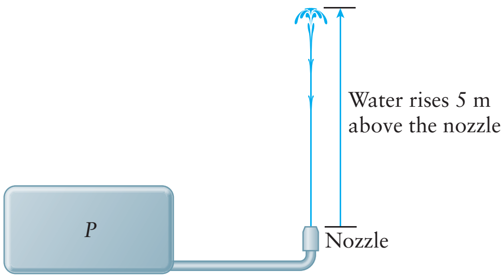


Figure 11-10 Problem 86

SOLVE

Speed of the water as it comes out of the nozzle:

$$K_{\text{i}} = U_{\text{f}}$$

$$\frac{1}{2}\rho v^2 = \rho g h$$

$$v = \sqrt{2gh} = \sqrt{2\left(9.8\frac{\text{m}}{\text{s}^2}\right)(5 \text{ m})} = 9.90\frac{\text{m}}{\text{s}}$$

Pressure inside the tank:

$$\frac{1}{2}\rho v_1^2 + P_1 + \rho g y_1 = \frac{1}{2}\rho v_2^2 + P_2 + \rho g y_2$$

$$0 + P_1 + 0 = \frac{1}{2}\rho v_2^2 + P_{\text{atm}} + 0$$

$$P_1 = \frac{1}{2}\rho v_2^2 + P_{\text{atm}} = \frac{1}{2}\left(1000\frac{\text{kg}}{\text{m}^3}\right)\left(9.90\frac{\text{m}}{\text{s}}\right)^2 + (1.01 \times 10^5\ \text{Pa}) = 1.50 \times 10^5\ \text{Pa}$$

Height of the water stream in a hurricane:

$$\frac{1}{2}\rho v_1^2 + P_1 + \rho g y_1 = \frac{1}{2}\rho v_2^2 + P_2 + \rho g y_2$$

$$0 + P_1 + 0 = \frac{1}{2}\rho v_2^2 + P_{\text{hurricane}} + 0$$

$$v_2 = \sqrt{\frac{2(P_1 - P_{\text{hurricane}})}{\rho}} = \sqrt{\frac{2\left((1.50 \times 10^5\ \text{Pa}) - \left(0.877\ \text{atm} \times \frac{1.01 \times 10^5\ \text{Pa}}{1\ \text{atm}}\right)\right)}{\left(1000\frac{\text{kg}}{\text{m}^3}\right)}} = 11.1\frac{\text{m}}{\text{s}}$$

$$K_{\text{i}} = U_{\text{f}}$$

$$\frac{1}{2}\rho v_2^2 = \rho g h$$

$$h = \frac{v_2^2}{2g} = \frac{\left(11.1\frac{\text{m}}{\text{s}}\right)^2}{2\left(9.8\frac{\text{m}}{\text{s}^2}\right)} = \boxed{6.29\ \text{m}}$$

REFLECT

There is a larger pressure difference between the tank and the atmosphere when the hurricane is coming through, so it makes sense that the speed (and therefore the final height of the stream) should be larger in that case.

11.87

SET UP

A cylinder that is 20 cm tall is filled with water. We will assume that the cylinder is open to the atmosphere. A hole is made in the side of the cylinder 5 cm down from the top and water begins to rush out of it. We can use Bernoulli's equation to compare the energies of the water at the top of the cylinder to the water exiting the hole. Since the cylinder and hole are open to the atmosphere, the pressure at both of these places will be equal to P_{atm}. We can also assume that the cylinder is large enough that the water level does not move; that is, $v_1 = 0$. As the water leaves the cylinder through the hole, its initial velocity will be completely in the horizontal direction. We can use kinematics to calculate the time it takes the stream of water

to fall 15 cm (from the hole to the ground) and then calculate the distance in the x direction that it travels in that time.

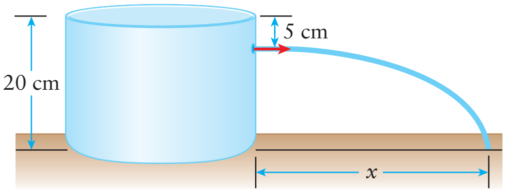


Figure 11-11 Problem 87

SOLVE

Speed of the water leaving the cylinder:

$$\frac{1}{2}\rho v_1^2 + P_1 + \rho g y_1 = \frac{1}{2}\rho v_2^2 + P_2 + \rho g y_2$$

$$0 + P_{\text{atm}} + \rho g y_1 = \frac{1}{2}\rho v_2^2 + P_{\text{atm}} + \rho g y_2$$

$$\rho g y_1 = \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$v_2 = \sqrt{2g(y_1 - y_2)} = \sqrt{2\left(9.8\frac{\text{m}}{\text{s}^2}\right)(0.05\text{ m})} = 0.99\frac{\text{m}}{\text{s}}$$

Kinematics:

$$\Delta y = v_{0,y}t + \frac{1}{2}a_y t^2 = 0 - \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2\Delta y}{g}}$$

$$\Delta x = v_{0,x}t + \frac{1}{2}a_x t^2 = v_{0,x}t + 0 = v_{0,x}\left(\sqrt{\frac{2\Delta y}{g}}\right) = \left(0.99\frac{\text{m}}{\text{s}}\right)\sqrt{\frac{2(0.15\text{ m})}{\left(9.8\frac{\text{m}}{\text{s}^2}\right)}} = \boxed{0.173\text{ m}}$$

REFLECT

A distance of 17 cm seems reasonable.

11.88

SET UP

Water flows laminarly through a horizontal tube that is 0.300-m-long with an inner diameter of 1.5×10^{-3} m at a volume flow rate of 0.500×10^{-3} L/s. We can use Poiseuille's law to calculate the necessary pressure difference to drive this flow. Recall that the viscosity of water at room temperature is $1.00 \times 10^{-3}\text{ Pa}\cdot\text{s}$.

SOLVE

$$Q = \frac{\Delta P \pi r^4}{8\eta L}$$

$$\Delta P = \frac{8\eta L Q}{\pi r^4} = \frac{8(1.0 \times 10^{-3}\ \text{Pa}\cdot\text{s})(0.300\ \text{m})\left(0.500 \times 10^{-3}\frac{\text{L}}{\text{s}} \times \frac{1\ \text{m}^3}{1000\ \text{L}}\right)}{\pi\left(\frac{1.50 \times 10^{-3}\ \text{m}}{2}\right)^4} = \boxed{1210\ \text{Pa}}$$

REFLECT

A volume flow rate of 0.5 mL/s (or 500 μL/s) is reasonably small, so we would expect the pressure difference over the tube to be small as well, since they are directly proportional to one another for a given tube.

11.89

SET UP

Blood takes about 1.50 s to pass through a capillary ($L = 2.00 \times 10^{-3}$ m) in the human circulatory system. The capillary has a radius of $r = 2.50 \times 10^{-6}$ m. The pressure drop across the capillary is $\Delta P = 2.60 \times 10^3$ Pa. We can use Poiseuille's law in order to calculate the viscosity of the blood. The volumetric flow rate is equal to the volume of the blood in the capillary, which is essentially equal to the volume of the capillary, divided by the time it takes the blood to pass through the capillary. For simplicity, we'll assume the capillary is a cylinder.

SOLVE

$$Q = \frac{\Delta P \pi r^4}{8\eta L}$$

$$\eta = \frac{\Delta P \pi r^4}{8LQ} = \frac{\Delta P \pi r^4}{8L\left(\frac{V}{t}\right)} = \frac{\Delta P \pi r^4 t}{8L(\pi r^2 L)} = \frac{\Delta P r^2 t}{8L^2} = \frac{\Delta P\left(\frac{D}{2}\right)^2 t}{8L^2}$$

$$= \frac{\Delta P D^2 t}{32L^2} = \frac{(2.60 \times 10^3\ \text{Pa})(5.00 \times 10^{-6}\ \text{m})^2(1.50\ \text{s})}{32(2.00 \times 10^{-3}\ \text{m})^2} = \boxed{7.62 \times 10^{-4}\frac{\text{N}\cdot\text{s}}{\text{m}^2}}$$

REFLECT

This is a little less than the value listed in Table 11-2 for the viscosity of blood at body temperature $\left(\sim 3 \times 10^{-4}\frac{\text{N}\cdot\text{s}}{\text{m}^2}\right)$.

11.90

SET UP

A large tank is filled to a depth of $d = 0.250$ m with oil ($\rho = 860\ \text{kg/m}^3$, $\eta = 0.180\ \text{Pa}\cdot\text{s}$. A hole is drilled in the bottom of the tank to form a small opening of radius

$r = 0.750 \times 10^{-2}$ m and length $L = 5.00 \times 10^{-2}$ m through which the oil can flow. We can use Poiseuille's equation to calculate the volume flow rate of the oil through the hole. The pressure difference across the length of the hole is equal to $\rho g d$ since the hole is located at a depth of d and is also open to the atmosphere.

SOLVE

$$Q = \frac{\Delta P \pi r^4}{8\eta L} = \frac{(\rho g d)\pi r^4}{8\eta L} = \frac{\left(860\frac{\text{kg}}{\text{m}^3}\right)\left(9.8\frac{\text{m}}{\text{s}^2}\right)(0.250\text{ m})\pi(0.750 \times 10^{-2}\text{ m})^4}{8(0.180\text{ Pa}\cdot\text{s})(5.00 \times 10^{-2}\text{ m})}$$

$$= 2.91 \times 10^{-4}\frac{\text{m}^3}{\text{s}} \times \frac{1000\text{ L}}{1\text{ m}^3} = \boxed{0.291\frac{\text{L}}{\text{s}}}$$

REFLECT

The pressure at the bottom of the tank is equal to $P_{\text{atm}} + \rho g d$, assuming the tank is open to the atmosphere.

11.91

SET UP

A pumping station pumps water $\left(\eta = 0.001\frac{\text{N}\cdot\text{s}}{\text{m}^2}\right)$ through a pipe of length $L = 5.00 \times 10^3$ m and diameter $d = 0.0100$ m. The pressure difference across the pipe is $\Delta P = 520 \times 10^3$ Pa. The volume flow rate of the water exiting the pipe is given by Poiseuille's equation, $Q = \frac{\Delta P \pi r^4}{8\eta L}$.

SOLVE

$$Q = \frac{\Delta P \pi r^4}{8\eta L} = \frac{(520 \times 10^3\text{ Pa})\pi\left(\frac{0.0100\text{ m}}{2}\right)^4}{8\left(0.001\frac{\text{N}\cdot\text{s}}{\text{m}^2}\right)(5.00 \times 10^3\text{ m})} = 2.6 \times 10^{-5}\frac{\text{m}^3}{\text{s}} \times \frac{1000\text{ L}}{1\text{ m}^3} = \boxed{2.6 \times 10^{-2}\frac{\text{L}}{\text{s}}}$$

REFLECT

The vertical rise of the pipe (19 m) is so much smaller than the length of the pipe (5000 m) that we can safely ignore it.

11.92

SET UP

The human body contains about 5.0 L of blood ($\rho_{\text{blood}} = 1060\text{ kg/m}^3$). The mass of the blood is equal to the density of blood multiplied by the total volume of blood. Of this total mass, about 45% of it is blood cells, 1% of which are white and the rest are red. Because of this, we'll assume that all of the blood cells are red blood cells. Furthermore, we'll treat the red blood cells as spheres of diameter $D = 7.5 \times 10^{-6}$ m. We can estimate the total number of blood cells in the blood by first determining the total volume of blood cells from the mass and density and dividing this by the volume of one blood cell.

SOLVE

Part a)

$$\rho_{\text{blood}} = \frac{m_{\text{blood}}}{V_{\text{blood}}}$$

$$m_{\text{blood}} = \rho_{\text{blood}} V_{\text{blood}} = \left(1060 \frac{\text{kg}}{\text{m}^3}\right)\left(5.0 \text{ L} \times \frac{1 \text{ m}^3}{1000 \text{ L}}\right) = \boxed{5.3 \text{ kg}}$$

Part b)

Mass of the blood cells:

$$m_{\text{cells}} = (0.45)m_{\text{blood}} = (0.45)(5.3 \text{ kg}) = 2.39 \text{ kg}$$

Total volume of these cells:

$$V_{\text{cells}} = \frac{m_{\text{cells}}}{\rho_{\text{cells}}} = \frac{2.39 \text{ kg}}{1125 \frac{\text{kg}}{\text{m}^3}} = 0.00212 \text{ m}^3$$

Number of cells:

$$V_{\text{cells}} = N_{\text{cells}} V_{1 \text{ cell}}$$

$$N_{\text{cells}} = \frac{V_{\text{cells}}}{V_{1 \text{ cell}}} = \frac{V_{\text{cells}}}{\left(\frac{4}{3}\pi R^3\right)} = \frac{3V_{\text{cells}}}{4\pi\left(\frac{D}{2}\right)^3} = \frac{6V_{\text{cells}}}{\pi D^3} = \frac{6(0.00212 \text{ m}^3)}{\pi(7.5 \times 10^{-6} \text{ m})^3} = \boxed{9.6 \times 10^{12}}$$

REFLECT

Since 1% of the total number of blood cells is white, there are approximately 9.6×10^{10} white blood cells.

11.93

SET UP

A person won a \$211.8 million prize. We can calculate the equivalent volume of gold by using the cost per ounce (C_{gold} = \$973/troy ounce) and the density of gold ($\rho_{\text{gold}} = 19.3 \text{ g/cm}^3$). (The conversion between troy ounces and grams is 1 troy ounce = 31.1035 g.) Assuming the gold is in the shape of a cube, the height of the cube is equal to the cube root of the volume. We can determine the height of an equivalent cube made of silver by setting up a proportionality between the gold cube and the silver cube: The volume is equal to the mass divided by the density, but the mass is inversely proportional to the cost per ounce.

SOLVE

Part a)

Height of gold cube:

$$V_{\text{gold}} = \$211.8 \times 10^6 \times \frac{1 \text{ troy ounce}}{\$953} \times \frac{31.1035 \text{ g}}{1 \text{ troy ounce}} \times \frac{1 \text{ cm}^3}{19.3 \text{ g}} = 3.58 \times 10^5 \text{ cm}^3$$

$$V_{\text{gold}} = L^3$$

$$L = \sqrt[3]{V_{gold}} = \sqrt[3]{3.58 \times 10^5 \text{ cm}^3} = \boxed{71 \text{ cm}}$$

Weight of gold cube:

$$m_{gold} = \$211.8 \times 10^6 \times \frac{1 \text{ troy ounce}}{\$953} \times \frac{31.1035 \text{ g}}{1 \text{ troy ounce}} \times \frac{1 \text{ kg}}{1000 \text{ g}} = 6913 \text{ kg}$$

$$F_g = m_{gold}g = (6913 \text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right) = \boxed{67{,}744 \text{ N}}$$

There is $\boxed{\text{no way a person can carry this much weight}}$ unaided.

Part b)

$$L^3 = \frac{m}{\rho}, \text{ but } m \propto \frac{1}{C}, \text{ so:}$$

$$\frac{L_{silver}}{L_{gold}} = \sqrt[3]{\frac{C_{gold}\rho_{gold}}{C_{silver}\rho_{silver}}} = \sqrt[3]{\frac{\left(\frac{\$953}{\text{troy ounce}}\right)\left(19.3\frac{\text{g}}{\text{cm}^3}\right)}{\left(\frac{\$14.16}{\text{troy ounce}}\right)\left(10.5\frac{\text{g}}{\text{cm}^3}\right)}} = 4.983$$

$$L_{silver} = (4.983)L_{gold} = (4.983)(71 \text{ cm}) = \boxed{354 \text{ cm} = 3.54 \text{ m}}$$

REFLECT

Silver is worth less than gold, so we need a larger amount of silver in order to reach a worth of \$211.8 million.

11.94

SET UP

We can convert the gas mileage of a car from mi/gal into mi/kg of gasoline through dimensional analysis. The density of gasoline is 737 kg/m^3. We will also need the following conversions: 1 gal = 3.788 L and 1000 L = 1 m^3.

SOLVE

$$50\frac{\text{mi}}{\text{gal}} \times \frac{1 \text{ gal}}{3.788 \text{ L}} \times \frac{1000 \text{ L}}{1 \text{ m}^3} \times \frac{1 \text{ m}^3}{737 \text{ kg}} = \boxed{18\frac{\text{mi}}{\text{kg}}}$$

REFLECT

A typical gas mileage for a fully gasoline-powered passenger car is around 20 mi/gal.

11.95

SET UP

A normal blood pressure is reported as 120/80, where the top number is the systolic pressure and the bottom number is the diastolic pressure. Both of these values are given in mmHg. We can rewrite these pressures in terms of other pressure units by applying the following conversion factors: 760 mmHg = 1.01×10^5 Pa = 1 atm = 14.7 psi. Blood pressure is an

example of a gauge pressure, as 120 mmHg and 80 mmHg refer to the pressures *above* atmospheric pressure.

SOLVE

Part a)

Systolic:

$$120\text{ mmHg} \times \frac{1.01 \times 10^5\text{ Pa}}{760\text{ mmHg}} = 1.59 \times 10^4\text{ Pa}$$

Diastolic:

$$80\text{ mmHg} \times \frac{1.01 \times 10^5\text{ Pa}}{760\text{ mmHg}} = 1.06 \times 10^4\text{ Pa}$$

Blood pressure:

$$\boxed{\frac{1.59 \times 10^4\text{ Pa}}{1.06 \times 10^4\text{ Pa}}}$$

Part b)

Systolic:

$$120\text{ mmHg} \times \frac{1\text{ atm}}{760\text{ mmHg}} = 0.158\text{ atm}$$

Diastolic:

$$80\text{ mmHg} \times \frac{1\text{ atm}}{760\text{ mmHg}} = 0.105\text{ atm}$$

Blood pressure:

$$\boxed{\frac{0.158\text{ atm}}{0.105\text{ atm}}}$$

Part c)

Systolic:

$$120\text{ mmHg} \times \frac{14.7\text{ psi}}{760\text{ mmHg}} = 2.32\text{ psi}$$

Diastolic:

$$80\text{ mmHg} \times \frac{14.7\text{ psi}}{760\text{ mmHg}} = 1.55\text{ psi}$$

Blood pressure:

$$\boxed{\frac{2.32\text{ psi}}{1.55\text{ psi}}}$$

Part d) Blood pressure is reported as a gauge pressure. When you're cut, blood comes out of your arteries; the air doesn't rush in.

REFLECT

If the blood pressures reported were absolute pressures, these would be much smaller than atmospheric pressure. In that case, the pressure difference between the vessels and the outside air would compress all of our blood vessels shut, which luckily does not happen.

11.96

SET UP

There is evidence that there may have been oceans as deep as $d = 500$ m on Mars. We can calculate the pressure an organism would feel on the bottom of the Martian ocean from the atmospheric pressure on Mars ($P_{\text{atm, Mars}} = 650$ Pa), the acceleration due to gravity on Mars ($g_{\text{Mars}} = 0.379g$), the density of seawater ($\rho_{\text{seawater}} = 1025\ \text{kg/m}^3$), and d. We can then use this pressure to calculate the depth in Earth's oceans with the corresponding pressure.

SOLVE

Part a)

$$P = P_{\text{atm, Mars}} + \rho_{\text{seawater}} g_{\text{Mars}} d = (650\ \text{Pa}) + \left(1025\frac{\text{kg}}{\text{m}^3}\right)(0.379)\left(9.8\frac{\text{m}}{\text{s}^2}\right)(500\ \text{m})$$

$$= \boxed{1.90 \times 10^6\ \text{Pa}}$$

Part b)

$$P_{\text{Mars}} = P_{\text{atm}} + \rho_{\text{seawater}} g d$$

$$d = \frac{P_{\text{Mars}} - P_{\text{atm}}}{\rho_{\text{water}} g} = \frac{(1.90 \times 10^6\ \text{Pa}) - (1.01 \times 10^5\ \text{Pa})}{\left(1025\frac{\text{kg}}{\text{m}^3}\right)\left(9.8\frac{\text{m}}{\text{s}^2}\right)} = \boxed{179\ \text{m}}$$

REFLECT

The atmospheric pressure is much larger on Earth and, correspondingly, has a much larger effect on the absolute pressure in the ocean as a function of depth.

11.97

SET UP

A freshwater dam has a length $L = 100$ m and a height $H = 50$ m. The force of the lake acting on the dam is equal to the total pressure of the lake acting over the cross-sectional area of the dam. Since the pressure is a function of depth, we will need to perform an integral over the height of the dam in order to calculate the total pressure. We'll call the bottom of the dam $y = 0$ and the top of the dam is $y = H$, which means the pressure as a function of y is $P(y) = \rho g(H - y)$. The infinitesimal area dA is equal to Ldy.

SOLVE

$$F = \int P(y)dA = \int_0^H \rho g(H - y)(Ldy) = \rho gL\int_0^H (H - y)dy = \rho gL\left[Hy - \frac{y^2}{2}\right]_0^H = \rho gL\left[H^2 - \frac{H^2}{2}\right]$$

$$= \frac{\rho gLH^2}{2} = \frac{\left(1000\frac{\text{kg}}{\text{m}^3}\right)\left(9.8\frac{\text{m}}{\text{s}^2}\right)(100\ \text{m})(50\ \text{m})^2}{2} = \boxed{1.2 \times 10^9\ \text{N} = 1.2\ \text{GN}}$$

REFLECT

For comparison, a pressure on a 100 m × 50 m piece of the lake floor would be

$$F = PA = (P_{\text{atm}} + \rho gH)LH$$

$$= \left((1.01 \times 10^5\ \text{Pa}) + \left(1000\frac{\text{kg}}{\text{m}^3}\right)\left(9.8\frac{\text{m}}{\text{s}^2}\right)(50\ \text{m})\right)(100\ \text{m})(50\ \text{m}) = 3\ \text{GN}$$

11.98

SET UP

A patient's arms are in a cast, so his blood pressure cannot be measured in the usual way. Instead, you have him stand and take the blood pressure at his calf, which is $h = 0.950$ m below his heart. We can determine the increase in pressure (in mmHg) due to the height difference from the density of blood ($\rho_{\text{blood}} = 1060\ \text{kg/m}^3$), g, and h. We can then add on this contribution to his normal blood pressure of 120/80.

SOLVE

Increase in pressure due to height:

$$\Delta P = \rho_{\text{blood}}gh = \left(1060\frac{\text{kg}}{\text{m}^3}\right)\left(9.8\frac{\text{m}}{\text{s}^2}\right)(0.950\ \text{m}) = 9869\ \text{Pa} \times \frac{760\ \text{mmHg}}{1.01 \times 10^5\ \text{Pa}} = 74\ \text{mmHg}$$

New blood pressure:

$$\frac{(120\ \text{mmHg}) + (74\ \text{mmHg})}{(80\ \text{mmHg}) + (74\ \text{mmHg})} = \boxed{\frac{194\ \text{mmHg}}{154\ \text{mmHg}}\ \text{or}\ \frac{194}{154}}$$

REFLECT

Accordingly, if we were to measure the blood pressure at a point above his heart, say, around his neck, we would expect it to be smaller than 120/80.

11.99

SET UP

A syringe has an inner diameter of 0.6×10^{-3} m. A nurse uses this syringe to inject fluid into a patient's artery where the blood pressure is 140/100. The minimum force the nurse needs to apply to the syringe during the injection is equal to the diastolic blood pressure multiplied by the cross-sectional area of the syringe.

SOLVE

$$F = PA = \left(100\text{ mmHg} \times \frac{1.01 \times 10^5\text{ Pa}}{760\text{ mmHg}}\right)(\pi)\left(\frac{0.6 \times 10^{-3}\text{ m}}{2}\right)^2 = \boxed{0.0038\text{ N}}$$

REFLECT

The minimum force required corresponds to the minimum pressure in the artery, which is the diastolic pressure.

11.100

SET UP

A spherical water tank is being filled from the top by a 1-cm-diameter hose. The water in the hose is flowing at a speed of $v_{in} = 0.15$ m/s. All of a sudden, the tank springs a leak at its bottom—a spherical hole that is 0.5 cm in diameter. We want to know the equilibrium level of the water in the tank; this occurs when the volume flow rate of the water into the tank is exactly balanced by the volume flow rate of the water out of the tank. We are given the diameters (and, thus, the cross-sectional areas) of the holes and v_{in}. We can use Bernoulli's equation to calculate the speed of the water through the leak v_{out} by comparing the energy at the top of the water (which is located at a height y relative to the leak) to the energy of the water exiting the leak. Since both locations are open to the atmosphere, the pressures will be the same. We can then find y by plugging the expression we find for v_{out} into the volume flow rate and solving.

SOLVE

Outgoing speed:

$$P_{top} + \frac{1}{2}\rho v_{top}^2 + \rho g y_{top} = P_{out} + \frac{1}{2}\rho v_{out}^2 + \rho g y_{out}$$

$$P_{atm} + \frac{1}{2}\rho(0)^2 + \rho g y = P_{atm} + \frac{1}{2}\rho v_{out}^2 + \rho g(0)$$

$$v_{out} = \sqrt{2gy}$$

Volume flow rates:

$$Q_{in} = A_{in}v_{in} = \pi(5 \times 10^{-3}\text{ m})^2\left(0.15\frac{\text{m}}{\text{s}}\right) = 1.18 \times 10^{-5}\frac{\text{m}^3}{\text{s}}$$

$$Q_{out} = A_{out}v_{out} = \pi(2.5 \times 10^{-3}\text{ m})^2\sqrt{2gy} = (1.96 \times 10^{-5}\text{ m}^2)\sqrt{2\left(9.8\frac{\text{m}}{\text{s}^2}\right)y}$$

$$Q_{in} = Q_{out}$$

$$1.18 \times 10^{-5}\frac{\text{m}^3}{\text{s}} = (1.96 \times 10^{-5}\text{ m}^2)\sqrt{2\left(9.8\frac{\text{m}}{\text{s}^2}\right)y}$$

$$y = \frac{1}{2\left(9.8\frac{\text{m}}{\text{s}^2}\right)}\left(\frac{1.18 \times 10^{-5}\frac{\text{m}^3}{\text{s}}}{1.96 \times 10^{-5}\text{ m}^2}\right)^2 = \boxed{0.0185\text{ m} = 1.85\text{ cm}}$$

REFLECT
We would not expect the tank to fill appreciably with a hole in the bottom.

11.101

SET UP
A large water tank is a height $h = 18$ m above the ground. A pipe is connected to the tank and leads down to the ground. We can use Bernoulli's equation to calculate the speed of the water as it exits the pipe by comparing the energy at the top of the tank (which is located at a height h relative to the ground) to the energy of the water exiting the pipe. Since both locations are open to the atmosphere, the pressures will be the same.

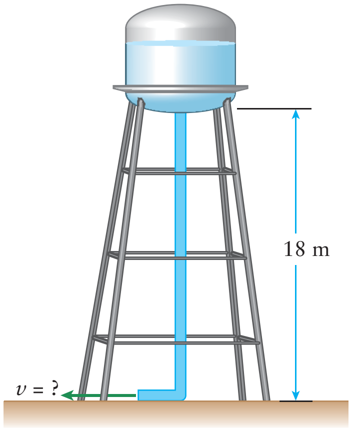


Figure 11-12 Problem 101

SOLVE

$$P_{\text{top}} + \frac{1}{2}\rho v_{\text{top}}^2 + \rho g y_{\text{top}} = P_{\text{bottom}} + \frac{1}{2}\rho v_{\text{bottom}}^2 + \rho g y_{\text{bottom}}$$

$$P_{\text{atm}} + \frac{1}{2}\rho(0)^2 + \rho g h = P_{\text{atm}} + \frac{1}{2}\rho v_{\text{bottom}}^2 + \rho g(0)$$

$$v_{\text{bottom}} = \sqrt{2gh} = \sqrt{2\left(9.8\frac{\text{m}}{\text{s}^2}\right)(18\text{ m})} = \boxed{18.8\frac{\text{m}}{\text{s}}}$$

REFLECT
The diameter of the pipe does not factor into our calculation.

11.102

SET UP
The mean blood pressure in the aorta (cross-sectional area A_1) is $P_1 = 100$ mmHg. The blood flows through the aorta at a speed of $v_1 = 0.60$ m/s. At a point in the aorta, the cross-sectional area $A_2 = (3/4)A_1$. We can use the equation of continuity to calculate the speed of the blood in this constricted region. We can calculate the blood pressure in the constricted region using Bernoulli's equation and the information regarding the unconstricted region. We'll assume the height of these two locations is the same. Recall that the density of blood is 1060 kg/m^3.

SOLVE
Part a)

$$A_1 v_1 = A_2 v_2$$

$$A_1 v_1 = \left(\frac{3}{4}A_1\right)v_2$$

$$v_2 = \frac{4}{3}v_1 = \frac{4}{3}\left(60\frac{\text{cm}}{\text{s}}\right) = \boxed{80\frac{\text{cm}}{\text{s}}}$$

Part b)

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho\left(\frac{4}{3}\right)^2 v_1^2$$

$$P_2 = P_1 + \frac{1}{2}\rho v_1^2\left(1 - \frac{16}{9}\right)$$

$$= \left(100\text{ mmHg} \times \frac{1.01 \times 10^5\text{ Pa}}{760\text{ mmHg}}\right) + \frac{1}{2}\left(1060\frac{\text{kg}}{\text{m}^3}\right)\left(0.60\frac{\text{m}}{\text{s}}\right)^2\left(-\frac{7}{9}\right)$$

$$= 13{,}141\text{ Pa} \times \frac{760\text{ mmHg}}{1.01 \times 10^5\text{ Pa}} = \boxed{98.8\text{ mmHg}}$$

REFLECT

The speed of the blood should increase as it passes through a constriction and the pressure should drop accordingly.

11.103

SET UP

A treasure chest (m_{chest} = 200 kg) that is 0.2 m × 0.4 m × 0.1 m is located 60 m below the surface of the ocean, which has a density of $\rho_{\text{seawater}} = 1025\text{ kg/m}^3$. A diver inflates a spherical buoy with air ($\rho_{\text{seawater}} = 1.217\text{ kg/m}^3$) to a radius of 0.40 m and attaches it to the chest. Since they are attached to one another, the buoy and the chest will move as one. We can use Newton's second law to calculate the initial acceleration of the buoy + chest system. The forces acting on the system are the buoyant force (pointing up toward $+y$) and the force due to gravity (pointing down toward $-y$). As the buoy ascends in the water, the external pressure decreases, which allows the air in the buoy to spread out more. Using the expression for the bulk modulus of air, we can compare the pressures and volumes at the surface of the ocean and the bottom of the ocean in order to calculate the radius of the buoy at the surface; the bulk modulus of air is $B_{\text{air}} = 1.01 \times 10^5$ Pa.

SOLVE

Part a)

Free-body diagram of the buoy + chest:

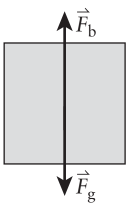


Figure 11-13 Problem 103

Newton's second law:

$$\sum F_y = F_b - F_g = m_{\text{total}} a_y$$

$$(\rho_{\text{seawater}} V_{\text{chest}} g + \rho_{\text{seawater}} V_{\text{buoy}} g) - (m_{\text{chest}} g + \rho_{\text{air}} V_{\text{buoy}} g) = (m_{\text{chest}} + \rho_{\text{air}} V_{\text{buoy}}) a_y$$

$$a_y = \frac{\rho_{\text{seawater}} g (V_{\text{chest}} + V_{\text{buoy}}) - g(m_{\text{chest}} + \rho_{\text{air}} V_{\text{buoy}})}{(m_{\text{chest}} + \rho_{\text{air}} V_{\text{buoy}})}$$

$$= \frac{\left(1025\frac{\text{kg}}{\text{m}^3}\right)\left(9.8\frac{\text{m}}{\text{s}^2}\right)\left((0.20\text{ m})(0.10\text{ m})(0.40\text{ m}) + \frac{4}{3}\pi(0.40\text{ m})^3\right) - \left(9.8\frac{\text{m}}{\text{s}^2}\right)\left((200\text{ kg}) + \left(1.217\frac{\text{kg}}{\text{m}^3}\right)\left(\frac{4}{3}\pi(0.40\text{ m})^3\right)\right)}{(200\text{ kg}) + \left(1.217\frac{\text{kg}}{\text{m}^3}\right)\left(\frac{4}{3}\pi(0.40\text{ m})^3\right)}$$

$$= \boxed{4.04\frac{\text{m}}{\text{s}^2}}$$

Part b)

$$\Delta P = -B_{\text{water}} \frac{\Delta V}{V_{\text{bottom}}}$$

$$\Delta V = -\frac{\Delta P}{B_{\text{water}}} V_{\text{bottom}} = \frac{\rho g d}{B_{\text{water}}}\left(\frac{4}{3}\pi r_{\text{bottom}}^3\right)$$

$$= \frac{4\pi\left(1025\frac{\text{kg}}{\text{m}^3}\right)\left(9.8\frac{\text{m}}{\text{s}^2}\right)(60\text{ m})(0.40\text{ m})^3}{3(1.01\times 10^5\text{ Pa})} = 1.60\text{ m}^3$$

$$\Delta V = V_{\text{final}} - V_{\text{inital}} = \frac{4}{3}\pi R_{\text{top}}^3 - \frac{4}{3}\pi R_{\text{bottom}}^3 = \frac{4}{3}\pi(R_{\text{top}}^3 - R_{\text{bottom}}^3)$$

$$R_{\text{top}} = \sqrt[3]{\frac{3\Delta V}{4\pi} + R_{\text{bottom}}^3} = \sqrt[3]{\frac{3(1.60\text{ m}^3)}{4\pi} + (0.40\text{ m})^3} = \boxed{0.76\text{ m}^3}$$

The radius of the buoy increases as it moves closer to the surface. This increase in the volume of the buoy means the magnitude of the buoyant force (and, to a lesser extent, the weight of the buoy) will also increase. This will cause the $\boxed{\text{acceleration to increase}}$ as the buoy rises.

REFLECT

Since we treat the buoy and chest as one object, we need to be sure to include the contributions from *both* the buoy *and* the chest to the buoyant force and the force due to gravity.

11.104

SET UP

The air around a spinning baseball (mass $m = 0.142$ kg, radius $r = 0.0355$ m) experiences a faster speed on the right than the left side and, hence, a smaller pressure on the right than

the left. The ball is initially traveling at a constant speed of $v_{0,x} = 38$ m/s. After a traveling a distance of 18.44 m in the x direction, the ball has shifted 0.15 m to the right in the $+y$ direction. The pressure difference between the two sides of the ball is equal to the net force acting on the ball in the y direction divided by the cross-sectional area of the ball. The net force in the y direction is equal to the mass of the ball multiplied by the acceleration in the y direction. Assuming the net force on the ball in the y direction is constant, we can calculate the acceleration in the y direction from kinematics by calculating the time it takes the ball to travel 18.44 m in the x direction; during this time, the ball has also moved 0.15 m in the y direction.

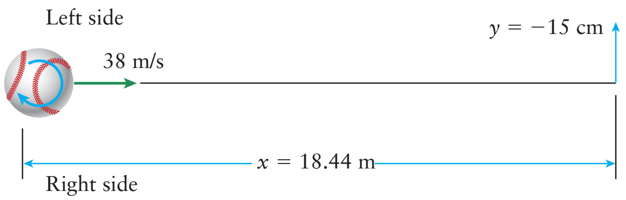


Figure 11-14 Problem 104

SOLVE

Time to travel 18.44 m:

$$v_{0,x} = \frac{\Delta x}{t}$$

$$t = \frac{\Delta x}{v_{0,x}} = \frac{18.44\ \text{m}}{38\frac{\text{m}}{\text{s}}} = 0.485\ \text{s}$$

Acceleration in the y direction:

$$\Delta y = v_{0,y}t + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}a_y t^2$$

$$a_y = \frac{2\Delta y}{t^2} = \frac{2(0.15\ \text{m})}{(0.485\ \text{s})^2} = 1.28\frac{\text{m}}{\text{s}^2}$$

Pressure difference:

$$\Delta P = \frac{F_y}{\pi r^2} = \frac{ma_y}{\pi r^2} = \frac{(0.142\ \text{kg})\left(1.28\frac{\text{m}}{\text{s}^2}\right)}{\pi(0.0355\ \text{m})^2} = \boxed{45.9\ \text{Pa}}$$

REFLECT

The projection of the surface area of the baseball in the y direction looks like a circle of radius r, which has an area of πr^2. This phenomenon is known as the Magnus effect.

11.105

SET UP

A faucet is turned on, and the stream of water falls into the sink. The radius and the speed of the stream as it first leaves the faucet are r_0 and v_0, respectively. We can use the Bernoulli equation to find an expression for the speed of the stream v as a function of y, $v(y) = v_1$,

where y is the distance the faucet is above the point in the stream. Once we have the speed at an arbitrary point, we can use the continuity equation to find the radius of the stream for an arbitrary point, $r(y) = r_1$. The rate at which the radius changes in terms of y is equal to the time derivative of r_1. Since y is a function of t, we will need to use the chain rule. After we find the expressions for r_1 and dr_1/dt, we can plug in $v_0 = 1.5$ m/s, $r_0 = 0.01$ m, to find the radius and its rate of change at $y = 0.10$ m.

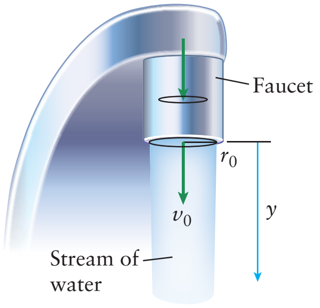


Figure 11-15 Problem 105

SOLVE

Part a)

Bernoulli's equation:

$$P_0 + \frac{1}{2}\rho v_0^2 + \rho g y_0 = P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1$$

$$P_{\text{atm}} + \frac{1}{2}\rho v_0^2 + \rho g y_0 = P_{\text{atm}} + \frac{1}{2}\rho v_1^2 + \rho g y_1$$

$$v_1 = \sqrt{v_0^2 + 2g(y_0 - y_1)} = \sqrt{v_0^2 + 2gy}$$

Continuity equation:

$$A_0 v_0 = A_1 v_1$$

$$(\pi r_0^2)v_0 = (\pi r_1^2)(\sqrt{v_0^2 + 2gy})$$

$$r_1 = \sqrt{\frac{v_0 r_0^2}{\sqrt{v_0^2 + 2gy}}} = \left(\frac{v_0 r_0^2}{\sqrt{v_0^2 + 2gy}}\right)^{\frac{1}{2}}$$

Speed as a function of y:

$$\frac{dr_1}{dt}(y) = \frac{dr_1}{dy}\frac{dy}{dt} = \frac{dr_1}{dy}(v_1) = \frac{d}{dy}\left[\left(\frac{v_0 r_0^2}{\sqrt{v_0^2 + 2gy}}\right)^{\frac{1}{2}}\right]v_1 = \frac{d}{dy}\left[\left(\frac{1}{\sqrt{v_0^2 + 2gy}}\right)^{\frac{1}{2}}\right]v_1\sqrt{v_0 r_0^2}$$

$$= v_1\sqrt{v_0 r_0^2}\frac{d}{dy}[(v_0^2 + 2gy)^{-\frac{1}{4}}] = v_1\sqrt{v_0 r_0^2}\left[\left(-\frac{1}{4}\right)(v_0^2 + 2gy)^{-\frac{5}{4}}(2g)\right]$$

$$= (v_0^2 + 2gy)^{\frac{1}{2}}\sqrt{v_0 r_0^2}\left[\left(-\frac{g}{2}\right)(v_0^2 + 2gy)^{-\frac{5}{4}}\right] = \boxed{\frac{-g\sqrt{v_0 r_0^2}}{2(v_0^2 + 2gy)^{\frac{3}{4}}}}$$

Part b)

Radius:

$$r_1 = \left(\frac{v_0 r_0^2}{\sqrt{v_0^2 + 2gy}}\right)^{\frac{1}{2}} = \left(\frac{\left(1.5\frac{\text{m}}{\text{s}}\right)(0.01\text{ m})^2}{\sqrt{\left(1.5\frac{\text{m}}{\text{s}}\right)^2 + 2\left(9.8\frac{\text{m}}{\text{s}^2}\right)(0.1\text{ m})}}\right)^{\frac{1}{2}} = \boxed{0.00855\text{ m} = 8.55\text{ mm}}$$

Rate of change of the radius:

$$\frac{dr_1}{dt} = \frac{-g\sqrt{v_0 r_0^2}}{2(v_0^2 + 2gy)^{\frac{3}{4}}} = \frac{-\left(9.8\frac{\text{m}}{\text{s}^2}\right)\sqrt{\left(1.5\frac{\text{m}}{\text{s}}\right)(0.01\text{ m})^2}}{2\left(\left(1.5\frac{\text{m}}{\text{s}}\right)^2 + 2\left(9.8\frac{\text{m}}{\text{s}^2}\right)(0.1\text{ m})\right)^{\frac{3}{4}}} = \boxed{-0.020\frac{\text{m}}{\text{s}}}$$

REFLECT

The minus sign in the rate of change of the radius means the water stream is getting narrower the farther it is from the faucet. You can observe this effect in your own kitchen!

11.106

SET UP

The pressure P is constant over the surface of a hemisphere of radius R. The net force due to P in the vertical direction, which we'll call y, and the direction out of the page, which we'll call z, is equal to zero due to symmetry. The net force due to P is nonzero in the horizontal, or x, direction. The magnitude of this component of the force is equal to the integral of the component of the pressure along this axis ($P_x = P\cos(\theta)$) multiplied by dA. Spherical coordinates will be easiest to use due to the symmetry of the situation, which means $dA = R^2\sin(\theta)d\theta d\phi$. Since it is a hemisphere, the integral over θ will be from $\theta = 0$ to $\theta = \pi/2$.

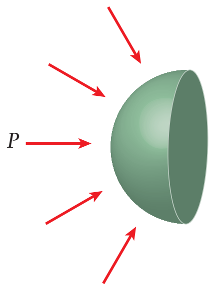


Figure 11-16 Problem 106

SOLVE

$$F_x = \int P\cos(\theta)dA = \iint P\cos(\theta)(R^2\sin(\theta)d\theta d\phi) = PR^2\int_0^{\frac{\pi}{2}}\cos(\theta)\sin(\theta)d\theta\int_0^{2\pi}d\phi$$

$$= PR^2[\phi]_0^{2\pi}\int_0^{\frac{\pi}{2}}\cos(\theta)\sin(\theta)d\theta$$

Letting $u = \sin(\theta)$ and $du = \cos(\theta)d\theta$, the integral becomes

$$F_x = PR^2[2\pi]\int_0^1 udu = 2\pi PR^2\left[\frac{u^2}{2}\right]_0^1 = 2\pi PR^2\left[\frac{1}{2}\right] = \pi PR^2$$

The net force has a magnitude of $\boxed{\pi \text{PR}^2 \text{ and points toward the right}}$.

REFLECT

The magnitude of the force is equal to the pressure P multiplied by the projection of the surface area onto the yz plane (that is, perpendicular to the x axis), which is equal to πR^2.

11.107

SET UP

An equilateral prism made of wood ($SG_{wood} = 0.6$) with sides s and length L is placed into a pool of water. In the first case the prism is pointed down, and in the second case the prism is pointed up. We are asked to find the ratio between the depths of submersion for the two cases, y_d/y_u. When the prism comes to rest, the buoyant force will exactly equal the force due to gravity. The volume underwater when the prism is pointed down is a smaller equilateral prism with sides x_d, a height y_d, and a length L. The volume underwater when the prism is pointed up is a trapezoidal prism, but it is easier mathematically to subtract the volume of the smaller equilateral prism above the water from the total volume of the original prism. The equilateral prism above the water has sides x_u, height $s\cos(30°) - y_u$, and length L. In both cases we can apply geometry and algebra to find a simplified expression for the depth in terms of s.

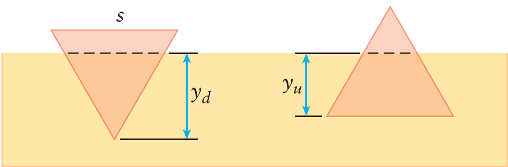


Figure 11-17 Problem 107

SOLVE

Pointed down:

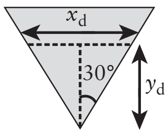


Figure 11-18 Problem 107

$$F_b = F_g$$

$$\rho_{water} V_d g = \rho_{wood} V_{prism} g$$

$$\rho_{water}\left(\frac{1}{2}x_d y_d L\right) = \rho_{wood}\left(\frac{1}{2}s(s\cos(30°))L\right)$$

$$x_d y_d = \left(\frac{\rho_{wood}}{\rho_{water}}\right) s^2 \cos(30°)$$

$$(2y_d \tan(30°))y_d = (SG_{wood})s^2 \cos(30°)$$

$$y_d = \sqrt{\frac{(SG_{wood})s^2\cos(30°)}{2\tan(30°)}} = s\sqrt{\frac{(0.6)\cos(30°)}{2\tan(30°)}} = 0.671s$$

Pointing up:

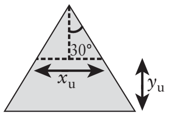


Figure 11-19 Problem 107

$$F_b = F_g$$

$$\rho_{water} V_u g = \rho_{wood} V_{prism} g$$

$$\rho_{water}\left(\frac{1}{2}s^2\cos(30°)L - \frac{1}{2}x_u(s\cos(30°) - y_u)L\right) = \rho_{wood}\left(\frac{1}{2}s^2\cos(30°)L\right)$$

$$\rho_{water}(s^2\cos(30°) - (2(s\cos(30°) - y_u)\tan(30°))(s\cos(30°) - y_u)) = \rho_{wood}s^2\cos(30°)$$

$$s^2\cos(30°) - 2(s\cos(30°) - y_u)^2\tan(30°) = \left(\frac{\rho_{wood}}{\rho_{water}}\right)s^2\cos(30°)$$

$$s^2\cos(30°) - (SG_{wood})s^2\cos(30°) = 2(s\cos(30°) - y_u)^2\tan(30°)$$

$$\frac{(1 - (SG_{wood}))s^2\cos(30°)}{2\tan(30°)} = (s\cos(30°) - y_u)^2$$

$$y_u = s\cos(30°) - s\sqrt{\frac{(1 - (SG_{wood}))\cos(30°)}{2\tan(30°)}} = s\left(\cos(30°) - \sqrt{\frac{(1 - 0.6)\cos(30°)}{2\tan(30°)}}\right)$$

$$= 0.318s$$

Ratio:

$$\frac{y_d}{y_u} = \frac{0.671s}{0.318s} = \boxed{2.11}$$

REFLECT

The buoyant force is the same whether the prism is pointed down or up, which means the volume of the prism underwater must be the same in both cases. This volume is equal to the cross-sectional area of the portion underwater multiplied by the length L. Therefore, the area of the triangle underwater (prism pointing down) must equal the area of the trapezoid underwater (prism pointing up). With this in mind, it makes sense that y_d should be larger than y_u.

Chapter 12
Oscillations

Conceptual Questions

12.1 Oscillatory motion not only includes simple harmonic motion, but it also includes circular motion, decaying oscillations, and oscillations that have shapes other than pure sine waves.

12.2 Simple harmonic motion also requires that the displacement point be in a direction that is opposite to the force.

12.3 Examples of simple harmonic motion include a child on a swing and any resonant musical instrument.

12.4 A simple pendulum is made from a long, thin string that is tied to a small mass. We can treat it as a point mass attached to the end of a string. A physical pendulum has a massive connecting structure from the pivot point to the end; that is, there is a continuously varying mass spread over the length of the pendulum.

12.5 Breathing rate (breaths per minute) is a frequency. The period is its reciprocal.

12.6 Sine is essentially the same mathematical function as cosine; it is just shifted to a different starting point.

12.7 Part a) The SI units for ω are radians/second.

Part b) The SI units for ωt are radians.

12.8 The time it takes a pendulum to repeat its motion is the same regardless of the initial starting angle. This only works if the angle is "small" (less than 15 degrees) so that the small angle approximation is valid.

12.9 It is an offset of where in the oscillation cycle we choose to set the zero time. If, for example, the phase angle is π rather than zero, the solution starts at $-A$ instead of $+A$. In either case, however, the initial velocity is zero and the motion is the same.

12.10 (1) The mass is concentrated at the bob for a simple pendulum, whereas it is spread over the entire length for the physical pendulum. (2) The simple pendulum has a period that is larger than the period of a physical pendulum of the same length. (3) The mass of the string can be neglected with a simple pendulum.

12.11 The force of gravity depends on elevation and determines the period of the pendulum. Measuring the period and length of the pendulum will yield enough information

to estimate the strength of gravity; doing so very precisely would enable useful comparison to established values.

12.12 For a mass on a spring, b has SI units of kg/s.

12.13 The amplitude will increase over time until whatever damping there is in the system is sufficient to counteract the driving force.

12.14 The basic case of simple harmonic motion occurs when $b = 0$ and $F(t) = $ constant (especially when this is zero).

12.15 The frequency of the driving force is just how often the applied force repeats. That depends on things outside the oscillator. The natural frequency of the oscillator is the frequency at which it oscillates most readily or the frequency at which it will oscillate if displaced from equilibrium and released.

Multiple-Choice Questions

12.16 **D** (2).

$$\omega_1 = \sqrt{\frac{k}{m_1}}$$

$$\omega_2 = \sqrt{\frac{k}{m_2}} = \sqrt{\frac{k}{\left(\frac{m_1}{4}\right)}} = 2\sqrt{\frac{k}{m_1}} = 2\omega_1$$

12.17 **B** ($x = 0$). The kinetic energy of the block is a maximum when the spring is at its equilibrium position.

12.18 **A** ($x = A$ and $x = -A$). The maximum acceleration occurs when the net horizontal force on the block is the largest.

12.19 **E** ($4A$). The object travels from $x = -A$ through $x = 0$ to $x = +A$ where it turns around and returns to $x = -A$.

12.20 **D** ($3T/2$). In one full period T the object travels a distance of $4A$. To reach a total distance of $6A$, an additional half period is needed.

12.21 **A** (The period will increase).

$$T_1 = \frac{2\pi}{\omega_1} = 2\pi\sqrt{\frac{m_1}{k}}$$

$$T_2 = \frac{2\pi}{\omega_2} = 2\pi\sqrt{\frac{m_2}{k}} = 2\pi\sqrt{\frac{(2m_1)}{k}} = \sqrt{2}\left(2\pi\sqrt{\frac{m_1}{k}}\right) = \sqrt{2}T_1$$

12.22 **A** (increases). The total energy of an object–spring system is proportional to the square of the amplitude.

12.23 **D** (All of the above).

$$\frac{1}{2}mv_{\max}^2 = \frac{1}{2}kA^2$$

$$v_{\max} = A\sqrt{\frac{k}{m}}$$

12.24 **C** ($2T$).

$$T_1 = 2\pi\sqrt{\frac{L_1}{g}}$$

$$T_2 = 2\pi\sqrt{\frac{L_2}{g}} = 2\pi\sqrt{\frac{(4L_1)}{g}} = 2\left(2\pi\sqrt{\frac{L_1}{g}}\right) = 2T_1$$

12.25 **B** $\left(2\pi\sqrt{\frac{2L}{3g}}\right)$.

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{\left(\frac{1}{3}ML^2\right)}{Mg\left(\frac{L}{2}\right)}} = 2\pi\sqrt{\frac{2L}{3g}}$$

Estimation Questions

12.26 Part a) On the low setting, the windshield wipers complete about 45 cycles in a minute, or $T_{\text{low}} = \frac{1}{f} = \frac{1}{\left(45\frac{\text{cycles}}{\text{min}}\right)} = 0.022 \text{ min} \times \frac{60 \text{ s}}{1 \text{ min}} = 1.33 \text{ s}.$

Part b) On the high setting, the windshield wipers complete about 60 cycles in a minute, or $T_{\text{low}} = \frac{1}{f} = \frac{1}{\left(60\frac{\text{cycles}}{\text{min}}\right)} = 0.017 \text{ min} \times \frac{60 \text{ s}}{1 \text{ min}} = 1 \text{ s}.$

12.27 Swings are typically around 2 m in length, so we get a period of around 3 s.

12.28 An average person blinks about 15 times in a minute, so
$T = \frac{1}{f} = \frac{1}{\left(15\frac{\text{blinks}}{\text{min}}\right)} = 0.067 \text{ min} \times \frac{60 \text{ s}}{1 \text{ min}} = 4 \text{ s}.$

12.29 The frequency of a hummingbird's wings is about 50 Hz.

12.30 A panting dog takes about 40 breaths per minute, or 0.67 Hz.

12.31 A shoelace is about a meter long, so the period of a simple pendulum of $L = 1$ m is

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{1\text{ m}}{10\frac{\text{m}}{\text{s}^2}}} \approx 2\text{ s.}$$

12.32 Part a) A photocopier can make about 30 one-sided copies per minute, which corresponds to a frequency of 0.5 Hz.

Part b) A photocopier can make about 20 double-sided copies per minute, which corresponds to a frequency of 0.33 Hz.

12.33 A 0.01-g fly clinging to the bottom of a pendulum that has a disk bob that is 1 kg, has a radius of 10 cm, and is centered at 1 m from the axis would produce a frequency reduction of around 1 part in 4×10^7.

12.34 The maximum potential energy (and kinetic energy) associated with the pendulum of a ticking grandfather clock is about 0.15 J.

12.35 We can model the broom as a long rod of mass M and length $L = 1$ m pivoting about one end. The center of mass of the rod is located about $2L/3$ from the pivot:

$$T = 2\pi\sqrt{\frac{I}{Mgh}} = 2\pi\sqrt{\frac{\left(\frac{ML^2}{3}\right)}{Mg\left(\frac{2L}{3}\right)}} = 2\pi\sqrt{\frac{L}{2g}} = 2\pi\sqrt{\frac{1\text{ m}}{2\left(10\frac{\text{m}}{\text{s}^2}\right)}} = 1.4\text{ s}$$

12.36

t (s)	F (N)
0	−20
0.1	−10
0.2	0
0.3	10
0.4	20
0.5	10
0.6	0
0.7	−10
0.8	−20
0.9	−10
1	0
1.1	10
1.2	20
1.3	10
1.4	0
1.5	−10

1.6	−20
1.7	−10
1.8	0
1.9	10
2	20
2.1	10
2.2	0
2.3	−10
2.4	−20
2.5	−10

Part a)

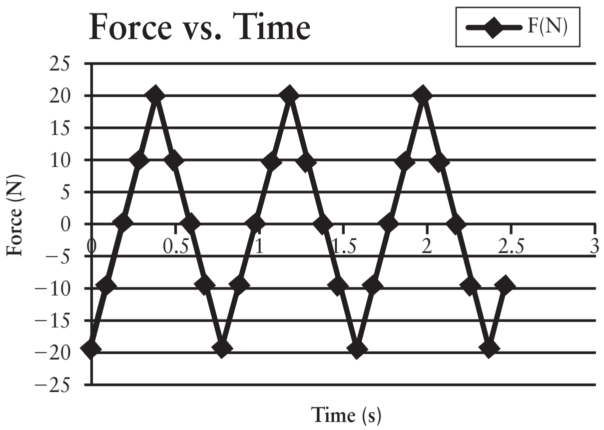


Figure 12-1 Problem 36

The plot of force versus time looks sinusoidal, so the force does obey simple harmonic motion.

Part b) The motion repeats every 0.8 s.

12.37 Part a)

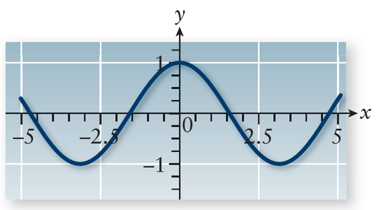


Figure 12-2 Problem 37

Part b) The graph is shifted 30 degrees ($\pi/6$ radians) to the left.

12.38 Part a) $e^{-1} = 0.368$

Part b) $e^{-2} = 0.135$

Part c) $e^{-3} = 0.0498$

Part d) $e^{-0.5} = 0.607$

Part e) $e^{-0.25} = 0.779$

12.39 On average, the period looks to be about 4 years.

12.40 A simple pendulum that has a length of $L = 1.25$ m and a bob with a mass $m = 4$ kg is pulled an angle θ with respect to the vertical. The potential energy of the bob as a function of θ is $U = mgL(1 - \cos(\theta))$. We can plug in the initial angle to find the maximum potential energy. The maximum speed occurs at the maximum kinetic energy, where $U_{max} = K_{max}$.

Angle (deg)	U_{max} (J)	v_{max} (m/s)
5	0.186459794	0.305335712
10	0.744420102	0.610090199
15	1.669634512	0.913683346
20	2.955061581	1.215537244
25	4.590918435	1.515077298

According to the small angle approximation, $\cos(\theta) \approx 1 - \frac{\theta^2}{2}$, which means $U_{max} \approx mgL\frac{\theta^2}{2}$. The maximum kinetic energy is still equal to the maximum potential energy.

Angle (deg)	Small Angle U_{max}	Small Angle v_{max}
5	0.18657817	0.305432619
10	0.746312678	0.610865238
15	1.679203527	0.916297857
20	2.985250714	1.221730476
25	4.66445424	1.527163095

The values for U_{max} and v_{max} are approximately the same for angles less than 15 degrees.

Problems

12.41

SET UP

We are given the second-order differential equation that describes simple harmonic motion as well as a list of functions that are possible solutions to this differential equation. We can plug each function into the differential equation and check which ones satisfy the equation.

SOLVE

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

Part a)

$$x(t) = A\cos(\omega t)$$

$$\frac{d^2}{dt^2}[A\cos(\omega t)] + \omega^2[A\cos(\omega t)] \stackrel{?}{=} 0$$

$$A[-\omega^2\cos(\omega t)] + \omega^2[A\cos(\omega t)] = 0$$

$\boxed{\text{Yes}}$, $x(t) = A\cos(\omega t)$ satisfies the differential equation.

Part b)

$$x(t) = A\sin(\omega t)$$

$$\frac{d^2}{dt^2}[A\sin(\omega t)] + \omega^2[A\sin(\omega t)] \stackrel{?}{=} 0$$

$$A[-\omega^2\sin(\omega t)] + \omega^2[A\sin(\omega t)] = 0$$

$\boxed{\text{Yes}}$, $x(t) = A\sin(\omega t)$ satisfies the differential equation.

Part c)

$$x(t) = A\cos(\omega t) + A\sin(\omega t)$$

$$\frac{d^2}{dt^2}[A\cos(\omega t) + A\sin(\omega t)] + \omega^2[A\cos(\omega t) + A\sin(\omega t)] \stackrel{?}{=} 0$$

$$A[-\omega^2\cos(\omega t) - \omega^2\sin(\omega t)] + \omega^2[A\cos(\omega t) + A\sin(\omega t)] = 0$$

$\boxed{\text{Yes}}$, $x(t) = A\cos(\omega t) + A\sin(\omega t)$ satisfies the differential equation.

Part d)

$$x(t) = Ae^{\omega t}$$

$$\frac{d^2}{dt^2}[Ae^{\omega t}] + \omega^2[Ae^{\omega t}] \stackrel{?}{=} 0$$

$$A[\omega^2 e^{\omega t}] + \omega^2[Ae^{\omega t}] \neq 0$$

$\boxed{\text{No}}$, $x(t) = Ae^{\omega t}$ does not satisfy the differential equation.

Part e)

$$x(t) = Ae^{\omega t} + Ae^{-\omega t}$$

$$\frac{d^2}{dt^2}[Ae^{\omega t} + Ae^{-\omega t}] + \omega^2[Ae^{\omega t} + Ae^{-\omega t}] \stackrel{?}{=} 0$$

$$A[\omega^2 e^{\omega t} + \omega^2 e^{-\omega t}] + \omega^2[Ae^{\omega t} + Ae^{-\omega t}] \neq 0$$

$\boxed{\text{No}}$, $x(t) = Ae^{\omega t} + Ae^{-\omega t}$ does not satisfy the differential equation.

Part f)

$$x(t) = Ae^{i\omega t}$$

$$\frac{d^2}{dt^2}[Ae^{i\omega t}] + \omega^2[Ae^{i\omega t}] \stackrel{?}{=} 0$$

$$A[-\omega^2 e^{i\omega t}] + \omega^2[Ae^{i\omega t}] = 0$$

$\boxed{\text{Yes}}$, $x(t) = Ae^{i\omega t}$ satisfies the differential equation.

Part g)

$$x(t) = Ae^{i\omega t} + Ae^{-i\omega t}$$

$$\frac{d^2}{dt^2}[Ae^{i\omega t} + Ae^{-i\omega t}] + \omega^2[Ae^{i\omega t} + Ae^{-i\omega t}] \stackrel{?}{=} 0$$

$$A[-\omega^2 e^{i\omega t} - \omega^2 e^{-i\omega t}] + \omega^2[Ae^{i\omega t} + Ae^{-i\omega t}] = 0$$

$\boxed{\text{Yes}}$, $x(t) = Ae^{i\omega t} + Ae^{-i\omega t}$ satisfies the differential equation.

REFLECT

A linear combination of two solutions of a given differential equation is *also* a solution to that differential equation.

12.42

SET UP

A mass M is attached to a spring with a spring constant k. We can use Hooke's law and Newton's second law to derive the general formula for simple harmonic motion, $\frac{d^2x}{dt^2} + \omega^2 x = 0$. Recall that the acceleration is the second derivative of the position with respect to time.

SOLVE

$$\sum F_x = ma_x$$

$$-kx = m\frac{d^2x}{dt^2}$$

$$\boxed{\frac{d^2x}{dt^2} + \omega^2 x = 0, \text{ where } \omega^2 = \frac{k}{m}}$$

REFLECT

In simple harmonic motion, the restoring force is the only force acting on the object along the direction of its motion.

12.43

SET UP

An object of mass M is attached to a spring with spring constant k and hung vertically. When the mass is initially hung on the spring, the spring will stretch, and the system will eventually come to rest at a new equilibrium position, which we'll call y_0. When the mass is slightly displaced, it will oscillate about this new equilibrium position. If we displace the mass downward a distance $(y - y_0)$, the forces acting on it are the spring pulling it back up and gravity pulling it down. We can rearrange Newton's second law for the mass and compare it with the standard differential equation for simple harmonic motion to find an expression for the motion of the mass as a function of time.

SOLVE

Newton's second law:

$$\sum F_y = Mg - k(y - y_0) = Ma = M\frac{d^2y}{dt^2}$$

$$\frac{d^2y}{dt^2} + \frac{k}{M}(y - y_0) - g = 0$$

$$\frac{d^2y}{dt^2} + \frac{k}{M}\left(y - y_0 - \frac{Mg}{k}\right) = 0$$

Change of variables; let $u = y - y_0 - \frac{Mg}{k}$:

$$\frac{d^2u}{dt^2} + \frac{k}{M}u = 0$$

$$\frac{d^2u}{dt^2} + \omega^2 u = 0$$

$$u = A\cos(\omega t) = y - y_0 - \frac{Mg}{k}$$

$$y = A\cos(\omega t) + \left(y_0 + \frac{Mg}{k}\right)$$

REFLECT

The force of gravity effectively changes the equilibrium position of a spring–mass system undergoing simple harmonic motion in the vertical direction. Gravity has no effect on the frequency of the oscillation.

12.44

SET UP

Your friend suggests that the solutions to the simple harmonic motion differential equation can be multiplied together to yield another solution. You can explicitly test this by using a trial function of $x(t) = [A_1\cos(\omega t)][A_2\sin(\omega t)]$ and plugging it into the differential equation to see if it works.

SOLVE

$$x(t) = [A_1\cos(\omega t)][A_2\sin(\omega t)] = A_1A_2\sin(\omega t)\cos(\omega t)$$

$$\frac{d^2}{dt^2}[A_1A_2\sin(\omega t)\cos(\omega t)] + \omega^2[A_1A_2\sin(\omega t)\cos(\omega t)] \stackrel{?}{=} 0$$

$$A_1A_2\frac{d}{dt}[\omega\cos(\omega t)\cos(\omega t) - \omega\sin(\omega t)\sin(\omega t)] + A_1A_2\omega^2[\sin(\omega t)\cos(\omega t)] \stackrel{?}{=} 0$$

$$A_1A_2\omega\frac{d}{dt}[\cos^2(\omega t) - \sin^2(\omega t)] + A_1A_2\omega^2[\sin(\omega t)\cos(\omega t)] \stackrel{?}{=} 0$$

$$A_1A_2\omega[-2\omega\cos(\omega t)\sin(\omega t) - 2\omega\sin(\omega t)\cos(\omega t)] + A_1A_2\omega^2[\sin(\omega t)\cos(\omega t)] \stackrel{?}{=} 0$$

$$A_1A_2\omega^2[-4\cos(\omega t)\sin(\omega t)] + A_1A_2\omega^2[\sin(\omega t)\cos(\omega t)] \neq 0$$

REFLECT

Although $x(t) = A_1A_2\sin(\omega t)\cos(\omega t) = \frac{A_1A_2}{2}\sin(2\omega t)$ looks like a solution, the amplitude and frequency of this function are not the same as the amplitude and frequency associated with simple harmonic motion.

12.45

SET UP

We are given a function—$x(t) = Ae^{i\omega t}$—and asked if it satisfies simple harmonic motion. We can plug it directly into the second-order differential equation that describes simple harmonic motion to check whether or not it is a solution. The function $x(t)$ has dimensions of length, which means A must also have dimensions of length.

SOLVE

Part a)

$$x(t) = Ae^{i\omega t}$$

$$\frac{d^2}{dt^2}[Ae^{i\omega t}] + \omega^2[Ae^{i\omega t}] \stackrel{?}{=} 0$$

$$A[-\omega^2e^{i\omega t}] + \omega^2[Ae^{i\omega t}] = 0$$

Part b) The SI units of A are meters.

REFLECT

Not only is $x(t) = Ae^{i\omega t}$ a solution to the simple harmonic motion differential equation, but also $x(t) = Ae^{i\omega t} + Ae^{-i\omega t}$.

12.46

SET UP

We are given a function—$x(t) = A\cos(\alpha t^2)$—and asked if it satisfies simple harmonic motion. We can plug it directly into the second-order differential equation that describes simple harmonic motion to check whether or not it is a solution.

SOLVE

$$x(t) = A\cos(\alpha t^2)$$

$$\frac{d^2}{dt^2}[A\cos(\alpha t^2)] + \omega^2[A\cos(\alpha t^2)] \stackrel{?}{=} 0$$

$$A\frac{d}{dt}[-\sin(\alpha t^2)(2\alpha t)] + A\omega^2\cos(\alpha t^2) \stackrel{?}{=} 0$$

$$-A(2\alpha)[\sin(\alpha t^2) + t\cos(\alpha t^2)(2\alpha t)] + A\omega^2\cos(\alpha t^2) \neq 0$$

$\boxed{\text{No}}$, $x(t) = A\cos(\alpha t^2)$ does not satisfy simple harmonic motion.

REFLECT

We had to invoke both the chain rule and the product rule in order to solve this problem.

12.47

SET UP

We are asked to plot three different functions:

$x_1(t) = A\cos(2\pi t)$, $x_2(t) = A\cos\left(2\pi t + \dfrac{\pi}{2}\right)$, $x_3(t) = A\cos\left(2\pi t + \dfrac{\pi}{4}\right)$. All three will have the same amplitude and frequency but will be shifted along the time axis with respect to one another due to the differences in phase.

SOLVE

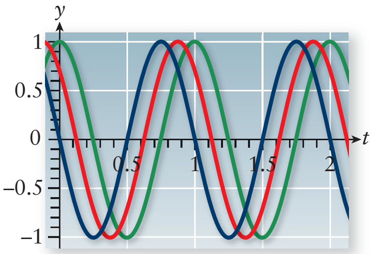


Figure 12-3 Problem 47

x_1 is green, x_2 is blue, x_3 is red. In the plot, A is assumed to be 1.

REFLECT

The phase of a wave affects its starting position.

12.48

SET UP

A simple harmonic oscillator is observed to start its oscillations at the maximum amplitude at $t = 0$. The mathematical function that would best describe the motion of this oscillator would be the cosine since it starts at its maximum value at $t = 0$. If the oscillations start at the equilibrium position of $x = 0$ at $t = 0$, we should use the sine function. We will assume the oscillations have anamplitude of A and an angular frequency of ω.

SOLVE

At maximum amplitude at $t = 0$:

$$x(t) = A\cos(\omega t)$$

At equilibrium at $t = 0$:

$$x(t) = A\sin(\omega t)$$

REFLECT

Technically, $-A\cos(\omega t)$ and $-A\sin(\omega t)$ are also valid solutions to the question, since we don't know if the object is at its maximum amplitude above or below equilibrium or if the object is traveling toward $+A$ or toward $-A$.

12.49

SET UP

The mathematical description of the motion of a particular simple harmonic oscillator is $x(t) = (0.15\text{ m})\cos\left(\pi t + \frac{\pi}{3}\right)$. The velocity of the oscillator is equal to the first derivative of the position with respect to time, and the acceleration of the oscillator is equal to the first derivative of the velocity with respect to time. We will assume that all of the values are given in SI units.

SOLVE

$$x(t) = (0.15\text{ m})\cos\left(\pi t + \frac{\pi}{3}\right)$$

Part a)

$$v(t) = \frac{dx}{dt} = \frac{d}{dt}\left[(0.15\text{ m})\cos\left(\pi t + \frac{\pi}{3}\right)\right] = (0.15\text{ m})\left[-\pi\sin\left(\pi t + \frac{\pi}{3}\right)\right]$$

$$v(t = 1\text{ s}) = (0.15\text{ m})\left[-\pi\sin\left(\pi(1\text{ s}) + \frac{\pi}{3}\right)\right] = \boxed{0.41\frac{\text{m}}{\text{s}}}$$

Part b)

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}\left[-(0.15\text{ m})\pi\sin\left(\pi t + \frac{\pi}{3}\right)\right] = -(0.15\text{ m})\pi^2\cos\left(\pi t + \frac{\pi}{3}\right)$$

$$a(t = 2\text{ s}) = -(0.15\text{ m})\pi^2\cos\left(\pi(2\text{ s}) + \frac{\pi}{3}\right) = \boxed{-0.74\frac{\text{m}}{\text{s}^2}}$$

REFLECT

The acceleration of the oscillator is also equal to the second derivative of the position with respect to time. Since we already calculated the velocity, it's easier to differentiate this function once than go back to the position and differentiate it twice.

12.50

SET UP

A 2-kg object is attached to a spring ($k = 75$ N/m) and set into simple harmonic motion. At $t = 0$, the object is at its maximum amplitude $A = 10$ cm. From this information, we can write the function describing the motion of this object $x(t)$. We will use a cosine function since the object starts at its maximum amplitude at $t = 0$. The angular frequency of the motion is $\omega = \sqrt{\frac{k}{m}}$. Once we have the functional form for $x(t)$ we can differentiate it once with respect to time to get the velocity of the object with respect to time, $v(t)$. The time derivative of $v(t)$ will give the acceleration of the object with respect to time, $a(t)$. The maximum values of the speed and acceleration are equal to the absolute value of the amplitudes of these functions. To find the velocity at a given time, we can plug that directly into the equation for $v(t)$.

SOLVE

$$x(t) = A\cos(\omega t)$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{\left(75\frac{\text{N}}{\text{m}}\right)}{2\text{ kg}}} = 6.1\frac{\text{rad}}{\text{s}}$$

Part a)

$$v(t) = \frac{dx}{dt} = \frac{d}{dt}[A\cos(\omega t)] = -A\omega\sin(\omega t)$$

$$v_{\text{max}} = |-A\omega| = (10\text{ cm})\left(6.1\frac{\text{rad}}{\text{s}}\right) = \boxed{61\frac{\text{cm}}{\text{s}} = 0.61\frac{\text{m}}{\text{s}}}$$

Part b)

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}[-A\omega\sin(\omega t)] = -A\omega^2\cos(\omega t)$$

$$a_{\text{max}} = |-A\omega^2| = (10\text{ cm})\left(6.1\frac{\text{rad}}{\text{s}}\right)^2 = \boxed{372\frac{\text{cm}}{\text{s}^2} = 3.72\frac{\text{m}}{\text{s}^2}}$$

Part c)

$$v(t) = -A\omega\sin(\omega t)$$

$$v(t = 5\text{ s}) = -(10\text{ cm})\left(6.1\frac{\text{rad}}{\text{s}}\right)\sin\left(\left(6.1\frac{\text{rad}}{\text{s}}\right)(5\text{ s})\right) = \boxed{48\frac{\text{cm}}{\text{s}} = 0.48\frac{\text{m}}{\text{s}}}$$

REFLECT

The period of the motion is about 1 s. The velocity at $t = 5$ s is less than the maximum speed, which makes sense, and points toward $+x$.

12.51

SET UP

The period of a simple harmonic oscillator is $T = 0.0125$. The frequency f of the oscillator is the reciprocal of the period.

SOLVE

$$f = \frac{1}{T} = \frac{1}{0.0125\text{ s}} = \boxed{80\text{ Hz}}$$

REFLECT

Remember that frequency and angular frequency are different quantities.

12.52

SET UP

A simple harmonic oscillator completes 1250 cycles in 20 minutes. The period is the time it takes to complete just one cycle, so we need to divide the total time by the total number of cycles. The frequency is equal to the reciprocal of the period.

SOLVE

Part a)

$$T = \frac{20\text{ min}}{1250\text{ cycles}} \times \frac{60\text{ s}}{1\text{ min}} = \boxed{0.96\text{ s}}$$

Part b)

$$f = \frac{1}{T} = \frac{1}{0.96\text{ s}} = \boxed{1.04\text{ Hz}}$$

REFLECT

The units of hertz (Hz) are equivalent to s^{-1}.

12.53

SET UP

An object undergoing simple harmonic motion has a frequency of $f = 15$ Hz. The period of the object's motion is equal to the reciprocal of its frequency. Multiplying the frequency of the motion by 2 min will give us the number of cycles the object undergoes in that time period.

SOLVE

Part a)

$$T = \frac{1}{f} = \frac{1}{15\text{ Hz}} = \boxed{0.067\text{ s}}$$

Part b)

$$N_{\text{cycles}} = 2\text{ min} \times \frac{60\text{ s}}{1\text{ min}} \times \frac{15\text{ cycles}}{1\text{ s}} = \boxed{1800\text{ cycles}}$$

REFLECT

A frequency of 15 Hz is reasonably fast for a mass on the end of a spring.

12.54

SET UP

The amplitude of an object undergoing simple harmonic motion is $A = 20$ cm. At $t = 0$ the object is located at a position of $x(0) = -12$ cm. Since the initial position is in between its maximum displacement (that is, the amplitude) and 0 we will need to calculate the phase angle ϕ for this description. Assuming a general mathematical form of $x(t) = A\cos(\omega t + \phi)$, we can plug in the above values for $t = 0$ and solve for ϕ. In order to write the full mathematical description of the motion, we need not only the phase angle but also the angular frequency. We are told that the period of the motion is $T = 2$ s, which we can use to find the angular frequency through $\omega = \dfrac{2\pi}{T}$.

SOLVE

Part a)

$$x(t) = A\cos(\omega t + \phi)$$

$$(-12\text{ cm}) = (20\text{ cm})\cos(\omega(0) + \phi)$$

$$\cos(\phi) = -\frac{3}{5}$$

$$\phi = \arccos\left(-\frac{3}{5}\right) = \boxed{2.21\text{ rad}}$$

Part b)

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{(2\text{ s})} = \pi\frac{\text{rad}}{\text{s}}$$

$$\boxed{x(t) = (20\text{ cm})\cos\left(\left(\pi\frac{\text{rad}}{\text{s}}\right)t + (2.21\text{ rad})\right)}$$

REFLECT

An angle of 2.21 rad is 127 degrees. We could have also assumed a general form of $x(t) = A\sin(\omega t + \phi)$, which would change the phase by $\pi/2$ rad.

12.55

SET UP

A 0.200-kg object is attached to the end of a spring with a spring constant $k = 55$ N/m. We can relate the period of the motion to the angular frequency, which, in turn, is related to the spring constant and the mass.

SOLVE

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\left(\sqrt{\frac{k}{m}}\right)} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.200\text{ kg}}{\left(55\frac{\text{N}}{\text{m}}\right)}} = \boxed{0.38\text{ s}}$$

REFLECT

In simple harmonic motion, the period is independent of the amplitude.

12.56

SET UP

An object of unknown mass is attached to a spring ($k = 200$ N/m) and completes 14 cycles in 16 s. We can first relate the mass to the angular frequency and then the angular frequency to the period.

SOLVE

$$\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T}$$

$$m = k\left(\frac{T}{2\pi}\right)^2 = \left(200\frac{\text{N}}{\text{m}}\right)\left(\frac{\left(\frac{16\text{ s}}{14\text{ cycles}}\right)}{2\pi}\right)^2 = \boxed{6.6\text{ kg}}$$

REFLECT

A mass of 6.6 kg seems reasonable.

12.57

SET UP

Two springs with spring constants k_1 and k_2, respectively, are attached end-to-end to a box of mass M. We can find the effective spring constant of this setup by comparing the two-spring setup with an equivalent one-spring setup. If the box is moved a distance Δx, each spring will stretch a different amount (Δx_1 and Δx_2) but their sum must be Δx. In the figure, we see that only spring 2 is attached to the mass, which means only the force due to spring 2 acts on the mass. Newton's third law tells us that the magnitude of the force of spring 1 on spring 2 must equal the magnitude of the force of spring 2 on spring 1: $k_1\Delta x_1 = k_2\Delta x_2$; we can use this fact to eliminate Δx_1 from our equation. The period of the motion of the box is equal to $T = 2\pi\sqrt{\frac{M}{k_{\text{eff}}}}$.

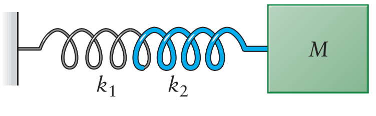


Figure 12-4 Problem 57

SOLVE

$$\Delta x = \Delta x_1 + \Delta x_2 = \left(\frac{k_2 \Delta x_2}{k_1}\right) + \Delta x_2 = \left(\frac{k_2}{k_1} + 1\right)\Delta x_2$$

$$\Delta x_2 = \frac{\Delta x}{\left(\frac{k_2}{k_1} + 1\right)}$$

Effective spring constant:

$$F_2 = F_{\text{eff}}$$

$$k_2 \Delta x_2 = k_{\text{eff}} \Delta x$$

$$k_2\left(\frac{\Delta x}{\left(\frac{k_2}{k_1} + 1\right)}\right) = k_{\text{eff}} \Delta x$$

$$k_{\text{eff}} = \frac{k_2}{\left(\frac{k_2}{k_1} + 1\right)} = \frac{k_2}{\left(\frac{k_2 + k_1}{k_1}\right)} = \frac{k_1 k_2}{(k_2 + k_1)}$$

Period:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{M}{k_{\text{eff}}}} = \boxed{2\pi\sqrt{\frac{M(k_1 + k_2)}{k_1 k_2}} = 2\pi\sqrt{M\left(\frac{1}{k_1} + \frac{1}{k_2}\right)}}$$

REFLECT

The springs in this configuration are described as being in "series." The effective spring constant in this case is *smaller* than the individual spring constants.

12.58

SET UP

Two springs with spring constants k_1 and k_2, respectively, are attached side-by-side to a box of mass M. We can find the effective spring constant of this setup by looking at the combined force due to these two springs. If the box is moved a distance Δx, both springs will be stretched that same distance. The period of the motion of the box is equal to $T = 2\pi\sqrt{\frac{M}{k_{\text{eff}}}}$.

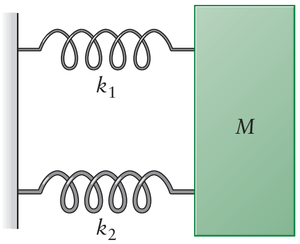


Figure 12-5 Problem 58

SOLVE

Effective spring constant:

$$F_1 + F_2 = F_{\text{eff}}$$

$$k_1\Delta x + k_2\Delta x = k_{\text{eff}}\Delta x$$

$$k_{\text{eff}} = k_1 + k_2$$

Period:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{M}{k_{\text{eff}}}} = \boxed{2\pi\sqrt{\frac{M}{k_1 + k_2}}}$$

REFLECT

The springs in this configuration are described as being in "parallel." The effective spring constant in this case is *larger* than the individual spring constants.

12.59

SET UP

We are given the expression for the corrected period of an oscillating object of mass m when attached to a spring of appreciable mass m_s. We can calculate the ratio of this corrected period to the idealized period and then determine the exact value for a 375-g mass and a 50-g spring.

SOLVE

$$\frac{T_{\text{corrected}}}{T} = \frac{\left(2\pi\sqrt{\dfrac{m + \frac{1}{3}m_s}{k}}\right)}{\left(2\pi\sqrt{\dfrac{m}{k}}\right)} = \frac{\sqrt{m + \frac{1}{3}m_s}}{\sqrt{m}} = \sqrt{1 + \frac{m_s}{3m}} = \sqrt{1 + \frac{50\text{ g}}{3(375\text{ g})}} = \boxed{1.022}$$

REFLECT

In this case, the mass of the spring increases the period by about 2%, which is not that much. Note that the ratio is independent of the spring constant.

12.60

SET UP

We are asked to write a mathematical expression describing the oscillation of the ocean level; we'll use the general form $x(t) = A\cos(\omega t + \phi)$, where t will be written in terms of military time in hours. We will take the equilibrium position to be exactly sea level, which means the amplitude of the motion is 1 m. High tide occurs at 8 A.M. and is 1 m above sea level. Six hours later, at 2 P.M., low tide is 1 m below sea level. Six hours after that, at 8 P.M., high tide occurs again; the period of the tides is, therefore, equal to 12 hr. Because the ocean level does not start at a maximum at $t = 0$, we need to determine the phase angle from the fact that $x(t = 8\text{ hr}) = 1\text{ m}$. Once we have the full mathematical description, we can set it equal to zero and solve for the times of the day when the ocean level is exactly at sea level.

SOLVE

Part a)

$$x(t) = A\cos(\omega t + \phi)$$

Amplitude:

$$A = 1 \text{ m}$$

Angular frequency:

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{12 \text{ hr}}$$

Phase angle:

$$(1 \text{ m}) = (1 \text{ m})\cos\left(\left(\frac{2\pi}{12 \text{ hr}}\right)(8 \text{ hr}) + \phi\right)$$

$$\phi = \arccos(1) - \frac{2\pi(8)}{12} = -\frac{4\pi}{3}$$

Putting it all together:

$$\boxed{x(t) = (1 \text{ m})\cos\left(\left(\frac{2\pi}{12 \text{ hr}}\right)t - \frac{4\pi}{3}\right)}$$

Part b)

$$x(t) = (1 \text{ m})\cos\left(\left(\frac{2\pi}{12 \text{ hr}}\right)t - \frac{4\pi}{3}\right) = 0$$

$$\cos\left(\left(\frac{2\pi}{12 \text{ hr}}\right)t - \frac{4\pi}{3}\right) = 0$$

$$\left(\frac{2\pi}{12 \text{ hr}}\right)t - \frac{4\pi}{3} = (2n + 1)\frac{\pi}{2}$$

$$t = \left((2n + 1)\frac{\pi}{2} + \frac{4\pi}{3}\right)\left(\frac{12 \text{ hr}}{2\pi}\right) = ((2n + 1)(3) + 8) \text{ hr}$$

$$t = 11, 17, 23$$

which correspond to $\boxed{\text{11 A.M., 5 P.M., and 11 P.M.}}$

REFLECT

We could have also written the mathematical expression as

$$x(t) = (1 \text{ m})\cos\left(\frac{2\pi}{12 \text{ hr}}(t - (8 \text{ hr}))\right)$$

12.61

SET UP

The total energy of a simple harmonic oscillator is given by $E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$. We are told

that $x(t) = A\cos(\omega t + \phi)$; we can differentiate $x(t)$ with respect to time to find $v(t)$. We can plug both of these in and do some algebra in order to show explicitly that E is a constant. We will need to invoke the Pythagorean identity: $\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi) = 1$.

SOLVE

$$x(t) = A\cos(\omega t + \phi)$$

$$v(t) = \frac{dx}{dt} = \frac{d}{dt}[A\cos(\omega t + \phi)] = -A\omega\sin(\omega t + \phi)$$

Conservation of energy:

$$\begin{aligned} E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 &= \frac{1}{2}m[-A\omega\sin(\omega t + \phi)]^2 + \frac{1}{2}k[A\cos(\omega t + \phi)]^2 \\ &= \frac{1}{2}mA^2\omega^2\sin^2(\omega t + \phi) + \frac{1}{2}kA^2\cos^2(\omega t + \phi) \\ &= \frac{1}{2}mA^2\left(\frac{k}{m}\right)\sin^2(\omega t + \phi) + \frac{1}{2}kA^2\cos^2(\omega t + \phi) \\ &= \frac{1}{2}kA^2\sin^2(\omega t + \phi) + \frac{1}{2}kA^2\cos^2(\omega t + \phi) \\ &= \frac{1}{2}kA^2[\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)] = \boxed{\frac{1}{2}kA^2} \end{aligned}$$

This is constant for a given k and A.

REFLECT

As expected, the total energy of a simple harmonic oscillator is proportional to the square of its amplitude.

12.62

SET UP

An object on a spring slides on a horizontal frictionless surface with simple harmonic motion, which means the total energy of the object is the elastic potential energy stored in the spring plus the kinetic energy of the object. We know the total energy of the object also equals $\frac{1}{2}kA^2$. The position(s) where the potential energy and kinetic energy are the same occur at $E_{\text{total}} = U + K = 2U$.

SOLVE

$$E_{\text{total}} = U + K = 2U$$

$$\frac{1}{2}kA^2 = 2\left(\frac{1}{2}kx^2\right)$$

$$\boxed{x = \pm\frac{A}{\sqrt{2}}}$$

REFLECT

We could have also solved this by setting $U = K$, plugging in $x(t) = A\cos(\omega t + \phi)$, solving for t, and then plugging these back into $x(t)$ to find the positions.

12.63

SET UP

The mathematical description of an object attached to a spring undergoing simple harmonic motion on a horizontal frictionless surface is $x(t) = A\cos(\omega t + \phi)$. The potential energy of the object is equal to the potential energy stored in the spring, $U = \frac{1}{2}kx^2$. The kinetic energy of the object is $K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2$. The total energy of a simple harmonic oscillator is given by $E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$. We can use our answers from parts (a) and (b) and do some algebra in order to show explicitly that E is a constant. We will need to invoke the Pythagorean identity: $\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi) = 1$.

SOLVE

Part a)

$$U = \frac{1}{2}kx^2 = \frac{1}{2}k[A\cos(\omega t + \phi)]^2 = \boxed{\frac{kA^2}{2}\cos^2(\omega t + \phi)}$$

Part b)

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 = \frac{1}{2}m\left(\frac{d}{dt}[A\cos(\omega t + \phi)]\right)^2 = \frac{1}{2}m(-\omega A\sin(\omega t + \phi))^2$$

$$= \boxed{\frac{m\omega^2A^2}{2}\sin^2(\omega t + \phi)}$$

Part c)

$$E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{kA^2}{2}\cos^2(\omega t + \phi) + \frac{m\omega^2A^2}{2}\sin^2(\omega t + \phi)$$

$$= \frac{kA^2}{2}\cos^2(\omega t + \phi) + \frac{m\left(\frac{k}{m}\right)A^2}{2}\sin^2(\omega t + \phi)$$

$$= \frac{kA^2}{2}\sin^2(\omega t + \phi) + \frac{kA^2}{2}\cos^2(\omega t + \phi)$$

$$= \frac{kA^2}{2}[\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)] = \boxed{\frac{1}{2}kA^2}$$

This is constant for a given k and A.

REFLECT

Because the object is sliding along a horizontal, frictionless surface we can ignore its gravitational potential energy.

12.64

SET UP

Starting with $x(t) = A\cos(\omega t + \phi)$, we can differentiate this with respect to time and evaluate the resulting function at $t = 0$ and $t = T/2$ to find the velocity at those points. We can also use conservation of energy $E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$ to solve for $v(0)$ and $v(T/2)$ given $x(0)$ and $x(T/2)$. We should get the same answer regardless of the method.

SOLVE

Part a)

$$x(t) = A\cos(\omega t + \phi)$$

$$v(t) = \frac{dx}{dt} = \frac{d}{dt}[A\cos(\omega t + \phi)] = -A\omega\sin(\omega t + \phi)$$

$$v(0) = -A\omega\sin(\omega(0) + \phi) = -A\omega\sin(\phi)$$

$$v\left(\frac{T}{2}\right) = -A\omega\sin\left(\omega\left(\frac{T}{2}\right) + \phi\right) = -A\omega\sin\left(\omega\left(\frac{1}{2}\left(\frac{2\pi}{\omega}\right)\right) + \phi\right)$$

$$= -A\omega\sin\left(\omega\left(\left(\frac{\pi}{\omega}\right)\right) + \phi\right) = -A\omega\sin(\pi + \phi) = \boxed{A\omega\sin(\phi)}$$

Part b)

$t = 0$:

$$E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

$$\frac{1}{2}kA^2 = \frac{1}{2}kA^2\cos^2(\phi) + \frac{1}{2}mv^2$$

$$kA^2(1 - \cos^2(\phi)) = mv^2$$

$$v = \sqrt{\frac{kA^2(1 - \cos^2(\phi))}{m}} = \sqrt{\frac{kA^2\sin^2(\phi)}{m}} = \pm A\sin(\phi)\sqrt{\frac{k}{m}} = \boxed{\pm A\omega\sin(\phi)}$$

$t = T/2$:

$$E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

$$\frac{1}{2}kA^2 = \frac{1}{2}kA^2\cos^2(\pi - \phi) + \frac{1}{2}mv^2$$

$$kA^2(1 - \cos^2(\pi - \phi)) = mv^2$$

$$v = \sqrt{\frac{kA^2(1 - \cos^2(\pi - \phi))}{m}} = \sqrt{\frac{kA^2\sin^2(\pi - \phi)}{m}} = \pm A\sin(\pi - \phi)\sqrt{\frac{k}{m}} = \boxed{\mp A\omega\sin(\phi)}$$

REFLECT

The velocity of the object should have the same magnitude but point in the opposite direction after one-half period.

12.65

SET UP

A 0.250-kg object attached to a spring oscillates on a frictionless horizontal table with a frequency of $f = 4$ Hz and an amplitude of $A = 0.20$ m. Although we were not given it explicitly, we can rewrite the spring constant in terms of the frequency and mass. The maximum potential energy of the system occurs when the kinetic energy is equal to zero, which means the maximum potential energy is equal to the total energy of the harmonic oscillator. We can then divide this quantity by two and solve for x to solve for the displacement of the object when the potential energy is one-half of the maximum. Finally, we can plug $x = 0.10$ m into the elastic potential energy to find the potential energy of the object at that displacement.

SOLVE

Part a)

$$U_{\text{max}} = \frac{1}{2}kA^2 = \frac{1}{2}\omega^2 mA^2 = \frac{1}{2}(2\pi f)^2 mA^2 = 2\pi^2 mf^2 A^2$$

$$= 2\pi^2(0.250 \text{ kg})(4 \text{ Hz})^2(0.20 \text{ m})^2 = \boxed{3.2 \text{ J}}$$

Part b)

$$U_{\text{half max}} = \frac{1}{2}kx^2 = \frac{1}{2}\left(\frac{1}{2}kA^2\right)$$

$$x = \frac{A}{\sqrt{2}} = \frac{20 \text{ cm}}{\sqrt{2}} = \boxed{14 \text{ cm}}$$

Part c)

$$U(x) = \frac{1}{2}kx^2 = \frac{1}{2}\omega^2 mx^2 = \frac{1}{2}(2\pi f)^2 mx^2 = 2\pi^2 mf^2 x^2$$

$$U(x = 0.10 \text{ m}) = 2\pi^2(0.250 \text{ kg})(4 \text{ Hz})^2(0.10 \text{ m})^2 = \boxed{0.79 \text{ J}}$$

REFLECT

The displacement where the potential energy is one-half of the maximum value does *not* occur at one-half the amplitude.

12.66

SET UP

The potential energy of an object on a spring is 2.4 J at a location where the kinetic energy is 1.6 J. The sum of these will give the total energy of the system, which is also equal to $\frac{1}{2}kA^2$, where $A = 0.20$ m. From this information, we can calculate the spring constant. The maximum force that the object experiences due to the spring occurs when the displacement is equal to the amplitude.

SOLVE

Part a)

$$E = U + K = \frac{1}{2}kA^2$$

$$k = \frac{2(U + K)}{A^2} = \frac{2((2.4\text{ J}) + (1.6\text{ J}))}{(0.20\text{ m})^2} = \boxed{200\frac{\text{N}}{\text{m}}}$$

Part b)

$$F_{\text{max}} = kA = \left(200\frac{\text{N}}{\text{m}}\right)(0.20\text{ m}) = \boxed{40\text{ N}}$$

REFLECT

We can ignore the minus sign in Hooke's law because we are interested only in the magnitude of the force, not its direction.

12.67

SET UP

The equation of motion for a given simple harmonic oscillator is $x(t) = A\sin(\omega t)$. The velocity of this oscillator is equal to the first derivative of $x(t)$ with respect to time. The potential energy of a simple harmonic oscillator is given by $U(t) = \frac{1}{2}kx^2$ and the kinetic energy is given by $K(t) = \frac{1}{2}mv^2$. The plots of $U(t)$ and $K(t)$ should look like $\sin^2(\omega t)$ and $\cos^2(\omega t)$, respectively.

SOLVE

Part a)

$$U(t) = \frac{1}{2}kx^2 = \frac{1}{2}k[A\sin(\omega t)]^2 = \frac{1}{2}kA^2\sin^2(\omega t)$$

For simplicity, the vertical axis of the following plot is $\frac{U}{\left(\frac{1}{2}kA^2\right)}$, and the horizontal axis is ωt:

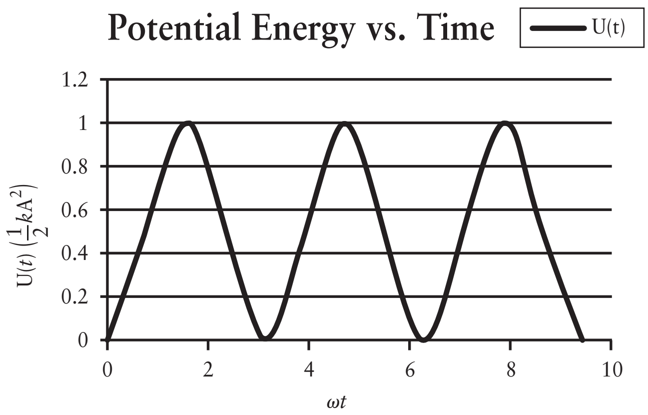


Figure 12-6 Problem 67

Part b)

$$v(t) = \frac{dx}{dt} = \frac{d}{dt}[A\sin(\omega t)] = A\omega\cos(\omega t)$$

Part c)

$$K(t) = \frac{1}{2}mv^2 = \frac{1}{2}mA^2\omega^2\cos^2(\omega t) = \frac{1}{2}kA^2\cos^2(\omega t)$$

For simplicity, the vertical axis of the following plot is $\frac{U}{\left(\frac{1}{2}kA^2\right)}, \frac{K}{\left(\frac{1}{2}kA^2\right)}$ and the horizontal axis is ωt:

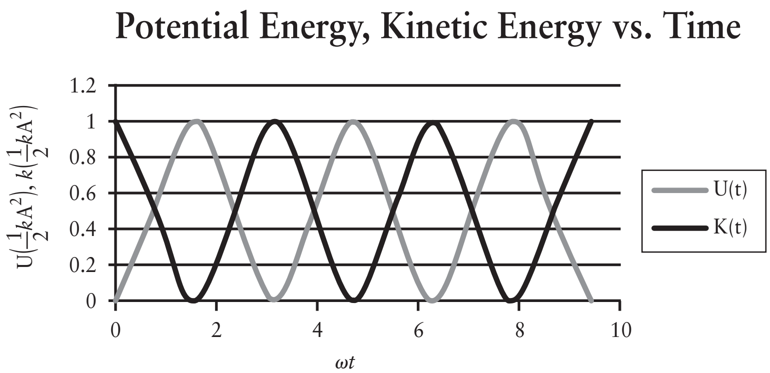


Figure 12-7 Problem 67

REFLECT

Although U and K oscillate in time, their sum is constant for all time.

12.68

SET UP

A simple pendulum is 1.24 m long. The period of its oscillation is $T = 2\pi\sqrt{\frac{L}{g}}$.

SOLVE

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{1.24\text{ m}}{\left(9.8\frac{\text{m}}{\text{s}^2}\right)}} = \boxed{2.24\text{ s}}$$

REFLECT

The period of a simple pendulum is independent of the bob's mass.

12.69

SET UP

A simple pendulum has a period of 2.25 s. We can use the definition of the period of a simple pendulum $T = 2\pi\sqrt{\frac{L}{g}}$ and solve for its length.

SOLVE

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$L = g\left(\frac{T}{2\pi}\right)^2 = \left(9.8\frac{\text{m}}{\text{s}^2}\right)\left(\frac{2.25\text{ s}}{2\pi}\right)^2 = \boxed{1.26\text{ m}}$$

REFLECT

The period of a simple pendulum is independent of the bob's mass.

12.70

SET UP

A simple pendulum has a length of 67.00 m. The acceleration due to gravity is 9.809 m/s^2. The period of the pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$.

SOLVE

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{67.00\text{ m}}{\left(9.809\frac{\text{m}}{\text{s}^2}\right)}} = \boxed{16.42\text{ s}}$$

REFLECT

The mass of the pendulum is irrelevant when calculating the period of a simple pendulum.

12.71

SET UP

A simple pendulum completes 14 cycles in 25 s. We can use the definition of the period of a simple pendulum $T = 2\pi\sqrt{\frac{L}{g}}$ and solve for its length.

SOLVE

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$L = g\left(\frac{T}{2\pi}\right)^2 = \left(9.8\frac{\text{m}}{\text{s}^2}\right)\left(\frac{\left(\frac{25\text{ s}}{14\text{ cycles}}\right)}{2\pi}\right)^2 = \boxed{0.792\text{ m}}$$

REFLECT

The period of a pendulum is the time it takes to make one full oscillation.

12.72

SET UP

We can use the definition of the period of a simple pendulum $T = 2\pi\sqrt{\frac{L}{g}}$ to predict the periods of a 1-m-long simple pendulum on each of the eight planets in our solar system. The values of g for each of the eight planets are $g_{\text{Mercury}} = 3.70\text{ m/s}^2$, $g_{\text{Venus}} = 8.87\text{ m/s}^2$, $g_{\text{Earth}} = 9.8\text{ m/s}^2$, $g_{\text{Mars}} = 3.69\text{ m/s}^2$, $g_{\text{Jupiter}} = 24.80\text{ m/s}^2$, $g_{\text{Saturn}} = 8.96\text{ m/s}^2$, $g_{\text{Uranus}} = 8.69\text{ m/s}^2$, and $g_{\text{Neptune}} = 11.20\text{ m/s}^2$.

SOLVE

Mercury:

$$T_{\text{Mercury}} = 2\pi\sqrt{\frac{L}{g_{\text{Mercury}}}} = 2\pi\sqrt{\frac{1\text{ m}}{\left(3.70\frac{\text{m}}{\text{s}^2}\right)}} = \boxed{3.27\text{ s}}$$

Venus:

$$T_{\text{Venus}} = 2\pi\sqrt{\frac{L}{g_{\text{Venus}}}} = 2\pi\sqrt{\frac{1\text{ m}}{\left(8.87\frac{\text{m}}{\text{s}^2}\right)}} = \boxed{2.11\text{ s}}$$

Earth:

$$T_{\text{Earth}} = 2\pi\sqrt{\frac{L}{g_{\text{Earth}}}} = 2\pi\sqrt{\frac{1\text{ m}}{\left(9.8\frac{\text{m}}{\text{s}^2}\right)}} = \boxed{2.01\text{ s}}$$

Mars:

$$T_{\text{Mars}} = 2\pi\sqrt{\frac{L}{g_{\text{Mars}}}} = 2\pi\sqrt{\frac{1\text{ m}}{\left(3.69\frac{\text{m}}{\text{s}^2}\right)}} = \boxed{3.27\text{ s}}$$

Jupiter:

$$T_{\text{Jupiter}} = 2\pi\sqrt{\frac{L}{g_{\text{Jupiter}}}} = 2\pi\sqrt{\frac{1\text{ m}}{\left(24.80\frac{\text{m}}{\text{s}^2}\right)}} = \boxed{1.26\text{ s}}$$

Saturn:

$$T_{\text{Saturn}} = 2\pi\sqrt{\frac{L}{g_{\text{Saturn}}}} = 2\pi\sqrt{\frac{1\text{ m}}{\left(8.96\frac{\text{m}}{\text{s}^2}\right)}} = \boxed{2.10\text{ s}}$$

Uranus:

$$T_{\text{Uranus}} = 2\pi\sqrt{\frac{L}{g_{\text{Uranus}}}} = 2\pi\sqrt{\frac{1\text{ m}}{\left(8.69\frac{\text{m}}{\text{s}^2}\right)}} = \boxed{2.13\text{ s}}$$

Neptune:

$$T_{\text{Neptune}} = 2\pi\sqrt{\frac{L}{g_{\text{Neptune}}}} = 2\pi\sqrt{\frac{1\text{ m}}{\left(11.20\frac{\text{m}}{\text{s}^2}\right)}} = \boxed{1.88\text{ s}}$$

REFLECT

As expected, larger values of g yield shorter periods—Jupiter has the shortest period and Mercury and Mars have the longest periods.

12.73

SET UP

The acceleration due to gravity g changes with elevation according to $g = g_0\left(\frac{R_E}{R_E + h}\right)^2$, where $g_0 = 9.800\text{ m/s}^2$, $R_E = 6.380 \times 10^6$ m, and $h = 0$ at sea level. We can use this expression to find g at the top of Mt. Everest ($h = 8848$ m) and then the period of a 1-m-long pendulum at this elevation. Defining T_0 to be the period of the pendulum at sea level, we can set up a proportionality to solve for the period on Mt. Everest as a function of the period at sea level.

SOLVE

Part a)

Acceleration due to gravity on Mt. Everest:

$$g = g_0\left(\frac{R_E}{R_E + h}\right)^2 = \left(9.800\frac{\text{m}}{\text{s}^2}\right)\left(\frac{6.380 \times 10^6\text{ m}}{(6.380 \times 10^6\text{ m}) + (8848\text{ m})}\right)^2 = 9.773\frac{\text{m}}{\text{s}^2}$$

Period of the pendulum:

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{1\ \text{m}}{\left(9.773\frac{\text{m}}{\text{s}^2}\right)}} = \boxed{2.010\ \text{s}}$$

Part b)

$$\frac{T_{\text{Everest}}}{T_{\text{sea level}}} = \frac{2\pi\sqrt{\frac{L}{g}}}{2\pi\sqrt{\frac{L}{g_0}}} = \sqrt{\frac{g_0}{g}}$$

$$T_{\text{Everest}} = T_{\text{sea level}}\sqrt{\frac{g_0}{g}} = T_0\sqrt{\frac{\left(9.800\frac{\text{m}}{\text{s}^2}\right)}{\left(9.773\frac{\text{m}}{\text{s}^2}\right)}} = \boxed{1.001\ T_0}$$

REFLECT

We expect g on Mt. Everest to be smaller than g at sea level. Accordingly, the period of the pendulum should be slightly larger on Mt. Everest than at sea level.

12.74

SET UP

A simple pendulum of length $L = 0.350$ m starts from a maximum angular displacement of 10 degrees from its equilibrium position. Defining $t = 0$ as the time when the pendulum is set into motion, the equation of motion describing the angular position of this pendulum is $\theta(t) = \theta_0\cos(\omega t)$, where $\theta_0 = 10$ degrees and $\omega = \sqrt{\frac{g}{L}}$. We can solve for the time at which the pendulum will be at an angle of 8 degrees and 5 degrees by rearranging the equation of motion. The pendulum will return to its starting position after one full period.

SOLVE

Angular frequency:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$\omega = \frac{2\pi}{T} = \sqrt{\frac{g}{L}} = \sqrt{\frac{\left(9.8\frac{\text{m}}{\text{s}^2}\right)}{0.350\ \text{m}}} = 5.29\frac{\text{rad}}{\text{s}}$$

Time:

$$\theta(t) = \theta_0\cos(\omega t)$$

$$t = \frac{1}{\omega}\arccos\left(\frac{\theta(t)}{\theta_0}\right)$$

Part a)

$$t = \frac{1}{\left(5.29\frac{\text{rad}}{\text{s}}\right)}\arccos\left(\frac{8°}{10°}\right) = \boxed{0.122\ \text{s}}$$

Part b)

$$t = \frac{1}{\left(5.29\frac{\text{rad}}{\text{s}}\right)}\arccos\left(\frac{5°}{10°}\right) = \boxed{0.198\ \text{s}}$$

Part c)

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\left(5.29\frac{\text{rad}}{\text{s}}\right)} = \boxed{1.19\ \text{s}}$$

REFLECT

The equation of motion of a pendulum is usually described in terms of the angular displacement rather than the linear displacement.

12.75

SET UP

A simple pendulum has a length of $L = 0.50$ m and oscillates with an amplitude of 8 degrees. From these data we can calculate the period of the motion and write down the equation of motion for the pendulum. The time it takes the pendulum to swing from $+8$ degrees to -8 degrees is half of a period. To find the time interval between $+4$ degrees and -4 degrees, we will need to find the times at which the pendulum is at each of these angular positions; we can do this by solving the equation of motion.

SOLVE

Period:

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{0.50\ \text{m}}{\left(9.8\frac{\text{m}}{\text{s}^2}\right)}} = 1.42\ \text{s}$$

Equation of motion:

$$\theta(t) = \theta_0\cos(\omega t) = \theta_0\cos\left(\frac{2\pi t}{T}\right) = (8°)\cos\left(\left(4.42\frac{\text{rad}}{\text{s}}\right)t\right)$$

Time interval between $+8°$ and $-8°$:

$$\frac{T}{2} = \boxed{0.71\ \text{s}}$$

Time interval between $+4°$ and $-4°$:

$$4° = (8°)\cos\left(\left(4.42\frac{\text{rad}}{\text{s}}\right)t_1\right)$$

$$t_1 = \frac{\arccos\left(\frac{4°}{8°}\right)}{\left(4.42\frac{\text{rad}}{\text{s}}\right)} = 0.237\text{ s}$$

$$-4° = (8°)\cos\left(\left(4.42\frac{\text{rad}}{\text{s}}\right)t_2\right)$$

$$t_2 = \frac{\arccos\left(\frac{-4°}{8°}\right)}{\left(4.42\frac{\text{rad}}{\text{s}}\right)} = 0.474\text{ s}$$

$$\Delta t = t_2 - t_1 = (0.474\text{ s}) - (0.237\text{ s}) = \boxed{0.237\text{ s}}$$

REFLECT

The time interval between $+4°$ and $-4°$ is one-third of the period.

12.76

SET UP

A rod of length $L = 0.30$ m and mass M is set into harmonic motion about one end. The period of a physical pendulum is $T = 2\pi\sqrt{\frac{I}{mgh}}$, where I is the moment of inertia, m is the mass of the rod, and h is the distance from the pivot to the center of mass. The moment of inertia of a rod of length L and mass M rotating about one end is $I = \frac{1}{3}ML^2$. The center of mass is located a distance $h = L/2$ from the pivot.

SOLVE

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{\left(\frac{1}{3}ML^2\right)}{Mg\left(\frac{L}{2}\right)}} = 2\pi\sqrt{\frac{2L}{3g}} = 2\pi\sqrt{\frac{2(0.30\text{ m})}{3\left(9.8\frac{\text{m}}{\text{s}^2}\right)}} = \boxed{0.90\text{ s}}$$

REFLECT

The period of a simple pendulum of the same length is 1.1 s, which is larger than that of the physical pendulum, as expected.

12.77

SET UP

A spherical bob of mass M and radius $R = 0.50$ m is suspended from a massless string of length $L = 1.0$ m. The period of a physical pendulum is related to the moment of inertia of the pendulum. Since the center of mass of the sphere is located a distance $h = (L + R)$ from the pivot point, we will need to use the parallel-axis theorem. As a reminder, the moment of inertia of a sphere is $\frac{2}{5}MR^2$.

SOLVE

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{I_{\text{sphere}} + I_{\text{from axis}}}{mgh}} = 2\pi\sqrt{\frac{\left(\frac{2}{5}MR^2\right) + M(L+R)^2}{Mg(L+R)}} = 2\pi\sqrt{\frac{\left(\frac{2}{5}R^2\right) + (L+R)^2}{g(L+R)}}$$

$$= 2\pi\sqrt{\frac{\left(\frac{2}{5}(0.50\text{ m})^2\right) + ((1.0\text{ m}) + (0.50\text{ m}))^2}{\left(9.8\frac{\text{m}}{\text{s}^2}\right)((1.0\text{ m}) + (0.50\text{ m}))}} = \boxed{2.51\text{ s}}$$

REFLECT

If we modeled this as a simple pendulum, the period would be 2.46 s.

12.78

SET UP

A hole is drilled through a thin, round disk of mass M and radius $R = 0.10$ m. The hole is located $h = 0.08$ m from the center of mass of the disk. The disk is hung on a nail and set into simple harmonic motion. The period of a physical pendulum is related to the moment of inertia of the pendulum. Since the center of mass of the disk is located a distance h from the pivot point, we will need to use the parallel-axis theorem. As a reminder, the moment of inertia of a thin disk is $\frac{1}{2}MR^2$.

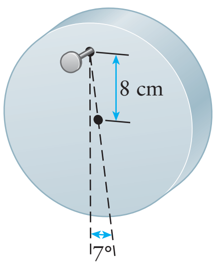


Figure 12-8 Problem 78

SOLVE

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{I_{\text{disk}} + I_{\text{from axis}}}{mgh}} = 2\pi\sqrt{\frac{\left(\frac{1}{2}MR^2\right) + Mh^2}{Mgh}} = 2\pi\sqrt{\frac{\left(\frac{1}{2}R^2\right) + h^2}{gh}}$$

$$= 2\pi\sqrt{\frac{\left(\frac{1}{2}(0.10\text{ m})^2\right) + (0.08\text{ m})^2}{\left(9.8\frac{\text{m}}{\text{s}^2}\right)(0.08\text{ m})}} = \boxed{0.76\text{ s}}$$

REFLECT

We didn't need the exact value of the amplitude of the oscillation, just the fact that it was small. Because the amplitude of the motion is small, the small angle approximation holds and we can use the above expression for the period.

12.79

SET UP

A solid sphere of mass M and radius $R = 0.05$ m is suspended from an eyelet attached to its surface. The sphere is displaced slightly from equilibrium and set into simple harmonic motion. The period of a physical pendulum is related to the moment of inertia of the pendulum. Since the center of mass of the sphere is located a distance $h = R$ from the pivot point, we will need to use the parallel-axis theorem. As a reminder, the moment of inertia of a sphere is $\frac{2}{5}MR^2$.

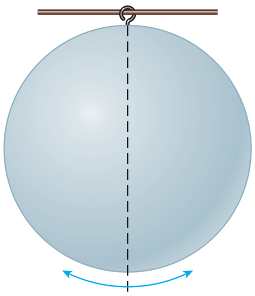


Figure 12-9 Problem 79

SOLVE

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{I_{\text{sphere}} + I_{\text{from axis}}}{mgh}} = 2\pi\sqrt{\frac{\left(\frac{2}{5}MR^2\right) + MR^2}{MgR}} = 2\pi\sqrt{\frac{7R}{5g}}$$

$$= 2\pi\sqrt{\frac{7(0.05\text{ m})}{5\left(9.8\frac{\text{m}}{\text{s}^2}\right)}} = \boxed{0.53\text{ s}}$$

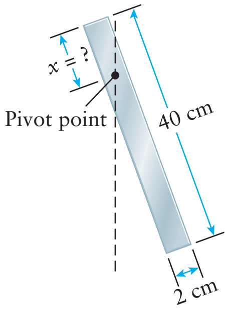
REFLECT

The density of the material is irrelevant since the mass cancels out in our expression for the period of the oscillation.

12.80

SET UP

We want to drill a hole in a thin, rectangular piece of metal of length $L = 40$ cm such that the period of its harmonic motion is minimized. The period of a physical pendulum is related to the moment of inertia of the pendulum. Since the center of mass of the rod is located a distance h from the pivot point, we will need to use the parallel-axis theorem. As a reminder, the moment of inertia of a thin rod rotating about its center of mass is $\frac{1}{12}ML^2$. To find the distance h from the center of mass where we should drill the hole, we need to differentiate the expression for the period with respect to h, set it equal to zero, and solve for h. The distance x from the end of the rod is equal to $x = \frac{L}{2} - h$.



Note: This is not drawn to scale.

Figure 12-10 Problem 80

SOLVE

Period:

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{I_{\text{rod}} + I_{\text{from axis}}}{mgh}} = 2\pi\sqrt{\frac{\left(\frac{1}{12}ML^2\right) + Mh^2}{Mgh}} = \frac{2\pi}{\sqrt{g}}\left(\frac{L^2}{12h} + h\right)^{\frac{1}{2}}$$

Minimum period:

$$\frac{dT}{dh} = 0$$

$$\frac{d}{dh}\left[\frac{2\pi}{\sqrt{g}}\left(\frac{L^2}{12h} + h\right)^{\frac{1}{2}}\right] = 0$$

$$\frac{d}{dh}\left(\frac{L^2}{12h} + h\right)^{\frac{1}{2}} = \frac{1}{2}\left(\frac{L^2}{12h} + h\right)^{-\frac{1}{2}}\left(-\frac{L^2}{12h^2} + 1\right) = 0$$

The only physical solution to this equation is when $\left(-\frac{L^2}{12h^2} + 1\right) = 0$:

$$-\frac{L^2}{12h^2} + 1 = 0$$

$$L^2 = 12h^2$$

The minimum occurs at $h = \frac{L}{\sqrt{12}}$.

$$x = \frac{L}{2} - h = \frac{L}{2} - \frac{L}{\sqrt{12}} = L\left(\frac{1}{2} - \frac{1}{\sqrt{12}}\right) = (40\text{ cm})\left(\frac{1}{2} - \frac{1}{\sqrt{12}}\right) = 8.45\text{ cm}.$$

The hole should be drilled $\boxed{8.45\text{ cm from the end of the rod}}$.

REFLECT

This answer depends only on the length of the rod and not its mass or volume.

12.81

SET UP

We are asked to explicitly show that $x(t) = Ae^{-\frac{b}{2m}t}\sin(\omega_1 t)$ is a solution for the damped harmonic oscillator, where $\omega_1 = \sqrt{\omega_0^2 - \frac{b^2}{4m^2}}$. The differential equation describing the damped harmonic oscillator is $m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$. We will need to invoke both the chain rule and the product rule.

SOLVE

$$x(t) = Ae^{-\frac{b}{2m}t}\sin(\omega_1 t)$$

$$\frac{dx}{dt} = \frac{d}{dt}[Ae^{-\frac{b}{2m}t}\sin(\omega_1 t)] = A\left[\left(-\frac{b}{2m}\right)e^{-\frac{b}{2m}t}\sin(\omega_1 t) + \omega_1 e^{-\frac{b}{2m}t}\cos(\omega_1 t)\right]$$

$$\frac{d^2x}{dt^2} = A\frac{d}{dt}\left[\left(-\frac{b}{2m}\right)e^{-\frac{b}{2m}t}\sin(\omega_1 t) + \omega_1 e^{-\frac{b}{2m}t}\cos(\omega_1 t)\right]$$

$$= A\left[\left(-\frac{b}{2m}\right)^2 e^{-\frac{b}{2m}t}\sin(\omega_1 t) + 2\left(-\frac{b}{2m}\right)\omega_1 e^{-\frac{b}{2m}t}\cos(\omega_1 t) - \omega_1^2 e^{-\frac{b}{2m}t}\sin(\omega_1 t)\right]$$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx \stackrel{?}{=} 0$$

$$m\left(A\left[\left(-\frac{b}{2m}\right)^2 e^{-\frac{b}{2m}t}\sin(\omega_1 t) + 2\left(-\frac{b}{2m}\right)\omega_1 e^{-\frac{b}{2m}t}\cos(\omega_1 t) - \omega_1^2 e^{-\frac{b}{2m}t}\sin(\omega_1 t)\right]\right)$$
$$+ b\left(A\left[\left(-\frac{b}{2m}\right)e^{-\frac{b}{2m}t}\sin(\omega_1 t) + \omega_1 e^{-\frac{b}{2m}t}\cos(\omega_1 t)\right]\right) + k(Ae^{-\frac{b}{2m}t}\sin(\omega_1 t)) \stackrel{?}{=} 0$$

$$A\left[\left(\frac{b^2}{4m}\right)e^{-\frac{b}{2m}t}\sin(\omega_1 t) - b\omega_1 e^{-\frac{b}{2m}t}\cos(\omega_1 t) - m\omega_1^2 e^{-\frac{b}{2m}t}\sin(\omega_1 t)\right]$$

$$+ A\left[\left(-\frac{b^2}{2m}\right)e^{-\frac{b}{2m}t}\sin(\omega_1 t) + b\omega_1 e^{-\frac{b}{2m}t}\cos(\omega_1 t)\right] + k\left(Ae^{-\frac{b}{2m}t}\sin(\omega_1 t)\right) \stackrel{?}{=} 0$$

$$A\left[\left(\frac{b^2}{4m}\right)e^{-\frac{b}{2m}t}\sin(\omega_1 t) - m\omega_1^2 e^{-\frac{b}{2m}t}\sin(\omega_1 t) - \left(\frac{b^2}{2m}\right)e^{-\frac{b}{2m}t}\sin(\omega_1 t) + ke^{-\frac{b}{2m}t}\sin(\omega_1 t)\right] \stackrel{?}{=} 0$$

$$Ae^{-\frac{b}{2m}t}\sin(\omega_1 t)\left[-\left(\frac{b^2}{4m}\right) - m\omega_1^2 + k\right] \stackrel{?}{=} 0$$

$$Ae^{-\frac{b}{2m}t}\sin(\omega_1 t)\left[-\left(\frac{b^2}{4m}\right) - m\omega_1^2 + m\omega_0^2\right] \stackrel{?}{=} 0$$

$$Ae^{-\frac{b}{2m}t}\sin(\omega_1 t)\left[-m\omega_1^2 + m\left(\omega_0^2 - \left(\frac{b^2}{4m^2}\right)\right)\right] \stackrel{?}{=} 0$$

$$Ae^{-\frac{b}{2m}t}\sin(\omega_1 t)\left[-m\omega_1^2 + m\omega_1^2\right] = 0$$

REFLECT

We needed to use the definition of $\omega_0 = \sqrt{\frac{k}{m}}$.

12.82

SET UP

A damped oscillator has a mass of 0.100 kg and starts with an initial amplitude A. After 10.0 s, the amplitude is equal to $0.368A$. We can rearrange the expression describing the amplitude as a function of time, $A(t) = Ae^{-\frac{b}{2m}t}$, and solve for the damping coefficient b.

SOLVE

$$A(t) = Ae^{-\frac{b}{2m}t}$$

$$b = -\frac{2m}{t}\ln\left(\frac{A(t)}{A}\right) = -\frac{2m}{t}\ln\left(\frac{0.368A}{A}\right) = -\frac{2(0.100\text{ kg})}{10.0\text{ s}}\ln(0.368) = \boxed{0.02\frac{\text{kg}}{\text{s}}}$$

REFLECT

The argument of e must be dimensionless, so the dimensions of b are mass per time.

12.83

SET UP

A pendulum bob of mass $m = 0.110$ kg swings at the end of a wire of length $L = 15.0$ m. The damping coefficient of the pendulum is $b = 0.010$ kg/s. We can use the definition of the lightly damped and undamped oscillation frequencies to calculate the period of this damped harmonic oscillator. The amplitude of a damped harmonic oscillator is a function of time:

$A(t) = A_0 e^{-\frac{b}{2m}t}$. We can use this relationship to find the amplitude of the pendulum after a time interval of three periods.

SOLVE

Period:

$$T = \frac{2\pi}{\omega_1} = \frac{2\pi}{\sqrt{\omega_0^2 - \frac{b^2}{4m^2}}} = \frac{2\pi}{\sqrt{\frac{g}{L} - \frac{b^2}{4m^2}}} = \frac{2\pi}{\sqrt{\frac{\left(9.8\frac{\text{m}}{\text{s}^2}\right)}{(15.0\text{ m})} - \frac{\left(0.010\frac{\text{kg}}{\text{s}}\right)^2}{4(0.110\text{ kg})^2}}} = 7.79\text{ s}$$

Amplitude:

$$A(t) = A_0 e^{-\frac{b}{2m}t}$$

$$A(3T) = A_0 e^{-\frac{b}{2m}(3T)} = (1.5\text{ m})e^{-\frac{(0.010\frac{\text{kg}}{\text{s}})(3)(7.79\text{ s})}{2(0.110\text{ kg})}} = \boxed{0.52\text{ m}}$$

REFLECT

The amplitude of the pendulum exponentially decays with time and comes to rest after an infinitely long period of time.

12.84

SET UP

A 0.500-kg object is attached to a spring with a spring constant of $k = 2.5$ N/m. The object rests on a horizontal surface that is covered in a viscous substance. The object is pulled 15 cm from equilibrium and set into harmonic motion. After 3 s, the amplitude has decreased to 7 cm. The natural frequency of the system is $\omega_0 = \sqrt{\frac{k}{m}}$, while the frequency of oscillation is $\omega_1 = \sqrt{\omega_0^2 - \frac{b^2}{4m^2}}$. We can calculate the damping constant b and the time it takes for the amplitude to become one-tenth of its initial value from $A(t) = A_0 e^{-\frac{b}{2m}t}$.

SOLVE

Part a)

$$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{\left(2.5\frac{\text{N}}{\text{m}}\right)}{0.500\text{ kg}}} = \boxed{2.24\frac{\text{rad}}{\text{s}}}$$

Part b)

$$A(t) = A_0 e^{-\frac{b}{2m}t}$$

$$b = -\frac{2m}{t}\ln\left(\frac{A(t)}{A_0}\right) = -\frac{2(0.500\text{ kg})}{3\text{ s}}\ln\left(\frac{7\text{ cm}}{15\text{ cm}}\right) = \boxed{0.254\frac{\text{kg}}{\text{s}}}$$

Part c)

$$\omega_1 = \sqrt{\omega_0^2 - \frac{b^2}{4m^2}} = \sqrt{\left(2.24\frac{\text{rad}}{\text{s}}\right)^2 - \frac{\left(0.254\frac{\text{kg}}{\text{s}}\right)^2}{4(0.500\text{ kg})^2}} = \boxed{2.23\frac{\text{rad}}{\text{s}}}$$

Part d)

$$A(t) = A_0 e^{-\frac{b}{2m}t}$$

$$t = -\frac{2m}{b}\ln\left(\frac{A(t)}{A_0}\right) = -\frac{2(0.500\text{ kg})}{\left(0.254\frac{\text{kg}}{\text{s}}\right)}\ln(0.1) = \boxed{9.1\text{ s}}$$

REFLECT

After 3 s, the amplitude has decreased by a little over 50%, so 9 s seems like a reasonable time for the amplitude to decrease by 90%.

12.85

SET UP

The motion of a damped pendulum of length $L = 1.25$ m is described by $x(t) = Ae^{-\frac{a}{2L}t}\cos(\omega_1 t)$, where $\omega_1 = \sqrt{\frac{g}{L} - \frac{a^2}{4L^2}}$ and $a = 5$ m/s. We can directly calculate the percent difference between the natural frequency $\omega_0 = \sqrt{\frac{g}{L}}$ and the frequency of oscillation, ω_1. The amplitude of the oscillation as a function of time is $A(t) = Ae^{-\frac{a}{2L}t}$; comparing the amplitude at $t = 0$ s and $t = 2$ s will tell us the fraction by which the amplitude has decreased.

SOLVE

Part a)

$$\omega_0 = \sqrt{\frac{g}{L}} = \sqrt{\frac{\left(9.8\frac{\text{m}}{\text{s}^2}\right)}{1.25\text{ m}}} = 2.8\frac{\text{rad}}{\text{s}^2}$$

$$\omega_1 = \sqrt{\frac{g}{L} - \frac{a^2}{4L^2}} = \sqrt{\frac{\left(9.8\frac{\text{m}}{\text{s}^2}\right)}{1.25\text{ m}} - \frac{\left(5\frac{\text{m}}{\text{s}}\right)^2}{4(1.25\text{ m})^2}} = 1.96\frac{\text{rad}}{\text{s}}$$

$$\frac{\omega_1}{\omega_0} = \frac{\left(1.96\frac{\text{rad}}{\text{s}^2}\right)}{\left(2.8\frac{\text{rad}}{\text{s}^2}\right)} = 0.7$$

The $\boxed{\text{frequency of oscillation is 30\% less}}$ than the natural frequency.

Part b)

$$\frac{A(2\text{ s})}{A(0\text{ s})} = \frac{Ae^{-\frac{a}{2L}t}}{A} = e^{-\frac{(5\frac{\text{m}}{\text{s}})}{2(1.25\text{ m})}(2\text{ s})} = e^{-4} = \boxed{0.0183}$$

REFLECT

A damping constant of $a = 5$ m/s is reasonably large, so we expect the amplitude to be much smaller after 2 s.

12.86

SET UP

A forced oscillator is driven at $f = 30$ Hz with a peak force $F_0 = 16.5$ N. The natural frequency of the system is $f_0 = 28$ Hz. The damping constant is $b = 1.25$ kg/s and the mass of the object is $m = 0.75$ kg. The amplitude of a driven oscillator is given by $A = \dfrac{F_0}{m\sqrt{(\omega_0^2 - \omega^2)^2 + \dfrac{b^2\omega^2}{m^2}}}$, where $\omega_0 = 2\pi f_0$ and $\omega = 2\pi f$.

SOLVE

Angular frequencies:

$$\omega = 2\pi f = 2\pi(30\text{ Hz}) = 188.5\frac{\text{rad}}{\text{s}}$$

$$\omega_0 = 2\pi f_0 = 2\pi(28\text{ Hz}) = 175.9\frac{\text{rad}}{\text{s}}$$

Amplitude:

$$A = \frac{F_0}{m\sqrt{(\omega_0^2 - \omega^2)^2 + \dfrac{b^2\omega^2}{m^2}}}$$

$$= \frac{(16.5\text{ N})}{(0.75\text{ kg})\sqrt{\left(\left(175.9\frac{\text{rad}}{\text{s}}\right)^2 - \left(188.5\frac{\text{rad}}{\text{s}}\right)^2\right)^2 + \dfrac{\left(1.25\frac{\text{kg}}{\text{s}}\right)^2\left(188.5\frac{\text{rad}}{\text{s}}\right)^2}{(0.75\text{ kg})^2}}}$$

$$= \boxed{4.8 \times 10^{-3}\text{ m} = 4.8\text{ mm}}$$

REFLECT

The amplitude of the motion will always be finite because this is a damped forced harmonic oscillator.

12.87

SET UP

An oscillating system has a natural frequency of $\omega_0 = 50$ rad/s and a damping coefficient of $b = 2.0$ kg/s. The system is driven by an oscillating force, $F(t) = (100\text{ N})\cos\left(\left(50\frac{\text{rad}}{\text{s}}\right)t\right)$.

From this expression, we can see that the frequency of the applied force is equal to the natural frequency of the system. The amplitude of a damped, driven oscillator is

$A = \dfrac{F_0}{m\sqrt{(\omega_0^2 - \omega^2)^2 + \dfrac{b^2\omega^2}{m^2}}}$.

SOLVE

$$A = \frac{F_0}{m\sqrt{(\omega_0^2 - \omega^2)^2 + \left(\dfrac{b\omega}{m}\right)^2}} = \frac{F_0}{m\sqrt{(\omega_0^2 - \omega_0^2)^2 + \left(\dfrac{b\omega_0}{m}\right)^2}} = \frac{F_0}{b\omega_0}$$

$$= \frac{100\text{ N}}{\left(2.0\dfrac{\text{kg}}{\text{s}}\right)\left(50\dfrac{\text{rad}}{\text{s}}\right)} = \boxed{1.0\text{ m}}$$

REFLECT

This is the maximum amplitude of the system's motion because the applied force oscillates at the natural frequency of the system.

12.88

SET UP

An object ($m = 5.0$ kg) is attached to a spring ($k = 180$ N/m). The damping coefficient of the system is $b = 0.20$ kg/s. The system is driven at a frequency of $\omega = 20$ rad/s with a peak force $F_0 = 50$ N. The amplitude of a damped, driven oscillator is $A = \dfrac{F_0}{m\sqrt{(\omega_0^2 - \omega^2)^2 + \dfrac{b^2\omega^2}{m^2}}}$. The natural frequency of a mass–spring system is given by $\omega_0 = \sqrt{\dfrac{k}{m}}$. Resonance occurs when the system is driven at the natural frequency.

SOLVE

Part a)

$$A = \frac{F_0}{m\sqrt{(\omega_0^2 - \omega^2)^2 + \dfrac{b^2\omega^2}{m^2}}} = \frac{F_0}{m\sqrt{\left(\dfrac{k}{m} - \omega^2\right)^2 + \dfrac{b^2\omega^2}{m^2}}}$$

$$= \frac{50\text{ N}}{(5.0\text{ kg})\sqrt{\left(\frac{\left(180\frac{\text{N}}{\text{m}}\right)}{5.0\text{ kg}} - \left(20\frac{\text{rad}}{\text{s}}\right)^2\right)^2 + \frac{\left(0.20\frac{\text{kg}}{\text{s}}\right)^2\left(20\frac{\text{rad}}{\text{s}}\right)^2}{(5.0\text{ kg})^2}}} = \boxed{0.027\text{ m}}$$

Part b) Resonance will occur when the oscillator is driven at the natural frequency of the system, $\boxed{\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{\left(180\frac{\text{N}}{\text{m}}\right)}{5.0\text{ kg}}} = 6\frac{\text{rad}}{\text{s}}}$.

REFLECT

The maximum amplitude at resonance is $A_{\text{max}} = \frac{F_0}{b\omega_0} = \frac{50\text{ N}}{\left(0.20\frac{\text{kg}}{\text{s}}\right)\left(6\frac{\text{rad}}{\text{s}}\right)} = 42\text{ m}.$

12.89

SET UP

The differential equation for a damped driven harmonic oscillator is $m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F(t)$. We need to show that $x(t) = A\sin(\omega t + \phi)$ is a valid solution given that $F(t) = F_0\cos(\omega t)$ by taking derivatives and simplifying the equation. In order for $x(t)$ to be a solution, it must satisfy the differential equation for all time.We can find expressions for both A and ϕ by enforcing this condition.

SOLVE

Part a)

Taking derivatives:

$$x(t) = A\sin(\omega t + \phi)$$

$$\frac{dx}{dt} = \frac{d}{dt}[A\sin(\omega t + \phi)] = A\omega\cos(\omega t + \phi)$$

$$\frac{d^2x}{dt^2} = \frac{d}{dt}[A\omega\cos(\omega t + \phi)] = -A\omega^2\sin(\omega t + \phi)$$

Differential equation:

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx \stackrel{?}{=} F(t)$$

$$m(-A\omega^2\sin(\omega t + \phi)) + b(A\omega\cos(\omega t + \phi)) + k(A\sin(\omega t + \phi)) \stackrel{?}{=} F_0\cos(\omega t)$$

$$-A\omega^2\sin(\omega t + \phi) + A\frac{b\omega}{m}\cos(\omega t + \phi) + A\frac{k}{m}\sin(\omega t + \phi) \stackrel{?}{=} \frac{F_0}{m}\cos(\omega t)$$

$$-A\omega^2\sin(\omega t+\phi)+A\frac{b\omega}{m}\cos(\omega t+\phi)+A\omega_0^2\sin(\omega t+\phi)\stackrel{?}{=}\frac{F_0}{m}\cos(\omega t)$$

$$A(\omega_0^2-\omega^2)\sin(\omega t+\phi)+A\frac{b\omega}{m}[\cos(\omega t)\cos(\phi)-\sin(\omega t)\sin(\phi)]\stackrel{?}{=}\frac{F_0}{m}\cos(\omega t)$$

$$A(\omega_0^2-\omega^2)[\sin(\omega t)\cos(\phi)+\cos(\omega t)\sin(\phi)]+A\frac{b\omega}{m}[\cos(\omega t)\cos(\phi)-\sin(\omega t)\sin(\phi)]\stackrel{?}{=}\frac{F_0}{m}\cos(\omega t)$$

$$\left[A(\omega_0^2-\omega^2)\sin(\phi)+A\frac{b\omega}{m}\cos(\phi)-\frac{F_0}{m}\right]\cos(\omega t)+A\left[(\omega_0^2-\omega^2)\cos(\phi)-\frac{b\omega}{m}\sin(\phi)\right]\sin(\omega t)\stackrel{?}{=}0$$

This will only be true for all time if the coefficients to $\cos(\omega t)$ and $\sin(\omega t)$ are equal to zero.

Part b)

$$(\omega_0^2-\omega^2)\cos(\phi)-\frac{b\omega}{m}\sin(\phi)=0$$

$$\tan(\phi)=\frac{\omega_0^2-\omega^2}{\left(\frac{b\omega}{m}\right)}$$

$$\boxed{\phi=\arctan\left(\frac{\omega_0^2-\omega^2}{\left(\frac{b\omega}{m}\right)}\right)}$$

$$A(\omega_0^2-\omega^2)\sin(\phi)+A\frac{b\omega}{m}\cos(\phi)-\frac{F_0}{m}=0$$

$$A\left[(\omega_0^2-\omega^2)\sin(\phi)+\frac{b\omega}{m}\cos(\phi)\right]=\frac{F_0}{m}$$

$$A\left[(\omega_0^2-\omega^2)\left(\frac{\tan(\phi)}{\sqrt{1+\tan^2(\phi)}}\right)+\frac{b\omega}{m}\left(\frac{1}{\sqrt{1+\tan^2(\phi)}}\right)\right]=\frac{F_0}{m}$$

$$\frac{A}{\sqrt{1+\tan^2(\phi)}}\left[(\omega_0^2-\omega^2)\tan(\phi)+\frac{b\omega}{m}\right]=\frac{F_0}{m}$$

$$\frac{A}{\sqrt{1+\left(\frac{\omega_0^2-\omega^2}{\left(\frac{b\omega}{m}\right)}\right)^2}}\left[(\omega_0^2-\omega^2)\left(\frac{\omega_0^2-\omega^2}{\left(\frac{b\omega}{m}\right)}\right)+\frac{b\omega}{m}\right]=\frac{F_0}{m}$$

$$\frac{A}{\left(\frac{b\omega}{m}\right)\sqrt{1+\left(\frac{\omega_0^2-\omega^2}{\left(\frac{b\omega}{m}\right)}\right)^2}}\left[(\omega_0^2-\omega^2)^2+\left(\frac{b\omega}{m}\right)^2\right]=\frac{F_0}{m}$$

$$\frac{A}{\sqrt{\left(\frac{b\omega}{m}\right)^2 + (\omega_0^2 - \omega^2)^2}} = \frac{F_0}{m\left[(\omega_0^2 - \omega^2)^2 + \left(\frac{b\omega}{m}\right)^2\right]}$$

$$\boxed{A = \frac{F_0}{m\sqrt{(\omega_0^2 - \omega^2)^2 + \left(\frac{b\omega}{m}\right)^2}}}$$

REFLECT

The angular frequency ω is the frequency of the driving force, while $\omega_0 = \sqrt{\frac{k}{m}}$ is the natural frequency of the oscillator.

12.90

SET UP

The quality factor $Q = \frac{m\omega_0}{b}$ is a parameter that describes the width and height of the resonant peak for a driven harmonic oscillator. The "width" of the resonant peak, which we'll call the full-width half-max (FWHM), is given by $\Delta\omega = \frac{\omega_0}{Q}$. The natural frequency of a mass–spring system is given by $\omega_0 = \sqrt{\frac{k}{m}}$. The numerical values given are $m = 0.100$ kg, $b =$ 0.2 kg/s, $F_0 = 7.5$ N, and $k = 25$ N/m.

SOLVE

Quality factor:

$$Q = \frac{m\omega_0}{b} = \frac{m}{b}\sqrt{\frac{k}{m}} = \frac{\sqrt{km}}{b} = \frac{\sqrt{\left(25\frac{\text{N}}{\text{m}}\right)(0.100\text{ kg})}}{\left(0.2\frac{\text{kg}}{\text{s}}\right)} = \boxed{7.91}$$

Full-width half-max:

$$\text{FWHM} = \Delta\omega = \frac{\omega_0}{Q} = \frac{1}{Q}\sqrt{\frac{k}{m}} = \frac{1}{7.91}\sqrt{\frac{\left(25\frac{\text{N}}{\text{m}}\right)}{0.100\text{ kg}}} = \boxed{2.00\frac{\text{rad}}{\text{s}}}$$

REFLECT

A large Q means the effects of damping are small and it will take longer for the oscillations to die out.

12.91

SET UP

The acceleration of an object ($m = 0.025$ kg) that exhibits simple harmonic motion is given by $a(t) = \left(10\frac{\text{m}}{\text{s}^2}\right)\cos\left(\pi t + \frac{\pi}{2}\right)$. We can determine the expression for the object's velocity $v(t)$ by integrating $a(t)$ with respect to time. We are told that the object's speed is a maximum at $t = 0$. Once we have the algebraic expression for $v(t)$, we can calculate $v(t = 2\text{ s})$.

SOLVE

$$v(t) = \int a(t)dt = \int 10\cos\left(\pi t + \frac{\pi}{2}\right)dt = \frac{10}{\pi}\sin\left(\pi t + \frac{\pi}{2}\right) + C \text{ (SI units)}$$

The maximum speed of an object undergoing simple harmonic motion is equal to its amplitude, which is $\frac{10}{\pi}\frac{\text{m}}{\text{s}}$ in this case. Therefore, $C = 0$, so

$$v(t) = \frac{10}{\pi}\sin\left(\pi t + \frac{\pi}{2}\right) \text{ (SI units)}$$

Velocity at $t = 2$ s:

$$v(2) = \frac{10}{\pi}\sin\left(2\pi + \frac{\pi}{2}\right) = \boxed{\frac{10}{\pi}\frac{\text{m}}{\text{s}} = 3.18\frac{\text{m}}{\text{s}}}$$

REFLECT

The general equation of the acceleration in terms of the period of the oscillation is $a(t) = a_0\cos\left(\frac{2\pi t}{T} + \phi\right)$. The period of the oscillation in this problem is $T = 2$ s, which means the velocity at $t = 0$ s will be equal to the velocity at $t = 2$ s.

12.92

SET UP

We are asked to explicitly show that the formulas for the period of an object on a spring and a simple pendulum are dimensionally correct. The dimensions of the spring constant k are mass per time squared.

SOLVE

Spring:

$$[T] = 2\pi\sqrt{\frac{[m]}{[k]}}$$

$$[\text{T}] \stackrel{?}{=} \sqrt{\frac{[\text{M}]}{\left(\frac{[\text{M}]}{[\text{T}]^2}\right)}}$$

$$[\text{T}] \stackrel{?}{=} \sqrt{[\text{T}]^2} = [\text{T}]$$

Pendulum:

$$[T] = 2\pi\sqrt{\frac{[L]}{[g]}}$$

$$[\mathrm{T}] \stackrel{?}{=} \sqrt{\frac{[\mathrm{L}]}{\left(\frac{[\mathrm{L}]}{[\mathrm{T}]^2}\right)}}$$

$$[\mathrm{T}] \stackrel{?}{=} \sqrt{[\mathrm{T}]^2} = [\mathrm{T}]$$

REFLECT

The dimensions of the spring constant can also be written as force per distance.

12.93

SET UP

An object ($m = 0.200$ kg) attached to a spring ($k = 75$ N/m) is pulled $A = 0.08$ m to the right of equilibrium and released from rest. It begins to oscillate on a horizontal, frictionless table. The general equation for the motion of the object as a function of time is $x(t) = A\cos(\omega t)$, where $\omega = \sqrt{\frac{k}{m}}$. The velocity of the object as a function of time is equal to the first derivative of $x(t)$ with respect to time; the maximum speed is the amplitude of this function. Using the general form for $v(t)$, we can find the time when the speed is equal to one-third of its maximum value and then plug this time into the expression for $x(t)$ to find the object's position.

SOLVE

Part a)

$$x(t) = A\cos(\omega t)$$

$$v = \frac{dx}{dt} = \frac{d}{dt}[A\cos(\omega t)] = -A\omega\sin(\omega t)$$

$$v_{\max} = A\omega = A\sqrt{\frac{k}{m}} = (0.08\ \mathrm{m})\sqrt{\frac{\left(75\frac{\mathrm{N}}{\mathrm{m}}\right)}{0.200\ \mathrm{kg}}} = \boxed{1.55\frac{\mathrm{m}}{\mathrm{s}}}$$

Part b)

Finding the time:

$$v(t) = \frac{v_{\max}}{3} = \frac{A\omega}{3} = -A\omega\sin(\omega t)$$

$$\sin(\omega t) = -\frac{1}{3}$$

$$t = \frac{1}{\omega}\arcsin\left(-\frac{1}{3}\right) = \sqrt{\frac{m}{k}}\arcsin\left(-\frac{1}{3}\right) = \sqrt{\frac{0.200\text{ kg}}{\left(75\frac{\text{N}}{\text{m}}\right)}}(3.48\text{ rad}) = 0.180\text{ s}$$

Finding the position:

$$x(0.180\text{ s}) = (0.08\text{ m})\cos\left(\sqrt{\frac{\left(75\frac{\text{N}}{\text{m}}\right)}{0.200\text{ kg}}}(0.180\text{ s})\right) = \boxed{-0.0754\text{ m}}$$

REFLECT

We could have also used conservation of mechanical energy to find the answer to part (b). The kinetic energy of the object when the speed is one-third its maximum value is $K = \frac{1}{2}m\left(\frac{1}{3}v_{\text{max}}\right)^2 = \frac{1}{9}\left(\frac{1}{2}mv_{\text{max}}^2\right) = \frac{1}{9}E_{\text{total}}$, which means the potential energy is $U = \frac{8}{9}E_{\text{total}} = \frac{8}{9}\left(\frac{1}{2}kA^2\right)$. By setting this equal to $\frac{1}{2}kx^2$, we can solve for the position of the object.

12.94

SET UP

A 0.100-kg object attached to the end of a spring ($k = 15$ N/m) is pulled 15 cm to the right and released from rest at $t = 0$. The object then undergoes simple harmonic motion on a horizontal, frictionless table. The motion of the object can be described by $x(t) = A\cos(\omega t)$, where $A = 15$ cm and $\omega = \sqrt{\frac{k}{m}}$. We can solve for the times when the object is at the equilibrium position by setting $x(t) = 0$. We can follow a similar process to find the times when the object is 10 cm to the left of equilibrium and 5 cm to the right of equilibrium. Positions to the right of the equilibrium position correspond to positive values of x, while positions to the left of the equilibrium position correspond to negative values of x.

SOLVE

Angular frequency:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{\left(15\frac{\text{N}}{\text{m}}\right)}{0.100\text{ kg}}} = 12.2\frac{\text{rad}}{\text{s}}$$

Equation of motion:

$$x(t) = A\cos(\omega t) = (15\text{ cm})\cos\left(\left(12.2\frac{\text{rad}}{\text{s}}\right)t\right)$$

Part a)

$$x(t) = 0 = (15\text{ cm})\cos\left(\left(12.2\frac{\text{rad}}{\text{s}}\right)t\right)$$

$$\cos\left(\left(12.2\frac{\text{rad}}{\text{s}}\right)t\right) = 0$$

$$t = \frac{\arccos(0)}{\left(12.2\frac{\text{rad}}{\text{s}}\right)} = \frac{\left(\frac{\pi}{2}\right)}{\left(12.2\frac{\text{rad}}{\text{s}}\right)}, \frac{\left(\frac{3\pi}{2}\right)}{\left(12.2\frac{\text{rad}}{\text{s}}\right)}, \frac{\left(\frac{5\pi}{2}\right)}{\left(12.2\frac{\text{rad}}{\text{s}}\right)}$$

$$\boxed{t = 0.129\text{ s}, 0.386\text{ s}, 0.644\text{ s}}$$

Part b)

$$x(t) = (-10\text{ cm}) = (15\text{ cm})\cos\left(\left(12.2\frac{\text{rad}}{\text{s}}\right)t\right)$$

$$t = \frac{1}{\left(12.2\frac{\text{rad}}{\text{s}}\right)}\arccos\left(\frac{-2}{3}\right) = \frac{2.30\text{ rad}}{\left(12.2\frac{\text{rad}}{\text{s}}\right)}, \frac{3.98\text{ rad}}{\left(12.2\frac{\text{rad}}{\text{s}}\right)}, \frac{8.58\text{ rad}}{\left(12.2\frac{\text{rad}}{\text{s}}\right)}$$

$$\boxed{t = 0.189\text{ s}, 0.326\text{ s}, 0.704\text{ s}}$$

Part c)

$$x(t) = (5\text{ cm}) = (15\text{ cm})\cos\left(\left(12.2\frac{\text{rad}}{\text{s}}\right)t\right)$$

$$t = \frac{1}{\left(12.2\frac{\text{rad}}{\text{s}}\right)}\arccos\left(\frac{1}{3}\right) = \boxed{0.101\text{ s}}$$

REFLECT

According to our results, the object is at $x = -10$ cm after it passes through the equilibrium position once but before it passes through it a second time, which makes sense. Recall that cosine is positive in quadrants I and IV and negative in II and III.

12.95

SET UP

A damped oscillator shows a reduction of 30% in amplitude after two periods. We can use the amplitude of a damped oscillator as a function of time, $A(t) = A_0 e^{-\frac{b}{2m}t}$, to find the reduction in amplitude and the mechanical energy loss after one period.

SOLVE

Amplitude after one period:

$$\frac{A_{2T}}{A_0} = e^{-\frac{b}{2m}(2T)} = \left(e^{-\frac{b}{2m}(T)}\right)^2 = \left(\frac{A_T}{A_0}\right)^2 = 0.7$$

Percent loss in mechanical energy per cycle:

$$\left(\frac{E_T - E_0}{E_0}\right) = \frac{\left(\frac{1}{2}kA_T^2\right) - \left(\frac{1}{2}kA_0^2\right)}{\left(\frac{1}{2}kA_0^2\right)} = \left(\frac{A_T}{A_0}\right)^2 - 1 = (0.837)^2 - 1 = -0.3$$

There is a $\boxed{30\% \text{ loss}}$ of mechanical energy per cycle.

REFLECT

The change in mechanical energy of the system is equal to the work done by all nonconservative forces on the system.

12.96

SET UP

A 1.20-kg object attached to a massless spring oscillates on a frictionless, horizontal surface. We are given a graph of its position as a function of time. The motion for the object, $x(t)$, can be modeled using a sine function since the object starts at its equilibrium position at $t = 0$. The time it takes to repeat the motion is equal to the period, which we can use to find the angular frequency and frequency of the oscillation. From the angular frequency and the mass of the object we can calculate the spring constant. The maximum speed of the object is equal to the amplitude of $v(t)$, the first derivative of $x(t)$ with respect to time. The maximum acceleration of the object is equal to the amplitude of $a(t)$, the second derivative of $x(t)$ with respect to time.

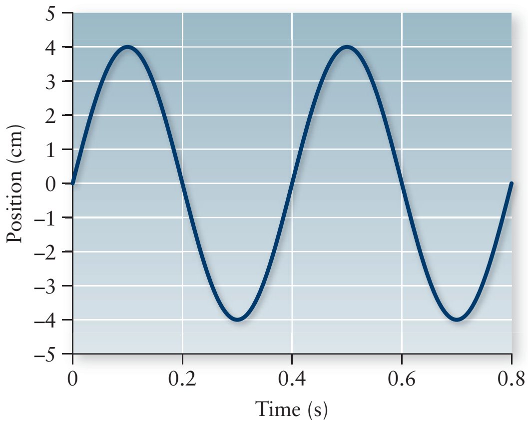


Figure 12-11 Problem 96

SOLVE

Part a)

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.4\ \text{s}} = \boxed{15.7\frac{\text{rad}}{\text{s}}}$$

$$f = \frac{1}{T} = \frac{1}{0.4\ \text{s}} = \boxed{2.5\frac{\text{rad}}{\text{s}}}$$

Part b)

$$\omega = \sqrt{\frac{k}{m}}$$

$$k = \omega^2 m = \left(15.7\frac{\text{rad}}{\text{s}}\right)^2 (1.20\ \text{kg}) = \boxed{296\frac{\text{kg}}{\text{s}^2}}$$

Part c)

$$x(t) = A\sin(\omega t)$$

$$v(t) = \frac{dx}{dt} = \frac{d}{dt}[A\sin(\omega t)] = A\omega\cos(\omega t)$$

$$v_{\text{max}} = A\omega = (4\ \text{cm})\left(15.7\frac{\text{rad}}{\text{s}}\right) = \boxed{62.8\frac{\text{cm}}{\text{s}}}$$

Part d)

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}[A\omega\cos(\omega t)] = -A\omega^2\sin(\omega t)$$

$$a_{\text{max}} = A\omega^2 = (4\ \text{cm})\left(15.7\frac{\text{rad}}{\text{s}}\right)^2 = \boxed{986\frac{\text{cm}}{\text{s}^2} = 9.86\frac{\text{m}}{\text{s}^2}}$$

REFLECT

Since we are looking for the maximum speed, we can ignore the minus sign. The maximum speeds occur when the object is passing through its equilibrium position. The maximum accelerations occur when the object is at its maximum distance from equilibrium.

12.97

SET UP

A rod pendulum has a mass m_0 and length $L_0 = 0.85$ m and hangs from one end. We can equate the expressions for the period of a physical pendulum and the period of a simple pendulum in order to solve for the length L_1 of a simple pendulum that has the same period.

A second rod pendulum of length L_2 is hanging from a point that is 5 cm from its end. We want to find L_2 such that the two rod pendulums have the same period. Since the center of mass of the rod is located a distance $h = \frac{L}{2} - (5\text{ cm})$ from the pivot point, we will need to use the parallel-axis theorem. The moment of inertia of a rod rotating about its end is $\frac{mL}{3}$ while the moment of inertia of a rod rotating about its center of mass is $\frac{mL}{12}$.

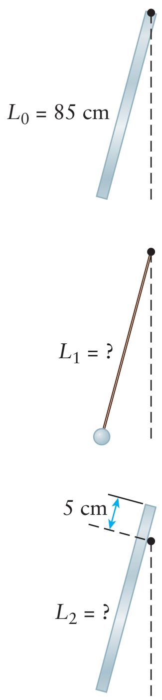


Figure 12-12 Problem 97

SOLVE

Part a)

$$2\pi\sqrt{\frac{I_0}{m_0 g h_0}} = 2\pi\sqrt{\frac{L_1}{g}}$$

$$\sqrt{\frac{\left(\frac{m_0 L_0^2}{3}\right)}{m_0\left(\frac{L_0}{2}\right)}} = \sqrt{L_1}$$

$$\sqrt{\frac{2L_0}{3}} = \sqrt{L_1}$$

$$L_1 = \frac{2}{3}L_0 = \frac{2}{3}(0.85\text{ m}) = \boxed{0.57\text{ m}}$$

Part b)

$$2\pi\sqrt{\frac{I_0}{m_0gh_0}} = 2\pi\sqrt{\frac{I_2}{m_2gh_2}}$$

$$\sqrt{\frac{\left(\frac{m_0L_0^2}{3}\right)}{m_0\left(\frac{L_0}{2}\right)}} = \sqrt{\frac{\left(\frac{m_2L_2^2}{12} + m_2h_2^2\right)}{m_2h_2}}$$

$$\sqrt{\frac{2L_0}{3}} = \sqrt{\frac{\left(\frac{L_2^2}{12} + \left(\left(\frac{L_2}{2}\right) - (0.05\text{ m})\right)^2\right)}{\left(\left(\frac{L_2}{2}\right) - (0.05\text{ m})\right)}}$$

Dropping the units and plugging in L_0 for simplicity:

$$\frac{2(0.85)}{3} = \frac{\left(\frac{L_2^2}{12} + \frac{L_2^2}{4} + (0.05)^2 - 0.05L_2\right)}{\left(\left(\frac{L_2}{2}\right) - (0.05\text{ m})\right)}$$

$$0.283L_2 - 0.0283 = \frac{L_2^2}{3} + (0.05)^2 - 0.05L_2$$

$$\frac{L_2^2}{3} - 0.333L_2 + 0.0308 = 0$$

$$L_2^2 - L_2 + 0.0924 = 0$$

$$L_2 = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(0.0924)}}{2(1)} = \frac{1 \pm \sqrt{0.6304}}{2}$$

Taking the positive root:

$$L_2 = \frac{1 + \sqrt{0.6304}}{2} = \boxed{0.90\text{ m}}$$

REFLECT

The negative root in the quadratic formula gives $L_2 = 0.103$ m, which is unphysical since it would hardly be long enough to pivot about a point 5 cm from its end.

12.98

SET UP

A block of wood of cross-sectional area A and mass m floats at rest in a basin of water (density ρ_w). If the block is pushed down a small distance Δx and released, it will undergo oscillatory motion. When the block is pushed down, it displaces more water and the

magnitude of the buoyant force increases by an amount $\rho_w(\Delta V)g$, where $\Delta V = A\Delta x$; the direction of this additional buoyant force is opposite the direction of Δx. We can rearrange Newton's second law for the block and compare it with the standard differential equation for simple harmonic motion to find an expression for ω and then the period T.

SOLVE

$$\sum F = F_{\text{b, additional}} = -\rho_w(\Delta V)g = -\rho_w(A\Delta x)g = ma = m\frac{d^2x}{dt^2}$$

$$m\frac{d^2x}{dt^2} + \rho_w(A\Delta x)g = 0$$

$$\frac{d^2x}{dt^2} + \frac{\rho_w A g}{m}(\Delta x) = 0$$

$$\omega^2 = \frac{\rho_w A g}{m}$$

$$T = \frac{2\pi}{\omega} = \boxed{2\pi\sqrt{\frac{m}{\rho_w A g}}}$$

REFLECT

We can check the dimensions of our algebraic expression to make sure our answer is reasonable:

$$[T] \stackrel{?}{=} \sqrt{\frac{[m]}{[\rho_w][A][g]}}$$

$$[\mathrm{T}] \stackrel{?}{=} \sqrt{\frac{[\mathrm{M}]}{\frac{[\mathrm{M}]}{[\mathrm{L}]^3}[\mathrm{L}]^2\frac{[\mathrm{L}]}{[\mathrm{T}]^2}}}$$

$$[\mathrm{T}] \stackrel{?}{=} \sqrt{[\mathrm{T}]^2} = [\mathrm{T}]$$

12.99

SET UP

The frequency of a hummingbird's wing flaps was measured to be 53 flaps per second; the number of flaps per second is equal to the frequency of the wing motion in hertz. A typical wing is 0.045 m long and rotates through a total angle of $\pi/2$ radians. We will assume the wings undergo simple harmonic motion while flapping. The period of the wing motion is

equal to the reciprocal of the frequency, and the angular frequency ω_0 of the motion is equal to the frequency multiplied by 2π. Once we have the angular frequency of the motion, we can write the equation describing the angular position of the wing as a function of time, $\theta(t)$. Differentiating this expression with respect to time will give the angular velocity as a function of time, the amplitude of which will be the maximum angular speed. The maximum linear speed of the tip of the wing is equal to the maximum angular speed multiplied by the length of the wing.

SOLVE

Part a)

$$T = \frac{1}{f} = \frac{1}{(53\ \text{Hz})} = \boxed{0.019\ \text{s} = 19\ \text{ms}}$$

Part b) The frequency was given as $\boxed{f = 53\ \text{Hz.}}$

Part c)

$$\omega_0 = 2\pi f = 2\pi(53\ \text{Hz}) = \boxed{333\frac{\text{rad}}{\text{s}}}$$

Part d)

$$\theta(t) = \theta_0\cos(\omega_0 t)$$

$$\omega(t) = \frac{d\theta}{dt} = \frac{d}{dt}[\theta_0\cos(\omega_0 t)] = -\theta_0\omega_0\sin(\omega_0 t)$$

$$\omega_{\text{max}} = \theta_0\omega_0$$

$$v_{\text{max}} = L\omega_{\text{max}} = L\theta_0\omega_0 = (0.045\ \text{m})\left(\frac{\pi}{4}\text{rad}\right)\left(333\frac{\text{rad}}{\text{s}}\right)$$

$$= \boxed{12\frac{\text{m}}{\text{s}}} \times \frac{2.2\ \text{mph}}{\left(1\frac{\text{m}}{\text{s}}\right)} = \boxed{26\ \text{mph}}$$

REFLECT

If the wings sweep through a total angle of 90 degrees ($\pi/2$ radians), then the amplitude of the motion is 45 degrees ($\pi/4$ radians).

12.100

SET UP

The total energy of an oscillating spring–mass system is equal to $\frac{1}{2}kA^2$. It is also equal to the elastic potential energy stored in the spring plus the kinetic energy of the moving object. Setting these equal, we can solve for an algebraic expression for the speed v as a function of x. In part (b) we are told the values of the physical parameters ($M = 0.250$ kg, $k = 85$ N/m, $A = 0.10$ m) in order to calculate the speed at various given positions.

SOLVE

Part a)

$$U + K = E$$

$$\frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kA^2$$

$$\boxed{v = \sqrt{\frac{k}{m}(A^2 - x^2)}}$$

Part b)

$$v(0\text{ m}) = \sqrt{\frac{\left(85\frac{\text{N}}{\text{m}}\right)}{0.250\text{ kg}}((0.10\text{ m})^2 - (0)^2)} = \boxed{1.84\frac{\text{m}}{\text{s}}}$$

$$v(0.02\text{ m}) = \sqrt{\frac{\left(85\frac{\text{N}}{\text{m}}\right)}{0.250\text{ kg}}((0.10\text{ m})^2 - (0.02\text{ m})^2)} = \boxed{1.81\frac{\text{m}}{\text{s}}}$$

$$v(0.05\text{ m}) = \sqrt{\frac{\left(85\frac{\text{N}}{\text{m}}\right)}{0.250\text{ kg}}((0.10\text{ m})^2 - (0.05\text{ m})^2)} = \boxed{1.60\frac{\text{m}}{\text{s}}}$$

$$v(0.08\text{ m}) = \sqrt{\frac{\left(85\frac{\text{N}}{\text{m}}\right)}{0.250\text{ kg}}((0.10\text{ m})^2 - (0.08\text{ m})^2)} = \boxed{1.11\frac{\text{m}}{\text{s}}}$$

$$v(0.10\text{ m}) = \sqrt{\frac{\left(85\frac{\text{N}}{\text{m}}\right)}{0.250\text{ kg}}((0.10\text{ m})^2 - (0.10\text{ m})^2)} = \boxed{0\frac{\text{m}}{\text{s}}}$$

REFLECT

The speed is the magnitude of the velocity, which must be positive. The block is traveling the fastest as it passes through the equilibrium position ($x = 0$) and has a speed of zero when it turns around ($x = A$).

12.101

SET UP

A spinning golf ball of radius R can be suspended in a stream of high-velocity air. The ball is in equilibrium at the vertical center of the stream, which we'll call $y = 0$. The forces acting on the ball are the net pressure acting up and the force of gravity pointing down; at equilibrium, these forces balance. There is a difference in pressure due to the flowing air, which we can relate to the speed of the air above and below the ball using Bernoulli's equation. The ball is spinning at a speed v_{rot}. This will cause the air to be dragged along with the spinning ball, which affects the speed of the air above and below the ball. Also, the speed of the air drops off linearly from the center of the stream, that is, $v(y) = v_0 - b|y|$, where v_0 is the speed of the air

in the center and b is a constant. If the ball is displaced a small amount Δy below equilibrium, the ball will no longer be in equilibrium, and there will be a net force upward on the ball since the ball is located in a different part of the air stream now. In order to show that the ball undergoes simple harmonic motion in this case, we need to show that the net force on the ball has the form of Hooke's law: The net force is proportional to the distance the ball is displaced from equilibrium.

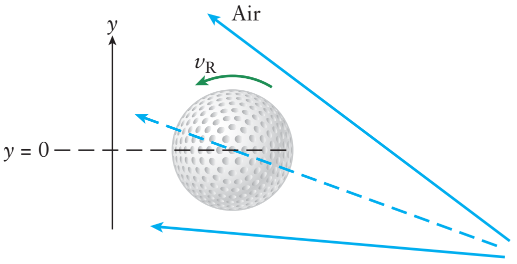


Figure 12-13 Problem 101

SOLVE

Free-body diagram of the spinning golf ball:

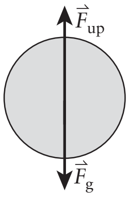


Figure 12-14 Problem 101

Magnitude of $\vec{F}_{up}$:

$$F_{up} = (P_{below} - P_{above})A$$

Using Bernoulli's equation, $P_{above} + \frac{1}{2}\rho v_{above}^2 = P_{below} + \frac{1}{2}\rho v_{below}^2$:

$$F_{up} = (P_{below} - P_{above})A = \left(\frac{1}{2}\rho v_{above}^2 - \frac{1}{2}\rho v_{below}^2\right)A = \frac{1}{2}\rho A(v_{above}^2 - v_{below}^2)$$

Newton's second law at equilibrium:

$$\sum F_y = F_{up} - F_g = \frac{1}{2}\rho A(v_{above}^2 - v_{below}^2) - mg = ma_y = 0$$

$$\frac{1}{2}\rho A(v_{above}^2 - v_{below}^2) = mg$$

But the speeds above and below the ball are $v_{above} = v(+R) + v_{rot} = (v_0 - bR) + v_{rot}$ and $v_{below} = v(-R) - v_{rot} = (v_0 - b|-R|) - v_{rot} = (v_0 - bR) - v_{rot}$. Plugging these into our expression from Newton's second law:

$$\frac{1}{2}\rho A(v_{above}^2 - v_{below}^2) = \frac{1}{2}\rho A((v_0 - bR + v_{rot})^2 - (v_0 - bR - v_{rot})^2) = mg$$

$$\frac{1}{2}\rho A(v_0^2 - 2v_0bR - 2v_{\text{rot}}bR + 2v_0v_{\text{rot}} + b^2R^2 + v_{\text{rot}}^2 - (v_0^2 - 2v_0bR + 2v_{\text{rot}}bR - 2v_0v_{\text{rot}} + b^2R^2 + v_{\text{rot}}^2)) = mg$$

$$\frac{1}{2}\rho A(4v_0v_{\text{rot}} - 4v_{\text{rot}}bR) = mg$$

If the ball is displaced downward an amount Δy, the upward force no longer equals mg, which results in a vertical net force upward:

$$\sum F_y = F_{\text{up}} - mg = \frac{1}{2}\rho A(v_{\text{above}}^2 - v_{\text{below}}^2) - mg$$

But the speeds above and below the ball are $v_{\text{above}} = v(R - \Delta y) + v_{\text{rot}} = (v_0 - b(R - \Delta y)) + v_{\text{rot}}$ and $v_{\text{below}} = v(-R - \Delta y) - v_{\text{rot}} = (v_0 - b|-R - \Delta y|) - v_{\text{rot}} = (v_0 - b(R + \Delta y)) - v_{\text{rot}}$. Plugging these into our expression from Newton's second law:

$$\frac{1}{2}\rho A((v_0 - bR + b(\Delta y) + v_{\text{rot}})^2 - (v_0 - bR - b(\Delta y) - v_{\text{rot}})^2) - mg$$

$$= \frac{1}{2}\rho A(4v_0b(\Delta y) + 4v_0v_{\text{rot}} - 4v_{\text{rot}}bR - 4b^2R(\Delta y)) - mg$$

$$= \frac{1}{2}\rho A(4v_0b(\Delta y) + 4v_0v_{\text{rot}} - 4v_{\text{rot}}bR - 4b^2R(\Delta y)) - \left[\frac{1}{2}\rho A(4v_0v_{\text{rot}} - 4v_{\text{rot}}bR)\right]$$

$$= \frac{1}{2}\rho A(4v_0b(\Delta y) - 4b^2R(\Delta y)) = -2\rho Ab(bR - v_0)(\Delta y)$$

This has the form $\sum F_y = -k(\Delta y)$, which is Hooke's law.

REFLECT

As with a mass on a spring, we can interpret the coefficient in front of Δy as a spring constant of sorts and relate it to the frequency of oscillation of the ball.

12.102

SET UP

We are asked to find a "correction" to the period of a pendulum due to each 1-km change in altitude. A change in altitude will correspond to a change in the local gravitational field. In general, the period of a pendulum is inversely proportional to the gravitational acceleration at that height $\left(T \propto \sqrt{\frac{1}{g}}\right)$. We will define the altitude to be y and the period at sea level $(y = 0)$ as T_0. The gravitational acceleration g as a function of y is given by $g(y) = g_0\left(\frac{R_{\text{E}}}{R_{\text{E}} + y}\right)^2$.

SOLVE

Difference in period:

$$\Delta T = T_y - T_0$$

Percent difference:

$$\frac{\Delta T}{T_0} = \frac{T_y}{T_0} - 1 = \frac{\sqrt{\frac{1}{g_y}}}{\sqrt{\frac{1}{g_0}}} - 1 = \sqrt{\frac{g_0}{g_y}} - 1 = \sqrt{\frac{g_0}{g_0\left(\frac{R_E}{R_E + y}\right)^2}} - 1 = \frac{R_E + y}{R_E} - 1 = \frac{y}{R_E}$$

Percent difference for each 1-km increase or decrease in elevation:

$$\boxed{\frac{\Delta T}{T_0} = \frac{y}{(6380\text{ km})}}$$

REFLECT

This expression should hold for both simple and physical pendulums since the period is inversely proportional to the square root of g in both cases. As expected, the correction is small; on top of Mt. Everest ($y = 8.8$ km), the period increases by 0.1%.

12.103

SET UP

A simple pendulum made of a thin bar of length L and a small ball of mass M has a period of $T_0 = 2.00$ s. We will treat the ball as a point mass. We can use this information to calculate L. In part (a) the bar has the same mass as the ball, which means we need to treat it as a physical pendulum. The center of mass of the pendulum is located a distance $h = (3L/4)$ from the pivot. The moment of inertia of the pendulum is the sum of the moments of inertia for the ball (ML^2) and the thin bar rotating about one end $\left(\frac{1}{3}ML^2\right)$. In part (b), we remove the ball, which changes the total mass of the pendulum; the location of the center of mass, and, thus, h; and the moment of inertia of the pendulum.

SOLVE

Length of the bar:

$$T_0 = 2\pi\sqrt{\frac{L}{g}}$$

$$L = \frac{T_0^2 g}{4\pi^2}$$

Part a)

$$T = 2\pi\sqrt{\frac{I_{\text{total}}}{m_{\text{total}}gh}} = 2\pi\sqrt{\frac{I_{\text{ball}} + I_{\text{bar}}}{(2M)g\left(\frac{3L}{4}\right)}} = 2\pi\sqrt{\frac{(ML^2) + \left(\frac{1}{3}ML^2\right)}{Mg\left(\frac{3L}{2}\right)}}$$

$$= 2\pi\sqrt{\frac{8L}{9g}} = 2\pi\sqrt{\frac{8\left(\frac{T_0^2 g}{4\pi^2}\right)}{9g}} = T_0\sqrt{\frac{8}{9}} = (2.00\text{ s})\sqrt{\frac{8}{9}} = \boxed{1.89\text{ s}}$$

Part b)

$$T = 2\pi\sqrt{\frac{\left(\frac{1}{3}ML^2\right)}{Mg\left(\frac{L}{2}\right)}} = 2\pi\sqrt{\frac{2L}{3g}} = 2\pi\sqrt{\frac{2\left(\frac{T_0^2 g}{4\pi^2}\right)}{3g}} = T_0\sqrt{\frac{2}{3}} = \boxed{1.63\text{ s}}$$

REFLECT

The periods of the physical pendulums are less than the period of the simple pendulum, which makes sense.

12.104

SET UP

A block ($m_{\text{block}} = 2.2$ kg) is resting on a horizontal frictionless surface and attached to a spring ($k = 80$ N/m). A bullet ($m_{\text{bullet}} = 0.010$ kg) is fired at an initial speed of $v_{\text{i}} = 150$ m/s and embeds itself into the block. The period of the resulting simple harmonic motion is equal to $T = 2\pi\sqrt{\frac{m_{\text{bullet}} + m_{\text{block}}}{k}}$. We can use conservation of momentum to find the initial speed of the bullet + block system and then conservation of mechanical energy to calculate the maximum compression of the spring. The maximum compression of the spring is equal to the amplitude of the simple harmonic motion.

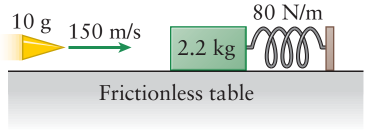


Figure 12-15 Problem 104

SOLVE

Period of motion:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m_{\text{bullet}} + m_{\text{block}}}{k}} = 2\pi\sqrt{\frac{(0.010\text{ kg}) + (2.2\text{ kg})}{\left(80\frac{\text{N}}{\text{m}}\right)}} = \boxed{1.04\text{ s}}$$

Conservation of momentum:

$$m_{\text{bullet}}v_{\text{i}} = (m_{\text{bullet}} + m_{\text{block}})v_{\text{f}}$$

$$v_{\text{f}} = \frac{m_{\text{bullet}}v_{\text{i}}}{m_{\text{bullet}} + m_{\text{block}}}$$

Conservation of energy:

$$K_{\text{i}} = U_{\text{elastic, f}}$$

$$\frac{1}{2}(m_{\text{bullet}} + m_{\text{block}})\left(\frac{m_{\text{bullet}}v_{\text{i}}}{m_{\text{bullet}} + m_{\text{block}}}\right)^2 = \frac{1}{2}kA^2$$

$$A = m_{\text{bullet}}v_{\text{i}}\sqrt{\frac{1}{k(m_{\text{bullet}} + m_{\text{block}})}}$$

$$= (0.010\ \text{kg})\left(150\frac{\text{m}}{\text{s}}\right)\sqrt{\frac{1}{\left(80\frac{\text{N}}{\text{m}}\right)((0.010\ \text{kg}) + (2.2\ \text{kg}))}} = \boxed{0.11\ \text{m}}$$

REFLECT
The period of the motion is independent of the speed of the bullet. The speed of the bullet only affects the amplitude of the motion.

12.105

SET UP
Two identical objects are released from rest at the same height y_0 on either side of a symmetric, frictionless ramp. They collide elastically at a distance $x_0/2$ from the bottom of each ramp. The time from when the objects are released to when they collide is equal to half of a period. We can split this time interval into two parts: the time it takes the object to travel from its initial position to the bottom of the ramp and the time from the bottom of the ramp until the collision. The distance d the object travels down the ramp is related to y_0 through the sine of the angle θ. We can also relate d to the time it takes the object to travel down the ramp through kinematics; the acceleration parallel to the ramp has a magnitude of $g\sin(\theta)$. For the second portion of the trip, we can use conservation of energy to find the speed of the object at the bottom of the ramp and use this speed to find the time it takes the object to travel a distance of $x_0/2$.

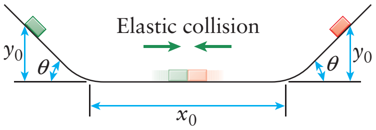


Figure 12-16 Problem 105

SOLVE
Distance traveled down the ramp:

$$d = \frac{y_0}{\sin(\theta)}$$

Time from initial position to the bottom of the ramp:

$$d = v_0 + \frac{1}{2}(g\sin(\theta))t_1^2 = 0 + \frac{1}{2}(g\sin(\theta))t_1^2$$

$$t_1 = \sqrt{\frac{2d}{g\sin(\theta)}} = \sqrt{\frac{2\left(\frac{y_0}{\sin(\theta)}\right)}{g\sin(\theta)}} = \sqrt{\frac{2y_0}{g\sin^2(\theta)}}$$

Speed at the bottom of the ramp:

$$mgy_0 = \frac{1}{2}mv^2$$

$$v = \sqrt{2gy_0}$$

Time from the bottom of the ramp to the point of collision:

$$t_2 = \frac{\left(\frac{x_0}{2}\right)}{v} = \frac{x_0}{2\sqrt{2gy_0}}$$

Period of cyclic process:

$$T = 2(t_1 + t_2) = \boxed{2\left(\sqrt{\frac{2y_0}{g\sin^2(\theta)}} + \frac{x_0}{2\sqrt{2gy_0}}\right)}$$

Part b)

$$T = 2\left(\sqrt{\frac{2(2\text{ m})}{\left(9.8\frac{\text{m}}{\text{s}^2}\right)\sin^2(30°)}} + \frac{(5\text{ m})}{2\sqrt{2\left(9.8\frac{\text{m}}{\text{s}^2}\right)(2\text{ m})}}\right) = \boxed{3.35\text{ s}}$$

REFLECT

Because the left and right sides are symmetric, the blocks must collide exactly halfway between the two. The motion of each block before the collision must be the same as the motion of each block after each collision due to both symmetry and the frictionless surface.

12.106

SET UP

You measure the frequency of a simple pendulum on Earth to be $f_{\text{Earth}} = 3.50$ Hz. You bring this pendulum to a satellite of unknown mass that has a diameter $D_{\text{satellite}} = 5.48 \times 10^6$ m and measure its frequency to be $f_{\text{satellite}} = 1.82$ Hz. From the definition of the period of a simple pendulum, we can find the ratio of the accelerations due to gravity on Earth and the satellite in terms of the frequencies of oscillation. The acceleration due to gravity on the satellite is proportional to the mass of the satellite, $g_{\text{satellite}} = \frac{Gm_{\text{satellite}}}{R^2_{\text{satellite}}}$. As a reminder, the mass of the Moon is 7.35×10^{22} kg.

SOLVE

Part a)

Frequency of the pendulum on Earth:

$$T_{\text{Earth}} = \frac{1}{f_{\text{Earth}}} = 2\pi\sqrt{\frac{L}{g_{\text{Earth}}}}$$

$$g_{\text{Earth}} = 4\pi^2 L f^2_{\text{Earth}}$$

Frequency of the pendulum on the satellite:

$$g_{\text{satellite}} = 4\pi^2 L f_{\text{satellite}}^2$$

Acceleration due to gravity on the satellite:

$$\frac{g_{\text{satellite}}}{g_{\text{Earth}}} = \frac{4\pi^2 L f_{\text{satellite}}^2}{4\pi^2 L f_{\text{Earth}}^2} = \frac{f_{\text{satellite}}^2}{f_{\text{Earth}}^2}$$

$$g_{\text{satellite}} = \frac{f_{\text{satellite}}^2}{f_{\text{Earth}}^2} g_{\text{Earth}}$$

Mass of the satellite:

$$g_{\text{satellite}} = \frac{Gm_{\text{satellite}}}{R_{\text{satellite}}^2} = \frac{Gm_{\text{satellite}}}{\left(\frac{D_{\text{satellite}}}{2}\right)^2} = \frac{4Gm_{\text{satellite}}}{D_{\text{satellite}}^2}$$

$$m_{\text{satellite}} = \frac{g_{\text{satellite}} D_{\text{satellite}}^2}{4G} = \frac{\left(\frac{f_{\text{satellite}}^2}{f_{\text{Earth}}^2} g_{\text{Earth}}\right) D_{\text{satellite}}^2}{4G} = \frac{f_{\text{satellite}}^2 g_{\text{Earth}} D_{\text{satellite}}^2}{4G f_{\text{Earth}}^2}$$

$$= \frac{(1.82\text{ Hz})^2\left(9.8\frac{\text{m}}{\text{s}^2}\right)(5.48 \times 10^6\text{ m})^2}{4\left(6.67 \times 10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(3.50\text{ Hz})^2} = \boxed{2.98 \times 10^{23}\text{ kg}}$$

$$\frac{m_{\text{satellite}}}{m_{\text{Moon}}} = \frac{2.98 \times 10^{23}\text{ kg}}{7.35 \times 10^{22}\text{ kg}} = 4.05$$

$$\boxed{m_{\text{satellite}} = 4.05\, m_{\text{Moon}}}$$

Part b) $\boxed{\text{No}}$, we can't use the vibrations of a spring–object system to determine the satellite's mass.

Part c) The frequency of oscillation of a spring–object system is independent of gravity.

REFLECT

The frequency of the pendulum is smaller on the satellite than on Earth. The frequency is proportional to the acceleration due to gravity, which is also proportional to the mass of the satellite. Therefore, we expect the mass of the satellite to be less than the mass of the Earth (5.98×10^{24} kg).

12.107

SET UP

We are modeling the jump of an 80-kg bungee jumper as simple harmonic motion with a period of $T = 9.5$ s. Assuming the bungee cord does not go slack during the trip, we can relate the period of the motion to the effective spring constant of the bungee cord. When the jumper finally comes to rest 40 m below the starting point, the elastic force of the bungee cord pulling

up must equal the force of gravity pulling down. We can calculate the unstretched length of the cord from this force balance and the final position of the jumper.

SOLVE

Part a)

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2(80\ \text{kg})}{(9.5\ \text{s})^2} = \boxed{35\frac{\text{N}}{\text{m}}}$$

Part b)

$$F_\text{g} = F_\text{spring}$$

$$mg = k\Delta L$$

$$\Delta L = \frac{mg}{k} = \frac{(80\ \text{kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)}{\left(35\frac{\text{N}}{\text{m}}\right)} = 22.4\ \text{m}$$

$$L_0 = L - \Delta L = (40\ \text{m}) - (22.4\ \text{m}) = \boxed{17.6\ \text{m}}$$

REFLECT

In actuality, a bungee cord does not compress like a spring but goes slack like a rope. A better model would describe half of the motion using simple harmonic motion and the other half as free fall.

12.108

SET UP

When a bug lands in a spiderweb, it causes the web to vibrate. We can model the spiderweb as a spring with an effective spring constant k. The frequency of the resulting oscillations will be related to k and the mass of the insect. When a 15×10^{-6} kg insect finally comes to rest 4.5×10^{-3} m below the equilibrium position of the web, the elastic force of the web pushing up on the insect must equal the force of gravity pulling down on it. We can use the force balance to calculate k for the web and then the frequency of the oscillations. For a spring–mass system, the frequency is independent of gravity, so it should remain constant regardless of the orientation of the web.

SOLVE

Part a)

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

The frequency is inversely proportional to the square root of the mass, which means a high frequency would indicate a $\boxed{\text{small insect.}}$

Part b)

$$F_g = F_{\text{spring}}$$

$$mg = k\Delta x$$

$$k = \frac{mg}{\Delta x} = \frac{(15 \times 10^{-6}\ \text{kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)}{(4.5 \times 10^{-3}\ \text{m})} = \boxed{0.033\frac{\text{N}}{\text{m}}}$$

Part c)

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{\left(0.033\frac{\text{N}}{\text{m}}\right)}{(15 \times 10^{-6})}} = \boxed{7.5\ \text{Hz}}$$

Part d) The frequency depends only on the mass of the insect and the effective spring constant of the web, not on gravity, so the $\boxed{\text{vibration rate would not change}}$ if the web were not horizontal.

REFLECT

A vibration frequency of 7.5 Hz seems reasonable for an insect caught in a spiderweb.

12.109

SET UP

A string ($L_0 = 0.200$ m) is made by combining 10 parallel strands of hair that are each 1.25×10^{-4} m thick. The Young's modulus of the hair is 4.50×10^9 Pa. We can calculate the effective spring constant k for an individual hair from the Young's modulus and its dimensions. The effective spring constant for the string made up of 10 parallel hairs is $k_{\text{eff}} = 10k$. A 0.175-kg utensil is hung from one end of the string. When it comes to rest, the elastic force of the string pulling up must equal the force of gravity pulling down. We can calculate the amount the string stretches from this force balance. The utensil is pulled down slightly and set into simple harmonic motion. The amount of time it takes the utensil to first return to its initial position is equal to one period, which we can calculate from the mass of the utensil and the effective spring constant of the string.

SOLVE

Part a)

$$F_g = F_{\text{hair}}$$

$$mg = k_{\text{eff}}\Delta L = (10k)\Delta L = 10\left(\frac{AY}{L_0}\right)\Delta L = 10(\pi R^2)\left(\frac{Y}{L_0}\right)\Delta L = 10\pi\left(\frac{D}{2}\right)^2\left(\frac{Y}{L_0}\right)\Delta L$$

$$\Delta L = \frac{2mgL_0}{5\pi D^2 Y} = \frac{2(0.175\ \text{kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)(0.200\ \text{m})}{5\pi(125 \times 10^{-6}\ \text{m})^2(4.50 \times 10^9\ \text{Pa})} = \boxed{6.21 \times 10^{-4}\ \text{m} = 621\ \mu\text{m}}$$

Part b)

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k_{\text{eff}}}} = 2\pi\sqrt{\frac{m}{10k}} = 2\pi\sqrt{\frac{m}{10\left(\frac{AY}{L_0}\right)}} = 2\pi\sqrt{\frac{mL_0}{10(\pi R^2)Y}} = 2\sqrt{\frac{\pi m L_0}{10\left(\frac{D}{2}\right)^2 Y}}$$

$$= 2\sqrt{\frac{2\pi m L_0}{5D^2 Y}} = 2\sqrt{\frac{2\pi(0.175\text{ kg})(0.200\text{ m})}{5(125\times10^{-6}\text{ m})^2(4.50\times10^9\text{ Pa})}} = \boxed{0.050\text{ s} = 50.0\text{ ms}}$$

REFLECT

Spring constants add normally for springs in parallel, while they add in reciprocal for springs in series.

12.110

SET UP

A piece of equipment ($m = 475$ kg) is to be hung by a steel cable ($L_0 = 2.80$ m). The maximum allowed vibrational frequency is 25.0 Hz. We can represent the spring constant of the steel cable in terms of its Young's modulus, $k = \frac{AY}{L_0}$, where A is its cross-sectional area; this allows us to solve for the maximum diameter of the wire. The Young's modulus of steel, from Table 9-1, is 200×10^9 Pa. When the piece of equipment comes to rest, the elastic force of the cable pulling up must equal the force of gravity pulling down. We can calculate the amount the cable stretches from this force balance and the spring constant in terms of the Young's modulus.

SOLVE

Part a)

$$k = \frac{AY}{L_0}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{\left(\frac{AY}{L_0}\right)}{m}} = \frac{1}{2\pi}\sqrt{\frac{(\pi R^2)Y}{mL_0}} = \frac{1}{2\pi}\sqrt{\frac{\pi\left(\frac{D}{2}\right)^2 Y}{mL_0}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{\pi D^2 Y}{4mL_0}} = \frac{D}{4}\sqrt{\frac{Y}{\pi m L_0}}$$

$$D = 4f\sqrt{\frac{\pi m L_0}{Y}} = 4(25.0\text{ Hz})\sqrt{\frac{\pi(475\text{ kg})(2.80\text{ m})}{(200\times10^9\text{ Pa})}} = \boxed{0.0145\text{ m} = 1.45\text{ cm}}$$

Part b)

$$F = k\Delta L = \left(\frac{AY}{L_0}\right)\Delta L = \frac{(\pi R^2)Y}{L_0}\Delta L = \frac{\pi\left(\frac{D}{2}\right)^2 Y}{L_0}\Delta L = \frac{\pi D^2 Y}{4L_0}\Delta L$$

$$\Delta L = \frac{4L_0F}{\pi D^2 Y} = \frac{4L_0(mg)}{\pi D^2 Y} = \frac{4(2.80\text{ m})(475\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)}{\pi(0.0145\text{ m})^2(200 \times 10^9\text{ Pa})} = 3.95 \times 10^{-4}\text{ m} = 3.95\text{ mm}$$

The 2.80-m-long cable stretches about 4 mm, which is $\boxed{\text{negligible}}$ compared to its unstretched length.

REFLECT

When deciding if a quantity is large or small, you should always ask yourself, "This number is large *compared to what*?" For example, 4 mm is small compared to 2.8 m but large compared to 2.8 nm.

12.111

SET UP

A baseball of mass m is dropped through a narrow tunnel that was drilled through the center of the Earth. The only force acting on the baseball while inside the Earth is the force due to gravity. According to the Shell Theorem, the gravitational field inside of a sphere is $g(r) = -g_0\left(\frac{r}{R}\right)$, where $g_0 = 9.8\text{ m/s}^2$, R is the radius of the Earth (6.38×10^6 m), and r is the distance from the center of the Earth. Because the gravitation field is proportional to the distance, we can rearrange Newton's second law such that it resembles the simple harmonic motion differential equation. This allows us to find the angular frequency and, thus, the period for the motion.

SOLVE

Free-body diagram for the ball above the center of the Earth:

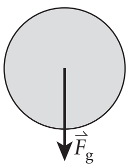


Figure 12-17 Problem 111

Free-body diagram for the ball below the center of the Earth:

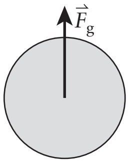


Figure 12-18 Problem 111

Newton's second law for the ball:

$$\sum F_r = F_g = -mg = -mg_0\left(\frac{r}{R}\right) = ma = m\frac{d^2r}{dt^2}$$

$$\frac{d^2r}{dt^2} + \left(\frac{g_0}{R}\right)r = 0$$

$$\omega^2 = \frac{g_0}{R}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R}{g_0}} = 2\pi\sqrt{\frac{(6.38 \times 10^6 \text{ m})}{\left(9.8\frac{\text{m}}{\text{s}^2}\right)}} = \boxed{5070 \text{ s}}$$

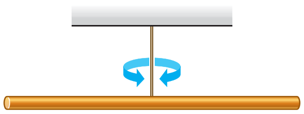
REFLECT

It takes about 1 hour and 25 minutes for the ball to return to its initial position.

12.112

SET UP

A long thin copper rod (length $L_{rod} = 0.60$ m, radius $r_{rod} = 4 \times 10^{-3}$ m, density $\rho = 8920$ kg/m^3) is suspended from a thin wire (length $L_{wire} = 0.20$ m, radius $r_{wire} = 5 \times 10^{-4}$ m). The thin wire is welded to the exact center of the rod. If the rod is slightly displaced from equilibrium, it will undergo simple harmonic motion. The net torque acting on the rod is due to the torsion in the wire and is equal to $-K\theta$, where $K = \frac{\pi G r_{wire}^4}{2l_{wire}}$ and $G = 45 \times 10^9$ Pa. Using Newton's second law for rotation, we can write a differential equation in terms of θ. The differential equation has the same form as a linear simple harmonic oscillator, which means we can relate the coefficient in front of the θ term to the angular frequency and then the period. As a reminder, the moment of inertia of a thin rod rotating about its center of mass is $I = \frac{1}{12}ML^2$.



The rod oscillates in/out of the plane of the page

Figure 12-19 Problem 112

SOLVE

Modulus of rigidity of the thin wire:

$$K = \frac{\pi G r_{wire}^4}{2l_{wire}} = \frac{\pi(45 \times 10^9 \text{ Pa})(5 \times 10^{-4} \text{ m})^4}{2(0.20 \text{ m})} = 0.0221\frac{\text{N}\cdot\text{m}}{\text{rad}}$$

Moment of inertia of the rod:

$$I = \frac{1}{12}ML_{rod}^2 = \frac{1}{12}(\rho V)L_{rod}^2 = \frac{1}{12}\rho(\pi r_{rod}^2 L_{rod})L_{rod}^2 = \frac{\pi}{12}\rho r_{rod}^2 L_{rod}^3$$

$$= \frac{\pi}{12}\left(8920\frac{\text{kg}}{\text{m}^3}\right)(4 \times 10^{-3} \text{ m})^2(0.60 \text{ m})^3 = 0.00807 \text{ kg}\cdot\text{m}^2$$

Newton's second law for rotation:

$$\sum \tau = -K\theta = I\alpha = I\frac{d^2\theta}{dt^2}$$

$$\frac{d^2\theta}{dt^2} + \frac{K}{I}\theta = 0$$

$$\omega = \sqrt{\frac{K}{I}}$$

$$T = 2\pi\sqrt{\frac{I}{K}} = 2\pi\sqrt{\frac{0.00807\ \mathrm{kg \cdot m^2}}{\left(0.0221\frac{\mathrm{N \cdot m}}{\mathrm{rad}}\right)}} = \boxed{3.80\ \mathrm{s}}$$

REFLECT

The negative sign in the torque on the rod implies that it is a "restoring torque," just as the negative sign in Hooke's law implies that it is a "restoring force." Because the differential equation in terms of θ has the same form as in x, the solutions will also be the same (for example, $\theta(t) = \theta_0\cos(\omega t)$).

12.113

SET UP

We are asked to explicitly show that $x(t) = A(\sin(\omega t) + \cos(\omega t))$ is a solution for the simple harmonic oscillator. The differential equation describing the simple harmonic oscillator is $\frac{d^2x}{dt^2} + \omega^2 x = 0$.

SOLVE

$$x(t) = A(\sin(\omega t) + \cos(\omega t))$$

$$\frac{dx}{dt} = \frac{d}{dt}[A(\sin(\omega t) + \cos(\omega t))] = A\omega(\cos(\omega t) - \sin(\omega t))$$

$$\frac{d^2x}{dt^2} = \frac{d}{dt}[A\omega(\cos(\omega t) - \sin(\omega t))] = -A\omega^2(\sin(\omega t) + \cos(\omega t))$$

$$\frac{d^2x}{dt^2} + \omega^2 x \stackrel{?}{=} 0$$

$$-A\omega^2(\sin(\omega t) + \cos(\omega t)) + \omega^2[A(\sin(\omega t) + \cos(\omega t))] = 0$$

REFLECT

A superposition of any two solutions to this differential equation will also be a solution to the differential equation.

12.114

SET UP

The quality factor Q is a parameter that describes the width and height of the resonant peak for a driven harmonic oscillator. One definition of the quality factor for a damped, driven oscillator at resonance (that is, $\omega = \omega_0$) is $Q = \frac{m\omega_0}{b}$. Another definition is in terms of energy loss per period, $Q = 2\pi\frac{\text{total energy}}{\text{energy loss per period}}$. We can combine the two definitions of Q to solve for the energy loss per period of a damped, driven oscillator. The total energy of the system is equal to $\frac{1}{2}kA^2$, where $A = \frac{F_0}{m\sqrt{(\omega_0^2 - \omega^2)^2 + \left(\frac{b\omega}{m}\right)^2}}$. The numerical values given for this problem are $m = 0.100\text{ kg}$, $b = 0.2\text{ kg/s}$, $F_0 = 7.5\text{ N}$, and $k = 25\text{ N/m}$.

SOLVE

Amplitude of a damped harmonic oscillator at resonance:

$$A = \frac{F_0}{m\sqrt{(\omega_0^2 - \omega^2)^2 + \left(\frac{b\omega}{m}\right)^2}} = \frac{F_0}{m\left(\frac{b\omega}{m}\right)} = \frac{F_0}{b\sqrt{\frac{k}{m}}} = \frac{F_0}{b}\sqrt{\frac{m}{k}}$$

Energy loss per period:

$$Q = \frac{m\omega_0}{b} = \frac{m\left(\sqrt{\frac{k}{m}}\right)}{b} = \frac{\sqrt{km}}{b} = 2\pi\frac{\text{total energy}}{\text{energy loss per period}} = 2\pi\frac{\left(\frac{1}{2}kA^2\right)}{\text{energy loss per period}}$$

$$\text{energy loss per period} = \pi\frac{bk\left(\frac{F_0}{b}\sqrt{\frac{m}{k}}\right)^2}{\sqrt{km}} = \frac{\pi F_0^2}{b}\sqrt{\frac{m}{k}} = \frac{\pi(7.5\text{ N})^2}{\left(0.2\frac{\text{kg}}{\text{s}}\right)}\sqrt{\frac{(0.100\text{ kg})}{\left(25\frac{\text{N}}{\text{m}}\right)}} = \boxed{56\text{ J}}$$

REFLECT

These are the same numerical values as in Problem 12.90. A large Q means the energy loss per period is small, and it will take longer for the oscillations to die out.

Chapter 13
Waves

Conceptual Questions

13.1 Longitudinal waves are those motions where the displacements are along the axis of wave propagation; transverse waves are those where the displacements are perpendicular to that axis. Longitudinal waves include sound and waves along a spring produced by extending and contracting it. Transverse waves include the vibrations of string instruments and "the wave" in a stadium.

13.2 The major organ that responds to sound waves is the ear (especially the eardrum). The voice box (or larynx) is the major organ that produces sound waves. There are also peristaltic waves associated with swallowing and digestion of food. Very low frequency waves (below the threshold of waves that humans can hear) are sometimes quite deleterious, creating severe disease if chronic exposure occurs.

13.3 A very few of them might be the same molecules, especially if he is close by. The air molecules collide many times on the way from place to place. The collisions transmit the wave motion without requiring the individual molecules to go the full distance.

13.4

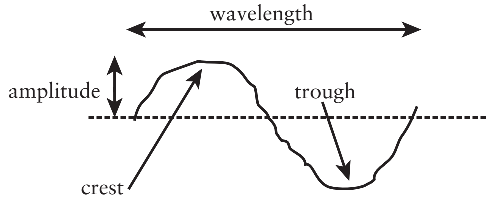


Figure 13-1 Problem 4

13.5 Yes. Sound is still produced even if there are no humans around to hear it.

13.6 Water waves are a superposition of both longitudinal waves and transverse waves. Together they result in the familiar shape of waves that one sees at the beach or in a lake.

13.7 Part a) The wave speed depends entirely on the medium so wave speed increases as the wave passes from air into water.

Part b) The frequency is determined by the actual vibrations of the source, so the frequency does not change.

Part c) The wavelength depends on changes in the wave speed, so the wavelength increases, too.

13.8 Part a) The rod with the smaller density.

Part b) The speed of longitudinal waves in a solid is inversely proportional to the square root of the density of the material:

$$v_{\text{longitudinal}} = \sqrt{\frac{Y}{\rho}}$$

13.9 A P wave moving through rock moves along the direction of propagation much like a sound wave through air. This push and pull can cause tall buildings to sway and perhaps topple. S waves cause up–down displacements, which could cause objects to fall off shelves. In earthquakes, the most damage results from waves that propagate along the surface, in a way similar to waves on the surface of the ocean. Surface waves can be transverse undulations or, like ocean waves crashing near the shore, rolling waves that have both transverse and longitudinal components.

13.10 Part a) This formula expresses the mathematical description of the shape of a wave that is moving through a medium. In the formula, y (SI unit: m) stands for the displacement of the wave from the equilibrium of the medium; x (SI unit: m) corresponds to the horizontal position; and t (SI unit: s) is the time variable. On the right-hand side, A (SI unit: m) is the amplitude (the maximum value of y); k (SI unit: m^{-1}) stands for the wavenumber, $k = \frac{2\pi}{\lambda}$; v_p (SI unit: m/s) is the propagation speed of the wave; and ω (SI unit: rad/s) is the angular frequency, $\omega = 2\pi f$.

Part b) Since the wave must repeat after one full cycle, the argument of the sine function must repeat every 2π rad:

$$k(x_1 - v_\text{p}t_1) = k(x_1 - v_\text{p}t_2) + 2\pi$$

$$kx_1 - kv_\text{p}t_1 = kx_1 - kv_\text{p}t_2 + 2\pi$$

$$kv_\text{p}(t_2 - t_1) = 2\pi$$

$$v_\text{p} = \frac{2\pi}{(t_2 - t_1)}\left(\frac{1}{k}\right) = \frac{2\pi}{T}\left(\frac{1}{k}\right) = \frac{\omega}{k}$$

where we've defined $\omega = \frac{2\pi}{T}$.

13.11 The frequency counts the number of oscillations per time. The angular frequency counts the oscillations per time in units of 2π; one complete oscillation is equivalent to 2π rad. The angular frequency is more natural from a mathematical point of view.

13.12

$$1 \text{ nautical league} \times \frac{5.556 \text{ km}}{1 \text{ nautical league}} \times \frac{3 \text{ s}}{1 \text{ km}} = 16.7 \text{ s}$$

The lightning strike is 1 nautical league away for every 16.7 s.

13.13 Part a) A transverse standing wave is a pair of transverse waves with opposite wave vectors, with the result that the wave doesn't move anywhere. Indeed, its shape does not even change except by growing and shrinking.

Part b) So long as one particular pair of opposite wave vectors dominates, the result will be a standing wave. This can happen if the string is, say, plucked in the middle. The even modes will not be excited by this motion, and the higher odd modes will decay faster than the first. At that time, the result will be a standing wave. Alternately, the string could be shaken periodically at the frequency of the first mode for a time.

13.14 *Rarefaction* is defined as "the reduction of a medium's density below the equilibrium value, or the opposite of compression." Rarefaction is the region of a longitudinal wave where the medium is "stretched out" from the normal equilibrium pattern. *Compression* is "the enlargement of a medium's density above the equilibrium value, or the opposite of rarefaction." Compression is the region of a longitudinal wave where the medium is "bunched up." A *phonon* is the normal mode of vibration that is repeated in longitudinal waves (that is, the fundamental pattern of rarefaction/compression that is seen over and over again.)

13.15 A rider on the train, moving at the same speed as the horn was when it emitted the sound, will not experience a Doppler effect on the whistle because of that shared velocity. There can be slight differences in the two velocities, though, from building up speed, braking, or going around a curve. In such cases, there will be a very slight Doppler effect.

13.16 Part a) A sonic boom occurs when the speed of the source of sound exceeds the speed of sound for the medium in question.

Part b) The Doppler formula does not make physical sense when the speed of the source is greater than the speed of sound in air. For the case where the source is approaching, the term in the denominator is $(v - v_s)$, which will lead to a *negative* frequency. Since this is impossible, it indicates that a different explanation must be employed. Basically, the source of the sound is already past the observer by the time the sound is transmitted.

13.17 The time derivative of the transverse position y is how far from equilibrium any one component is. In particular, the velocity is proportional to the amplitude. The wave speed is not. It is how fast any part of the wave (the pattern, not the constituent parts) moves.

13.18

$$\left[\frac{\partial^2 y}{\partial x^2}\right] \stackrel{?}{=} \left[\frac{1}{v^2}\right]\left[\frac{\partial^2 y}{\partial t^2}\right]$$

$$\frac{[\mathrm{L}]}{[\mathrm{L}]^2} \stackrel{?}{=} \left(\frac{[\mathrm{T}]^2}{[\mathrm{L}]^2}\right)\left(\frac{[\mathrm{L}]}{[\mathrm{T}]^2}\right)$$

$$\frac{1}{[\mathrm{L}]} = \frac{1}{[\mathrm{L}]}$$

13.19 The phase difference is how two waves line up. If it's 0, then they line up perfectly and add to each other directly, resulting in a wave with double the amplitude. If it's 180 degrees, the two waves cancel perfectly, resulting in no wave. If it's 90 degrees or 270 degrees, then they add to each other resulting in a wave with $\sqrt{2}$ times the amplitude and phase directly in between that of the original two waves. If it's 300 degrees, then again they add differently, but the result is a wave with $\sqrt{3}$ times the amplitude and a phase again directly between the two original waves.

13.20 Part a) The path difference is related to the phase difference between two waves. If a wave is exactly out of phase with a second wave, they will cancel each other out. If the second wave is one wavelength ahead or behind the first wave, the waves will constructively interfere.

Part b) The condition $n\lambda$ for constructive interference only works if the waves begin in phase.

13.21 The time it takes for the wave to make it from one end to the other and back determines the phase difference between one pass and the next. The lowest frequency that produces zero phase difference and, thus, total reinforcement is the fundamental frequency.

13.22 There is no physical way that an antinode on the open end can have another antinode on the closed end, so there are only odd numbers of ¼ waves allowed in the standing wave patterns for open–closed pipes.

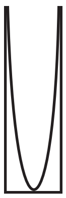


Figure 13-2 Problem 22

13.23 Part a) Intensity is the power per unit area that is emitted by a source of sound. It has nothing to do with the reception of the sound through the process of "hearing." The SI units are W/m^2. Sound level is a mathematical relationship that puts the intensity onto a logarithmic scale. The units are decibels (dB). Loudness is the physiological response

to sound waves. Different frequencies with the same intensity will be more sensitively heard by humans. Power is the energy per unit time emitted by the source.

Part b) Intensity depends on the inverse of the distance squared, so the intensity increases as the source of sound moves closer.

Part c) Sound level depends on the log of intensity, so sound level increases as the source of sound moves closer.

Part d) The total power emitted by the source does not change as it moves closer to the observer.

13.24 If you know that one string is in tune, you can hold the next string above or below on a fret such that the notes should be the same. If you sound the strings together and hear zero beat frequency, you know the second string is in tune and you can repeat this process for all of the strings.

13.25 The car receives the signal that is coming directly from the broadcasting tower and from many other reflections (buildings, mountains, low-lying clouds, etc.). When the signals are not directly out of phase, some signal gets through and you will hear the station's broadcast. However, sometimes the path difference between the direct wave and the reflected wave is an integral multiple of a half-wavelength, leading to an "out-of-phase" or destructive interference pattern. As the car comes to a stop, the dead zones are traversed slowly enough to be noticeable.

13.26 George's string was incorrectly adjusted to Elaine's frequency (214 Hz). Although both pianos are now out of tune, they are in tune to one another, and the beat frequency is zero.

Multiple-Choice Questions

13.27 **B** (transverse wave). The motion of "the wave" is perpendicular to the motion of the individual fans.

13.28 **A** (constructive interference). The waves will be exactly in phase at a point halfway between the two sources because the path difference is equal to zero.

13.29 **D** (5).

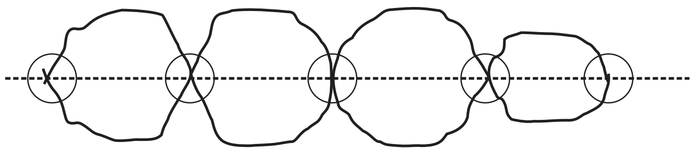


Figure 13-3 Problem 29

13.30 **C** (160 Hz). The frequencies of the higher harmonics are integral multiples of the fundamental frequency.

13.31 **B** (The frequency will increase). As the length of the horn is made shorter, the allowed wavelengths will also get shorter. The speed of sound, which is equal to the product of the wavelength and frequency, is constant in air, so the frequency will increase.

13.32 **C** (4 Hz). The beat frequency is equal to the difference in frequency between the two tuning forks.

13.33 **D** (The sound intensity drops to 1/9 its original value). The sound intensity is inversely proportional to the square of the distance.

13.34 **E** (increase by a factor of 9). The sound intensity is proportional to the square of the amplitude.

13.35 **C** (The intensity increases by a factor of 100). The sound level is logarithmic. An increase of 20 dB corresponds to an increase by a factor of 10^2.

13.36 **C** ($f_2 < f < f_1$). The perceived frequency of the ambulance as it approaches the parked car (f_1) will be higher than the actual frequency due to the Doppler effect. The perceived frequency of the ambulance as it moves away from the parked car (f_2) will be lower than the actual frequency due to the Doppler effect.

Estimation Questions

13.37 The wavelength of sound with a frequency of 3000 Hz is (343 m/s)/(3000 Hz) = 0.114 m = 11.4 cm. If the voice box is considered an open–closed organ pipe, $L = \frac{\lambda}{4} = 2.8$ cm.

13.38 Part a)

$$\text{Distance} \approx \left(343\frac{\text{m}}{\text{s}}\right)(10\text{ s}) = 3430\text{ m}$$

Part b) The speed of sound depends upon the temperature of air. For example, at 0 degrees Celsius, the speed of sound is about 331 m/s, at 10 degrees Celsius, the speed is 337 m/s. So, if the temperature starts at 20 degrees Celsius and drops 10 degrees, the distance will decrease by about 60 m (or about 1.75%). If the temperature drops another 10 degrees, the distance will decrease another 60 m for a total of about 3.5%. Note that this does not take into account the changes due to wind.

13.39 A hummingbird's wings flap at a frequency of about 50 flaps/s.

13.40 Part a) If the perimeter of the arena is around 500 m, it will take about 30 s for the human wave to make it around the entire arena. This corresponds to a speed of about 17 m/s.

Part b) The wavelength might be related to the size of the group of people who stand or raise their hands at any one time. The frequency would be the number of times people raise their hands per unit time interval. The amplitude might be related to how high their hands are raised.

13.41 We can measure the pitch of the fundamental compared to a known standard (like the note A 440 Hz) and measure the length of the string. The speed is twice the length divided by the frequency.

13.42 The wavelength is equal to the speed multiplied by the period. In this case it should be on the order of a few centimeters.

13.43 We will assume the distance between home plate and the left field bleachers is 150 m and that the wind is moving at 10 m/s. With no wind, $t = (150\text{ m})/(343\text{ m/s}) = 0.44\text{ s}$; with wind, $t = (150\text{ m})/(353\text{ m/s}) = 0.43\text{ s}$.

13.44 Part a) Typically, waves in the open water of the ocean have amplitudes of about 5 m, a frequency of about 0.05 Hz, and wavelengths of about 150 m.

Part b)

$$v = \lambda f = (150\text{ m})(0.05\text{ Hz}) = 7.5\frac{\text{m}}{\text{s}}$$

13.45

x (m)	y (m)
0	0
1	4.5
2	7.8
3	9
4	7.8
5	4.5
6	0
7	−4.5
8	−7.8
9	−9
10	−7.8
11	−4.5
12	0

Part a)

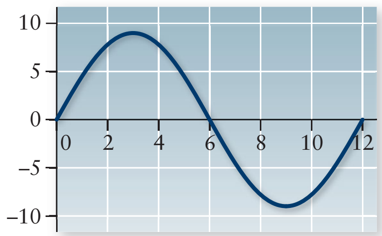


Figure 13-4 Problem 45

Part b) The general form for a wave traveling to the right is

$$y(x, t) = A\sin\left(\left(\frac{2\pi}{\lambda}\right)x - \left(\frac{2\pi}{T}\right)t\right)$$

Using the data:

$$y(x, t) = (9\text{ m})\sin\left(\left(\frac{2\pi}{12\text{ m}}\right)x - \left(\frac{2\pi}{2\text{ s}}\right)t\right) = \boxed{(9\text{ m})\sin\left(\left(\frac{\pi}{6}\text{m}^{-1}\right)x - (\pi\text{ s}^{-1})t\right)}$$

Problems

13.46

SET UP

Sometimes during an earthquake, a dog can hear the P wave before the shaking that accompanies the S wave's arrival. Humans, on the other hand, cannot hear this wave; this means it is outside the normal range of our hearing but within the normal range for dogs. Humans can hear frequencies up to 23,000 Hz. We can use the speed of the P wave (v = 4000 m/s) and the maximum frequency perceived by humans to find the minimum wavelength of the seismic waves that can be heard by a dog but not by a human.

SOLVE

$$v = \lambda f$$

$$\lambda = \frac{v}{f} = \frac{\left(4000\frac{\text{m}}{\text{s}}\right)}{23{,}000\text{ Hz}} = \boxed{0.174\text{ m} = 17.4\text{ cm}}$$

REFLECT

The smallest wavelength that a dog can hear is 8.9 cm.

13.47

SET UP

A wave on a string propagates at v = 22 m/s. The frequency of the wave is f = 24 Hz. We can use the speed of the wave and the frequency to calculate the wavelength. The wavenumber is $k = \frac{2\pi}{\lambda}$.

SOLVE

Wavelength:

$$\lambda = \frac{v}{f} = \frac{\left(22\frac{\text{m}}{\text{s}}\right)}{24\text{ Hz}} = \boxed{0.92\text{ m} = 92\text{ cm}}$$

Wavenumber:

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{\left(\frac{v}{f}\right)} = \frac{2\pi f}{v} = \frac{2\pi(24\ \text{Hz})}{\left(22\frac{\text{m}}{\text{s}}\right)} = \boxed{6.85\frac{\text{rad}}{\text{m}}}$$

REFLECT

Be careful when rounding intermediate values. You should carry along one more significant figure than you are allowed in your final answer throughout your calculations.

13.48

SET UP

The period of a sound wave is $T = 0.01$ s. The frequency f is equal to the reciprocal of the period. The angular frequency is $\omega = \frac{2\pi}{T}$ or $\omega = 2\pi f$.

SOLVE

Frequency:

$$f = \frac{1}{T} = \frac{1}{0.01\ \text{s}} = \boxed{100\ \text{Hz}}$$

Angular frequency:

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.01\ \text{s}} = \boxed{628\frac{\text{rad}}{\text{s}}}$$

REFLECT

Humans can hear frequencies ranging from about 20 Hz to 20 kHz. A frequency of 100 Hz corresponds to a sound in the bass frequencies.

13.49

SET UP

We are asked to show that two different expressions for the propagation speed of a wave—$v = \frac{\omega}{k}$ and $v = \lambda f$—are dimensionally correct. Speed has dimensions of length per time; angular frequency has dimensions of 1 over time, wavenumber has dimensions of 1 over length; wavelength has dimensions of length; and frequency has dimensions of 1 over time.

SOLVE

$$[v] \stackrel{?}{=} \frac{[\omega]}{[k]}$$

$$\frac{[\text{L}]}{[\text{T}]} \stackrel{?}{=} \frac{\left(\frac{1}{[\text{T}]}\right)}{\left(\frac{1}{[\text{L}]}\right)}$$

$$\frac{[\mathrm{L}]}{[\mathrm{T}]} = \frac{[\mathrm{L}]}{[\mathrm{T}]}$$

$$[v] \stackrel{?}{=} [\lambda][f]$$

$$\frac{[\mathrm{L}]}{[\mathrm{T}]} \stackrel{?}{=} [\mathrm{L}]\frac{1}{[\mathrm{T}]}$$

$$\frac{[\mathrm{L}]}{[\mathrm{T}]} = \frac{[\mathrm{L}]}{[\mathrm{T}]}$$

REFLECT

The SI units of radians are dimensionless.

13.50

SET UP

A transverse wave on a string has an amplitude of $A = 0.20$ m, a wavelength $\lambda = 0.35$ m, and a frequency $f = 2$ Hz. We are asked to provide a mathematical description of the displacement from equilibrium for the wave for four different initial conditions. The general mathematical description has the form $y(x, t) = A\sin(kx - \omega t + \phi)$ or $y(x, t) = A\cos(kx - \omega t + \phi)$, depending on the initial conditions. We can calculate k and ω from λ and f, respectively.

SOLVE

Wavenumber:

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.35\ \mathrm{m}} = 17.95\ \mathrm{m}^{-1}$$

Angular frequency:

$$\omega = 2\pi f = 2\pi(2\ \mathrm{Hz}) = 12.56\frac{\mathrm{rad}}{\mathrm{s}}$$

Part a) $y(0, 0) = 0$

$$\boxed{y(x, t) = (0.20\ \mathrm{m})\sin\left((17.95\ \mathrm{m}^{-1})x - \left(12.56\frac{\mathrm{rad}}{\mathrm{s}}\right)t\right)}$$

Part b) $y(0, 0) = +20$ cm

$$\boxed{y(x, t) = (0.20\ \mathrm{m})\cos\left((17.95\ \mathrm{m}^{-1})x - \left(12.56\frac{\mathrm{rad}}{\mathrm{s}}\right)t\right)}$$

Part c) $y(0, 0) = -20$ cm

$$\boxed{y(x, t) = -(0.20\ \mathrm{m})\cos\left((17.95\ \mathrm{m}^{-1})x - \left(12.56\frac{\mathrm{rad}}{\mathrm{s}}\right)t\right)}$$

Part d) $y(0, 0) = +12$ cm

$$y(0, 0) = (0.12\ \mathrm{m}) = (0.20\ \mathrm{m})\cos\left((17.95\ \mathrm{m}^{-1})(0) - \left(12.56\frac{\mathrm{rad}}{\mathrm{s}}\right)(0) + \phi\right)$$

$$\cos(\phi) = \frac{0.12 \text{ m}}{0.20 \text{ m}}$$

$$\phi = \arccos(0.6) = 0.927$$

$$\boxed{y(x, t) = (0.20 \text{ m})\cos\left((17.95 \text{ m}^{-1})x - \left(12.56\frac{\text{rad}}{\text{s}}\right)t + 0.927\right)}$$

REFLECT

The sine and cosine functions are essentially interchangeable in this application since they are related by a phase difference of $\pi/2$. For example, if we were to use cosine, we would need to include a phase of $-\pi/2$.

13.51

SET UP

A wave on a string is described by $y(x, t) = 0.05\sin(x - 10t)$ (SI units). We can compare this equation to the general form of a transverse wave, $y(x, t) = A\sin(kx - \omega t + \phi)$, along with the definitions of the wavenumber and angular frequency in order to determine the frequency, the wavelength, and the speed of the wave.

SOLVE

Part a)

$$f = \frac{\omega}{2\pi} = \frac{\left(10\frac{\text{rad}}{\text{s}}\right)}{2\pi} = \boxed{1.6 \text{ Hz}}$$

Part b)

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{1 \text{ m}^{-1}} = \boxed{6.3 \text{ m}}$$

Part c)

$$v = \frac{\omega}{k} = \frac{\left(10\frac{\text{rad}}{\text{s}}\right)}{1 \text{ m}^{-1}} = \boxed{10\frac{\text{m}}{\text{s}}}$$

REFLECT

We could have also used $v = \lambda f$ to find the speed, but it's better to use the values provided rather than intermediate values that you've calculated.

13.52

SET UP

A transverse wave is traveling in the $+x$ direction at a speed of $v = 20$ m/s, has a frequency of $f = 10$ Hz, and has an amplitude of $A = 0.10$ m. We can use these data and the general form of a transverse wave traveling toward $+x$, $y(x, t) = A\sin(kx - \omega t + \phi)$, to write the mathematical description for this specific wave. We will also need the definitions of the wavenumber and angular frequency. For simplicity, we will assume $y(0, 0) = 0$.

SOLVE

Wavenumber:

$$v = \lambda f = \left(\frac{2\pi}{k}\right)f$$

$$k = \frac{2\pi f}{v} = \frac{2\pi(10 \text{ Hz})}{\left(20\frac{\text{m}}{\text{s}}\right)} = \pi \text{ m}^{-1}$$

Angular frequency:

$$\omega = 2\pi f = 2\pi(10 \text{ Hz}) = 20\pi\frac{\text{rad}}{\text{s}}$$

Mathematical description:

$$\boxed{y(x, t) = (0.10 \text{ m})\sin\left((\pi \text{ m}^{-1})x - \left(20\pi\frac{\text{rad}}{\text{s}}\right)t\right)}$$

REFLECT

The phase is equal to zero because we assumed an initial condition of $y(0, 0) = 0$.

13.53

SET UP

The equation for a particular wave is $y(x, t) = 0.10\sin(kx - \omega t)$ (SI units). The frequency of this wave is $f = 2.0$ Hz; we can use this to find ω and then the value of the wave at $x = 0$, $t = 4.0$ s.

SOLVE

All quantities are in SI units:

$$\omega = 2\pi f = 2\pi(2.0) = 4\pi$$

$$y(0{,}4.0) = 0.10\sin(k(0) - (4\pi)(4.0 \text{ s})) = 0.10\sin(-16\pi) = \boxed{0}$$

REFLECT

An equivalent form of the wave would be $y(x, t) = 0.10\sin\left(\frac{2\pi x}{\lambda} - (2\pi f)t\right)$. It'd be good practice to prove this to yourself and that you get the same answer as above.

13.54

SET UP

The pressure wave that travels along the inside of an organ pipe has the following mathematical description: $p(x, t) = (1 \text{ atm}) - (1 \text{ atm})\cos(6x - 4t)$, where the missing units are assumed to be SI. The general form of this wave is $p(x, t) = p_0 - A\cos(kx - \omega t)$, where p_0 is a pressure offset. We can compare the given wave to the general form to determine the amplitude and wavenumber of the wave. We can calculate the frequency and speed of the wave from the definitions of those quantities. The spatial displacement $s(x, t)$ has an

amplitude of 0.02 m and will oscillate with the same wavenumber and angular frequency as the pressure wave. The spatial displacement wave will be 90 degrees out of phase with the pressure wave, though, so we should use a sine function rather than a cosine.

SOLVE

Part a) Amplitude is $\boxed{1\text{ atm}}$.

Part b) Wavenumber is $\boxed{6\text{ m}^{-1}}$.

Part c)

$$f=\frac{\omega}{2\pi}=\frac{\left(4\frac{\text{rad}}{\text{s}}\right)}{2\pi}=\boxed{0.64\text{ Hz}}$$

Part d)

$$v=\frac{\omega}{k}=\frac{\left(4\frac{\text{rad}}{\text{s}}\right)}{6\text{ m}^{-1}}=\boxed{0.67\frac{\text{m}}{\text{s}}}$$

Part e)

$$\boxed{s(x,t)=(0.02)\sin(6x-4t)}$$

REFLECT

When the pressure is at a maximum, the individual molecules in the air will be at their equilibrium positions (that is, their spatial displacement is zero).

13.55

SET UP

We are given plots of a wave at $t=0$ and $x=0$. Since the plots look like sine functions, we will use this in our mathematical description. We can read the amplitude, wavelength, and period directly from the graphs. The wavenumber and angular frequency can be calculated from the wavelength and period. Because the wave starts at $y=0$, the phase is also equal to 0.

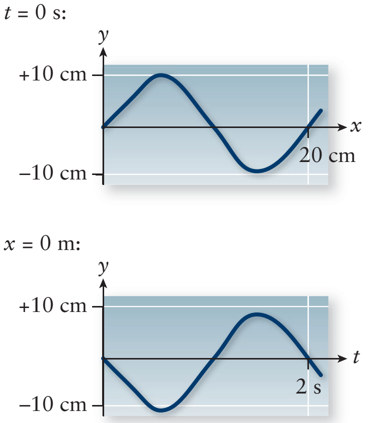


Figure 13-5 Problem 55

SOLVE

Amplitude, $A = 0.10$ m

Wavelength, $\lambda = 0.20$ m

Period, $T = 2$ s

Wavenumber:

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{(0.20 \text{ m})} = 10\pi \text{ m}^{-1}$$

Angular frequency:

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2 \text{ s}} = \pi\frac{\text{rad}}{\text{s}}$$

Mathematical description:

$$\boxed{y(x, t) = 0.10\sin(10\pi x - \pi t)} \text{ (SI units)}$$

REFLECT

The plot y versus t looks like $-\sin(\omega t)$ because sine is an odd function; that is, $\sin(-\omega t) = -\sin(\omega t)$.

13.56

SET UP

We are given a plot of a wave at $t = 0$ and asked to write an equation describing its displacement. We will use a sine function in our mathematical description. We can read the amplitude and wavelength directly from the graph. We are told that the period is $T = 4$ s. The wavenumber and angular frequency can be calculated from the wavelength and period. Because the wave does not start exactly at $y = 0$ or $y = A$, we will need to find the phase angle ϕ of the wave. This can be easily done by solving for ϕ at $y(0, 0)$.

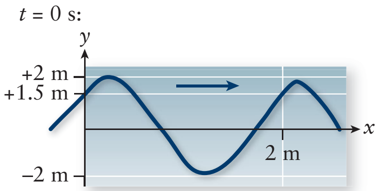


Figure 13-6 Problem 56

SOLVE

Amplitude, $A = 2$ m

Wavelength, $\lambda = 2$ m

Period, $T = 4$ s

Wavenumber:

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{(2 \text{ m})} = \pi \text{ m}^{-1}$$

Angular frequency:

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{4\text{ s}} = \frac{\pi}{2}\frac{\text{rad}}{\text{s}}$$

Phase:

$$y(0, 0) = (1.5\text{ m}) = (2\text{ m})\sin(k(0) - \omega(0) + \phi)$$

$$\sin(\phi) = \frac{(1.5\text{ m})}{(2\text{ m})}$$

$$\phi = \arcsin(0.75) = 0.85$$

Mathematical description:

$$\boxed{y(x, t) = 2\sin\left(\pi x - \frac{\pi}{2}t + 0.85\right)}\ \text{(SI units)}$$

REFLECT

We could have used a cosine function to describe the wave instead.

13.57

SET UP

A string has a mass of 5.0×10^{-3} kg and length of 2.2 m. The string is pulled taut with a tension of $T = 78$ N. The propagation speed of a transverse wave on a string is given by $v_{\text{p}} = \sqrt{\frac{T}{\mu}}$, where T is the tension and μ is the linear mass density.

SOLVE

$$v_{\text{p}} = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{78\text{ N}}{\left(\frac{5.0 \times 10^{-3}\text{ kg}}{2.2\text{ m}}\right)}} = \boxed{185\frac{\text{m}}{\text{s}}}$$

REFLECT

The propagation speed is how fast the wave travels down the rope, not how fast a small piece of the rope moves up and down.

13.58

SET UP

A transverse wave is set in motion on a long, taut rope. The tension in the rope is 80 N. The waves travel 12.8 m in 2.1 s. We can use the propagation speed of a transverse wave on a string, $v_{\text{p}} = \sqrt{\frac{T}{\mu}}$, to find the linear mass density μ.

SOLVE

$$v_{\text{p}} = \sqrt{\frac{T}{\mu}}$$

$$\mu = \frac{T}{v_{\mathrm{p}}^2} = \frac{80\ \mathrm{N}}{\left(\frac{12.8\ \mathrm{m}}{2.1\ \mathrm{s}}\right)^2} = \boxed{2.2\frac{\mathrm{kg}}{\mathrm{m}}}$$

REFLECT

A linear mass density of 2.2 kg/m is reasonable for a heavy rope. A mass of 2.2 kg corresponds to almost 5 lb.

13.59

SET UP

The ratio of the higher frequency to the lower frequency of neighboring strings on a violin is 3/2. The highest-frequency string, the E string, has a diameter of $D_{\mathrm{E}} = 0.25$ mm. We can calculate the diameters of the remaining strings (A, D, and G, from high to low frequency) by setting up ratios of the frequencies and relating them to the diameters through the speed and linear mass density. The strings are all the same length, made out of the same material, and under the same tension, so the wavelengths, densities, and tensions are all equal.

SOLVE

E string to A string:

$$\frac{f_{\mathrm{E}}}{f_{\mathrm{A}}} = \frac{\left(\frac{v_{\mathrm{E}}}{\lambda}\right)}{\left(\frac{v_{\mathrm{A}}}{\lambda}\right)} = \frac{\left(\sqrt{\frac{T}{\mu_{\mathrm{E}}}}\right)}{\left(\sqrt{\frac{T}{\mu_{\mathrm{A}}}}\right)} = \sqrt{\frac{\mu_{\mathrm{A}}}{\mu_{\mathrm{E}}}} = \sqrt{\frac{\rho\pi R_{\mathrm{A}}^2}{\rho\pi R_{\mathrm{E}}^2}} = \sqrt{\frac{\left(\frac{D_{\mathrm{A}}}{2}\right)^2}{\left(\frac{D_{\mathrm{E}}}{2}\right)^2}} = \frac{D_{\mathrm{A}}}{D_{\mathrm{E}}}$$

$$D_{\mathrm{A}} = \left(\frac{f_{\mathrm{E}}}{f_{\mathrm{A}}}\right)D_{\mathrm{E}} = \left(\frac{3}{2}\right)(0.25\ \mathrm{mm}) = \boxed{0.38\ \mathrm{mm}}$$

A string to D string:

$$\frac{f_{\mathrm{A}}}{f_{\mathrm{D}}} = \frac{D_{\mathrm{D}}}{D_{\mathrm{A}}}$$

$$D_{\mathrm{D}} = \left(\frac{f_{\mathrm{A}}}{f_{\mathrm{D}}}\right)D_{\mathrm{A}} = \left(\frac{3}{2}\right)(0.38\ \mathrm{mm}) = \boxed{0.56\ \mathrm{mm}}$$

D string to G string:

$$\frac{f_{\mathrm{D}}}{f_{\mathrm{G}}} = \frac{D_{\mathrm{G}}}{D_{\mathrm{D}}}$$

$$D_{\mathrm{G}} = \left(\frac{f_{\mathrm{D}}}{f_{\mathrm{G}}}\right)D_{\mathrm{D}} = \left(\frac{3}{2}\right)(0.56\ \mathrm{mm}) = \boxed{0.84\ \mathrm{mm}}$$

REFLECT

It makes sense that the thicker the string, the lower the note.

13.60

SET UP

At room temperature, the bulk modulus of glycerine is $B = 4.35 \times 10^9$ Pa and its density is $\rho = 1260\ \text{kg/m}^3$. We can use this information to find the speed of sound in glycerine at room temperature using $v_p = \sqrt{\frac{B}{\rho}}$.

SOLVE

$$v_p = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{4.35 \times 10^9\ \text{Pa}}{\left(1260\frac{\text{kg}}{\text{m}^3}\right)}} = \boxed{1858\frac{\text{m}}{\text{s}}}$$

REFLECT

For comparison, the speed of sound in air is 343 m/s. The bulk modulus is a function of temperature, which is why we were explicitly told that we are working at room temperature (25 degrees C).

13.61

SET UP

The bulk modulus and density of water are $B = 2.2 \times 10^9$ Pa and $\rho = 1000\ \text{kg/m}^3$, respectively. We can use this information to find the speed of sound in water using $v_p = \sqrt{\frac{B}{\rho}}$.

SOLVE

$$v_p = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2.2 \times 10^9\ \text{Pa}}{\left(1000\frac{\text{kg}}{\text{m}^3}\right)}} = \boxed{1483\frac{\text{m}}{\text{s}}}$$

REFLECT

For comparison, the speed of sound in air is 343 m/s.

13.62

SET UP

The bulk modulus of seawater is $B = 2.34 \times 10^9$ Pa. In the ocean, 1000-Hz sound waves have a wavelength of 1.51 m. In order to find the density of seawater, we first need to find the speed of sound in seawater from the frequency and wavelength of the sound wave and then we can solve for the density using $v_p = \sqrt{\frac{B}{\rho}}$.

SOLVE

Speed of sound:

$$v = \lambda f = (1.51\ \text{m})(1000\ \text{Hz}) = 1510\frac{\text{m}}{\text{s}}$$

Density:

$$v_p = \sqrt{\frac{B}{\rho}}$$

$$\rho = \frac{B}{v_p^2} = \frac{2.34 \times 10^9 \text{ Pa}}{\left(1510\frac{\text{m}}{\text{s}}\right)^2} = \boxed{1026\frac{\text{kg}}{\text{m}^3}}$$

REFLECT

This value listed in Table 11-1 for the density of seawater at 4 degrees Celsius is 1025 kg/m^3, so our value is reasonable.

13.63

SET UP

The bulk modulus and density of gasoline are $B = 1.3 \times 10^9$ Pa and $\rho = 0.74$ kg/L, respectively. We can use $v_p = \sqrt{\frac{B}{\rho}}$ to find the speed of sound in gasoline. As a reminder, the conversion factor between L and m^3 is 1000 L = 1 m^3.

SOLVE

$$v_p = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{1.3 \times 10^9 \text{ Pa}}{\left(0.74\frac{\text{kg}}{\text{L}} \times \frac{1000 \text{ L}}{1 \text{ m}^3}\right)}} = \boxed{1325\frac{\text{m}}{\text{s}}}$$

REFLECT

This is similar to the speed of sound in water that we calculated in Problem 13.61 (v_p = 1483 m/s), which seems reasonable since the bulk moduli and densities are similar.

13.64

SET UP

The bulk modulus of liquid A is twice the bulk modulus of liquid B. The density of liquid A is one-half the density of liquid B. We can use the definition of the propagation speed $v_p = \sqrt{\frac{B}{\rho}}$ to find the ratio between the two speeds.

SOLVE

$$\frac{v_A}{v_B} = \frac{\sqrt{\frac{B_A}{\rho_A}}}{\sqrt{\frac{B_B}{\rho_B}}} = \sqrt{\frac{B_A\rho_B}{\rho_A B_B}} = \sqrt{\frac{(2B_B)\rho_B}{\left(\frac{\rho_B}{2}\right)B_B}} = \sqrt{4} = \boxed{2}$$

REFLECT

Increasing the bulk modulus and decreasing the density have the same effect on the propagation speed.

13.65

SET UP

The speed of a longitudinal wave in a solid is given by $v_{\text{longitudinal}} = \sqrt{\frac{Y}{\rho}}$, while the speed of a transverse wave in a solid is given by $v_{\text{transverse}} = \sqrt{\frac{G}{\rho}}$, where G is the shear modulus. We can calculate the wave speeds for steel ($Y = 210 \times 10^9$ Pa, $G = 84 \times 10^9$ Pa, $\rho = 7800$ kg/m^3) directly from these formulas.

SOLVE

Longitudinal:

$$v_{\text{longitudinal}} = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{210 \times 10^9\text{ Pa}}{\left(7800\frac{\text{kg}}{\text{m}^3}\right)}} = \boxed{5189\frac{\text{m}}{\text{s}}}$$

Transverse:

$$v_{\text{transverse}} = \sqrt{\frac{G}{\rho}} = \sqrt{\frac{84 \times 10^9\text{ Pa}}{\left(7800\frac{\text{kg}}{\text{m}^3}\right)}} = \boxed{3282\frac{\text{m}}{\text{s}}}$$

REFLECT

For most common materials, the Young's modulus is larger than the shear modulus, so the longitudinal speed will be larger than the transverse speed.

13.66

SET UP

We want to find an expression that shows how the speed of sound in air varies with temperature (in Celsius). Starting from $v = \sqrt{\frac{B}{\rho}}$, we can use the relationship between the bulk modulus and pressure ($B = \gamma P$), the ideal gas law ($PV = nRT$), the definition of molar mass $\left(\frac{M}{n}\right)$, and the conversion between Kelvin and Celsius ($T = T_C + 273$). The molar mass of air is 0.02895 kg/mol; the γ for air is 1.4; and $R = 8.314$ J/(mol · K).

SOLVE

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma\left(\frac{nRT}{V}\right)}{\rho}} = \sqrt{\frac{\gamma nR(T_C + 273)}{\rho V}} = \sqrt{\frac{\gamma nR(T_C + 273)}{M}}$$

$$= \sqrt{\frac{\gamma R(T_C + 273)}{\left(\frac{M}{n}\right)}} = \sqrt{\frac{(1.4)(8.314)(T_C + 273)}{(0.02895)}} = \boxed{\sqrt{402(T_C + 273)}} \text{ (SI units)}$$

REFLECT
For temperatures near 0 Celsius, we can use a Taylor expansion to write this as $v = 331 + 0.6T_C$.

13.67

SET UP
We are shown four different scenarios where square waveforms of different widths and amplitudes are approaching one another (see figure). When they overlap in time and space, they will interfere with one another. If the waveforms are both upright, they will constructively interfere; if the waveforms are inverted relative to one another, they will destructively interfere.

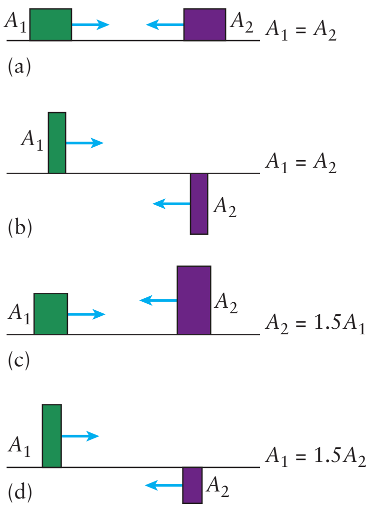


Figure 13-7 Problem 67

SOLVE
Part a) The resulting waveform will have the same width as the incoming waveforms but an amplitude of $A = A_1 + A_2 = \boxed{2A_1}$.

Part b) The resulting waveform will have an amplitude of $A = A_1 - A_2 = \boxed{0}$.

Part c) The resulting waveform will have the same width as the incoming waveforms but an amplitude of $A = A_1 + A_2 = \boxed{2.5A_1}$.

Part d) The resulting waveform will have the same width as the incoming waveforms but an amplitude of $A = A_1 - A_2 = \boxed{A_1/3}$.

REFLECT
The waveforms will pass through one another unaffected. Only in the exact moment when they are coincident will we observe another square waveform with the amplitudes we found.

13.68

SET UP

We are shown the resultant waveform caused by two waves interfering at a point x. One of the waves is moving toward the right, while the other is moving toward the left. Let's say the resultant waveform has an amplitude of $2A$ at x. Three possible situations that could give us this result are (1) two waves of equal amplitude A; (2) one wave of amplitude $3A$, one of amplitude $-A$; and (3) one wave of amplitude $0.5A$, one of amplitude $1.5A$. All of the waves will have the same general shape as the resultant waveform.

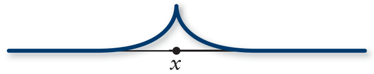


Figure 13-8 Problem 68

SOLVE

1) two waves of equal amplitude A:

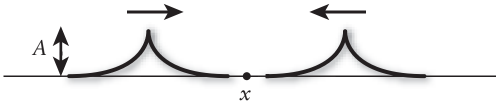


Figure 13-9 Problem 68

2) one wave of amplitude $3A$, one of amplitude $-A$:

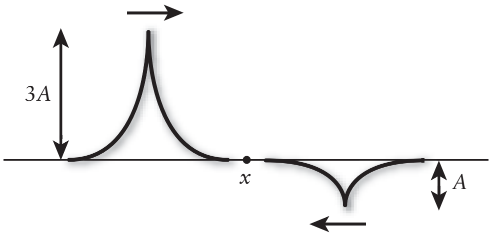


Figure 13-10 Problem 68

3) one wave of amplitude $0.5A$, one of amplitude $1.5A$:

Figure 13-11 Problem 68

REFLECT

There is an infinite number of answers to this question.

13.69

SET UP

We are shown four different scenarios where waveforms of different amplitudes are approaching one another (see figure). When they overlap in time and space, they will interfere

with one another. If the waveforms are both upright, they will constructively interfere; if the waveforms are inverted relative to one another, they will destructively interfere.

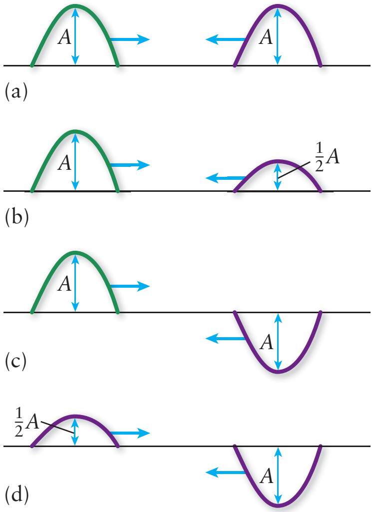


Figure 13-12 Problem 69

SOLVE

Part a) The resulting waveform will have the same width and general shape as the incoming waveforms but an amplitude of $A = A_1 + A_2 = \boxed{2A}$.

Part b) The resulting waveform will have the same width and general shape as the incoming waveforms but an amplitude of $A = A_1 + A_2 = \boxed{1.5A}$.

Part c) The resulting waveform will have an amplitude of $A = A_1 - A_2 = \boxed{0}$.

Part d) The resulting waveform will have the same width and general shape as the incoming waveforms but an amplitude of $A = A_1 - A_2 = \boxed{-A/2}$.

REFLECT

The waveforms will pass through one another unaffected. Only in the exact moment when they are coincident will we observe another waveform with the amplitudes we found and the same shape as the incoming waveforms.

13.70

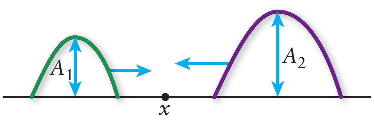


Figure 13-13 Problem 70

SET UP

Two waveforms of different widths and amplitudes are traveling toward one another. When they overlap in time and space at point x, they will constructively interfere with one another

since both waveforms are upright. The resulting waveform will have a "bump" in the middle because the incoming waveforms have different widths.

SOLVE

The general form of the resulting waveform at x:

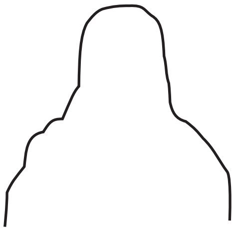


Figure 13-14 Problem 70

REFLECT

The maximum amplitude in the center of the waveform is equal to $A_1 + A_2$.

13.71

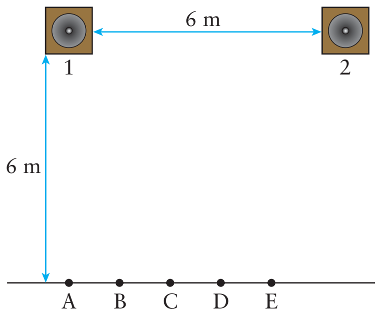


Figure 13-15 Problem 71

SET UP

Two speakers are 6 m apart and playing the same tone ($f = 171.5$ Hz) in phase. The speed of sound ($v = 343$ m/s) divided by the frequency of the tone will give us its wavelength. In order to determine which of the labeled points (A–E) will experience fully constructive interference, we need to use geometry to determine the path length difference between the two waves. The path length for each wave is equal to the hypotenuse of a triangle made by the 6-m distance between the speakers and the line of points and the distance to the point. Keep in mind that each of the labeled points is 1 m from its neighbors. If the quotient of the path length difference and the wavelength of the tone is an integer, then that point will experience fully constructive interference; if not, there will be partial destructive interference.

SOLVE

Wavelength:

$$\lambda = \frac{v}{f} = \frac{\left(343\frac{\text{m}}{\text{s}}\right)}{171.5\text{ Hz}} = 2\text{ m}$$

Point A)

$$\Delta_{\text{pl}} = D_2 - D_1 = (\sqrt{(6\text{ m})^2 + (6\text{ m})^2}) - (6\text{ m}) = 2.49\text{ m}$$

$$\Delta_{\text{pl}} = n\lambda$$

$$n = \frac{\Delta_{\text{pl}}}{\lambda} = \frac{2.49\text{ m}}{2\text{ m}} = 1.25$$

Since n is not an integer, point A $\boxed{\text{will not experience constructive interference}}$.

Point B)

$$\Delta_{\text{pl}} = D_2 - D_1 = (\sqrt{(5\text{ m})^2 + (6\text{ m})^2}) - (\sqrt{(1\text{ m})^2 + (6\text{ m})^2}) = 1.73\text{ m}$$

$$\Delta_{\text{pl}} = n\lambda$$

$$n = \frac{\Delta_{\text{pl}}}{\lambda} = \frac{1.73\text{ m}}{2\text{ m}} = 0.865$$

Since n is not an integer, point B $\boxed{\text{will not experience constructive interference}}$.

Point C)

$$\Delta_{\text{pl}} = D_2 - D_1 = (\sqrt{(4\text{ m})^2 + (6\text{ m})^2}) - (\sqrt{(2\text{ m})^2 + (6\text{ m})^2}) = 0.887\text{ m}$$

$$\Delta_{\text{pl}} = n\lambda$$

$$n = \frac{\Delta_{\text{pl}}}{\lambda} = \frac{0.887\text{ m}}{2\text{ m}} = 0.444$$

Since n is not an integer, point C $\boxed{\text{will not experience constructive interference}}$.

Point D)

$$\Delta_{\text{pl}} = D_2 - D_1 = (\sqrt{(3\text{ m})^2 + (6\text{ m})^2}) - (\sqrt{(3\text{ m})^2 + (6\text{ m})^2}) = 0\text{ m}$$

$$\Delta_{\text{pl}} = n\lambda$$

$$n = \frac{\Delta_{\text{pl}}}{\lambda} = \frac{0\text{ m}}{2\text{ m}} = 0$$

Since n is an integer, point D $\boxed{\text{will experience constructive interference}}$.

Point E)

$$\Delta_{\text{pl}} = D_2 - D_1 = (\sqrt{(2\text{ m})^2 + (6\text{ m})^2}) - (\sqrt{(4\text{ m})^2 + (6\text{ m})^2}) = -0.887\text{ m}$$

$$\Delta_{\text{pl}} = n\lambda$$

$$n = \frac{\Delta_{\text{pl}}}{\lambda} = \frac{-0.887\text{ m}}{2\text{ m}} = -0.444$$

Since n is not an integer, point E $\boxed{\text{will not experience constructive interference}}$.

REFLECT

The speakers are symmetric about a vertical line through point D, which means the distances from each speaker to that point will be equal.

13.72

SET UP

Two waves with the same amplitude ($A = 15$ cm), wavelength ($\lambda = 2$ m), and speed ($v = 3$ m/s) are moving toward one another. The wave that is traveling to the left is out of phase by $\pi/6$ rad. We can use this information and the general form of a wave traveling to the right—$y(x, t) = A\sin(kx - \omega t + \phi)$—in order to write down the mathematical description of each traveling wave. The resultant wave upon interference is just the sum of these two waves. We can find the resultant displacement at $x = 2$ m and $t = 3$ s by evaluating this new function.

SOLVE

Wavenumber:

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{(2\text{ m})} = \pi\text{ m}^{-1}$$

Angular frequency:

$$\omega = vk = \left(3\frac{\text{m}}{\text{s}}\right)(\pi\text{ m}^{-1}) = 3\pi\frac{\text{rad}}{\text{s}}$$

Wave to the right:

$$y_1(x, t) = (15\text{ cm})\sin(\pi x - 3\pi t)\text{ (Omitted units are in SI)}$$

Wave to the left:

$$y_2(x, t) = (15\text{ cm})\sin\left(\pi x + 3\pi t + \frac{\pi}{6}\right)\text{ (Omitted units are in SI)}$$

Part a)

$$y_3(x, t) = y_1(x, t) + y_2(x, t) = [(15\text{ cm})\sin(\pi x - 3\pi t)] + \left[(15\text{ cm})\sin\left(\pi x + 3\pi t + \frac{\pi}{6}\right)\right]$$

$$= \boxed{(15\text{ cm})\left[\sin(\pi x - 3\pi t) + \sin\left(\pi x + 3\pi t + \frac{\pi}{6}\right)\right]}\text{ (Omitted units are in SI)}$$

Part b)

$$y_3(2,3) = (15\text{ cm})\left[\sin(\pi(2) - 3\pi(3)) + \sin\left(\pi(2) + 3\pi(3) + \frac{\pi}{6}\right)\right]$$

$$= (15\text{ cm})\left[\sin(7\pi) + \sin\left(11\pi + \frac{\pi}{6}\right)\right] = (15\text{ cm})\left[0 - \frac{1}{2}\right] = \boxed{-7.5\text{ cm}}$$

REFLECT

Your answers to parts (a) and (b) will differ if you choose to use cosine instead of sine; the answer to part (b) will be -28 cm.

13.73

SET UP

A string is fixed on both ends with a standing wave vibrating in the fourth harmonic. The standing wave pattern for the first harmonic will have two nodes and one antinode. The fourth harmonic will have three more nodes and antinodes, for a total of five nodes and four antinodes.

SOLVE

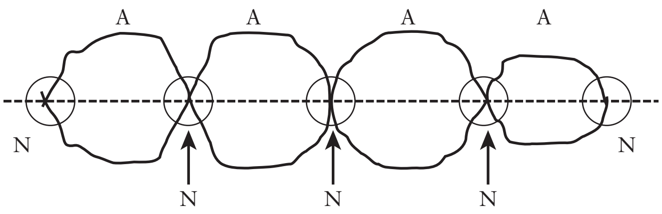


Figure 13-16 Problem 73

REFLECT

The frequency of the fourth harmonic is four times the fundamental frequency.

13.74

SET UP

A 2-m-long string is tied at both ends and set into vibration at the fundamental frequency (that is, $n = 1$). The standing wave wavelengths are given by $\lambda = \frac{2L}{n}$. The frequency is equal to the speed of the wave ($v = 60$ m/s) divided by the wavelength.

SOLVE

Wavelength:

$$\lambda = 2L = 2(2 \text{ m}) = \boxed{4 \text{ m}}$$

Speed:

$$f = \frac{v}{\lambda} = \frac{\left(60\frac{\text{m}}{\text{s}}\right)}{4 \text{ m}} = \boxed{15 \text{ Hz}}$$

REFLECT

The speed of the wave depends only on the tension in the string and its linear mass density. Therefore, the product $v = \lambda f$ will remain constant for a given setup.

13.75

SET UP

A string with a length $L = 1.25$ m and mass $m = 0.0548$ kg has a tension of $T = 200$ N. It is fixed at both ends. We want to calculate the frequencies for the first four harmonics. The

speed of the waves on the string is equal to $v = \sqrt{\frac{T}{\mu}}$, where μ is the linear mass density. The allowed frequencies are given by $f = \frac{v}{\lambda} = \frac{vn}{2L}$.

SOLVE

Part a)

Standing wave frequencies:

$$f = \frac{vn}{2L} = \frac{n}{2L}\sqrt{\frac{T}{\mu}} = \frac{n}{2L}\sqrt{\frac{T}{\left(\frac{m}{L}\right)}} = \frac{n}{2}\sqrt{\frac{T}{Lm}}$$

First harmonic ($n = 1$):

$$f_1 = \frac{1}{2}\sqrt{\frac{200\text{ N}}{(1.25\text{ m})(0.0548\text{ kg})}} = \boxed{27.0\text{ Hz}}$$

Second harmonic ($n = 2$):

$$f_1 = \sqrt{\frac{200\text{ N}}{(1.25\text{ m})(0.0548\text{ kg})}} = \boxed{54.0\text{ Hz}}$$

Third harmonic ($n = 3$):

$$f_1 = \frac{3}{2}\sqrt{\frac{200\text{ N}}{(1.25\text{ m})(0.0548\text{ kg})}} = \boxed{81.0\text{ Hz}}$$

Fourth harmonic ($n = 4$):

$$f_1 = 2\sqrt{\frac{200\text{ N}}{(1.25\text{ m})(0.0548\text{ kg})}} = \boxed{108\text{ Hz}}$$

Part b)

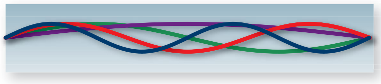


Figure 13-17 Problem 75

First harmonic = purple ($n = 1$)

Second harmonic = green ($n = 2$)

Third harmonic = red ($n = 3$)

Fourth harmonic = blue ($n = 4$)

REFLECT

The frequencies of the higher harmonics are integer multiples of the fundamental frequency, $f_n = nf_1$.

13.76

SET UP

A string ($L = 2.35$ m) that is fixed at both ends vibrates at its fundamental frequency of $f_1 = 24$ Hz. The frequencies of the higher harmonics are integer multiples of this fundamental frequency, $f_n = nf_1$. The standing wave pattern for the first harmonic will have two nodes and one antinode; the second harmonic will have an additional node and antinode; the third harmonic will have yet another node and antinode, and so on. The speed of the wave is equal to the fundamental frequency multiplied by $2L$.

SOLVE

Second harmonic ($n = 2$):

$$f_2 = 2f_1 = 2(24\text{ Hz}) = \boxed{48\text{ Hz}}$$

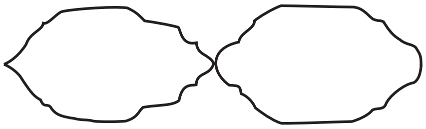


Figure 13-18 Problem 76

Third harmonic ($n = 3$):

$$f_3 = 3f_1 = 3(24\text{ Hz}) = \boxed{72\text{ Hz}}$$

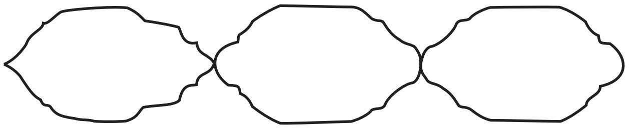


Figure 13-19 Problem 76

Fourth harmonic ($n = 4$):

$$f_4 = 4f_1 = 4(24\text{ Hz}) = \boxed{96\text{ Hz}}$$

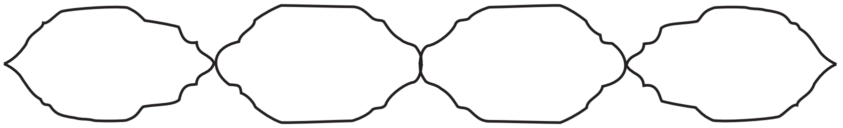


Figure 13-20 Problem 76

Fifth harmonic ($n = 5$):

$$f_5 = 5f_1 = 5(24\text{ Hz}) = \boxed{120\text{ Hz}}$$

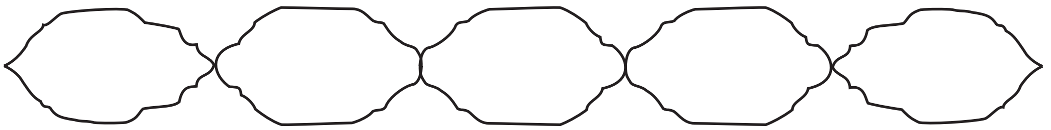


Figure 13-21 Problem 76

Speed:

$$v = \lambda f = \left(\frac{2L}{n}\right)f_n = \left(\frac{2L}{1}\right)f_1 = 2(2.35\text{ m})(24\text{ Hz}) = \boxed{113\frac{\text{m}}{\text{s}}}$$

REFLECT

The wavelength of the fundamental frequency is equal to $2L$, or 2.35 m.

13.77

SET UP

A string of length L is fixed at both ends and is vibrating at a harmonic frequency of f_n = 40 Hz. The next successive harmonic frequency is f_{n+1} = 48 Hz. Taking the ratio of these two frequencies will tell us the value of n to which 40 Hz corresponds. Once we know n, we can use the speed (v = 56 m/s) and the algebraic form for the wavelength for that mode to find L.

SOLVE

Harmonic:

$$\frac{f_{n+1}}{f_n} = \frac{(n+1)f_1}{nf_1} = \frac{n+1}{n} = \frac{48\text{ Hz}}{40\text{ Hz}} = \frac{6}{5}$$

$$n = 5$$

Length of the string:

$$v = 56\frac{\text{m}}{\text{s}} = \lambda_5 f_5 = \left(\frac{2L}{5}\right)(40\text{ Hz})$$

$$L = \frac{\left(56\frac{\text{m}}{\text{s}}\right)}{40\text{ Hz}}\left(\frac{5}{2}\right) = \boxed{3.5\text{ m}}$$

REFLECT

The fundamental frequency is equal to the difference between successive harmonic frequencies: $f_{n+1} - f_n = (n+1)f_1 - nf_1 = f_1$.

13.78

SET UP

A string of length L is vibrating at a harmonic frequency of f_n = 170 Hz. The next successive harmonic frequency is f_{n+1} = 204 Hz. Taking the ratio of these two frequencies will tell us the values of n to which 170 Hz and 204 Hz correspond. If they differ by 1, then we know the string is tied at both ends. If they differ by 2, then the string is only tied at one end. Once we know n, we can use the speed (v = 218 m/s) and the algebraic form for the wavelength for that mode to find L.

SOLVE

One end or both ends:

$$\frac{f_{n+1}}{f_n} = \frac{(n+1)f_1}{nf_1} = \frac{n+1}{n} = \frac{204\text{ Hz}}{170\text{ Hz}} = \frac{6}{5}$$

The string must be $\boxed{\text{tied on both ends}}$ because the successive harmonic frequencies differ by $\Delta n = 1$.

Length of the string:

$$v = 218\frac{\text{m}}{\text{s}} = \lambda_5 f_5 = \left(\frac{2L}{5}\right)(170\ \text{Hz})$$

$$L = \frac{\left(218\frac{\text{m}}{\text{s}}\right)}{170\ \text{Hz}}\left(\frac{5}{2}\right) = \boxed{3.21\ \text{m}}$$

REFLECT

For a string tied on only one end, the successive harmonic frequencies differ by $\Delta n = 2$ since there are no even harmonics in that case.

13.79

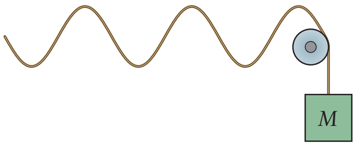


Figure 13-22 Problem 79

SET UP

A mass M provides tension in a 4.5-m-long string. The mass of the string is 0.252 kg. The string vibrates at a frequency of 30 Hz, which sets up a standing wave with a wavelength of 1.5 m. If the mass M remains at rest, the tension in the string will be equal in magnitude to the weight of the mass. We can set the two expressions for the speed of the wave on a string, $v = \sqrt{\frac{T}{\mu}}$ and $v = \lambda f$, equal to one another and solve for M.

SOLVE

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{Mg}{\mu}} = \lambda f$$

$$M = \frac{\mu}{g}(\lambda f)^2 = \frac{\left(\frac{0.252\ \text{kg}}{4.5\ \text{m}}\right)}{\left(9.8\frac{\text{m}}{\text{s}^2}\right)}((1.5\ \text{m})(30\ \text{Hz}))^2 = \boxed{11.6\ \text{kg}}$$

REFLECT

The standing wave produced is the sixth harmonic:

$$n = \frac{2L}{\lambda_n} = \frac{2(4.5\ \text{m})}{1.5\ \text{m}} = 6$$

13.80

SET UP

A guitar string ($\mu = 2.35 \times 10^{-3}$ kg/m) vibrates over a distance $L = 0.60$ m. Its fundamental frequency is $f = 440$ Hz. By setting the two expressions for the speed of a wave on a string, $v = \sqrt{\frac{T}{\mu}}$ and $v = \lambda f$, equal to one another, we can solve for the tension in the string. The wavelength of the fundamental frequency is $2L$.

SOLVE

$$v = \sqrt{\frac{T}{\mu}} = \lambda_1 f_1 = (2L)f$$

$$T = 4L^2 f^2 \mu = 4(0.60\text{ m})^2(440\text{ Hz})^2\left(2.35 \times 10^{-3}\frac{\text{kg}}{\text{m}}\right) = \boxed{655\text{ N}}$$

REFLECT

The phrase "designed to play a note of ______" lets us know the fundamental frequency of that string.

13.81

SET UP

A steel string ($\rho_{\text{steel}} = 7800\text{ kg/m}^3$) has an unstretched length of $L = 0.325$ m and a diameter of $D = 0.25 \times 10^{-3}$ m. By setting the two expressions for the speed of a wave on a string, $v = \sqrt{\frac{T}{\mu}}$ and $v = \lambda f$, equal to one another, we can solve for the tension in the string. We can rewrite the linear mass density in terms of the density of steel, the volume of the string, and L. The fundamental frequency of this string is $f = 660$ Hz and the fundamental wavelength is $2L$.

SOLVE

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\left(\frac{M}{L}\right)}} = \sqrt{\frac{TL}{\rho V}} = \sqrt{\frac{TL}{\rho\left(\pi\left(\frac{D}{2}\right)^2 L\right)}} = \sqrt{\frac{4T}{\rho\pi D^2}} = \lambda_1 f_1 = (2L)f$$

$$T = \pi L^2 f^2 \rho D^2 = \pi(0.325\text{ m})^2(660\text{ Hz})^2\left(7800\frac{\text{kg}}{\text{m}^3}\right)(0.25 \times 10^{-3}\text{ m})^2 = \boxed{70.5\text{ N}}$$

REFLECT

The Young's modulus of steel is 200×10^9 Pa. The strain due to the tensile stress on the string is 0.7%, so we can safely assume that the stretched length of the string is approximately equal to its unstretched length.

13.82

SET UP

An organ pipe of length L is open on both ends. The fundamental frequency of this pipe is $f_1 = 40$ Hz. The allowed wavelength for longitudinal standing waves in air in a pipe open on both ends is $v = \left(\frac{2L}{n}\right)f_n$, where $v = 343$ m/s; we can use this expression to calculate the length of the pipe. The first three overtones refer to the next three harmonics above the fundamental frequency; that is, $n = 2$, 3, and 4. The frequencies of these harmonics are integer multiples of the fundamental frequency: $f_n = nf_1$.

SOLVE

Part a)

$$v = \left(\frac{2L}{n}\right)f_n = \left(\frac{2L}{1}\right)f_1$$

$$L = \frac{v}{2f_1} = \frac{\left(343\frac{\text{m}}{\text{s}}\right)}{2(40\text{ Hz})} = \boxed{4.3\text{ m}}$$

Part b)

$n = 2$:

$$f_2 = 2f_1 = 2(40\text{ Hz}) = \boxed{80\text{ Hz}}$$

$n = 3$:

$$f_3 = 3f_1 = 3(40\text{ Hz}) = \boxed{120\text{ Hz}}$$

$n = 4$:

$$f_4 = 4f_1 = 4(40\text{ Hz}) = \boxed{160\text{ Hz}}$$

REFLECT

We can calculate the frequencies of the first three overtones without knowing the length of the pipe because we are given the frequency of the fundamental tone.

13.83

SET UP

Two successive harmonics of an organ pipe are 228.6 Hz and 274.3 Hz. We are asked to determine whether the pipe is open on both ends or open on one end and closed at the other end. Taking the ratio of these two frequencies will tell us the values of n to which these frequencies correspond. Only odd harmonics are allowed in an open–closed pipe, which means the integers in this ratio must differ by 2. An open–open pipe has harmonics with successive values of n.

SOLVE

Open–closed:

$$\frac{f_{n+2}}{f_n} = \frac{(n+2)f_1}{nf_1} = \frac{n+2}{n}$$

$$\frac{274.3 \text{ Hz}}{228.6 \text{ Hz}} = \frac{6}{5} \stackrel{?}{=} \frac{n+2}{n}$$

The integers in this fraction must both be odd if the pipe is open on one end and closed on the other end. Therefore, it is $\boxed{\text{not an open–closed pipe}}$.

Open–open:

$$\frac{f_{n+1}}{f_n} = \frac{(n+1)f_1}{nf_1} = \frac{n+1}{n}$$

$$\frac{274.3 \text{ Hz}}{228.6 \text{ Hz}} = \frac{6}{5} \stackrel{?}{=} \frac{n+1}{n}$$

The integers in this fraction must be successive if the pipe is open on both ends, which 5 and 6 are. Therefore, it is $\boxed{\text{an open–open pipe}}$.

REFLECT

We didn't have to calculate out both cases. Once we divided the frequencies and saw that the fraction was 6/5, we knew right away that it had to be an open–open pipe.

13.84

SET UP

An open–closed tube ($L = 0.40$ m) resonates in its fundamental frequency (that is, $n = 1$) at 220 Hz. The allowed wavelengths for longitudinal standing waves in an open–closed pipe are $\lambda_n = \frac{4L}{n}$ and only the odd harmonics are allowed. The speed of sound is equal to the product of the wavelength and the frequency of the fundamental tone.

SOLVE

$$v = \lambda_n f_n = \left(\frac{4L}{n}\right)f_n = \left(\frac{4L}{1}\right)f_1 = 4(0.40 \text{ m})(220 \text{ Hz}) = \boxed{352\frac{\text{m}}{\text{s}}}$$

REFLECT

The speed of sound at room temperature (20 degrees Celsius) is 343 m/s. The speed increases with temperature, so the temperature in the room is higher than room temperature.

13.85

SET UP

The second overtone of an organ pipe ($L = 2.25$ m) that is open at both ends excites the third overtone in another organ pipe. The second overtone, which is the third allowed harmonic, of

an open–open pipe has a frequency of $f_3 = 3f_1 = 3\left(\frac{v}{2L}\right)$. It turns out that the original pipe can excite the third overtone in *either* an open–open pipe or an open–closed pipe, so we can set this frequency equal to the frequency of the third overtone for both cases and solve for their respective lengths. The frequency of the third overtone in either case is equal to the frequency of the fourth allowed harmonic. For an open–open pipe the fourth allowed harmonic has a frequency of $f_4 = 4f_1 = 4\left(\frac{v}{2L_{\text{open–open}}}\right)$. Since only odd harmonics are allowed in an open–closed pipe, the fourth allowed harmonic has a frequency of $f_7 = 7f_1 = 7\left(\frac{v}{4L_{\text{open–closed}}}\right)$. We will assume a speed of sound of $v = 340$ m/s throughout this problem.

SOLVE

Frequency emitted by the original pipe:

$$f_3 = 3f_1 = 3\left(\frac{v}{2L}\right) = \frac{3\left(340\frac{\text{m}}{\text{s}}\right)}{2(2.25\text{ m})} = 226.7\text{ Hz}$$

Excited open–open pipe:

$$f_4 = 226.7\text{ Hz} = 4\left(\frac{v}{2L_{\text{open–open}}}\right)$$

$$L_{\text{open–open}} = \frac{2v}{226.7\text{ Hz}} = \frac{2\left(340\frac{\text{m}}{\text{s}}\right)}{226.7\text{ Hz}} = \boxed{3.00\text{ m}}$$

Excited open–closed pipe:

$$f_7 = 226.7\text{ Hz} = 7\left(\frac{v}{4L_{\text{open–closed}}}\right)$$

$$L_{\text{open–closed}} = \frac{7v}{4(226.7\text{ Hz})} = \frac{7\left(340\frac{\text{m}}{\text{s}}\right)}{4(226.7\text{ Hz})} = \boxed{2.62\text{ m}}$$

REFLECT

Be careful when interpreting the term "overtone." To get around the counting confusion for an open–closed pipe, we can define $n = 2m - 1$ (where $m = 1, 2, 3$, etc.) and represent the harmonics of an open–closed pipe as $f_{2m-1} = \frac{nv}{\lambda_n} = \frac{(2m-1)v}{\lambda_{2m-1}}$. In this system, $m = 2$ is the first overtone, which corresponds to f_3 as expected.

13.86

SET UP

We can model the human ear canal as an open–closed pipe of length $L = 0.028$ m. The allowed wavelengths for longitudinal standing waves in an open–closed pipe are $\lambda_n = \frac{4L}{n}$

and only the odd harmonics are allowed. The speed of sound is equal to the product of the wavelength and the frequency of the fundamental tone.

SOLVE

$$v = \lambda_n f_n = \left(\frac{4L}{n}\right) f_n = \left(\frac{4L}{1}\right) f_1$$

$$f_1 = \frac{v}{4L} = \frac{\left(343 \frac{\text{m}}{\text{s}}\right)}{4(0.028\ \text{m})} = \boxed{3100\ \text{Hz}}$$

REFLECT

The range of normal human hearing is about 20 Hz to 20 kHz, with the most sensitive frequencies occurring in the range of 2–5 kHz, which is consistent with our answer.

13.87

SET UP

An alligator emits a sound at 18 Hz. We can estimate the length of the alligator L by modeling it as an open–closed tube of length L with a fundamental frequency of $f_1 = 18$ Hz. The speed of sound in air at room temperature is 343 m/s.

SOLVE

$$v = \lambda_n f_n = \left(\frac{4L}{n}\right) f_n = \left(\frac{4L}{1}\right) f_1$$

$$L = \frac{v}{4f_1} = \frac{\left(343 \frac{\text{m}}{\text{s}}\right)}{4(18\ \text{Hz})} = \boxed{4.8\ \text{m}}$$

This is a little large but $\boxed{\text{not completely unreasonable}}$ given our crude assumption that the alligator is just an open–closed tube.

REFLECT

An average alligator is about 4 m long with most of that length consisting of its tail.

13.88

SET UP

An elephant's trunk is 3 m long. We can determine the fundamental frequency of the sound from the elephant's trunk by modeling it as an open–closed tube of length $L = 3$ m. The speed of sound in air at room temperature is 343 m/s. The first overtone refers to the next allowed harmonic, which is $n = 3$; therefore, the frequency of the first overtone is $3f_1$.

SOLVE

Part a)

$$v = \lambda_n f_n = \left(\frac{4L}{n}\right) f_n = \left(\frac{4L}{1}\right) f_1$$

$$f_1 = \frac{v}{4L} = \frac{\left(343\frac{\text{m}}{\text{s}}\right)}{4(3\text{ m})} = \boxed{28.6\text{ Hz}}$$

Part b)

$$f_3 = 3f_1 = 3(28.6\text{ Hz}) = \boxed{85.8\text{ Hz}}$$

REFLECT

Only odd harmonics are allowed in an open–closed pipe.

13.89

SET UP

The longest pipe in a pipe organ has a length of 9.75 m and the shortest is 0.0191 m. Assuming the speed of air is 343 m/s, we can find the range of the organ by calculating the fundamental frequencies of these pipes. We need to treat them as both open–open pipes $\left(f_1 = \frac{v}{2L}\right)$ and open–closed pipes $\left(f_1 = \frac{v}{4L}\right)$.

SOLVE

Fundamental frequencies for open–open pipes:

$$f_{\text{low}} = \frac{v}{2L} = \frac{\left(343\frac{\text{m}}{\text{s}}\right)}{2(9.75\text{ m})} = \boxed{17.6\text{ Hz}}$$

$$f_{\text{high}} = \frac{v}{2L} = \frac{\left(343\frac{\text{m}}{\text{s}}\right)}{2(0.0191\text{ m})} = \boxed{8980\text{ Hz}}$$

Fundamental frequencies for open–closed pipes:

$$f_{\text{low}} = \frac{v}{4L} = \frac{\left(343\frac{\text{m}}{\text{s}}\right)}{4(9.75\text{ m})} = \boxed{8.79\text{ Hz}}$$

$$f_{\text{high}} = \frac{v}{4L} = \frac{\left(343\frac{\text{m}}{\text{s}}\right)}{4(0.0191\text{ m})} = \boxed{4490\text{ Hz}}$$

REFLECT

The largest range of frequencies occurs when the long pipe acts as an open–closed pipe and the short pipe acts as an open–open one.

13.90

SET UP

The beat frequency produced by the sound from a string vibrating at an unknown frequency and a 440-Hz tuning fork is 4 Hz. The beat frequency is equal to the absolute value of the difference between the two frequencies, $f_{\text{beats}} = |f_2 - f_1|$.

SOLVE

$$f_{\text{beats}} = |f_2 - f_1|$$

$$f_2 = f_1 \pm f_{\text{beats}} = (440\text{ Hz}) \pm (4\text{ Hz}) = \boxed{444\text{ Hz or }436\text{ Hz}}$$

REFLECT

It doesn't matter whether the string is vibrating above or below 440 Hz. All that matters is that the *difference* between the frequencies is 4 Hz.

13.91

SET UP

When a 440-Hz tuning fork is sounded with a guitar string that is nominally "in tune" at 440 Hz, a beat frequency of 5 Hz is heard. This means the guitar string is actually out of tune by 5 Hz. A guitar string is tuned by increasing or decreasing the tension in the string by turning the tuning pegs at the top of the guitar neck. If the beats speed up, then the beat frequency is increasing, which means the difference in the frequencies of the guitar string and the tuning fork is getting larger. The guitar string is in tune when the beats disappear.

SOLVE

Part a) The string is out of tune by $\boxed{5\text{ Hz}}$.

Part b) Try tightening the string just a little tiny bit. If that makes the beats faster, loosen it a bit.

REFLECT

The length of the string and, thus, the wavelength are fixed for a guitar, so the only way to change the frequency of the string is to change the speed of the wave. The speed of a wave on a string is related to the tension.

13.92

SET UP

The beat frequency produced by a 256-Hz tuning fork and an out-of-tune tuning fork is 4 Hz. The beat frequency is equal to the absolute value of the difference between the two frequencies, $f_{\text{beats}} = |f_2 - f_1|$.

SOLVE

$$f_{\text{beats}} = |f_2 - f_1|$$

$$f_2 = f_1 \pm f_{\text{beats}} = (256\text{ Hz}) \pm (4\text{ Hz}) = \boxed{260\text{ Hz or }252\text{ Hz}}$$

REFLECT

We cannot determine the exact frequency of the out-of-tune fork from the beats alone. We would need to play it separately and determine if it was higher or lower in pitch compared with the in-tune fork.

13.93

SET UP

A guitar string under 100 N of tension is supposed to have a frequency of 110 Hz. But, when played at the same time as a reference tone at 110 Hz, beats at 2 Hz are heard. The tension in the string is decreased, and the beat frequency increases, which means the string is becoming more out of tune. For a guitar, the length of the string and, thus, the wavelength are fixed. Therefore, the frequency is proportional to the square root of the tension. When the tension is decreased, the frequency of the sound will also decrease. Because the beat frequency increases when the tension in the string decreases, the initial frequency of the guitar string must be below the desired frequency. Once we know the initial frequency of the guitar, we can set up a ratio between the tensions and the frequencies and solve for the final desired tension when the string is in tune.

SOLVE

Initial frequency:

$$f_{\text{beats}} = |f_2 - f_1|$$

$$f_2 = f_1 \pm f_{\text{beats}} = (110\text{ Hz}) \pm (2\text{ Hz}) = 112\text{ Hz or }108\text{ Hz}$$

Because the beat frequency increases when the tension in the string decreases, the initial frequency of the guitar string must be below the desired frequency, or $f_1 = 108$ Hz.

Tension:

$$\frac{v_1}{v_2} = \frac{\lambda f_1}{\lambda f_2} = \frac{\sqrt{\dfrac{T_1}{\mu}}}{\sqrt{\dfrac{T_2}{\mu}}}$$

$$\frac{f_1}{f_2} = \sqrt{\frac{T_1}{T_2}}$$

$$T_2 = T_1\left(\frac{f_2}{f_1}\right)^2 = (100\text{ N})\left(\frac{110\text{ Hz}}{108\text{ Hz}}\right)^2 = \boxed{103.7\text{ N}}$$

REFLECT

We are told that decreasing the tension increases the beat frequency. Therefore, the tension should be higher than 100 N when the guitar is in tune.

13.94

SET UP

Two identical guitar strings under 200 N produce a sound at $f_1 = 290$ Hz. The tension in one string is then dropped to 196 N, which will decrease the speed of the waves on the string. Since the string remains the same length, the frequency of this string f_2 will also decrease. We can set up a ratio between the speeds on the two strings and solve for the value of f_2 in terms of the tensions and the original frequency. The resulting beat frequency is the absolute value of the difference between f_2 and f_1.

SOLVE

New frequency:

$$\frac{v_2}{v_1} = \frac{\sqrt{\frac{T_2}{\mu}}}{\sqrt{\frac{T_1}{\mu}}} = \frac{\lambda f_2}{\lambda f_1}$$

$$\sqrt{\frac{T_2}{T_1}} = \frac{f_2}{f_1}$$

$$f_2 = f_1\sqrt{\frac{T_2}{T_1}} = (290\ \text{Hz})\sqrt{\frac{196\ \text{N}}{200\ \text{N}}} = 287\ \text{Hz}$$

Beat frequency:

$$f_{\text{beats}} = |f_2 - f_1| = |(287\ \text{Hz}) - (290\ \text{Hz})| = \boxed{3\ \text{Hz}}$$

REFLECT

It makes sense that a small decrease in the tension would cause a small decrease in the frequency, which means the beat frequency should be reasonably small.

13.95

SET UP

We need to determine the factor we need to move from a point source to experience a decrease in the intensity by a factor of 10, 3, and 2. The intensity is inversely proportional to the square of the distance from the source. We can set up a ratio to solve for r_2 in terms of r_1. The problem shouldn't change if the point source were radiating in only one-half of the isotropic medium.

SOLVE

$$\frac{I_2}{I_1} = \frac{\left(\frac{1}{r_2^2}\right)}{\left(\frac{1}{r_1^2}\right)} = \left(\frac{r_1}{r_2}\right)^2$$

$$r_2 = r_1\sqrt{\frac{I_1}{I_2}}$$

Part a)

$$r_2 = r_1\sqrt{\frac{I_1}{\left(\frac{I_1}{10}\right)}} = r_1\sqrt{10} = \boxed{3.16r_1}$$

Part b)

$$r_2 = r_1\sqrt{\frac{I_1}{\left(\frac{I_1}{3}\right)}} = r_1\sqrt{3} = \boxed{1.73r_1}$$

Part c)

$$r_2 = r_1\sqrt{\frac{I_1}{\left(\frac{I_1}{2}\right)}} = r_1\sqrt{2} = \boxed{1.41r_1}$$

Part d) No difference, except that you may be able to get a larger factor decrease by moving to the side rather than directly away from the source.

REFLECT

Since we want the intensity to decrease, it makes sense that r_2 must be larger than r_1.

13.96

SET UP

The sound level of a blue whale is $\beta_{\text{whale}} = 190$ dB and the sound level of a jackhammer is $\beta_{\text{jackhammer}} = 105$ dB. We can solve for the intensity of each sound in terms of the sound level using the definition of the sound level in decibels, $\beta = 10\log\left(\frac{I}{I_0}\right)$. The ratio of the acoustic power generated by the jackhammer to the acoustic power generated by the whale will be equal to the ratio of their intensities for a given distance from the sources and a given time interval.

SOLVE

Intensities:

$$\beta_{\text{whale}} = 10\log\left(\frac{I_{\text{whale}}}{I_0}\right)$$

$$I_{\text{whale}} = \left(10^{\frac{\beta_{\text{whale}}}{10}}\right)I_0 = \left(10^{\frac{190}{10}}\right)I_0 = (10^{19})I_0$$

$$\beta_{\text{jackhammer}} = 10\log\left(\frac{I_{\text{jackhammer}}}{I_0}\right)$$

$$I_{\text{jackhammer}} = \left(10^{\frac{\beta_{\text{jackhammer}}}{10}}\right)I_0 = \left(10^{\frac{105}{10}}\right)I_0 = (10^{10.5})I_0$$

Energies:

$$\frac{I_{\text{jackhammer}}}{I_{\text{whale}}} = \frac{P_{\text{jackhammer}}}{P_{\text{whale}}} = \frac{(10^{10.5})I_0}{(10^{19})I_0} = 10^{-8.5}$$

$$P_{\text{jackhammer}} = (10^{-8.5})P_{\text{whale}} = \boxed{(3.16 \times 10^{-9})P_{\text{whale}}}$$

REFLECT

We can do a quick mental check of the arithmetic to make sure our answer is correct:

$$10^{-8.5} = 10^{-\frac{17}{2}} = \sqrt{10^{-17}} = \sqrt{(10^{-18})(10^{1})} = (10^{-9})\sqrt{10} = (10^{-9})(3.16) = 3.16 \times 10^{-9}$$

13.97

SET UP

Rush hour traffic lasts for 4 hr each day. A nearby resident plans to convert this sound energy into a more useful form of energy by collecting the sound with a 1-m^2 microphone that has an efficiency of 30%. The sound level of the traffic at the location of the microphone is β = 100 dB. The definition of the sound level relates the intensity of the sound at a given position to the reference intensity, $I_0 = 10^{-12}$ W/m^2. The total power incident the microphone is equal to the intensity at the microphone multiplied by the surface area of the microphone. The total sound energy due to the traffic is equal to the power multiplied by the time interval the traffic is active, or 4 hr. Because the microphone only absorbs 30% of the total energy that hits it, we need to multiply the total energy by 0.30 to find the total energy collected by the microphone.

SOLVE

Power:

$$\beta = 10\log\left(\frac{I}{I_0}\right)$$

$$I = I_0 10^{\frac{\beta}{10}} = \left(10^{-12}\frac{\text{W}}{\text{m}^2}\right)10^{\frac{100}{10}} = 10^{-2}\frac{\text{W}}{\text{m}^2}$$

$$I = \frac{P}{A}$$

$$P = IA = \left(10^{-2}\frac{\text{W}}{\text{m}^2}\right)(1\text{ m}^2) = 10^{-2}\text{ W}$$

Energy collected:

$$E_{\text{total}} = P\Delta t = (10^{-2}\text{ W})\left(4\text{ hr} \times \frac{3600\text{ s}}{1\text{ hr}}\right) = 144\text{ J}$$

$$E_{\text{collected}} = (0.30)E_{\text{total}} = (0.30)(144\text{ J}) = \boxed{43\text{ J}}$$

She would do better by completely replacing the plan with something else. Barring that, she could move the collector closer to the highway (above the sound-blocking barrier would be best) or make it much larger (much of the cost would be in electronics, not the collector). Focusing wouldn't do much in the horizontal direction as the source is diffuse, but vertical focusing reflectors could help.

REFLECT

An energy amount of 43 J is not very much. If this amount of energy were in the form of heat, it would increase the temperature of 1 kg of water by only 0.01 degrees C.

13.98

SET UP

The sound level of a longitudinal wave emitted from a speaker at a distance of $r = 3$ m is $\beta = 85$ dB. The definition of the sound level relates the intensity of the sound at a given position to the reference intensity, $I_0 = 10^{-12}$ W/m^2. The intensity at a distance r from a point source is equal to the total power divided by the surface area of a sphere of radius r.

SOLVE

Part a)

$$\beta = 10\log\left(\frac{I}{I_0}\right)$$

$$I = I_0 10^{\frac{\beta}{10}} = \left(10^{-12}\frac{\text{W}}{\text{m}^2}\right)10^{\frac{85}{10}} = \boxed{3.2 \times 10^{-4}\frac{\text{W}}{\text{m}^2}}$$

Part b)

$$I = \frac{P}{4\pi r^2}$$

$$P = 4\pi r^2 I = 4\pi(3\text{ m})^2\left(3.2 \times 10^{-4}\frac{\text{W}}{\text{m}^2}\right) = \boxed{3.6 \times 10^{-2}\text{ W}}$$

REFLECT

A point source emits spherical waves in all directions as long as the medium is isotropic.

13.99

SET UP

At your location, the sound level of a single goose honking is 88 dB. We can use the definition of the sound level to find the intensity of the honk from a single goose $I_{1\text{ goose}}$. The intensity of 30 geese honking is 30 $I_{1\text{ goose}}$; the sound level for 30 geese is calculated from this total intensity.

SOLVE

$$\beta_{1\text{ goose}} = 10\log\left(\frac{I_{1\text{ goose}}}{I_0}\right)$$

$$I_{1\text{ goose}} = I_0\left(10^{\frac{\beta_{1\text{ goose}}}{10}}\right)$$

$$\beta_{30\text{ geese}} = 10\log\left(\frac{I_{30\text{ geese}}}{I_0}\right) = 10\log\left(\frac{30I_{1\text{ goose}}}{I_0}\right) = 10\log\left(\frac{30(I_0)\left(10^{\frac{\beta_{1\text{ goose}}}{10}}\right)}{I_0}\right)$$

$$= 10\log\left(30\left(10^{\frac{\beta_{1\text{ goose}}}{10}}\right)\right) = 10\log\left(30\left(10^{\frac{88}{10}}\right)\right) = \boxed{102.8\text{ dB}}$$

REFLECT
The sound level of 30 geese honking should be larger than the sound of one goose by $10\log(30) = 14.7$ dB.

13.100

SET UP
We are asked to find a formula for comparing two sound levels, $\Delta\beta = \beta_2 - \beta_1$. We will need to invoke the quotient rule for logarithms, $\log(a) - \log(b) = \log\left(\frac{a}{b}\right)$.

SOLVE

$$\Delta\beta = \beta_2 - \beta_1 = 10\log\left(\frac{I_2}{I_0}\right) - 10\log\left(\frac{I_1}{I_0}\right) = 10[(\log(I_2) - \log(I_0)) - (\log(I_1) - \log(I_0))]$$

$$= 10[\log(I_2) - \log(I_1)] = \boxed{10\log\left(\frac{I_2}{I_1}\right)}$$

REFLECT
Rather than comparing the intensity of the sound to a known reference, we are comparing the two intensities to one another in the logarithm.

13.101

SET UP
We can use the formula we derived in Problem 13.100 to relate an increase in sound level to an increase in intensity.

SOLVE
Part a)

$$\Delta\beta = 10\log\left(\frac{I_2}{I_1}\right) = 1 \text{ dB}$$

$$I_2 = I_1 10^{\frac{1}{10}} = \boxed{1.26I_1}$$

Part b)

$$\Delta\beta = 10\log\left(\frac{I_2}{I_1}\right) = 20 \text{ dB}$$

$$I_2 = I_1 10^2 = \boxed{100I_1}$$

REFLECT
We would expect the intensity to increase if there is an increase in the sound level.

13.102

SET UP

The intensity of a sound at your eardrum, which has a cross-sectional area of 55 mm^2, is 0.003 W/m^2. The rate at which sound energy hits your eardrum is equal to the power, which is related to the intensity $I = \frac{P}{A}$. A point source emits spherical waves with a surface area of $4\pi r^2$, where r is the distance from the source.

SOLVE

Part a)

$$P = IA = \left(0.003\frac{\text{W}}{\text{m}^2}\right)\left(55\text{ mm}^2 \times \left(\frac{1\text{ m}}{10^3\text{ mm}}\right)^2\right) = \boxed{1.65 \times 10^{-7}\text{ W}}$$

Part b)

$$P = IA = I(4\pi r^2) = 4\pi\left(0.003\frac{\text{W}}{\text{m}^2}\right)(2\text{ m})^2 = \boxed{0.151\text{ W}}$$

REFLECT

For a constant intensity, the power is directly proportional to the surface area; if the area increases by six orders of magnitude, the power must also increase by six orders of magnitude.

13.103

SET UP

The cross-sectional area of a typical eardrum is $5.0 \times 10^{-5}\text{ m}^2$, and the intensity at the threshold of pain is 1 W/m^2. The sound power that hits an eardrum is related to the intensity $I = \frac{P}{A}$.

SOLVE

$$P = IA = \left(1\frac{\text{W}}{\text{m}^2}\right)(5.0 \times 10^{-5}\text{ m}^2) = \boxed{5.0 \times 10^{-5}\text{ W} = 50\ \mu\text{W}}$$

REFLECT

A circle with a radius of 4 mm has a cross-sectional area of about $5.0 \times 10^{-5}\text{ m}^2$.

13.104

SET UP

A fire engine's siren has a frequency of $f = 1600$ Hz when at rest. We can use the Doppler shift equation to calculate the observed frequency when moving toward or away from the stationary source at a speed of $v_{\text{obs}} = 28$ m/s. The plus sign corresponds to the observer moving toward the source, whereas the minus sign corresponds to the observer moving away from the source.

SOLVE

$$f_{obs} = \left(\frac{v_s \pm v_{obs}}{v_s \mp v_{src}}\right)f$$

Part a)

$$f_{obs} = \left(\frac{v_s + v_{obs}}{v_s}\right)f = \left(1 + \frac{v_{obs}}{v_s}\right)f = \left(1 + \frac{\left(28\frac{\text{m}}{\text{s}}\right)}{\left(343\frac{\text{m}}{\text{s}}\right)}\right)(1600\text{ Hz}) = \boxed{1731\text{ Hz}}$$

Part b)

$$f_{obs} = \left(\frac{v_s - v_{obs}}{v_s}\right)f = \left(1 - \frac{v_{obs}}{v_s}\right)f = \left(1 - \frac{\left(28\frac{\text{m}}{\text{s}}\right)}{\left(343\frac{\text{m}}{\text{s}}\right)}\right)(1600\text{ Hz}) = \boxed{1469\text{ Hz}}$$

REFLECT

The observed frequency should be higher when moving toward the source and lower when moving away.

13.105

SET UP

An ultrasound machine can measure blood flow speeds by emitting a sound wave at $f_0 = 2.0 \times 10^6$ Hz. The sound wave bounces off a stream of blood flowing at $v = 0.020$ m/s away from the probe. Since the initial sound wave emitted from the probe hits the moving blood and is reflected back toward the probe, we can use $f'' = \left(\frac{v_s \pm v}{v_s \mp v}\right)f_0$ to calculate the observed frequency of the reflected wave, f''; the speed of sound in human tissue is $v_s = 1500$ m/s. The difference in frequency between the transmitted and reflected waves is equal to the beat frequency.

SOLVE

Reflected frequency:

$$f'' = \left(\frac{v_s \pm v}{v_s \mp v}\right)f_0$$

$$f_{obs} = \left(\frac{v_s - v_{obs}}{v_s + v_{src}}\right)f = \left(\frac{\left(1500\frac{\text{m}}{\text{s}}\right) - \left(0.020\frac{\text{m}}{\text{s}}\right)}{\left(1500\frac{\text{m}}{\text{s}}\right) + \left(0.020\frac{\text{m}}{\text{s}}\right)}\right)(2.0 \times 10^6\text{ Hz}) = 1.999947 \times 10^6\text{ Hz}$$

Beat frequency:

$$f_{beats} = |f_2 - f_1| = |(1.999947 \times 10^6\text{ Hz}) - (2.0 \times 10^6\text{ Hz})| = \boxed{53\text{ Hz}}$$

REFLECT

The observed frequency should be lower because the stream of blood is moving away from the source.

13.106

SET UP

A car sounding its horn ($f = 600$ Hz) is moving at $v_{src} = 20$ m/s toward the east. A stationary observer is first standing due east of the car and then due west of the car. We can use the expression for the Doppler shift to calculate the frequency that the stationary observer heard. The speed of sound in air is $v = 343$ m/s.

SOLVE

$$f_{obs} = \left(\frac{v_s \pm v_{obs}}{v_s \mp v_{src}}\right)f$$

Part a)

$$f_{obs} = \left(\frac{v_s}{v_s - v_{src}}\right)f = \left(\frac{\left(343\frac{\text{m}}{\text{s}}\right)}{\left(343\frac{\text{m}}{\text{s}}\right) - \left(20\frac{\text{m}}{\text{s}}\right)}\right)(600 \text{ Hz}) = \boxed{637 \text{ Hz}}$$

Part b)

$$f_{obs} = \left(\frac{v_s}{v_s + v_{src}}\right)f = \left(\frac{\left(343\frac{\text{m}}{\text{s}}\right)}{\left(343\frac{\text{m}}{\text{s}}\right) + \left(20\frac{\text{m}}{\text{s}}\right)}\right)(600 \text{ Hz}) = \boxed{567 \text{ Hz}}$$

REFLECT

According to the observer, the frequency of the horn will continue to increase as it approaches and will decrease once it passes, which is consistent with our everyday observations.

13.107

SET UP

A bicyclist is moving toward a sheer wall at an unknown speed v while holding a tuning fork ringing at $f_0 = 484$ Hz. The bicyclist detects a beat frequency of 6 Hz between the sound from the tuning fork and the sound reflecting off the wall. Because the bicycle is moving toward the wall, the reflected frequency will be higher than f_0. Once we know the reflected frequency, we can use $f'' = \left(\frac{v_s + v}{v_s - v}\right)f_0$ to calculate the speed of the bicycle. We'll assume the speed of sound in air to be 343 m/s.

SOLVE

Observed reflected frequency:

$$f_{beats} = 6 \text{ Hz} = |f'' - (484 \text{ Hz})|$$

The bicycle is moving toward the wall, so we expect the observed reflected frequency to be higher than the original frequency. Therefore:

$$f'' = (484\text{ Hz}) + (6\text{ Hz}) = 490\text{ Hz}$$

Speed of the bicycle:

$$f'' = \left(\frac{v_s + v}{v_s - v}\right) f_0$$

$$\frac{f''}{f_0}(v_s - v) = v_s + v$$

$$\left(\frac{f''}{f_0} - 1\right) v_s = \left(\frac{f''}{f_0} + 1\right) v$$

$$v = \frac{\left(\frac{f''}{f_0} - 1\right)}{\left(\frac{f''}{f_0} + 1\right)} v_s = \frac{\left(\frac{490\text{ Hz}}{484\text{ Hz}} - 1\right)}{\left(\frac{490\text{ Hz}}{484\text{ Hz}} + 1\right)}\left(343\frac{\text{m}}{\text{s}}\right) = \boxed{2.11\frac{\text{m}}{\text{s}}}$$

REFLECT

A speed of 2.11 m/s is a little under 5 mph, which is a reasonable speed for a person cycling while trying to hold a tuning fork.

13.108

SET UP

An ultrasonic scan can measure the pumping heart of a fetus by emitting a sound wave at $f_0 = 5.0 \times 10^6$ Hz. The sound wave bounces off the heart wall moving toward the probe at a speed of $v = 0.10$ m/s. Since the initial sound wave emitted from the probe hits the moving heart wall and is reflected back toward the probe, we can use $f'' = \left(\frac{v_s \pm v}{v_s \mp v}\right) f_0$ to calculate the observed frequency of the reflected wave, f''; the speed of ultrasound in human tissue is $v_s = 1540$ m/s. The difference in frequency between the transmitted and reflected waves is equal to the beat frequency.

SOLVE

Reflected frequency:

$$f'' = \left(\frac{v_s \pm v}{v_s \mp v}\right) f_0$$

$$f_{\text{obs}} = \left(\frac{v_s + v_{\text{obs}}}{v_s - v_{\text{src}}}\right) f = \left(\frac{\left(1500\frac{\text{m}}{\text{s}}\right) + \left(0.10\frac{\text{m}}{\text{s}}\right)}{\left(1500\frac{\text{m}}{\text{s}}\right) - \left(0.10\frac{\text{m}}{\text{s}}\right)}\right)(5.0 \times 10^6\text{ Hz}) = 5.000645 \times 10^6\text{ Hz}$$

Beat frequency:

$$f_{\text{beats}} = |f_2 - f_1| = |(5.000645 \times 10^6\text{ Hz}) - (5.0 \times 10^6\text{ Hz})| = \boxed{645\text{ Hz}}$$

REFLECT

The observed frequency should be higher because the heart wall is moving toward the source.

13.109

SET UP

A bat emits a high-pitched sound at $f = 50{,}000$ Hz while traveling at 10 m/s toward an insect that is traveling away from it. The bat observes the reflected wave that echoes off the insect at a frequency of $f_{obs} = 50{,}050$ Hz. The general Doppler shift is given by $f_{obs} = \left(\frac{v_s \pm v_{obs}}{v_s \mp v_{src}}\right)f$, where the top signs correspond to motion "toward" and the bottom signs correspond to motion "away." The bat acts as a moving source traveling toward the observer at $v_{src} = 10$ m/s. The insect acts as a moving observer traveling away from the source at v_{obs}. This means we'll use the top sign $(-)$ for the source term and the bottom sign for the observer term $(-)$.

SOLVE

$$f_{obs} = \left(\frac{v_s - v_{obs}}{v_s - v_{src}}\right)f$$

$$v_{obs} = v_s - \frac{f_{obs}}{f}(v_s - v_{src}) = \left(343\frac{\text{m}}{\text{s}}\right) - \frac{50{,}050\text{ Hz}}{50{,}000\text{ Hz}}\left(\left(343\frac{\text{m}}{\text{s}}\right) - \left(10\frac{\text{m}}{\text{s}}\right)\right) = \boxed{9.67\frac{\text{m}}{\text{s}}}$$

REFLECT

If the mosquito doesn't change its path, the bat traveling at 10 m/s in the same direction will catch up to it and assuredly eat it.

13.110

SET UP

The mathematical description of a transverse wave is $y(x, t) = (20\text{ cm})\cos\left(5x - 4t + \frac{\pi}{4}\right)$. (Missing units are assumed to be SI.) To find the value of y at a specific time and place, we just have to evaluate the function for the given x and t. In order to find the transverse velocity $v(x, t)$ of any point along the disturbance of the wave, we can evaluate $y(x, t)$ at an arbitrary fixed position x_0 and take the derivative of $y(x_0, t)$ with respect to time; the *maximum* transverse speed is the amplitude of this function. Once we have the functional form for $v(x, t)$, we can evaluate it for the given values of x and t.

SOLVE

Part a)

$$y(0, 0) = (20\text{ cm})\cos\left(5(0) - 4(0) + \frac{\pi}{4}\right) = (20\text{ cm})\cos\left(\frac{\pi}{4}\right) = \frac{20\text{ cm}}{\sqrt{2}} = \boxed{14\text{ cm}}$$

Part b)

$$y(1,1) = (20\text{ cm})\cos\left(5(1) - 4(1) + \frac{\pi}{4}\right) = (20\text{ cm})\cos\left(1 + \frac{\pi}{4}\right) = \boxed{-4.3\text{ cm}}$$

Part c)

$$v(x_0,t) = \frac{\partial y(x_0,t)}{\partial t} = \frac{\partial}{\partial t}\left[(20\text{ cm})\cos\left(5x_0 - 4t + \frac{\pi}{4}\right)\right] = -(20\text{ cm})(4\text{ s}^{-1})\sin\left(5x_0 - 4t + \frac{\pi}{4}\right)$$

$$= \boxed{-\left(80\frac{\text{cm}}{\text{s}}\right)\sin\left(5x_0 - 4t + \frac{\pi}{4}\right)}$$

Part d) Maximum transverse speed is $\boxed{80\text{ cm/s}}$.

Part e)

$$v(0,0) = -\left(80\frac{\text{cm}}{\text{s}}\right)\sin\left(5(0) - 4(0) + \frac{\pi}{4}\right) = -\left(80\frac{\text{cm}}{\text{s}}\right)\sin\left(\frac{\pi}{4}\right) = -\frac{\left(80\frac{\text{cm}}{\text{s}}\right)}{\sqrt{2}} = \boxed{-57\frac{\text{cm}}{\text{s}}}$$

$$v(1,1) = -\left(80\frac{\text{cm}}{\text{s}}\right)\sin\left(5(1) - 4(1) + \frac{\pi}{4}\right) = -\left(80\frac{\text{cm}}{\text{s}}\right)\sin\left(1 + \frac{\pi}{4}\right) = \boxed{-78\frac{\text{cm}}{\text{s}}}$$

REFLECT

The wave velocity is different from the transverse velocity. The wave velocity or propagation velocity ($v = \omega/k = 0.8$ m/s, in this case) describes the motion of the disturbance as a whole. The transverse velocity (which is sinusoidal with a *maximum* speed of $v = 0.8$ m/s) describes the motion of an individual point along the wave. The shared value of 0.8 m/s is purely coincidental.

13.111

SET UP

A volcanic eruption triggers a tsunami. We need to determine whether the sound from the eruption will arrive first through the water or through the air. The speed of sound in air is 343 m/s. We can calculate the speed of sound in water from the bulk modulus ($B = 2.0 \times 10^9$ Pa) and density ($\rho = 1000\text{ kg/m}^3$) of water. The time the first sound wave reaches a seismic station 250×10^3 m away is equal to the distance divided by the speed. The time it takes the tsunami to arrive is the distance divided by the speed of the tsunami, $v_{\text{tsunami}} = 800$ km/hr.

SOLVE

Part a)

$$v_{\text{water}} = \sqrt{\frac{B_{\text{water}}}{\rho_{\text{water}}}} = \sqrt{\frac{2.0 \times 10^9\text{ Pa}}{\left(1000\frac{\text{kg}}{\text{m}^3}\right)}} = 1414\frac{\text{m}}{\text{s}}$$

The speed of sound in air is $v_{\text{air}} = 343$ m/s. The $\boxed{\text{sound in the water}}$ will arrive first.

Part b)

$$t_{\text{water}} = \frac{x}{v_{\text{water}}} = \frac{250 \times 10^3\text{ m}}{\left(1414\frac{\text{m}}{\text{s}}\right)} = 177\text{ s} \times \frac{1\text{ min}}{60\text{ s}} = \boxed{3.0\text{ min}}$$

Part c)

$$t_{\text{tsunami}} = \frac{x}{v_{\text{tsunami}}} = \frac{250 \times 10^3 \text{ m}}{\left(\frac{800 \text{ km}}{1 \text{ hr}} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ hr}}{3600 \text{ s}}\right)} = 1125 \text{ s} \times \frac{1 \text{ min}}{60 \text{ s}} = \boxed{19 \text{ min}}$$

REFLECT

The information regarding the time difference between the tidal wave and the sound is irrelevant. The bulk modulus and density of seawater will be slightly different form the values for freshwater but not enough to radically change our answers.

13.112

SET UP

A transverse wave is described according to $y(x, t) = 0.1\sin(1.25x - 3.5t)$ (SI units). Since we have the functional form of the wave, we can evaluate it at the given position and time to find the displacement of the wave. The wave oscillates in time, so there will be multiple times when the wave has a specific displacement, each related by 2π. The transverse velocity and transverse acceleration of a point on the wave are the first and second partial derivatives of $y(x, t)$ with respect to time, respectively. The maximum values of these quantities are the amplitudes of the resulting sinusoidal functions.

SOLVE

Part a)

$$y(1, 2) = 0.1\sin(1.25(1) - 3.5(2)) = 0.1\sin(-5.75) = \boxed{0.051 \text{ m}}$$

Part b)

$$y(3,t) = 0.1\sin(1.25(3) - 3.5t) = 0.08$$

$$t = \frac{1.25(3) - \arcsin\left(\frac{0.08}{0.1}\right) + 2\pi n}{3.5} = \boxed{0.81 \text{ s}, 2.60 \text{ s}, 4.40 \text{ s}, 6.19 \text{ s, etc.}}$$

Part c)

$$v(x, t) = \frac{\partial y}{\partial t} = \frac{\partial}{\partial t}[0.1\sin(1.25x - 3.5t)] = 0.1(-3.5)\cos(1.25x - 3.5t)$$
$$= -0.35\cos(1.25x - 3.5t)$$

Maximum transverse speed: $\boxed{v_{\text{max}} = 0.35 \text{ m/s}}$

Part d)

$$a(x, t) = \frac{\partial v}{\partial t} = \frac{\partial}{\partial t}[-0.35\cos(1.25x - 3.5t)] = -(-0.35)(-3.5)\sin(1.25x - 3.5t)$$
$$= -1.23\sin(1.25x - 3.5t)$$

Maximum transverse acceleration: $\boxed{a_{\text{max}} = 1.23 \text{ m/s}^2}$

Part e)

$$v(2{,}4) = -0.35\cos(1.25(2) - 3.5(4)) = \boxed{-0.17\frac{\text{m}}{\text{s}}}$$

Part f)

$$a(2{,}4) = -1.23\sin(1.25(2) - 3.5(4)) = \boxed{-1.1\frac{\text{m}}{\text{s}^2}}$$

REFLECT

The wavelength is about 5 m and the period is about 1.8 s, so our values of $v(x = 2\text{ m}, t = 4\text{ s})$ and $a(x = 2\text{ m}, t = 4\text{ s})$ make sense.

13.113

SET UP

A transverse wave is described by $y(x, t) = (1.25\text{ m})\cos(5x - 4t)$ (SI units). We are asked to plot y versus x for $t = 0$ s and $t = 1$ s. In both cases, the amplitude of the wave is 1.25 m and the wavelength is $\lambda = \frac{2\pi}{k} = \frac{2\pi}{(5\text{ m}^{-1})} = 1.3$ m.

SOLVE

Part a)

$$y(x, 0) = (1.25\text{ m})\cos(5x)$$

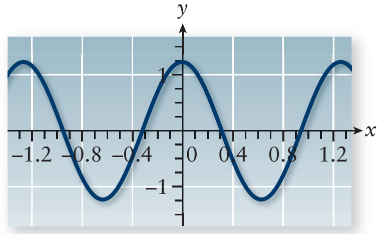


Figure 13-23a Problem 113

Part b)

$$y(x, 1) = (1.25\text{ m})\cos(5x - 4)$$

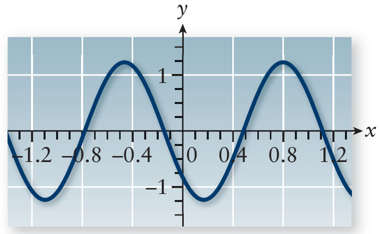


Figure 13-23b Problem 113

REFLECT

The wave is moving toward the right at a speed of 0.8 m/s. The period of the wave is $T = 1.6$ s.

13.114

SET UP

The speed of sound in air as a function of temperature in Celsius is given by $v(T) = \sqrt{109{,}700 + 402T}$. The temperature in the atmosphere increases by 1 degree Celsius for every 150-m decrease in elevation. (We will define positive to point toward higher elevation.) The acceleration of a sound wave is equal to the first time derivative of the speed of sound with respect to time. Since we are given v as a function of T and T as a function of altitude, we will need to use the chain rule. Recall that the first time derivative of the height is equal to the speed v.

SOLVE

$$a = \frac{dv}{dt} = \left(\frac{dv}{dT}\right)\left(\frac{dT}{dy}\right)\left(\frac{dy}{dt}\right) = \left[\frac{d}{dT}(109{,}700 + 402T)^{\frac{1}{2}}\right]\left(\frac{1°\text{C}}{-150\text{ m}}\right)(v)$$

$$= \left[\frac{1}{2}(109{,}700 + 402T)^{-\frac{1}{2}}(402)\right]\left(\frac{1}{-150}\right)(v) = \left[\frac{402}{2v}\right]\left(\frac{1}{-150}\right)(v)$$

$$= \boxed{-1.34\frac{\text{m}}{\text{s}^2}}$$

REFLECT

The acceleration of the sound wave is constant!

13.115

SET UP

The speed of sound in helium and air at 0 degrees Celsius is $v_{\text{He}} = 972$ m/s and $v_{\text{air}} = 331$ m/s, respectively. We can use the speed of sound and density of helium ($\rho_{\text{He}} = 0.1786$ kg/m^3) to find the bulk modulus of helium. A person producing a frequency of 0.500 kHz in air will produce a different frequency if his respiratory tract is filled with helium. Since the effective length of the resonator remains constant, the wavelengths of the standing waves will also be constant, regardless of the gas in the resonator. Therefore, the ratio of the speeds of sound in helium and air will equal the ratio of the frequencies in helium and air. The addition of helium to the respiratory tract will affect the mix of frequencies excited in the resonator and, thus, the sound produced.

SOLVE

Part a)

$$v_{\text{He}} = \sqrt{\frac{B_{\text{He}}}{\rho_{\text{He}}}}$$

$$B_{\text{He}} = \rho_{\text{He}}v_{\text{He}}^2 = \left(0.1786\frac{\text{kg}}{\text{m}^3}\right)\left(972\frac{\text{m}}{\text{s}}\right)^2 = \boxed{1.69 \times 10^5\text{ Pa}}$$

Part b)

$$\frac{v_{\text{He}}}{v_{\text{air}}} = \frac{\lambda f_{\text{He}}}{\lambda f_{\text{air}}}$$

$$f_{\text{He}} = \left(\frac{v_{\text{He}}}{v_{\text{air}}}\right)f_{\text{air}} = \frac{\left(972\frac{\text{m}}{\text{s}}\right)}{\left(331\frac{\text{m}}{\text{s}}\right)}(0.500\text{ kHz}) = \boxed{1.47\text{ kHz}}$$

Part c) The parts of the voice resonator that are in the air all have a frequency increase from the substitution of some helium into the air; the parts of the resonator that are in the body do not. This produces a different mix of frequencies and a different general sound.

REFLECT

The speed of sound in sulfur hexafluoride (SF_6) at room temperature is about $0.44v_{\text{air}}$. Introducing it into the respiratory tract will have the opposite effect of helium on the person's voice by making it deeper.

13.116

SET UP

A beetle lands on a web made of spider silk that is under a tension of $T = 0.50$ N. The density and diameter of a strand of spider silk are approximately $\rho = 1.3\ \text{g/cm}^3$ and $D = 3.0 \times 10^{-6}$ m, respectively. The propagation speed of the transverse waves on the silk is given by $v_{\text{p}} = \sqrt{\frac{T}{\mu}}$; the linear mass density μ is equal to ρA. We can use this speed to calculate the time it takes a wave to travel a distance of 0.25 m.

SOLVE

Speed on the string:

$$v_{\text{p}} = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\rho A}} = \sqrt{\frac{T}{\rho\left(\pi\left(\frac{D}{2}\right)^2\right)}} = \sqrt{\frac{4T}{\pi \rho D^2}}$$

$$= \sqrt{\frac{4(0.50\ \text{N})}{\pi\left(1.3\frac{\text{g}}{\text{cm}^3} \times \frac{1\ \text{kg}}{1000\ \text{g}} \times \left(\frac{100\ \text{cm}}{1\ \text{m}}\right)^3\right)(3.0 \times 10^{-6}\ \text{m})^2}} = 7376\frac{\text{m}}{\text{s}}$$

Time:

$$t = \frac{x}{v} = \frac{0.25\ \text{m}}{\left(7376\frac{\text{m}}{\text{s}}\right)} = \boxed{3.4 \times 10^{-5}\ \text{s} = 34\ \mu\text{s}}$$

REFLECT

Since the waves only take 34 μs to reach the spider, the spider can start descending on the helpless beetle almost immediately after it lands in the web.

13.117

SET UP

The frequency of a musical note that is exactly an octave higher than another note is twice that of the lower note; correspondingly, the frequency of a note that is lower by an octave is half as big. A note that is two octaves above concert A (440 Hz) will, therefore, be four times higher. The ratio of the frequency of middle C to a note one octave below it will be equal to the square root of the ratio between the tensions in the string while playing those notes.

SOLVE

Part a)

One octave:

$$A' = 2A$$

Two octaves:

$$A'' = 2A' = 2(2A) = 4A = 4(440\text{ Hz}) = \boxed{1760\text{ Hz}}$$

Part b)

$$\frac{f_{\text{middle C}}}{f_{\text{octave below}}} = \frac{\left(\frac{v_{\text{middle C}}}{\lambda}\right)}{\left(\frac{v_{\text{octave below}}}{\lambda}\right)} = \frac{\left(\sqrt{\frac{T_{\text{middle C}}}{\mu}}\right)}{\left(\sqrt{\frac{T_{\text{octave below}}}{\mu}}\right)} = \sqrt{\frac{T_{\text{middle C}}}{T_{\text{octave below}}}}$$

$$T_{\text{octave below}} = (T_{\text{middle C}})\left(\frac{f_{\text{octave below}}}{f_{\text{middle C}}}\right)^2 = (T)\left(\frac{f_{\text{octave below}}}{2f_{\text{octave below}}}\right)^2 = \boxed{\frac{T}{4}}$$

REFLECT

We can safely assume the length of the vibrating string on the viola does not change, which means the wavelengths of the standing wave will not change. The frequency of the note corresponds to the pitch that we hear.

13.118

SET UP

An octave is divided into 12 notes: C, $C^{\#}/D^{b}$, D, $D^{\#}/E^{b}$, E, F,$F^{\#}/G^{b}$, G, $G^{\#}/A^{b}$, A, $A^{\#}/B^{b}$, B, C′. We are told in Problem 13.117 that two musical notes that are an octave apart differ in frequency by a factor of 2. Since the tempered scale is split into 12 equal intervals, the ratio r between the frequencies of neighboring notes is $2^{1/12}$. There are three intervals between $F^{\#}$ and A, which has a frequency of 440 Hz, so the ratio between their frequencies should be r^3. Finally, a string on a musical instrument is tuned from B^{b} to B. Since the length of the string and, thus, the wavelength remain fixed, we can relate the ratio of the frequencies to the ratio of the speeds and relate that to the tension in the string.

SOLVE

Part a)

$$\frac{f_{C'}}{f_C} = \left(\frac{f_{C'}}{f_B}\right)\left(\frac{f_B}{f_{A^\#}}\right)\left(\frac{f_{A^\#}}{f_A}\right)\left(\frac{f_A}{f_{G^\#}}\right)\left(\frac{f_{G^\#}}{f_G}\right)\left(\frac{f_G}{f_{F^\#}}\right)\left(\frac{f_{F^\#}}{f_F}\right)\left(\frac{f_F}{f_E}\right)\left(\frac{f_E}{f_{D^\#}}\right)\left(\frac{f_{D^\#}}{f_D}\right)\left(\frac{f_D}{f_{C^\#}}\right)\left(\frac{f_{C^\#}}{f_C}\right) = r^{12} = 2$$

$$\boxed{r = 2^{\frac{1}{12}}}$$

Part b)

$$\frac{f_A}{f_{F^\#}} = \left(\frac{f_A}{f_{G^\#}}\right)\left(\frac{f_{G^\#}}{f_G}\right)\left(\frac{f_G}{f_{F^\#}}\right) = \left(2^{\frac{1}{12}}\right)^3 = 2^{\frac{1}{4}}$$

$$f_{F^\#} = \left(2^{-\frac{1}{4}}\right) f_A = \left(2^{-\frac{1}{4}}\right)(440 \text{ Hz}) = \boxed{370 \text{ Hz}}$$

Part c)

$$\frac{f_B}{f_{B^\flat}} = \frac{\left(\frac{v_B}{\lambda}\right)}{\left(\frac{v_{B^\flat}}{\lambda}\right)} = \frac{\left(\sqrt{\frac{T_B}{\mu}}\right)}{\left(\sqrt{\frac{T_{B^\flat}}{\mu}}\right)} = \sqrt{\frac{T_B}{T_{B^\flat}}} = 2^{\frac{1}{12}}$$

$$\frac{T_B}{T_{B^\flat}} = \left(2^{\frac{1}{12}}\right)^2 = 2^{\frac{1}{6}}$$

$$T_B = \left(2^{\frac{1}{6}}\right) T_{B^\flat} = 1.12 T_{B^\flat}$$

The tension needs to be $\boxed{\text{increased by a factor of 1.12}}$.

REFLECT

The frequency and speed are directly proportional for a constant wavelength. If the tension increases, the speed will also increase, which means the frequency must increase.

13.119

SET UP

The 5^{th} overtone of a 0.7-m-long organ pipe is 1500 Hz. We are not told whether this is an open–open or an open–closed pipe. For an open–open pipe the 5^{th} overtone is f_6, whereas the 5^{th} overtone for an open–closed pipe is f_{11}. We can use the relationship for the speed of sound as a function of temperature (in Celsius) to determine the temperature of the location of the organ. We will need to calculate the temperature twice—once for an open–open pipe and again for an open–closed pipe—and determine which is more reasonable for a pipe organ located in Vancouver, British Columbia.

SOLVE

Open–open:

$$f_6 = 6f_1 = 6\left(\frac{v}{2L}\right) = \frac{3v}{L}$$

$$v = \frac{Lf_6}{3} = \sqrt{109{,}700 + 402T}$$

$$T = \frac{\left(\frac{Lf_6}{3}\right)^2 - 109{,}700}{402} = \frac{\left(\frac{(0.7 \text{ m})(1500 \text{ Hz})}{3}\right)^2 - 109{,}700}{402} = 31.8°\text{C}$$

Open–closed:

$$f_{11} = 11f_1 = 11\left(\frac{v}{4L}\right)$$

$$v = \frac{4Lf_{11}}{11} = \sqrt{109{,}700 + 402T}$$

$$T = \frac{\left(\frac{4Lf_6}{11}\right)^2 - 109{,}700}{402} = \frac{\left(\frac{4(0.7\text{ m})(1500\text{ Hz})}{11}\right)^2 - 109{,}700}{402} = 89.8^\circ\text{C}$$

Since an air temperature of 89.8 degrees Celsius is not reasonable, the temperature in the organ loft is 31.8 degrees Celsius.

REFLECT

A temperature of 31.8 degrees Celsius is about 89 degrees Fahrenheit, which is on the high end of reasonable.

13.120

SET UP

An adult female duck has a typical length of $L_{\text{female}} = 16$ in and the length of her bill plus neck is 0.050 m. Assuming sound is only produced in the neck and bill, we can use that length to determine the fundamental frequency of her quack. The bill plus neck will act as an open–closed pipe, since the bill is open to the atmosphere. An adult male duck has a typical length of $L_{\text{male}} = 18$ in; we will assume all of the dimensions of the male duck scale up in the same ratio as the female. We can set up a ratio of the male to female lengths in order to find the male's fundamental frequency in terms of the female's fundamental frequency. A higher pitch corresponds to a higher frequency.

SOLVE

Part a)

$$L = \frac{n\lambda_n}{4} = \frac{n}{4}\left(\frac{v}{f_n}\right)$$

$$f_1 = \frac{v}{4L} = \frac{\left(343\frac{\text{m}}{\text{s}}\right)}{4(0.050\text{ m})} = \boxed{1715\text{ Hz}}$$

Part b)

$$\frac{L_{\text{male}}}{L_{\text{female}}} = \frac{\left(\frac{v}{4f_{\text{male}}}\right)}{\left(\frac{v}{4f_{\text{female}}}\right)} = \frac{f_{\text{female}}}{f_{\text{male}}}$$

$$f_{\text{male}} = f_{\text{female}}\left(\frac{L_{\text{female}}}{L_{\text{male}}}\right) = (1715\text{ Hz})\left(\frac{16\text{ in}}{18\text{ in}}\right) = \boxed{1524\text{ Hz}}$$

Part c) The $\boxed{\text{male}}$ duck produces a lower-pitched quack.

REFLECT

The frequency of a standing wave is inversely proportional to its wavelength. Since the male duck is larger, we would expect it to have a deeper quack.

13.121

SET UP

A worker puts in ear plugs, and the sound level of a jackhammer decreases from 105 dB to 75 dB. The worker then moves twice as far from the sound. Putting all of this together, we can calculate the decrease in intensity caused by each effect, putting in ear plugs and moving away from the source. The decrease in total overall intensity compared to the initial intensity I_1 is equal to the product of the two reductions. Using this information, we can calculate the reduction in the sound level compared to the initial sound level. Recall that the difference between two noise levels is equal to $\Delta\beta = 10\log\left(\frac{I_2}{I_1}\right)$.

SOLVE

Intensity reduction due to ear plugs:

$$\Delta\beta = \beta_{\text{ear plugs}} - \beta_1 = 10\log\left(\frac{I_{\text{ear plugs}}}{I_1}\right)$$

$$I_{\text{ear plugs}} = \left(10^{\frac{\Delta\beta}{10}}\right)I_1 = \left(10^{\frac{-30}{10}}\right)I_1 = (10^{-3})I_1$$

Intensity reduction due to moving away from source:

$$I \propto \frac{1}{r^2}$$

$$\frac{I_{\text{distance}}}{I_1} = \frac{\left(\frac{1}{2r_1}\right)^2}{\left(\frac{1}{r_1}\right)^2} = \frac{1}{4}$$

$$I_{\text{distance}} = \frac{1}{4}I_1$$

Sound level due to both effects:

$$I_{\text{total}} = \left(\frac{1}{4}\right)(10^{-3})I_1 = (2.5 \times 10^{-4})I_1$$

$$\Delta\beta = \beta_{\text{total}} - \beta_1 = 10\log\left(\frac{I_{\text{total}}}{I_1}\right)$$

$$\beta_{\text{total}} = \beta_1 + 10\log\left(\frac{I_{\text{total}}}{I_1}\right) = (105\text{ dB}) + 10\log(2.5 \times 10^{-4}) = \boxed{69\text{ dB}}$$

REFLECT

The ear plugs have a larger effect than simply moving away from the source, which makes sense.

13.122

SET UP

A cancer treatment uses focused ultrasonic waves to heat up and kill a tumor. During a specific procedure, a beam of ultrasound ($f = 4.0 \times 10^6$ Hz, $I = 1500$ W/cm^2) is delivered in 2.5-s pulses to a 0.14 cm $\times$ 0.56 cm area. The speed of sound in the tissue is 1540 m/s, and the density of the tissue is 1058 kg/m^3. We can calculate the wavelength of the ultrasound directly from the frequency and the speed of sound. The energy delivered during each 2.5-s pulse is related to the intensity of the beam and the cross-sectional area of the treatment location. The intensity of the beam is also related to the maximum displacement of the molecules s_{max}: $I = 2\pi^2 \rho v_p f^2 s_{max}^2$.

SOLVE

Part a)

$$\lambda = \frac{v}{f} = \frac{\left(1540\frac{\text{m}}{\text{s}}\right)}{4.0 \times 10^6\text{ Hz}} = \boxed{3.85 \times 10^{-4}\text{ m} = 0.385\text{ mm}}$$

Part b)

$$I = \frac{P}{A} = \frac{\Delta E}{A(\Delta t)}$$

$$\Delta E = IA(\Delta t) = \left(1500\frac{\text{W}}{\text{cm}^2}\right)((0.14\text{ cm})(0.56\text{ cm}))(2.5\text{ s}) = \boxed{294\text{ J}}$$

Part c)

$$I = 2\pi^2 \rho v_p f^2 s_{max}^2$$

$$s_{max} = \sqrt{\frac{I}{2\pi^2 \rho v_p f^2}} = \sqrt{\frac{\left(1500\frac{\text{W}}{\text{cm}^2} \times \left(\frac{100\text{ cm}}{1\text{ m}}\right)^2\right)}{2\pi^2\left(1058\frac{\text{kg}}{\text{m}^3}\right)\left(1540\frac{\text{m}}{\text{s}}\right)(4.0 \times 10^6\text{ Hz})^2}}$$

$$= \boxed{1.7 \times 10^{-7}\text{ m} = 1.7\ \mu\text{m}}$$

REFLECT

Ultrasound wavelengths are typically on the order of millimeters, so our answers seem reasonable. Be sure to use a consistent set of units in your calculations.

13.123

SET UP

A sound level of 120 dB was recorded from 10-Hz sound waves emitted from a volcanic eruption. The intensity of the sound at the location of the detector is related both to the sound level $\left(\beta = 10\log\left(\frac{I}{I_0}\right)\right)$ and to the maximum displacement of the air molecules

s_{max} $(I = 2\pi^2 \rho v_p f^2 s_{max}^2)$, where $\rho = 1.2\ \mathrm{kg/m^3}$. We can also relate the intensity to the amount of energy the wave would deliver to a $2.0\ \mathrm{m} \times 3.0\ \mathrm{m}$ wall located at the site of the detector within 1.0 min, $I = \frac{P}{A} = \frac{\Delta E}{A(\Delta t)}$; recall that power is energy transferred per time.

SOLVE

Part a)

Intensity:

$$\beta = 10\log\left(\frac{I}{I_0}\right)$$

$$I = \left(10^{\frac{\beta}{10}}\right)I_0$$

Maximum displacement:

$$I = 2\pi^2 \rho v_p f^2 s_{max}^2 = \left(10^{\frac{\beta}{10}}\right)I_0$$

$$s_{max} = \sqrt{\frac{\left(10^{\frac{\beta}{10}}\right)I_0}{2\pi^2 \rho v_p f^2}} = \sqrt{\frac{\left(10^{\frac{120}{10}}\right)\left(10^{-12}\frac{\mathrm{W}}{\mathrm{m^2}}\right)}{2\pi^2\left(1.2\frac{\mathrm{kg}}{\mathrm{m^3}}\right)\left(343\frac{\mathrm{m}}{\mathrm{s}}\right)(10\ \mathrm{Hz})^2}} = \boxed{0.0011\ \mathrm{m} = 1.1\ \mathrm{mm}}$$

Part b)

$$I = \frac{P}{A} = \frac{\Delta E}{A(\Delta t)} = \left(10^{\frac{\beta}{10}}\right)I_0$$

$$\Delta E = \left(10^{\frac{\beta}{10}}\right)I_0 A(\Delta t) = \left(10^{\frac{120}{10}}\right)\left(10^{-12}\frac{\mathrm{W}}{\mathrm{m^2}}\right)((2.0\ \mathrm{m})(3.0\ \mathrm{m}))(60\ \mathrm{s}) = \boxed{360\ \mathrm{J}}$$

REFLECT

Since we don't know the distance between the volcano and the sound detector, we have to assume the wall is located at the same distance away.

13.124

SET UP

A diagnostic sonogram uses ultrasound with a frequency of 2.0×10^6 Hz to image a prostate gland. The speed of sound in the prostate is 1540 m/s and its density is $1060\ \mathrm{kg/m^3}$. We can calculate the wavelength of the ultrasound directly from the frequency and the speed of sound. The speed of sound in the prostate is related to the Young's modulus and the density of the tissue.

SOLVE

Part a)

$$\lambda = \frac{v}{f} = \frac{\left(1540\frac{\mathrm{m}}{\mathrm{s}}\right)}{2.0 \times 10^6\ \mathrm{Hz}} = \boxed{7.7 \times 10^{-4}\ \mathrm{m} = 0.77\ \mathrm{mm}}$$

Part b)

$$v_p = \sqrt{\frac{Y}{\rho}}$$

$$Y = \rho v_p^2 = \left(1060\frac{\text{kg}}{\text{m}^3}\right)\left(1540\frac{\text{m}}{\text{s}}\right)^2 = \boxed{2.5 \times 10^9 \text{ Pa}}$$

REFLECT

Comparing with values from Table 9-1, our value for the Young's modulus of the prostate is an order of magnitude larger than that of the ACL (0.1×10^9 Pa), on the same order as nylon (3×10^9 Pa), and an order of magnitude smaller than concrete (30×10^9 Pa), which all seems reasonable.

13.125

SET UP

A jogger is running toward a car with its alarm blaring and hears a frequency of $f_{\text{toward}} =$ 869.5 Hz. After passing it, she hears a frequency of $f_{\text{away}} = 854.5$ Hz. We can set up a ratio between these two frequencies and the expressions describing their Doppler shifts in order to calculate the speed of the jogger. We will use 343 m/s for the speed of sound.

SOLVE

Running toward the car:

$$f_{\text{towards}} = \left(\frac{v_s + v_{\text{obs}}}{v_s}\right)f_0$$

Running from the car:

$$f_{\text{away}} = \left(\frac{v_s - v_{\text{obs}}}{v_s}\right)f_0$$

Speed of the jogger:

$$\frac{f_{\text{towards}}}{f_{\text{away}}} = \frac{\left(\frac{v_s + v_{\text{obs}}}{v_s}\right)f_0}{\left(\frac{v_s - v_{\text{obs}}}{v_s}\right)f_0} = \frac{v_s + v_{\text{obs}}}{v_s - v_{\text{obs}}}$$

$$f_{\text{towards}}(v_s - v_{\text{obs}}) = (v_s + v_{\text{obs}})f_{\text{away}}$$

$$v_{\text{obs}} = \left(\frac{f_{\text{towards}} - f_{\text{away}}}{f_{\text{towards}} + f_{\text{away}}}\right)v_s = \left(\frac{(869.5\text{ Hz}) - (854.5\text{ Hz})}{(869.5\text{ Hz}) + (854.5\text{ Hz})}\right)\left(343\frac{\text{m}}{\text{s}}\right) = \boxed{2.98\frac{\text{m}}{\text{s}}}$$

REFLECT

This corresponds to a speed of about 6.7 mph, which is a reasonable jogging pace.

13.126

SET UP

Two identical speakers ($m = 0.375$ kg) are attached to parallel springs ($k = 5000$ N/m). The speakers both face in the same direction and play the same tone at $f = 10.0$ Hz. The speakers are set into simple harmonic motion with an amplitude of $A = 0.350$ m but oscillate 180 degrees out of phase with one another. We are interested in the highest beat frequency a person will hear when she is standing in front of the speakers. The person will perceive a Doppler shift for each frequency because each speaker is moving back and forth. The speakers are 180 degrees out of phase, so one speaker will be approaching the person at its maximum speed while the other speaker will be receding from the person at the same maximum speed. The maximum Doppler shift will occur when the speakers are moving the fastest, which occurs as they pass through the equilibrium position. We can use conservation of energy to calculate the maximum speed of the speaker. The beat frequency is equal to the difference in the observed frequencies.

SOLVE

Maximum speed of the speakers:

$$\frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}kA^2$$

$$v_{\text{max}} = A\sqrt{\frac{k}{m}} = (0.350\text{ m})\sqrt{\frac{\left(5000\frac{\text{N}}{\text{m}}\right)}{0.375\text{ kg}}} = 40.4\frac{\text{m}}{\text{s}}$$

Observed frequency for approaching speaker:

$$f_{\text{approaching}} = \left(\frac{v_{\text{s}}}{v_{\text{s}} - v_{\text{src}}}\right)f_0 = \left(\frac{\left(343\frac{\text{m}}{\text{s}}\right)}{\left(343\frac{\text{m}}{\text{s}}\right) - \left(40.4\frac{\text{m}}{\text{s}}\right)}\right)(10.0\text{ Hz}) = 11.3\text{ Hz}$$

Observed frequency for receding speaker:

$$f_{\text{receding}} = \left(\frac{v_{\text{s}}}{v_{\text{s}} + v_{\text{src}}}\right)f_0 = \left(\frac{\left(343\frac{\text{m}}{\text{s}}\right)}{\left(343\frac{\text{m}}{\text{s}}\right) + \left(40.4\frac{\text{m}}{\text{s}}\right)}\right)(10.0\text{ Hz}) = 8.95\text{ Hz}$$

Beat frequency:

$$f_{\text{beats}} = |f_{\text{approaching}} - f_{\text{receding}}| = |(11.3\text{ Hz}) - (8.95\text{ Hz})| = \boxed{2.35\text{ Hz}}$$

REFLECT

The beat frequency is momentarily equal to zero when the speakers come to rest at the turning points of their motion.

13.127

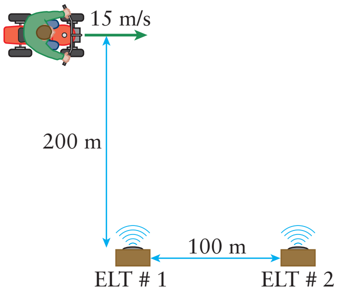


Figure 13-24 Problem 127

SET UP

A rescuer is driving an ATV due east at a speed of 15 m/s. There are two emergency beacons, separated by 100 m and each emitting a radio signal of frequency $f = 121.5 \times 10^6$ Hz in phase, 200 m to the south of the ATV. (The speed of radio waves is 3×10^8 m/s.) There is a detector on the ATV tuned to that frequency. Because the beacons are separated in space, the signals will interfere with one another as the ATV moves toward the east. The positions of constructive interference will be regularly spaced from one another along the straight-line path of the ATV. The easiest position of constructive interference to calculate occurs for $n = 0$; this will be exactly between the two speakers at a position of 50 m to the right of speaker 1. To calculate the position of the $n = 1$ constructive interference will require a lot of geometry and algebra to determine the lengths of the paths (see figure below). The speed of the ATV divided by the distance between the $n = 1$ mode and $n = 0$ mode will tell us how many times the driver will detect constructive interference per second.

SOLVE

Wavelength:

$$\lambda = \frac{v}{f} = \frac{\left(3 \times 10^8 \frac{\text{m}}{\text{s}}\right)}{121.5 \times 10^6\ \text{Hz}} = 2.47\ \text{m}$$

Path length difference for constructive interference for $n = 1$:

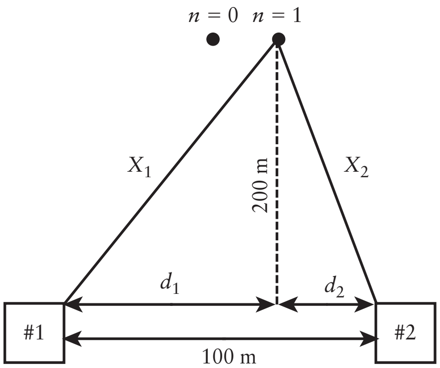


Figure 13-25 Problem 127

$$\Delta_{\text{pl}} = x_1 - x_2 = (1)\lambda = 2.47\ \text{m}$$

$$d_1 + d_2 = 100\ \text{m}$$

Using the Pythagorean theorem:

$$x_1^2 = d_1^2 + 200^2$$

$$x_2^2 = d_2^2 + 200^2$$

Therefore,

$$x_1^2 - d_1^2 = x_2^2 - d_2^2$$

Solving for d_1:

$$x_1^2 = x_2^2 + d_1^2 - d_2^2 = (x_1 - 2.47)^2 + d_1^2 - (100 - d_1)^2$$

$$x_1^2 = x_1^2 + 2.47^2 - 4.94x_1 + d_1^2 - 100^2 - d_1^2 + 200d_1$$

$$4.94x_1 = 200d_1 - 9993.8991$$

$$(x_1 = 40.486d_1 - 2023.1)^2$$

$$x_1^2 = 1639.1d_1^2 - 163{,}813.8d_1 + 4{,}092{,}933.6$$

$$200^2 + d_1^2 = 1639.1d_1^2 - 163{,}813.8d_1 + 4{,}092{,}933.6$$

$$1638.1d_1^2 - 163{,}813.8d_1 + 4{,}052{,}933.6 = 0$$

Solving the quadratic equation:

$$d_1 = \frac{-(-163{,}813.8) \pm \sqrt{(-163{,}813.8)^2 - 4(1638.1)(4{,}052{,}933.6)}}{2(1638.1)} = \frac{163{,}813.8 \pm 16{,}688.9}{3276.2}$$

Taking the positive root:

$$d_1 = \frac{163{,}813.8 + 16{,}688.9}{3276.2} = 55.1$$

Difference along the path of the ATV between locations of constructive interference:

$$(55.1\text{ m}) - (50\text{ m}) = 5.1\text{ m}$$

Constructive interference per second:

$$15\frac{\text{m}}{\text{s}} \times \frac{1\text{ constructive interference}}{5.1\text{ m}} = \boxed{2.94\frac{\text{constructive interference}}{\text{s}}}$$

REFLECT

Be careful when rounding. Do not round intermediate values; only round your final answer.

13.128

SET UP

A string that is fixed on both ends has a mass M, length L, and a tension T; it is set into its fundamental harmonic of oscillation $f_1 = \frac{v}{2L}$, where $v = \sqrt{\frac{T}{\mu}}$. The length of the string is then

increased to $L + \Delta L$ while holding v constant, which will change the frequency of oscillation. We can explicitly calculate the shift in the fundamental frequency $\Delta f = f_1(L + \Delta L) - f_1(L)$ in terms of the speed, original length, and change in length. We expect the change in length to be much smaller than the original length of the string, which means $\frac{\Delta L}{L} \ll 1$. If we rewrite our expression for Δf in terms of $\frac{\Delta L}{L}$, we will be able to approximate the frequency shift by using the binomial expansion, $(1 + x)^n \approx 1 + nx$ for small x. Finally, we are given the values $M = 0.004$ kg, $L = 0.80$ m, $T = 120$ N, and $\Delta L = 0.01$ m and asked to compare our approximation to the actual value for the frequency shift.

SOLVE

Part a)

$$\Delta f = f_1(L + \Delta L) - f_1(L) = \frac{v}{2(L + \Delta L)} - \frac{v}{2L} = \frac{v}{2}\left(\frac{1}{L + \Delta L} - \frac{1}{L}\right)$$

$$= \frac{v}{2}\left(\frac{L - L - \Delta L}{L(L + \Delta L)}\right) = \frac{v}{2}\left(\frac{L - L - \Delta L}{L(L + \Delta L)}\right) = \boxed{\frac{v}{2L}\left(\frac{-\Delta L}{L + \Delta L}\right)}$$

Part b)

$$\Delta f = \frac{v}{2L}\left(\frac{-\Delta L}{L + \Delta L}\right) = \frac{v}{2L}\left(\frac{-\left(\frac{\Delta L}{L}\right)}{\left(1 + \frac{\Delta L}{L}\right)}\right)$$

Defining $x = \frac{\Delta L}{L}$ and applying the binomial expansion:

$$\Delta f = \frac{v}{2L}\left(\frac{-\left(\frac{\Delta L}{L}\right)}{\left(1 + \frac{\Delta L}{L}\right)}\right) = \frac{v}{2L}(-x)(1 + x)^{-1} \approx -\frac{vx}{2L}(1 - x) \approx -\frac{vx}{2L} = \boxed{-\frac{v\Delta L}{2L^2}}$$

Part c)

$$\Delta f = f_{\text{new}} - f_{\text{old}} = \frac{v}{2L}\left(\frac{-\Delta L}{L + \Delta L}\right)$$

$$f_{\text{new}} = f_{\text{old}} + \frac{v}{2L}\left(\frac{-\Delta L}{L + \Delta L}\right) = \frac{v}{2L} + \frac{v}{2L}\left(\frac{-\Delta L}{L + \Delta L}\right) = \frac{v}{2L}\left(1 - \frac{\Delta L}{L + \Delta L}\right) = \frac{v}{2L}\left(\frac{L}{L + \Delta L}\right)$$

$$= \frac{v}{2(L + \Delta L)} = \frac{1}{2(L + \Delta L)}\sqrt{\frac{T}{\mu}} = \frac{1}{2(L + \Delta L)}\sqrt{\frac{T}{\left(\frac{M}{L}\right)}} = \frac{1}{2(L + \Delta L)}\sqrt{\frac{TL}{M}}$$

$$= \frac{1}{2((0.80\text{ m}) + (0.01\text{ m}))}\sqrt{\frac{(120\text{ N})(0.80\text{ m})}{0.004\text{ kg}}} = \boxed{95.6\text{ Hz}}$$

Approximation:

$$\Delta f = f_{\text{new}} - f_{\text{old}} = -\frac{v\Delta L}{2L^2}$$

$$f_{\text{new}} = f_{\text{old}} - \frac{v\Delta L}{2L^2} = \frac{v}{2L} - \frac{v\Delta L}{2L^2} = \frac{v}{2L}\left(1 - \frac{\Delta L}{L}\right) = \frac{\left(\sqrt{\frac{T}{\mu}}\right)}{2L}\left(1 - \frac{\Delta L}{L}\right) = \frac{\left(\sqrt{\frac{T}{\left(\frac{M}{L}\right)}}\right)}{2L}\left(1 - \frac{\Delta L}{L}\right)$$

$$= \frac{\left(\sqrt{\frac{LT}{M}}\right)}{2L}\left(1 - \frac{\Delta L}{L}\right) = \frac{1}{2}\sqrt{\frac{T}{LM}}\left(1 - \frac{\Delta L}{L}\right) = \frac{1}{2}\sqrt{\frac{120\text{ N}}{(0.80\text{ m})(0.004\text{ kg})}}\left(1 - \frac{0.01\text{ m}}{0.80\text{ m}}\right)$$

$$= \boxed{95.6\text{ Hz}}$$

REFLECT

If the tension and the linear mass density remain constant for the string, the speed of transverse waves on the string remains constant. The speed is equal to the wavelength multiplied by the frequency; increasing the length of the string increases the wavelength, which would decrease the frequency. This explains the explicit minus sign in our expression for Δf.

13.129

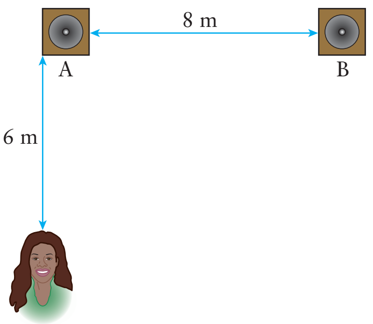


Figure 13-26 Problem 129

SET UP

Two 40-W speakers, labeled A and B, are playing the same 857.5-Hz tone in phase. The speakers are 8 m apart, and speaker A is 6 m in front of you. In order to determine the intensity where you are, we need to determine whether or not there is constructive interference by calculating the path difference between the sound waves from A and B and compare it to the wavelength. If there is fully constructive interference, the total intensity is the sum of the intensity of each speaker at that position, where the intensity of each speaker is $I = \frac{P}{4\pi r^2}$. The sound level is equal to $\beta = 10\log\left(\frac{I_{\text{total}}}{I_0}\right)$.

SOLVE

Wavelength:

$$\lambda = \frac{v}{f} = \frac{\left(343\frac{\text{m}}{\text{s}}\right)}{857.5\ \text{Hz}} = 0.4\ \text{m}$$

Interference:

$$\Delta_{\text{pl}} = D_{\text{B}} - D_{\text{A}} = \left(\sqrt{(8\ \text{m})^2 + (6\ \text{m})^2}\right) - (6\ \text{m}) = (10\ \text{m}) - (6\ \text{m}) = 4\ \text{m}$$

$$\frac{\Delta_{\text{pl}}}{\lambda} = \frac{4\ \text{m}}{0.4\ \text{m}} = 10$$

which means there will be fully constructive interference.

Total Intensity:

$$I_{\text{total}} = I_{\text{A}} + I_{\text{B}} = \frac{P}{4\pi r_{\text{A}}^2} + \frac{P}{4\pi r_{\text{B}}^2} = \frac{P}{4\pi}\left(\frac{1}{r_{\text{A}}^2} + \frac{1}{r_{\text{B}}^2}\right) = \frac{40\ \text{W}}{4\pi}\left(\frac{1}{(6\ \text{m})^2} + \frac{1}{(10\ \text{m})^2}\right) = \boxed{0.12\frac{\text{W}}{\text{m}^2}}$$

Sound level:

$$\beta = 10\log\left(\frac{I_{\text{total}}}{I_0}\right) = 10\log\left(\frac{\left(0.12\frac{\text{W}}{\text{m}^2}\right)}{\left(10^{-12}\frac{\text{W}}{\text{m}^2}\right)}\right) = \boxed{111\ \text{dB}}$$

REFLECT

A sound level of 111 dB is equivalent to a car horn at a distance of 1 m.

Chapter 14
Thermodynamics I

Conceptual Questions

14.1 It makes sense because the average kinetic energy in each degree of freedom (position, rotation, deformation) is proportional to the temperature. It is a bit contradictory because the individual components can have more or less kinetic energy than the average.

14.2 Since the two gases are in thermal contact for a long time, they will eventually reach thermal equilibrium, which means $T_A = T_B$. From the ideal gas law, $PV = NkT$, so $P_A V_A = P_B V_B$. We cannot say anything about the individual values of the pressure and volume, only their product.

14.3 The warmer the day, the faster the molecules will move, so the sooner the smell will spread.

14.4 It is true that a 1-degree Celsius (or Kelvin) change is equal to a 1.8-degree Fahrenheit change, but the length of a thermometer is based on the maximum and minimum values it is designed to read, not the temperature scale it employs.

14.5 If the answer is a temperature *difference*, then it could well be correct. Otherwise, a temperature of -508 degrees F is less than absolute zero. The student's only other chance to be correct is if the question asks her to prove something through contradiction.

14.6 There are five that usually come up: Kelvin, Celsius, Fahrenheit, Rankine, and Réaumur.

14.7 Part a) The temperature is different in the various accessible places on the body, and most peripheral temperatures are noticeably cooler than the core temperature. For example, if the armpit temperature gets as high as 37 degrees C, that's a fever.

Part b) In addition to the normal measurement variations, the body's temperature is homeostatically regulated only to a range, and the set point of that regulation can shift. (It tends to rise in the evening.)

14.8 This idea is best expressed in the more modern concept known as the heat index, which combines air temperature and relative humidity in order to determine the equivalent perceived temperature. The human body cools itself through the evaporation of sweat, which carries heat away from the body. When the relative humidity is high, the evaporation rate and, therefore, the rate at which the body dissipates heat are reduced.

14.9 Part a) They are the same temperature.

Part b) The steam can cause a more severe burn because of its latent heat of vaporization.

14.10 Thermometer A (200–270 K) can be used as a freezer gauge. Thermometer B (230–270 K) can be used as a meteorological gauge (for outdoor temperatures). Thermometer C (300–550 K) can be used as an oven gauge. Thermometer D (300–315 K) can be used as a medical thermometer for humans.

14.11 Supposing that a person understands what "thermometer" means and how to use one, the definition is not too bad. One limitation is that temperatures are achievable that cannot be measured with thermometers. Another limitation of the definition is that it doesn't incorporate anything about the underlying phenomenon, though defining temperature in a way that does is difficult.

14.12 Place the thermometer in an ice bath and let it come to equilibrium. The level of the mercury corresponds to 0 degrees C. Then boil some water, place the thermometer in it, and let it reach equilibrium. The level of the mercury in this case corresponds to 100 degrees C. Then we can divide the distance between the 0 mark and the 100 mark by 10 to get the marks for 10 degrees C, 20 degrees C, and so on. Dividing these regions into 10 again will give the individual degree markings.

14.13 The reason that so many people live near coastal regions (versus in landlocked countries) is related to this thermodynamic argument. The temperature range in landlocked regions is much larger because the specific heat of dirt is so much lower than that of water. The average temperature of Kansas City is about 54 degrees F, and the average temperature in San Francisco is 55 degrees F. However, maximum and minimum temperatures range from below 0 degrees F to above 100 degrees F in the Midwest, while in the San Francisco area, the range is from 30 degrees F to less than 100 degrees F. The water that surrounds coastal cities tends to balance the temperature changes due to the large value for water's specific heat. The average temperature of a coastal city is basically the average temperature of the ocean right off the coast.

14.14 Part a) $K_{\text{avg}} = \frac{1}{2}kT$

Part b) $K_{\text{avg}} = 5kT$

14.15 It makes sense to make highway repairs in the summertime because the material (concrete, asphalt, etc.) will expand to its maximum level when it is first put in place. If you repair a pothole, for example, in the winter months, the asphalt will expand beyond the pothole's capacity when the warmer summertime comes around. This causes buckling and bumpy conditions.

14.16 In addition to making repairs easier, the gaps between smaller sections allow for more expansion during warmer months.

14.17 The two metals must be chosen to have different thermal expansion coefficients. Because the metals are attached directly, thermal expansion will bend the bimetallic strip, which in turn pushes or pulls on the switch.

14.18 Part a) joule, calorie, BTU (British thermal units)

Part b) Joules are used in physics; calories are used in chemistry and food chemistry; and BTUs are used in engineering.

14.19 Part a) $PV = NkT$ is most appropriate if one is counting molecules (such as when deriving a quantity). $PV = nRT$ is most appropriate if one is counting moles (such as when analyzing lab results).

Part b) P is the pressure of the gas, V is the volume the gas occupies, N is the number of molecules, n is the number of moles of molecules, k is the Boltzmann constant, R is the universal gas constant, and T is the temperature of the gas.

14.20 The word *latent* means "hidden." In the context of latent heat, a system is absorbing heat, but the temperature is not changing while a phase change is taking place.

14.21 First, the temperature of the ice rises to 0 degrees C. (For reference, the specific heat of ice is $2090\frac{\text{J}}{\text{kg}\cdot{}^\circ\text{C}}$.) Second, all of the ice melts. (The heat of fusion L_f is $333{,}000\frac{\text{J}}{\text{kg}}$.) Third, the water warms up to 100 degrees C. (The specific heat of water is $4186\frac{\text{J}}{\text{kg}\cdot{}^\circ\text{C}}$.) Fourth, all the water boils. (The heat of vaporization L_v is $2{,}260{,}000\frac{\text{J}}{\text{kg}}$.) Finally, the temperature of the steam rises. (The specific heat of steam is $2010\frac{\text{J}}{\text{kg}\cdot{}^\circ\text{C}}$.)

14.22 Radiation is the movement of heat energy from a high-temperature source to a lower-temperature environment. It will occur without any medium between the source and the environment. Convection is the movement of heat energy to a higher location due to its lessened density (on top of the denser, hence, cooler, regions). Conduction is the movement of heat energy from a hotter region toward a cooler region due to physical contact.

14.23 Heated air expands and becomes more buoyant. Cooler, denser air near the ceiling descends. Thus, the warm fluid rises and the cooler fluid falls. The same effect will be found in a pot of heated water as well.

14.24 Fiberglass insulation resists the flow of heat energy because it provides a maximum amount of air pockets in the space between interior and exterior walls. Since air has a

very low thermal conductivity, this inhibits the warmth from leaving our homes in the cold months and entering during the hot months.

14.25 If A is in thermal equilibrium with B, and B is in thermal equilibrium with C, then A is in thermal equilibrium with C. This is used to establish that temperature is actually a property of objects.

Multiple-Choice Questions

14.26 **C** (from the object that has the higher temperature to the object that has the lower temperature). Heat moves from a region of higher temperature to a region of lower temperature.

14.27 **B** (reduced to one-quarter its original value).

$$v^2 \propto T$$

$$\frac{v_{\text{rms, 2}}}{v_{\text{rms, 1}}} = \frac{1}{2} = \sqrt{\frac{T_2}{T_1}}$$

$$T_2 = \frac{1}{4}T_1$$

14.28 **C** (The mean free path of gas A is the same as the gas B). The mean free path is independent of the atomic mass.

14.29 **A** (becomes larger). Nearly all substances and objects expand when heated.

14.30 **D** (4 degrees C). The maximum density of pure water occurs at 4 degrees C.

14.31 **D** (the temperature of each object will be the same). The two objects will eventually reach thermal equilibrium.

14.32 **E** (sublimation). The opposite process (from gas to solid) is called deposition.

14.33 **A** (radiation). Radiation is the movement of heat energy from a high-temperature source to a lower-temperature environment.

14.34 **D** (16 times). Power, or heat per second, is proportional to the fourth power of the temperature:

$$\frac{P_2}{P_1} = \frac{T_2^4}{T_1^4} = \frac{(2T_1)^4}{T_1^4} = \frac{16T_1}{T_1} = 16$$

14.35 **C** (halved). The rate of heat transfer is inversely proportional to the thickness of the wall L.

$$\frac{H_2}{H_1} = \frac{\left(\frac{1}{L_2}\right)}{\left(\frac{1}{L_1}\right)} = \frac{L_1}{L_2} = \frac{L_1}{(2L_1)} = \frac{1}{2}$$

Estimation Questions

14.36 On Earth, the coldest temperature was −89 degrees C (−128.2 degrees F) in July 1983, and the hottest temperature was 57 degrees C (134.6 degrees F) in September 1922. In Boston, the hottest temperature ever was 40 degrees C (104 degrees F) on July 4, 1911, and the coldest was −34.4 degrees C (−30 degrees F) in 1946.

14.37 About 100 degrees C—from about 50 degrees C (relatively hot) to about −50 degrees C (relatively cold).

14.38 The hottest ocean temperature is around 38 degrees C and the coldest is 0 degrees C.

14.39 Along the coast the temperature changes are about 5–10 degrees C. In landlocked areas the temperature changes are about 20–25 degrees C.

14.40 It takes about 30,000 J to melt an ice cube.

14.41 After about 10 min, your muscles will shut down and your heart will stop after about an hour.

14.42 It takes a kettle of water about 2 to 3 min to boil.

14.43 Although most people say that half of your body heat is lost through your head, the actual value is around 10%.

14.44 The temperature should increase by about 5 degrees C.

14.45 The radiated power is proportional to the temperature to the fourth power, so a decrease in power by 50% corresponds to a decrease in temperature of about 84%.

14.46 The African elephant's ears are about three times the size of those of the Asian elephant. The radiated power is proportional to both the surface area of the ears and the fourth power of the temperature:

$$\frac{A_{\text{Asian}}}{A_{\text{African}}} = \frac{T^4_{\text{Asian}}}{T^4_{\text{African}}}$$

$$\frac{A_{\text{Asian}}}{(3A_{\text{Asian}})} = \frac{1}{3} = \frac{T^4_{\text{Asian}}}{T^4_{\text{African}}}$$

$$T_{\text{African}} = T_{\text{Asian}}\sqrt[4]{3} = 1.32T_{\text{Asian}}$$

14.47 About 10 times more heat is lost to radiation compared to conduction.

14.48

Heat (J)	Temperature Change (°C)
0	0
3.9	10
7.9	20
20	50
40	100

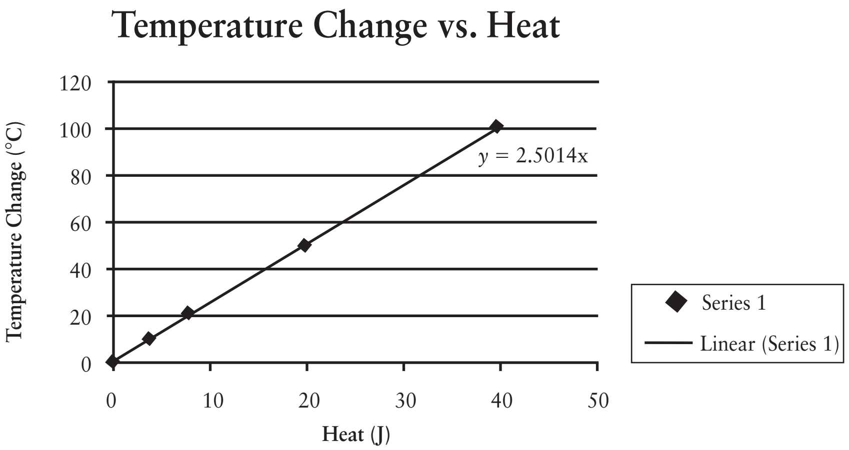


Figure 14-1 Problem 48

We can calculate the specific heat from the slope of our line of best fit:

$$\Delta T = \frac{1}{mc}Q$$

$$\frac{1}{mc} = 2.5014\frac{°\text{C}}{\text{J}}$$

$$c = \frac{1}{(1\ \text{g})\left(2.5014\frac{°\text{C}}{\text{J}}\right)} = \boxed{0.400\frac{\text{J}}{\text{g}\cdot°\text{C}} = 400\frac{\text{J}}{\text{kg}\cdot°\text{C}}}$$

Comparing this to the values listed in Table 14-3, the metal is most likely copper $\left(c = 387\frac{\text{J}}{\text{kg}\cdot°\text{C}}\right)$ or iron/steel $\left(c = 452\frac{\text{J}}{\text{kg}\cdot°\text{C}}\right)$.

14.49

	Temperature (C)	Temperature (K)	Price ($/kg)
Helium	−268	5	100
Nitrogen	−196	77	2
Dry Ice	−78.5	194.5	1

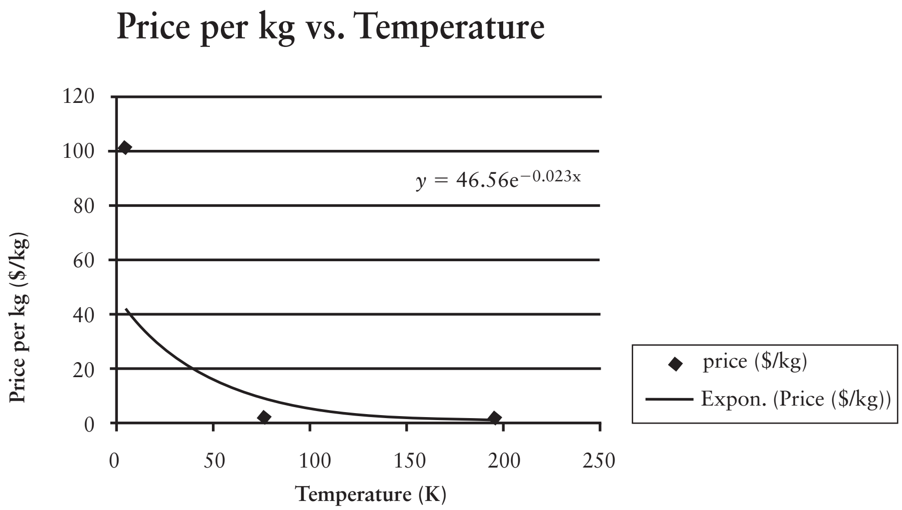


Figure 14-2 Problem 49

$$\frac{\text{Price}}{\text{kg}} = 46.56e^{-0.023T}$$

Part a) The temperature of "ordinary" ice is 273 K:

$$\frac{\text{Price}}{\text{kg}} = 46.56e^{-0.023(273)} = \boxed{\frac{0.09}{\text{kg}}}$$

Part b) A 10-lb (4.5-kg) bag of ice is a few dollars, much larger than the $0.41 our model suggests.

14.50

Density (g/cm^3)	Temperature (°C)
0.99990	8.0
0.99996	6.0
1.00000	4.0
0.99996	2.0
0.99988	0.0

Part a)

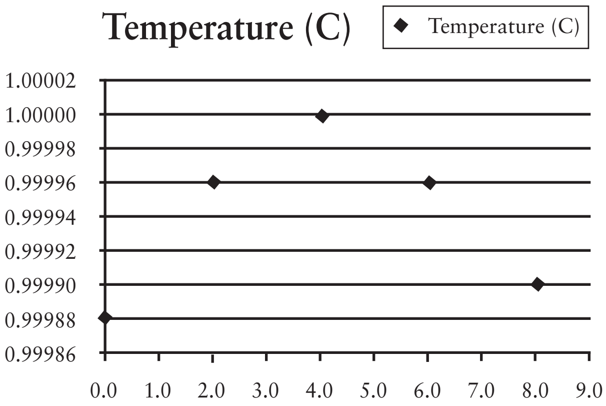


Figure 14-3 Problem 50

Part b) When a lake freezes, as the water progresses from above 4 degrees C to 0 degrees C, the lighter, colder water will start to float. This means the water in lakes freezes from the surface first downward toward the bottom.

Problems

14.51

SET UP

We are asked to convert between different temperature scales. The conversions we'll need to employ are $T_F = \left(\frac{9}{5}\right)T_C + 32$, $T_C = \left(\frac{5}{9}\right)(T_F - 32)$, and $T_K = T_C + 273.15$, where T_F, T_C, and T_K are the temperatures in Fahrenheit, Celsius, and Kelvin, respectively.

SOLVE

Part a)

$$T_F = \left(\frac{9}{5}\right)(28) + 32 = \boxed{82°F}$$

Part b)

$$T_C = \left(\frac{5}{9}\right)(58 - 32) = \boxed{14°C}$$

Part c)

$$T_C = 128 - 273.15 = -145.15$$

$$T_F = \left(\frac{9}{5}\right)(-145.15) + 32 = \boxed{-229°F}$$

Part d)

$$T_C = \left(\frac{5}{9}\right)(78 - 32) = 25.6°C$$

$$T_K = 25.6 + 273.15 = \boxed{299\text{ K}}$$

Part e)

$$T_F = \left(\frac{9}{5}\right)(37) + 32 = \boxed{99°F}$$

REFLECT

Remember that temperatures in Fahrenheit and Celsius can be negative, but temperatures in Kelvin cannot be negative.

14.52

SET UP

We are asked to convert between different temperature scales and comment on the physical significance of each temperature. The conversions we need to employ are the following: $T_F = \left(\frac{9}{5}\right)T_C + 32$, $T_C = \left(\frac{5}{9}\right)(T_F - 32)$, and $T_K = T_C + 273.15$, where T_F, T_C, and T_K are the temperatures in Fahrenheit, Celsius, and Kelvin, respectively.

SOLVE

Part a) The temperature at which water freezes at 1 atm:

$$T_F = \left(\frac{9}{5}\right)(0) + 32 = \boxed{32°F}$$

Part b) The temperature at which water boils at 1 atm:

$$T_C = \left(\frac{5}{9}\right)(212 - 32) = \boxed{100°C}$$

Part c) The temperature at which water freezes at 1 atm:

$$T_C = 273 - 273.15 = -0.15°C$$

$$T_F = \left(\frac{9}{5}\right)(-0.15) + 32 = \boxed{32°F}$$

Part d) Room temperature:

$$T_C = \left(\frac{5}{9}\right)(68 - 32) = 20°C$$

$$T_K = 20 + 273.15 = \boxed{293\text{ K}}$$

REFLECT

Room temperature is 20 degrees Celsius.

14.53

SET UP

We are asked to explain the physical significance of the temperature −273.15 degrees C. This temperature is equal to 0 K, or absolute zero, which is the temperature at which the pressure of all gases becomes zero.

SOLVE

The temperature at which the pressure of all gases becomes zero is −273.15 degrees C. This value, the same for all gases, sets the zero point of the Kelvin temperature scale.

REFLECT

Absolute zero is equal to −459.67 degrees Fahrenheit.

14.54

SET UP

We can use algebra to rearrange $T_C = \left(\frac{5}{9}\right)(T_F - 32)$ to derive a formula for converting from Celsius to Fahrenheit.

SOLVE

$$T_C = \left(\frac{5}{9}\right)(T_F - 32)$$

$$\boxed{T_F = \left(\frac{9}{5}\right)T_C + 32}$$

It's more common for the multiplicative factor to be written as a fraction because it's easier to remember 5/9 or 9/5 rather than 1/1.8 or 1.8.

REFLECT

Remember to subtract 32 from the Fahrenheit temperature *before* multiplying by 5/9.

14.55

SET UP

The highest temperature ever recorded on Earth is 56.7 degrees Celsius and the lowest is −89.2 degrees Celsius. We are asked to convert these temperatures into Fahrenheit and Kelvin. The conversions we need to use are $T_F = \left(\frac{9}{5}\right)T_C + 32$ and $T_K = T_C + 273.15$, where T_F, T_C, and T_K are the temperatures in Fahrenheit, Celsius, and Kelvin, respectively.

SOLVE

Death Valley:

$$T_F = \left(\frac{9}{5}\right)(56.7) + 32 = \boxed{134°\text{F}}$$

$$T_K = 56.7 + 273.15 = \boxed{329.9\text{ K}}$$

Vostok:

$$T_F = \left(\frac{9}{5}\right)(-89.2) + 32 = \boxed{-129°\text{F}}$$

$$T_K = -89.2 + 273.15 = \boxed{184.0\text{ K}}$$

REFLECT

For comparison, dry ice has a temperature around −78.5 degrees C, or −109.3 degrees F.

14.56

SET UP

The normal range for oral temperature in adults is 36.7–37.0 degrees Celsius. We can use $T_F = \left(\frac{9}{5}\right)T_C + 32$ and $T_K = T_C + 273.15$ to convert these temperatures into Fahrenheit and Kelvin.

SOLVE

Fahrenheit:

$$T_{\mathrm{F,\,low}} = \left(\frac{9}{5}\right)(36.7) + 32 = \boxed{98.1°\mathrm{F}}$$

$$T_{\mathrm{F,\,high}} = \left(\frac{9}{5}\right)(37.0) + 32 = \boxed{98.6°\mathrm{F}}$$

Kelvin:

$$T_{\mathrm{K,\,low}} = 36.7 + 273.15 = \boxed{309.9\ \mathrm{K}}$$

$$T_{\mathrm{K,\,high}} = 37.0 + 273.15 = \boxed{310.2\ \mathrm{K}}$$

REFLECT

An oral temperature above 100 degrees F (37.8 degrees C) is considered a fever.

14.57

SET UP

There exists a numerical value that is the same on both the Celsius and Fahrenheit scales. The heat energy transfers must be constant regardless of the temperature scale used, but the degree units on the Celsius and Fahrenheit scales are different. The relationship to the temperature is a fixed amount of energy transfer and the degree units on the Celsius and Fahrenheit scales are not the same. While a plot of average kinetic energy per molecule in a gas versus temperature yields a straight line using either scale, the lines have a different slope when plotted using Fahrenheit temperatures or Celsius temperatures. Therefore, these lines must cross.

SOLVE

Special value where $T_{\mathrm{C}} = T_{\mathrm{F}}$:

$$T_{\mathrm{C}} = \frac{5}{9}(T_{\mathrm{F}} - 32) = \frac{5}{9}(T_{\mathrm{C}} - 32)$$

$$\frac{4}{9}T_{\mathrm{C}} = (-32)\left(\frac{5}{9}\right)$$

$$\boxed{T_{\mathrm{C}} = -40 = T_{\mathrm{F}}}$$

REFLECT

The Celsius and Kelvin scales will never share a numerical value.

14.58

SET UP

Two thermally isolated systems have a temperature T_{A} and T_{B}, respectively. The systems are brought into thermal contact and reach an equilibrium temperature of T_{C}. Heat will flow from a system with a high temperature to a system with a lower temperature. With this in

mind, we can determine the relative magnitudes of T_A and T_B when given some information about the relative magnitude of T_C to T_A.

SOLVE

Part a) If $T_C < T_A$, then heat flowed from A to B, which means $\boxed{T_B < T_A}$.

Part b) If $T_C > T_A$, then heat flowed from B to A, which means $\boxed{T_B > T_A}$.

REFLECT

Heat will flow until both systems reach the same temperature.

14.59

SET UP

One mole of an ideal gas is at a pressure $P = 1$ atm and occupies a volume $V = 1$ L. Before using the ideal gas law $PV = nRT$ to calculate the temperature of the gas in Kelvin, we need to convert P and V to SI units in order to use $R = 8.314$ J/mol · K. The relationships $T_C = T_K - 273.15$ and $T_F = \left(\frac{9}{5}\right)T_C + 32$ will allow us to convert from Kelvin into Celsius and Fahrenheit.

SOLVE

Part a)

$$T = \frac{PV}{nR} = \frac{\left(1\text{ atm} \times \frac{1.01 \times 10^5\text{ Pa}}{1\text{ atm}}\right)\left(1\text{ L} \times \frac{1\text{ m}^3}{1000\text{ L}}\right)}{(1\text{ mol})\left(8.314\frac{\text{J}}{\text{mol} \cdot \text{K}}\right)} = \boxed{12.1\text{ K}}$$

Part b)

$$T_C = T_K - 273.15 = 12.1 - 273.15 = \boxed{-261^\circ\text{C}}$$

$$T_F = \left(\frac{9}{5}\right)T_C + 32 = \left(\frac{9}{5}\right)(-261) + 32 = \boxed{-438^\circ\text{F}}$$

REFLECT

Another useful form of the gas constant exists when P is in atm and V is in L, $R = 0.0821\frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}}$.

14.60

SET UP

A diatomic ideal gas has five degrees of freedom—three translational and two rotational. Each degree of freedom contributes an average energy of $\frac{1}{2}RT$ to the total. Therefore, an average energy per mole of ideal oxygen (O_2) gas is $\frac{5}{2}RT$.

SOLVE

$$E_{avg} = \frac{5}{2}RT = \frac{5}{2}\left(8.314\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(300\text{ K}) = 6236\frac{\text{J}}{\text{mol}}$$

One mole of ideal oxygen gas will have an average energy of $\boxed{6236\text{ J}}$ at 300 K.

REFLECT

There are only two rotational degrees of freedom for a diatomic molecule—rotation about the axes perpendicular to the bond. The molecule is symmetric upon rotation about an axis through its bond. The same reasoning holds for linear polyatomic molecules (for example, carbon dioxide, CO_2).

14.61

SET UP

An unknown gas has a volume $V = 4.13$ L and pressure $P = 10$ atm at a temperature $T =$ 293 K. We can use the ideal gas law, with $R = 0.0821\frac{\text{L}\cdot\text{atm}}{\text{mol}\cdot\text{K}}$, to find the number of moles of gas. The gas sample has a mass of 55 g; dividing this by the number of moles yields the molar mass of the unknown gas. We can use the molar mass and a periodic table to determine the identity of the unknown gas.

SOLVE

Number of moles:

$$PV = nRT$$

$$n = \frac{PV}{RT} = \frac{(10\text{ atm})(4.13\text{ L})}{\left(0.0821\frac{\text{L}\cdot\text{atm}}{\text{mol}\cdot\text{K}}\right)(293\text{ K})} = 1.72\text{ mol}$$

Molar mass:

$$\frac{55\text{ g}}{1.72\text{ mol}} = 32.0\frac{\text{g}}{\text{mol}}$$

This is the molar mass of $\boxed{O_2}$.

REFLECT

Be sure all of your units are consistent when performing your calculation. The pressure and volume are given in non-SI units, so it makes sense to use $R = 0.0821\frac{\text{L}\cdot\text{atm}}{\text{mol}\cdot\text{K}}$.

14.62

SET UP

At 373 K and 1 atm, 10 g of liquid water evaporates into water vapor. The volume the water vapor occupies can be found through the ideal gas law. The number of moles of water is equal to the mass of the sample divided by the molar mass of water, which is 18 g/mol.

SOLVE

$$PV = nRT$$

$$V = \frac{nRT}{P} = \frac{\left(\frac{m}{\text{MM}}\right)RT}{P} = \frac{\left(\frac{10\text{ g}}{\left(\frac{18\text{ g}}{\text{mol}}\right)}\right)\left(0.0821\frac{\text{L}\cdot\text{atm}}{\text{mol}\cdot\text{K}}\right)(373\text{ K})}{1\text{ atm}} = \boxed{17\text{ L}}$$

REFLECT

We can perform a quick calculation to make sure our answer is reasonable: 0.0821 is a little under 0.1, so 10% of 373 is around 37. The fraction $\frac{10}{18} = \frac{5}{9}$ is a little over 0.5, so half of 37 is 18.5. Comparing this to our actual answer, 17 L seems reasonable.

14.63

SET UP

An ideal gas is confined to an insulated container at a temperature of 300 K. Since the gas is allowed to travel in three dimensions, the average kinetic energy of a single atom of the gas will be $K_{\text{avg}} = \frac{3}{2}kT$.

SOLVE

$$K_{\text{avg}} = \frac{3}{2}kT = \frac{3}{2}\left(1.38 \times 10^{-23}\frac{\text{J}}{\text{K}}\right)(300\text{ K}) = \boxed{6.21 \times 10^{-21}\text{ J}}$$

REFLECT

The problem asks for the average kinetic energy per atom of the gas, which means we are working with a monatomic ideal gas.

14.64

SET UP

One mole of helium gas is confined to a volume $V = 1$ L at a pressure $P = 10$ atm. The temperature of the gas can be calculated from the ideal gas law. Since the gas is allowed to travel in three dimensions, the average kinetic energy of a single atom of the gas is $K_{\text{avg}} = \frac{1}{2}mv^2 = \frac{3}{2}kT$, and the square root of v^2 is v_{rms}. The atomic mass of helium is 4.00 amu and the conversion between amu and kg is 1 amu $= 1.66 \times 10^{-27}$ kg.

SOLVE

Ideal gas law:

$$PV = nRT$$

$$T = \frac{PV}{nR} = \frac{(10\text{ atm})(1\text{ L})}{(1\text{ mol})\left(0.0821\frac{\text{L}\cdot\text{atm}}{\text{mol}\cdot\text{K}}\right)} = 121.8\text{ K}$$

Root-mean-square speed:

$$\frac{1}{2}mv^2 = \frac{3}{2}kT$$

$$v_{\text{rms}} = \sqrt{v^2} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3\left(1.38 \times 10^{-23}\frac{\text{J}}{\text{K}}\right)(121.8\text{ K})}{\left(4.00\text{ amu} \times \frac{1.66 \times 10^{-27}\text{ kg}}{1\text{ amu}}\right)}} = \boxed{871\frac{\text{m}}{\text{s}}}$$

REFLECT

This seems like a reasonable speed for a tiny atom zipping around.

14.65

SET UP

Water vapor (H_2O, molecular mass = 18 amu) and oxygen (O_2, molecular mass = 32 amu) are both present in the atmosphere. Since the gases are allowed to travel in three dimensions, the average kinetic energy of a single molecule of the gas is $K_{\text{avg}} = \frac{1}{2}mv^2 = \frac{3}{2}kT$, and the square root of v^2 is v_{rms}. We assume the temperature is constant.

SOLVE

Root-mean-square speed:

$$K_{\text{avg}} = \frac{1}{2}mv^2 = \frac{3}{2}kT$$

$$v_{\text{rms}} = \sqrt{v^2} = \sqrt{\frac{3kT}{m}}$$

Ratio:

$$\frac{v_{\text{rms, water}}}{v_{\text{rms, oxygen}}} = \frac{\left(\sqrt{\frac{3kT}{m_{\text{water}}}}\right)}{\left(\sqrt{\frac{3kT}{m_{\text{oxygen}}}}\right)} = \sqrt{\frac{m_{\text{oxygen}}}{m_{\text{water}}}} = \sqrt{\frac{32\text{ amu}}{18\text{ amu}}} = \sqrt{\frac{16}{9}} = \frac{4}{3}$$

REFLECT

The lighter molecule (water) has a higher v_{rms} at a given temperature, which makes sense.

14.66

SET UP

A sample of air ($V = 1$ L, $n = 0.009$ mol) has a pressure $P = 1.05$ atm. We assume that our air sample has the same makeup as air, in general: 78% N_2, 21% O_2, and 1% Ar. The average speed of the O_2 molecules can be calculated from their average three-dimensional translational kinetic energy, which is equal to $K_{\text{avg, tr}} = \frac{1}{2}(\text{MM})v^2 = \frac{3}{2}RT$. The temperature of

the O_2 molecules is the same as the air sample, which can be calculated from the ideal gas law. The molar mass of O_2 is MM = 0.032 kg/mol.

SOLVE

Temperature:

$$P_{O_2}V = n_{O_2}RT$$

$$(0.21P)V = (0.21n)RT$$

$$PV = nRT$$

$$T = \frac{PV}{nR} = \frac{(1.05\text{ atm})(1\text{ L})}{(0.009\text{ mol})\left(0.0821\frac{\text{L}\cdot\text{atm}}{\text{mol}\cdot\text{K}}\right)} = 1421\text{ K}$$

Average speed:

$$\frac{1}{2}(\text{MM})v^2 = \frac{3}{2}RT$$

$$v = \sqrt{\frac{3RT}{(\text{MM})}} = \sqrt{\frac{3\left(8.314\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(1421\text{ K})}{\left(0.032\frac{\text{kg}}{\text{mol}}\right)}} = \boxed{1052\frac{\text{m}}{\text{s}}}$$

REFLECT

At thermal equilibrium, the temperature must be constant throughout the entire sample regardless of the makeup of the sample.

14.67

SET UP

The average speed of 1 mol of ideal oxygen gas molecules (MM = 0.032 kg/mol) is 450 m/s. The average three-dimensional translational kinetic energy is equal to $K_{\text{avg, tr}} = \frac{1}{2}(\text{MM})v^2 = \frac{3}{2}RT$.

SOLVE

$$\frac{1}{2}(\text{MM})v^2 = \frac{3}{2}RT$$

$$T = \frac{mv^2}{3R} = \frac{\left(0.032\frac{\text{kg}}{\text{mol}}\right)\left(450\frac{\text{m}}{\text{s}}\right)^2}{3\left(8.314\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)} = \boxed{260\text{ K}}$$

REFLECT

The average speed of oxygen molecules at room temperature ($T = 298$ K) is 482 m/s. Since the average speed of the oxygen molecules is less than 482 m/s in this problem, the temperature must be lower than 298 K.

14.68

SET UP

Air is made up of 78.09% nitrogen, 20.95% oxygen, 0.03% carbon dioxide, and 0.93% argon. If the atmospheric pressure on a given day is $P = 0.97$ atm, the partial pressure of each component gas is equal to the fraction of that gas in air multiplied by P.

SOLVE

$$P_{N_2} = (0.7809)P = (0.7809)(0.97\text{ atm}) = \boxed{0.76\text{ atm}}$$

$$P_{O_2} = (0.2095)P = (0.2095)(0.97\text{ atm}) = \boxed{0.20\text{ atm}}$$

$$P_{CO_2} = (0.0003)P = (0.0003)(0.97\text{ atm}) = \boxed{2.9 \times 10^{-4}\text{ atm}}$$

$$P_{Ar} = (0.0093)P = (0.0093)(0.97\text{ atm}) = \boxed{9.0 \times 10^{-3}\text{ atm}}$$

REFLECT

The sum of the partial pressures equals the total pressure:

$$\begin{aligned} P_{N_2} + P_{O_2} + P_{CO_2} + P_{Ar} &= (0.76\text{ atm}) + (0.20\text{ atm}) + (2.9 \times 10^{-4}\text{ atm}) + (9.0 \times 10^{-3}\text{ atm}) \\ &= 0.97\text{ atm} \end{aligned}$$

14.69

SET UP

As a percentage of volume, the components of air are 78.09% N_2, 20.95% O_2, 0.03% CO_2, and 0.93% Ar. From the ideal gas law, the relative volumes of each component will remain constant with changes in temperature at constant pressure. We can use this fact to rewrite these values as a percentage of mass as opposed to volume by calculating a weighted average of the masses. To four significant figures, the molar masses are N_2, 28.02 g/mol; O_2, 32.00 g/mol; CO_2, 44.01 g/mol; and Ar, 39.94 g/mol.

SOLVE

Gas	% by Volume	Molar Mass	Weighted Molar Mass	% by Mass
N_2	78.09	28.02	21.880818	75.53
O_2	20.95	32.00	6.704	23.14
CO_2	0.03	44.01	0.013203	0.05
Ar	0.93	39.94	0.371442	1.28

REFLECT

The molar masses of O_2, CO_2, and Ar are all larger than the molar mass of N_2; therefore, we expect the individual mass percentages of these three components to be larger than their volume percentages.

14.70

SET UP

A chamber contains helium at a pressure of $P = 7.0 \times 10^{-11}$ Pa at a temperature $T =$ 300 K. The diameter of a helium atom is 1.0×10^{-10} m. The mean free path is given by $\lambda = \dfrac{kT}{4\sqrt{2}\pi r^2 P}$. In order to find the time between collisions, we first need to calculate the speed of the helium atoms from their average translational kinetic energy and the temperature. The collision time is the mean free path divided by the average speed of the helium atoms.

SOLVE

Mean free path:

$$\lambda = \frac{kT}{4\sqrt{2}\pi r^2 P} = \frac{\left(1.38 \times 10^{-23}\frac{\text{J}}{\text{K}}\right)(300\text{ K})}{4\sqrt{2}\pi\left(\frac{1.0 \times 10^{-10}\text{ m}}{2}\right)^2 (7.0 \times 10^{-11}\text{ Pa})} = \boxed{1.33 \times 10^{9}\text{ m}}$$

Speed:

$$\frac{1}{2}mv^2 = \frac{3}{2}kT$$

$$v = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3\left(1.38 \times 10^{-23}\frac{\text{J}}{\text{K}}\right)(300\text{ K})}{\left(4.00\text{ amu} \times \frac{1.66 \times 10^{-27}\text{ kg}}{1\text{ amu}}\right)}} = 1368\frac{\text{m}}{\text{s}}$$

Collision time:

$$t = \frac{\lambda}{v} = \frac{1.33 \times 10^{9}\text{ m}}{\left(1368\frac{\text{m}}{\text{s}}\right)} = \boxed{9.72 \times 10^{5}\text{ s}}$$

REFLECT

We would expect collisions to be extremely rare in a vacuum. A collision time of 9.72×10^5 s corresponds to 11.25 days.

14.71

SET UP

The mean free path for O_2 molecules at a temperature of 300 K and a pressure of 1.00 atm is 7.10×10^{-8} m. We can rearrange the expression for the mean free path, $\lambda = \dfrac{kT}{4\sqrt{2}\pi r^2 P}$, in order to solve for the radius of an oxygen molecule.

SOLVE

$$\lambda = \frac{kT}{4\sqrt{2}\pi r^2 P}$$

$$r = \sqrt{\frac{kT}{4\sqrt{2}\pi\lambda P}} = \sqrt{\frac{\left(1.38 \times 10^{-23}\frac{\text{J}}{\text{K}}\right)(300\text{ K})}{4\sqrt{2}\pi(7.10 \times 10^{-8}\text{ m})\left(1.00\text{ atm} \times \frac{1.01 \times 10^5\text{ Pa}}{1\text{ atm}}\right)}} = \boxed{1.80 \times 10^{-10}\text{ m}}$$

REFLECT

The radius of an oxygen atom is 0.6×10^{-10} m and the length of an oxygen–oxygen double bond is 1.21×10^{-10} m. If we model O_2 as two solid spheres 0.6×10^{-10} m in radius separated by 1.21×10^{-10} m, we will get a diameter of 3.61×10^{-10} m, which agrees well with our answer.

14.72

SET UP

The temperature change necessary for a cylinder of gold to increase in length by 0.1% is given by $\Delta L = \alpha L_0 \Delta T$, where the coefficient of linear expansion for gold is $14.2 \times 10^{-6}\text{ K}^{-1}$.

SOLVE

$$\Delta L = \alpha L_0 \Delta T$$

$$\Delta T = \left(\frac{\Delta L}{L_0}\right)\frac{1}{\alpha} = (0.001)\frac{1}{(14.2 \times 10^{-6}\text{ K}^{-1})} = \boxed{70.4\text{ K}}$$

REFLECT

Percent change in length is given by $\left(\frac{\Delta L}{L_0}\right)(100\%)$.

14.73

SET UP

A metal bar with an initial length $L_0 = 10$ m shortens by 0.5×10^{-2} m when the temperature decreases by 15 K. The coefficient of linear expansion is given by $\Delta L = \alpha L_0 \Delta T$. Both ΔL and ΔT will be negative since the length and temperature decrease.

SOLVE

$$\Delta L = \alpha L_0 \Delta T$$

$$\alpha = \frac{\Delta L}{L_0 \Delta T} = \frac{(-0.5 \times 10^{-2}\text{ m})}{(10\text{ m})(-15\text{ K})} = \boxed{3.3 \times 10^{-5}\text{ K}^{-1} = 33 \times 10^{-6}\text{ K}^{-1}}$$

REFLECT

Our answer is consistent with the coefficients of linear expansion for other metals listed in Table 14-1.

14.74

SET UP

The temperature of a 1.00-m-long aluminum rod is increased by 8.00 K. The coefficient of linear expansion for aluminum is $22.2 \times 10^{-6}\ \mathrm{K}^{-1}$. The amount the rod increases in length is given by $\Delta L = \alpha L_0 \Delta T$.

SOLVE

$$\Delta L = \alpha L_0 \Delta T = (22.2 \times 10^{-6}\ \mathrm{K}^{-1})(1.00\ \mathrm{m})(8.00\ \mathrm{K}) = \boxed{1.78 \times 10^{-4}\ \mathrm{m}}$$

REFLECT

The length of the rod increases by 178 μm, which seems reasonable for an increase in temperature of only 8 K.

14.75

SET UP

The temperature of a silver pin that is 5.00 cm long decreases by 152 K. The coefficient of linear expansion of silver is 19.5×10^{-6} K. The final length of the pin after cooling is equal to the initial length L_0 plus the change in the length due to the temperature change, $\Delta L = \alpha L_0 \Delta T$.

SOLVE

$$L_f = L_0 + \Delta L = L_0 + (\alpha L_0 \Delta T) = L_0(1 + \alpha \Delta T)$$

$$= (5.00\ \mathrm{cm})(1 + (19.5 \times 10^{-6}\ \mathrm{K}^{-1})(-152\ \mathrm{K})) = \boxed{4.985\ \mathrm{cm}}$$

REFLECT

We expect the pin to get smaller as it cools down, so $L_f < L_0$.

14.76

SET UP

A hole with diameter $L_0 = 8.00$ cm is drilled in a sheet of lead. The temperature of the sheet is then increased by 200 K. The coefficient of linear expansion of lead is 28.0×10^{-6} K. The final diameter of the hole after heating is equal to the initial diameter L_0 plus the change in the diameter due to the temperature change, $\Delta L = \alpha L_0 \Delta T$.

SOLVE

$$L_f = L_0 + \Delta L = L_0 + (\alpha L_0 \Delta T) = L_0(1 + \alpha \Delta T)$$

$$= (8.00\ \mathrm{cm})(1 + (28.0 \times 10^{-6}\ \mathrm{K}^{-1})(200\ \mathrm{K})) = \boxed{8.04\ \mathrm{cm}}$$

REFLECT

As the metal heats up and each of the dimensions expands, the hole will also expand, which means the final diameter will be larger than the initial diameter. The hole is circularly symmetric so we only need to calculate one expansion (along the diameter).

14.77

SET UP

A sheet of copper is initially 80.00 cm by 100.00 cm. Its temperature is then increased by 200 K. The length and width of the sheet both increase independently due to the temperature increase according to $\Delta L = \alpha L_0 \Delta T$, where the coefficient of linear expansion of copper is $16.5 \times 10^{-6}\ \text{K}^{-1}$. The final length is equal to the initial length plus the change in length, same for the width. The new area of the sheet is the product of the final length and the final width.

SOLVE

Length:

$$L_f = L_0 + \Delta L = L_0 + (\alpha L_0 \Delta T) = L_0(1 + \alpha \Delta T)$$

$$= (80.00\ \text{cm})(1 + (16.5 \times 10^{-6}\ \text{K}^{-1})(200\ \text{K})) = 80.264\ \text{cm}$$

Width:

$$w_f = w_0 + \Delta w = w_0 + (\alpha w_0 \Delta T) = w_0(1 + \alpha \Delta T)$$

$$= (100.00\ \text{cm})(1 + (16.5 \times 10^{-6}\ \text{K}^{-1})(200\ \text{K})) = 100.33\ \text{cm}$$

New area:

$$A_f = L_f w_f = (80.264\ \text{cm})(100.3\ \text{cm}) = \boxed{8053\ \text{cm}^2}$$

REFLECT

The final area is *not* equal to the initial area plus the product of the changes in the length and width. Instead, the final area is equal to

$$A_f = L_f w_f = (L_0 + \Delta L)(w_0 + \Delta w) = L_0 w_0 + w_0 \Delta L + L_0 \Delta w + \Delta L \Delta w$$

Since the changes in the length and width are small relative to the initial length and width, we can approximate the change in area as $\Delta A \approx w_0 \Delta L + L_0 \Delta w$.

14.78

SET UP

The initial dimensions of a sheet of copper are 20 cm by 30 cm. The temperature is then increased by 45 K. The length and width of the sheet both increase independently due to the temperature increase according to $\Delta L = \alpha L_0 \Delta T$, where the coefficient of linear expansion of copper is $16.5 \times 10^{-6}\ \text{K}^{-1}$. The final length is equal to the initial length plus the change in length, same for the width. The percent by which the area increases can be found by dividing the final area by the initial area; the final area of the sheet is the product of the final length and the final width.

SOLVE

Part a)

Length:

$$L_f = L_0 + \Delta L = L_0 + (\alpha L_0 \Delta T) = L_0(1 + \alpha \Delta T)$$

$$= (20\ \text{cm})(1 + (16.5 \times 10^{-6}\ \text{K}^{-1})(45\ \text{K})) = \boxed{20.015\ \text{cm}}$$

Width:

$$w_f = w_0 + \Delta w = w_0 + (\alpha w_0 \Delta T) = w_0(1 + \alpha\Delta T)$$

$$= (30\text{ cm})(1 + (16.5 \times 10^{-6}\text{ K}^{-1})(45\text{ K})) = \boxed{30.022\text{ cm}}$$

Part b)

Initial area:

$$A_0 = L_0 w_0 = (20\text{ cm})(30\text{ cm}) = 600\text{ cm}^2$$

Final area:

$$A_f = L_f w_f = (20.015\text{ cm})(30.022\text{ cm}) = 600.89\text{ cm}^2$$

Percent increase:

$$\frac{A_f}{A_0} = \frac{600.89\text{ cm}^2}{600\text{ cm}^2} = 1.0015$$

The area of the sheet $\boxed{\text{increases by } 0.15\%}$.

REFLECT

We can also approximate the fractional increase in the area as

$$\frac{\Delta A}{A_0} \approx \frac{w_0\Delta L + L_0\Delta w}{L_0 w_0} = \frac{\Delta L}{L_0} + \frac{\Delta w}{w_0} = \left(\frac{0.015\text{ cm}}{20\text{ cm}}\right) + \left(\frac{0.022\text{ cm}}{30\text{ cm}}\right) = 0.0015$$

14.79

SET UP

A cylinder of solid aluminum has an initial length $L_0 = 5$ m and an initial radius $R_0 = 2$ cm. The temperature of the aluminum is increased by 25 K. The length and radius of the cylinder both increase independently due to the temperature increase according to $\Delta L = \alpha L_0 \Delta T$, where the coefficient of linear expansion of aluminum is $22.2 \times 10^{-6}\text{ K}^{-1}$. Since the mass of the cylinder remains constant but the volume increases, the density of the aluminum cylinder will decrease. We can set up a ratio of the densities in order to calculate the factor by which the density decreases and the volume increases.

SOLVE

Part a)

$$\Delta L = \alpha L_0 \Delta T = (22.2 \times 10^{-6}\text{ K}^{-1})(5\text{ m})(25\text{ K}) = \boxed{0.00278\text{ m}}$$

Part b)

Final radius:

$$R_f = R_0 + \Delta R = R_0 + (\alpha R_0 \Delta T) = R_0(1 + \alpha\Delta T)$$

$$= (2\text{ cm})(1 + (22.2 \times 10^{-6}\text{ K}^{-1})(25\text{ K})) = 2.00111\text{ cm}$$

Change in density:

$$\frac{\rho_{\mathrm{f}}}{\rho_0} = \frac{\left(\frac{M}{V_{\mathrm{f}}}\right)}{\left(\frac{M}{V_0}\right)} = \frac{V_0}{V_{\mathrm{f}}} = \frac{\pi R_0^2 L_0}{\pi R_{\mathrm{f}}^2 L_{\mathrm{f}}} = \frac{R_0^2 L_0}{R_{\mathrm{f}}^2 L_{\mathrm{f}}} = \frac{(2\text{ cm})^2(5\text{ m})}{(2.00111\text{ cm})^2(5.00278\text{ m})} = 0.998$$

The density $\boxed{\text{decreases by } 0.2\%}$ upon heating.

Part c)

$$\frac{V_{\mathrm{f}}}{V_0} = \left(\frac{\rho_{\mathrm{f}}}{\rho_0}\right)^{-1} = (0.998)^{-1} = 1.002$$

The volume $\boxed{\text{increases by } 0.2\%}$ upon heating.

REFLECT

The mass of the cylinder must remain constant since we are not adding or removing anything from it; all we are doing is increasing its size. We could have also used the approximate relationship for the thermal expansion in three dimensions: $V_{\mathrm{f}} = V_0 + \Delta V = V_0(1 + 3\alpha\Delta T)$.

14.80

SET UP

At a temperature of 373 K, a cube of pure iron has a volume of 20 cm^3. The length of each side of the cube is just the cube root of the volume. The cube is then cooled to 293 K. The change in volume of the cube due to cooling is equal to $\Delta V = 3\alpha V_0 \Delta T$, where the coefficient of linear expansion for pure iron is $12.0 \times 10^{-6}\text{ K}^{-1}$. We can calculate the final volume and then the final length of the cube once we know the change in volume.

SOLVE

Part a)

$$V = L_0^3$$

$$L_0 = \sqrt[3]{V} = \sqrt[3]{20\text{ cm}^3} = \boxed{2.714\text{ cm}}$$

Part b)

New volume:

$$\Delta V = 3\alpha V_0 \Delta T$$

$$V_{\mathrm{f}} = V_0 + \Delta V = V_0 + 3\alpha V_0 \Delta T = V_0(1 + 3\alpha\Delta T)$$

$$= (20\text{ cm}^3)(1 + 3(12.0 \times 10^{-6}\text{ K}^{-1})(-80\text{ K})) = 19.9424\text{ cm}^3$$

New length:

$$V_{\mathrm{f}} = L_{\mathrm{f}}^3$$

$$L_{\mathrm{f}} = \sqrt[3]{V_{\mathrm{f}}} = \sqrt[3]{19.9424\text{ cm}^3} = \boxed{2.712\text{ cm}}$$

REFLECT
Because the cube is cooling down, we would expect the volume and, thus, the length of each side to decrease.

14.81

SET UP
A sphere of gold has an initial radius $R_0 = 1$ cm. The temperature is increased by 60 K, which means the radius of the sphere also increases by $\Delta L = \alpha L_0 \Delta T$, where the coefficient of linear expansion of gold is $14.2 \times 10^{-6}\ \mathrm{K}^{-1}$. We can calculate the percent change in the volume of the sphere from $\Delta V = 3\alpha V_0 \Delta T$.

SOLVE

Part a)

$$R_{\mathrm{f}} = R_0 + \Delta R = R_0 + (\alpha R_0 \Delta T) = R_0(1 + \alpha \Delta T)$$

$$= (1\ \mathrm{cm})(1 + (14.2 \times 10^{-6}\ \mathrm{K}^{-1})(60\ \mathrm{K})) = \boxed{1.00085\ \mathrm{cm}}$$

Part b)

$$\Delta V = 3\alpha V_0 \Delta T$$

$$\frac{\Delta V}{V_0} = 3\alpha \Delta T = 3(14.2 \times 10^{-6}\ \mathrm{K}^{-1})(60\ \mathrm{K}) = 0.002556$$

The volume $\boxed{\text{increases by } 0.2556\%}$.

REFLECT
The volume of the sphere should increase upon heating.

14.82

SET UP
The temperature of a 0.500-kg metal sample increases by 4.8 K when 307 J of heat was added to it. The specific heat of the material can be calculated from $\Delta T = \frac{1}{mc}Q$.

SOLVE

$$\Delta T = \frac{1}{mc}Q$$

$$c = \frac{Q}{m\Delta T} = \frac{307\ \mathrm{J}}{(0.500\ \mathrm{kg})(4.8\ \mathrm{K})} = \boxed{128 \frac{\mathrm{J}}{\mathrm{kg} \cdot \mathrm{K}}}$$

REFLECT
Comparing this result to the values of specific heats in Table 14-3, the sample is most likely lead.

14.83

SET UP

You want to heat up 0.250 kg of water from 20 degrees C to 95 degrees C. The minimum amount of heat required to accomplish that is given by $Q = mc(\Delta T)$, where c is the specific heat of water $c = 4186\frac{\text{J}}{\text{kg}\cdot\text{K}}$.

SOLVE

$$\Delta T = \frac{1}{mc}Q$$

$$Q = mc(\Delta T) = (0.250\text{ kg})\left(4186\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(75\text{ K}) = \boxed{7.8 \times 10^{4}\text{ J} = 78\text{ kJ}}$$

REFLECT

A temperature difference of 75 degrees C is the same as a temperature difference of 75 K. Accordingly, sometimes you'll see the specific heat of water as $c = 4186\frac{\text{J}}{\text{kg}\cdot{}^{\circ}\text{C}}$.

14.84

SET UP

Two samples of water are mixed in a thermodynamically isolated chamber; one has a mass of $m_{\text{C}} = 20$ g at $T_{\text{C}} = 15$ degrees C, the other mass of $m_{\text{H}} = 40$ g at $T_{\text{H}} = 60$ degrees C. Heat will transfer once the two samples are mixed together and the combined sample will eventually reach thermal equilibrium. We can use Equation 14-27 from the text, $T_{\text{f}} = \frac{m_{\text{H}}c_{\text{H}}T_{\text{H, i}} + m_{\text{C}}c_{\text{C}}T_{\text{C, i}}}{m_{\text{H}}c_{\text{H}} + m_{\text{C}}c_{\text{C}}}$, to calculate the final equilibrium temperature of the mixture. Since we're mixing two samples of water, the specific heat is the same for both and will cancel from the equation.

SOLVE

$$T_{\text{f}} = \frac{m_{\text{H}}c_{\text{H}}T_{\text{H, i}} + m_{\text{C}}c_{\text{C}}T_{\text{C, i}}}{m_{\text{H}}c_{\text{H}} + m_{\text{C}}c_{\text{C}}} = \frac{m_{\text{H}}T_{\text{H, i}} + m_{\text{C}}T_{\text{C, i}}}{m_{\text{H}} + m_{\text{C}}} = \frac{(40\text{ g})(60^{\circ}\text{C}) + (20\text{ g})(15^{\circ}\text{C})}{(40\text{ g}) + (20\text{ g})} = \boxed{45^{\circ}\text{C}}$$

REFLECT

There is more "hot" water than "cold" water, so the final equilibrium temperature should be closer to T_{H} than T_{C}.

14.85

SET UP

The temperature of 2.00 kg of water decreased by 46 K. The amount of heat transferred from the water to the environment is equal to $-mc\Delta T$. The specific heat of water is $4186\frac{\text{J}}{\text{kg}\cdot\text{K}}$.

SOLVE

$$Q_{\text{environment}} = -Q_{\text{water}} = -mc\Delta T = -(2.00\text{ kg})\left(4186\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(-46\text{ K}) = \boxed{3.9 \times 10^5\text{ J}}$$

REFLECT

The amount of heat leaving the water is equal to the heat gained by the environment due to conservation of energy.

14.86

SET UP

The temperature of a 1.0-kg copper pot decreased by 100 K. The amount of heat transferred from the pot to achieve this temperature drop is equal to $mc\Delta T$. The specific heat of copper is $387\frac{\text{J}}{\text{kg}\cdot\text{K}}$.

SOLVE

$$Q = mc\Delta T = (1.0\text{ kg})\left(387\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(-100\text{ K}) = \boxed{-3.87 \times 10^4\text{ J}}$$

REFLECT

If this heat were transferred to 1.0 kg of water, the temperature of the water would only increase by 9.2 K. This illustrates just how large the specific heat of water is.

14.87

SET UP

The temperature of a lake was increased by 5 K when 1.7×10^{14} J was transferred to it. We can calculate the mass of the lake from $\Delta T = \frac{1}{mc}Q$ and the specific heat of the lake $\left(4186\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)$.

SOLVE

$$\Delta T = \frac{1}{mc}Q$$

$$m = \frac{Q}{c\Delta T} = \frac{1.7 \times 10^{14}\text{ J}}{\left(4186\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(5\text{ K})} = \boxed{8.1 \times 10^9\text{ kg}}$$

REFLECT

The lake has a volume of approximately 8.1×10^9 L. For a comparison, the volume of Lake Superior, the largest of the Great Lakes, is 1.2×10^{16} L.

14.88

SET UP

We are asked to calculate the temperature increase in a 1-kg sample of water assuming that its initial gravitational potential energy is completely converted into heat after it falls. Upon setting the expressions for the gravitational potential energy of the water equal to the heat, we can solve for the temperature increase. The water falls a distance of 807 m and has a specific heat of $4186\frac{\text{J}}{\text{kg}\cdot\text{K}}$.

SOLVE

$$U_g = Q$$

$$mgy = mc\Delta T$$

$$\Delta T = \frac{gy}{c} = \frac{\left(9.8\frac{\text{m}}{\text{s}^2}\right)(807\text{ m})}{\left(4186\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)} = \boxed{1.89\text{ K}}$$

REFLECT

This is the maximum possible temperature increase since we assumed that the gravitational potential energy was completely converted into heat. In actuality there will be other forms of energy present.

14.89

SET UP

A 5.00-kg iron bar absorbs 2.5×10^6 J of heat. The temperature increase of the rod can be found from $\Delta T = \frac{1}{mc}Q$, where the specific heat of iron is $452\frac{\text{J}}{\text{kg}\cdot\text{K}}$. Assuming the initial temperature of the rod was room temperature (around 300 K), we can calculate the final temperature upon heating and determine whether the iron would be glowing yellow or red at that point by referring to Figure 26-1 in the text (reproduced as Figure 14-4 here).

SOLVE

Part a)

$$\Delta T = \frac{Q}{mc} = \frac{(2.5 \times 10^6\text{ J})}{(5.00\text{ kg})\left(452\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)} = \boxed{1110\text{ K}}$$

Part b)

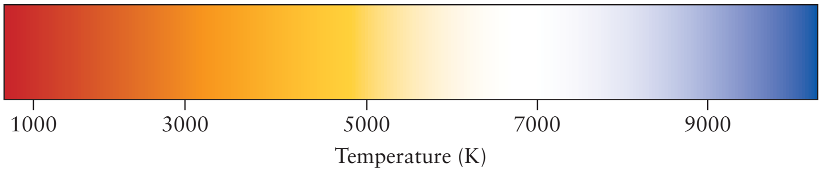


Figure 14-4 Problem 89

A temperature increase of about 1110 K puts the temperature of the iron bar at around 1400 K. According to Figure 14-4, at that temperature the iron bar will be glowing red hot, so it should be put $\boxed{\text{back in the fire}}$.

REFLECT
As the bar heats up, it will start out black, then turn red, yellow, and eventually white as the temperature increases.

14.90

SET UP
The temperature of a 0.200-kg block of ice fell 30 K. The amount of heat removed from the ice is equal to $Q = mc\Delta T$, where the specific heat of ice is $2093\frac{\text{J}}{\text{kg}\cdot\text{K}}$.

SOLVE

$$Q = mc\Delta T = (0.200\text{ kg})\left(2093\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(-30\text{ K}) = \boxed{-1.3\times 10^4\text{ J}}$$

REFLECT
This is the amount of heat that is transferred to the environment due to conservation of energy.

14.91

SET UP
A 250-g sample of copper has an initial temperature of 100 degrees C. It is placed in 300 g of water that is at a temperature of 30 degrees C. We can use Equation 14-27 from the text, $T_f = \frac{m_H c_H T_{H,i} + m_C c_C T_{C,i}}{m_H c_H + m_C c_C}$, to calculate the final equilibrium temperature of the mixture. In part (b), the copper loses 5% of its initial heat when it is dropped into the water. Since the heat is directly proportional to the temperature change, the temperature of the copper will decrease by 5% as well before it is introduced to the water. In this case, $T_{H,i} = 95$ degrees C. The specific heat of copper is $387\frac{\text{J}}{\text{kg}\cdot\text{K}}$ and the specific heat of water is $4186\frac{\text{J}}{\text{kg}\cdot\text{K}}$.

SOLVE

Part a)

$$T_f = \frac{m_H c_H T_{H,i} + m_C c_C T_{C,i}}{m_H c_H + m_C c_C}$$

$$= \frac{(250\text{ g})\left(387\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(100°\text{C}) + (300\text{ g})\left(4186\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(30°\text{C})}{(250\text{ g})\left(387\frac{\text{J}}{\text{kg}\cdot\text{K}}\right) + (300\text{ g})\left(4186\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)} = \boxed{35.0°\text{C}}$$

Part b)

$$T_f = \frac{m_H c_H T_{H,i} + m_C c_C T_{C,i}}{m_H c_H + m_C c_C}$$

$$= \frac{(250\text{ g})\left(387\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(95°\text{C}) + (300\text{ g})\left(4186\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(30°\text{C})}{(250\text{ g})\left(387\frac{\text{J}}{\text{kg}\cdot\text{K}}\right) + (300\text{ g})\left(4186\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)} = \boxed{34.6°\text{C}}$$

REFLECT

The mass and specific heat of the water are larger than that of the copper, so the equilibrium temperature should be much closer to the water's initial temperature than that of the copper. In part (b), the copper is initially cooler than in part (a), so the equilibrium temperature should also be lower.

14.92

SET UP

A 50-g calorimeter cup contains 100 g of water all at the same temperature. A heated 300-g sample of an unknown metal is placed in the calorimeter. At equilibrium, the temperature of the water and aluminum cup increased by 16 K and the temperature of the unknown metal decreased by 109 K. Assuming the calorimeter and the sample form an isolated system, conservation of energy states that the heat lost by the sample must be absorbed by the water and aluminum, or that the net heat transfer is equal to zero. Using this and all of the provided data, we can calculate the specific heat of the sample and compare it to the values listed in Table 14-3 in order to identify it. The specific heat of aluminum is $910\frac{\text{J}}{\text{kg}\cdot\text{K}}$ and the specific heat of water is $4186\frac{\text{J}}{\text{kg}\cdot\text{K}}$.

SOLVE

$$Q_{\text{unknown}} + Q_{\text{water}} + Q_{\text{Al}} = 0$$

$$m_{\text{unknown}} c_{\text{unknown}} \Delta T_{\text{unknown}} + m_{\text{water}} c_{\text{water}} \Delta T_{\text{water}} + m_{\text{Al}} c_{\text{Al}} \Delta T_{\text{Al}} = 0$$

$$c_{\text{unknown}} = \frac{-m_{\text{water}}c_{\text{water}}\Delta T_{\text{water}} - m_{\text{Al}}c_{\text{Al}}\Delta T_{\text{Al}}}{m_{\text{unknown}}\Delta T_{\text{unknown}}}$$

$$= \frac{-(100\text{ g})\left(4186\dfrac{\text{J}}{\text{kg}\cdot\text{K}}\right)(16\text{ K}) - (50\text{ g})\left(910\dfrac{\text{J}}{\text{kg}\cdot\text{K}}\right)(16\text{ K})}{(300\text{ g})(-109\text{ K})} = \boxed{227\dfrac{\text{J}}{\text{kg}\cdot\text{K}}}$$

This is close to the value of $\boxed{\text{silver}}$ listed in Table 14-3 $\left(236\dfrac{\text{J}}{\text{kg}\cdot\text{K}}\right)$.

REFLECT

The mass of the unknown metal is larger than the water and aluminum. The temperature change of the unknown metal is also larger than that of the water and aluminum. Therefore, it makes sense that the specific heat of the metal is less than the specific heat of aluminum.

14.93

SET UP

Each stroke of a hacksaw supplies 30 J of heat to a 20-g steel bolt. We can calculate the total heat necessary to raise the temperature of the bolt by 60 K from $Q = mc\Delta T$, where $c = 452\dfrac{\text{J}}{\text{kg}\cdot\text{K}}$. The total heat divided by the heat supplied per stroke will give the total number of strokes required.

SOLVE

Total heat required:

$$Q = mc\Delta T = (0.020\text{ kg})\left(452\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(60\text{ K}) = 542.4\text{ J}$$

Number of strokes:

$$n_{\text{strokes}} = \frac{\text{Total heat}}{\text{Heat supplied by one stroke}} = \frac{542.4\text{ J}}{30\text{ J}} = 18.1 = \boxed{19\text{ strokes}}$$

REFLECT

The 19th stroke will cause an overall temperature increase of more than 60 K, but 18 strokes will only increase the temperature by 59.7 K.

14.94

SET UP

The amount of heat required to convert 0.025 kg of ice at 0 degrees C to 0.025 kg of water at 0 degrees C is equal to $Q = mL_F$, where the latent heat of fusion for water is 334 kJ/kg.

SOLVE

$$Q = mL_F = (0.025\text{ kg})\left(334\frac{\text{kJ}}{\text{kg}}\right) = \boxed{8.35\text{ kJ}}$$

REFLECT

We can quickly check our answer by realizing 0.025 is $0.25 \cdot 0.10$, or $\left(\frac{1}{4}\right)\left(\frac{1}{10}\right)$. Dividing 334 by 10 gives 33.4; dividing this by 4 should give a little over 8, which agrees with our answer.

14.95

SET UP

We are asked to calculate the total heat required to change 0.025 kg of ice at −40 degrees C to 0.025 kg of steam at 140 degrees C. We can split the process into five separate stages and calculate the heat transfer for each stage. The stages are (1) increasing the temperature of the ice from −40 degrees C to 0 degrees C, (2) converting all of the ice to water at 0 degrees C, (3) increasing the temperature of the water from 0 degrees C to 100 degrees C, (4) converting all of the water to steam at 100 degrees C, and (5) increasing the temperature of the steam from 100 degrees C to 140 degrees C. The heat necessary to increase the temperature of the object is given by $Q = mc\Delta T$; the heat necessary to convert from solid to liquid is related to the latent heat of fusion, $Q = mL_F$; and the heat necessary to convert from liquid to gas is related to the latent heat of vaporization, $Q = mL_V$. The specific heats of ice, liquid water, and steam are $c_{ice} = 2093\frac{J}{kg \cdot K}$, $c_{water} = 4186\frac{J}{kg \cdot K}$, and $c_{steam} = 2009\frac{J}{kg \cdot K}$, respectively. The latent heats of fusion and vaporization for water are $L_F = 334 \times 10^3\frac{J}{kg}$ and $L_V = 2260 \times 10^3\frac{J}{kg}$, respectively.

SOLVE

$$
\begin{aligned}
Q_{total} &= Q_{ice,\ -40°C\to 0°C} + Q_{ice\to water,\ 0°C} + Q_{water,\ 0°C\to 100°C} + Q_{water\to steam,\ 100°C} + Q_{steam,\ 100°C\to 140°C} \\
&= mc_{ice}\Delta T_{-40°C\to 0°C} + mL_F + mc_{water}\Delta T_{0°C\to 100°C} + mL_V + mc_{steam}\Delta T_{100°C\to 140°C} \\
&= m(c_{ice}\Delta T_{-40°C\to 0°C} + L_F + c_{water}\Delta T_{0°C\to 100°C} + L_V + c_{steam}\Delta T_{100°C\to 140°C}) \\
&= (0.025\text{ kg})\left(\left(2093\frac{J}{kg \cdot K}\right)(40\text{ K}) + \left(334 \times 10^3\frac{J}{kg}\right) + \left(4186\frac{J}{kg \cdot K}\right)(100\text{ K})\right. \\
&\quad \left. + \left(2260 \times 10^3\frac{J}{kg}\right) + \left(2009\frac{J}{kg \cdot K}\right)(40\text{ K})\right) \\
&= \boxed{79{,}400\text{ J} = 79.4\text{ kJ}}
\end{aligned}
$$

REFLECT

Most of the heat (almost 82%) required for the process goes into changing the phase of the object from solid to gas.

14.96

SET UP

A sealed container initially holds 0.030 kg of steam at a temperature of 120 degrees C when 100,000 J of heat is removed from the steam. In order to determine the final state of the

system, we need to calculate the heat transfer when the steam cools from 120 degrees C to 100 degrees C, the steam condenses to water at 100 degrees C, the water cools from 100 degrees C to 0 degrees C, the water freezes to ice at 0 degrees C, and the ice cools from 0 degrees C. We need to keep calculating these heats until the total heat removed is equal to 100,000 J. The heat necessary to decrease the temperature of the object is given by $Q = mc\Delta T$; the heat necessary to convert from gas to liquid is related to the latent heat of vaporization, $Q = mL_V$; and the heat necessary to convert from liquid to solid is related to the latent heat of fusion, $Q = mL_F$. The specific heats of ice, liquid water, and steam are $c_{\text{ice}} = 2093\frac{\text{J}}{\text{kg}\cdot\text{K}}$, $c_{\text{water}} = 4186\frac{\text{J}}{\text{kg}\cdot\text{K}}$, and $c_{\text{steam}} = 2009\frac{\text{J}}{\text{kg}\cdot\text{K}}$, respectively. The latent heats of fusion and vaporization for water are $L_F = 334 \times 10^3\frac{\text{J}}{\text{kg}}$ and $L_V = 2260 \times 10^3\frac{\text{J}}{\text{kg}}$, respectively.

SOLVE

Heat lost when temperature cools to 100 degrees C:

$$Q = mc\Delta T = (0.030\text{ kg})\left(2009\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(-20\text{ K}) = -1205.4\text{ J}$$

Heat lost when steam is converted to liquid water:

$$Q = mL_V = (0.030\text{ kg})\left(-2260 \times 10^3\frac{\text{J}}{\text{kg}}\right) = -67{,}800\text{ J}$$

Heat lost when temperature cools to 0 degrees C:

$$Q = mc\Delta T = (0.030\text{ kg})\left(4186\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(-100\text{ K}) = -12{,}558\text{ J}$$

Heat lost when liquid water is converted into ice:

$$Q = mL_F = (0.030\text{ kg})\left(-334 \times 10^3\frac{\text{J}}{\text{kg}}\right) = -10{,}020\text{ J}$$

Total heat lost up to this point:

$$Q_{\text{so far}} = (-1205.4\text{ J}) + (-67{,}800\text{ J}) + (-12{,}558\text{ J}) + (-10{,}020\text{ J}) = -91{,}583.4\text{ J}$$

Temperature after an additional 8146.6 J of heat is lost:

$$\Delta T = \frac{Q}{mc} = \frac{(-8146.6\text{ J})}{(0.030\text{ kg})\left(2093\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)} = -134\text{ K}$$

The final state of the system will be $\boxed{\text{0.030 kg of ice at a temperature of } -134 \text{ degrees C}}$.

REFLECT

Qualitatively comparing this problem to Problem 14.96, we find that it takes about 79,000 J of heat to change a similar amount of ice at −40 degrees C to steam at 140 degrees C. Therefore, it makes sense that by removing 100,000 J from steam at 120 degrees C we should end up with ice colder than −40 degrees C.

14.97

SET UP

The total heat required to melt 0.400-kg of copper initially at 293 K is equal to the sum of the heat necessary to raise the temperature of the sample to the melting point ($T_f = 1357$ K) and the heat necessary to convert the entire solid into liquid. The heat necessary to increase the temperature of the object is given by $Q = mc\Delta T$, and the heat necessary to convert from solid to liquid is related to the latent heat of fusion, $Q = mL_F$. The specific heat of copper is $c = 387\frac{\text{J}}{\text{kg}\cdot\text{K}}$, and the latent heat of fusion for copper in this problem is $L_F = 205{,}350\frac{\text{J}}{\text{kg}}$.

SOLVE

Increase temperature to melting point:

$$Q = mc\Delta T = (0.400\text{ kg})\left(387\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(1064\text{ K}) = 164{,}707\text{ J}$$

Melting the copper:

$$Q = mL_F = (0.400\text{ kg})\left(205{,}350\frac{\text{J}}{\text{kg}}\right) = 82{,}140\text{ J}$$

Total heat:

$$Q_{\text{total}} = (164{,}707\text{ J}) + (82{,}140\text{ J}) = 246{,}847\text{ J} = \boxed{247\text{ kJ}}$$

REFLECT

We have to increase the temperature of the copper by over 1300 K so it makes sense that most of the heat added goes toward this process rather than melting.

14.98

SET UP

A calorimeter is made up of a 0.200-kg copper cup and 0.300 kg of water at an unknown initial temperature. A 0.020-kg sample of ice at −10 degrees C is placed into the calorimeter. After all of the ice melts, the final equilibrium temperature is 18 degrees C. The total heat gained by the ice is equal to the heat necessary to raise its temperature from −10 degrees C to 0 degrees C, completely melt the ice, and increase the temperature of the melted ice from 0 degrees C to 18 degrees C. From conservation of energy, the heat gained by the ice is equal to the heat lost by the calorimeter (that is, water + copper cup). The loss of heat from the calorimeter goes into cooling its common temperature. The heat necessary to change the temperature of an object is given by $Q = mc\Delta T$, and the heat necessary to convert from solid to liquid is related to the latent heat of fusion, $Q = mL_F$. The specific heats of ice, water, and copper are $c_{\text{ice}} = 2093\frac{\text{J}}{\text{kg}\cdot\text{K}}$, $c_{\text{water}} = 4186\frac{\text{J}}{\text{kg}\cdot\text{K}}$, and $c_{\text{Cu}} = 387\frac{\text{J}}{\text{kg}\cdot\text{K}}$, respectively. The latent heat of fusion for water is $L_F = 334 \times 10^3\frac{\text{J}}{\text{kg}}$.

SOLVE

Heat gained by the ice:

$$Q_{\text{ice}} = m_{\text{ice}}c_{\text{ice}}\Delta T_{-10°\text{C}\to 0°\text{C}} + m_{\text{ice}}L_{\text{f}} + m_{\text{ice}}c_{\text{water}}\Delta T_{0°\text{C}\to 18°\text{C}}$$

$$= m_{\text{ice}}(c_{\text{ice}}\Delta T_{-10°\text{C}\to 0°\text{C}} + L_{\text{f}} + c_{\text{water}}\Delta T_{0°\text{C}\to 18°\text{C}})$$

$$= (0.020\text{ kg})\left(\left(2093\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(10\text{ K}) + \left(334\times 10^3\frac{\text{J}}{\text{kg}}\right) + \left(4186\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(18\text{ K})\right)$$

$$= 8605.56\text{ J}$$

Temperature change of calorimeter:

$$Q_{\text{calorimeter}} = -Q_{\text{ice}} = m_{\text{water}}c_{\text{water}}\Delta T + m_{\text{Cu}}c_{\text{Cu}}\Delta T = \Delta T(m_{\text{water}}c_{\text{water}} + m_{\text{Cu}}c_{\text{Cu}})$$

$$\Delta T = \frac{-Q_{\text{ice}}}{m_{\text{water}}c_{\text{water}} + m_{\text{Cu}}c_{\text{Cu}}} = \frac{-8605.56\text{ J}}{(0.300\text{ kg})\left(4186\frac{\text{J}}{\text{kg}\cdot\text{K}}\right) + (0.200\text{ kg})\left(387\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)} = -6.5\text{ K}$$

Since the temperature fell 6.5 degrees to 18 degrees C, the initial temperature of the water and copper was $\boxed{24.5\text{ degrees C}}$.

REFLECT

We would not expect the relatively large bath of water and copper to have its temperature change as much as the small ice cube.

14.99

SET UP

An unknown amount of ice at −20 degrees C is added to 0.050 kg of steam at 120 degrees C. The equilibrium temperature of the system is 40 degrees C. Assuming the steam and ice form a thermally isolated system, the amount of heat leaving the steam is equal to the heat gained by the ice due to conservation of energy. The total heat lost by the steam is equal to the heat lost to lower its temperature from 120 degrees C to 100 degrees C, completely condense the steam into water, and decrease the temperature of the condensed steam from 100 degrees C to 40 degrees C. The heat gained by the ice goes into heating it from −20 degrees C to 0 degrees C, melting the ice, and raising the temperature of the melted ice from 0 degrees C to 40 degrees C. The heat necessary to change the temperature of an object is given by $Q = mc\Delta T$; the heat necessary to convert from gas to liquid is related to the latent heat of vaporization, $Q = mL_{\text{V}}$; and the heat necessary to convert from liquid to solid is related to the latent heat of fusion, $Q = mL_{\text{F}}$. The specific heats of ice, liquid water, and steam are $c_{\text{ice}} = 2093\frac{\text{J}}{\text{kg}\cdot\text{K}}$, $c_{\text{water}} = 4186\frac{\text{J}}{\text{kg}\cdot\text{K}}$, and $c_{\text{steam}} = 2009\frac{\text{J}}{\text{kg}\cdot\text{K}}$, respectively. The latent heats of fusion and vaporization for water are $L_{\text{F}} = 334\times 10^3\frac{\text{J}}{\text{kg}}$ and $L_{\text{V}} = 2260\times 10^3\frac{\text{J}}{\text{kg}}$, respectively.

SOLVE

Heat required to condense steam at 120 degrees C to water at 40 degrees C:

$$Q_{\text{steam}} = m_{\text{steam}} c_{\text{steam}} \Delta T_{120°\text{C}\to 100°\text{C}} + m_{\text{steam}} L_{\text{V}} + m_{\text{steam}} c_{\text{water}} \Delta T_{100°\text{C}\to 40°\text{C}}$$

$$= m_{\text{steam}} \left(c_{\text{steam}} \Delta T_{120°\text{C}\to 100°\text{C}} + L_{\text{V}} + c_{\text{water}} \Delta T_{100°\text{C}\to 40°\text{C}} \right)$$

$$= (0.050\ \text{kg})\left(\left(2009\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(-20\ \text{K}) + \left(-2260 \times 10^3 \frac{\text{J}}{\text{kg}}\right) + \left(4186\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(-60\ \text{K})\right)$$

$$= -127{,}567\ \text{J}$$

Mass of ice at −20 degrees C:

$$Q_{\text{ice}} = -Q_{\text{steam}} = m_{\text{ice}} c_{\text{ice}} \Delta T_{-20°\text{C}\to 0°\text{C}} + m_{\text{ice}} L_{\text{F}} + m_{\text{ice}} c_{\text{water}} \Delta T_{0°\text{C}\to 40°\text{C}}$$

$$-Q_{\text{steam}} = m_{\text{ice}} \left(c_{\text{ice}} \Delta T_{-20°\text{C}\to 0°\text{C}} + L_{\text{F}} + c_{\text{water}} \Delta T_{0°\text{C}\to 40°\text{C}} \right)$$

$$m_{\text{ice}} = \frac{-Q_{\text{steam}}}{c_{\text{ice}} \Delta T_{-20°\text{C}\to 0°\text{C}} + L_{\text{F}} + c_{\text{water}} \Delta T_{0°\text{C}\to 40°\text{C}}}$$

$$= \frac{-(-127{,}567\ \text{J})}{\left(2093\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(20\ \text{K}) + \left(334 \times 10^3 \frac{\text{J}}{\text{kg}}\right) + \left(4186\frac{\text{J}}{\text{kg}}\right)(40\ \text{K})} = \boxed{0.235\ \text{kg} = 235\ \text{g}}$$

REFLECT

Don't forget the negative sign in front of the latent heats when the sample is condensing or freezing. It's often a good idea to ask yourself "do I expect heat to absorbed (+) or released (−)?" with respect to the sign every time you calculate heat.

14.100

SET UP

A 60-kg ice hockey player initially moving at a speed of 8 m/s skids to a complete stop; 40% of his initial kinetic energy is converted to heat, which goes into melting some of the ice. The amount of ice melted is related to the latent heat of fusion of water $L_{\text{F}} = 334 \times 10^3 \frac{\text{J}}{\text{kg}}$ by $Q = mL_{\text{F}}$. We are assuming that the temperature of the ice is already at 0 degrees C.

SOLVE

Kinetic energy of the skater:

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(60\ \text{kg})\left(8\frac{\text{m}}{\text{s}}\right)^2 = 1920\ \text{J}$$

Mass of the melted ice:

$$Q = mL_{\text{F}}$$

$$m = \frac{Q}{L_{\text{F}}} = \frac{0.4K}{L_{\text{F}}} = \frac{(0.4)(1920\ \text{J})}{\left(334 \times 10^3 \frac{\text{J}}{\text{kg}}\right)} = \boxed{0.00230\ \text{kg} = 2.30\ \text{g}}$$

REFLECT
We would not expect a large amount of the ice to melt when a skater stops on an ice rink.

14.101

SET UP
We want to see which sample of iron, copper, or water will melt the most from a huge block of ice. The iron sample has a mass of $m_{\text{Fe}} = 50$ g and an initial temperature of 120 degrees C; the copper sample has a mass of $m_{\text{Cu}} = 60$ g and an initial temperature of 150 degrees C; and the water has a mass of $m_{\text{water}} = 30$ g and an initial temperature of 40 degrees C. Assuming the sample and the ice form a thermally isolated system, the amount of heat leaving the sample is equal to the heat gained by the ice due to conservation of energy. The total heat lost by the sample is equal to the heat lost to lower its temperature from its initial value to 0 degrees C. The heat gained by the ice goes into heating it from -5 degrees C to 0 degrees C and then melting the ice. The heat necessary to change the temperature of an object is given by $Q = mc\Delta T$, and the heat necessary to convert from liquid to solid is related to the latent heat of fusion, $Q = mL_{\text{F}}$. The specific heats of iron, copper, liquid water, and ice are $c_{\text{Fe}} = 452\dfrac{\text{J}}{\text{kg}\cdot\text{K}}$, $c_{\text{Cu}} = 387\dfrac{\text{J}}{\text{kg}\cdot\text{K}}$, $c_{\text{water}} = 4186\dfrac{\text{J}}{\text{kg}\cdot\text{K}}$, and $c_{\text{ice}} = 2093\dfrac{\text{J}}{\text{kg}\cdot\text{K}}$, respectively. The latent heat of fusion for water is $L_{\text{F}} = 334\times10^{3}\dfrac{\text{J}}{\text{kg}}$. We can calculate the mass of ice melted in each case and compare these amounts to determine which sample melts the most.

SOLVE
Iron:

$$Q_{\text{ice},\,-5^\circ\text{C}\to0^\circ\text{C}} + Q_{\text{ice}\to\text{water}} + Q_{\text{Fe},\,120^\circ\text{C}\to0^\circ\text{C}} = 0$$

$$m_{\text{ice}}c_{\text{ice}}\Delta T_{-5^\circ\text{C}\to0^\circ\text{C}} + m_{\text{ice}}L_{\text{F}} + m_{\text{Fe}}c_{\text{Fe}}\Delta T_{120^\circ\text{C}\to0^\circ\text{C}} = 0$$

$$m_{\text{ice}} = \frac{-m_{\text{Fe}}c_{\text{Fe}}\Delta T_{120^\circ\text{C}\to0^\circ\text{C}}}{c_{\text{ice}}\Delta T_{-5^\circ\text{C}\to0^\circ\text{C}} + L_{\text{F}}} = \frac{-(50\text{ g})\left(452\dfrac{\text{J}}{\text{kg}\cdot\text{K}}\right)(-120\text{ K})}{\left(2093\dfrac{\text{J}}{\text{kg}\cdot\text{K}}\right)(5\text{ K}) + \left(334\times10^{3}\dfrac{\text{J}}{\text{kg}}\right)} = \boxed{7.87\text{ g}}$$

Copper:

$$Q_{\text{ice},\,-5^\circ\text{C}\to0^\circ\text{C}} + Q_{\text{ice}\to\text{water}} + Q_{\text{Cu},\,150^\circ\text{C}\to0^\circ\text{C}} = 0$$

$$m_{\text{ice}}c_{\text{ice}}\Delta T_{-5^\circ\text{C}\to0^\circ\text{C}} + m_{\text{ice}}L_{\text{F}} + m_{\text{Cu}}c_{\text{Cu}}\Delta T_{150^\circ\text{C}\to0^\circ\text{C}} = 0$$

$$m_{\text{ice}} = \frac{-m_{\text{Cu}}c_{\text{Cu}}\Delta T_{150^\circ\text{C}\to0^\circ\text{C}}}{c_{\text{ice}}\Delta T_{-5^\circ\text{C}\to0^\circ\text{C}} + L_{\text{F}}} = \frac{-(60\text{ g})\left(387\dfrac{\text{J}}{\text{kg}\cdot\text{K}}\right)(-150\text{ K})}{\left(2093\dfrac{\text{J}}{\text{kg}\cdot\text{K}}\right)(5\text{ K}) + \left(334\times10^{3}\dfrac{\text{J}}{\text{kg}}\right)} = \boxed{10.1\text{ g}}$$

Water:

$$Q_{\text{ice},\,-5°\text{C}\to 0°\text{C}} + Q_{\text{ice}\to\text{water}} + Q_{\text{water},\,40°\text{C}\to 0°\text{C}} = 0$$

$$m_{\text{ice}}c_{\text{ice}}\Delta T_{-5°\text{C}\to 0°\text{C}} + m_{\text{ice}}L_{\text{F}} + m_{\text{water}}c_{\text{water}}\Delta T_{40°\text{C}\to 0°\text{C}} = 0$$

$$m_{\text{ice}} = \frac{-m_{\text{water}}c_{\text{water}}\Delta T_{40°\text{C}\to 0°\text{C}}}{c_{\text{ice}}\Delta T_{-5°\text{C}\to 0°\text{C}} + L_{\text{F}}} = \frac{-(30\text{ g})\left(4186\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(-40\text{ K})}{\left(2093\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(5\text{ K}) + \left(334\times 10^{3}\frac{\text{J}}{\text{kg}}\right)} = \boxed{14.6\text{ g}}$$

The $\boxed{\text{water will melt the most ice}}$.

REFLECT

Since we assume the ice block was really large, we do not have to worry about the sample and melted ice reaching an equilibrium temperature higher than 0 degrees C. Even though the mass of the water was the least, it melted the most ice mainly due to water's large specific heat.

14.102

SET UP

A heated bar of gold radiates at a temperature of 573 K (300 degrees C). We want to know the factor by which the radiated heat increases if we heat the bar up to 873 K (600 degrees C) and then 1173 K (900 degrees C) and also if we double the surface area. The radiated power is proportional to both the surface area of the bar and the temperature to the fourth power: $P = \sigma\varepsilon AT^4$.

SOLVE

$$\frac{P_2}{P_1} = \frac{\sigma\varepsilon AT_2^4}{\sigma\varepsilon AT_1^4} = \frac{T_2^4}{T_1^4}$$

Part a)

$$\frac{P_{873\text{ K}}}{P_{573\text{ K}}} = \frac{(873\text{ K})^4}{(573\text{ K})^4} = \boxed{5.39}$$

Part b)

$$\frac{P_{1173\text{ K}}}{P_{573\text{ K}}} = \frac{(1173\text{ K})^4}{(573\text{ K})^4} = \boxed{17.6}$$

Part c)

$$\frac{P_2}{P_1} = \frac{\sigma\varepsilon A_2T_2^4}{\sigma\varepsilon A_1T_1^4} = \frac{(2A_1)T_2^4}{A_1T_1^4} = 2\left(\frac{T_2^4}{T_1^4}\right)$$

Our answers will be twice as large as earlier: $\boxed{10.8}$, $\boxed{35.2}$.

REFLECT
Be sure to convert the temperatures into Kelvin before performing your calculations.

14.103

SET UP
We want to know the factor by which the radiated heat increases if the temperature of the Sun increases from 5800 K to 12,000 K. The radiated power is proportional to the temperature to the fourth power: $P = \sigma \varepsilon A T^4$.

SOLVE

$$\frac{P_{12{,}000\text{ K}}}{P_{5800\text{ K}}} = \frac{\sigma \varepsilon A T_2^4}{\sigma \varepsilon A T_1^4} = \frac{T_2^4}{T_1^4} = \frac{(12{,}000\text{ K})^4}{(5800\text{ K})^4} = \boxed{18.33}$$

REFLECT
We're assuming the size of the Sun doesn't change, just its temperature.

14.104

SET UP
A star radiates 3.75 times less heat than our Sun. The radiated power is proportional to the temperature to the fourth power: $P = \sigma \varepsilon A T^4$. We can set up a ratio between the power radiated by each star in order to calculate the temperature of the star in terms of the temperature of the Sun. We assume the star has the same surface area as the Sun.

SOLVE

$$\frac{P_{\text{star}}}{P_{\text{Sun}}} = \frac{\sigma \varepsilon A T_{\text{star}}^4}{\sigma \varepsilon A T_{\text{Sun}}^4}$$

$$\frac{T_{\text{star}}}{T_{\text{Sun}}} = \sqrt[4]{\frac{P_{\text{star}}}{P_{\text{Sun}}}} = \sqrt[4]{\frac{\left(\frac{P_{\text{Sun}}}{3.75}\right)}{P_{\text{Sun}}}} = \sqrt[4]{\frac{1}{3.75}} = \boxed{0.719}$$

REFLECT
The temperature of the star will be around 4170 K.

14.105

SET UP
A star radiates 1000 times more power than our Sun, but its temperature is only 70% of the Sun's temperature. The radiated power is proportional to both the surface area and the temperature to the fourth power: $P = \sigma \varepsilon A T^4$. We can set up a ratio between the power radiated by each star in order to calculate the radius of the star in terms of the radius of the Sun, $R_{\text{Sun}} = 6.96 \times 10^8$ m. We assume that both stars are perfect emitters and that they are both perfectly spherical.

SOLVE

$$\frac{P_{\text{star}}}{P_{\text{Sun}}} = \frac{\sigma\varepsilon A_{\text{star}} T_{\text{star}}^4}{\sigma\varepsilon A_{\text{star}} T_{\text{Sun}}^4} = \frac{(4\pi R_{\text{star}}^2)T_{\text{star}}^4}{(4\pi R_{\text{Sun}}^2)T_{\text{Sun}}^4} = \frac{R_{\text{star}}^2 T_{\text{star}}^4}{R_{\text{Sun}}^2 T_{\text{Sun}}^4}$$

$$R_{\text{star}} = R_{\text{Sun}}\sqrt{\frac{P_{\text{star}} T_{\text{Sun}}^4}{P_{\text{Sun}} T_{\text{star}}^4}} = R_{\text{Sun}}\sqrt{\frac{(1000 P_{\text{Sun}})T_{\text{Sun}}^4}{P_{\text{Sun}}(0.7T_{\text{Sun}})^4}} = (6.96 \times 10^8\text{ m})\sqrt{\frac{1000}{(0.7)^4}} = \boxed{4.5 \times 10^{10}\text{ m}}$$

REFLECT

This is about 65 times larger than the radius of the Sun. Changing the temperature has a much larger effect on the power than changing the surface area since it is raised to the fourth power.

14.106

SET UP

The temperature of a person's skin is 307 K, the emissivity of the skin is 0.900, and the surface area of the skin is 1.50 m^2. The rate at which energy radiates from the skin is equal to $P = \sigma\varepsilon A T^4$, where $\sigma = 5.6703 \times 10^{-8}\frac{\text{W}}{\text{m}^2 \cdot \text{K}^4}$. We can calculate the net energy loss from the body due to radiation in 1 min by multiplying the power by 1 min.

SOLVE

Part a)

$$P = \sigma\varepsilon A T^4 = \left(5.6703 \times 10^{-8}\frac{\text{W}}{\text{m}^2 \cdot \text{K}^4}\right)(0.900)(1.50\text{ m}^2)(307\text{ K})^4 = \boxed{680\text{ W}}$$

Part b)

$$E = Pt = (680\text{ W})\left(1\text{ min} \times \frac{60\text{ s}}{1\text{ min}}\right) = \boxed{40{,}800\text{ J}}$$

REFLECT

The temperature of the surroundings is not necessary when calculating the energy dissipated through radiation.

14.107

SET UP

A glass window is 0.30 m × 1.50 m and 1.2×10^{-3} m thick. We can calculate the rate of heat transfer through the window by conduction by $H = k\frac{A}{L}(T_{\text{H}} - T_{\text{C}})$. The temperature difference across the window is 17 K, and the thermal conductivity of window glass is $k = 0.96\frac{\text{W}}{\text{m} \cdot \text{K}}$.

SOLVE

$$H = k\frac{A}{L}(T_{\text{H}} - T_{\text{C}}) = \left(0.96\frac{\text{W}}{\text{m} \cdot \text{K}}\right)\frac{(0.30\text{ m})(1.50\text{ m})}{1.2 \times 10^{-3}\text{ m}}(17\text{ K}) = \boxed{6100\text{ W} = 6.1\text{ kW}}$$

REFLECT

Unlike radiation, the rate of heat transfer by conduction depends on the temperature difference between two objects.

14.108

SET UP

A two-dimensional ideal gas made up of diatomic molecules is at a temperature of 75 degrees F. Each degree of freedom contributes $\frac{1}{2}kT$ to the average kinetic energy of the gas molecules. In this case, there are four degrees of freedom: two translational because we are told it's a two-dimensional gas and two rotational because the molecules are diatomic. Therefore, the average kinetic energy of the gas molecules is $2kT$, where the temperature is in Kelvin. The conversion between Celsius and Fahrenheit is $T_C = \frac{5}{9}(T_F - 32)$, and the conversion between Kelvin and Celsius is $T_K = T_C + 273.15$.

SOLVE

Temperature:

$$T_C = \frac{5}{9}(T_F - 32) = \frac{5}{9}(75 - 32) = 23.9°C$$

$$T_K = T_C + 273.15 = 23.9 + 273.15 = 297 \text{ K}$$

Average kinetic energy:

$$K_{avg} = 2kT = 2\left(1.38 \times 10^{-23}\frac{\text{J}}{\text{K}}\right)(297 \text{ K}) = \boxed{8.20 \times 10^{-21} \text{ J}}$$

REFLECT

The total kinetic energy is equal to the translational kinetic energy plus the rotational kinetic energy.

14.109

SET UP

In order to determine if the Sun is likely to lose its atomic hydrogen, we need to calculate the speed necessary to escape the Sun's gravity and compare it to the average speed of the hydrogen atoms on the Sun's surface. The escape velocity can be found by equating the kinetic energy of the hydrogen atoms traveling at the escape speed to the gravitational potential energy of the Sun at its surface. Assuming the root-mean-square speed of the hydrogen atoms is equal to the escape velocity, we can use the average kinetic energy to solve for the temperature at the Sun's surface. The actual temperature of the Sun's surface is 5800 K; as long as the "escape temperature" is larger than the actual temperature, the Sun is not likely to lose its hydrogen. The quantities we'll need to solve this problem are the mass of the Sun, $m_S = 1.99 \times 10^{30}$ kg; the radius of the Sun, $R_S = 6.96 \times 10^8$ m; and the mass of hydrogen, $m_H = 1.68 \times 10^{-27}$ kg.

SOLVE

Part a)

$$\frac{1}{2}m_{\text{H}}v_{\text{esc}}^2 = \frac{Gm_{\text{H}}m_{\text{S}}}{R_{\text{S}}}$$

$$v_{\text{esc}} = \sqrt{\frac{2Gm_{\text{S}}}{R_{\text{S}}}} = \sqrt{\frac{2\left(6.67 \times 10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(1.99 \times 10^{30}\text{ kg})}{(6.96 \times 10^{8}\text{ m})}} = \boxed{6.18 \times 10^{5}\frac{\text{m}}{\text{s}} = 618\frac{\text{km}}{\text{s}}}$$

Part b)

$$K_{\text{avg}} = \frac{1}{2}m_{\text{H}}v_{\text{rms}}^2 = \frac{3}{2}kT$$

$$T = \frac{m_{\text{H}}v_{\text{rms}}^2}{3k} = \frac{(1.68 \times 10^{-27}\text{ kg})\left(6.18 \times 10^{5}\frac{\text{m}}{\text{s}}\right)^2}{3\left(1.38 \times 10^{-23}\frac{\text{J}}{\text{K}}\right)} = \boxed{1.55 \times 10^{7}\text{ K}}$$

Part c) Since 5800 K is much less than 1.53×10^7 K, the Sun is unlikely to lose its hydrogen, at least on a reasonable time scale.

REFLECT

The solar wind is not driven by thermal escape from the photosphere. Our calculation centers on hydrogen atoms, but there are also smaller particles (for example, electrons) that can achieve higher speeds. These charged particles can also interact with the magnetic fields present.

14.110

SET UP

An ideal gas has a constant bulk modulus B. We can use the expression for bulk modulus, $\Delta P = -B\frac{\Delta V}{V_0}$, to determine what will happen to the pressure if we decrease the volume.

SOLVE

$$\Delta P = -B\frac{\Delta V}{V_0}$$

The magnitude of the change in pressure is directly proportional the magnitude of the change in volume, but the overall minus sign means that a decrease in one will correspond to an increase in the other. Therefore, decreasing the volume by a factor of ½ will $\boxed{\text{increase the pressure by a factor of } {}^1/_2}$.

REFLECT

We are assuming the temperature of the gas remains constant during the compression.

14.111

SET UP

The main constituents of the atmosphere on Mars are carbon dioxide (95.4%), nitrogen (2.7%), argon (1.6%), oxygen (0.13%), and carbon monoxide (0.007%). The atmospheric pressure on Mars is $P_{\text{atm, Mars}} = 0.675$ kPa. The partial pressure of a constituent gas is equal to the fraction of that gas in the mixture multiplied by the total atmospheric pressure.

SOLVE

Carbon dioxide:

$$P_{CO_2} = (0.954)P_{\text{atm, Mars}} = (0.954)(0.675\text{ kPa}) = \boxed{0.644\text{ kPa}}$$

Nitrogen:

$$P_{N_2} = (0.027)P_{\text{atm, Mars}} = (0.027)(0.675\text{ kPa}) = \boxed{0.018\text{ kPa}}$$

Argon:

$$P_{Ar} = (0.016)P_{\text{atm, Mars}} = (0.016)(0.675\text{ kPa}) = \boxed{0.011\text{ kPa}}$$

Oxygen:

$$P_{O_2} = (0.0013)P_{\text{atm, Mars}} = (0.0013)(0.675\text{ kPa}) = \boxed{0.00088\text{ kPa}}$$

Carbon monoxide:

$$P_{CO} = (0.0007)P_{\text{atm, Mars}} = (0.0007)(0.675\text{ kPa}) = \boxed{0.0005\text{ kPa}}$$

REFLECT

The sum of the partial pressures of *all* of the constituents of the atmosphere will equal $P_{\text{atm, Mars}}$. The above gases make up 99.9% of the atmosphere for a total pressure of 0.674 kPa.

14.112

SET UP

The atmosphere of Jupiter is made up of 89% H_2 and 11% He. The partial pressure of H_2 on Jupiter is 0.45 atm. The partial pressure of a constituent gas is equal to the fraction of that gas in the mixture multiplied by the total atmospheric pressure; this will allow us to calculate the atmospheric pressure on Jupiter. Once we have that, we can multiply it by the fraction of helium in the atmosphere to find the partial pressure of helium on Jupiter.

SOLVE

Part a)

$$P_{H_2} = (0.89)P_{\text{atm, Jupiter}}$$

$$P_{\text{atm, Jupiter}} = \frac{P_{H_2}}{0.89} = \frac{0.45\text{ atm}}{0.89} = \boxed{0.51\text{ atm}}$$

Part b)

$$P_{He} = (0.11)P_{\text{atm, Jupiter}} = (0.11)(0.51\text{ atm}) = \boxed{0.056\text{ atm}}$$

REFLECT

We can add up the partial pressures of hydrogen and helium to double-check our answers: 0.45 atm + 0.056 atm = 0.51 atm.

14.113

SET UP

Inexperienced scuba divers are taken to a reef 25 m below the ocean's surface, but experienced divers are allowed to visit depths 40 m below the surface. This precaution is due to the dangers surrounding the increased partial pressure of nitrogen in the air they breathe due to the increased pressure with depth. We can calculate the pressure at depths of 25 m and 40 m using the density of seawater (1030 kg/m^3). The partial pressure of nitrogen is equal to the fraction of nitrogen in the air (approximately 79%) multiplied by the total pressure at that depth. Divers can experience adverse effects when the partial pressure of nitrogen is larger than 3.5×10^5 Pa and need to exercise added caution when traveling to lower depths.

SOLVE

Part a)

Depth of 25 m:

$$P_{25\text{ m}} = P_{\text{atm}} + \rho g h = (101{,}300\text{ Pa}) + \left(1030\frac{\text{kg}}{\text{m}^3}\right)\left(9.8\frac{\text{m}}{\text{s}^2}\right)(25\text{ m}) = 353{,}650\text{ Pa}$$

$$P_{\text{N}_2} = (0.79)P_{25\text{ m}} = (0.79)(353{,}650\text{ Pa}) = \boxed{280{,}000\text{ Pa} = 280\text{ kPa}}$$

Depth of 40 m:

$$P_{40\text{ m}} = P_{\text{atm}} + \rho g h = (101{,}300\text{ Pa}) + \left(1030\frac{\text{kg}}{\text{m}^3}\right)\left(9.8\frac{\text{m}}{\text{s}^2}\right)(40\text{ m}) = 505{,}060\text{ Pa}$$

$$P_{\text{N}_2} = (0.79)P_{25\text{ m}} = (0.79)(505{,}060\text{ Pa}) = \boxed{400{,}000\text{ Pa} = 400\text{ kPa}}$$

Part b) Experienced divers know how to ascend from such depths safely or can at least be counted on to learn properly before trying.

REFLECT

We see that the partial pressure of nitrogen at a depth of 25 m is less than 3.5×10^5 Pa, whereas the partial pressure of nitrogen at a depth of 40 m is more than 3.5×10^5 Pa. These calculations agree with the fact that only experienced divers should be allowed to visit depths lower than about 34 m.

14.114

SET UP

With each breath, a person breathes in about 0.50 L of air containing 20.9% oxygen and breathes out about 0.50 L of air containing 16.3% oxygen, regardless of the altitude, but people feel "out of breath" at higher altitudes. The atmospheric pressure at sea level is 1.00 atm, and the atmospheric pressure at an altitude of 3048 m is 0.695 atm. We can use the ideal gas law to calculate the number of molecules of oxygen inhaled and exhaled from the

partial pressure of oxygen in the air and the temperature (293 K at both locations). The net number of oxygen molecules "consumed" by the body is the difference in these values. We can compare these values for the two altitudes and try to understand why people feel out of breath at high altitudes.

SOLVE

Part a)

Breathing in:

$$P_{O_2}V = N_{O_2}kT$$

$$N_{O_2\text{ in}} = \frac{P_{O_2}V}{kT} = \frac{(0.209)P_{\text{atm}}V}{kT}$$

$$= \frac{(0.209)\left(1.00\text{ atm} \times \dfrac{101{,}300\text{ Pa}}{1\text{ atm}}\right)\left(0.50\text{ L} \times \dfrac{1\text{ m}^3}{1000\text{ L}}\right)}{\left(1.38 \times 10^{-23}\dfrac{\text{J}}{\text{K}}\right)(293\text{ K})} = 2.62 \times 10^{21}\text{ molecules O}_2$$

Breathing out:

$$P_{O_2}V = N_{O_2}kT$$

$$N_{O_2\text{ out}} = \frac{P_{O_2}V}{kT} = \frac{(0.163)P_{\text{atm}}V}{kT}$$

$$= \frac{(0.163)\left(1.00\text{ atm} \times \dfrac{101{,}300\text{ Pa}}{1\text{ atm}}\right)\left(0.50\text{ L} \times \dfrac{1\text{ m}^3}{1000\text{ L}}\right)}{\left(1.38 \times 10^{-23}\dfrac{\text{J}}{\text{K}}\right)(293\text{ K})} = 2.04 \times 10^{21}\text{ molecules O}_2$$

Net number of O_2 molecules used per breath:

$$N_{\text{breath}} = N_{O_2\text{ in}} - N_{O_2\text{ out}}$$

$$= (2.62 \times 10^{21}\text{ molecules}) - (2.04 \times 10^{21}\text{ molecules}) = \boxed{5.80 \times 10^{20}\text{ molecules}}$$

Part b)

Breathing in:

$$P_{O_2}V = N_{O_2}kT$$

$$N_{O_2\text{ in}} = \frac{P_{O_2}V}{kT} = \frac{(0.209)P_{\text{atm}}V}{kT}$$

$$= \frac{(0.209)\left(0.695\text{ atm} \times \dfrac{101{,}300\text{ Pa}}{1\text{ atm}}\right)\left(0.50\text{ L} \times \dfrac{1\text{ m}^3}{1000\text{ L}}\right)}{\left(1.38 \times 10^{-23}\dfrac{\text{J}}{\text{K}}\right)(293\text{ K})} = 1.82 \times 10^{21}\text{ molecules O}_2$$

Breathing out:

$$P_{O_2}V = N_{O_2}kT$$

$$N_{O_2\text{ out}} = \frac{P_{O_2}V}{kT} = \frac{(0.163)P_{\text{atm}}V}{kT}$$

$$= \frac{(0.163)\left(0.695\text{ atm} \times \frac{101{,}300\text{ Pa}}{1\text{ atm}}\right)\left(0.50\text{ L} \times \frac{1\text{ m}^3}{1000\text{ L}}\right)}{\left(1.38 \times 10^{-23}\frac{\text{J}}{\text{K}}\right)(293\text{ K})} = 1.42 \times 10^{21}\text{ molecules O}_2$$

Net number of O_2 molecules used per breath:

$$N_{\text{breath}} = N_{O_2\text{ in}} - N_{O_2\text{ out}}$$

$$= (1.82 \times 10^{21}\text{ molecules}) - (1.42 \times 10^{21}\text{ molecules}) = \boxed{4.00 \times 10^{20}\text{ molecules}}$$

Part c) Each breath at higher altitudes contains fewer oxygen molecules than at sea level. To make up for this reduction, people take more frequent breaths.

REFLECT

Some of the symptoms of altitude sickness are fatigue, headache, dizziness, and shortness of breath.

14.115

SET UP

A sample of gas is made up of many different monatomic gases, where the percent of the xth component in the gas is F_x, and the partial pressure due to the xth component is P_x. The atoms of the xth component have a mass m_x, and there are n_x moles of the gas. We can use the ideal gas law to first determine the temperature of the gas and then plug this into the expression for the average kinetic energy of a three-dimensional monatomic gas $\left(K_{\text{avg}} = \frac{3}{2}kT\right)$ in order to solve for the average speed of the atoms of the xth component.

SOLVE

Temperature of the gas:

$$P_xV = n_xRT$$

$$F_xPV = F_xnRT$$

$$T = \frac{PV}{nR}$$

Average speed of x:

$$\frac{1}{2}m_xv_x^2 = \frac{3}{2}kT$$

$$v_x = \sqrt{\frac{3kT}{m_x}} = \sqrt{\frac{3k}{m_x}\left(\frac{PV}{nR}\right)} = \boxed{\sqrt{\left(\frac{3k}{R}\right)\left(\frac{PV}{nm_x}\right)}}$$

REFLECT

According to our expression, the lighter the atoms of the component gas, the faster their average speed, which makes sense.

14.116

SET UP

Titan has a nitrogen atmosphere with a surface temperature of 94 K and pressure of 1.5 atm. The particle density of Titan's atmosphere is equal to the number of particles divided by the volume, which we can find through the ideal gas law. We can determine whether Titan or the Earth has a denser atmosphere by taking a ratio between their particle densities. The Earth's atmosphere has a pressure of 1.0 atm and a temperature around 283 K. Finally, we can calculate the mean free path of a nitrogen molecule in Titan's atmosphere directly from $\lambda = \dfrac{1}{4\sqrt{2}\pi r^2\left(\frac{N}{V}\right)}$. We will model a nitrogen molecule as a sphere of diameter 2.4×10^{-10} m. For a comparison, in Section 14-3 of the text, the mean free path of carbon dioxide at STP was 1.6×10^{-7} m.

SOLVE

Part a)

$$PV = NkT$$

$$\frac{N}{V} = \frac{P}{kT} = \frac{\left(1.5 \text{ atm} \times \dfrac{101{,}300 \text{ Pa}}{1 \text{ atm}}\right)}{\left(1.38 \times 10^{-23}\dfrac{\text{J}}{\text{K}}\right)(94 \text{ K})} = \boxed{1.2 \times 10^{26}\frac{\text{particles}}{\text{m}^3}}$$

Part b)

$$\frac{\left(\frac{N}{V}\right)_{\text{Titan}}}{\left(\frac{N}{V}\right)_{\text{Earth}}} = \frac{\left(\frac{P}{kT}\right)_{\text{Titan}}}{\left(\frac{P}{kT}\right)_{\text{Earth}}} = \left(\frac{P_{\text{Titan}}}{P_{\text{Earth}}}\right)\left(\frac{T_{\text{Earth}}}{T_{\text{Titan}}}\right) = \left(\frac{1.5 \text{ atm}}{1.0 \text{ atm}}\right)\left(\frac{283 \text{ K}}{94 \text{ K}}\right) = 4.5$$

$\boxed{\text{Titan}}$ has a denser atmosphere.

Part c)

$$\lambda = \frac{1}{4\sqrt{2}\pi r^2\left(\frac{N}{V}\right)} = \frac{1}{4\sqrt{2}\pi\left(\dfrac{2.4 \times 10^{-10} \text{ m}}{2}\right)^2\left(1.2 \times 10^{26}\dfrac{\text{particles}}{\text{m}^3}\right)} = \boxed{3.3 \times 10^{-8} \text{ m}}$$

The mean free path calculated for carbon dioxide at STP is 1.6×10^{-7} m. Our answer is about one-third as large, which makes sense given that the particle density is much higher on Titan than on Earth.

REFLECT

The mass of a nitrogen molecule was irrelevant in this problem. We would use it if we were interested in the root-mean-square speed of a molecule in Titan's atmosphere.

14.117

SET UP

A brick wall is composed of 19.0-cm-long bricks ($\alpha_{\text{brick}} = 5.5 \times 10^{-6}\ {}^\circ\text{C}^{-1}$) and 1.00-cm-long sections of mortar ($\alpha_{\text{mortar}} = 8.0 \times 10^{-6}\ {}^\circ\text{C}^{-1}$). A 20-m-long wall will be made up of 100 of these repeating brick + mortar units. In order to calculate the expansion on this wall due to a temperature increase of 25 degrees C, we need to calculate the expansion of one brick and one piece of mortar due to this temperature increase. The total expansion of the wall will be equal to 100 times the expansion of one brick + mortar unit.

SOLVE

One brick + mortar unit:

$$\Delta L_{\text{brick}} = \alpha_{\text{brick}} L_{0,\text{ brick}} \Delta T = (5.5 \times 10^{-6}\ {}^\circ\text{C}^{-1})(19.0\text{ cm})(25^\circ\text{C}) = 0.00261\text{ cm}$$

$$\Delta L_{\text{mortar}} = \alpha_{\text{mortar}} L_{0,\text{ mortar}} \Delta T = (8.0 \times 10^{-6}\ {}^\circ\text{C}^{-1})(1.00\text{ cm})(25^\circ\text{C}) = 0.000200\text{ cm}$$

$$\Delta L_{\text{total}} = \Delta L_{\text{brick}} + \Delta L_{\text{mortar}} = (0.00261\text{ cm}) + (0.000200\text{ cm}) = 0.00281\text{ cm}$$

There are 100 brick + mortar units in a 20-m-long wall, so:

$$\Delta L_{\text{wall}} = 100 \Delta L_{\text{total}} = 100(0.00281\text{ cm}) = 0.281\text{ cm}$$

The length of the wall $\boxed{\text{increases by 0.281 cm}}$.

REFLECT

A value of 2.81 mm seems to be a reasonable value for a wall to expand.

14.118

SET UP

We want to derive an approximate formula for the area expansion that a sheet of material (linear expansion coefficient α) experiences as a result of a temperature change ΔT. For simplicity, we assume the sheet is square with sides L_0, which means the initial area is $A_0 = L_0^2$. After the expansion, the final sides of the sheet are $L_f = L_0 + \Delta L$ in length, where $\Delta L = \alpha L_0 \Delta T$. The change in area is equal to the final area minus the initial area.

SOLVE

$$\Delta A = L_f^2 - L_0^2 = (L_0 + \Delta L)^2 - L_0^2 = L_0^2 + 2L_0\Delta L + (\Delta L)^2 - L_0^2 = 2L_0\Delta L + (\Delta L)^2$$

Neglecting the small $(\Delta L)^2$ term:

$$\Delta A \approx 2L_0\Delta L = 2L_0(\alpha L_0 \Delta T) = 2\alpha L_0^2 \Delta T = \boxed{2\alpha A_0 \Delta T}$$

REFLECT
This is the same argument that was made in Section 14-4 in the text for approximating the volume change.

14.119

SET UP
At 15 degrees C, the diameter of a copper sphere is $D_{0,\,\mathrm{Cu}} = 1$ cm and the diameter of a hole in an aluminum sheet is $D_{0,\,\mathrm{Al}} = 0.99$ cm. We want to determine a set of temperatures for the sphere and sheet where the sphere passes through the hole in the aluminum. One way we can accomplish this is by heating up the aluminum sheet such that the diameter of the hole increases by 0.01 cm. The thermal expansion of the diameter of the hole is given by $\Delta D_{\mathrm{Al}} = \alpha_{\mathrm{Al}} D_{0,\,\mathrm{Al}} \Delta T$, where $\alpha_{\mathrm{Al}} = 22.2 \times 10^{-6}\ \mathrm{K}^{-1}$.

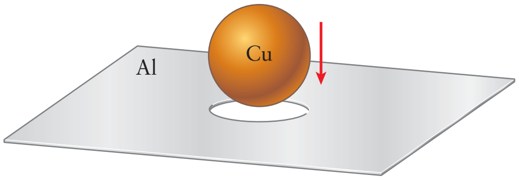


Figure 14-5 Problem 119

SOLVE

$$\Delta D_{\mathrm{Al}} = \alpha_{\mathrm{Al}} D_{0,\,\mathrm{Al}} \Delta T$$

$$\Delta T = \frac{\Delta D_{\mathrm{Al}}}{\alpha_{\mathrm{Al}} D_{0,\,\mathrm{Al}}} = \frac{0.01\ \mathrm{cm}}{(22.2 \times 10^{-6}\ \mathrm{K}^{-1})(0.99\ \mathrm{cm})} = 455\ \mathrm{K}$$

$$T_{\mathrm{f,\,Al}} = T_{0,\,\mathrm{Al}} + \Delta T = (15^\circ\mathrm{C}) + (455^\circ\mathrm{C}) = 470^\circ\mathrm{C}$$

The sphere will pass through the hole when the $\boxed{\text{temperature of the sphere is kept at 15 degrees C}}$ and the $\boxed{\text{temperature of the aluminum sheet is higher than 470 degrees C}}$.

REFLECT
Another set of conditions is the temperature of the copper sphere is cooled to −135.6 degrees C and the aluminum sheet is heated up to 326 degrees C. This corresponds to a decrease of 0.0025 cm in the sphere's diameter and an increase of 0.0075 cm in the hole's diameter. Be careful when searching for other conditions since the temperature of the sphere cannot go below absolute zero.

14.120

SET UP
Two bars—a 20.0-m-long steel bar and a 10.0-m-long copper bar—are separated by a gap of 0.015 m. The bars are subject to the same temperature increase, which causes them to expand and eventually close the gap between them. We can use $\Delta L = \alpha L_0 \Delta T$ to calculate

the amount that each bar expands. We can solve for the temperature increase by setting the sum of these two amounts equal to 0.015 m. Once we have the value of the temperature change, we can plug it back into $\Delta L = \alpha L_0 \Delta T$ to calculate the amount that each bar expands. The linear expansion coefficients for steel and copper are $\alpha_{\text{steel}} = 13.0 \times 10^{-6}\ \text{K}^{-1}$ and $\alpha_{\text{Cu}} = 16.5 \times 10^{-6}\ \text{K}^{-1}$, respectively.

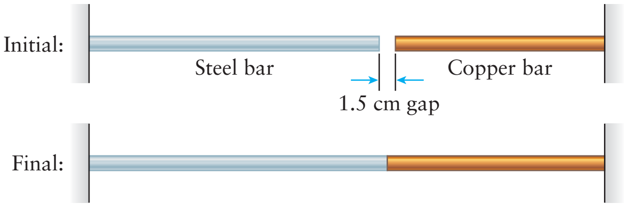


Figure 14-6 Problem 120

SOLVE

Temperature change:

$$\Delta L_{\text{steel}} = \alpha_{\text{steel}} L_{0,\ \text{steel}} \Delta T$$

$$\Delta L_{\text{Cu}} = \alpha_{\text{Cu}} L_{0,\ \text{Cu}} \Delta T$$

$$\Delta L_{\text{total}} = \Delta L_{\text{steel}} + \Delta L_{\text{Cu}} = \alpha_{\text{Cu}} L_{0,\ \text{Cu}} \Delta T + \alpha_{\text{steel}} L_{0,\ \text{steel}} \Delta T = \Delta T(\alpha_{\text{Cu}} L_{0,\ \text{Cu}} + \alpha_{\text{steel}} L_{0,\ \text{steel}})$$

$$\Delta T = \frac{\Delta L_{\text{total}}}{\alpha_{\text{Cu}} L_{0,\ \text{Cu}} + \alpha_{\text{steel}} L_{0,\ \text{steel}}}$$

$$= \frac{(0.015\ \text{m})}{(16.5 \times 10^{-6}\ \text{K}^{-1})(10.0\ \text{m}) + (13.0 \times 10^{-6}\ \text{K}^{-1})(20.0\ \text{m})} = \boxed{35\ \text{K}}$$

Expansions:

$$\Delta L_{\text{steel}} = \alpha_{\text{steel}} L_{0,\ \text{steel}} \Delta T = (13.0 \times 10^{-6}\ \text{K}^{-1})(20.0\ \text{m})(35\ \text{K}) = \boxed{0.0091\ \text{m} = 9.1\ \text{mm}}$$

$$\Delta L_{\text{Cu}} = \alpha_{\text{Cu}} L_{0,\ \text{Cu}} \Delta T = (16.5 \times 10^{-6}\ \text{K}^{-1})(10.0\ \text{m})(35\ \text{K}) = \boxed{0.0058\ \text{m} = 5.8\ \text{mm}}$$

REFLECT

Each bar is fixed on one end, so it can only expand at its "free" end.

14.121

SET UP

A person collects sound waves from traffic and converts them into an electrical signal used to heat 5 kg of water. After 7 days the temperature of the water increased 0.01 K. Using the specific heat of water $\left(c = 4186 \frac{\text{J}}{\text{kg} \cdot \text{K}}\right)$, we can calculate the energy (in the form of heat) required to cause this temperature increase. The total acoustic power "caught" by the transducer is equal to this energy divided by 7 days.

SOLVE

Heat required to raise the temperature:

$$Q = mc\Delta T = (5\text{ kg})\left(4186\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(0.01\text{ K}) = 209.3\text{ J}$$

Acoustic power collected:

$$P = \frac{E}{t} = \frac{209.3\text{ J}}{\left(7\text{ days} \times \frac{24\text{ hr}}{1\text{ day}} \times \frac{3600\text{ s}}{1\text{ hr}}\right)} = \boxed{3.46 \times 10^{-4}\text{ W} = 346\ \mu\text{W}}$$

REFLECT

An efficiency of 100% means all of the acoustic energy is converted into electrical energy. In actuality, the efficiency would be (much) lower than 100%, so 346 μW is the minimum power that the transducer collects.

14.122

SET UP

The specific heat for a sealed system of mass m is a function of temperature, $c(T) = c_0 + c_1 T$. The heat added to the system when the temperature rises from T_0 to T_f is equal to the integral $Q = \int_{T_0}^{T_f} mc(T)dT$. In part (b) we can calculate the heat added for given set of values: $c_0 = 2000\frac{\text{J}}{\text{kg}\cdot{}^\circ\text{C}}$, $c_1 = 40\frac{\text{J}}{\text{kg}\cdot{}^\circ\text{C}^2}$, $m = 0.100$ kg, $T_0 = 20$ degrees C, and $T_f = 60$ degrees C.

SOLVE

Part a)

$$Q = \int_{T_0}^{T_f} mc(T)dT = m\int_{T_0}^{T_f}(c_0 + c_1 T)dT = m\left[c_0 T + \frac{c_1}{2}T^2\right]_{T_0}^{T_f}$$

$$= m\left[c_0 T_f - c_0 T_0 + \frac{c_1}{2}T_f^2 - \frac{c_1}{2}T_0^2\right]_{T_0}^{T_f} = \boxed{mc_0(T_f - T_0) + \frac{mc_1}{2}(T_f^2 - T_0^2)}$$

Part b)

$$Q = (0.100\text{ kg})\left(2000\frac{\text{J}}{\text{kg}\cdot{}^\circ\text{C}}\right)((60^\circ\text{C}) - (20^\circ\text{C})) + \frac{(0.100\text{ kg})\left(40\frac{\text{J}}{\text{kg}\cdot{}^\circ\text{C}^2}\right)}{2}((60^\circ\text{C})^2 - (20^\circ\text{C})^2)$$

$$= \boxed{14{,}400\text{ J}}$$

REFLECT

Even though c_1 is smaller than c_0, the second term is on the same order as the first due to squaring the temperatures.

14.123

SET UP

The specific heat for a sealed system of mass $m = 1$ kg is a function of temperature, $c(T) = 3000 + 9T^2$ (SI units, temperature in Celsius). The initial temperature of the system is 0 degrees C and 15,000 J of heat is added. The final temperature of the system can be calculated from the integral $Q = \int_{T_0}^{T_\mathrm{f}} mc(T)dT$. We assume that no phase changes occur.

SOLVE

$$Q = \int_{T_0}^{T_\mathrm{f}} mc(T)dT = m\int_{0°\mathrm{C}}^{T_\mathrm{f}} (3000 + 9T^2)dT = m[3000T + 3T^3]_{0°\mathrm{C}}^{T_\mathrm{f}} = m[3000T_\mathrm{f} + 3T_\mathrm{f}^3]$$

$$15{,}000 = (1)[3000T_\mathrm{f} + 3T_\mathrm{f}^3]$$

$$5000 = 1000T_\mathrm{f} + T_\mathrm{f}^3$$

To solve this cubic, we define two variables s and t, where $T_\mathrm{f} = s - t$. Therefore,

$$3st = 1000$$

$$s = \frac{1000}{3t}$$

$$s^3 - t^3 = 5000$$

$$\left(\frac{1000}{3t}\right)^3 - t^3 = 5000$$

$$\frac{10^9}{27t^3} - t^3 = 5000$$

$$t^6 + 5000t^3 - \frac{10^9}{27} = 0$$

This is a quadratic in t^3:

$$t^3 = \frac{-5000 \pm \sqrt{(5000)^2 - 4(1)\left(-\frac{10^9}{27}\right)}}{2(1)} = \frac{-5000 \pm \sqrt{(5000)^2 + 4\left(\frac{10^9}{27}\right)}}{2}$$

Taking the positive root:

$$t^3 = \frac{-5000 + \sqrt{(5000)^2 + 4\left(\frac{10^9}{27}\right)}}{2} = 4079$$

$$t = \sqrt[3]{4079} = 15.98$$

$$s^3 = t^3 + 5000 = 4079 + 5000 = 9079$$

$$s = \sqrt[3]{9079} = 20.86$$

$$T_\mathrm{f} = s - t = 20.86 - 15.98 = \boxed{4.88°\mathrm{C}}$$

REFLECT

If the specific heat were constant with respect to temperature and equal to $c = 3000$, the temperature increase would be 5 degrees C. In actuality, the specific heat increases with temperature, so it makes sense that the value we calculated should be less than 5 degrees C.

14.124

SET UP

A 60-kg woman eats about 2.000×10^6 cal of food a day. Typically, 80% of the calories eaten are converted into heat. Assuming the woman is made completely of water and that she has no way of getting rid of the heat produced, we can use the specific heat of water to calculate the increase in her body temperature $\Delta T = \frac{Q}{mc}$; the specific heat of water is $c = 4186 \frac{\text{J}}{\text{kg} \cdot \text{K}}$ and the conversion between calories and joules is 1 cal = 4.184 J. The human body removes extra heat through radiation and evaporation of sweat.

SOLVE

Part a)

$$\Delta T = \frac{Q}{mc} = \frac{0.8\left(2.000 \times 10^6 \text{ cal} \times \dfrac{4.184 \text{ J}}{1 \text{ cal}}\right)}{(60 \text{ kg})\left(4186 \dfrac{\text{J}}{\text{kg} \cdot \text{K}}\right)} = 26.7 \text{ K} = \boxed{26.7°\text{C}}$$

Part b) $\boxed{\text{Yes}}$, this is a noticeable temperature increase. In fact, a temperature increase of this magnitude would kill her!

Part c) Her body prevents this increase by $\boxed{\text{radiating heat and evaporating sweat}}$.

REFLECT

A body temperature over 41.5 degrees C is considered a severe medical emergency; this is known as hyperpyrexia.

14.125

SET UP

The total heat required to condense 5×10^{-3} kg of helium initially at 30 degrees C is equal to the sum of the heat necessary to lower the temperature of the sample to the boiling point ($T_V = -268.93$ degrees C) and the heat necessary to convert the entire gas into liquid. The heat transfer necessary to decrease the temperature of the helium is given by $Q = mc\Delta T$, and the heat necessary to convert from gas to liquid is related to the latent heat of vaporization, $Q = mL_V$. The specific heat of helium is $c = 5193 \frac{\text{J}}{\text{kg} \cdot °\text{C}}$, and the latent heat of vaporization for helium is $L_V = 21{,}000 \frac{\text{J}}{\text{kg}}$.

SOLVE

$$Q_{\text{total}} = Q_{30°\text{C}\to -268.93°\text{C}} + Q_{\text{gas}\to\text{liquid}} = mc\Delta T_{30°\text{C}\to -268.93°\text{C}} + mL_{\text{V}} = m(c\Delta T_{30°\text{C}\to -268.93°\text{C}} + L_{\text{V}})$$

$$= (5 \times 10^{-3}\ \text{kg})\left(\left(5193\frac{\text{J}}{\text{kg}\cdot°\text{C}}\right)(-298.93°\text{C}) + \left(-21{,}000\frac{\text{J}}{\text{kg}}\right)\right) = \boxed{-7867\ \text{J}}$$

REFLECT

The majority of the heat lost by the helium goes into cooling the gas by almost 300 degrees C.

14.126

SET UP

The total heat required to convert 1 kg of solid carbon dioxide initially at 194.65 K is equal to the sum of the heat necessary to increase the temperature of the sample to 293.15 K and the heat necessary to convert the entire solid into gas. Since the heat capacity is a function of temperature, the heat transfer necessary to increase the temperature of the carbon dioxide is given by the integral $\int_{194.65\text{ K}}^{293.15\text{ K}} mc(T)dT$, where $c(T) = 0.001112T + 0.5128$ (SI units). The heat necessary to convert from solid to gas is related to the latent heat of sublimation, $Q = mL_{\text{S}}$. The latent heat of sublimation for carbon dioxide is $L_{\text{S}} = 573{,}700\frac{\text{J}}{\text{kg}}$.

SOLVE

$$Q_{\text{total}} = Q_{194.65\text{ K}\to 293.15\text{ K}} + Q_{\text{solid}\to\text{gas}} = \int_{194.65\text{ K}}^{293.15\text{ K}} mc(T)dT + mL_{\text{S}}$$

$$= m\int_{194.65\text{ K}}^{293.15\text{ K}} (0.001112T + 0.5128)dT + mL_{\text{S}}$$

$$= m\left[\frac{0.001112}{2}T^2 + 0.5128T\right]_{194.65\text{ K}}^{293.15\text{ K}} + mL_{\text{S}}$$

$$= (1)\left[\frac{0.001112}{2}(293.15^2 - 194.65^2) + 0.5128(293.15 - 194.65)\right] + (1)(573{,}700)$$

$$= \boxed{573{,}800\ \text{J}}$$

REFLECT

The majority of the heat absorbed goes into subliming the solid carbon dioxide. The opposite of sublimation—going directly from gas to solid—is known as deposition.

14.127

SET UP

We want to know the heat required to melt a large amount of ice that was 3.0 m thick with an area of 720,000 km^2. The density of the ice is 92% that of pure water. The density multiplied by the volume of the ice sheet will give us the mass of the ice that melted. The ice, which was initially at −10.0 degrees C, first needs to heat up to 0 degrees C before it can

melt. Therefore, the total heat required to melt the ice that was initially at −10 degrees C is equal to the sum of the heat necessary to increase the temperature of the sample to the melting point ($T_F = 0$ degrees C) and the heat necessary to convert the entire solid into liquid. The heat transfer necessary to increase the temperature of the ice is given by $Q = mc\Delta T$, and the heat necessary to convert from solid to liquid is related to the latent heat of fusion, $Q = mL_F$. The specific heat of ice is $c = 2093 \frac{\text{J}}{\text{kg} \cdot {}^\circ\text{C}}$, and the latent heat of fusion for water is $L_F = 334 \times 10^3 \frac{\text{J}}{\text{kg}}$. Once we calculate the total heat, we can compare it to the energy released by 1 gal of gasoline (1.3×10^8 J) and 1 ton of coal (21.5×10^9 J).

SOLVE

Part a)

Mass of melted ice:

$$m = \rho V = \rho A d = (0.92)\left(1000 \frac{\text{kg}}{\text{m}^3}\right)\left(720{,}000\ \text{km}^2 \times \left(\frac{1000\ \text{m}}{1\ \text{km}}\right)^2\right)(3.0\ \text{m}) = 1.987 \times 10^{15}\ \text{kg}$$

Heat required to melt the ice:

$$Q_{\text{total}} = Q_{-10.0^\circ\text{C}\to 0^\circ\text{C}} + Q_{\text{solid}\to\text{liquid}} = mc\Delta T_{-10.0^\circ\text{C}\to 0^\circ\text{C}} + mL_f = m(c\Delta T_{-10.0^\circ\text{C}\to 0^\circ\text{C}} + L_f)$$

$$= (1.987 \times 10^{15}\ \text{kg})\left(\left(2093 \frac{\text{J}}{\text{kg} \cdot {}^\circ\text{C}}\right)(10^\circ\text{C}) + \left(334 \times 10^3 \frac{\text{J}}{\text{kg}}\right)\right)$$

$$= 7.05 \times 10^{20}\ \text{J} = \boxed{7.1 \times 10^{20}\ \text{J}}$$

Part b)

$$7.05 \times 10^{20}\ \text{J} \times \frac{1\ \text{gal gasoline}}{1.3 \times 10^8\ \text{J}} = \boxed{5.4 \times 10^{12}\ \text{gal gasoline}}$$

Part c)

$$7.05 \times 10^{20}\ \text{J} \times \frac{1\ \text{ton coal}}{21.5 \times 10^9\ \text{J}} = \boxed{3.3 \times 10^{10}\ \text{tons of coal}}$$

REFLECT

For comparison, an average-sized car's gas tank can hold about 15 gal of gasoline, so 5.4×10^{12} gal of gas will fill 3.6×10^{11} cars. The population of the Earth is around 7×10^9 people, which means each person in the world could fill up 51 cars.

14.128

SET UP

The surface area of a person's skin is about 1.5 m^2 and is about 1×10^{-3} m thick. We can calculate the rate of heat transfer through the skin by conduction by $H = k\frac{A}{L}(T_H - T_C)$. The temperature difference across the skin is 1 K, and the thermal conductivity of human fat is $k = 0.2 \frac{\text{W}}{\text{m} \cdot \text{K}}$.

SOLVE

$$H = k\frac{A}{L}(T_H - T_C) = \left(0.2\frac{\text{W}}{\text{m}\cdot\text{K}}\right)\left(\frac{1.5\ \text{m}^2}{1 \times 10^{-3}\ \text{m}}\right)(1\ \text{K}) = \boxed{300\ \text{W}}$$

REFLECT

As you would imagine, the rate of heat transfer through the skin is larger in colder weather than in warmer weather due to the increased temperature difference between the body and the environment.

14.129

SET UP

Building insulation is rated in R-value, the resistance to the transfer of thermal energy, and is defined as $R = \frac{L}{k}$, where L is the thickness of the insulation and k is the thermal conductivity of the material. The wall of a certain home with a cross-sectional area of 25 m^2 consists of brick $\left(R_B = 0.6\frac{\text{ft}^2\cdot\text{hr}\cdot{}^\circ\text{F}}{\text{BTU}}\right)$, plywood $\left(R_P = 0.6\frac{\text{ft}^2\cdot\text{hr}\cdot{}^\circ\text{F}}{\text{BTU}}\right)$, and R-12 insulation $\left(R_I = 12\frac{\text{ft}^2\cdot\text{hr}\cdot{}^\circ\text{F}}{\text{BTU}}\right)$. The temperature difference between inside and outside is 5 degrees C, which is 9 degrees F. Realizing the rate of heat transfer must be constant through each material, we can determine the effective R-value R_{eff} for the combination of the insulation + plywood + brick by writing the expressions for H in each region and solving for R_{eff} in terms of R_I, R_P, and R_B. We define the temperature at the insulation–plywood interface to be T_{IP} and the temperature at the plywood–brick interface to be T_{PB}. Once we convert the R-value into more suitable units, we can solve for H_{eff} through the wall. The conversions we need are 1 ft = 12 in, 1 in = 2.54 cm, and 1 BTU = 1055 J.

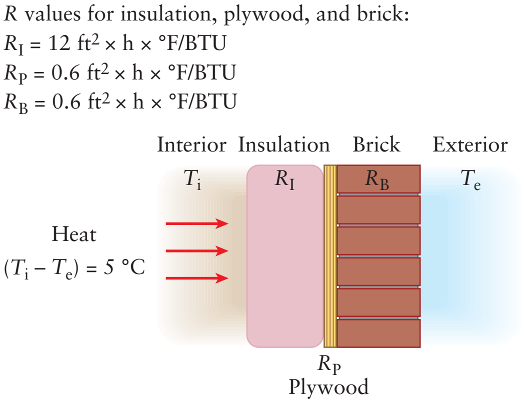


Figure 14-7 Problem 129

SOLVE

Effective R-value:

$$H = k\frac{A}{L}\Delta T = \frac{A}{R}\Delta T$$

$$H_{\text{eff}} = H_{\text{I}} = H_{\text{P}} = H_{\text{B}}$$

$$\frac{A}{R_{\text{eff}}}(T_{\text{i}} - T_{\text{e}}) = \frac{A}{R_{\text{I}}}(T_{\text{i}} - T_{\text{IP}}) = \frac{A}{R_{\text{P}}}(T_{\text{IP}} - T_{\text{PB}}) = \frac{A}{R_{\text{B}}}(T_{\text{PB}} - T_{\text{e}})$$

$$\frac{(T_{\text{i}} - T_{\text{e}})}{R_{\text{eff}}} = \frac{(T_{\text{i}} - T_{\text{IP}})}{R_{\text{I}}} = \frac{(T_{\text{IP}} - T_{\text{PB}})}{R_{\text{P}}} = \frac{(T_{\text{PB}} - T_{\text{e}})}{R_{\text{B}}}$$

Relating R_{eff} to each R-value:

$$R_{\text{I}}(T_{\text{i}} - T_{\text{e}}) = R_{\text{eff}}(T_{\text{i}} - T_{\text{IP}})$$

$$R_{\text{P}}(T_{\text{i}} - T_{\text{e}}) = R_{\text{eff}}(T_{\text{IP}} - T_{\text{PB}})$$

$$R_{\text{B}}(T_{\text{i}} - T_{\text{e}}) = R_{\text{eff}}(T_{\text{PB}} - T_{\text{e}})$$

Adding the three equations:

$$R_{\text{I}}(T_{\text{i}} - T_{\text{e}}) + R_{\text{P}}(T_{\text{i}} - T_{\text{e}}) + R_{\text{B}}(T_{\text{i}} - T_{\text{e}}) = R_{\text{eff}}(T_{\text{i}} - T_{\text{IP}}) + R_{\text{eff}}(T_{\text{IP}} - T_{\text{PB}}) + R_{\text{eff}}(T_{\text{PB}} - T_{\text{e}})$$

$$(R_{\text{I}} + R_{\text{P}} + R_{\text{B}}) = R_{\text{eff}}(T_{\text{i}} - T_{\text{e}})$$

$$R_{\text{eff}} = R_{\text{I}} + R_{\text{P}} + R_{\text{B}} = \left(12\frac{\text{ft}^2 \cdot \text{hr} \cdot {}^\circ\text{F}}{\text{BTU}}\right) + \left(0.6\frac{\text{ft}^2 \cdot \text{hr} \cdot {}^\circ\text{F}}{\text{BTU}}\right) + \left(0.6\frac{\text{ft}^2 \cdot \text{hr} \cdot {}^\circ\text{F}}{\text{BTU}}\right)$$

$$= 13.2\frac{\text{ft}^2 \cdot \text{hr} \cdot {}^\circ\text{F}}{\text{BTU}} \times \frac{3600\ \text{s}}{1\ \text{hr}} \times \frac{1\ \text{BTU}}{1055\ \text{J}} \times \left(\frac{12\ \text{in}}{1\ \text{ft}}\right)^2 \times \left(\frac{2.54\ \text{cm}}{1\ \text{in}}\right)^2 \times \left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^2$$

$$= 4.185\frac{\text{m}^2 \cdot \text{s} \cdot {}^\circ\text{F}}{\text{J}}$$

Rate of heat loss:

$$H_{\text{eff}} = \frac{A}{R_{\text{eff}}}\Delta T = \frac{(25\ \text{m}^2)}{\left(4.185\frac{\text{m}^2 \cdot \text{s} \cdot {}^\circ\text{F}}{\text{J}}\right)}(9{}^\circ\text{F}) = \boxed{54\ \text{W}}$$

REFLECT

We do not need to use the thicknesses of the materials since we were given the R-values. A temperature change in Fahrenheit is equal to (9/5) times the temperature change in Celsius:

$$T_{\text{C}_2} - T_{\text{C}_1} = \Delta T_{\text{C}} = \frac{5}{9}(T_{\text{F}_2} - 32) - \frac{5}{9}(T_{\text{F}_1} - 32) = \frac{5}{9}(T_{\text{F}_2} - T_{\text{F}_1}) = \frac{5}{9}\Delta T_{\text{F}_2}$$

$$\Delta T_{\text{F}_2} = \frac{9}{5}(\Delta T_{\text{C}})$$

14.130

SET UP

A 70-kg person can generate about 300 W of power on a treadmill that is inclined at 3%, which means the rise increases by 3 m for every 100 m traveled horizontally. If the person runs on the treadmill at a speed of 3 m/s for 45 min, he effectively runs a distance $d = 8100$ m. We can use trigonometry to determine the effective height increase of the person. The total energy required of the person to stay on the treadmill is equal to this increase in gravitational potential energy plus his kinetic energy. Dividing this energy by the total energy he generates gives the percent required to keep him on the treadmill; the total energy he generates is equal to the time interval (45 min) multiplied by his power (300 W). The remaining energy goes into heating up his body. If all of this heat were absorbed by a sample of water, we can calculate the mass of water that evaporates from the specific heat of water $\left(c = 4186\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)$ and the latent heat of vaporization of water $\left(L_V = 2260 \times 10^3\frac{\text{J}}{\text{kg}}\right)$. We assume that the water starts at 37 degrees C, which is body temperature.

SOLVE

Part a)

Distance person runs in 45 min:

$$d = \left(45\text{ min} \times \frac{60\text{ s}}{1\text{ min}}\right)\left(3\frac{\text{m}}{\text{s}}\right) = 8100\text{ m}$$

Angle of incline:

$$\theta = \arctan(0.03) = 0.03\text{ rad}$$

Effective height increase:

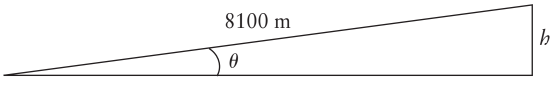


Figure 14-8 Problem 130

$$\sin(\theta) = \frac{h}{8100\text{ m}}$$

$$h = (8100\text{ m})\sin(\theta) = (8100\text{ m})\sin(0.03) = 243\text{ m}$$

Energy required of the person to stay on the treadmill:

$$E_{\text{treadmill}} = K + U_g = \frac{1}{2}mv^2 + mgh$$

$$= \frac{1}{2}(70\text{ kg})\left(3\frac{\text{m}}{\text{s}}\right)^2 + (70\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)(243\text{ m}) = 167{,}013\text{ J}$$

Total energy output of the person:

$$E_{\text{person}} = \left(45\text{ min} \times \frac{60\text{ s}}{1\text{ min}}\right)(300\text{ W}) = 810{,}000\text{ J}$$

Percent of energy used to keep the person on the treadmill:

$$\frac{E_{\text{treadmill}}}{E_{\text{person}}} = \frac{167{,}013\text{ J}}{810{,}000\text{ J}} = 0.206 = \boxed{20.6\%}$$

Remainder of energy ($Q = E_{\text{person}} - E_{\text{treadmill}} = (810{,}000\text{ J}) - (167{,}013\text{ J}) = 642{,}987\text{ J}$) goes into heating up the body: $\boxed{79.4\%}$.

Part b)

$$Q = mc\Delta T + mL_{\text{V}} = m(c\Delta T + L_{\text{V}})$$

$$m = \frac{Q}{c\Delta T + L_{\text{V}}} = \frac{642{,}987\text{ J}}{\left(4186\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(63\text{ K}) + \left(2260 \times 10^3\frac{\text{J}}{\text{kg}}\right)} = \boxed{0.2548\text{ kg}}$$

REFLECT

If we assume the water is already at 100 degrees C, then 0.2845 kg of water evaporates. If we assume the water is at room temperature, then 0.2478 kg of water evaporates.

14.131

SET UP

A man has a surface area of $A = 2.1\text{ m}^2$ and a skin temperature of 303 K. Normally 80% of the food calories (that is, kcal) he eats are converted into heat. The man is in a 293-K room and wants to know how many food calories x he needs to eat in a 24-hr period in order to maintain his current body temperature. The net radiated energy is equal to the energy he radiates minus the energy the air radiates back into him. The power radiated is given by $P = \sigma\varepsilon AT^4$, where we assume $\varepsilon = 1$. The conversion between kcal and joules is 1 kcal = 4186 J.

SOLVE

$$E_{\text{food}} = E_{\text{radiated}} = P_{\text{radiated}}t$$

$$(0.80x)\left(\frac{4186\text{ J}}{1\text{ kcal}}\right) = (\sigma\epsilon AT_{\text{body}}^4 - \sigma\epsilon AT_{\text{air}}^4)t = (T_{\text{body}}^4 - T_{\text{air}}^4)\sigma\epsilon At$$

$$x = \frac{(T_{\text{body}}^4 - T_{\text{air}}^4)\sigma\epsilon At}{(0.80)\left(\frac{4186\text{ J}}{1\text{ kcal}}\right)}$$

$$= \frac{((303\text{ K})^4 - (293\text{ K})^4)\left(5.6703 \times 10^{-8}\frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\right)(1)(2.1\text{ m}^2)\left(24\text{ hr} \times \frac{3600\text{ s}}{1\text{ hr}}\right)}{(0.80)\left(\frac{4186\text{ J}}{1\text{ kcal}}\right)}$$

$$= \boxed{3250\text{ kcal}}$$

A value of 3250 kcal seems $\boxed{\text{reasonable}}$ for a man who is 1.88m (6 ft. 2 in) tall with a mass of 80 kg (176 lb).

REFLECT

The recommended caloric intake for an adult male is around 2550 kcal. It would make sense that additional, more complicated processes are occurring in the body than our simple model takes into account.

14.132

SET UP

The temperature of the Sun is 5800 K and the radius of the Sun is 6.96×10^8 m. The rate at which energy radiates from the sun is equal to $P = \sigma \varepsilon A T^4$, where $\sigma = 5.6703 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \cdot \text{K}^4}$. We will assume that the Sun is a perfect emitter of radiation (that is, $\varepsilon = 1$) and that it is spherical.

SOLVE

$$P = \sigma \varepsilon A T^4 = \left(5.6703 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \cdot \text{K}^4}\right)(1)(4\pi(6.96 \times 10^8 \text{m})^2)(5800\text{ K})^4 = \boxed{3.91 \times 10^{26}\text{ W}}$$

REFLECT

We would expect the Sun to radiate a huge amount of power.

14.133

SET UP

We will investigate the observation that small mammals seem to be constantly eating, while large mammals eat much less frequently. This means that smaller mammals constantly need to replenish their energy supplies with more food. The energy stored by a mammal as, say, fat, is proportional to its mass (and thus its volume), while the power radiated by the mammal is proportional to the surface area. The fraction of the animal's stored energy radiated away per second is equal to the power divided by the total stored energy. For simplicity, we'll treat the animals as spheres.

SOLVE

Part a) The stored energy is proportional to the animal's mass, which is proportional to the animal's volume since the density is approximately constant. The volume is proportional to the cube of the radius $\left(V_{\text{sphere}} = \frac{4}{3}\pi R^3\right)$. The rate at which the animal radiates energy is proportional to its surface area $(P = \sigma \varepsilon A T^4)$, which is proportional to the square of its radius $(A_{\text{sphere}} = 4\pi R^2)$.

Part b) The fraction of the animal's stored energy that it radiates away per second:

$$\frac{\left(\frac{\Delta E}{\Delta t}\right)}{E} = \frac{P}{E} \propto \frac{R^2}{R^3} \propto \boxed{\frac{1}{R}}$$

Part c) For a small animal (small R), the fraction of its energy radiated per second is larger than it is for a large animal. To replace this energy in order to maintain a constant body temperature, the small animal must eat more frequently than the large animal.

REFLECT

Using proportional reasoning is an invaluable way of gleaning some insight and gaining some intuition regarding complex and seemingly intractable problems.

14.134

SET UP

An asteroid of radius $R_a = 5 \times 10^3$ m and density $\rho_a = 2.0\ \mathrm{g/cm^3}$ crashed into the Earth at a speed of $v = 11 \times 10^3$ m/s. We will assume the ocean water has the same density as freshwater and an initial temperature of 20 degrees C. In order to calculate the maximum amount of ocean water evaporated by the impact, we will assume that all of the kinetic energy of the asteroid was converted into heat upon impact and absorbed by the water. The total heat required to evaporate the water that was initially at 20 degrees C is equal to the sum of the heat necessary to increase the temperature of the water to the boiling point ($T_V = 100$ degrees C) and the heat necessary to convert the entire liquid into gas. The heat transfer necessary to increase the temperature of the water is given by $Q = m_w c\Delta T$, and the heat necessary to convert from liquid to gas is related to the latent heat of vaporization, $Q = mL_V$. The specific heat of water is $c = 4186\frac{\mathrm{J}}{\mathrm{kg\cdot K}}$, and the latent heat of fusion for water is $L_F = 2260 \times 10^3\frac{\mathrm{J}}{\mathrm{kg}}$. We can use the density of water to convert the mass to a volume; taking the cube root of this volume will give the height of the cube.

SOLVE

$$K = Q$$

$$\frac{1}{2}m_a v^2 = m_w c\Delta T_{20^\circ\mathrm{C}\to 100^\circ\mathrm{C}} + m_w L_V = m_w(c\Delta T_{20^\circ\mathrm{C}\to 100^\circ\mathrm{C}} + L_V)$$

$$m_w = \frac{m_a v^2}{2(c\Delta T_{20^\circ\mathrm{C}\to 100^\circ\mathrm{C}} + L_V)} = \frac{(\rho_a V_a)v^2}{2(c\Delta T_{20^\circ\mathrm{C}\to 100^\circ\mathrm{C}} + L_V)}$$

$$= \frac{\rho_a\left(\frac{4}{3}\pi R_a^3\right)v^2}{2(c\Delta T_{20^\circ\mathrm{C}\to 100^\circ\mathrm{C}} + L_V)} = \frac{2\rho_a \pi R_a^3 v^2}{3(c\Delta T_{20^\circ\mathrm{C}\to 100^\circ\mathrm{C}} + L_V)}$$

$$= \frac{2\left(2.0\frac{\mathrm{g}}{\mathrm{cm^3}} \times \frac{1\ \mathrm{kg}}{1000\ \mathrm{g}} \times \left(\frac{100\ \mathrm{cm}}{1\ \mathrm{m}}\right)^3\right)\pi(5 \times 10^3\ \mathrm{m})^3\left(11 \times 10^3\frac{\mathrm{m}}{\mathrm{s}}\right)^2}{3\left(\left(4186\frac{\mathrm{J}}{\mathrm{kg\cdot K}}\right)(80\ \mathrm{K}) + \left(2260 \times 10^3\frac{\mathrm{J}}{\mathrm{kg}}\right)\right)} = \boxed{2.4 \times 10^{16}\ \mathrm{kg}}$$

Part b)

$$V = \frac{m}{\rho} = L^3$$

$$L = \sqrt[3]{\frac{m}{\rho}} = \sqrt[3]{\frac{2.4 \times 10^{16}\ \text{kg}}{\left(1000\dfrac{\text{kg}}{\text{m}^3}\right)}} = \boxed{2.9 \times 10^4\ \text{m}}$$

REFLECT

This is equivalent to a cube that is 18 mi tall!

14.135

SET UP

We can model chemical reactions as occurring when molecules collide, so it makes sense that the reaction rate r should depend on the mean free time τ. We would expect the faster the collisions occur, the faster the reaction rate, so r should be inversely proportional to τ. An average molecule will travel a distance equal to the mean free path λ at the root-mean-square speed v_{rms} before colliding with another molecule. Using the definition of speed, the mean free time will be equal to the mean free path divided by the root-mean-square speed. We can use the expressions for $v_{\text{rms}} = \sqrt{\dfrac{3kT}{m}}$ and $\lambda = \dfrac{1}{4\sqrt{2}\pi r^2\left(\dfrac{N}{V}\right)}$, along with the ideal gas law, to determine the ratio between the reaction rate on Mars (pressure $P_{\text{Mars}} = 650\ \text{N/m}^2$, temperature $T_{\text{Mars}} = 273\ \text{K}$) and the reaction rate r on Earth (pressure $P_{\text{Earth}} = 101{,}300\ \text{N/m}^2$, temperature $T_{\text{Earth}} = 278\ \text{K}$).

SOLVE

Part a) Two molecules need to collide in order to react, so the faster the collisions occur, the faster the reaction rate. We would expect the reaction rate r to be $\boxed{\text{inversely proportional}}$ to the mean free time τ because a small τ corresponds to a large r.

Part b)

$$v_{\text{rms}} = \frac{\lambda}{\tau}$$

$$\boxed{\tau = \frac{\lambda}{v_{\text{rms}}}}$$

Part c)

In general

$$r \propto \frac{1}{\tau} = \frac{v_{\text{rms}}}{\lambda} = \frac{\left(\sqrt{\dfrac{3kT}{m}}\right)}{\left(\dfrac{1}{4\sqrt{2}\pi r^2\left(\dfrac{N}{V}\right)}\right)} = \frac{4\sqrt{2}\pi r^2\left(\dfrac{N}{V}\right)\sqrt{3kT}}{\sqrt{m}} = \frac{4\sqrt{2}\pi r^2\left(\dfrac{P}{kT}\right)\sqrt{3kT}}{\sqrt{m}} = \frac{4\sqrt{6}\pi r^2 P}{\sqrt{mkT}}$$

Ratio between Mars and Earth:

$$\frac{r_{\text{Mars}}}{r_{\text{Earth}}} = \frac{\left(\dfrac{4\sqrt{6}\pi r^2 P_{\text{Mars}}}{\sqrt{mkT_{\text{Mars}}}}\right)}{\left(\dfrac{4\sqrt{6}\pi r^2 P_{\text{Earth}}}{\sqrt{mkT_{\text{Earth}}}}\right)} = \left(\frac{P_{\text{Mars}}}{P_{\text{Earth}}}\right)\left(\frac{\sqrt{T_{\text{Earth}}}}{\sqrt{T_{\text{Mars}}}}\right) = \left(\frac{650\dfrac{\text{N}}{\text{m}^2}}{101{,}300\dfrac{\text{N}}{\text{m}^2}}\right)\left(\frac{\sqrt{278\text{ K}}}{\sqrt{273\text{ K}}}\right) = 6.5 \times 10^{-3}$$

$$\boxed{r_{\text{Mars}} = (6.5 \times 10^{-3})r_{\text{Earth}} = (6.5 \times 10^{-3})r}$$

REFLECT

The temperature and pressure are lower on Mars than on Earth, so it makes sense that the collision rate and, thus, the reaction rate should also be lower.

14.136

SET UP

An unsealed spherical container with a radius of 2 m is made of steel and filled completely with water. The container sits in the Sun all day and its temperature increases by 23 K. Due to the increase in temperature, the volumes of both the steel and the water will increase. We can explicitly calculate these increases in volume in order to describe what happens to the system after the temperature increase. The volume increase of the steel is approximately $\Delta V_{\text{steel}} = 3\alpha_{\text{steel}}V_0\Delta T$, where the coefficient of linear expansion for steel is $\alpha_{\text{steel}} = 13.0 \times 10^{-6}\text{ K}^{-1}$. The volume increase of the water is $\Delta V_{\text{water}} = \beta_{\text{water}}V_0\Delta T$, where the coefficient of volume expansion for water is $\beta_{\text{water}} = 207 \times 10^{-6}\text{ K}^{-1}$.

SOLVE

Volume expansion of steel:

$$\Delta V_{\text{steel}} = 3\alpha_{\text{steel}}V_0\Delta T = 3(13.0 \times 10^{-6}\text{ K}^{-1})\left(\frac{4}{3}\pi(2\text{ m})^3\right)(23\text{ K}) = 0.03\text{ m}^3$$

Volume expansion of water:

$$\Delta V_{\text{water}} = \beta_{\text{water}}V_0\Delta T = (207 \times 10^{-6}\text{ K}^{-1})\left(\frac{4}{3}\pi(2\text{ m})^3\right)(23\text{ K}) = 0.16\text{ m}^3$$

Since the water expands more than the steel, some of the $\boxed{\text{water will spill out}}$.

REFLECT

If the container were sealed, the water would be unable to spill out. The force of the water on the walls of the vessel would increase. Eventually the vessel would rupture when this force becomes large enough.

14.137

SET UP

A method for detecting planets (radius r) orbiting a larger star (radius R) is to monitor the brightness of the star. If a planet passes in front of a star, it will block some of the emitted light and the star will appear less bright. We assume the planet does not radiate visible light. To determine the factor by which the intensity of the light is reduced when a planet is in front

of the star, we can take a ratio between the change in intensity $\Delta I = I_0 - I_p$, where I_0 is the initial intensity and I_p is the intensity when the planet is in front of the star, and the initial intensity. The intensity for any observer is given by the power emitted divided by the area, which is related to the distance between the emitter and the observer; although these areas are the same, the effective emitted powers are not. The power emitted by the star is $P_0 = \sigma\varepsilon A_0 T^4$, where $A_0 = \pi R^2$ since the amount of light reaching an observer is equivalent to the power radiated by a disk of radius R. (Thinking about this another way, the observer can't see the light emitted from the backside of the star and effectively only sees light emitted from the cross section of the front, which is a circle.) The effective power emitted when the planet is in the way is $P_p = \sigma\varepsilon A_p T^4$, where $A_p = \pi R^2 - \pi r^2$ since the planet blocks the middle of the star. By simplifying the ratio, we find that $\dfrac{\Delta I}{I_0} = \left(\dfrac{r}{R}\right)^2$. Using this expression and the radii of Jupiter ($r_{\text{Jupiter}} = 6.91 \times 10^7$ m), the Earth($r_{\text{Earth}} = 6.38 \times 10^6$ m), and the Sun ($R_{\text{Sun}} = 6.96 \times 10^8$ m), we can look at the intensity reductions caused by the two planets.

SOLVE

Part a)

$$\frac{\Delta I}{I_0} = \frac{I_0 - I_p}{I_0} = 1 - \frac{I_p}{I_0} = 1 - \frac{\left(\dfrac{P_p}{A}\right)}{\left(\dfrac{P_p}{A}\right)} = 1 - \frac{P_p}{P_0} = 1 - \frac{\sigma\varepsilon A_p T^4}{\sigma\varepsilon A_0 T^4} = 1 - \frac{\pi R^2 - \pi r^2}{\pi R^2}$$

$$= 1 - \frac{R^2 - r^2}{R^2} = 1 - 1 + \frac{r^2}{R^2} = \boxed{\left(\frac{r}{R}\right)^2}$$

Part b)

Jupiter:

$$\left(\frac{\Delta I}{I_0}\right)_{\text{Jupiter}} = \left(\frac{r_{\text{Jupiter}}}{R_{\text{Sun}}}\right)^2 = \left(\frac{6.91 \times 10^7 \text{ m}}{6.96 \times 10^8 \text{ m}}\right)^2 = 0.0099 \approx \boxed{1\%}$$

Earth:

$$\left(\frac{\Delta I}{I_0}\right)_{\text{Earth}} = \left(\frac{r_{\text{Earth}}}{R_{\text{Sun}}}\right)^2 = \left(\frac{6.38 \times 10^6 \text{ m}}{6.96 \times 10^8 \text{ m}}\right)^2 = 8.4 \times 10^{-5} \approx \boxed{0.0084\%}$$

Part c) Jupiter should be easy to find by measuring the reduction in intensity. Earth is much harder to pick out but still possible if they are determined to keep looking.

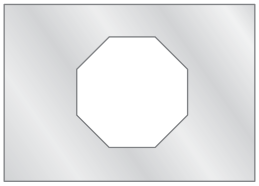
REFLECT

Be careful not to confuse the different areas. For the intensity of a point source that creates spherical waves, we're interested in the surface area of the sphere with a radius equal to the distance between the source and the observer. The area we need for the radiated power is equal to the surface area (or effective area, in this case) of the emitter itself.

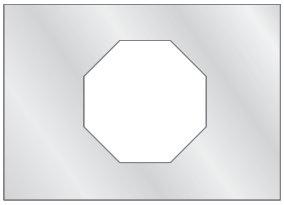
14.138

SET UP

An octagon with sides $s_0 = 0.10$ m is cut out of a sheet of aluminum (linear expansion coefficient $\alpha = 22.2 \times 10^{-6}\ \text{K}^{-1}$). The initial area of the octagon is given by $A_0 = 2\cot\left(\frac{\pi}{8}\right)s_0^2$. The temperature of the aluminum is increased by 20 K, which means it will expand in each direction. We can use our result from Problem 14.118, $\Delta A \approx 2\alpha A_0 \Delta T$, in order to calculate the change in the area. The final area is equal to this change plus A_0.



Originally, the octagon has sides of length $s_0 = 10$ cm.



After a temperature increase of $\Delta T = 20$ °C, the new length of the sides is s_f.

Figure 14-9 Problem 138

SOLVE

Initial area:

$$A_0 = 2\cot\left(\frac{\pi}{8}\right)s_0^2 = 2\cot\left(\frac{\pi}{8}\right)(0.10\ \text{m})^2 = 0.04828\ \text{m}^2$$

Final area:

$$A_f = A_0 + \Delta A = A_0 + 2\alpha A_0 \Delta T = A_0(1 + 2\alpha\Delta T)$$

$$= (0.04828\ \text{m}^2)(1 + 2(22.2 \times 10^{-6}\ \text{K}^{-1})(20\ \text{K})) = \boxed{0.04832\ \text{m}^2}$$

REFLECT

This change corresponds to a 0.08% increase in the area, which is very small. The final size of the sides is $s = 0.10004$ m; they increase only by about 40 microns.

Chapter 15
Thermodynamics II

Conceptual Questions

15.1 Part a) Isothermal processes are those in which the temperature remains constant throughout. An example of an isothermal process is when an acid is slowly poured into a base and allowed to equilibrate with a water bath. The work done by isothermal processes (for an ideal gas) is given by $nRT\ln\left(\frac{V_f}{V_i}\right)$.

Part b) Adiabatic processes are those in which there is no heat lost or gained. An example of an adiabatic process is a bicycle pump that is used to increase the pressure and temperature of the air that is inside as it goes into the tire. The work done by an adiabatic process is simply the change in internal energy (because $Q = 0$).

Part c) Isobaric processes are those in which the pressure remains constant. An example of an isobaric process is a gas in a cylinder that is sealed with a movable piston. As the gas is heated, the volume will expand to balance out the increase in temperature but the pressure remains constant.

Part d) Isochoric processes are those in which the volume does not change. An example of an isochoric process is a gas inside a cylinder that cannot expand or contract. If you change the temperature of the gas, it will respond by changing the pressure. No work is done by isochoric processes.

15.2 Yes, a system can absorb heat without increasing its internal energy, for example, if the heat absorbed by the system is equal to the work done by the system.

15.3 The work done on the gas to compress it goes into increasing the temperature.

15.4 The temperature will remain constant when there is an equal amount of work done on or by the system.

15.5 Kinetic friction saps bulk kinetic energy and converts it into heat. Unless the engine was attempting to generate heat, the heat from friction is energy that won't be going to whatever it was the engine was supposed to accomplish.

15.6 Energy is transferred to the refrigerator's working substance both as heat from the things inside the box and as work done on it by the compressor. The refrigerator then transfers all of this energy as heat to the room air. With the refrigerator door kept open, the compressor has to work even harder. This extra energy is transferred by the refrigerator to the room air as heat, causing the room to grow even hotter.

15.7 A coefficient of performance only gauges how efficiently existing energy is shifted from one place to another; it does not indicate violation of conservation of energy.

15.8 More water takes longer to freeze. If we use a higher-power refrigerator, then it will take less time to freeze the water. If the ambient air is warmer, then it will take longer to freeze the water.

15.9 It increases by the amount of the heat divided by the temperature for the cool object minus the amount of heat divided by the temperature for the hot object: $\Delta S = \frac{Q}{T_c} - \frac{Q}{T_h}$. When there was a temperature difference, that difference could be harnessed to do work. Once the two objects have equilibrated, that method of extracting work is no longer possible because the difference no longer exists.

15.10 Yes, this is consistent with the first law of thermodynamics because there is no change in internal energy for an isothermal process. Therefore, the work done by the gas must equal the heat put into the system.

15.11 Before the gas was released, one knew that the gas was entirely in the smaller volume. This is a state of greater order. After the release, there is no such claim that can be made to simplify the representation of the exact state of the gas. This corresponds to an increase in entropy.

15.12 High efficiency is almost always a primary objective for a steam-electric generating plant. In such a plant, the steam is the working substance for the heat engine that drives the electric generator. The temperature of the steam is the temperature of the high-temperature heat source. The temperature of the low-temperature reservoir is usually fixed by circumstances, such as the temperature of a nearby lake. Thus, increasing the feed-steam temperature is the only way to increase the Carnot efficiency limit for the generator.

15.13 The irreversible processes are a bit like one-way streets—you can go around the block but not take a U-turn. More concretely, general heat loss is undone not by letting the heat back into the hot object but by finding something even hotter or by doing work on the object to heat it; the halting of a ball that rolled to a stop is not undone by letting the grass kick it back into motion but by going and fetching the ball. Note that the reversible adiabatic cooling does not let heat escape to the environment but rather stores the thermal energy as mechanical energy in another part of the system.

15.14 During an irreversible process, energy equal to $T\Delta S_{\text{total}}$, where ΔS_{total} is the change in the entropy of the universe, becomes unavailable to do work.

15.15 No. The term *surroundings* is a boundary we draw at our convenience. If we choose the system as Earth, and imagine enclosing it in a perfectly reflecting balloon to prevent heat exchanges with its surroundings, would we suddenly be unable to break teacups?

15.16 No. Although they are pretty close, there is always some irreversible thermodynamic change that accompanies the running of real-world engines: A bit of the metal wears away, there is an increase in the friction between the piston and the cylinder, and so on. We spend lots of effort trying to make sure these losses are minimized, especially with lubrication, but we will never create the "perfect" engine that is not degraded by continued use.

15.17 As the piston recedes, it will sap energy from the gas molecules striking it. This lowers the gas temperature, requiring it to be reheated by the hot reservoir—but that is then not isothermal. The slower the piston recedes, the less energy it saps, but if the engine is to actually run at all it does have to move. By not acting isothermally, there is heat, which means Q/T will be nonzero, and thus there is an entropy increase.

15.18 The entropy of the water decreases as its temperature is lowered and heat is removed.

15.19 There are relatively few configurations of the cup's components that are a cup, compared to the number of configurations that are not. This ratio is so large that it is difficult to express even with scientific notation. Even if the dropping-on-floor process is changing the state of the glass in random ways instead of preferentially destructive ones, the random drawing will be from the full set of states; cup states are a vanishingly small portion of the set.

15.20 The entropy of 1 kg of liquid iron is larger than that of 1 kg of solid iron because the atoms in liquid iron are less ordered than those in solid iron. Also, heat had to be added to solid iron in order to melt it.

Multiple-Choice Questions

15.21 **B** (decreases). $Q = 0$ because it is thermally isolated. Work is being done by the gas on the piston, which means the internal energy and, thus, the temperature of the gas decreases.

15.22 **B** (+40 J). $Q = 0$ because the process is adiabatic. Work is being done on the gas, so the internal energy will increase by an amount $(800\text{ N})(5.0 \times 10^{-2}\text{ m}) = +40\text{ J}$.

15.23 **B** (0 J). The walls of the container don't move.

15.24 **B** (temperature). An isobaric process is one in which the pressure remains constant throughout.

15.25 **A** (pressure). An isothermal process is one in which the temperature remains constant throughout.

15.26 **C** (volume). An isochoric process is one in which the volume remains constant throughout.

15.27 **D** (adiabatic). An adiabatic process is one in which no heat is transferred to or from the system.

15.28 **D** (the second law of thermodynamics). This statement is referred to as the Clausius form of the second law of thermodynamics.

15.29 **A** (two adiabatic processes and two isothermal processes). The processes are isothermal expansion, adiabatic expansion, isothermal compression, and adiabatic compression.

15.30 **A** (Lower T_C by 10 K). The efficiency is given by $e = 1 - \frac{T_C}{T_H}$.

Estimation Questions

15.31 Part a) If the book is 1 kg and you raise it 1 m, then the work you have performed is about $mgy = 10$ J.

Part b) The temperature of a glass containing 100 mL of water that absorbs 10 J of heat would increase by 0.02 degree C.

15.32 Part a) Say you need to fill up your gas tank 10 times throughout the trip. This is approximately 150 gallons of gasoline. Each gallon of gasoline provides 1.3×10^8 J of energy, so the total work required to drive across America would be about 2×10^{10} J.

Part b) A 25 m $\times$ 50 m $\times$ 3 m swimming pool will contain about 3.75×10^6 kg of water. The temperature will increase by 1.3 degrees C.

15.33 If we treat the gas as ideal, then $U = \frac{3}{2}nRT$. One liter of oxygen is about 0.05 mol, since 22.4 L = 1 mol of gas. At $T = 20$ degrees C (or 293 K), $U_0 = \frac{3}{2}(0.05 \text{ mol})\left(8.31\frac{\text{J}}{\text{mol} \cdot \text{K}}\right)(293 \text{ K}) \approx 183$ J and at $T = 100$ degrees C (or 373 K), $U_f = \frac{3}{2}(0.05 \text{ mol})\left(8.31\frac{\text{J}}{\text{mol} \cdot \text{K}}\right)(373 \text{ K}) \approx 232$ J. Therefore, the change in internal energy is 40 K.

15.34

$$PV = nRT$$

$$P = \frac{nRT}{V} = \frac{\left(2 \text{ L} \times \frac{1 \text{ mol}}{22.4 \text{ L}}\right)\left(0.0821\frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}}\right)(300 \text{ K})}{2 \text{ L}} \approx 1.1 \text{ atm}$$

15.35 A 70-kg student using 2000 kcal of energy can climb up to a height of 12 km, which is not reasonable. Chances are both the metabolic efficiency and the efficiency of her climbing itself are much lower than 50% and 100%, respectively.

15.36 Most internal combustion engines operate with around 30–35% efficiency.

15.37 Assuming a 20-gallon tank and that 1 gallon of gasoline releases 125,000 BTU, the gasoline can release 2.64×10^9 J of total energy. Most internal combustion engines are around 30% efficient, so 7.9×10^8 J would be available to move the car. The force of the tires on the road is equal to the force of static friction, $F = \mu_s mg$. The coefficient of static friction is around 0.4 on wet pavement and 0.7 on dry pavement, so we'll choose $\mu_s = 0.5$ as a nice, round number. The work done by this force on a 1000-kg car is $W = Fd = \mu_s mgd$. Solving for the distance, $d = \dfrac{7.9 \times 10^8 \text{ J}}{(0.5)(1000 \text{ kg})\left(9.8\dfrac{\text{m}}{\text{s}^2}\right)} = 160$ km, or 200 km to one significant figure.

15.38 There are about 125 million cars on the road. Assume that each of them drives 25 miles per day. This gives about 31,000,000 miles driven each day. If 25% of the cars are not in good repair, that means about 8 million miles are driven with this extra 5% inefficiency each day. Using our solution from Problem 15.37 (1 tank of gas = 320 km = 200 mi), it would take an extra 3.3×10^{13} J to drive these 8 million miles. That's about 250,000 extra gallons of gas!

15.39 Using the definition of efficiency,

$$e = \frac{T_{\text{body}} - T_{\text{environment}}}{T_{\text{body}}} = \frac{(310 \text{ K}) - (293 \text{ K})}{310 \text{ K}} = 0.055 = 5.5\%$$

This is far lower than the real human body efficiency; the body is not well modeled as a heat engine.

15.40 Part a) A coefficient of performance of around 2.0–2.5 is a typical value.

Part b) Since nearly 10% of the energy demands of the United States are associated with residential and commercial refrigeration needs, this is a very important value to consider. One way of thinking about coefficients of performance of 2.0 is that 2 units of heat are removed from the system for every 1 unit of energy consumed. Refrigerators with larger coefficients of performance would be more cost-effective to run.

15.41 Number of microstates:

$$W_n = \frac{(n_{\text{H}} + n_{\text{T}})!}{n_{\text{H}}!n_{\text{T}}!}$$

Probability of any given macrostate:

$$P_n = \frac{W_n}{\sum_n W_n}$$

Part a)

Macrostate	Microstates	Number of Microstates	Probability	Entropy
(5H,0T) n = 5	(HHHHH)	1	1/32	$k_B\ln(1)=0$
(4H,1T) n = 4	(HHHHT) (HHHTH) (HHTHH) (HTHHH) (THHHH)	5	5/32	$k_B\ln(5) = 1.61\ k_B$
(3H,2T) n = 3	(HHHTT) (HHTHT) (HTHHT) (THHHT) (HHTTH) (HTHTH) (THHTH) (HTTHH) (THTHH) (TTHHH)	10	10/32	$k_B\ln(10) = 2.30\ k_B$
(2H,3T) n = 2	(TTTHH) (TTHTH) (THTTH) (HTTTH) (TTHHT) (THTHT) (HTTHT) (THHTT) (HTHTT) (HHTTT)	10	10/32	$k_B\ln(10)=2.30\ k_B$
(1H,4T) n = 1	(TTTTH) (TTTHT) (TTHTT) (THTTT) (HTTTT)	5	5/32	$k_B\ln(5) = 1.61\ k_B$
(0H,5T) n = 0	(TTTTT)	1	1/32	$k_B\ln(l) = 0$

Part b)

Macrostate	#Microstates	Probability	Entropy
(20H,0T)	1	1/616,666 = 0.0000016%	$k_B\ln(l) = 0$
(19H,1T)	20	20/616,666 = 0.0000324%	$k_B\ln(20) = 3\ k_B$
(18H,2T)	190	190/616,666 = 0.000308%	$k_B\ln(190) = 5.25\ k_B$
(17H,3T)	1140	1140/616,666 = 0.00185%	$k_B\ln(1140) = 7.04\ k_B$
(16H,4T)	4845	4845/616,666 = 0.00786%	$k_B\ln(4845) = 8.49\ k_B$
(15H,5T)	15,504	15,504/616,666 = 0.0251%	$k_B\ln(15{,}504) = 9.65\ k_B$
(14H,6T)	38,760	38,760/616,666 = 0.0629%	$k_B\ln(38{,}760) = 10.6\ k_B$
(13H,7T)	77,520	77,520/616,666 = 0.126%	$k_B\ln(77{,}520) = 11.3\ k_B$
(12H,8T)	125,970	125,970/616,666 = 0.204%	$k_B\ln(125{,}970) = 11.7\ k_B$
(11H,9T)	167,960	167,960/616,666 = 0.272%	$k_B\ln(167{,}960) = 12.0\ k_B$
(10H,10T)	184,756	184,756/616,666 = 0.3%	$k_B\ln(184{,}756) = 12.1\ k_B$
(9H,11T)	167,960	167,960/616,666 = 0.272%	$k_B\ln(167{,}960) = 12\ k_B$
(8H,12T)	125,970	125,970/616,666 = 0.204%	$k_B\ln(125{,}970) = 11.7\ k_B$
(7H, 13T)	77,520	77,520/616,666 = 0.126%	$k_B\ln(77{,}520) = 11.3\ k_B$
(6H,14T)	38,760	38,760/616,666 = 0.0629%	$k_B\ln(38{,}760) = 10.6\ k_B$
(5H,15T)	15,504	15,504/616,666 = 0.0251%	$k_B\ln(15{,}504) = 9.65\ k_B$
(4H,16T)	4845	4845/616,666 = 0.00786%	$k_B\ln(4845) = 8.49\ k_B$
(3H,17T)	1140	1140/616,666 = 0.00185%	$k_B\ln(1140) = 7.04\ k_B$
(2H,18T)	190	190/616,666 = 0.000308%	$k_B\ln(190) = 5.25\ k_B$
(1H,19T)	20	20/616,666 = 0.0000324%	$k_B\ln(20) = 3\ k_B$
(0H,20T)	1	1/616,666 = 0.0000016%	$k_B\ln(l) = 0$

Part c) The distribution is peaked very sharply compared to its width, though in absolute terms (for the number of heads) far more broadly than the previous two distributions, just from the sheer numbers we're working with.

Problems

15.42

SET UP

A PV curve for a gas is given. The work done *by* the gas is equal to the area under the curve. The work done *on* the gas will be equal in magnitude but opposite in sign.

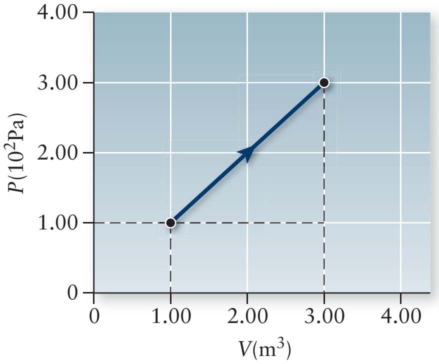


Figure 15-1 Problem 42

SOLVE

$$W_{\text{by gas}} = \frac{1}{2}((1.00 \times 10^2 \text{ Pa}) + (3.00 \times 10^2 \text{ Pa}))(2.00 \text{ m}^3) = 4.00 \text{ J}$$

$$W_{\text{on gas}} = -W_{\text{by gas}} = \boxed{-400 \text{ J}}$$

REFLECT

The prepositions *by* and *on* are easy to gloss over but are very important in thermodynamics problems.

15.43

SET UP

A PV curve for a gas is given. The work done *by* the gas is equal to the area under the curve. The work done *on* the gas will be equal in magnitude but opposite in sign.

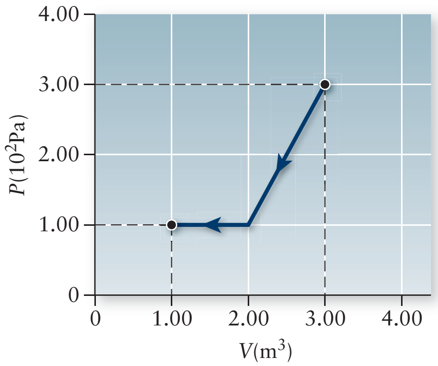


Figure 15-2 Problem 43

SOLVE

$$W_{\text{by gas}} = \frac{1}{2}((1.00 \times 10^2\ \text{Pa}) + (3.00 \times 10^2\ \text{Pa}))(-1.00\ \text{m}^3) + (1.00 \times 10^2\ \text{Pa})(-1.00\ \text{m}^3)$$
$$= -300\ \text{J}$$

$$W_{\text{on gas}} = -W_{\text{by gas}} = \boxed{+300\ \text{J}}$$

REFLECT

The work done by the gas is negative because its final volume is less than its initial volume.

15.44

SET UP

A gas is heated and allowed to expand such that it follows a horizontal line path on a PV diagram. The initial pressure and volume are 1.0×10^5 Pa and $1.0\ \text{m}^3$, respectively. The final pressure and volume are 1.0×10^5 Pa and $2.0\ \text{m}^3$, respectively. The work done by the gas on its surroundings is equal to the area under the PV curve.

SOLVE

$$W = (1.0 \times 10^5\ \text{Pa})(1.0\ \text{m}^3) = \boxed{1.0 \times 10^5\ \text{J}}$$

REFLECT

The work done by the gas should be positive because the gas expands. Since the pressure remains constant throughout the process, this is an isobaric process.

15.45

SET UP

A gas is heated such that it follows a vertical line path on a PV diagram. The initial pressure and volume are 1.0×10^5 Pa and $3.0\ \text{m}^3$, respectively. The final pressure and volume are 2.0×10^5 Pa and $3.0\ \text{m}^3$, respectively. The work done by the gas on its surroundings is equal to zero because the gas neither expands nor contracts (that is, its volume remains constant).

SOLVE

$$W = 0 \text{ because } \Delta V = 0$$

REFLECT

Another way of thinking about this is that the area under a horizontal line is zero. Since the volume remains constant throughout the process, this is an isochoric process.

15.46

SET UP

A system does not do any external work but absorbs 800 J of heat. The change in the internal energy of the gas is given by $\Delta U = Q - W$. Heat added to the system is considered positive.

SOLVE

$$\Delta U = Q - W = (+800\text{ J}) - (0) = \boxed{+800\text{ J}}$$

REFLECT

It makes sense that the internal energy of the system should increase if we add heat to it.

15.47

SET UP

A system absorbs 500 J of heat and does 200 J of work on its surroundings. The change in the internal energy of the gas is given by $\Delta U = Q - W$. Heat added to the system is considered positive, and work done by the system is considered positive, as well.

SOLVE

$$\Delta U = Q - W = (+500\text{ J}) - (+200\text{ J}) = \boxed{+300\text{ J}}$$

REFLECT

The system absorbs more energy (as heat) than it expends (as work), so the overall change in internal energy should be positive.

15.48

SET UP

A sealed container has a piston and initially contains $V_i = 8000\text{ cm}^3$ of an ideal gas at a pressure $P_i = 8.0$ atm. The gas is heated slowly and the gas is allowed to expand isothermally to a final volume of $V_f = 16{,}000\text{ cm}^3$. The work done by an ideal gas in an isothermal process is equal to $W = nRT\ln\left(\frac{V_f}{V_i}\right)$. We do not know the exact values of n and T but we do know that they remain constant throughout the whole process: n is constant because the container is sealed; T is constant because the process is isothermal. Therefore, the product nRT is constant. From the ideal gas law, the products P_iV_i and P_fV_f must also be constant. We can replace nRT with P_iV_i and solve for the work done by the gas on the piston.

SOLVE

$$W = nRT\ln\left(\frac{V_f}{V_i}\right) = P_iV_i\ln\left(\frac{V_f}{V_i}\right)$$

$$= \left(8.0\text{ atm} \times \frac{101{,}300\text{ Pa}}{1\text{ atm}}\right)\left(8000\text{ cm}^3 \times \left(\frac{1\text{ m}}{100\text{ cm}}\right)^3\right)\ln\left(\frac{16{,}000\text{ cm}^3}{8000\text{ cm}^3}\right) = \boxed{4500\text{ J} = 4.5\text{ kJ}}$$

REFLECT

Be careful when choosing equations to use in thermodynamics questions as they all come with a series of caveats. For example, the formula we used in this problem, $W = nRT\ln\left(\frac{V_f}{V_i}\right)$, only works for the work done by an ideal gas in an isothermal process.

15.49

SET UP

An expanding ideal gas does 8.8 kJ of work. The process occurs isothermally, which means the change in the internal energy of the gas is zero. From the first law of thermodynamics, this means the heat absorbed by the gas is equal to the work done by the gas.

SOLVE

$$\Delta U = 0 = Q - W$$

$$Q = W = \boxed{+8.8\text{ kJ}}$$

REFLECT

The work done by a gas on its surroundings is considered to be positive, as is the heat absorbed by a gas.

15.50

SET UP

An expandable container initially contains $V_{\text{i}} = 8.0\text{ m}^3$ of helium gas. The gas undergoes an isothermal expansion to a final volume of $V_{\text{f}} = 10.0\text{ m}^3$. The work done by an ideal gas in an isothermal process is equal to $W = nRT\ln\left(\frac{V_{\text{f}}}{V_{\text{i}}}\right)$. Assuming the number of moles of gas remains constant, the product nRT is constant for an isothermal process. From the ideal gas law, the products $P_{\text{i}}V_{\text{i}}$ and $P_{\text{f}}V_{\text{f}}$ must also be constant. We can replace nRT with $P_{\text{i}}V_{\text{i}}$ and solve for the initial pressure of the gas.

SOLVE

$$W = nRT\ln\left(\frac{V_{\text{f}}}{V_{\text{i}}}\right) = P_{\text{i}}V_{\text{i}}\ln\left(\frac{V_{\text{f}}}{V_{\text{i}}}\right)$$

$$P_{\text{i}} = \frac{W}{V_{\text{i}}\ln\left(\frac{V_{\text{f}}}{V_{\text{i}}}\right)} = \frac{(2.0 \times 10^3\text{ J})}{(8.0\text{ m}^3)\ln\left(\frac{10.0\ m^3}{8.0\text{ m}^3}\right)} = \boxed{1.1 \times 10^3\text{ Pa} = 1.1\text{ kPa}}$$

REFLECT

The final pressure should be less than 1.1 kPa since the gas has expanded. In fact, $P_{\text{f}} = \frac{P_{\text{i}}V_{\text{i}}}{V_{\text{f}}} = \frac{(1.1\text{ kPa})(8.0\text{ m}^3)}{10.0\text{ m}^3} = 0.88\text{ kPa}.$

15.51

SET UP

An expandable container contains 2.00 mol of an ideal gas and is initially at a pressure $P_{\text{i}} =$ 12 atm. The gas undergoes a reversible isothermal expansion to a final pressure of $P_{\text{f}} = 3$ atm. The work done by an ideal gas in an isothermal process is equal to $W = nRT\ln\left(\frac{V_{\text{f}}}{V_{\text{i}}}\right)$.

Assuming the number of moles of gas remains constant, the product nRT is constant for an isothermal process. From the ideal gas law, the products $P_i V_i$ and $P_f V_f$ must also be constant. From this we can solve for the ratio of the final to initial volumes in terms of the pressures and then the work done by the gas.

SOLVE

$$W = nRT\ln\left(\frac{V_f}{V_i}\right) = nRT\ln\left(\frac{P_i}{P_f}\right)$$

$$= (2.00\text{ mol})\left(8.314\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(400\text{ K})\ln\left(\frac{12\text{ atm}}{3\text{ atm}}\right) = \boxed{9200\text{ J} = 9.2\text{ kJ}}$$

REFLECT

Because the pressure of the gas decreases, the volume must increase for an isothermal process. An expanding gas does positive work.

15.52

SET UP

A gas contained within an expandable container is kept at a constant pressure $P = 2.8 \times 10^5$ atm. The gas undergoes an expansion from its initial volume of $V_i = 0.5\text{ m}^3$ to a final volume of $V_f = 1.5\text{ m}^3$ when 300 kJ of heat is added to the system. The work done by an ideal gas at a constant pressure is equal to $P\Delta V$. The change in the internal energy of the gas can be found using the first law of thermodynamics, $\Delta U = Q - W$.

SOLVE

Work:

$$W = P\Delta V = (2.8 \times 10^5\text{ Pa})((1.5\text{ m}^3) - (0.5\text{ m}^3)) = 2.8 \times 10^5\text{ J} = 280\text{ kJ}$$

Internal energy:

$$\Delta U = Q - W = (300\text{ kJ}) - (280\text{ kJ}) = \boxed{20\text{ kJ}}$$

REFLECT

This is an isobaric process because the pressure remains constant. The gas expands, so it makes sense that the work done by the gas should be positive.

15.53

SET UP

An ideal gas is contained in a vessel with fixed walls. While its pressure is decreased, 800 kJ of heat leaves the gas. Since the volume is fixed, the work done by the gas is equal to zero. The change in the internal energy of the gas is equal to Q in this case from the first law of thermodynamics. Heat leaving the system corresponds to a negative value, so $Q = -800$ kJ.

SOLVE

$$\Delta U = Q - W = (-800\text{ kJ}) - 0 = \boxed{-800\text{ kJ}}$$

REFLECT
This is an isochoric process because the volume is constant. Since the pressure of the gas decreases while the volume remains constant, the temperature of the gas must decrease. Therefore, we would expect heat to leave the system.

15.54

SET UP
An ideal gas is compressed adiabatically. In accomplishing this, 1888 J of work is done on the gas. Since the process is adiabatic, the heat flow in or out of the system is equal to zero. The change in the internal energy of the gas is equal to $-W$ in this case from the first law of thermodynamics. Work being done on the gas corresponds to a negative value, so $W = -1888$ J.

SOLVE

$$\Delta U = Q - W = 0 - (-1888\text{ J}) = \boxed{1888\text{ J}}$$

REFLECT
The only form of energy transfer is the work being done on the gas, so the internal energy of the gas should increase.

15.55

SET UP
An ideal monatomic gas ($n = 2.0$ mol) does 8.0 kJ of work as it expands adiabatically. Since the process is adiabatic, the heat flow in or out of the system is equal to zero. The change in the internal energy of the gas is equal to $-W$ in this case from the first law of thermodynamics; work being done by the gas corresponds to a positive value, so $W = +8.0$ kJ. For a monatomic ideal gas, the change in the internal energy is also equal to $\Delta U = \frac{3}{2}nR\Delta T$. The change in temperature of the gas can be found by setting these two expressions equal.

SOLVE
First law of thermodynamics:

$$\Delta U = Q - W = 0 - W = -W$$

Temperature change:

$$\Delta U = \frac{3}{2}nR\Delta T = -W$$

$$\Delta T = -\frac{2W}{3nR} = -\frac{2(+8.0 \times 10^3\text{ J})}{3(2.0\text{ mol})\left(8.314\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)} = \boxed{-320\text{ K}}$$

REFLECT
Because heat cannot flow into or out of the system, the work done by the gas must come at the expense of its own internal energy.

15.56

SET UP

We are shown the PV curve for a monatomic gas that underwent a thermodynamic process where both its volume and pressure tripled. The work done by the gas is equal to the area under the PV curve.

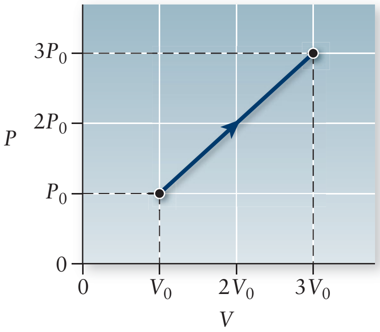


Figure 15-3 Problem 56

SOLVE

$$W = \frac{1}{2}(P_0 + 3P_0)(3V_0 - V_0) = \frac{1}{2}(4P_0)(2V_0) = \boxed{4P_0V_0}$$

REFLECT

The work should be positive because the volume of the gas has increased.

15.57

SET UP

An engine takes in 10 kJ and exhausts 6 kJ. The efficiency of the engine is equal to the difference in the energy it takes in and the energy it exhausts divided by the amount of energy it takes in: $e = \frac{W}{Q_H} = \frac{Q_H - |Q_C|}{Q_H}$.

SOLVE

$$e = \frac{W}{Q_H} = \frac{Q_H - |Q_C|}{Q_H} = \frac{(10\text{ kJ}) - (6\text{ kJ})}{10\text{ kJ}} = 0.4 = \boxed{40\%}$$

REFLECT

Efficiency is most commonly given as a percentage.

15.58

SET UP

The theoretical maximum efficiency of an engine is related to the temperatures of the reservoirs by $e = 1 - \frac{T_C}{T_H}$, here $T_C = 373$ K and $T_H = 773$ K.

SOLVE

$$e = 1 - \frac{T_C}{T_H} = 1 - \frac{373\text{ K}}{773\text{ K}} = 0.517 = \boxed{51.7\%}$$

REFLECT
The maximum efficiency will occur only when the cold reservoir is at absolute zero.

15.59

SET UP
A heat engine operates between $T_C = 373$ K and $T_H = 473$ K with an efficiency equal to 70% of its theoretical maximum efficiency. The theoretical maximum efficiency of a heat engine is given by $e = 1 - \frac{T_C}{T_H}$.

SOLVE
Theoretical maximum efficiency:

$$e_{max} = 1 - \frac{T_C}{T_H} = 1 - \frac{373\text{ K}}{473\text{ K}} = 0.2114$$

Actual efficiency:

$$e = 0.70e_{max} = 0.70(0.2114) = 0.148 = \boxed{14.8\%}$$

REFLECT
The actual efficiency can also be calculated by the work output divided by the heat input.

15.60

SET UP
An engine operates between $T_C = 283$ K and $T_H = 473$ K. The theoretical maximum efficiency of a heat engine is given by $e = 1 - \frac{T_C}{T_H}$. The actual efficiency is equal to the work output divided by the heat input, $e = \frac{W}{Q_H}$. Assuming the actual efficiency is equal to the theoretical maximum efficiency, we can solve for Q_H given $W = 1000$ J.

SOLVE
Theoretical maximum efficiency:

$$e = 1 - \frac{T_C}{T_H} = 1 - \frac{283\text{ K}}{473\text{ K}} = 0.4017$$

Heat supplied:

$$e = \frac{W}{Q_H}$$

$$Q_H = \frac{W}{e} = \frac{1000\text{ J}}{0.4017} = \boxed{2490\text{ J}}$$

REFLECT

This is the smallest amount of heat that can be supplied since we've assumed the actual efficiency is equal to the theoretical maximum efficiency.

15.61

SET UP

A furnace supplies 28 kW of thermal power at $T_H = 573$ K and exhausts energy at $T_C = 293$ K. The theoretical maximum efficiency of a heat engine is given by $e = 1 - \frac{T_C}{T_H}$. The actual efficiency is equal to the work output divided by the heat input, $e = \frac{W}{Q_H}$. We can divide the numerator and denominator by time, to convert the work and heat to work per second and thermal power, respectively. Assuming the actual efficiency is equal to the theoretical maximum efficiency, we can solve for the work per second $\frac{W}{\Delta t}$ given $\frac{Q_H}{\Delta t} = 28$ kW.

SOLVE

Theoretical maximum efficiency:

$$e = 1 - \frac{T_C}{T_H} = 1 - \frac{293 \text{ K}}{573 \text{ K}} = 0.4887$$

Work per second:

$$e = \frac{\left(\frac{W}{\Delta t}\right)}{\left(\frac{Q_H}{\Delta t}\right)}$$

$$\frac{W}{\Delta t} = e\left(\frac{Q_H}{\Delta t}\right) = (0.4887)(28 \text{ kW}) = \boxed{13.7 \text{ kW}}$$

REFLECT

This is the largest amount of work that can be expected since we've assumed the actual efficiency is equal to the theoretical maximum efficiency.

15.62

SET UP

In one second, a kitchen refrigerator extracts $Q_C = 75$ kJ while exhausting $Q_H = 100$ kJ. The coefficient of performance of this refrigerator is $\text{CP} = \frac{Q_C}{Q_H - Q_C}$.

SOLVE

$$\text{CP} = \frac{Q_C}{Q_H - Q_C} = \frac{75 \text{ kJ}}{(100 \text{ kJ}) - (75 \text{ kJ})} = \boxed{3.0}$$

REFLECT
The coefficient of performance is another measure of "how much you get for how much you put in."

15.63

SET UP
A Carnot refrigerator operates at temperatures $T_C = 273$ K and $T_H = 353$ K. The coefficient of performance for this refrigerator is equal to $\text{CP} = \frac{T_C}{T_H - T_C}$.

SOLVE
$$\text{CP} = \frac{T_C}{T_H - T_C} = \frac{273\text{ K}}{(353\text{ K}) - (273\text{ K})} = \boxed{3.4}$$

REFLECT
The maximum coefficient of performance of a refrigerator is obtained when the system is based on the Carnot cycle. We can only use this alternate form for the coefficient of performance for refrigerators based on the Carnot cycle.

15.64

SET UP
An electric refrigerator removes $Q_C = 13.0$ MJ of heat when $W = 1$ kWh. The coefficient of performance for this refrigerator is $\text{CP} = \frac{Q_C}{W}$. We first need to convert the work from kWh to MJ in order to achieve a consistent set of units.

SOLVE
Kilowatt-hour conversion:

$$1\text{ kWh} \times \frac{3600\text{ s}}{1\text{ hr}} = 3600\text{ kJ} = 3.60\text{ MJ}$$

Coefficient of performance:

$$\text{CP} = \frac{Q_C}{W} = \frac{13.0\text{ MJ}}{3.60\text{ MJ}} = \boxed{3.61}$$

REFLECT
The kilowatt-hour is a unit of energy commonly seen on utility bills and meters.

15.65

SET UP
A refrigerator requires $W = 35$ J to remove $Q_C = 190$ J from its interior. The temperature of the surroundings is $T_H = 295$ K. The coefficient of performance of the refrigerator is given by $\text{CP} = \frac{Q_C}{W}$. We can then use the other expressions for the coefficient of performance,

$CP = \dfrac{Q_C}{Q_H - Q_C}$ and $CP = \dfrac{T_C}{T_H - T_C}$, to calculate heat ejected to the surroundings Q_H and the internal temperature of the refrigerator, respectively.

SOLVE

Part a)

$$CP = \frac{Q_C}{W} = \frac{190\ \text{J}}{35\ \text{J}} = \boxed{5.4}$$

Part b)

$$CP = \frac{Q_C}{Q_H - Q_C}$$

$$(CP)(Q_H - Q_C) = Q_C$$

$$Q_H = \frac{Q_C + (CP)Q_c}{CP} = Q_C\left(\frac{1}{CP} + 1\right) = (190\ \text{J})\left(\frac{1}{5.4} + 1\right) = \boxed{225\ \text{J}}$$

Part c)

$$CP = \frac{T_C}{T_H - T_C}$$

$$(CP)(T_H - T_C) = T_C$$

$$T_C = \frac{(CP)T_H}{1 + CP} = \frac{(5.4)(295\ \text{K})}{1 + 5.4} = 249\ \text{K} = \boxed{-24°\text{C}}$$

REFLECT

The fact the refrigerator cycle is reversible means it follows the Carnot cycle, which allows us to use the expression for the coefficient of performance based on the temperatures.

15.66

SET UP

A refrigerator consumes 370 W of electrical power. Its interior temperature is $T_C = 273$ K and the temperature of the surroundings is $T_H = 293$ K. The ratio of the actual coefficient of performance when it operates relative to the theoretical maximum coefficient of performance is 66% for this refrigerator. The theoretical maximum coefficient of performance is achieved when the refrigerator acts reversibly (that is, according to the Carnot cycle) and is equal to $CP_{max} = \dfrac{T_C}{T_H - T_C}$. The heat removed from the interior of the refrigerator in one minute can be calculated from the actual coefficient of performance and the electrical power consumed by the appliance, $CP = \dfrac{Q_C}{W} = \dfrac{Q_C}{(P\Delta t)}$.

SOLVE

Theoretical maximum coefficient of performance:

$$\mathrm{CP}_{\max} = \frac{T_\mathrm{C}}{T_\mathrm{H} - T_\mathrm{C}} = \frac{273\ \mathrm{K}}{(293\ \mathrm{K}) - (273\ \mathrm{K})} = 13.65$$

Heat removed in 1 min:

$$\mathrm{CP} = \frac{Q_\mathrm{C}}{W} = \frac{Q_\mathrm{C}}{(P\Delta t)}$$

$$Q_\mathrm{C} = (\mathrm{CP})P\Delta t = 0.66(\mathrm{CP}_{\max})P\Delta t = 0.66(13.65)(370\ \mathrm{W})(60\ \mathrm{s}) = \boxed{2.00 \times 10^5\ \mathrm{J}}$$

REFLECT

We can quickly check the reasonability of our solution by using round numbers. In 1 min the refrigerator consumes about 2×10^4 J of energy. The actual CP in this case would be 10, which is close to 2/3 of 13.65.

15.67

SET UP

A container holds 32 g of O_2 gas, which has a molar mass of 32 g/mol, under a constant pressure of 8.0 atm. The heat required to increase the temperature of the gas by 100 K at constant pressure is equal to $Q_P = nC_P\Delta T$. We will assume that the molar specific heat of oxygen at 8.0 atm is equal to the molar specific heat at 1.0 atm, $C_P = 29.4\frac{\mathrm{J}}{\mathrm{mol \cdot K}}$.

SOLVE

$$Q_P = nC_P\Delta T = \left(32\ \mathrm{g} \times \frac{1\ \mathrm{mol}}{32\ \mathrm{g}}\right)\left(29.4\frac{\mathrm{J}}{\mathrm{mol \cdot K}}\right)(100\ \mathrm{K}) = \boxed{2900\ \mathrm{J} = 2.9\ \mathrm{kJ}}$$

REFLECT

The molar heat capacity for an ideal diatomic gas at constant pressure should be equal to $\frac{7}{2}R = 29.1\frac{\mathrm{J}}{\mathrm{mol \cdot K}}$ regardless of the pressure, so our assumption that the molar heat capacity at 8.0 atm is equal to its value at 1.0 atm is a reasonable one.

15.68

SET UP

A container holds 32 g of O_2 gas, which has a molar mass of 32 g/mol, under a pressure of 8.0 atm. The heat required to increase the temperature of the gas by 100 K at constant volume is equal to $Q_V = nC_V\Delta T$. We will assume that the molar specific heat of oxygen at 8.0 atm is equal to the molar specific heat at 1.0 atm, $C_V = 21.1\frac{\mathrm{J}}{\mathrm{mol \cdot K}}$.

SOLVE

$$Q_V = nC_V\Delta T = \left(32\ \mathrm{g} \times \frac{1\ \mathrm{mol}}{32\ \mathrm{g}}\right)\left(21.1\frac{\mathrm{J}}{\mathrm{mol \cdot K}}\right)(100\ \mathrm{K}) = \boxed{2100\ \mathrm{J} = 2.1\ \mathrm{kJ}}$$

REFLECT

The molar heat capacity for an ideal diatomic gas at constant volume should be equal to $\frac{5}{2}R = 20.8\frac{\text{J}}{\text{mol} \cdot \text{K}}$ regardless of the pressure, so our assumption that the molar heat capacity at 8.0 atm is equal to its value at 1.0 atm is a reasonable one.

15.69

SET UP

The temperature of 4 g of helium gas is increased by 1 K at constant volume. We want to know the mass of oxygen gas that, using the same amount of heat, will experience a temperature increase of 1 K as well. The heat required to change the temperature of a gas at constant volume is given by $Q_V = nC_V\Delta T$. Using this expression, we can set up a ratio between the heats required by each gas, which are equal in this case, in terms of the numbers of moles and molar heat capacities at constant volume. The number of moles is equal to the mass divided by the molar mass of the gas. The data we'll need are the molar masses and the molar heat capacities at constant volume for both helium and oxygen: $\text{MM}_{\text{He}} = 4\frac{\text{g}}{\text{mol}}$, $\text{MM}_{\text{O}_2} = 32\frac{\text{g}}{\text{mol}}$, $C_{V,\,\text{He}} = 12.5\frac{\text{J}}{\text{mol} \cdot \text{K}}$, and $C_{V,\,\text{O}_2} = 21.1\frac{\text{J}}{\text{mol} \cdot \text{K}}$.

SOLVE

$$\frac{Q_{V,\,\text{O}_2}}{Q_{V,\,\text{He}}} = \frac{n_{\text{O}_2}C_{V,\,\text{O}_2}\Delta T}{n_{\text{He}}C_{V,\,\text{He}}\Delta T}$$

$$1 = \frac{\left(\frac{m_{\text{O}_2}}{\text{MM}_{\text{O}_2}}\right)C_{V,\,\text{O}_2}}{\left(\frac{m_{\text{He}}}{\text{MM}_{\text{He}}}\right)C_{V,\,\text{He}}} = \left(\frac{m_{\text{O}_2}}{m_{\text{He}}}\right)\left(\frac{\text{MM}_{\text{He}}}{\text{MM}_{\text{O}_2}}\right)\left(\frac{C_{V,\,\text{O}_2}}{C_{V,\,\text{He}}}\right)$$

$$m_{\text{O}_2} = m_{\text{He}}\left(\frac{\text{MM}_{\text{O}_2}}{\text{MM}_{\text{He}}}\right)\left(\frac{C_{V,\,\text{He}}}{C_{V,\,\text{O}_2}}\right) = (4\text{ g})\frac{\left(32\frac{\text{g}}{\text{mol}}\right)\left(12.5\frac{\text{J}}{\text{mol} \cdot \text{K}}\right)}{\left(4\frac{\text{g}}{\text{mol}}\right)\left(21.1\frac{\text{J}}{\text{mol} \cdot \text{K}}\right)} = \boxed{19\text{ g}}$$

REFLECT

This is about 0.6 mol of oxygen. For constant heat and temperature change, the product of the number of moles and the molar heat capacity is constant. Since the molar heat capacity of oxygen is larger than that of helium, the number of moles of oxygen must be less than the number of moles of helium under these circumstances.

15.70

SET UP

Heat is added to 1 mol of air at constant pressure resulting in a temperature increase of 100 degrees C. The same amount of heat is then added to 1 mol of air at constant volume, and we are asked to calculate the resulting temperature increase. The heat required to change the

temperature of a gas at constant pressure is given by $Q_P = nC_P\Delta T$, whereas the heat required to change the temperature of a gas at constant volume is given by $Q_V = nC_V\Delta T$. Using these expressions, we can set up a ratio between the heats required at constant pressure and volume, which are equal in this case, in terms of the number of moles and molar heat capacities in order to solve for the temperature increase at constant volume. The molar specific heat ratio for air is $\gamma = \frac{C_P}{C_V} = 1.4$.

SOLVE

$$\frac{Q_V}{Q_P} = \frac{nC_V\Delta T_V}{nC_P\Delta T_P}$$

$$1 = \frac{C_V\Delta T_V}{C_P\Delta T_P} = \frac{1}{\gamma}\frac{\Delta T_V}{\Delta T_P}$$

$$\Delta T_V = \gamma\Delta T_P = (1.4)(100°\text{C}) = \boxed{140°\text{C}}$$

REFLECT

The molar heat capacity at constant pressure will be larger than the molar heat capacity at constant volume. Using proportional reasoning, the temperature increase at constant pressure should then be smaller than the temperature increase at constant volume.

15.71

SET UP

The volume of a gas is halved during an adiabatic compression that increases the pressure by a factor of 2.6. An ideal gas that undergoes an adiabatic expansion or compression obeys PV^γ = constant. Since this product must remain constant for all time, we can write the final pressure and volume in terms of the initial pressure and volume and solve for the molar specific heat ratio γ.

SOLVE

$$P_1V_1^\gamma = P_2V_2^\gamma$$

$$P_1V_1^\gamma = (2.6P_1)\left(\frac{V_1}{2}\right)^\gamma$$

$$2^\gamma = 2.6$$

$$\gamma = \log_2(2.6) = \frac{\log(2.6)}{\log(2)} = \boxed{1.4}$$

REFLECT

We invoked the change of base formula for logarithms to change from log base 2 to log base 10:

$$\log_b a = \frac{\log a}{\log b}$$

15.72

SET UP

The volume of a gas is halved during an adiabatic compression that increases the pressure by a factor of 2.5. An ideal gas that undergoes an adiabatic expansion or compression obeys PV^{γ} = constant. Since this product must remain constant for all time, we can write the final pressure and volume in terms of the initial pressure and volume and solve for the molar specific heat ratio γ. We can use the value of γ we find along with $TV^{\gamma-1}$ = constant in order to determine the factor by which the temperature of the gas increases.

SOLVE

Molar specific heat ratio:

$$P_1V_1^{\gamma} = P_2V_2^{\gamma}$$

$$P_1V_1^{\gamma} = (2.5P_1)\left(\frac{V_1}{2}\right)^{\gamma}$$

$$2^{\gamma} = 2.5$$

$$\gamma = \log_2(2.5) = \frac{\log(2.5)}{\log(2)} = 1.32$$

Temperature increase:

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$\frac{T_2}{T_1} = \frac{V_1^{\gamma-1}}{V_2^{\gamma-1}} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{V_1}{\left(\frac{V_1}{2}\right)}\right)^{\gamma-1} = 2^{\gamma-1} = 2^{1.32-1} = \boxed{1.25}$$

REFLECT

Thinking about the process in general, we expect the temperature of the gas to increase upon adiabatic compression since there is no heat flow in or out of the system. In terms of the first law of thermodynamics, work is being done on the gas by the piston, which corresponds to negative work:

$$\Delta U = Q - W = 0 - (-W) = W > 0$$

The internal energy of the gas increases, which means the temperature of the gas increases.

15.73

SET UP

The temperature of air increases from $T_i = 300$ K to $T_f = 1130$ K. An ideal gas that undergoes an adiabatic expansion or compression obeys $TV^{\gamma-1}$ = constant. We can use this expression and $\gamma_{\text{air}} = 1.4$ to solve for the ratio of initial volume to final volume.

SOLVE

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$$

$$\frac{T_f}{T_i} = \frac{V_i^{\gamma-1}}{V_f^{\gamma-1}} = \left(\frac{V_i}{V_f}\right)^{\gamma-1}$$

$$\frac{V_i}{V_f} = \left(\frac{T_f}{T_i}\right)^{\frac{1}{\gamma-1}} = \left(\frac{1130\text{ K}}{300\text{ K}}\right)^{\frac{1}{1.4-1}} = \boxed{27.5}$$

REFLECT

The final temperature is larger, so the adiabatic process will be compression. Therefore, $V_f < V_i$.

15.74

SET UP

A monatomic ideal gas expands adiabatically from $V_1 = 1.5\text{ m}^3$ to $V_2 = 3.0\text{ m}^3$ at an initial pressure of $P_1 = 1$ atm. An ideal gas that undergoes an adiabatic expansion or compression obeys $PV^\gamma = \text{constant}$, where $\gamma = \frac{C_P}{C_V}$. The molar specific heat of a monatomic ideal gas at constant volume is equal to $\frac{3}{2}R$. The molar specific heat of any ideal gas (not necessarily monatomic) at constant pressure is $C_P = C_V + R$. Since the product PV^γ must remain constant for all time, we can write the final pressure and volume in terms of the initial pressure and volume and solve for the final pressure.

SOLVE

Molar specific heat ratio:

$$\gamma = \frac{C_P}{C_V} = \frac{C_V + R}{C_V} = 1 + \frac{R}{C_V} = 1 + \frac{R}{\left(\frac{3}{2}R\right)} = \frac{5}{3}$$

Final pressure:

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$P_2 = \frac{P_1 V_1^\gamma}{V_2^\gamma} = P_1\left(\frac{V_1}{V_2}\right)^\gamma = (1\text{ atm})\left(\frac{1.5\text{ m}^3}{3.0\text{ m}^3}\right)^{\frac{5}{3}} = \boxed{0.3\text{ atm}}$$

REFLECT

The final volume is larger than the initial volume. The final pressure, therefore, should be smaller than the initial pressure since the gas has undergone an adiabatic expansion.

15.75

SET UP

A mole of an ideal monatomic gas ($\gamma = 1.66$) expands adiabatically from $P_1 = 10$ atm to $P_2 = 2$ atm at an initial temperature of $T_1 = 273.15$ K. An ideal gas that undergoes an adiabatic expansion or compression obeys $P^{1-\gamma}T^\gamma = \text{constant}$. Since the product $P^{1-\gamma}T^\gamma$ must remain constant for all time, we can write the final pressure and temperature in terms of the

initial pressure and temperature and solve for the final temperature of the gas. The first law of thermodynamics relates the heat and work done by the gas to the change in the internal energy of the gas. In general, the internal energy of a mole of monatomic ideal gas is equal to $\frac{3}{2}RT$. For an adiabatic thermodynamic process, $Q = 0$, so $\Delta U = -W_{\text{by gas}}$. The work done on the gas is equal to the opposite of the work done by the gas.

SOLVE

Final temperature:

$$P_1^{1-\gamma}T_1^{\gamma} = P_2^{1-\gamma}T_2^{\gamma}$$

$$T_2^{\gamma} = \frac{P_1^{1-\gamma}}{P_2^{1-\gamma}}T_1^{\gamma} = \left(\frac{P_1}{P_2}\right)^{1-\gamma}T_1^{\gamma}$$

$$T_2 = \left(\frac{P_1}{P_2}\right)^{\frac{1-\gamma}{\gamma}}T_1 = \left(\frac{10\text{ atm}}{2\text{ atm}}\right)^{\frac{1-1.66}{1.66}}(273.15\text{ K}) = 144.04\text{ K}$$

Work done on the gas:

$$\Delta U = Q - W = 0 - W_{\text{by gas}}$$

$$W_{\text{by gas}} = -\Delta U = -\frac{3}{2}R(T_2 - T_1) = -\frac{3}{2}\left(8.314\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)((144.04\text{ K}) - (273.15\text{ K})) = 1610\text{ J}$$

$$W_{\text{on gas}} = -W_{\text{by gas}} = -1610\text{ J} = \boxed{-1.6\text{ kJ}}$$

REFLECT

The gas expands, which means the work done by the gas is positive and the work done on the gas is negative. The temperature of the gas should decrease if the gas expands adiabatically.

15.76

SET UP

A reservoir at a temperature of 400 K absorbs 100 J of heat. The entropy change of the reservoir is given by $\Delta S = \frac{Q}{T}$.

SOLVE

$$\Delta S = \frac{Q}{T} = \frac{100\text{ J}}{400\text{ K}} = \boxed{0.25\frac{\text{J}}{\text{K}}}$$

REFLECT

The heat absorbed by a system is positive. We would expect the entropy of a system to increase when it absorbs heat.

15.77

SET UP

A sample of ice absorbs 6.68×10^4 J of heat at 273 K as it melts. The minimum entropy change of the system is given by $\Delta S = \frac{Q}{T}$.

SOLVE

$$\Delta S = \frac{Q}{T} = \frac{6.68 \times 10^4\ \text{J}}{273\ \text{K}} = \boxed{245\frac{\text{J}}{\text{K}}}$$

REFLECT

The entropy change is equal to $\frac{Q}{T}$ only in the case of a reversible process. We do not need to use the mass of the ice since we are given the amount of heat absorbed.

15.78

SET UP

A 0.500-kg block of ice absorbs enough heat reversibly to completely melt at 273 K. The heat required to melt the ice is equal to the mass of the ice multiplied by the heat of fusion of ice, $Q = mL_F$, where $L_F = 334 \times 10^3\frac{\text{J}}{\text{kg}}$. The entropy change for a reversible process is $\Delta S = \frac{Q}{T}$.

SOLVE

$$\Delta S = \frac{Q}{T} = \frac{mL_F}{T} = \frac{(0.500\ \text{kg})\left(334 \times 10^3\frac{\text{J}}{\text{kg}}\right)}{273\ \text{K}} = \boxed{612\frac{\text{J}}{\text{K}}}$$

REFLECT

The entropy of a melting system should increase throughout the process.

15.79

SET UP

An air conditioner maintains a room at a constant temperature of 295 K by removing heat from the system. The entropy change associated with 5.0×10^3 J of heat removed is given by $\Delta S = \frac{Q}{T}$.

SOLVE

$$\Delta S = \frac{Q}{T} = \frac{-5.0 \times 10^3\ \text{J}}{295\ \text{K}} = \boxed{-17\frac{\text{J}}{\text{K}}}$$

REFLECT

Remember that heat removed from a system is considered negative.

15.80

SET UP

An ideal gas ($n = 1.0$ mol) expands isothermally from $V_i = 1.0\ \text{m}^3$ to $V_f = 2.0\ \text{m}^3$. The internal energy of a system does not change when it undergoes an isothermal process. Using this fact, we can calculate the heat absorbed during the expansion from the first law of thermodynamics and the ideal gas law. The entropy change of the gas is equal to $\Delta S = \frac{Q}{T}$.

SOLVE

Heat:

$$\Delta U = 0 = Q - W$$

$$Q = W = \int_{V_i}^{V_f} P dV = \int_{V_i}^{V_f} \frac{nRT}{V} dV = nRT \int_{V_i}^{V_f} \frac{dV}{V} = nRT[\ln(V)]_{V_i}^{V_f} = nRT \ln\left(\frac{V_f}{V_i}\right)$$

Entropy change:

$$\Delta S = \frac{Q}{T} = \frac{nRT\ln\left(\frac{V_f}{V_i}\right)}{T} = nR\ln\left(\frac{V_f}{V_i}\right) = (1.0 \text{ mol})\left(8.314 \frac{\text{J}}{\text{mol}\cdot\text{K}}\right)\ln\left(\frac{2.0 \text{ m}^3}{1.0 \text{ m}^3}\right) = \boxed{5.8 \frac{\text{J}}{\text{K}}}$$

REFLECT

The temperature remains constant throughout the process and, even though we don't know the exact value of the temperature, it doesn't matter since it eventually cancels out in the expression for the entropy change.

15.81

SET UP

A car ($m = 1800$ kg) is traveling at a speed of $v_i = 80$ km/hr when it crashes into a wall. The temperature of the air is 300 K. The entropy change of the universe due to an irreversible process is equal to the total energy transferred divided by the temperature, $\Delta S = \frac{Q}{T}$.

SOLVE

$$\Delta S = \frac{Q}{T} = \frac{\left(\frac{1}{2}mv_i^2\right)}{T} = \frac{\left(\frac{1}{2}\right)(1800 \text{ kg})\left(80 \frac{\text{km}}{\text{hr}} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ hr}}{3600 \text{ s}}\right)^2}{300 \text{ K}} = \boxed{1500 \frac{\text{J}}{\text{K}} = 1.5 \frac{\text{kJ}}{\text{K}}}$$

REFLECT

We're assuming that all of the initial kinetic energy of the car is absorbed by the universe as heat.

15.82

SET UP

A rock ($m = 1000$ kg) falls 100 m into a large lake that is at a temperature $T = 293$ K. The rock's kinetic energy when it hits the lake is converted all into thermal energy absorbed by the lake. From conservation of mechanical energy, the rock's kinetic energy at this point is equal to its initial gravitational potential energy. The entropy change of the lake is equal to $\Delta S = \frac{Q}{T}$.

SOLVE

$$\Delta S = \frac{Q}{T} = \frac{\left(\frac{1}{2}mv_i^2\right)}{T} = \frac{(mgh)}{T} = \frac{(1000 \text{ kg})\left(9.8 \frac{\text{m}}{\text{s}^2}\right)(100 \text{ m})}{293 \text{ K}} = \boxed{3.3 \times 10^3 \frac{\text{J}}{\text{K}}}$$

REFLECT
The entropy of the lake increases due to the absorbed heat, as expected.

15.83

SET UP
The temperature at the Sun's surface is $T_H = 5700\text{ K}$, while the temperature of the Earth's surface is $T_C = 293\text{ K}$. The Sun transfers 8000 J of heat to the Earth, which means $Q_H = -8000\text{ J}$ and $Q_C = 8000\text{ J}$. The entropy change for the universe is equal to the sum of the entropy changes for the Sun and Earth, $\Delta S = \Delta S_H + \Delta S_C$, where $\Delta S_H = \frac{Q_H}{T_H}$ and $\Delta S_C = \frac{Q_C}{T_C}$.

SOLVE

$$\Delta S = \Delta S_H + \Delta S_C = \frac{Q_H}{T_H} + \frac{Q_C}{T_C} = \frac{-8000\text{ J}}{5700\text{ K}} + \frac{8000\text{ J}}{293\text{ K}} = \boxed{26\frac{\text{J}}{\text{K}}}$$

REFLECT
The transfer of heat from the Sun to the Earth (or from a hot place to a cold place) is a spontaneous process, as seen in the positive value for the entropy change of the universe.

15.84

SET UP
A sample of water ($m = 1\text{ kg}$) freezes at 273 K. The required amount of heat lost to freeze the water is equal to the mass of the water multiplied by the heat of fusion of ice, $Q = -mL_F$, where $L_F = 334 \times 10^3 \frac{\text{J}}{\text{kg}}$. Assuming the process is reversible, the entropy change is $\Delta S = \frac{Q}{T}$.

SOLVE

$$\Delta S = \frac{Q}{T} = \frac{mL_F}{T} = \frac{(1\text{ kg})\left(-334 \times 10^3 \frac{\text{J}}{\text{kg}}\right)}{273\text{ K}} = \boxed{-1.22 \times 10^3 \frac{\text{J}}{\text{K}}}$$

REFLECT
The heat associated with a liquid to solid transition must be negative; this causes the entropy change of the system to also be negative, which makes sense since a solid is more ordered than a liquid.

15.85

SET UP
In a poorly insulated bottle, water ($m_{water} = 0.350\text{ kg}$) and ice ($m_{ice} = 0.150\text{ kg}$) are initially at equilibrium at 273 K. Over time, the contents of the bottle come to thermal equilibrium with the outside air temperature, $T_{air} = 298\text{ K}$. Heat is transferred from the air to the ice and water—first, the ice melts and then all of the water heats up to 298 K. The heat necessary to melt the ice is related to its latent heat of fusion $\left(L_F = 334 \times 10^3 \frac{\text{J}}{\text{kg}}\right)$; the heat necessary to

increase the temperature of the water is related to its specific heat $\left(C_{\text{water}} = 4186\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)$. The entropy changes of the melting ice and air are equal to $\Delta S_{\text{melting}} = \frac{Q_{\text{melting}}}{T_{\text{ice}}}$ and $\Delta S_{\text{air}} = \frac{Q_{\text{air}}}{T_{\text{air}}}$, respectively. The temperature of the water is not constant, so we need to integrate $dS_{\text{warming}} = \frac{mC_{\text{water}}dT}{T}$ in order to calculate the change in entropy of the water as it warms from 273 K to 298 K. The total change in entropy of the universe is the sum of these three entropy changes.

SOLVE

Heat transferred:

$$Q_{\text{water}} = m_{\text{water}}C_{\text{water}}(T_{\text{eq}} - T_{\text{i}}) = (0.350\text{ kg})\left(4186\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)((298\text{ K}) - (273\text{ K})) = 36{,}627.5\text{ J}$$

$$Q_{\text{ice}} = Q_{\text{melting}} + Q_{\text{warming}} = m_{\text{ice}}L_{\text{F}} + m_{\text{ice}}C_{\text{water}}(T_{\text{eq}} - T_{\text{i}}) = m_{\text{ice}}(L_{\text{F}} + C_{\text{water}}(T_{\text{eq}} - T_{\text{i}}))$$

$$= (0.150\text{ kg})\left(\left(334\times 10^{3}\frac{\text{J}}{\text{K}}\right) + \left(4186\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)((298\text{ K}) - (273\text{ K}))\right) = 65{,}797.5\text{ J}$$

$$Q_{\text{air}} = -(Q_{\text{water}} + Q_{\text{ice}}) = -((36{,}627.5\text{ J}) + (65{,}797.5\text{ J})) = -102{,}425\text{ J}$$

Entropy change of the ice melting:

$$\Delta S_{\text{melting}} = \frac{Q_{\text{melting}}}{T_{\text{ice}}} = \frac{50{,}100\text{ J}}{273\text{ K}} = 183.5\frac{\text{J}}{\text{K}}$$

Entropy change of the water warming:

$$\Delta S_{\text{warming}} = \int_{273\text{ K}}^{298\text{ K}}\frac{dQ}{T} = \int_{273\text{ K}}^{298\text{ K}}\frac{mC_{\text{water}}dT}{T} = mC_{\text{water}}[\ln(T)]_{273\text{ K}}^{298\text{ K}}$$

$$= ((0.150\text{ kg}) + (0.350\text{ kg}))\left(4186\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)\ln\left(\frac{298\text{ K}}{273\text{ K}}\right) = 183.39\frac{\text{J}}{\text{K}}$$

Entropy change of the air:

$$\Delta S_{\text{air}} = \frac{Q_{\text{air}}}{T_{\text{air}}} = \frac{-102{,}425\text{ J}}{298\text{ K}} = -343.7\frac{\text{J}}{\text{K}}$$

Entropy change of the universe:

$$\Delta S = \Delta S_{\text{melting}} + \Delta S_{\text{warming}} + \Delta S_{\text{air}} = \left(183.5\frac{\text{J}}{\text{K}}\right) + \left(183.4\frac{\text{J}}{\text{K}}\right) + \left(-343.7\frac{\text{J}}{\text{K}}\right) = \boxed{23.1\frac{\text{J}}{\text{K}}}$$

REFLECT

We can only use $\Delta S = \frac{Q}{T}$ if the temperature is constant. This formula follows directly from the integral form: $\Delta S = \int ds = \int\frac{dQ}{T} = \frac{1}{T}\int dQ = \frac{Q}{T}$.

15.86

SET UP

Cold water ($m_C = 0.060$ kg, $T_C = 299$ K) is poured into a cup containing hot coffee ($m_H =$ 0.220 kg, $T_H = 348$ K). Once combined, heat will flow between the two samples of water until they reach a final equilibrium temperature T_{eq}; the amount of heat that left the hot water ($Q_H = m_H C \Delta T_H$) is equal to the amount of heat absorbed by the cold water ($Q_C = m_C C \Delta T_C$). We can set the absolute values of these two expressions equal to one another and solve for the final equilibrium temperature and then the heat transferred. The entropy change in the universe is equal to the sum of the entropy changes in the hot and cold water, $\Delta S = \Delta S_H + \Delta S_C$, where $\Delta S_H = \dfrac{Q_H}{T_H}$ and $\Delta S_C = \dfrac{Q_C}{T_C}$.

SOLVE

Heat:

$$Q_H = m_H C \Delta T_H = m_H C(T_{eq} - T_H)$$

$$Q_C = m_C C \Delta T_C = m_C C(T_{eq} - T_C)$$

$$|Q_H| = |Q_C|$$

$$|m_H C(T_{eq} - T_H)| = |m_C C(T_{eq} - T_C)|$$

$$m_H(T_H - T_{eq}) = m_C(T_{eq} - T_C)$$

$$T_{eq} = \frac{m_H T_H + m_C T_C}{m_H + m_C} = \frac{(0.220 \text{ kg})(348 \text{ K}) + (0.060 \text{ kg})(299 \text{ K})}{(0.220 \text{ kg}) + (0.060 \text{ kg})} = 337 \text{ K}$$

$$Q_C = m_C C(T_{eq} - T_C) = (0.060 \text{ kg})\left(4186 \frac{\text{J}}{\text{kg} \cdot \text{K}}\right)((337 \text{ K}) - (299 \text{ K})) = 9.54 \times 10^3 \text{ J}$$

Entropy change:

$$\Delta S = \Delta S_H + \Delta S_C = \frac{Q_H}{T_H} + \frac{Q_C}{T_C} = \frac{-9.54 \times 10^3 \text{ J}}{348 \text{ K}} + \frac{9.54 \times 10^3 \text{ J}}{299 \text{ K}} = \boxed{4.5 \frac{\text{J}}{\text{K}}}$$

REFLECT

A positive change in the entropy of the universe tells us the process is spontaneous. When we pour amounts of hot and cold water together, we expect them to spontaneously mix together. It would be very strange indeed if the hot part of the water remained hot and the cold part remained cold!

15.87

SET UP

Water ($m = 2.50$ kg, $T = 273$ K) is placed inside of a refrigerator ($T_C = 273$ K) to freeze. The amount of heat required to freeze the water is related to the latent heat of water, $Q = mL_F$. The temperature of the surroundings is $T_H = 299$ K. The coefficient of performance of a reversible refrigerator is $\text{CP} = \dfrac{T_C}{T_H - T_C}$. Once we know this, we can use another definition of

the coefficient of performance $\left(\mathrm{CP} = \frac{Q}{W}\right)$ to solve for the work required to transfer the heat. This work is equal to the time necessary to freeze the water multiplied by the power rating of the refrigerator ($P = 88.0\ \mathrm{W}$).

SOLVE

Heat loss required to freeze the water:

$$|Q| = |mL_\mathrm{F}| = (2.50\ \mathrm{kg})\left(334 \times 10^3 \frac{\mathrm{J}}{\mathrm{kg}}\right) = 8.35 \times 10^5\ \mathrm{J}$$

Theoretical maximum coefficient of performance:

$$\mathrm{CP} = \frac{T_\mathrm{C}}{T_\mathrm{H} - T_\mathrm{C}} = \frac{273\ \mathrm{K}}{(299\ \mathrm{K}) - (273\ \mathrm{K})} = 10.5$$

Time required to freeze the water:

$$\mathrm{CP} = \frac{Q}{W} = \frac{Q}{(P\Delta t)}$$

$$\Delta t = \frac{Q}{P(\mathrm{CP})} = \frac{8.35 \times 10^5\ \mathrm{J}}{(88.0\ \mathrm{W})(10.5)} = 904\ \mathrm{s} \times \frac{1\ \mathrm{min}}{60\ \mathrm{s}} = \boxed{15\ \mathrm{min}}$$

REFLECT

Fifteen minutes seems like a reasonable amount of time to freeze such a relatively large amount of water.

15.88

SET UP

A vertical metal cylinder contains an ideal gas. The top of the cylinder is closed off with a piston of mass m and cross-sectional area A that is initially located at a height h, which is free to move. The gas is compressed when a total mass m of sand is poured on the piston. The final height of the piston is h_2. Assuming there is no heat flow in or out of the cylinder, the gas will undergo an adiabatic compression, which means $PV^\gamma = \text{constant}$. Once the piston comes to rest, the pressure is equal to the weight of the piston (or piston + sand) divided by A. The volume of the gas is equal to the cross-sectional area A multiplied by the height of the piston. The molar specific heat ratio will depend on the nature of the gas. For a monatomic ideal gas like helium, $\gamma = \frac{5}{3}$; for a diatomic ideal gas like oxygen or hydrogen, $\gamma = \frac{7}{5}$. Since the product PV^γ must remain constant for all time, we can write the final pressure and volume in terms of the initial pressure and volume and solve for the final height h_2 in terms of h.

SOLVE

Final height:

$$P_1V_1^\gamma = P_2V_2^\gamma$$

$$\left(\frac{F_1}{A}\right)(Ah)^\gamma = \left(\frac{F_2}{A}\right)(Ah_2)^\gamma$$

$$\left(\frac{mg}{A}\right)h^{\gamma} = \left(\frac{2mg}{A}\right)h_2^{\gamma}$$

$$h_2 = \frac{h}{\left(2^{\frac{1}{\gamma}}\right)} = h\left(2^{-\frac{1}{\gamma}}\right)$$

Part a)

$$\gamma = \frac{C_P}{C_V} = \frac{\left(\frac{7}{2}R\right)}{\left(\frac{5}{2}R\right)} = \frac{7}{5}$$

$$h_2 = h\left(2^{-\frac{1}{\gamma}}\right) = h\left(2^{-\frac{5}{7}}\right) = \boxed{0.61h}$$

Part b)

$$\gamma = \frac{C_P}{C_V} = \frac{\left(\frac{5}{2}R\right)}{\left(\frac{3}{2}R\right)} = \frac{5}{3}$$

$$h_2 = h\left(2^{-\frac{1}{\gamma}}\right) = h\left(2^{-\frac{3}{5}}\right) = \boxed{0.66h}$$

Part c)

$$\gamma = \frac{C_P}{C_V} = \frac{\left(\frac{7}{2}R\right)}{\left(\frac{5}{2}R\right)} = \frac{7}{5}$$

$$h_2 = h\left(2^{-\frac{1}{\gamma}}\right) = h\left(2^{-\frac{5}{7}}\right) = \boxed{0.61h}$$

REFLECT

When we pour sand on top of the piston, the gas should be compressed; therefore, $h_2 < h$.

15.89

SET UP

A Carnot engine extracts heat Q_H from seawater ($T_H = 291$ K) to power a ship. The exhausted heat Q_C is used to sublimate a reserve of dry ice of mass m at a temperature $T_C = 195$ K. The ship's engines run at 8000 horsepower, where 1 hp = 746 W, for a full day; multiplying the engine's power by the time will give the work the engine is required to output to run the ship. We can use this value, the efficiency of a Carnot engine, and the fact that $W = Q_H - |Q_C|$ to find the amount of heat Q_C that will be exhausted. All of the exhausted heat will go to sublimating the dry ice, which means $Q_C = mL_F$; the latent heat of sublimation of carbon dioxide is $L_S = 573{,}700\frac{\text{J}}{\text{kg}}$.

SOLVE

Energy required for one day:

$$W = P\Delta t = \left(8000 \text{ hp} \times \frac{746 \text{ W}}{1 \text{ hp}}\right)\left(1 \text{ day} \times \frac{24 \text{ hr}}{1 \text{ day}} \times \frac{3600 \text{ s}}{1 \text{ hr}}\right) = 5.156 \times 10^{11} \text{ J}$$

Efficiency of the engine:

$$e = 1 - \frac{T_C}{T_H} = 1 - \frac{195 \text{ K}}{291 \text{ K}} = 0.3299$$

Total heat exhausted:

$$e = \frac{W}{Q_H} = \frac{W}{W + |Q_C|} = \frac{W}{W + |mL_S|}$$

$$m = \frac{W(1-e)}{e|L_S|} = \frac{(5.156 \times 10^{11} \text{ J})(1 - 0.3299)}{(0.3299)\left(573{,}700 \frac{\text{J}}{\text{kg}}\right)} = \boxed{1.83 \times 10^6 \text{ kg}}$$

REFLECT

An engine uses heat to do work; a refrigerator does work to remove heat.

15.90

SET UP

A Carnot engine removes $Q_H = 1200$ J from a hot reservoir and dumps $Q_C = 600$ J into the atmosphere ($T_C = 293$ K). The efficiency of the engine is equal to $e = 1 - \frac{Q_C}{Q_H}$. The temperature of the hot reservoir T_H can be found from the expression for the efficiency of a Carnot engine, $e = 1 - \frac{T_C}{T_H}$.

SOLVE

Part a)

$$e = 1 - \frac{Q_C}{Q_H} = 1 - \frac{600 \text{ J}}{1200 \text{ J}} = 0.500 = \boxed{50.0\%}$$

Part b)

$$e = 1 - \frac{T_C}{T_H}$$

$$T_H = \frac{T_C}{1-e} = \frac{293 \text{ K}}{1 - 0.500} = \boxed{586 \text{ K}}$$

REFLECT

The ratio of the heats Q_C and Q_H should equal the ratio of the temperatures in Kelvin. The heats are related by a factor of 2, so we would expect an efficiency of 50%.

15.91

SET UP

The actual efficiency of an engine when it operates, as opposed to the theoretical maximum efficiency when it operates reversibly, is 85.0%. During each cycle, the engine absorbs $Q_H = 480$ J from a reservoir at $T_H = 573$ K and dumps $Q_C = 300$ J into a cold reservoir. We can calculate the temperature of the cold reservoir T_C from the theoretical efficiency, $e_{max} = 1 - \frac{T_C}{T_H}$. Once we know the actual and reversible efficiencies, we can calculate the actual and reversible work done in one cycle from $e = \frac{W}{Q_H}$, assuming the engine extracts $Q_H = 480$ J in each case.

SOLVE

Part a)

Efficiency:

$$e = 1 - \frac{Q_C}{Q_H} = 1 - \frac{300\text{ J}}{480\text{ J}} = 0.375$$

$$e = 0.850e_{max} = 0.375$$

$$e_{max} = \frac{0.375}{0.850} = 0.441$$

Temperature of the cold reservoir:

$$e_{max} = 1 - \frac{T_C}{T_H}$$

$$T_C = T_H(1 - e_{max}) = (573\text{ K})(1 - 0.441) = \boxed{320\text{ K} = 47^\circ\text{C}}$$

Part b)

Work done by the engine:

$$e = \frac{W}{Q_H}$$

$$W = eQ_H = (0.375)(480\text{ J}) = 180\text{ J}$$

Carnot:

$$e_{max} = \frac{W}{Q_H}$$

$$W = e_{max}Q_H = (0.441)(480\text{ J}) = 212\text{ J}$$

Difference in work:

$$(212\text{ J}) - (180\text{ J}) = \boxed{32\text{ J}}$$

REFLECT

A Carnot engine acts reversibly and achieves a maximum efficiency of 100%; it will always provide the most work for a given heat transfer.

15.92

SET UP

The interior of a freezer is $T_C = 10$ degrees F and the temperature of the kitchen is $T_H =$ 78 degrees F. Heat leaks through the walls into the freezer at a rate of 70.0 cal/min. After converting the temperatures into Kelvin and the rate of heat flow into J/s, we can calculate the total entropy change of the universe in 1 hr using $\Delta S = \Delta S_H + \Delta S_C = \frac{Q_H}{T_H} + \frac{Q_C}{T_C}$, where the magnitudes of the heats are equal. The energy unavailable to do work when the heat leaks into the freezer is equal to the temperature of the freezer multiplied by the total entropy change in the universe, $T_C \Delta S$.

SOLVE

Part a)

Temperature conversions:

$$T_{C,\,°C} = \frac{5}{9}(T_{C,\,°F} - 32) = \frac{5}{9}(10 - 32) = -12.2°C$$

$$T_{C,\,K} = 273.15 + T_{C,\,°C} = 273.15 - 12.2 = 261\text{ K}$$

$$T_{H,\,°C} = \frac{5}{9}(T_{H,\,°F} - 32) = \frac{5}{9}(78 - 32) = 25.6°C$$

$$T_{H,\,K} = 273.15 + T_{H,\,°C} = 273.15 + 25.6 = 299\text{ K}$$

Heat transferred in 1 hr:

$$Q = \left(70.0\frac{\text{cal}}{\text{min}} \times \frac{4.184\text{ J}}{1\text{ cal}}\right)(60\text{ min}) = 1.76 \times 10^4\text{ J}$$

Entropy:

$$\Delta S = \Delta S_H + \Delta S_C = \frac{Q_H}{T_H} + \frac{Q_C}{T_C} = \frac{-1.76 \times 10^4\text{ J}}{299\text{ K}} + \frac{1.76 \times 10^4\text{ J}}{261\text{ K}} = \boxed{8.57\frac{\text{J}}{\text{K}}}$$

Part b)

$$W_{\text{unavailable}} = T_C \Delta S = (261\text{ K})\left(8.57\frac{\text{J}}{\text{K}}\right) = \boxed{2240\text{ J}}$$

REFLECT

The entropy change of the universe when heat leaks from the kitchen into the freezer is positive, which means it is a spontaneous process, as expected.

15.93

SET UP

When a 60-kg woman walks up five flights of stairs ($\Delta h = 20$ m), she normally releases 100 J of heat; if she has a high fever, she gives off 10% more heat. The work done by the person to climb the stairs is equal to the change in her gravitational potential energy. The efficiency of the woman's body is equal to the work done divided by the total energy she outputs, which is equal to the work done plus the heat emitted.

SOLVE

Work done:

$$W = \Delta U = mg\Delta h = (60 \text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)(20 \text{ m}) = 11{,}760 \text{ J} = 11.76 \text{ kJ}$$

Normal temperature:

$$Q_\text{C} = 100 \text{ kJ}$$

$$Q_\text{H} = W + Q_\text{C} = (11.76 \text{ kJ}) + (100 \text{ kJ}) = 111.76 \text{ kJ}$$

$$e = \frac{W}{Q_\text{H}} = \frac{11.76 \text{ kJ}}{111.76 \text{ kJ}} = \boxed{0.105 = 10.5\%}$$

Elevated temperature:

$$Q_\text{C} = 1.1(100 \text{ kJ}) = 110 \text{ kJ}$$

$$Q_\text{H} = W + Q_\text{C} = (11.76 \text{ kJ}) + (120 \text{ kJ}) = 121.76 \text{ kJ}$$

$$e = \frac{W}{Q_\text{H}} = \frac{11.76 \text{ kJ}}{121.76 \text{ kJ}} = \boxed{0.0966 = 9.66\%}$$

Your $\boxed{\text{efficiency drops when you have a fever}}$.

REFLECT

Since her efficiency is lower when she is sick, it will take her more energy to accomplish the same tasks she did when healthy.

15.94

SET UP

A rigid 5.50-L pressure cooker contains steam at $T_1 = 373$ K and $P_1 = 1.00$ atm. Since the volume of the vapor is constant, we can use the ideal gas law to set up a ratio between the initial and final temperatures and pressures in order to solve for the temperature T_2 of the steam when the pressure is $P_2 = 1.25$ atm. The heat required to raise the temperature of the gas from T_1 to T_2 at constant volume is $Q_V = nC_V\Delta T$, where $C_V = 25.9\frac{\text{J}}{\text{mol}\cdot\text{K}}$. We are not explicitly given the number of moles n but can solve for it using the ideal gas law and the volume, T_1, and P_1. We can use Avogadro's number (1 mol = 6.02×10^{23} molecules), the mass of a water molecule (2.99×10^{-26} kg), and the conversion between joules and calories

$(4.184\text{ J} = 1\text{ cal})$ in order to convert the specific heat of steam from $\frac{\text{J}}{\text{mol}\cdot\text{K}}$ to $\frac{\text{J}}{\text{kg}\cdot\text{K}}$ and $\frac{\text{cal}}{\text{g}\cdot\text{K}}$.

SOLVE

Part a)

$$\frac{T_1}{P_1} = \frac{T_2}{P_2}$$

$$T_2 = T_1\left(\frac{P_2}{P_1}\right) = (373\text{ K})\left(\frac{1.25\text{ atm}}{1.00\text{ atm}}\right) = 466\text{ K} = \boxed{193°\text{C}}$$

Part b)

$$Q_V = nC_V\Delta T = \left(\frac{P_1V_1}{RT_1}\right)C_V(T_2 - T_1)$$

$$= \frac{(101{,}300\text{ Pa})\left(5.50\text{ L}\times\frac{1\text{ m}^3}{1000\text{ L}}\right)}{\left(8.314\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(373\text{ K})}\left(25.9\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)((466\text{ K}) - (373\text{ K})) = \boxed{433\text{ J}}$$

Part c)

$$25.9\frac{\text{J}}{\text{mol}\cdot\text{K}} \times \frac{1\text{ mol}}{6.02\times10^{23}\text{ molecules}} \times \frac{1\text{ molecule}}{2.99\times10^{-26}\text{ kg}} = \boxed{1440\frac{\text{J}}{\text{kg}\cdot\text{K}}}$$

$$1440\frac{\text{J}}{\text{kg}\cdot\text{K}} \times \frac{1\text{ kg}}{1000\text{ g}} \times \frac{1\text{ cal}}{4.184\text{ J}} = \boxed{0.344\frac{\text{cal}}{\text{g}\cdot\text{K}}}$$

REFLECT

The pressure should increase when we raise the temperature of the steam in the pressure cooker. Also, we can safely assume the pressure cooker is a closed system, which means the number of moles will remain constant.

15.95

SET UP

A sample of 0.2 mol of an ideal gas undergoes an isothermal expansion at 320 K from $V_1 = 2$ L to $V_2 = 8$ L. Using the ideal gas law, we can calculate the initial pressure P_1 and the final pressure P_2. The PV curve of an isothermal expansion is shaped like a hyperbola. The work done by the gas for an isothermal process is $W = nRT\left[\ln\left(\frac{V_2}{V_1}\right)\right]$. The change in the internal energy of the gas is zero since the process is isothermal, which means the heat gained by the gas is equal to the work done by the gas.

SOLVE

Part a)

$$P_1 = \frac{nRT}{V_1} = \frac{(0.2\ \text{mol})\left(0.0821\dfrac{\text{L}\cdot\text{atm}}{\text{mol}\cdot\text{K}}\right)(320\ \text{K})}{2\ \text{L}} = 2.63\ \text{atm}$$

$$P_2 = \frac{nRT}{V_2} = \frac{(0.2\ \text{mol})\left(0.0821\dfrac{\text{L}\cdot\text{atm}}{\text{mol}\cdot\text{K}}\right)(320\ \text{K})}{8\ \text{L}} = 0.657\ \text{atm}$$

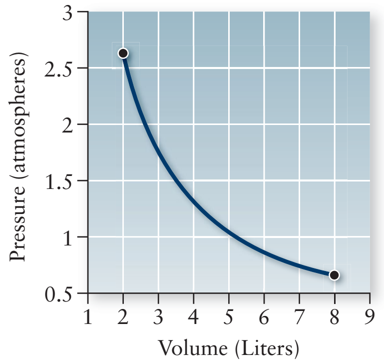


Figure 15-4 Problem 95

Part b)

$$W = \int_{V_1}^{V_2} P\,dV = \int_{V_1}^{V_2} \frac{nRT}{V}\,dV = nRT\int_{V_1}^{V_2}\frac{dV}{V} = nRT\left[\ln\left(\frac{V_2}{V_1}\right)\right]$$

$$= (0.2\ \text{mol})\left(8.314\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(320\ \text{K})\left[\ln\left(\frac{8\ \text{L}}{2\ \text{L}}\right)\right] = \boxed{738\ \text{J}}$$

Part c)

$$\Delta U = 0 = Q - W$$

$$Q = W = 738\ \text{J}$$

Part d)

$$\Delta U = 0$$

REFLECT

The gas expands so the work done by the gas is positive. The expansion is isothermal, so heat must be added to the gas in order to maintain its temperature; this is considered positive heat using our sign conventions.

15.96

SET UP

The temperatures of the hot and cold reservoirs of a power plant are $T_H = 593$ K and $T_C = 313$ K, respectively. We can use these values to calculate the theoretical maximum efficiency of the plant from $e_{max} = 1 - \frac{T_C}{T_H}$. In one year, the plant takes in 4.4×10^{16} J of heat and produces an average output of 600×10^6 W of power. The actual efficiency of the plant is given by $e = \frac{W}{Q_H}$. The second-law efficiency is equal to the ratio of the actual efficiency to the maximum efficiency.

SOLVE

Theoretical maximum efficiency:

$$e_{max} = 1 - \frac{T_C}{T_H} = 1 - \frac{313\text{ K}}{593\text{ K}} = 0.472$$

Actual efficiency:

$$e = \frac{W}{Q_H} = \frac{(600 \times 10^6\text{ W})\left(1\text{ yr} \times \frac{365.25\text{ day}}{1\text{ yr}} \times \frac{24\text{ hr}}{1\text{ day}} \times \frac{3600\text{ s}}{1\text{ hr}}\right)}{4.4 \times 10^{16}\text{ J}} = 0.430$$

Second-law efficiency:

$$\frac{e}{e_{max}} = \frac{0.430}{0.472} = \boxed{0.911 = 91.1\%}$$

REFLECT

A second-law efficiency of 100% can only be achieved for a reversible engine (for example, a Carnot engine).

15.97

SET UP

The temperature increases by 3.0 degrees C for every 100 m we drill into the Earth's crust. Water pumped into an oil well that is 1830 m deep is used as a heat engine. The temperature of the hot reservoir (that is, the bottom of the well) is equal to the depth multiplied by the relationship between the temperature increase and depth. The temperature of the cold reservoir (that is, the surface) is $T_C = 293$ K. The maximum efficiency occurs when the heat engine acts reversibly, so $e = 1 - \frac{T_C}{T_H}$. We want to use a combination of these wells to create a reversible power plant that provides 2.5×10^6 W of power. We can use a different expression for efficiency, $Q_H = \frac{W}{e}$, in order to calculate the total heat the power plant absorbs from the Earth in one day.

SOLVE

Part a)

Maximum temperature:

$$T_H = (1830\text{ m})\left(\frac{3.0°\text{C}}{100\text{ m}}\right) = 54.9°\text{C} = 328\text{ K}$$

Maximum efficiency:

$$e = 1 - \frac{T_C}{T_H} = 1 - \frac{293\text{ K}}{328\text{ K}} = 0.107 = \boxed{11\%}$$

Part b)

$$e = \frac{W}{Q_H}$$

$$Q_H = \frac{W}{e} = \frac{(2.5\times10^6\text{ W})\left(1\text{ day}\times\frac{24\text{ hr}}{1\text{ day}}\times\frac{3600\text{ s}}{1\text{ hr}}\right)}{0.107} = \boxed{2.0\times10^{12}\text{ J} = 2.0\text{ TJ}}$$

REFLECT

A terajoule (TJ) is equal to 1×10^{12} J. Because we used the maximum efficiency, 2.0 TJ is the minimum possible energy that is absorbed from the interior of the Earth in one day. If the actual efficiency is smaller than 11%, the power plant must absorb more than 2.0 TJ of heat in a day in order to deliver a power of 2.5 MW.

15.98

SET UP

The energy efficiency ratio (EER) is defined as the input rate of heat Q_C/t (in BTU/hr) divided by the output rate of work W/t (in W). This reduces to the coefficient of performance, albeit in mixed units, by dividing out the common factors of t. We can show that $\text{CP} = \frac{\text{EER}}{3.412}$ by converting from BTU and hr into J (1 BTU = 1055 J) and s (1 hr = 3600 s). The best possible CP for a typical freezer operating at $T_C = 273$ K and $T_H = 343$ K can be calculated from $\text{CP} = \frac{T_C}{T_H - T_C}$.

SOLVE

Part a)

$$\text{EER} = \frac{(Q_C/t)(\text{BTU/hr})}{(W/t)(\text{W})} = \frac{Q_C(\text{BTU})}{W(\text{W}\cdot\text{hr})}$$

$$\text{CP} = \frac{Q_C}{W}$$

The final ratio for EER is the same as the coefficient of performance (CP). The units are not the same, however—the heat and work in the $\boxed{\text{EER are expressed in mixed units, while they are expressed in identical units in the CP}}$.

Part b)

$$\mathrm{CP} = \frac{Q_C(\mathrm{BTU})}{W(\mathrm{W}\cdot\mathrm{hr})} \times \frac{1\ \mathrm{hr}}{3600\ \mathrm{s}} \times \frac{1055\ \mathrm{J}}{1\ \mathrm{BTU}} = 0.29306(\mathrm{EER}) = \boxed{\frac{\mathrm{EER}}{3.412}}$$

Part c)

Coefficient of performance:

$$\mathrm{CP} = \frac{\mathrm{EER}}{3.412} = \frac{5.1}{3.412} = \boxed{1.5}$$

Best possible coefficient of performance:

$$\mathrm{CP} = \frac{T_C}{T_H - T_C} = \frac{273\ \mathrm{K}}{(343\ \mathrm{K}) - (273\ \mathrm{K})} = \boxed{3.9}$$

Part d)

$$\mathrm{CP} = \frac{\mathrm{EER}}{3.412}$$

$$\mathrm{EER} = 3.412(\mathrm{CP}) = 3.412(3.9) = \boxed{13.3}$$

REFLECT

The best possible CP and EER occur for a reversible refrigerator. The values for Q_C and W in the CP expression need to be in the same units, but not necessarily SI units. For example, the calculation for CP would work if both were expressed in terms of BTU.

15.99

SET UP

You place 1.50 L of water, initially at 20 degrees C, into a 0-degree C freezer that has an EER of 6.50. The required heat loss to freeze the ice is equal to the heat necessary to decrease the temperature of the water from 20 degrees C to 0 degrees C ($Q_{20°\mathrm{C}\to 0°\mathrm{C}} = mc\Delta T_{20°\mathrm{C}\to 0°\mathrm{C}}$) plus the heat necessary to convert all of the water into ice ($Q_{\mathrm{liquid}\to\mathrm{solid}} = -mL_F$), where $c = 4186 \frac{\mathrm{J}}{\mathrm{kg}\cdot\mathrm{K}}$ and $L_F = 3.34 \times 10^5 \frac{\mathrm{J}}{\mathrm{kg}}$. The absolute value of this sum is equal to Q_C, the amount of heat removed from the water. The work required to make the ice is related to the coefficient of performance and Q_C, $\mathrm{CP} = \frac{Q_C}{W}$; using the result from Problem 15.98, we can relate the CP to the EER, $\mathrm{CP} = \frac{\mathrm{EER}}{3.412}$. The heat ejected into the kitchen during this process is equal to $Q_H = W + Q_C$. The entropy change of the kitchen due to the ejected heat is $\Delta S_H = \frac{Q_H}{T_H}$, where $T_H = 295$ K.

SOLVE

Part a)
Heat required to freeze the water:

$$Q_C = |Q_{20°C\to 0°C} + Q_{\text{liquid}\to\text{solid}}| = |mc\Delta T_{20°C\to 0°C} - mL_F|$$

$$= |m(c\Delta T_{20°C\to 0°C} - L_F)| = |(\rho V)(c\Delta T_{20°C\to 0°C} - L_F)|$$

$$= \left(1000\frac{\text{kg}}{\text{m}^3}\right)\left(1.50\text{ L} \times \frac{1\text{ m}^3}{1000\text{ L}}\right)\left|\left(4186\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(-20\text{ K}) - \left(3.34 \times 10^5\frac{\text{J}}{\text{kg}}\right)\right|$$

$$= 6.266 \times 10^5\text{ J}$$

Work required to make the ice:

$$\text{CP} = \frac{Q_C}{W} = \frac{\text{EER}}{3.412}$$

$$W = \frac{3.412 Q_C}{\text{EER}} = \frac{(3.412)(6.266 \times 10^5\text{ J})}{6.50} = \boxed{3.29 \times 10^5\text{ J} = 329\text{ kJ}}$$

$$1\text{ kWh} \times \frac{1000\text{ W}}{1\text{ kW}} \times \frac{3600\text{ s}}{1\text{ hr}} = 3.6 \times 10^6\text{ J}$$

$$3.29 \times 10^5\text{ J} \times \frac{1\text{ kWh}}{3.6 \times 10^6\text{ J}} = \boxed{0.0914\text{ kWh}}$$

Part b)

$$W = Q_H - Q_C$$

$$Q_H = W + Q_C = (3.29 \times 10^5\text{ J}) + (6.266 \times 10^5\text{ J}) = \boxed{9.56 \times 10^5\text{ J} = 956\text{ kJ}}$$

Part c)

$$\Delta S_H = \frac{Q_H}{T_H} = \frac{956\text{ kJ}}{295\text{ K}} = \boxed{3.24\frac{\text{kJ}}{\text{K}}}$$

REFLECT

The entropy change in the water is $\Delta S_C = \frac{-Q_C}{T_H} = \frac{-6.27\text{ kJ}}{293\text{ K}} = -2.14\frac{\text{kJ}}{\text{K}}$, which means the entropy change for the universe is positive. This agrees with our experience that the freezing of water in a freezer is a spontaneous process.

15.100

SET UP

The volume of air taken in during a typical breath is $V = 0.5$ L. The air is initially at $T_C = 293$ K and is heated up to $T_H = 310$ K as it enters the lungs. Since air is 80% N_2 we will model it as a diatomic ideal gas. The heat required to warm the air in one breath from T_C

to T_H at a constant pressure is $Q_P = nC_P\Delta T$, where the specific heat of a diatomic gas at constant pressure is equal to $C_P = \frac{7}{2}R$. We can find the number of moles in one breath of air at T_C through the ideal gas law. If we take two breaths every 3.0 s, we can calculate the total number of breaths we take in one day. This number multiplied by the energy necessary to heat one breath will give the total amount of energy necessary per day just to heat up the air we breathe. Finally, we can compare this total energy to the typical daily caloric intake of an adult, 2000 kcal, in order to make sure our answer is reasonable.

SOLVE

Part a)

$$Q_P = nC_P\Delta T = \left(\frac{PV}{RT_C}\right)\left(\frac{7}{2}R\right)(T_H - T_C) = \frac{7PV(T_H - T_C)}{2T_C}$$

$$= \frac{7(101{,}300\text{ Pa})\left(0.5\text{ L} \times \frac{1\text{ m}^3}{1000\text{ L}}\right)((310\text{ K}) - (293\text{ K}))}{2(293\text{ K})} = \boxed{10.3\text{ J}}$$

Part b)

Number of breaths in a day:

$$24\text{ hr} \times \frac{3600\text{ s}}{1\text{ day}} \times \frac{2\text{ breaths}}{3.0\text{ s}} = 5.76 \times 10^4\text{ breaths}$$

Total heat used for breathing:

$$(5.76 \times 10^4\text{ breaths})\left(10.3\frac{\text{J}}{\text{breath}}\right) = 5.93 \times 10^5\text{ J} \times \frac{1\text{ kcal}}{4184} = \boxed{142\text{ kcal}}$$

The typical daily caloric for an adult is around 2000 kcal, so this is $\boxed{\text{not a significant amount}}$ of energy.

REFLECT

There clearly needs to be a pressure difference for air to flow in and out of our lungs, but the pressure difference is small, so we can assume that the pressure remains essentially constant at 1 atm.

15.101

SET UP

A heat engine absorbs $Q_H = 1250$ J from a high-temperature reservoir ($T_H = 490$ K) and does 475 J of work. The temperature of the cold reservoir is $T_C = 273$ K. The efficiency of the engine is equal to the work done divided by the heat absorbed. The change in the entropy of the universe is equal to the sum of the entropy changes of the hot and cold reservoirs. The heat exhausted to the cold reservoir Q_C was not given but is equal to $Q_H - W$ from the first law of thermodynamics. Finally, the amount of energy unavailable for doing work after one full cycle is equal to $T_C\Delta S$.

SOLVE

Part a)

$$e = \frac{W}{Q_{\mathrm{H}}} = \frac{475\ \mathrm{J}}{1250\ \mathrm{J}} = \boxed{0.38 = 38\%}$$

Part b)

$$\Delta S = \Delta S_{\mathrm{H}} + \Delta S_{\mathrm{C}} = \frac{Q_{\mathrm{H}}}{T_{\mathrm{H}}} + \frac{Q_{\mathrm{C}}}{T_{\mathrm{C}}} = \frac{Q_{\mathrm{H}}}{T_{\mathrm{H}}} + \frac{Q_{\mathrm{H}} - W}{T_{\mathrm{C}}}$$

$$= \frac{-1250\ \mathrm{J}}{490\ \mathrm{K}} + \frac{(1250\ \mathrm{J}) - (475\ \mathrm{J})}{273\ \mathrm{K}} = \boxed{0.288\frac{\mathrm{J}}{\mathrm{K}}}$$

Part c)

$$W_{\mathrm{unavailable}} = T_{\mathrm{C}}\Delta S = (273\ \mathrm{K})\left(0.288\frac{\mathrm{J}}{\mathrm{K}}\right) = \boxed{78.6\ \mathrm{J}}$$

REFLECT

The amount of energy unavailable to do work is also equal to the difference between the maximum work possible in a reversible process and the actual work done by a system.

15.102

SET UP

A cabin is built on an 8.50-m × 12.5-m rectangular foundation with walls 3.00 m tall. The walls and flat roof are made of white pine (thermal conductivity $k = 0.12\frac{\mathrm{W}}{\mathrm{m \cdot K}}$) that is 9.00×10^{-2} m thick. The temperature inside the cabin is $T_{\mathrm{H}} = 70$ degrees F and the temperature outside is $T_{\mathrm{C}} = -10$ degrees F. Assuming heat is lost through only the walls and roof, the rate at which heat leaves the interior of the cabin is given by $H = \frac{\Delta Q}{\Delta t} = k\frac{A}{L}(T_{\mathrm{H}} - T_{\mathrm{C}})$. In order to maintain the cabin at a constant temperature T_{H}, a heat pump operating on the Carnot cycle must pump in an amount of heat equal to the amount lost through the walls and roof; that is, $Q_{\mathrm{H}} = \Delta Q$. The energy required to operate the pump is $W = Q_{\mathrm{H}} - Q_{\mathrm{C}}$. Since the pump operates according to the Carnot cycle, $\frac{Q_{\mathrm{H}}}{T_{\mathrm{H}}} = \frac{Q_{\mathrm{C}}}{T_{\mathrm{C}}}$. Combining all of this information, we can calculate the amount of energy W the pump consumes per second to maintain the interior temperature of the cabin.

SOLVE

Temperature conversions:

$$T_{\mathrm{C},\,^{\circ}\mathrm{C}} = \frac{5}{9}(T_{\mathrm{C},\,^{\circ}\mathrm{F}} - 32) = \frac{5}{9}(-10 - 32) = -23.3^{\circ}\mathrm{C}$$

$$T_{\mathrm{C},\,\mathrm{K}} = 273.15 + T_{\mathrm{C},\,^{\circ}\mathrm{C}} = 273.15 - 23.3 = 250\ \mathrm{K}$$

$$T_{\mathrm{H},\,^\circ\mathrm{C}} = \frac{5}{9}(T_{\mathrm{H},\,^\circ\mathrm{F}} - 32) = \frac{5}{9}(70 - 32) = 21.1^\circ\mathrm{C}$$

$$T_{\mathrm{H,\,K}} = 273.15 + T_{\mathrm{H},\,^\circ\mathrm{C}} = 273.15 + 21.1 = 294\ \mathrm{K}$$

Total surface area of the cabin:

$$A = A_{\mathrm{walls}} + A_{\mathrm{roof}}$$

$$= [2(8.50\ \mathrm{m})(3.00\ \mathrm{m}) + 2(12.5\ \mathrm{m})(3.00\ \mathrm{m})] + (8.50\ \mathrm{m})(12.5\ \mathrm{m}) = 232.25\ \mathrm{m}^2$$

Total heat lost:

$$H = \frac{\Delta Q}{\Delta t} = k\frac{A}{L}(T_\mathrm{H} - T_\mathrm{C})$$

$$\Delta Q = k\frac{A\Delta t}{L}(T_\mathrm{H} - T_\mathrm{C})$$

Heat transferred by the Carnot pump to maintain the temperature:

$$Q_\mathrm{H} = \Delta Q$$

$$Q_\mathrm{C} + W = k\frac{A\Delta t}{L}(T_\mathrm{H} - T_\mathrm{C})$$

$$\frac{Q_\mathrm{H}T_\mathrm{C}}{T_\mathrm{H}} + W = k\frac{A\Delta t}{L}(T_\mathrm{H} - T_\mathrm{C})$$

Electrical energy:

$$W = k\frac{A\Delta t}{L}(T_\mathrm{H} - T_\mathrm{C}) - \frac{Q_\mathrm{H}T_\mathrm{C}}{T_\mathrm{H}} = k\frac{A\Delta t}{L}(T_\mathrm{H} - T_\mathrm{C}) - \left(\frac{T_\mathrm{C}}{T_\mathrm{H}}\right)\left(k\frac{A\Delta t}{L}(T_\mathrm{H} - T_\mathrm{C})\right)$$

$$W = k\frac{A\Delta t}{L}(T_\mathrm{H} - T_\mathrm{C})\left[1 - \left(\frac{T_\mathrm{C}}{T_\mathrm{H}}\right)\right]$$

$$= \left(0.12\frac{\mathrm{W}}{\mathrm{m}\cdot\mathrm{K}}\right)\frac{(232.25\ \mathrm{m}^2)(1.00\ \mathrm{s})}{(9.00 \times 10^{-2}\ \mathrm{m})}((294\ \mathrm{K}) - (250\ \mathrm{K}))\left[1 - \left(\frac{250\ \mathrm{K}}{294\ \mathrm{K}}\right)\right]$$

$$= \boxed{2039\ \mathrm{J}}$$

REFLECT

This is the amount of energy required of the pump *per second*. If the pump were to run constantly for a full day, the total energy required would be 1.76×10^8 J, or about 49 kWh. The cost of electrical power is typically between 10 cents/kWh to 15 cents/kWh, so it will cost about $6 a day to run the pump. Keep in mind that we have made many assumptions throughout this problem (for example, ignoring radiation, Carnot pump); the actual cost will definitely be more than this. Heat pumps are usually used in houses in more temperate areas (for example, Florida) rather than areas that get extremely cold (for example, Massachusetts or Saskatchewan).

15.103

SET UP

A diver (mass $m = 75.0$ kg) jumps from a height $h = 10.0$ m into a pool of water. The water in the pool is at 298 K. In order to calculate the maximum entropy change of the water, we will assume that all of the diver's initial potential energy is transferred to the water as heat by the action of the drag force. The change in entropy of the water is equal to the heat transferred divided by the temperature.

SOLVE

$$\Delta S = \frac{Q}{T} = \frac{mgh}{T} = \frac{(75.0\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)(10.0\text{ m})}{298\text{ K}} = \boxed{24.7\frac{\text{J}}{\text{K}}}$$

The pool's entropy $\boxed{\text{increases}}$.

REFLECT

Heat is transferred to the pool, so we would expect an increase in entropy.

15.104

SET UP

Two samples of water—$m_\text{H} = 80$ kg at $T_\text{H} = 313$ K and $m_\text{C} = 80$ kg at $T_\text{C} = 305$ K—are mixed together. The heat lost by the warmer water is equal to the heat absorbed by the colder water; setting these expressions equal will allow us to find the final equilibrium temperature of the system. The entropy change of the universe due to this mixing is equal to the sum of the entropy changes of each sample of water, or $\Delta S = \Delta S_\text{H} + \Delta S_\text{C} = \frac{Q_\text{H}}{T_\text{H}} + \frac{Q_\text{C}}{T_\text{C}}$.

SOLVE

Heat:

$$Q_\text{H} = m_\text{H}C\Delta T_\text{H} = m_\text{H}C(T_\text{eq} - T_\text{H})$$

$$Q_\text{C} = m_\text{C}C\Delta T_\text{C} = m_\text{C}C(T_\text{eq} - T_\text{C})$$

$$|Q_\text{H}| = |Q_\text{C}|$$

$$|m_\text{H}C(T_\text{eq} - T_\text{H})| = |m_\text{C}C(T_\text{eq} - T_\text{C})|$$

$$m_\text{H}(T_\text{H} - T_\text{eq}) = m_\text{C}(T_\text{eq} - T_\text{C})$$

$$T_\text{eq} = \frac{m_\text{H}T_\text{H} + m_\text{C}T_\text{C}}{m_\text{H} + m_\text{C}} = \frac{(80\text{ kg})(313\text{ K}) + (80\text{ kg})(305\text{ K})}{(80\text{ kg}) + (80\text{ kg})} = 309\text{ K}$$

$$Q_\text{C} = m_\text{C}C(T_\text{eq} - T_\text{C}) = (80\text{ kg})\left(4186\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)((309\text{ K}) - (305\text{ K})) = 1.34 \times 10^6\text{ J}$$

Entropy change:

$$\Delta S = \Delta S_{H} + \Delta S_{C} = \frac{Q_{H}}{T_{H}} + \frac{Q_{C}}{T_{C}} = \frac{-1.34 \times 10^{6}\ \text{J}}{313\ \text{K}} + \frac{1.34 \times 10^{6}\ \text{J}}{305\ \text{K}} = \boxed{112\frac{\text{J}}{\text{K}}}$$

REFLECT

Since we are mixing equal masses of water at the beginning, the equilibrium temperature will simply be the midpoint between the two starting temperatures.

15.105

SET UP

A certain person typically eats about 2250 kcal per day, 80% of which is transferred to the surroundings as heat. The surroundings are at 295 K. The entropy change of the surroundings is equal to the heat absorbed divided by the temperature.

SOLVE

$$\Delta S = \frac{Q}{T} = \frac{(0.80)\left(2250 \times 10^{3}\ \text{cal} \times \dfrac{4.184\ \text{J}}{1\ \text{cal}}\right)}{295\ \text{K}} = \boxed{2.55 \times 10^{4}\frac{\text{J}}{\text{K}} = 25.5\frac{\text{kJ}}{\text{K}}}$$

The entropy of the apartment $\boxed{\text{increases}}$.

REFLECT

The surroundings absorb heat, so the entropy of the surroundings should increase.

15.106

SET UP

A heat engine absorbs $Q_{H} = 1250$ J from a high-temperature reservoir ($T_{H} = 490$ K) and does 475 J of work. The temperature of the cold reservoir is $T_{C} = 273$ K. The efficiency of the engine is equal to the work done divided by the heat absorbed. The change in the entropy of the universe is equal to the sum of the entropy changes of the hot and cold reservoirs. The heat exhausted to the cold reservoir Q_{C} is not given but is equal to $Q_{H} - W$ from the first law of thermodynamics. Finally, the amount of energy unavailable for doing work after one full cycle is equal to $T_{C}\Delta S$.

SOLVE

Part a)

$$e = \frac{W}{Q_{H}} = \frac{475\ \text{J}}{1250\ \text{J}} = \boxed{0.38 = 38\%}$$

Part b)

$$\Delta S = \Delta S_{H} + \Delta S_{C} = \frac{Q_{H}}{T_{H}} + \frac{Q_{C}}{T_{C}} = \frac{Q_{H}}{T_{H}} + \frac{Q_{H} - W}{T_{C}}$$

$$= \frac{-1250\ \text{J}}{490\ \text{K}} + \frac{(1250\ \text{J}) - (475\ \text{J})}{273\ \text{K}} = \boxed{0.288\frac{\text{J}}{\text{K}}}$$

Part c)

$$W_{\text{unavailable}} = T_C \Delta S = (273 \text{ K})\left(0.288 \frac{\text{J}}{\text{K}}\right) = \boxed{78.6 \text{ J}}$$

REFLECT

The amount of energy unavailable to do work is also equal to the difference between the maximum work possible in a reversible process and the actual work done by a system.

15.107

SET UP

A reversible engine employs 1.23 mol of an ideal gas for which $\gamma = 1.41$ and runs in a cycle according to the PV diagram as shown. In short, the gas undergoes an isobaric expansion at $P_1 = 15$ atm from V_1 and $T_1 = 300$ K to V_2 and $T_2 = 600$ K (step a), an isothermal expansion at $T_2 = 600$ K from V_2 and $P_1 = 15$ atm to V_3 and $P_2 = 3$ atm (step b), an isobaric compression at $P_2 = 3$ atm from V_3 and $T_2 = 600$ K to V_4 and $T_1 = 300$ K (step c), and, finally, an isothermal compression at $T_1 = 300$ K from V_4 and $P_2 = 3$ atm to V_1 and $P_1 =$ 15 atm (step d). We are asked to find, for each step and the complete cycle (which is just the sum of the values for each step), the work done by the gas, the heat absorbed by the gas, the internal energy change of the gas, and the entropy change for the gas. The work done by a gas in an isobaric process is $W_P = P\Delta V$; the work done by a gas in an isothermal process is $W_T = nRT\ln\left(\frac{V_2}{V_1}\right)$. The heat absorbed by a gas in an isobaric process is $Q_P = nC_P\Delta T$; since we only know γ, we will need to represent C_P in terms of γ alone. The heat absorbed by a gas in an isothermal process is equal to the work done by the gas since the change in internal energy is equal to zero. In general, the change in internal energy is given by the first law of thermodynamics, but we know that $\Delta U = 0$ for an isothermal process. The entropy change of a gas in an isobaric process is related to the heat at constant pressure, $\Delta S_P = \int_{T_1}^{T_2} \frac{dQ_P}{T} = \int_{T_1}^{T_2} \frac{nC_P dT}{T}$. The entropy change of a gas in an isothermal process is $\Delta S_T = \frac{Q}{T}$.

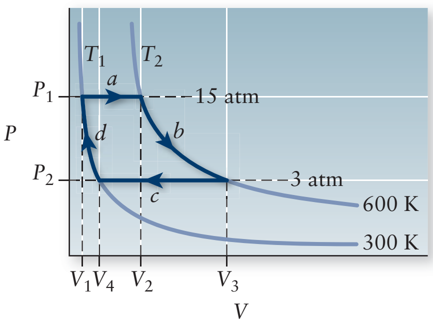


Figure 15-5 Problem 107

SOLVE

Work done by the gas

Isobaric expansion *a*:

$$W_a = P_1 \Delta V = P_1(V_2 - V_1) = P_1\left(\frac{nRT_2}{P_1} - \frac{nRT_1}{P_1}\right) = nR(T_2 - T_1)$$

$$= (1.23\ \text{mol})\left(8.314 \frac{\text{J}}{\text{mol} \cdot \text{K}}\right)((600\ \text{K}) - (300\ \text{K})) = \boxed{3070\ \text{J} = 3.07\ \text{kJ}}$$

Isothermal expansion *b*:

$$W_b = nRT_2 \ln\left(\frac{V_3}{V_2}\right) = nRT_2 \ln\left(\frac{\left(\frac{nRT_2}{P_2}\right)}{\left(\frac{nRT_2}{P_1}\right)}\right) = nRT_2 \ln\left(\frac{P_1}{P_2}\right)$$

$$= (1.23\ \text{mol})\left(8.314 \frac{\text{J}}{\text{mol} \cdot \text{K}}\right)(600\ \text{K}) \ln\left(\frac{15\ \text{atm}}{3\ \text{atm}}\right) = \boxed{9880\ \text{J} = 9.88\ \text{kJ}}$$

Isobaric compression *c*:

$$W_c = P_2 \Delta V = P_2(V_4 - V_3) = P_2\left(\frac{nRT_1}{P_2} - \frac{nRT_2}{P_2}\right) = nR(T_1 - T_2)$$

$$= (1.23\ \text{mol})\left(8.314 \frac{\text{J}}{\text{mol} \cdot \text{K}}\right)((300\ \text{K}) - (600\ \text{K})) = \boxed{-3070\ \text{J} = -3.07\ \text{kJ}}$$

Isothermal compression *d*:

$$W_d = nRT_1 \ln\left(\frac{V_1}{V_4}\right) = nRT_1 \ln\left(\frac{\left(\frac{nRT_1}{P_1}\right)}{\left(\frac{nRT_1}{P_2}\right)}\right) = nRT_1 \ln\left(\frac{P_2}{P_1}\right)$$

$$= (1.23\ \text{mol})\left(8.314 \frac{\text{J}}{\text{mol} \cdot \text{K}}\right)(300\ \text{K}) \ln\left(\frac{3\ \text{atm}}{15\ \text{atm}}\right) = \boxed{-4940\ \text{J} = -4.94\ \text{kJ}}$$

Total cycle:

$$W_{\text{total}} = W_a + W_b + W_c + W_d = (3.07\ \text{kJ}) + (9.88\ \text{kJ}) + (-3.07\ \text{kJ}) + (-4.94\ \text{kJ}) = \boxed{4.94\ \text{kJ}}$$

Heat absorbed by the gas

Representing C_P in terms of γ:

$$\gamma = \frac{C_P}{C_V}$$

$$C_P = \gamma C_V = \gamma(C_P - R)$$

$$C_P = \frac{\gamma R}{\gamma - 1}$$

Isobaric expansion *a*:

$$Q_a = nC_P\Delta T = n\left(\frac{\gamma R}{\gamma - 1}\right)(T_2 - T_1)$$

$$= (1.23\text{ mol})\left(\frac{(1.41)\left(8.314\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)}{1.41 - 1}\right)((600\text{ K}) - (300\text{ K})) = \boxed{10{,}550\text{ J} = 10.55\text{ kJ}}$$

Isothermal expansion *b*:

$$Q_b = W_b = \boxed{9.88\text{ kJ}}$$

Isobaric compression *c*:

$$Q_c = nC_P\Delta T = n\left(\frac{\gamma R}{\gamma - 1}\right)(T_1 - T_2)$$

$$= (1.23\text{ mol})\left(\frac{(1.41)\left(8.314\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)}{1.41 - 1}\right)((300\text{ K}) - (600\text{ K})) = \boxed{-10{,}550\text{ J} = -10.55\text{ kJ}}$$

Isothermal compression *d*:

$$Q_d = W_d = \boxed{-4.94\text{ kJ}}$$

Total cycle:

$$Q_{\text{total}} = Q_a + Q_b + Q_c + Q_d = (10.55\text{ kJ}) + (9.88\text{ kJ}) + (-10.55\text{ kJ}) + (-4.94\text{ kJ}) = \boxed{4.94\text{ kJ}}$$

<u>Internal energy change of the gas</u>

Isobaric expansion *a*:

$$\Delta U_a = Q_a - W_a = (10.55\text{ kJ}) - (3.07\text{ kJ}) = \boxed{7.48\text{ kJ}}$$

Isothermal expansion *b*:

$$\Delta U_b = \boxed{0}$$

Isobaric compression *c*:

$$\Delta U_c = Q_c - W_c = (-10.55\text{ kJ}) - (-3.07\text{ kJ}) = \boxed{-7.48\text{ kJ}}$$

Isothermal compression *d*:

$$\Delta U_d = \boxed{0}$$

Total cycle:

$$\Delta U_{\text{total}} = \Delta U_a + \Delta U_b + \Delta U_c + \Delta U_d = (7.48\text{ kJ}) + (0) + (-7.48\text{ kJ}) + (0) = \boxed{0}$$

Entropy change of the gas

Entropy change for an isobaric process:

$$\Delta S_P = \int_{T_1}^{T_2} \frac{dQ_P}{T} = \int_{T_1}^{T_2} \frac{nC_P dT}{T} = nC_P[\ln(T)]_{T_1}^{T_2} = nC_P \ln\left(\frac{T_2}{T_1}\right)$$

Isobaric expansion *a*:

$$\Delta S_a = nC_P \ln\left(\frac{T_2}{T_1}\right) = n\left(\frac{\gamma R}{\gamma - 1}\right)\ln\left(\frac{T_2}{T_1}\right)$$

$$= (1.23\text{ mol})\left(\frac{(1.41)\left(8.314\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)}{1.41 - 1}\right)\ln\left(\frac{600\text{ K}}{300\text{ K}}\right) = \boxed{24.4\frac{\text{J}}{\text{K}}}$$

Isothermal expansion *b*:

$$\Delta S_b = \frac{Q_b}{T_2} = \frac{9880\text{ J}}{600\text{ K}} = \boxed{16.5\frac{\text{J}}{\text{K}}}$$

Isobaric compression *c*:

$$\Delta S_c = nC_P \ln\left(\frac{T_1}{T_2}\right) = n\left(\frac{\gamma R}{\gamma - 1}\right)\ln\left(\frac{T_1}{T_2}\right)$$

$$= (1.23\text{ mol})\left(\frac{(1.41)\left(8.314\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)}{1.41 - 1}\right)\ln\left(\frac{300\text{ K}}{600\text{ K}}\right) = \boxed{-24.4\frac{\text{J}}{\text{K}}}$$

Isothermal compression *d*:

$$\Delta S_d = \frac{Q_d}{T_1} = \frac{-4940\text{ J}}{300\text{ K}} = \boxed{-16.5\frac{\text{J}}{\text{K}}}$$

Total cycle:

$$\Delta S_{\text{total}} = \Delta S_a + \Delta S_b + \Delta S_c + \Delta S_d = \left(24.4\frac{\text{J}}{\text{K}}\right) + \left(16.5\frac{\text{J}}{\text{K}}\right) + \left(-24.4\frac{\text{J}}{\text{K}}\right) + \left(-16.5\frac{\text{J}}{\text{K}}\right) = \boxed{0}$$

REFLECT

In short the gas absorbs heat and does work. The total change in entropy of the completed cycle is equal to zero because the engine runs reversibly.

15.108

SET UP

We are asked to derive the condition for the adiabatic expansion of an ideal gas, $PV^\gamma = \text{constant}$, where $\gamma = \dfrac{C_P}{C_V}$, from the differential form of the first law of thermodynamics, $dU = dQ - PdV$. In an adiabatic process, $dQ = 0$. We will also need the relationship between the change in internal energy and the specific heat at constant volume ($dU = nC_V dT$), the ideal gas law, and the fact that $C_P - C_V = R$.

SOLVE

First law of thermodynamics for an adiabatic process:

$$dU = dQ - PdV = 0 - PdV$$

$$nC_V dT = -PdV$$

$$ndT = -\frac{PdV}{C_V}$$

Differential form of the ideal gas law:

$$d(PV) = d(nRT)$$

$$VdP + PdV = nRdT$$

$$ndT = \frac{1}{R}(VdP + PdV)$$

But $ndT = -\frac{PdV}{C_V}$:

$$-\frac{PdV}{C_V} = \frac{1}{R}(VdP + PdV)$$

$$-\frac{PRdV}{C_V} = VdP + PdV$$

$$-\frac{P(C_P - C_V)dV}{C_V} = VdP + PdV$$

$$-\frac{PC_P dV}{C_V} + PdV = VdP + PdV$$

$$-P\gamma dV = VdP$$

$$\frac{(VdP + P\gamma dV)}{PV} = \frac{(0)}{PV}$$

$$\int \frac{dP}{P} + \int \frac{\gamma dV}{V} = \int 0$$

$$\ln(P) + \gamma \ln(V) = \text{constant}$$

$$\ln(P) + \ln(V^\gamma) = \text{constant}$$

$$\ln(PV^\gamma) = \text{constant}$$

$$\boxed{PV^\gamma = e^{\text{constant}} = \text{constant}}$$

REFLECT

Raising e to a number will just give us another number.

Chapter 16
Electrostatics I

Conceptual Questions

16.1 Similarities: The force varies as $\frac{1}{r^2}$; the force is directly proportional to the product of the masses or charges; the force is directed along the line that connects the two particles.

Differences: Newton's law includes a minus sign while Coulomb's law does not; like charges repel, like masses attract; the gravitational constant G is many orders of magnitude smaller than the Coulomb constant k; the gravitational force is many orders of magnitude weaker than the Coulomb force.

16.2 The gravitational force is ignored because it is many orders of magnitude weaker than the Coulomb force. The Coulomb force is about 10^{40} times stronger than the gravitational force on the scale of particles.

16.3 There would be no difference. The chargers were originally arbitrarily defined as positive and negative.

16.4 Either atoms would not be neutrally charged or the number of protons and electrons in a neutral atom would not be the same.

16.5 The mass decreases because electrons are removed from the object to make it positively charged.

16.6 Because the socks are identical, they acquire like charges as the result of sliding friction with clothes in the dryer; like charges repel.

16.7 Our current understanding is that like charges repel and opposite charges attract. If a charged insulating object either repels or attracts both charged glass (positive) and charged rubber (negative), then you may have discovered a new kind of charge. However, you may also just be observing polarization (to be discussed in later chapters).

16.8 The sliding of the slippers across the carpet rubs electrons from the fibers of the carpet onto the bottom of the slippers. As more electrons are added, they repel the previous ones and they start to spread out over the surface of the person's body. This is especially enhanced if the slippers have a synthetic covering and if the humidity is very low.

16.9 When the comb is run through your hair, electrons are transferred to the comb. The paper is polarized by the charged comb. The paper is then attracted to the comb. When they touch, a small amount of charge is transferred to the paper so now the paper and comb are similarly charged and repelled from each other.

16.10 When the plastic comb is run through your hair, electrons are transferred from your hair to it, so the comb acquires a net negative charge. This charge repels the free electrons in the aluminum, which results in some of the free electrons moving to the part of the can away from the comb. If the net charge on the can is zero, this leaves the part of the can next to the comb positively charged (with an electron deficit) and the part of the can away from the comb negatively charged (with an electron excess). This results in a net attractive force exerted by the comb on the can because the magnitude of the attractive force exerted on the nearby positive charge exceeds the magnitude of the repulsive force exerted on the more distant negative charge. When the comb touches the can, some electrons are transferred from the comb to the can, giving the can a net negative charge. When the negatively charged comb is again brought near the can, the force is again attractive, even though the net charge on the can is negative, and the attractive force on this positive charge exceeds the repulsive force on the larger, but considerably more distant, negative charge.

16.11 Part a) No, the object could be a conductor that is attracted after being polarized.

Part b) Yes, to be repelled the suspended object should be positively charged.

16.12 The main reason is humidity. The higher the humidity, the greater the rate at which static charges leak off charged objects into the air.

16.13 Part a) If the charges creating the field move, the fact that the field propagates at the speed of light also allows us to understand the changes in the field, and hence force on the other charged objects. With the electric field, we see that charged particles take some time to experience the effects of other charges moving near them.

Part b) In electrostatics the field is just a computational device, and using it is merely a matter of convenience. However, in electrodynamics the field is necessary for energy and momentum to be conserved, so it is more than just convenience that leads us to the electric field.

16.14 No. The electric field lines point in the direction of the electric force, but the force is in the direction of the acceleration, not the velocity.

16.15 The electric forces are equal in magnitude and opposite in direction. The force on the proton is downward, and the force on the electron is upward.

16.16 Electric fields obey the law of superposition—that is, to find the field at some point P due to a distribution of charges we simply add up (vectorally) the electric field due to each individual charge. We know from calculus that an integral is simply a sum of very small, continuously distributed quantities.

16.17 Only charges inside the Gaussian surface contribute a nonzero flux, so the electric field of charges outside the surface can be ignored.

16.18 The electric field a distance r from the center of a uniformly charged sphere is $\dfrac{kQ}{r^2}$, where Q is the charge inside a sphere of radius r. Inside the sphere, as one moves out

from the center Q increases with the cube of the distance from the center of the sphere. Thus, the electric field increases as r rather than decreases as $\frac{1}{r^2}$.

16.19 No, the charge density can be nonzero. All that is required by Gauss' law is that the total charge enclosed by the surface is zero.

16.20 Gauss' law, correctly applied, does not give the same result for the two cases. For the uniformly charged infinite plane, we consider a cylindrical Gaussian surface with the ends of the cylinder both parallel to and equidistant from the plane. From the symmetry we know that on the curved side of the cylinder the normal component of the electric field is zero everywhere, so the flux through this side is zero. Also from symmetry, we know that on the ends of the cylinder the normal component of the electric field is uniform and equal to its magnitude. Thus, the total flux out of the cylinder is $\Phi = 2EA = 4\pi k\sigma A$, so $E = 2\pi k\sigma$.

For the uniformly charged disk, we again consider a cylindrical Gaussian surface with the ends of the cylinder both parallel to and equidistant from the disk. There is insufficient symmetry to allow us to conclude either that on the curved side of the cylinder the normal component of the electric field is zero or that on the ends of the cylinder the normal component of the electric field is uniform and equal to its magnitude.

Multiple-Choice Questions

16.21 **B** (attract each other). Like charges repel; opposite charges attract.

16.22 **C** (acquires a positive charge). The combined net charge of the objects must be zero since they started uncharged.

16.23 **C** (F). The forces must have the same magnitude due to Newton's third law.

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16.24 **B** (insulator). Friction is required to add charge to the balloon.

16.25 **A** (positively). Charge can move easily in a conductor, so the mobile electrons will be attracted to the positively charged rod, leaving the other end positively charged.

16.26 **D** (accelerate in the same direction as the electric field). The electric field points in the direction of the force on a positive charge.

16.27 **C** (0). The net electric flux through the closed surface is proportional to the net charge enclosed by the surface, which is equal to zero.

16.28 **C** (Φ). The net electric flux through the closed surface is proportional to the net charge enclosed by the surface, which remains constant.

16.29 **B** ($-Q$). The electric field inside the conducting shell must be zero. The charge enclosed by a Gaussian sphere of radius r, where $R_1 < r < R_2$, must be zero. Therefore, the charge on the inner surface of the shell is $-Q$.

Estimation/Numerical Questions

16.30 Part a) For simplicity, we'll assume that the human body is made entirely out of water, which has a molar mass of 18 g/mol. Each water molecule has 10 protons—8 from oxygen, 1 from each hydrogen. The positive charge in a 70-kg person is:

$$70 \text{ kg} \times \frac{1000 \text{ g}}{1 \text{ kg}} \times \frac{1 \text{ mol}}{18 \text{ g}} \times \frac{6.02 \times 10^{23} \text{ molecules}}{1 \text{ mol}} \times \frac{10 \text{ protons}}{1 \text{ molecule}} \times \frac{1.6 \times 10^{-19} \text{ C}}{1 \text{ proton}}$$

$$= 4 \times 10^{9} \text{ C}$$

Part b) The net charge in your body is approximately zero because each atom has an equal number of positive and negative charges.

16.31 The electric force acting on the paper due to the comb must be equal in magnitude to the force of gravity acting on the paper. We'll assume the mass of the paper is on the order of 0.01 g. If the comb and paper have the same charge and are separated by 1 cm:

$$q = \sqrt{\frac{mgr^2}{k}} \sim \sqrt{\frac{(10^{-5})(10)(10^{-2})^2}{(10^{10})}} = 10^{-9} \text{ C} = 1 \text{ nC}$$

16.32 Part a)

$$F = \frac{kq_1q_2}{r^2} \sim \frac{(10^{10})(10^{-6})(10^{-6})}{(10^{-2})^2} = 10^{2} \text{ N}$$

Part b)

$$F = \frac{kq_1q_2}{r^2} \sim \frac{(10^{10})(10^{-9})(10^{-9})}{(10^{-2})^2} = 10^{-4} \text{ N}$$

16.33

$$F = \frac{kq_1q_2}{r^2}$$

$$q \sim \sqrt{\frac{(10 \text{ N})(10^{-1} \text{ m})^2}{\left(10^{10}\frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)}} = 10^{-5} \text{ C}$$

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16.34

$$1 \text{ mol e}^- \times \frac{6.02 \times 10^{23} \text{ e}^-}{1 \text{ mol e}^-} \times \frac{1.6 \times 10^{-19} \text{ C}}{1 \text{ e}^-} = 96{,}320 \text{ C}$$

16.35 The electric flux through the passenger window is about $0.05\frac{\text{N}\cdot\text{m}^2}{\text{C}}$.

16.36

$$F_g = F_E$$

$$mg = qE$$

$$E = \frac{mg}{q} = \frac{(10^{-3})(10)}{10^{-5}} = 10^3\frac{\text{N}}{\text{C}}$$

16.37

r (m)	F (N)
0.003	5500
0.004	3000
0.005	2000
0.010	600
0.020	175
0.040	30
0.050	10
0.080	6
0.100	5
0.200	1
0.300	0.5
0.400	0.35
0.500	0.25
0.600	0.15

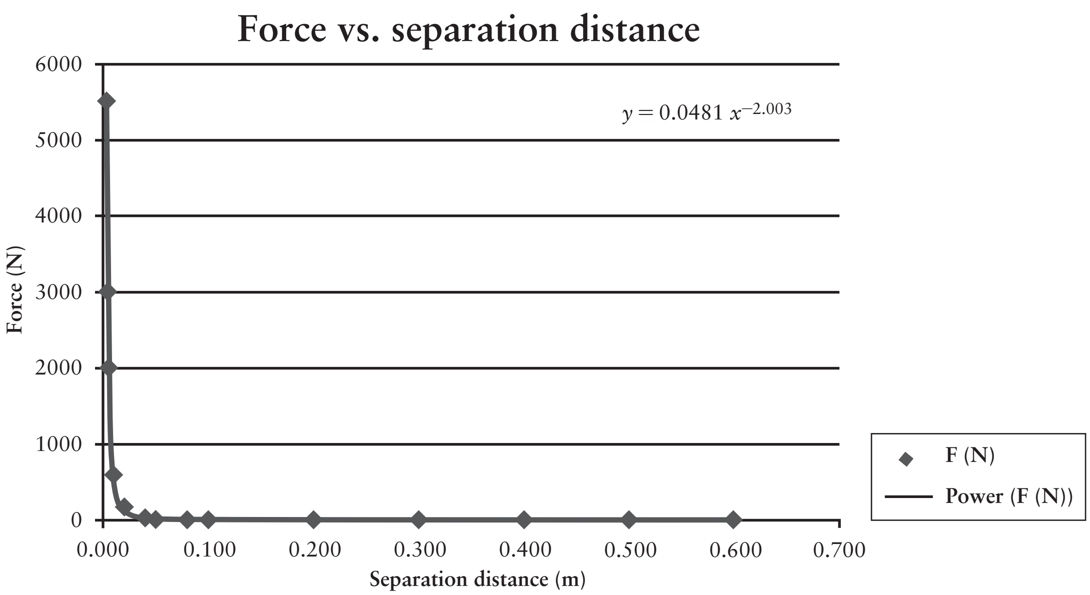

Figure 16-1 Problem 37

The curve of best fit goes as $r^{-2.003}$, which means $\boxed{n = 2}$.

Problems

16.38

SET UP

Five electrons are added to 1.0 C of positive charge. Each electron has a charge of -1.6×10^{-19} C. The net charge is the sum of the positive and negative charges.

SOLVE

$$q_{\text{net}} = (1.0\text{ C}) + 5(-1.6 \times 10^{-19}\text{ C}) = \boxed{1.0\text{ C}}$$

REFLECT

A charge of 1.0 C is very large, so adding 5 electrons to it will hardly change the total charge.

16.39

SET UP

A copper nucleus has 29 protons. Each proton has a charge of $+1.6 \times 10^{-19}$ C. The total charge of the nucleus is equal to the total charge due to the protons.

SOLVE

$$29\text{ protons} \times \frac{1.6 \times 10^{-19}\text{ C}}{1\text{ proton}} = \boxed{4.6 \times 10^{-18}\text{ C}}$$

REFLECT

A neutron has no charge.

16.40

SET UP

An ion has 17 protons and 18 electrons. The net charge on the ion is equal to the sum of the total positive charge due to the protons and the total negative charge due to the electrons.

SOLVE

$$q_{\text{net}} = 17(1.6 \times 10^{-19}\text{ C}) + 18(-1.6 \times 10^{-19}\text{ C}) = \boxed{-1.6 \times 10^{-19}\text{ C}}$$

REFLECT

An ion is an atom or molecule with a net charge. Since the charge on a proton and the charge on an electron are the same in magnitude but opposite in sign, 17 of the electrons effectively cancel out the 17 protons. The net charge is then just $-e = -1.6 \times 10^{-19}$ C.

16.41

SET UP

We are asked to calculate the number of coulombs of negative charge in 0.5 kg of water. To answer this, we need to know the total number of electrons in the sample. The molar mass of water is 18 g/mol, and 1 mol of water contains 6.02×10^{23} molecules of water. Each molecule

of water is made up of two hydrogen atoms and one oxygen atom. Hydrogen has one electron and oxygen has eight, which means each water molecule contains 10 electrons. The charge on one electron is -1.602×10^{19} C.

SOLVE

$$0.5\text{ kg} \times \frac{1000\text{ g}}{1\text{ kg}} \times \frac{1\text{ mol}}{18\text{ g}} \times \frac{6.02 \times 10^{23}\text{ molecules}}{1\text{ mol}} \times \frac{10\text{ electrons}}{1\text{ molecule}} \times \frac{-1.602 \times 10^{-19}\text{ C}}{1\text{ electron}}$$
$$= \boxed{-2.7 \times 10^{7}\text{ C}}$$

REFLECT

The net charge of 0.5 kg of water is zero since there are also 10 protons per molecule.

16.42

SET UP

The total number of electrons that must be transferred from an object to achieve a net charge of 1.6 C is equal to the desired net charge divided by the magnitude of the charge on one electron, $e = 1.6 \times 10^{-19}$ C.

SOLVE

$$1.6\text{ C} \times \frac{1\text{ electron}}{1.6 \times 10^{-19}\text{ C}} = \boxed{1.0 \times 10^{19}\text{ electrons}}$$

REFLECT

Since the charge is positive, the electrons must be removed from the object.

16.43

SET UP

The charge per unit length on a glass rod is 0.0050 C/m; the rod is 1×10^{-3} m long. Once we find the total charge on the rod, we can divide it by the charge on a single electron in order to calculate the number of electrons that were removed.

SOLVE

Total charge on the rod:

$$q = (1 \times 10^{-3}\text{ m})\left(0.0050\frac{\text{C}}{\text{m}}\right) = 5 \times 10^{-6}\text{ C}$$

Number of electrons:

$$n = \frac{q}{e} = \frac{5 \times 10^{-6}\text{ C}}{1.6 \times 10^{-19}\text{ C}} = \boxed{3.1 \times 10^{13}\text{ electrons}}$$

REFLECT

A constant charge per unit length means the charge is evenly distributed throughout the rod.

16.44

SET UP

Two point charges, q_1 and q_2, are separated by a distance $r = 20 \times 10^{-2}$ m. The magnitude of charge 2 is twice the magnitude of charge 1. Each charge exerts a force of $F = 45$ N on the other. Rearranging the algebraic expression for Coulomb's law, we can calculate the magnitude of the charges.

SOLVE

$$F = \frac{kq_1q_2}{r^2} = \frac{kq_1(2q_1)}{r^2}$$

$$q_1 = \sqrt{\frac{Fr^2}{2k}} = \sqrt{\frac{(45\text{ N})(20 \times 10^{-2}\text{ m})^2}{2\left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)}} = \boxed{1.0 \times 10^{-5}\text{ C}}$$

$$q_2 = 2q_1 = 2(1.0 \times 10^{-5}\text{ C}) = \boxed{2.0 \times 10^{-5}\text{ C}}$$

REFLECT

We weren't given any information on the direction of the force, so we don't know if these charges have like or unlike charges.

16.45

SET UP

We can use Coulomb's law to calculate the distance between two electrons such that the electrostatic force exerted by each on the other was equal in magnitude to the force of gravity on an electron. The mass of an electron is $m_e = 9.11 \times 10^{-31}$ kg; the magnitude of an electron's charge is $e = 1.602 \times 10^{-19}$ C.

SOLVE

$$\frac{k(e)(e)}{r^2} = m_e g$$

$$r = e\sqrt{\frac{k}{m_e g}} = (1.602 \times 10^{-19}\text{ C})\sqrt{\frac{\left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)}{(9.11 \times 10^{-31}\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)}} = \boxed{5.08\text{ m}}$$

REFLECT

The electric force between two electrons is repulsive, while the gravitational force between an electron and the Earth is attractive.

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16.46

SET UP

Charge A ($q_A = +5 \times 10^{-6}$ C) is located at the origin of a coordinate system. Charge B ($q_B = -3 \times 10^{-6}$ C) is fixed at $x = 3$ m. We can use Coulomb's law to calculate the magnitude and direction of the force that B exerts on A and the force that A exerts on B. The unit vector from B to A points in the $-x$ direction; the unit vector from A to B points in the $+x$ direction.

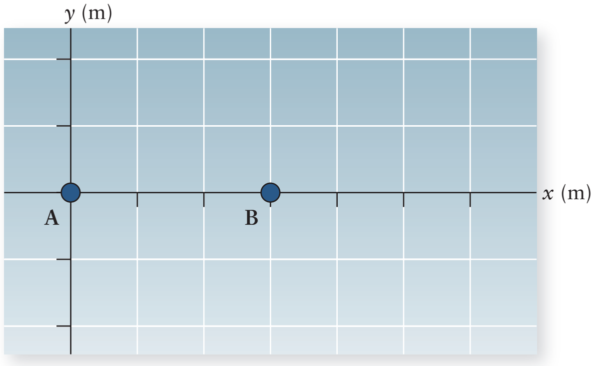

Figure 16-2 Problem 46

SOLVE

Part a)

$$\vec{F}_{B\to A} = \frac{kq_Aq_B}{x^2}(-\hat{x}) = -\frac{\left(8.99 \times 10^9 \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}\right)(5 \times 10^{-6}\ \mathrm{C})(-3 \times 10^{-6}\ \mathrm{C})}{(3\ \mathrm{m})^2}\hat{x}$$

$$= \boxed{(0.015\ \mathrm{N})\hat{x}}$$

Part b)

$$\vec{F}_{A\to B} = \frac{kq_Aq_B}{x^2}\hat{x} = \frac{\left(8.99 \times 10^9 \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}\right)(5 \times 10^{-6}\ \mathrm{C})(-3 \times 10^{-6}\ \mathrm{C})}{(3\ \mathrm{m})^2}\hat{x}$$

$$= \boxed{-(0.015\ \mathrm{N})\hat{x}}$$

REFLECT

The charges are opposite in sign, so they will attract one another, which means the force in part (a) should point to the right and the force in part (b) should point to the left. The force that charge A exerts on B must be equal in magnitude and opposite in direction to the force that charge B exerts on charge A due to Newton's third law.

16.47

SET UP

Charge A ($q_A = +3 \times 10^{-6}$ C) is located at the origin of a coordinate system. Charge B ($q_B = -4 \times 10^{-6}$ C) is fixed at $x = 3$ m; charge C ($q_C = -2 \times 10^{-6}$ C) is fixed at $x = 6$ m; and charge D ($q_D = +6 \times 10^{-6}$ C) is fixed at $x = 8$ m. We can use Coulomb's law to calculate the magnitude and direction of the net force on A due to the other charges. The unit vectors from B to A, C to A, and D to A all point in the $-x$ direction.

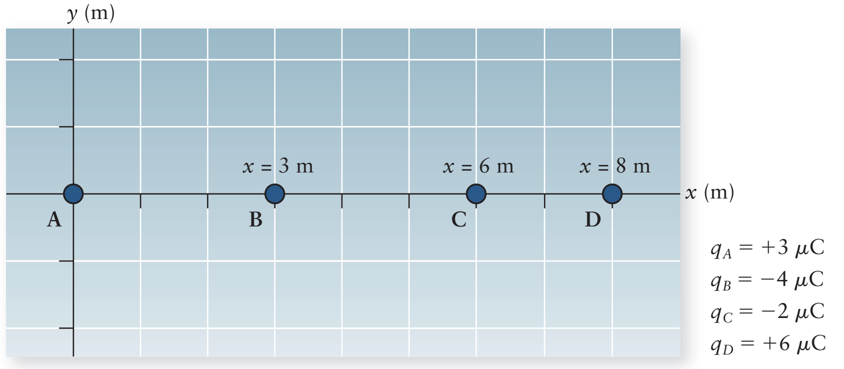

Figure 16-3 Problem 47

SOLVE

$$\sum \vec{F}_A = \vec{F}_{B\to A} + \vec{F}_{C\to A} + \vec{F}_{D\to A} = \frac{kq_Aq_B}{x_{AB}^2}(-\hat{x}) + \frac{kq_Aq_C}{x_{AC}^2}(-\hat{x}) + \frac{kq_Aq_D}{x_{AD}^2}(-\hat{x})$$

$$= -kq_A\left[\frac{q_B}{x_{AB}^2} + \frac{q_C}{x_{AC}^2} + \frac{q_D}{x_{AD}^2}\right]\hat{x}$$

$$= -\left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(3 \times 10^{-6}\ \text{C})\left[\frac{-4 \times 10^{-6}\ \text{C}}{(3\ \text{m})^2} + \frac{-2 \times 10^{-6}\ \text{C}}{(6\ \text{m})^2} + \frac{6 \times 10^{-6}\ \text{C}}{(8\ \text{m})^2}\right]\hat{x}$$

$$= \boxed{(0.0110\ \text{N})\hat{x}}$$

REFLECT

Even though q_D is larger in magnitude than both q_B and q_C, charges B and C are closer to A and should have a larger contribution to the net force. Since charges B and C are negative, it makes sense that the net force on A should point to the right.

16.48

SET UP

A charge ($q_1 = +3.00 \times 10^{-6}$ C) is located at the origin of a coordinate system. A second charge ($q_2 = -2.00 \times 10^{-6}$ C) is located at $x = 0.300$ m and $y = 0.200$ m. We can use Coulomb's law to calculate the electric force of charge 2 on charge 1.

SOLVE

Angle of the position vector for charge 2:

$$\tan(\theta) = \frac{0.200\ \text{m}}{0.300\ \text{m}}$$

$$\theta = \arctan\left(\frac{0.200\ \text{m}}{0.300\ \text{m}}\right) = 33.7^\circ$$

x component:

$$F_{2\to1,x} = -\frac{kq_1q_2}{r^2}\cos(\theta)$$

$$= -\frac{\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(3.00\times10^{-6}\ \text{C})(-2.00\times10^{-6}\ \text{C})}{(0.300\ \text{m})^2+(0.200\ \text{m})^2}\cos(33.7^\circ) = 0.345\ \text{N}$$

y component:

$$F_{2\to1,y} = -\frac{kq_1q_2}{r^2}\sin(\theta)$$

$$= -\frac{\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(3.00\times10^{-6}\ \text{C})(-2.00\times10^{-6}\ \text{C})}{(0.300\ \text{m})^2+(0.200\ \text{m})^2}\sin(33.7^\circ) = 0.230\ \text{N}$$

Force:

$$\boxed{\vec{F}_{2\to1} = (0.345\ \text{N})\hat{x} + (0.230\ \text{N})\hat{y}}$$

REFLECT

We could have also represented the force as a magnitude (0.415 N) and an angle (33.7 degrees). The charges are opposite in sign, so the force should be attractive and point up and to the right.

16.49

SET UP

Two charges, $q_A = +5$ nC and $q_B = -7$ nC, are located along the x-axis at $x = 0$ and $x = 5$ m, respectively. A third charge, $q_C = +2$ nC, is placed at a position x, where the net force acting on charge C is zero. To find this position, we can set the magnitude of the electric force of charge A on charge C equal to the magnitude of the electric force of charge B on charge C and solve for x.

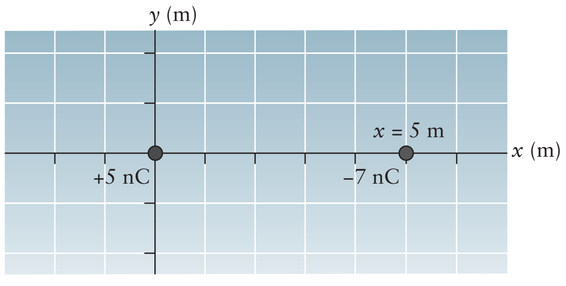

Figure 16-4 Problem 49

SOLVE

$$F_{A\to C} = F_{B\to C}$$

$$\left|\frac{kq_Aq_C}{x_{AC}^2}\right| = \left|\frac{kq_Bq_C}{x_{BC}^2}\right|$$

$$\frac{|q_A|}{x^2} = \frac{|q_B|}{(x-5)^2} \text{ (assume distances are in m, charges in nC)}$$

$$(|q_A| - |q_B|)x^2 - 10|q_A|x + 25|q_A| = 0$$

$$x = \frac{10|q_A| \pm \sqrt{(10|q_A|)^2 - 4(|q_A| - |q_B|)(25|q_A|)}}{2(|q_A| - |q_B|)}$$

$$x = \frac{50 \pm \sqrt{2500 - 4(-2)(125)}}{2(-2)} = \frac{50 \pm \sqrt{3500}}{-4}$$

Taking the positive root:

$$x = \frac{50 + \sqrt{3500}}{-4} = \boxed{-27.3 \text{ m}}$$

REFLECT

The $(x - 5)^2$ in the denominator of $F_{B\to C}$ represents the fact that charge B is located 5 m to the right of the origin. It makes sense that charge C must be placed to the left of charge A since the magnitude of q_B is larger than the magnitude of q_A and $\vec{F}_{A\to C}$ is repulsive, while the $\vec{F}_{B\to C}$ is attractive.

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16.50

SET UP

Two charges, $q_A = -2.00\ \mu C$ and $q_B = +3.00\ \mu C$, are located along the x-axis at $x = 0$ and $x = 0.100$ m, respectively. A third charge, $q_C = +4.00\ \mu C$, is placed at a position x, where the net force acting on charge C is zero. To find this position, we can set the magnitude of the electric force of charge A on charge C equal to the magnitude of the electric force of charge B on charge C and solve for x.

SOLVE

$$F_{A\to C} = F_{B\to C}$$

$$\left|\frac{kq_Aq_C}{x_{AC}^2}\right| = \left|\frac{kq_Bq_C}{x_{BC}^2}\right|$$

$$\frac{|q_A|}{x^2} = \frac{|q_B|}{(x-0.100)^2} \text{ (assume distances are in m, charges in } \mu\text{C)}$$

$$(|q_B| - |q_A|)x^2 + 0.200|q_A|x - 0.010|q_A| = 0$$

$$x = \frac{-0.200|q_A| \pm \sqrt{(0.200|q_A|)^2 - 4(|q_B| - |q_A|)(-0.010|q_A|)}}{2(|q_B| - |q_A|)}$$

$$= \frac{-0.400 \pm \sqrt{0.160 + 0.080}}{2} = \frac{-0.400 \pm \sqrt{0.240}}{2} = \frac{-0.400 \pm 0.490}{2}$$

Taking the negative root:

$$x = \frac{-0.400 - 0.490}{2} = \boxed{-0.445 \text{ m}}$$

REFLECT

The positive root gives $x = 0.045$ m, which is located in between charges A and B. Although the forces due to each charge on charge C are equal in magnitude, they point in the same direction in this region, which means the net force on charge C is nonzero. It makes sense that charge C must be placed to the left of charge A since the magnitude of q_B is larger than the magnitude of q_A and $\vec{F}_{A\to C}$ is attractive, while $\vec{F}_{B\to C}$ is repulsive.

16.51

SET UP

Charge 1 ($q_1 = +0.6 \times 10^{-6}$ C) is located at the origin of a coordinate system. Charge 2 ($q_2 = +0.8 \times 10^{-6}$ C) is fixed at $x = 5.0 \times 10^{-2}$ m. We can use Coulomb's law to calculate the magnitude and direction of the force that 1 exerts on 2 and the force that 2 exerts on 1. The unit vector from 1 to 2 points in the $+x$ direction; the unit vector from 2 to 1 points in the $-x$ direction. Since both charges are positive, they will repel one another. If charge 2 is changed to -0.8×10^{-6} C, then the magnitudes of the forces will be the same, but they will point in opposite directions.

SOLVE

Part a)

$$\vec{F}_{1\to 2} = \frac{kq_1q_2}{r^2}\hat{x} = \frac{\left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(0.6 \times 10^{-6}\text{ C})(0.8 \times 10^{-6}\text{ C})}{(5.0 \times 10^{-2}\text{ m})^2} = \boxed{(1.73 \text{ N})\hat{x}}$$

$$\vec{F}_{2\to 1} = \frac{kq_1q_2}{r^2}(-\hat{x}) = -\frac{\left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(0.6 \times 10^{-6}\text{ C})(0.8 \times 10^{-6}\text{ C})}{(5.0 \times 10^{-2}\text{ m})^2}$$

$$= \boxed{(-1.73 \text{ N})\hat{x}}$$

Part b) The magnitudes of the forces would remain the same, but the charges would attract rather than repel each other.

REFLECT

In part (a), the force of 1 on 2 points to the right and the force of 2 on 1 points to the left, as expected for two charges repelling one another.

16.52

SET UP

Two charges, q_1 and q_2, are equidistant to a positive charge q. The force vectors for $\vec{F}_1$ (q_1 on q) and $\vec{F}_2$ (q_2 on q) are shown. Charge q is being repelled by q_1, which means they have the same sign. The length of a vector corresponds to its magnitude; the longer the arrow, the larger the magnitude of the vector.

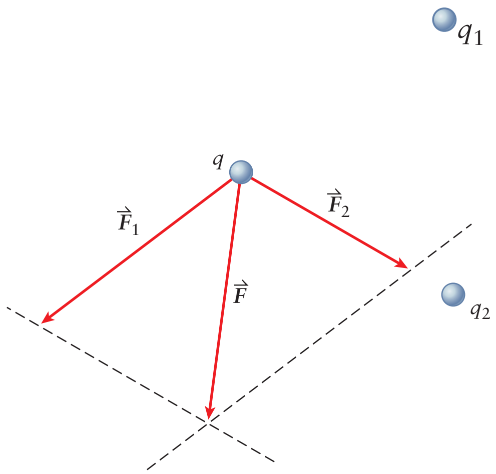


Figure 16-5 Problem 52

SOLVE

Part a) $\vec{F}_1$ points away from q_1, which means $\boxed{q_1 \text{ is a positive charge}}$.

Part b) The arrow denoting $\vec{F}_1$ is larger than the arrow denoting $\vec{F}_2$. Assuming q_1 and q_2 are equidistant to q, $\boxed{q_1 \text{ is larger}}$ in magnitude.

REFLECT

Charge q_2 must be negative if q is positive since $\vec{F}_2$ is attractive.

16.53

SET UP

Three positive charges ($q_A = 3 \times 10^{-6}$ C, $q_B = 6 \times 10^{-6}$ C, $q_C = 2 \times 10^{-6}$ C) are placed in a coordinate system (see figure). We can use Coulomb's law, $\vec{F} = \frac{kq_1q_2}{r^2}\hat{r}$, to calculate the net force acting on each charge due to the two others. Since all of the charges are positive, all of the forces will be repulsive.

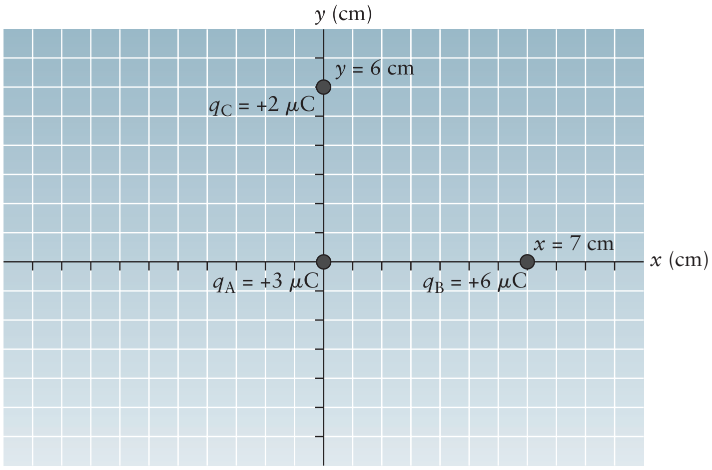


Figure 16-6 Problem 53

SOLVE

Charge A:

x component of net force on charge A:

$$\sum F_{q_A,x} = F_{q_B\to q_A,x} + F_{q_C\to q_A,x} = F_{q_B\to q_A,x} + 0 = -\frac{kq_Bq_A}{r_{AB}^2}$$

$$= -\frac{\left(8.99\times10^9\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(6\times10^{-6}\text{ C})(3\times10^{-6}\text{ C})}{(7\times10^{-2}\text{ m})^2} = -33.0\text{ N}$$

y component of net force on charge A:

$$\sum F_{q_A,y} = F_{q_B\to q_A,y} + F_{q_C\to q_A,y} = 0 + F_{q_C\to q_A,y} = -\frac{kq_Cq_A}{r_{AC}^2}$$

$$= -\frac{\left(8.99\times10^9\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(2\times10^{-6}\text{ C})(3\times10^{-6}\text{ C})}{(6\times10^{-2}\text{ m})^2} = -15.0\text{ N}$$

Magnitude of net force on charge A:

$$F_{q_A} = \sqrt{(-33.0\text{ N})^2 + (-15.0\text{ N})^2} = \boxed{36.3\text{ N}}$$

Direction of net force on charge A:

$$\theta_A = \arctan\left(\frac{-15.0\text{ N}}{-33.0\text{ N}}\right) = \boxed{204.4^\circ}\text{ (which is 24.4° below the }-x\text{-axis).}$$

Charge B:

x component of net force on charge B:

$$\sum F_{q_B,x} = F_{q_A\to q_B,x} + F_{q_C\to q_B,x} = \frac{kq_Aq_B}{r_{AB}^2} + \frac{kq_Cq_B}{r_{BC}^2}\cos(\theta_{C\to B})$$

$$= kq_B\left[\frac{q_A}{r_{AB}^2} + \frac{q_C}{r_{BC}^2}\left(\frac{7\text{ cm}}{\sqrt{(6\text{ cm})^2+(7\text{ cm})^2}}\right)\right]$$

$$= \left(8.99\times10^9\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(6\times10^{-6}\text{ C})\left[\frac{3\times10^{-6}\text{ C}}{(7\times10^{-2}\text{ m})^2} + \frac{2\times10^{-6}\text{ C}}{(6\times10^{-2}\text{ m})^2+(7\times10^{-2}\text{ m})^2}(0.759)\right]$$

$$= 42.7\text{ N}$$

y component of net force on charge B:

$$\sum F_{q_B,y} = F_{q_A\to q_B,y} + F_{q_C\to q_B,y} = 0 - \frac{kq_Cq_B}{r_{BC}^2}\sin(\theta_{C\to B}) = -\frac{kq_Bq_C}{r_{BC}^2}\left(\frac{6\text{ cm}}{\sqrt{(6\text{ cm})^2+(7\text{ cm})^2}}\right)$$

$$= -\frac{\left(8.99\times10^9\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(6\times10^{-6}\text{ C})(2\times10^{-6}\text{ C})}{(6\times10^{-2}\text{ m})^2+(7\times10^{-2}\text{ m})^2}(0.651)$$

$$= -8.26\text{ N}$$

Magnitude of net force on charge B:

$$F_{q_A} = \sqrt{(42.7\text{ N})^2 + (-8.26\text{ N})^2} = \boxed{43.5\text{ N}}$$

Direction of net force on charge B:

$$\theta_A = \arctan\left(\frac{-8.26\text{ N}}{42.7\text{ N}}\right) = \boxed{349.1°} \text{ (which is 10.9° below the } +x\text{-axis).}$$

<u>Charge C:</u>

x component of net force on charge C:

$$\sum F_{q_C,x} = F_{q_A \to q_C,x} + F_{q_B \to q_C,x} = 0 - \frac{kq_Bq_C}{r_{BC}^2}\cos(\theta_{B\to C}) = -\frac{kq_Bq_C}{r_{BC}^2}\left(\frac{7\text{ cm}}{\sqrt{(6\text{ cm})^2 + (7\text{ cm})^2}}\right)$$

$$= -\frac{\left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(6 \times 10^{-6}\text{ C})(2 \times 10^{-6}\text{ C})}{(6 \times 10^{-2}\text{ m})^2 + (7 \times 10^{-2}\text{ m})^2}(0.759)$$

$$= -9.63\text{ N}$$

y component of net force on charge C:

$$\sum F_{q_C,y} = F_{q_A \to q_C,y} + F_{q_B \to q_C,y} = \frac{kq_Aq_C}{r_{AC}^2} + \frac{kq_Bq_C}{r_{BC}^2}\sin(\theta_{B\to C}) = kq_C\left[\frac{q_A}{r_{AC}^2} + \frac{q_B}{r_{BC}^2}\left(\frac{6\text{ cm}}{\sqrt{(6\text{ cm})^2 + (7\text{ cm})^2}}\right)\right]$$

$$= \left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(2 \times 10^{-6}\text{ C})\left[\frac{(3 \times 10^{-6}\text{ C})}{(6 \times 10^{-2}\text{ m})^2} + \frac{(6 \times 10^{-6}\text{ C})}{(6 \times 10^{-2}\text{ m})^2 + (7 \times 10^{-2}\text{ m})^2}(0.651)\right]$$

$$= 23.2\text{ N}$$

Magnitude of net force on charge C:

$$F_{q_A} = \sqrt{(-9.63\text{ N})^2 + (23.2\text{ N})^2} = \boxed{25.2\text{ N}}$$

Direction of net force on charge C:

$$\theta_A = \arctan\left(\frac{23.2\text{ N}}{-9.63\text{ N}}\right) = \boxed{112.6°} \text{ (which is 67.4° above the } -x\text{-axis).}$$

REFLECT

We could have also represented all of the forces in terms of unit vectors: $\sum \vec{F}_{q_A} = -(33.0\text{ N})\hat{x} - (15.0\text{ N})\hat{y}$, $\sum \vec{F}_{q_B} = (42.7\text{ N})\hat{x} - (8.26\text{ N})\hat{y}$, and $\sum \vec{F}_{q_C} = -(9.63\text{ N})\hat{x} + (23.2\text{ N})\hat{y}$.

Get Help: P'Cast 16.2 – Red Blood Cells

16.54

SET UP

A square, thin, flat copper plate has dimensions of 4 cm × 4 cm and carries a total charge of -10×10^{-6} C. Copper is a conductor, so the electrons will redistribute themselves such that they are as far as possible from one another on the surface of the plate. The number of electrons added to the surface of the plate is equal to the total charge divided by the charge

of one electron, -1.6×10^{-19} C. Since the electrons are evenly spread out, the number of electrons on each side is the square root of the total; the distance between electrons is equal to the length of the side divided by the number of electrons per side.

SOLVE

Number of electrons:

$$-10 \times 10^{-6}\ \text{C} \times \frac{1\ \text{electron}}{-1.6 \times 10^{-19}\ \text{C}} = 6.25 \times 10^{13}\ \text{electrons}$$

Electrons per meter:

$$\frac{\sqrt{6.25 \times 10^{13}}\ \text{electrons}}{4\ \text{cm}} = 1.98 \times 10^{6}\frac{\text{electrons}}{\text{cm}} \times \frac{100\ \text{cm}}{1\ \text{m}} = 1.98 \times 10^{8}\frac{\text{electrons}}{\text{m}}$$

Distance between electrons:

$$\left[1.98 \times 10^{8}\frac{\text{electrons}}{\text{m}}\right]^{-1} = \boxed{5.06 \times 10^{-9}\frac{\text{m}}{\text{electron}}}$$

REFLECT

A separation distance of 5 nm seems reasonable.

16.55

SET UP

A sphere ($R = 0.0127$ m) has 2 C of positive charge distributed evenly throughout it. The charge per unit volume is equal to the total charge divided by the volume of the sphere. An insulator can have a net charge distributed throughout it, whereas a conductor in static equilibrium can only have a net charge on its surface.

SOLVE

Part a)

$$\rho = \frac{q}{V} = \frac{q}{\left(\frac{4}{3}\pi R^3\right)} = \frac{3q}{4\pi R^3} = \frac{3(2\ \text{C})}{4\pi(0.0127\ \text{m})^3} = \boxed{2.33 \times 10^{5}\frac{\text{C}}{\text{m}^3}}$$

Part b) The sphere is $\boxed{\text{insulating}}$. If it were conducting, none of the charge would be in the interior of the sphere; it would only be on the surface.

REFLECT

The charge per unit volume is also known as the volume charge density. If the sphere were conducting, we could calculate the charge per unit area by dividing the total charge by the surface area of the sphere.

16.56

SET UP

A sphere of diameter $D = 0.30$ m collects a charge of $q = 30 \times 10^{-6}$ C. The surface charge density is equal to the total charge divided by the surface area of the sphere.

SOLVE

$$\sigma = \frac{q}{A} = \frac{q}{4\pi R^2} = \frac{q}{4\pi\left(\frac{D}{2}\right)^2} = \frac{q}{\pi D^2} = \frac{30 \times 10^{-6}\ \mathrm{C}}{\pi(0.30\ \mathrm{m})^2} = \boxed{1.1 \times 10^{-4}\frac{\mathrm{C}}{\mathrm{m}^2}}$$

REFLECT

Since the sphere is conducting, the excess charge only exists on its surface.

16.57

SET UP

A person walking across a carpet accumulates -50 nC of charge in each step. Multiplying this value by 25 will tell us the amount of charge she builds up in 25 steps. Dividing the total charge on the person by the charge on a single electron (-1.602×10^{-19} C) yields the total number of excess electrons present due to static buildup. Finally, from these same values, we can calculate the number of steps required to accumulate a total of 10^{12} electrons.

SOLVE

Part a)

$$25\ \text{steps} \times \frac{-50\ \text{nC}}{1\ \text{step}} = \boxed{-1250\ \text{nC} = -1.25\ \mu\text{C}}$$

Part b)

$$-1.25 \times 10^{-6}\ \mathrm{C} \times \frac{1\ \text{electron}}{-1.602 \times 10^{-19}\ \mathrm{C}} = \boxed{7.80 \times 10^{12}\ \text{electrons}}$$

Part c)

$$10^{12}\ \text{electrons} \times \frac{-1.602 \times 10^{-19}\ \mathrm{C}}{1\ \text{electron}} \times \frac{1\ \text{step}}{-50 \times 10^{-9}\ \mathrm{C}} = 3.2\ \text{steps}$$

A worker $\boxed{\text{should not take more than 3 steps}}$ before touching the components.

REFLECT

People building electronics or working with precise instrumentation will "ground" themselves often by touching a metal conduit or wearing an antistatic wrist strap to rid themselves of excess charge buildup.

16.58

SET UP

We are given five charge distributions and asked to draw the electric field lines. Electric field lines start on positive charges and end on negative charges; they also never cross or split. The number of field lines associated with a charge is proportional to the strength of the charge. For example, a charge of $+2e$ will have twice as many field lines coming out of it as a charge of $+e$.

SOLVE

Part a)

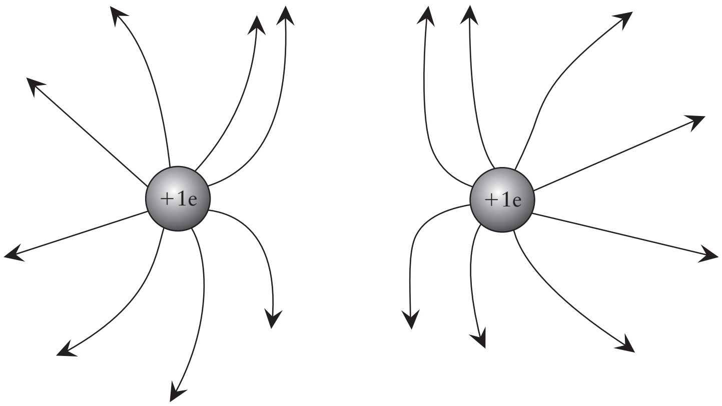


Figure 16-7 Problem 58

Part b)

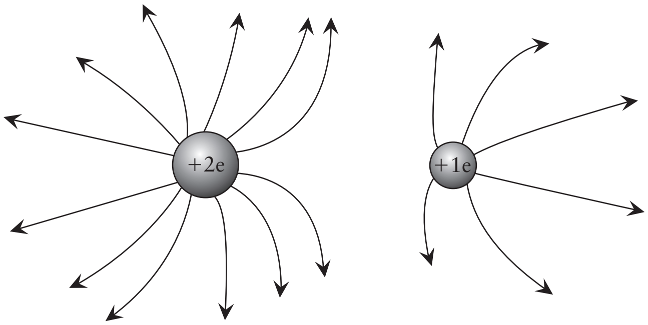


Figure 16-8 Problem 58

Part c)

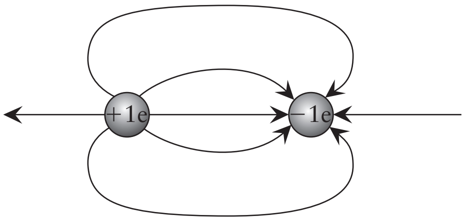


Figure 16-9 Problem 58

Part d)

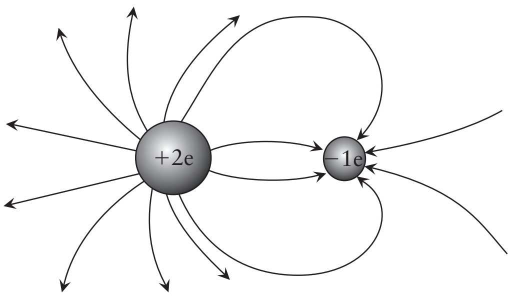


Figure 16-10 Problem 58

Part e)

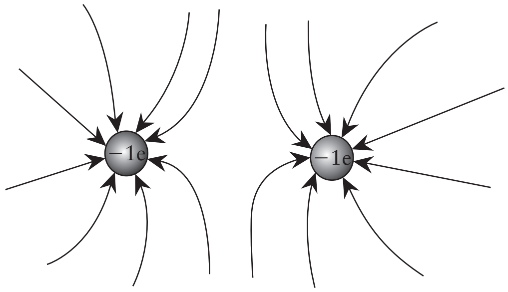


Figure 16-11 Problem 58

REFLECT

The field lines in parts (a) and (e) have the same shape, but the lines emanate from the positive charges and terminate on the negative charges.

16.59

SET UP

Point P is located to the left of two charges (see figure). The electric field at P is equal to zero, which means the fields due to q_1 and q_2 point in opposite directions. The magnitude of the electric field due to a point charge q is equal to $\frac{kq}{r^2}$. Because q_2 is farther away from point P than q_1, the magnitude of q_2 must be larger than that of q_1.

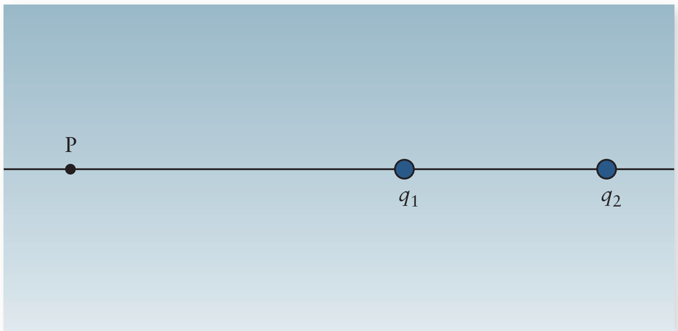


Figure 16-12 Problem 59

SOLVE

Part a) The charges q_1 and q_2 must have $\boxed{\text{opposite signs}}$.

Part b) Charge q_2 must have a larger magnitude to counter the fact that it is farther away from P.

REFLECT

From the information provided, we can only say that the charges have opposite signs and not which is positive and which is negative.

16.60

SET UP

An electric field near the surface of the Earth points straight down and has a magnitude of $E = 100$ N/C. Charge q is placed on a coin ($m = 3$ g), which causes it to rise straight up into the air at $a_y = 0.19$ m/s^2. We can use Newton's second law to calculate q. We'll define our coordinate system such that positive y points upward. The forces acting on the coin are the electric force and gravity.

SOLVE

$$\sum F_y = F_{E,y} + F_{g,y} = ma_y$$

$$-qE - mg = ma_y$$

$$q = -\frac{m(a_y + g)}{E} = -\frac{(3 \times 10^{-3}\ \text{kg})\left(\left(0.19\frac{\text{m}}{\text{s}^2}\right) + \left(9.8\frac{\text{m}}{\text{s}^2}\right)\right)}{\left(100\frac{\text{N}}{\text{C}}\right)} = \boxed{-3 \times 10^{-4}\ \text{C}}$$

REFLECT

The net force must point upward, which means the electric force acting on the coin must be upward. Since the electric field points downward, the charge on the coin must be negative.

16.61

SET UP

Two charges are placed on the x-axis: $q_1 = +5 \times 10^{-6}$ C at $x = 0$ and $q_2 = -10 \times 10^{-6}$ C at $x = 0.10$ m. The electric field due to a point charge is equal to $\vec{E} = \frac{kq}{r^2}\hat{r}$, where r is the distance from the charge. The electric field on the x-axis at $x = 0.06$ m is equal to the vector sum of the fields due to each charge at that point. At that point, the unit vector from q_1 points toward $+x$, and the unit vector from q_2 points toward $-x$. In order to determine where on the x-axis the electric field is zero, we need to write algebraic expressions for the magnitude of each field. The field due to q_1 on the x-axis is $\vec{E}_1 = \left(\frac{kq_1}{x^2}\right)\hat{x}$ and the field due to q_2 on the x-axis is $\vec{E}_2 = \left(\frac{kq_2}{(x-0.10)^2}\right)\hat{x}$ because it is centered at $x = 0.10$ m; note that SI units are assumed for both equations. By setting the sum of these expressions equal to zero and solving for x, we can find the position on the x-axis where $E = 0$.

SOLVE

Part a)

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \left(\frac{kq_1}{x_1^2}\right)\hat{x} + \left(\frac{kq_2}{x_2^2}\right)(-\hat{x}) = k\left[\frac{q_1}{x_1^2} - \frac{q_2}{x_2^2}\right]\hat{x}$$

$$= \left(8.99 \times 10^9\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left[\frac{5 \times 10^{-6}\ \text{C}}{(0.06\ \text{m})^2} - \frac{-10 \times 10^{-6}\text{C}}{(0.04\ \text{m})^2}\right]\hat{x} = \boxed{\left(6.9 \times 10^7\frac{\text{N}}{\text{C}}\right)\hat{x}}$$

Part b)

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \left(\frac{kq_1}{x^2}\right)\hat{x} + \left(\frac{kq_2}{(x-0.10)^2}\right)\hat{x} = 0 \text{ (SI units)}$$

$$\frac{q_1}{x^2} + \frac{q_2}{(x-0.10)^2} = 0$$

$$(q_1 + q_2)x^2 - 0.20q_1x + 0.01q_1 = 0$$

$$x = \frac{-(-0.20q_1) \pm \sqrt{(-0.20q_1)^2 - 4(q_1+q_2)(0.01q_1)}}{2(q_1+q_2)}$$

$$= \frac{(1.0 \times 10^{-6}) \pm (1.414 \times 10^{-6})}{-1.0 \times 10^{-5}}$$

Taking the positive root:

$$x = \frac{(1.0 \times 10^{-6}) + (1.414 \times 10^{-6})}{-1.0 \times 10^{-5}} = \boxed{-0.241 \text{ m} = -24.1 \text{ cm}}$$

REFLECT

The point where $E = 0$ must lie to the left of both charges, where it is closer to the smaller charge.

16.62

SET UP

Two charges are placed on the x-axis: $q_1 = +1 \times 10^{-7}$ C at $x_1 = 5$ cm and q_2 at $x_2 = 8$ cm. We are told that the electric field on the x-axis at $x = 0$ is equal to 0. The total electric field is equal to the vector sum of the fields due to each charge; the electric field due to a point charge is equal to $\vec{E} = \frac{kq}{r^2}\hat{r}$, where r is the distance from the charge. Since the field is equal to 0 at the origin, the x components of the field due to each charge must add to 0; from this, we can solve for q_2.

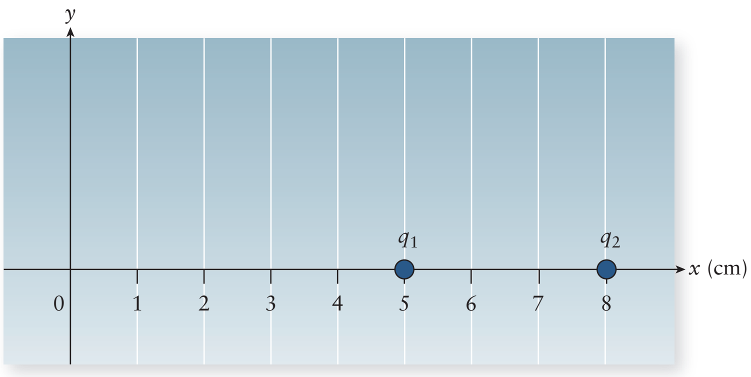


Figure 16-13 Problem 62

SOLVE

$$E_x = E_{1,x} + E_{2,x} = \frac{kq_1}{x_1^2} + \frac{kq_2}{x_2^2} = 0$$

$$\frac{q_1}{x_1^2} = -\frac{q_2}{x_2^2}$$

$$q_2 = -q_1\frac{x_2^2}{x_1^2} = -(10^{-7}\ \text{C})\frac{(8\ \text{cm})^2}{(5\ \text{cm})^2} = \boxed{-2.6 \times 10^{-7}\ \text{C}}$$

REFLECT

The magnitude of q_2 must be larger than the magnitude of q_1 since it is farther away from the origin. Also, q_2 must have the opposite sign of q_1 in order for their fields to cancel out.

16.63

SET UP

Two charges are placed on the x-axis: $q_1 = +1 \times 10^{-7}$ C at $x_1 = 5$ cm and q_2 at $x_2 = 8$ cm. The electric field at $x = 0$, $y = 3$ m is equal to the vector sum of the fields due to each charge; the electric field due to a point charge is equal to $\vec{E} = \frac{kq}{r^2}\hat{r}$, where r is the distance from the charge. Once we know the electric field at that location, the force on an electron there is equal to $\vec{F} = q\vec{E}$, where $q = -e$.

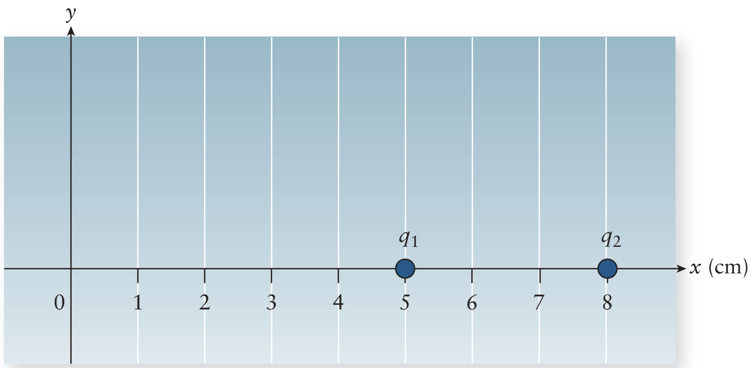


Figure 16-14 Problem 63

SOLVE

Part a)
x component:

$$E_x = E_{1,x} + E_{2,x} = \frac{kq_1}{r_1^2}\cos(\theta_1) + \frac{kq_2}{r_2^2}\cos(\theta_2) = k\left[\frac{q_1}{r_1^2}\cos(\theta_1) + \frac{q_2}{r_2^2}\cos(\theta_2)\right]$$

$$= \left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left[\frac{(10^{-7}\ \text{C})}{(0.05\ \text{m})^2 + (3\ \text{m})^2}\left(\frac{0.05\ \text{m}}{\sqrt{(0.05\ \text{m})^2 + (3\ \text{m})^2}}\right)\right.$$

$$\left. + \frac{(2 \times 10^{-7}\ \text{C})}{(0.08\ \text{m})^2 + (3\ \text{m})^2}\left(\frac{0.08\ \text{m}}{\sqrt{(0.08\ \text{m})^2 + (3\ \text{m})^2}}\right)\right] = 7\frac{\text{N}}{\text{C}}$$

y component:

$$E_y = E_{1,y} + E_{2,y} = \frac{kq_1}{r_1^2}\sin(\theta_1) + \frac{kq_2}{r_2^2}\sin(\theta_2) = k\left[\frac{q_1}{r_1^2}\sin(\theta_1) + \frac{q_2}{r_2^2}\sin(\theta_2)\right]$$

$$= \left(8.99 \times 10^9 \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}\right)\left[\frac{(10^{-7}\ \mathrm{C})}{(0.05\ \mathrm{m})^2 + (3\ \mathrm{m})^2}\left(\frac{3\ \mathrm{m}}{\sqrt{(0.05\ \mathrm{m})^2 + (3\ \mathrm{m})^2}}\right)\right.$$

$$\left. + \frac{(2 \times 10^{-7}\ \mathrm{C})}{(0.08\ \mathrm{m})^2 + (3\ \mathrm{m})^2}\left(\frac{3\ \mathrm{m}}{\sqrt{(0.08\ \mathrm{m})^2 + (3\ \mathrm{m})^2}}\right)\right] = 300\frac{\mathrm{N}}{\mathrm{C}}$$

Electric field vector:

$$\boxed{\vec{E} = \left(7\frac{\mathrm{N}}{\mathrm{C}}\right)\hat{x} + \left(300\frac{\mathrm{N}}{\mathrm{C}}\right)\hat{y}}$$

Part b)

$$\vec{F} = q\vec{E} = (-1.6 \times 10^{-19}\ \mathrm{C})\left[\left(7\frac{\mathrm{N}}{\mathrm{C}}\right)\hat{x} + \left(300\frac{\mathrm{N}}{\mathrm{C}}\right)\hat{y}\right]$$

$$= \boxed{-(1 \times 10^{-18}\ \mathrm{N})\hat{x} - (5 \times 10^{-17}\ \mathrm{N})\hat{y}}$$

REFLECT

The force on the electron points back toward the two positive charges as expected.

16.64

SET UP

Two charges are placed on the x-axis: $q_1 = +1.0 \times 10^{-7}$ C at $x_1 = 5.0$ cm and $q_2 = 2.0 \times 10^{-7}$ C at $x_2 = 8.0$ cm. The electric field at $x = 6.0$ m, $y = 3.0$ m is equal to the vector sum of the fields due to each charge; the electric field due to a point charge is equal to $\vec{E} = \frac{kq}{r^2}\hat{r}$, where r is the distance from the charge. Once we know the electric field at that location, the force on a proton there is equal to $\vec{F} = q\vec{E}$, where $q = +e$.

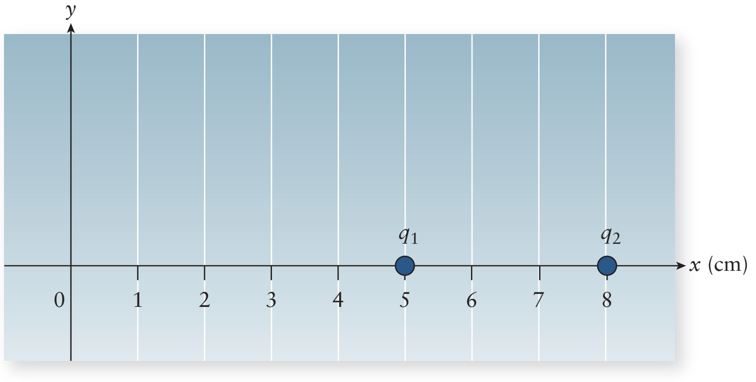


Figure 16-15 Problem 64

SOLVE

Part a)
x component:

$$E_x = E_{1,x} + E_{2,x} = \frac{kq_1}{r_1^2}\cos(\theta_1) + \frac{kq_2}{r_2^2}\cos(\theta_2) = k\left[\frac{q_1}{r_1^2}\cos(\theta_1) + \frac{q_2}{r_2^2}\cos(\theta_2)\right]$$

$$= \left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left[\frac{(10^{-7}\text{ C})}{(5.95\text{ m})^2 + (3.0\text{ m})^2}\left(\frac{5.95\text{ m}}{\sqrt{(5.95\text{ m})^2 + (3.0\text{ m})^2}}\right)\right.$$
$$\left. + \frac{(2 \times 10^{-7}\text{ C})}{(5.92\text{ m})^2 + (3.0\text{ m})^2}\left(\frac{5.92\text{ m}}{\sqrt{(5.92\text{ m})^2 + (3.0\text{ m})^2}}\right)\right] = 54\frac{\text{N}}{\text{C}}$$

y component:

$$E_y = E_{1,y} + E_{2,y} = \frac{kq_1}{r_1^2}\sin(\theta_1) + \frac{kq_2}{r_2^2}\sin(\theta_2) = k\left[\frac{q_1}{r_1^2}\sin(\theta_1) + \frac{q_2}{r_2^2}\sin(\theta_2)\right]$$

$$= \left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left[\frac{(10^{-7}\text{ C})}{(5.95\text{ m})^2 + (3.0\text{ m})^2}\left(\frac{3.0\text{ m}}{\sqrt{(5.95\text{ m})^2 + (3.0\text{ m})^2}}\right)\right.$$
$$\left. + \frac{(2.0 \times 10^{-7}\text{ C})}{(5.92\text{ m})^2 + (3.0\text{ m})^2}\left(\frac{3.0\text{ m}}{\sqrt{(5.92\text{ m})^2 + (3.0\text{ m})^2}}\right)\right] = 28\frac{\text{N}}{\text{C}}$$

Electric field vector:

$$\boxed{\vec{E} = \left(54\frac{\text{N}}{\text{C}}\right)\hat{x} + \left(28\frac{\text{N}}{\text{C}}\right)\hat{y}}$$

Part b)

$$\vec{F} = q\vec{E} = (1.6 \times 10^{-19}\text{ C})\left[\left(54\frac{\text{N}}{\text{C}}\right)\hat{x} + \left(28\frac{\text{N}}{\text{C}}\right)\hat{y}\right]$$
$$= \boxed{(8.7 \times 10^{-18}\text{ N})\hat{x} + (4.4 \times 10^{-18}\text{ N})\hat{y}}$$

REFLECT

The force on the protons points away from the two positive charges as expected.

16.65

SET UP

A hydrogen atom consists of an electron in a circular orbit of radius $a_0 = 5.29 \times 10^{-11}$ m around a proton. The force acting on the electron due to the proton causes the electron to undergo uniform circular motion. Using Newton's second law and the expression for the field due to a point charge, we can solve for the speed of the electron in orbit.

SOLVE

$$\sum F_x = F_E = m_e a_c$$

$$|qE| = \left|-e\left(\frac{ke}{a_0^2}\right)\right| = m_e\left(\frac{v^2}{a_0}\right)$$

$$v = \sqrt{\frac{ke^2}{a_0 m_e}} = \sqrt{\frac{\left(8.99 \times 10^9 \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}\right)(1.6 \times 10^{-19}\ \mathrm{C})^2}{(5.29 \times 10^{-11}\ \mathrm{m})(9.11 \times 10^{-31}\ \mathrm{kg})}} = \boxed{2.18 \times 10^6 \frac{\mathrm{m}}{\mathrm{s}}}$$

REFLECT

It's reasonable that the speed of the electron is much larger than 1 m/s.

16.66

SET UP

A rod of length L has a positive charge of Q evenly distributed throughout. We are asked to calculate the electric field strength at a point P that is a distance $x = D$ from the left end of the rod along the x-axis. We'll set up a coordinate system where the origin is at the left end of the rod with $+x$ pointing to the right and $+y$ pointing up toward P. Due to symmetry, the y component of the electric field at point P is equal to zero. Therefore, the electric field at P is equal to the magnitude of the x component of the electric field, $E_{P,x}$ and points toward the right. We can split the rod up into infinitesimal point charges dq and integrate over the field due to each point charge, $dE = \frac{kdq}{r^2}$, where r is the straight-line distance between dq and P. We can convert the integral from dq to dx by realizing the charge on the rod is uniformly distributed (that is, the linear charge density is constant). After we solve for an algebraic expression for the electric field, we can plug in $L = 1.25$ m, $Q = 20 \times 10^{-3}$ C, and $D = 3.25$ m, in order to calculate the field strength at P.

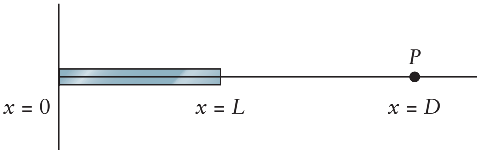


Figure 16-16 Problem 66

SOLVE

Part a)

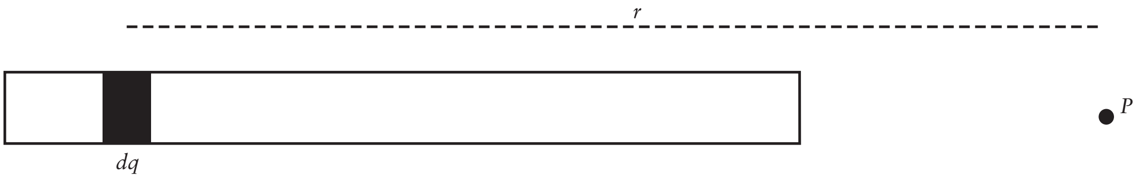


Figure 16-17 Problem 66

x component:

$$E_{P,x} = \int \frac{kdq}{r^2} = \int_0^L \frac{k}{r^2}\left(\frac{Q}{L}dx\right)$$

But $r = D - x$,

$$E_{P,x} = \frac{kQ}{L}\int_0^L \frac{dx}{(D-x)^2} = \frac{kQ}{L}\left[\frac{1}{D-x}\right]_0^L = \frac{kQ}{L}\left[\frac{1}{D-L} - \frac{1}{D}\right] = \frac{kQ}{L}\left[\frac{D-(D-L)}{D(D-L)}\right]$$

$$= \frac{kQ}{L}\left[\frac{L}{D(D-L)}\right] = \frac{kQ}{D(D-L)}$$

The electric field at point P is $\boxed{\vec{E}_P = \left[\frac{kQ}{D(D-L)}\right]\hat{x}}$.

Part b)

$$E_P = \frac{kQ}{D(D-L)} = \frac{\left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(20 \times 10^{-3}\ \text{C})}{(3.25\ \text{m})((3.25\ \text{m}) - (1.25\ \text{m}))} = \boxed{2.77 \times 10^7 \frac{\text{N}}{\text{C}}}$$

REFLECT

If we're very far away from the rod (that is, $D >> L$), the electric field reduces to the expression for the electric field due to a point charge, $\vec{E}_P = \left[\frac{kQ}{D^2}\right]\hat{x}$.

16.67

SET UP

A rod of length $L = 1.0$ m has a positive charge of $Q = 1.0$ C evenly distributed throughout. We are asked to calculate the electric field strength at a point P that is a distance $y_P = 0.50$ m from the middle of the rod along a line bisecting the rod (see figure). We'll set up a coordinate system where the origin is at the center of the rod with $+x$ pointing to the right and $+y$ pointing up towards P. Due to symmetry, the x component of the electric field at point P is equal to zero. Therefore, the electric field strength at P is equal to the magnitude of the y component of the electric field, E_y. We can split the rod up into infinitesimal point charges dq and integrate over the y components of the field due to each point charge, $dE_y = (dE)\cos(\theta)$ where θ is the angle made with the y-axis. The field dE due to the point charge dq is equal to $dE = \frac{kdq}{r^2}$, where r is the straight-line distance between dq and P. We can convert the integral from dq to dx by realizing the charge on the rod is uniformly distributed (*i.e.*, the linear charge density is constant).

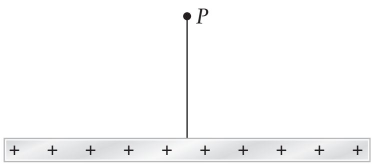


Figure 16-18 Problem 67

SOLVE

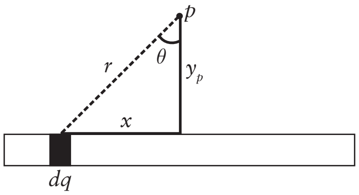


Figure 16-19 Problem 67

$$E_y = \int dE_y = \int (dE)\cos(\theta) = \int\left(\frac{kdq}{r^2}\right)\left(\frac{y_P}{r}\right)$$

Converting the integral from dq to dx using $\frac{dq}{Q} = \frac{dx}{L}$:

$$E_y = \int_{-L/2}^{L/2} \left(\frac{k}{r^2}\right)\left(\frac{y_P}{r}\right)\left(\frac{Q}{L}dx\right) = \frac{kQy_P}{L}\int_{-L/2}^{L/2} \frac{dx}{r^3}$$

But $r = \sqrt{x^2 + y_P^2} = (x^2 + y_P^2)^{1/2}$:

$$E_y = \frac{kQy_P}{L}\int_{-L/2}^{L/2} \frac{dx}{(x^2 + y_P^2)^{3/2}} = \frac{kQy_P}{L}\left[\frac{x}{y_P^2\sqrt{x^2 + y_P^2}}\right]_{-L/2}^{L/2} = \frac{kQ}{Ly_P}\left[\frac{\left(\frac{L}{2}\right)}{\sqrt{\left(\frac{L}{2}\right)^2 + y_P^2}} - \frac{\left(-\frac{L}{2}\right)}{\sqrt{\left(-\frac{L}{2}\right)^2 + y_P^2}}\right]$$

$$= \frac{kQ}{Ly_P}\left[\frac{L}{\sqrt{\left(\frac{L}{2}\right)^2 + y_P^2}}\right] = \frac{kQ}{y_P}\left[\frac{1}{\sqrt{\left(\frac{L}{2}\right)^2 + y_P^2}}\right]$$

$$= \frac{\left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(1.0\text{ C})}{0.50\text{ m}}\left[\frac{1}{\sqrt{\left(\frac{1.0\text{ m}}{2}\right)^2 + (0.50\text{ m})^2}}\right] = \boxed{2.5 \times 10^{10}\frac{\text{N}}{\text{C}}}$$

REFLECT

As L approaches infinity, our algebraic answer is equal to the electric field due to a very long, straight wire, $E = \frac{\lambda}{2\pi\epsilon_0 r}$.

Get Help: Interactive Example – Continuous Line of Charge

16.68

SET UP

A circular ring has a radius $R = 16$ cm and a uniformly distributed charge $Q = 0.5$ mC. The electric field at the center of the ring is equal to zero due to symmetry.

SOLVE

$$\boxed{E = 0}$$

REFLECT

The center of the ring is equidistant from every small dq of the ring. For every dq contributing to the electric field pointing one way, there is a dq on the opposite side contributing to the electric field pointing in the opposite way.

16.69

SET UP

A circular ring has a radius $R = 0.20$ m and a uniformly distributed charge of $Q = 0.5 \times 10^{-6}$ C. The magnitude of the electric field a distance $z = 0.10$ m on the axis of the ring is given by $E = \frac{kQz}{(R^2 + z^2)^{3/2}}$, as derived in Section 16–5 of the text.

SOLVE

$$E = \frac{kQz}{(R^2 + z^2)^{3/2}} = \frac{\left(8.99 \times 10^9 \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}\right)(0.5 \times 10^{-6}\ \text{C})(0.10\ \text{m})}{((0.20\ \text{m})^2 + (0.10\ \text{m}))^{3/2}} = \boxed{\left(4.0 \times 10^4 \frac{\text{N}}{\text{C}}\right)}$$

REFLECT

Due to symmetry, the electric field must point along the axis of the ring and away from it.

16.70

SET UP

A rod located on the x-axis from $x_1 = 1.00$ m to $x_2 = 3.00$ m has a uniform linear charge density of $\lambda = 4.00 \times 10^{-6}$ C/m. We are asked to calculate the electric field strength at a point P that is a distance $y_{\text{P}} = 3.00$ m on the y-axis. To find the electric field strength at P we can split the rod up into infinitesimal point charges dq and integrate over the x, $dE_x = -(dE)\sin(\theta)$, and y components, $dE_y = (dE)\cos(\theta)$, of the field due to each point charge where θ is the angle made with the y-axis. The field dE due to the point charge dq is equal to $dE = \frac{kdq}{r^2}$, where r is the straight-line distance between dq and P. We can convert the integral from dq to dx by realizing the charge on the rod is uniformly distributed (that is, the linear charge density is constant).

SOLVE

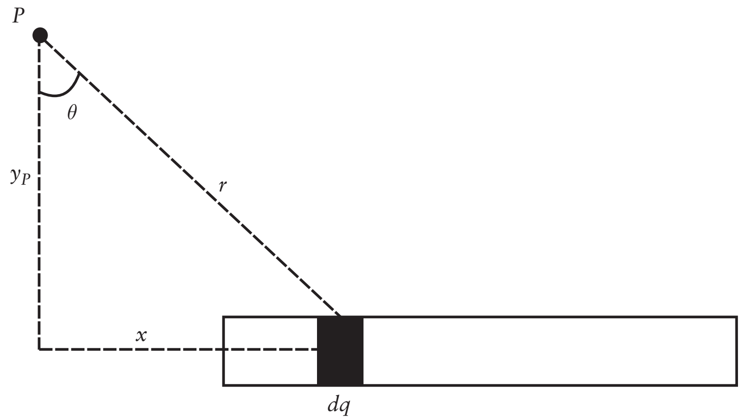


Figure 16-20 Problem 70

x component:

$$E_x = \int dE_x = -\int (dE)\sin(\theta) = -\int\left(\frac{kdq}{r^2}\right)\left(\frac{x}{r}\right)$$

Converting the integral from dq to dx using $\frac{dq}{Q} = \frac{dx}{L}$:

$$E_x = -\int_{x_1}^{x_2}\left(\frac{k}{r^2}\right)\left(\frac{x}{r}\right)\left(\frac{Q}{L}dx\right) = -\frac{kQ}{L}\int_{x_1}^{x_2}\frac{xdx}{r^3} = -k\lambda\int_{x_1}^{x_2}\frac{xdx}{r^3}$$

But $r = \sqrt{x^2 + y_P^2} = (x^2 + y_P^2)^{1/2}$:

$$E_x = -k\lambda\int_{x_1}^{x_2}\frac{xdx}{(x^2+y_P^2)^{3/2}} = -k\lambda\left[-\frac{1}{\sqrt{x^2+y_P^2}}\right]_{x_1}^{x_2} = k\lambda\left[\frac{1}{\sqrt{x_2^2+y_P^2}} - \frac{1}{\sqrt{x_1^2+y_P^2}}\right]$$

$$= \left(8.99\times 10^9\frac{\mathrm{C}^2}{\mathrm{N\cdot m^2}}\right)\left(4.00\times 10^{-6}\frac{\mathrm{C}}{\mathrm{m}}\right)\left[\frac{1}{\sqrt{(3.00\ \mathrm{m})^2+(3.00\ \mathrm{m})^2}} - \frac{1}{\sqrt{(1.00\ \mathrm{m})^2+(3.00\ \mathrm{m})^2}}\right]$$

$$= -2900\frac{\mathrm{N}}{\mathrm{C}}$$

y component:

$$E_y = \int dE_y = \int (dE)\cos(\theta) = \int\left(\frac{kdq}{r^2}\right)\left(\frac{y_P}{r}\right)$$

Converting the integral from dq to dx using $\frac{dq}{Q} = \frac{dx}{L}$:

$$E_y = \int_{x_1}^{x_2}\left(\frac{k}{r^2}\right)\left(\frac{y_P}{r}\right)\left(\frac{Q}{L}dx\right) = \frac{kQy_P}{L}\int_{x_1}^{x_2}\frac{dx}{r^3} = k\lambda y_P\int_{x_1}^{x_2}\frac{dx}{r^3}$$

But $r = \sqrt{x^2 + y_P^2} = (x^2 + y_P^2)^{1/2}$:

$$E_y = k\lambda y_P\int_{x_1}^{x_2}\frac{dx}{(x^2+y_P^2)^{3/2}} = k\lambda y_P\left[\frac{x}{y_P^2\sqrt{x^2+y_P^2}}\right]_{x_1}^{x_2} = \frac{k\lambda}{y_P}\left[\frac{x_2}{\sqrt{x_2^2+y_P^2}} - \frac{x_1}{\sqrt{x_1^2+y_P^2}}\right]$$

$$= \frac{k\lambda}{y_P}\left[\frac{x_2}{\sqrt{x_2^2+y_P^2}} - \frac{x_1}{\sqrt{x_1^2+y_P^2}}\right]$$

$$= \frac{\left(8.99\times 10^9\frac{\mathrm{C}^2}{\mathrm{N\cdot m^2}}\right)\left(4.00\times 10^{-6}\frac{\mathrm{C}}{\mathrm{m}}\right)}{3.00\ \mathrm{m}}\left[\frac{3.00\ \mathrm{m}}{\sqrt{(3.00\ \mathrm{m})^2+(3.00\ \mathrm{m})^2}} - \frac{1.00\ \mathrm{m}}{\sqrt{(1.00\ \mathrm{m})^2+(3.00\ \mathrm{m})^2}}\right]$$

$$= 4690\frac{\mathrm{N}}{\mathrm{C}}$$

Electric field at P:

$$\boxed{\vec{E}_P = \left(-2900\frac{\text{N}}{\text{C}}\right)\hat{x} + \left(4690\frac{\text{N}}{\text{C}}\right)\hat{y}}$$

REFLECT

Point P is located up and to the left of the positively charged rod. Therefore, the field at P should also point up and to the left, as we found.

16.71

SET UP

The electric field at a point along the central axis of a ring of charge Q is $E_y(y) = \dfrac{kQy}{(R^2+y^2)^{3/2}}$. In order to find the position where the field is a maximum, we need to set the derivative of E_y with respect to y equal to zero and solve for y. Evaluating E_y at this position will give us an expression for the maximum electric field.

SOLVE

Part a)

$$\frac{dE_y}{dy} = 0$$

$$\frac{d}{dy}\left[\frac{kQy}{(R^2+y^2)^{3/2}}\right] = kQ\frac{d}{dy}\left[y(R^2+y^2)^{-3/2}\right] = 0$$

$$\left[(R^2+y^2)^{-3/2} - \frac{3}{2}y(2y)\right] = (R^2+y^2)^{-5/2}\left[(R^2+y^2) - 3y^2\right] = (R^2+y^2)^{-5/2}\left[R^2 - 2y^2\right] = 0$$

$$y = \sqrt{\frac{R^2}{2}} = \boxed{\frac{R}{\sqrt{2}}}$$

Part b)

$$E_y\left(y = \frac{R}{\sqrt{2}}\right) = \frac{kQ\left(\dfrac{R}{\sqrt{2}}\right)}{\left(R^2 + \left(\dfrac{R}{\sqrt{2}}\right)^2\right)^{3/2}} = \frac{kQR}{\sqrt{2}\left(\dfrac{3R^2}{2}\right)^{3/2}} = \boxed{\frac{2kQ}{R^2\sqrt{27}}}$$

REFLECT

The magnitude of the field will be the same for an equal distance above and below the ring by symmetry.

Get Help: Interactive Example – Continuous Line of Charge

16.72

SET UP

A rectangular area A is rotated in a uniform electric field (magnitude E) from a position where the electric flux is a maximum to an orientation where the flux is one-half the maximum value. The electric flux is equal to the dot product between the electric field vector and the area vector, $\Phi = EA\cos(\theta)$. The maximum flux occurs when the electric field and area vector are parallel, that is, $\theta = 0$.

SOLVE

Maximum flux:

$$\Phi_{max} = EA\cos(0°) = EA$$

Angle of rotation:

$$\Phi = EA\cos(\theta) = \frac{\Phi_{max}}{2} = \frac{EA}{2}$$

$$\cos(\theta) = \frac{1}{2}$$

$$\theta = \arccos\left(\frac{1}{2}\right) = 60°$$

The rectangle was rotated through an angle of $\boxed{60 \text{ degrees}}$.

REFLECT

Due to symmetry, the rectangular area can be rotated 60 degrees clockwise or 60 degrees counterclockwise.

16.73

SET UP

A point charge of 4.0×10^{-12} C is located at the center of a cubical Gaussian surface. Gauss' law relates the total electric flux through a closed surface to the net charge enclosed by the surface. Since the point charge is located at the center of the cube, the flux will be equal through each of the six faces. Therefore, the flux through one face is equal to one-sixth of the total flux.

SOLVE

Total flux:

$$\Phi = \frac{q_{encl}}{\epsilon_0} = \frac{4.0 \times 10^{-12} \text{ C}}{\left(8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}\right)} = 0.452 \frac{\text{N} \cdot \text{m}^2}{\text{C}}$$

Flux through one face:

$$\Phi_{1\text{ face}} = \frac{\Phi}{6} = \frac{\left(0.452\frac{\text{N}\cdot\text{m}^2}{\text{C}}\right)}{6} = \boxed{0.075\frac{\text{N}\cdot\text{m}^2}{\text{C}}}$$

REFLECT

The total flux through the cube will be constant regardless of where inside the cube the charge is located. However, the flux through a single face will not.

16.74

SET UP

The net electric flux through a box is $\Phi = 4.80 \times 10^3\ \frac{\text{N}\cdot\text{m}^2}{\text{C}}$. The charge enclosed by the box is given by Gauss' law, $\Phi = \frac{q_{\text{encl}}}{\epsilon_0}$.

SOLVE

$$\Phi = \frac{q_{\text{encl}}}{\epsilon_0}$$

$$q_{\text{encl}} = \Phi\epsilon_0 = \left(4.80 \times 10^3\frac{\text{N}\cdot\text{m}^2}{\text{C}}\right)\left(8.85 \times 10^{-12}\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right) = \boxed{4.25 \times 10^{-8}\ \text{C}}$$

REFLECT

The size of the box is irrelevant in this problem since we are given the flux.

16.75

SET UP

A uniformly charged plastic rod ($L = 0.100$ m) is sealed inside of a plastic bag. The net electric flux through the bag is $\Phi = 7.5 \times 10^5\frac{\text{N}\cdot\text{m}^2}{\text{C}}$. We can use Gauss' law, $\Phi = \frac{q_{\text{encl}}}{\epsilon_0}$, to calculate the linear charge density of the rod.

SOLVE

$$\Phi = \frac{q_{\text{encl}}}{\epsilon_0} = \frac{\lambda L}{\epsilon_0}$$

$$\lambda = \frac{\Phi\epsilon_0}{L} = \frac{\left(7.5 \times 10^5\frac{\text{N}\cdot\text{m}^2}{\text{C}}\right)\left(8.85 \times 10^{-12}\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)}{0.100\ \text{m}} = \boxed{6.6 \times 10^{-5}\frac{\text{C}}{\text{m}}}$$

REFLECT

The net electric flux through the bag is positive, which means the rod is positively charged. The exact shape of the bag is irrelevant because we are given the flux.

16.76

SET UP

A prism-shaped object (see figure) is 0.400 m high, 0.300 m deep, and 0.800 m long. The hypotenuse of the right triangle is 0.500 m. The object is immersed in a uniform electric field $\vec{E} = \left(500\frac{\text{N}}{\text{C}}\right)\hat{x}$. The electric flux through each numbered surface is equal to the dot product between the electric field vector and the area vector for that surface, $\Phi = \vec{E} \cdot \vec{A}$. To make the calculation easier, remember that the area vector points "outward," the dot product of a unit vector with itself is 1, and the dot product between two unlike unit vectors is 0. We can also find the angle between the two vectors and use the alternate form of the dot product $\Phi = EA\cos(\theta)$, where θ is the angle between $\vec{E}$ and $\vec{A}$; the electric field $\vec{E}$ is parallel to the x-axis and the angle $\vec{A}$ makes with the x-axis is given by $\cos(\theta) = \dfrac{40\text{ cm}}{50\text{ cm}}$ from geometry. The net flux is equal to the sum of the flux through each surface. Inserting a negative charge into the object will create a net negative flux, according to Gauss' law.

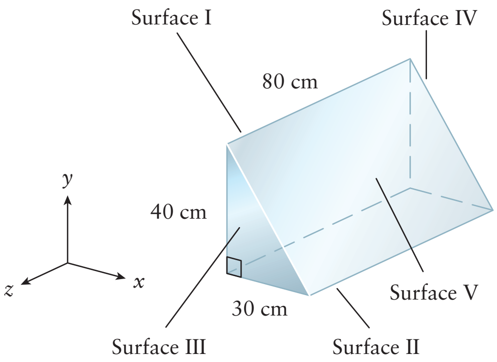


Figure 16-21 Problem 76

SOLVE

Part a)

Flux through Surface I:

$$\Phi_{\text{I}} = \vec{E} \cdot \vec{A}_{\text{I}} = \left(\left(500\frac{\text{N}}{\text{C}}\right)\hat{x}\right) \cdot ((0.400\text{ m})(0.800\text{ m})(-\hat{x})) = \boxed{-160\frac{\text{N}\cdot\text{m}^2}{\text{C}}}$$

Flux through Surface II:

$$\Phi_{\text{II}} = \vec{E} \cdot \vec{A}_{\text{II}} = \left(\left(500\frac{\text{N}}{\text{C}}\right)\hat{x}\right) \cdot (A_{\text{II}}(-\hat{y})) = \boxed{0}$$

Flux through Surface III:

$$\Phi_{\text{III}} = \vec{E} \cdot \vec{A}_{\text{III}} = \left(\left(500\frac{\text{N}}{\text{C}}\right)\hat{x}\right) \cdot (A_{\text{III}}(\hat{z})) = \boxed{0}$$

Flux through Surface IV:

$$\Phi_{\text{IV}} = \vec{E} \cdot \vec{A}_{\text{IV}} = \left(\left(500\frac{\text{N}}{\text{C}}\right)\hat{x}\right) \cdot (A_{\text{IV}}(-\hat{z}) = \boxed{0}$$

Flux through Surface V:

$$\Phi_{\text{V}} = \vec{E} \cdot \vec{A}_{\text{V}} = EA_{\text{V}}\cos(\theta) = \left(500\frac{\text{N}}{\text{C}}\right)(0.800\text{ m})(0.500\text{ m})\left(\frac{0.400\text{ m}}{0.500\text{ m}}\right) = \boxed{160\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}}$$

Part b)

$$\Phi_{total} = \Phi_I + \Phi_{II} + \Phi_{III} + \Phi_{IV} + \Phi_V$$
$$= \left(-160\,\frac{N \cdot m^2}{C}\right) + 0 + 0 + 0 + \left(160\,\frac{N \cdot m^2}{C}\right) = \boxed{0}$$

Part c) The net flux would now be negative since the object encloses a negative charge.

REFLECT

Since there is no charge inside the object in parts (a) and (b), the net flux through the object should equal zero from Gauss' law.

16.77

SET UP

A prism-shaped object (see figure) is 0.400 m high, 0.300 m deep, and 0.800 m long. The hypotenuse of the right triangle is 0.500 m. The object is immersed in an electric field $\vec{E} = \left(500\frac{N}{C}\right)\hat{x} + \left(400\frac{N}{C}\right)\hat{y}$. The electric flux through each numbered surface is equal to the dot product between the electric field vector and the area vector for that surface, $\Phi = \vec{E} \cdot \vec{A}$. To make the calculation easier, remember that the area vector points "outward," the dot product of a unit vector with itself is 1, and the dot product between two unlike unit vectors is 0. We can also find the angle between the two vectors and use the alternate form of the dot product $\Phi = EA\cos(\theta)$, where θ is the angle between $\vec{E}$ and $\vec{A}$; for example, the angle $\vec{E}$ makes with the x-axis is equal to $\arctan\left(\frac{E_y}{E_x}\right)$ and the angle Surface V (that is, $\vec{A}_v$) makes with the x-axis is given by $\cos(\theta) = \frac{40\text{ cm}}{50\text{ cm}}$ from geometry. The net flux is equal to the sum of the flux through each surface. Inserting a negative charge into the object will create a net negative flux, according to Gauss' law.

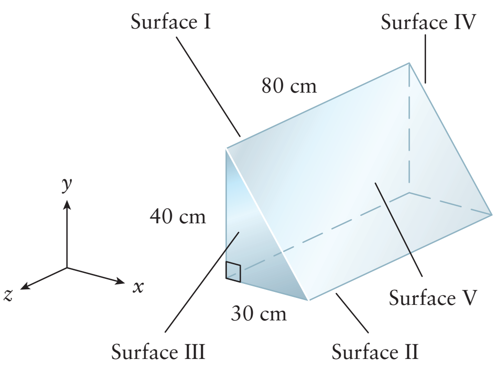


Figure 16-22 Problem 77

SOLVE

Part a)
Flux through Surface I:

$$\Phi_I = \vec{E} \cdot \vec{A}_I = \left(\left(500\frac{N}{C}\right)\hat{x} + \left(400\frac{N}{C}\right)\hat{y}\right) \cdot ((0.400\text{ m})(0.800\text{ m})(-\hat{x})) = \boxed{-160\frac{N \cdot m^2}{C}}$$

Flux through Surface II:

$$\Phi_{\mathrm{II}} = \vec{\mathrm{E}} \cdot \vec{\mathrm{A}}_{\mathrm{II}} = \left(\left(500\frac{\mathrm{N}}{\mathrm{C}}\right)\hat{x} + \left(400\frac{\mathrm{N}}{\mathrm{C}}\right)\hat{y}\right) \cdot ((0.30\ \mathrm{m})(0.80\ \mathrm{m})(-\hat{y})) = \boxed{-96\frac{\mathrm{N}\cdot\mathrm{m}^2}{\mathrm{C}}}$$

Flux through Surface III:

$$\Phi_{\mathrm{III}} = \vec{E} \cdot \vec{A}_{\mathrm{III}} = \left(\left(500\frac{\mathrm{N}}{\mathrm{C}}\right)\hat{x} + \left(400\frac{\mathrm{N}}{\mathrm{C}}\right)\hat{y}\right) \cdot (A_{\mathrm{III}}(\hat{z})) = \boxed{0}$$

Flux through Surface IV:

$$\Phi_{\mathrm{IV}} = \vec{E} \cdot \vec{A}_{\mathrm{IV}} = \left(\left(500\frac{\mathrm{N}}{\mathrm{C}}\right)\hat{x} + \left(400\frac{\mathrm{N}}{\mathrm{C}}\right)\hat{y}\right) \cdot (A_{\mathrm{IV}}(-\hat{z}) = \boxed{0}$$

Flux through Surface V:

$$\theta_E = \arctan\left(\frac{E_y}{E_x}\right) = \arctan\left(\frac{\left(400\frac{\mathrm{N}}{\mathrm{C}}\right)}{\left(500\frac{\mathrm{N}}{\mathrm{C}}\right)}\right) = 38.66^{\circ}$$

$$\theta_A = \arccos\left(\frac{40\ \mathrm{cm}}{50\ \mathrm{cm}}\right) = 36.87^{\circ}$$

$$\Phi_{\mathrm{V}} = \vec{E} \cdot \vec{A}_{\mathrm{V}} = \left(\sqrt{\left(500\frac{\mathrm{N}}{\mathrm{C}}\right)^2 + \left(400\frac{\mathrm{N}}{\mathrm{C}}\right)^2}\right)(0.80\ \mathrm{m})(0.50\ \mathrm{m})\cos((38.68^{\circ}) - (36.87^{\circ}))$$

$$= \boxed{256\frac{\mathrm{N}\cdot\mathrm{m}^2}{\mathrm{C}}}$$

Part b)

$$\Phi_{\mathrm{total}} = \Phi_{\mathrm{I}} + \Phi_{\mathrm{II}} + \Phi_{\mathrm{III}} + \Phi_{\mathrm{IV}} + \Phi_{\mathrm{V}}$$

$$= \left(-160\frac{\mathrm{N}\cdot\mathrm{m}^2}{\mathrm{C}}\right) + \left(-96\frac{\mathrm{N}\cdot\mathrm{m}^2}{\mathrm{C}}\right) + 0 + 0 + \left(256\frac{\mathrm{N}\cdot\mathrm{m}^2}{\mathrm{C}}\right) = \boxed{0}$$

REFLECT

Since there is no charge inside the object, the net flux through the object should equal zero from Gauss' law.

16.78

SET UP

Four objects—a solid cylinder, a flat plane, a solid sphere, and a hollow sphere—carry a charge $+Q$. The (volume) charge density ρ of a solid object is equal to its total charge divided

by its volume; the (surface) charge density of a plane and of hollow objects is equal to its total charge divided by its surface area.

SOLVE

Part a)

$$\rho = \frac{Q}{V} = \boxed{\frac{Q}{\pi R^2 L}}$$

Part b)

$$\sigma = \frac{Q}{A} = \boxed{\frac{Q}{WL}}$$

Part c)

$$\rho = \frac{Q}{V} = \frac{Q}{\left(\frac{4}{3}\pi R^3\right)} = \boxed{\frac{3Q}{4\pi R^3}}$$

Part d)

$$\sigma = \frac{Q}{A} = \boxed{\frac{Q}{4\pi R^2}}$$

REFLECT

If a charge is "uniformly distributed" then its charge density is constant.

16.79

SET UP

A sphere carries a uniform surface charge density (charge per unit area) σ. We can use Gauss' law to find an expression for the electric field just outside the surface of the sphere. At this location, the Gaussian sphere will have approximately the same surface area as the charged sphere, which we'll call A. Assuming σ is positive, the electric field will point radially outward from the sphere.

SOLVE

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\epsilon_0}$$

$$EA = \frac{(\sigma A)}{\epsilon_0}$$

$$\boxed{E = \frac{\sigma}{\epsilon_0}, \text{ pointing radially outward}}$$

REFLECT

The electric field is constant just outside the sphere.

16.80

SET UP

An infinite plane carries a uniformly distributed charge $+Q$; the surface charge density of the plane is $+\sigma$. We can use Gauss' law to calculate the electric field above and below the plane. We'll use a Gaussian cylinder of length L and cross-sectional area A. From symmetry, the electric field should point perpendicularly out of the surface of the plane, both above and below it; therefore, there will be no flux through the sides of the cylinder. The charge enclosed by the cylinder is equal to the charge located in the circular area A that intersects the plane. Once we have the expression for the electric field due to one plane, we can use superposition to find the electric field everywhere in space for two equal but oppositely charged parallel planes.

SOLVE

Part a)

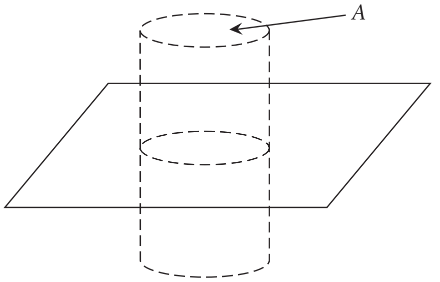


Figure 16-23 Problem 80

$$\Phi_{\text{total}} = \Phi_{\text{top}} + \Phi_{\text{bottom}} + \Phi_{\text{sides}} = \int_{\text{top}} \vec{E} \cdot d\vec{A} + \int_{\text{bottom}} \vec{E} \cdot d\vec{A} + \int_{\text{sides}} \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\epsilon_0}$$

$$EA + EA + 0 = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0}$$

The electric field has a magnitude of $\boxed{\dfrac{\sigma}{2\epsilon_0} \text{ and points outward perpendicularly to the plane}}$.

Part b)

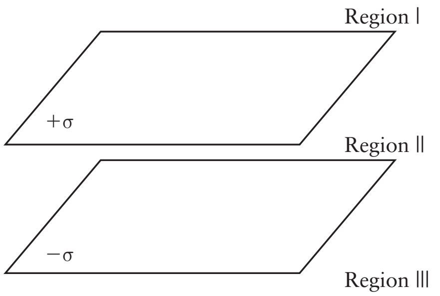


Figure 16-24 Problem 80

Region I: The field due to the top plane has a magnitude of $\frac{\sigma}{2\epsilon_0}$ and points up and the field due to the bottom plane has a magnitude of $\frac{\sigma}{2\epsilon_0}$ and points down. The net field in this region is $\boxed{0}$.

Region II: The field due to the top plane has a magnitude of $\frac{\sigma}{2\epsilon_0}$ and points down and the field due to the bottom plane has a magnitude of $\frac{\sigma}{2\epsilon_0}$ and also points down. The net field in this region is $\boxed{\frac{\sigma}{\epsilon_0} \text{ and points down}}$.

Region III: The field due to the top plane has a magnitude of $\frac{\sigma}{2\epsilon_0}$ and points down and the field due to the bottom plane has a magnitude of $\frac{\sigma}{2\epsilon_0}$ and points up. The net field in this region is $\boxed{0}$.

REFLECT

The electric field due to an infinite plane is constant everywhere in space. When applying superposition, the actual *net* field is the vector sum of the component fields, and we treat each component field as if the source were isolated. For example, when we add the fields due to the two infinite planes, the field due to the positive plane points away from the plane everywhere in space regardless of the presence of the second plane.

16.81

SET UP

A very long, hollow, charge cylinder has an inner radius $a = 3.00 \times 10^{-2}$ m, an outer radius $b = 5.00 \times 10^{-2}$ m, and a uniform charge density $\rho = +42.0 \times 10^{-6}\ \text{C/m}^3$. Since the charge distribution has cylindrical symmetry, we will use a cylindrical Gaussian surface of (constant) length L and (variable) radius r, split the problem into three regions—1) $r \leq a$, 2) $a \leq r \leq b$, and 3) $r \geq b$—and apply Gauss' law in order to find the electric field in each region. The charge enclosed by the Gaussian cylinder in region 1 is zero, which means the electric field in that region is also zero. The cylinder in region 2 encloses a fraction of the charge distributed throughout the thick, cylindrical shell; the volume of this shape can be calculated by subtracting the volume of the cylindrical hole from the volume of the Gaussian cylinder. Finally, the Gaussian cylinder in region 3 encloses the entire charged cylinder.

SOLVE

Part a)

$r \leq a$:

Looking down the cylinder:

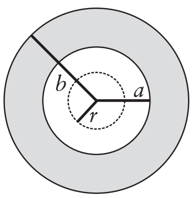


Figure 16-25 Problem 81

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\epsilon_0}$$

$$q_{\text{encl}} = 0; \quad \text{therefore, } \boxed{\vec{E} = 0}$$

Part b)

$a \leq r \leq b$:

Looking down the cylinder:

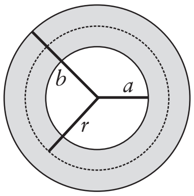


Figure 16-26 Problem 81

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\epsilon_0}$$

$$EA = \frac{(\rho V_{\text{encl}})}{\epsilon_0} = \frac{\rho(\pi r^2 - \pi a^2)L}{\epsilon_0}$$

$$E(2\pi rL) = \frac{\rho(\pi r^2 - \pi a^2)L}{\epsilon_0}$$

$$E = \frac{\rho(r^2 - a^2)}{2\epsilon_0 r} = \frac{\left(42.0 \times 10^{-6}\frac{\text{C}}{\text{m}^3}\right)(r^2 - a^2)}{2\left(8.85 \times 10^{-12}\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)r} = \left(2.37 \times 10^6\frac{\text{N}}{\text{C}\cdot\text{m}}\right)\frac{(r^2 - a^2)}{r}$$

The magnitude of the electric field in the region $a \leq r \leq b$ has a $\boxed{\text{magnitude of } \left(2.37 \times 10^6\frac{\text{N}}{\text{C}\cdot\text{m}}\right)\frac{(r^2 - a^2)}{r} \text{ and points radially outward}}$.

Part c)

$r \geq b$:

Looking down the cylinder:

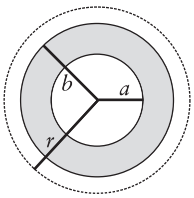


Figure 16-27 Problem 81

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\epsilon_0}$$

$$EA = \frac{(\rho V)}{\epsilon_0} = \frac{\rho(\pi b^2 - \pi a^2)L}{\epsilon_0}$$

$$E(2\pi rL) = \frac{\rho(\pi b^2 - \pi a^2)L}{\epsilon_0}$$

$$E = \frac{\rho(b^2 - a^2)}{2\epsilon_0 r} = \frac{\left(42.0 \times 10^{-6}\frac{\text{C}}{\text{m}^3}\right)((5.00 \times 10^{-2}\text{ m})^2 - (3.00 \times 10^{-2}\text{ m})^2)}{2\left(8.85 \times 10^{-12}\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)r} = \frac{\left(3800\frac{\text{N}\cdot\text{m}}{\text{C}}\right)}{r}$$

The magnitude of the electric field in the region $r \geq b$ has a $\boxed{\text{magnitude of } \frac{\left(3800\frac{\text{N}\cdot\text{m}}{\text{C}}\right)}{r} \text{ and}}$ $\boxed{\text{points radially outward}}$.

REFLECT

Problems that ask you to "determine the electric field for all radii" or "determine the electric field everywhere in space" will usually require you to invoke Gauss' law. The field should increase as r increases in region 2 since the Gaussian surface is enclosing more and more charge. The field should decrease as r increases in region 3 since we are moving farther and farther away from the source of charge.

Get Help: Interactive Example – Coaxial Cylindrical Conductors
Interactive Example – Spherical Shell Insulator

16.82

SET UP

A sphere of radius R carries a charge distribution that varies with the radius according to $\rho = \left(-23.0 \times 10^{-6}\frac{\text{C}}{\text{m}^4}\right)r$. We can use Gauss' law to derive an expression for the electric field everywhere in space. Since the charge distribution is spherically symmetric, our Gaussian surface will be a sphere of radius r. There are two main regions to consider: (1) $r < R$ and (2) $r \geq R$. In either case, we first need to calculate the charge enclosed by the Gaussian integral by performing an integral since the charge depends on r. The differential volume in spherical coordinates is $dV = r^2 dr \sin(\theta) d\theta d\phi$.

SOLVE

$r < R$:

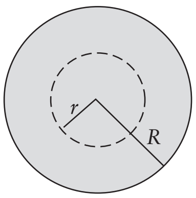


Figure 16-28 Problem 82

Total enclosed charge:

$$q_{\text{encl}} = \int \rho dV = \int_0^{2\pi}\int_0^{\pi}\int_0^{r}\left(-23.0 \times 10^{-6}\frac{\text{C}}{\text{m}^4}\right)r(r^2 dr \sin(\theta)d\theta d\phi)$$

$$= \left(-23.0 \times 10^{-6}\frac{\text{C}}{\text{m}^4}\right)\int_0^{r} r^3 dr\int_0^{\pi}\sin(\theta)d\theta\int_0^{2\pi}d\phi = \left(-23.0 \times 10^{-6}\frac{\text{C}}{\text{m}^4}\right)\left[\frac{r^4}{4}\right]_0^{r}[-\cos(\theta)]_0^{\pi}[\phi]_0^{2\pi}$$

$$= \left(-23.0 \times 10^{-6}\frac{\text{C}}{\text{m}^4}\right)\left[\frac{r^4}{4}\right][2][2\pi] = \left(-23.0 \times 10^{-6}\frac{\text{C}}{\text{m}^4}\right)[\pi r^4]$$

Electric field:

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{\text{encl}}}{\epsilon_0}$$

$$EA = \frac{q_{\text{encl}}}{\epsilon_0}$$

$$E = \frac{q_{\text{encl}}}{A\epsilon_0} = \frac{\left(-23.0 \times 10^{-6}\frac{\text{C}}{\text{m}^4}\right)[\pi r^4]}{(4\pi r^2)\epsilon_0} = \frac{\left(-23.0 \times 10^{-6}\frac{\text{C}}{\text{m}^4}\right)r^2}{4\left(8.85 \times 10^{-12}\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)} = \left(-6.50 \times 10^5\frac{\text{N}}{\text{C}\cdot\text{m}^2}\right)r^2$$

The electric field has a magnitude of $\boxed{\left(6.50 \times 10^5\frac{\text{N}}{\text{C}\cdot\text{m}^2}\right)r^2 \text{ and points radially inward}}$.

$r \geq R$:

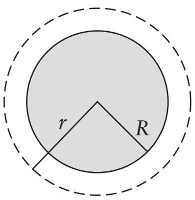


Figure 16-29 Problem 82

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{\text{encl}}}{\epsilon_0}$$

Total charge in the sphere:

$$q_{\text{encl}} = Q_{\text{total}} = \int \rho dV = \int_0^{2\pi}\int_0^{\pi}\int_0^{R}\left(-23.0 \times 10^{-6}\frac{\text{C}}{\text{m}^4}\right)r(r^2 dr \sin(\theta)d\theta d\phi)$$

$$= \left(-23.0 \times 10^{-6}\frac{\text{C}}{\text{m}^4}\right)\int_0^{R} r^3 dr\int_0^{\pi}\sin(\theta)d\theta\int_0^{2\pi}d\phi = \left(-23.0 \times 10^{-6}\frac{\text{C}}{\text{m}^4}\right)\left[\frac{r^4}{4}\right]_0^{R}[-\cos(\theta)]_0^{\pi}[\phi]_0^{2\pi}$$

$$= \left(-23.0 \times 10^{-6}\frac{\text{C}}{\text{m}^4}\right)\left[\frac{R^4}{4}\right][2][2\pi] = \left(-23.0 \times 10^{-6}\frac{\text{C}}{\text{m}^4}\right)[\pi R^4]$$

Electric field:

$$EA = \frac{Q_{\text{total}}}{\epsilon_0}$$

$$E = \frac{Q_{\text{total}}}{A\epsilon_0} = \frac{\left(-23.0 \times 10^{-6}\frac{\text{C}}{\text{m}^4}\right)[\pi R^4]}{(4\pi r^2)\epsilon_0} = \frac{\left(-23.0 \times 10^{-6}\frac{\text{C}}{\text{m}^4}\right)R^4}{4\left(8.85 \times 10^{-12}\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)r^2} = \left(-6.50 \times 10^5\frac{\text{N}}{\text{C}\cdot\text{m}^2}\right)\frac{R^4}{r^2}$$

The electric field has a magnitude of $\boxed{\left(6.50 \times 10^5\frac{\text{N}}{\text{C}\cdot\text{m}^2}\right)\frac{R^4}{r^2} \text{ and points radially inward}}$.

REFLECT

The electric field for $r < R$ increases in magnitude as r increases, until it reaches a maximum at $r = R$, when the Gaussian surface has finally enclosed all of the charge. For $r \geq R$ the field decreases as we move away from the charged sphere; the sphere looks like a point charge in this region and the fact that the magnitude decreases by a factor of r^2 confirms this.

16.83

SET UP

The electric field just outside the surface of a ball bearing of diameter $D = 2.00 \times 10^{-2}$ m is $E = 400$ N/C. We can draw a spherical Gaussian surface just outside the ball bearing and invoke Gauss' law to calculate its total charge. The surface charge density is equal to the charge on the ball bearing divided by its surface area.

SOLVE

Part a)

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\epsilon_0}$$

$$EA = \frac{Q}{\epsilon_0}$$

$$Q = EA\epsilon_0 = E(4\pi R^2)\epsilon_0 = 4\pi\epsilon_0 E\left(\frac{D}{2}\right)^2 = \pi\epsilon_0 ED^2$$

$$= \pi\left(8.85 \times 10^{-12}\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\left(400\frac{\text{N}}{\text{C}}\right)(2.00 \times 10^{-2}\text{ m})^2 = \boxed{4.45 \times 10^{-12}\text{ C}}$$

Part b)

$$\sigma = \frac{Q}{A} = E\epsilon_0 = \left(400\frac{\text{N}}{\text{C}}\right)\left(8.85 \times 10^{-12}\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right) = \boxed{3.54 \times 10^{-9}\frac{\text{C}}{\text{m}^2}}$$

REFLECT

Just outside the surface, we can treat the ball bearing as a point charge with charge Q. The electric field due to a point charge at a distance R from its center is $E = \frac{Q}{4\pi\epsilon_0 R^2}$, which is what we found through Gauss' law.

16.84

SET UP

A point charge ($Q_{\text{point charge}} = -3.20\ \mu\text{C}$) sits in static equilibrium in the center of a conducting spherical shell ($R_{\text{inner}} = 2.50 \times 10^{-2}$, $R_{\text{outer}} = 3.50 \times 10^{-2}$ cm). The shell has a net charge of $Q_{\text{shell}} = -5.80\ \mu\text{C}$. The electric field inside a conductor is zero, so the flux through a spherical Gaussian surface in this region is equal to 0. The total charge enclosed by this surface is equal to the sum of $Q_{\text{point charge}}$ and Q_{inner}, the charge on the inner surface. The net charge on the shell is equal to the charge on the inner surface Q_{inner} plus the charge on the outer surface Q_{outer}. Finally, we can draw a spherical Gaussian surface with $r = R_{\text{outer}}$ to calculate the electric field just outside the sphere. The total charge enclosed by this Gaussian surface is $Q_{\text{point charge}} + Q_{\text{shell}}$.

SOLVE

Part a)
Charge on inner surface of shell:

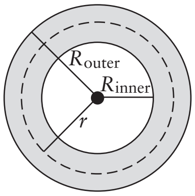


Figure 16-30 Problem 84

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\epsilon_0}$$

$$0 = \frac{Q_{\text{point charge}} + Q_{\text{inner}}}{\epsilon_0}$$

$$Q_{\text{inner}} = -Q_{\text{point charge}} = -(-3.20\ \mu\text{C}) = \boxed{3.20\ \mu\text{C}}$$

Charge on outer surface of shell:

$$Q_{\text{shell}} = Q_{\text{inner}} + Q_{\text{outer}}$$

$$Q_{\text{outer}} = Q_{\text{shell}} - Q_{\text{inner}} = (-5.80\ \mu\text{C}) - (3.20\ \mu\text{C}) = \boxed{-9.00\ \mu\text{C}}$$

Part b)
Electric field just outside the shell:

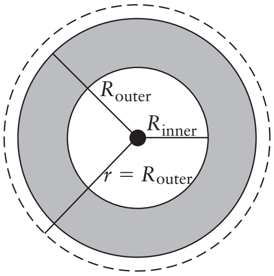


Figure 16-31 Problem 84

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{encl}}{\epsilon_0}$$

$$EA = \frac{Q_{point\ charge} + Q_{shell}}{\epsilon_0}$$

$$E = \frac{Q_{point\ charge} + Q_{shell}}{\epsilon_0 A} = \frac{Q_{point\ charge} + Q_{shell}}{\epsilon_0 (4\pi R_{outer}^2)}$$

$$= \frac{(-3.20 \times 10^{-6}\ \mathrm{C}) + (-5.80 \times 10^{-6}\ \mathrm{C})}{4\pi\left(8.85 \times 10^{-12} \frac{\mathrm{C}^2}{\mathrm{N \cdot m^2}}\right)(3.50 \times 10^{-2}\ \mathrm{m})^2} = -6.61 \times 10^7 \frac{\mathrm{N}}{\mathrm{C}}$$

The electric field just outside the shell has a magnitude of $\boxed{6.61 \times 10^7 \frac{\mathrm{N}}{\mathrm{C}} \text{ and points}}$ $\boxed{\text{radially inward}}$.

REFLECT

Since the point charge inside the shell is negative, we would expect the charge on the inner surface of the shell to be positive. In the region outside the shell, the charge distribution looks like a negative point charge of magnitude $-9.00\ \mu\mathrm{C}$.

16.85

SET UP

A long, straight rod has a linear charge density of $\lambda = 12 \times 10^{-6} \frac{\mathrm{C}}{\mathrm{m}}$. We can use Gauss' law to calculate the magnitude of the electric field at point P, which is a distance $r = 0.10$ m radially out from the central axis of the rod. We will use a cylindrical Gaussian surface of radius r and length L. The charge enclosed by this surface is equal to $q_{encl} = \lambda L$. The electric field points radially outward from the rod everywhere in space because the linear charge density is positive; we can represent this using $\hat{r}$.

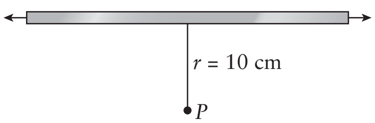


Figure 16-32 Problem 85

SOLVE

Magnitude of the electric field at P:

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\epsilon_0}$$

$$E(2\pi rL) = \frac{(\lambda L)}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{\left(12 \times 10^{-6}\frac{\text{C}}{\text{m}}\right)}{2\pi\left(8.85 \times 10^{-12}\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(0.10\text{ m})} = 2.2 \times 10^6\frac{\text{N}}{\text{C}}$$

Electric field vector:

$$\boxed{\vec{E} = \left(2.2 \times 10^6\frac{\text{N}}{\text{C}}\right)\hat{r}}$$

REFLECT

Your exact answer depends on how you've defined your coordinate system. For example, if we defined a Cartesian coordinate system in the drawing where point P lies along the $-y$-axis, we can represent the electric field at point P as $\vec{E} = -\left(2.2 \times 10^6\frac{\text{N}}{\text{C}}\right)\hat{y}$. Although convenient for point P, this coordinate system would not be as useful for describing the electric field at points that lie off axis.

16.86

SET UP

A spherical balloon that has a radius of 0.125 m contains helium at $T = 293$ K and $P = 1.3$ atm. The number of helium atoms can be calculated from the ideal gas law. One electron is removed from each helium atom and placed on a satellite that is 3.2187×10^7 m above the Earth. Coulomb's law will give the magnitude of the force between the two charged objects.

SOLVE

Number of helium atoms:

$$PV = NkT$$

$$N = \frac{PV}{kT} = \frac{\left(1.3\text{ atm} \times \frac{1.01 \times 10^5\text{ Pa}}{1\text{ atm}}\right)\left(\frac{4}{3}\pi\right)(0.125\text{ m})^3}{\left(1.38 \times 10^{-23}\frac{\text{J}}{\text{K}}\right)(293\text{ K})} = 2.66 \times 10^{23}\text{ He}$$

Force:

$$F = \left|\frac{kq_1q_2}{r^2}\right| = \left|\frac{k(2.66 \times 10^{23}e)(-2.66 \times 10^{23}e)}{r^2}\right| = \frac{(2.66 \times 10^{23})^2ke^2}{r^2}$$

$$= \frac{(2.66 \times 10^{23})^2\left(8.99 \times 10^9\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(1.6 \times 10^{-19}\text{ C})^2}{(3.2187 \times 10^7\text{ m})^2} = \boxed{1.57 \times 10^4\text{ N}}$$

REFLECT

The magnitude of the charge on each object is 4.25×10^4 C.

16.87

SET UP

The radius of a plutonium nucleus, which contains 94 protons, is $r_{\text{nuc}} = 7.5 \times 10^{-15}$ m. We can treat the nucleus as a point charge of magnitude $94e$ in the region outside the nucleus because the protons and neutrons will be concentrated in such a small region. The nucleus emits an alpha particle, which has 2 protons, that has a mass $m_\alpha = 6.6 \times 10^{-27}$ kg and a charge of $2e$; the plutonium nucleus now has a charge of $92e$. The only force acting on the alpha particle is the electric force due to the nucleus. We can use Newton's second law to calculate the greatest acceleration of the alpha particle.

SOLVE

Part a) With 94 protons confined to a small space, they are likely to be concentrated into an approximately uniform sphere of charge, which is equivalent to a point charge for points outside of it.

Part b)

$$\sum F = \frac{kq_1q_2}{r_{\text{nuc}}^2} = m_\alpha a$$

$$a = \frac{kq_1q_2}{m_\alpha r_{\text{nuc}}^2} = \frac{k(92e)(2e)}{m_\alpha r_{\text{nuc}}^2} = \frac{184ke^2}{m_\alpha r_{\text{nuc}}^2} = \frac{184\left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(1.6 \times 10^{-19}\text{ C})^2}{(6.6 \times 10^{-27}\text{ kg})(7.5 \times 10^{-15}\text{ m})^2} = \boxed{1.1 \times 10^{29}\frac{\text{m}}{\text{s}^2}}$$

REFLECT

The acceleration should be extremely large because the nucleus and alpha particle are initially separated by only 7.5 fm.

16.88

SET UP

When a test charge of $q_1 = +5 \times 10^{-9}$ C is placed at a certain point, it experiences a force of $F_1 = 0.08$ N directed northeast. We then replace q_1 with $q_2 = -2 \times 10^{-9}$ C. The electric field at that point will not change, so we can use the magnitudes of the charges and F_1 to calculate the magnitude of the force F_2 acting on q_2. The force F_2 will point southwest because q_2 is negatively charged. The magnitude of the electric field is equal to F_1 divided by q_1. The force acting on a positive test charge will point in the same direction as the electric field at that point, so the electric field points northeast.

SOLVE

Part a)

$$E = \frac{F_1}{q_1} = \frac{F_2}{q_2}$$

$$F_2 = F_1\frac{q_2}{q_1} = (0.08\text{ N})\left(\frac{2 \times 10^{-9}\text{ C}}{5 \times 10^{-9}\text{ C}}\right) = 0.032\text{ N}$$

The force acting on q_2 has a magnitude of $\boxed{0.032\ N\text{ and points southwest}}$.

Part b)

$$F_1 = q_1 E$$

$$E = \frac{F_1}{q_1} = \frac{0.08\text{ N}}{5 \times 10^{-9}\text{ C}} = 1.6 \times 10^7\frac{\text{N}}{\text{C}}$$

The electric field at the point has a magnitude of $\boxed{1.6 \times 10^7\frac{\text{N}}{\text{C}}\text{ and points northeast}}$.

REFLECT

We could have also used the information about q_2 to solve for the electric field:

$$E = \frac{F_2}{q_2} = \frac{0.032\text{ N}}{2 \times 10^{-9}\text{ C}} = 1.6 \times 10^7\frac{\text{N}}{\text{C}}$$

The direction of the field is opposite to the direction of the force on a negative charge.

16.89

SET UP

A red blood cell carries an excess charge of $Q = -2.5 \times 10^{-12}$ C distributed uniformly over its surface. We will model the cells as spheres of diameter $d = 7.5 \times 10^{-6}$ m and mass $m = 9.0 \times 10^{-14}$ kg. The number of excess electrons on the red blood cell is equal to Q divided by the charge on one electron, -1.602×10^{-19} C. In order to determine whether these extra electrons appreciably affect the mass of the cell, we can calculate the ratio of the mass of the excess electrons to the mass of the sphere; the mass of an electron is $m_e = 9.11 \times 10^{-31}$ kg. Finally, the surface charge density of the red blood cell is equal to the total charge divided by the surface area of the cell.

SOLVE

Part a)

$$-2.5 \times 10^{-12}\text{ C} \times \frac{1\text{ electron}}{-1.602 \times 10^{-19}\text{ C}} = \boxed{1.6 \times 10^7\text{ electrons}}$$

Part b)

$$\frac{m_{\text{electrons}}}{m_{\text{cell}}} = \frac{(1.6 \times 10^7)(9.11 \times 10^{-31}\text{ kg})}{9.0 \times 10^{-14}\text{ kg}} = \boxed{1.6 \times 10^{-10}}$$

The extra mass is $\boxed{\text{not significant}}$.

Part c)

$$\sigma = \frac{Q}{A} = \frac{Q}{4\pi R^2} = \frac{Q}{4\pi\left(\frac{d}{2}\right)^2} = \frac{Q}{\pi d^2} = \frac{-2.5 \times 10^{-12}\ \mathrm{C}}{\pi(7.5 \times 10^{-6}\ \mathrm{m})^2}$$

$$= \boxed{-1.4 \times 10^{-2}\frac{\mathrm{C}}{\mathrm{m}^2}} \times \frac{1\ \text{electron}}{-1.6 \times 10^{-19}\ \mathrm{C}} = \boxed{8.8 \times 10^{16}\frac{\text{electrons}}{\mathrm{m}^2}}$$

REFLECT

There would need to be about 10^{17} excess electrons, or an excess charge of around 10^{-2} C, for the mass of the excess electrons to be approximately equal to the mass of the cell.

16.90

SET UP

A red blood cell carries an excess charge of $Q = -2.5 \times 10^{-12}$ C distributed uniformly over its surface. We will model the cells as spheres of radius $r_0 = 3.75 \times 10^{-6}$ m. Since the excess charge is on the surface, the electric field at all points inside the red blood cell is 0. We can treat the cell as a point charge located at the center of the cell for all points outside of the cell; the magnitude of the electric field due to a point charge is $E = \frac{kQ}{r^2}$, where r is the distance from the charge. The charge on the cell is negative, which means the electric field outside the cell will point radially inward.

SOLVE

Part a) $\boxed{E = 0}$.

Part b) $\boxed{E = 0}$.

Part c)

$$E = \frac{kQ}{r_0^2} = \frac{\left(8.99 \times 10^9 \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)(2.5 \times 10^{-12}\ \mathrm{C})}{(3.75 \times 10^{-6}\ \mathrm{m})^2} = 1.60 \times 10^9 \frac{\mathrm{N}}{\mathrm{C}}$$

The electric field just outside the surface of the shell has a magnitude of

$\boxed{1.60 \times 10^9 \frac{\mathrm{N}}{\mathrm{C}} \text{ and points radially inward}}$.

Part d)

$$E = \frac{kQ}{r^2} = \frac{\left(8.99 \times 10^9 \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)(2.5 \times 10^{-12}\ \mathrm{C})}{((3.75 \times 10^{-6}\ \mathrm{m}) + (3.0 \times 10^{-6}\ \mathrm{m}))^2} = 4.93 \times 10^8 \frac{\mathrm{N}}{\mathrm{C}}$$

The electric field just outside the surface of the shell has a magnitude of $\boxed{4.93 \times 10^8 \frac{\text{N}}{\text{C}} \text{ and points radially inward}}$.

REFLECT

The electric field should decrease as we move farther away from the cell.

16.91

SET UP

Two small spheres, each with mass $m = 0.10 \times 10^{-3}$ kg, are suspended as pendulums by strings from a common point. The same charge q is given to each sphere. The spheres are in equilibrium when each string makes a 3.0-degree angle with the vertical. We can calculate the magnitude of the charge on each sphere from Newton's second law. We'll define our coordinate system such that $+x$ points to the right and $+y$ points upward. The forces acting on each ball are tension, gravity, and the force due to the electrical field of the other sphere. Because the spheres are small, we can model them as point charges.

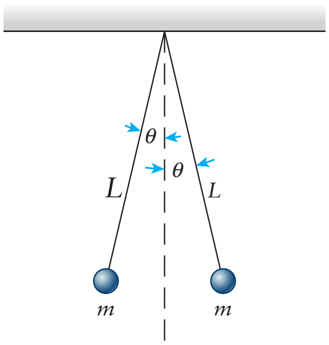


Figure 16-33 Problem 91

SOLVE

Free-body diagram of the ball on the right:

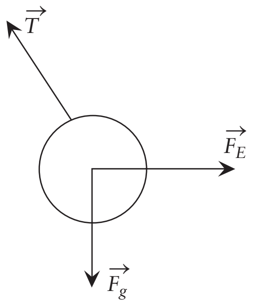


Figure 16-34 Problem 91

Newton's second law, x component:

$$\sum F_x = qE - T\sin(\theta) = ma_x = 0$$
$$qE = T\sin(\theta)$$

Newton's second law, y component:

$$\sum F_y = T\cos(\theta) - mg = ma_y = 0$$
$$mg = T\cos(\theta)$$

Magnitude of the charge:

$$\frac{T\sin(\theta)}{T\cos(\theta)} = \frac{qE}{mg}$$

$$\tan(\theta) = \frac{q}{mg}\left(\frac{kq}{r^2}\right) = \frac{kq^2}{mg(2L\sin(\theta))^2} = \frac{kq^2}{4mgL^2\sin^2(\theta)}$$

$$q = \sqrt{\frac{4mgL^2\sin^2(\theta)\tan(\theta)}{k}}$$

$$= \sqrt{\frac{4(0.10 \times 10^{-3}\ \text{kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)(1.0\ \text{m})^2\sin^2(3.0^\circ)\tan(3.0^\circ)}{\left(8.99 \times 10^9\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)}} = \boxed{7.9 \times 10^{-9}\ \text{C}}$$

REFLECT

Since the spheres are repelled by each other, they must have the same charge. However, we don't know whether this charge is positive or negative without more information.

16.92

SET UP

Three point charges are placed on the $x-y$ plane: $q_1 = +50.0 \times 10^{-9}$ C at the origin; $q_2 = -50.0 \times 10^{-9}$ C at $x = 0.100$ m, $y = 0$; and $q_3 = +150 \times 10^{-9}$ C at $x = 0.100$ m, $y = 0.0800$ m. The total electric force acting on q_3 is the vector sum of the electric force due to q_1 on q_3 and the electric force due to q_2 on q_3. The electric field at the location of q_3 due to the other two charges is equal to the net electric force acting on q_3 divided by q_3.

SOLVE

Part a)

x component:

$$F_{3,x} = F_{1\rightarrow 3,x} + F_{2\rightarrow 3,x} = +\left(\frac{kq_1q_3}{r_{13}^2}\right)\cos(\theta) + 0$$

$$= \frac{\left(8.99 \times 10^9\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(50.0 \times 10^{-9}\ \text{C})(150 \times 10^{-9}\ \text{C})}{(0.100\ \text{m})^2 + (0.0800\ \text{m})^2}\left(\frac{0.100\ \text{m}}{\sqrt{(0.100\ \text{m})^2 + (0.0800\ \text{m})^2}}\right)$$

$$= 0.00321\ \text{N}$$

y component:

$$F_{3,y} = F_{1\rightarrow 3,y} + F_{2\rightarrow 3,y} = +\left(\frac{kq_1q_3}{r_{13}^2}\right)\sin(\theta) - \left(\frac{kq_2q_3}{r_{23}^2}\right)$$

$$= \frac{\left(8.99 \times 10^9\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(50.0 \times 10^{-9}\ \text{C})(150 \times 10^{-9}\ \text{C})}{(0.100\ \text{m})^2 + (0.0800\ \text{m})^2}\left(\frac{0.0800\ \text{m}}{\sqrt{(0.100\ \text{m})^2 + (0.0800\ \text{m})^2}}\right)$$

$$- \frac{\left(8.99 \times 10^9\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(50.0 \times 10^{-9}\ \text{C})(150 \times 10^{-9}\ \text{C})}{(0.0800\ \text{m})^2} = -0.00797\ \text{N}$$

Net force acting on q_3:

$$\boxed{\vec{F}_3 = (0.00321\ \text{N})\hat{x} - (0.00797\ \text{N})\hat{y}}$$

Part b)

$$\vec{E} = \frac{\vec{F}_3}{q_3} = \frac{(0.00321\ \text{N})\hat{x} - (0.00797\ \text{N})\hat{y}}{150 \times 10^{-9}\ \text{C}}$$

$$= \boxed{\left(2.14 \times 10^4 \frac{\text{N}}{\text{C}}\right)\hat{x} - \left(5.31 \times 10^4 \frac{\text{N}}{\text{C}}\right)\hat{y}}$$

REFLECT

Charge q_2 is closer to charge q_3 than q_1 is so it should have a larger contribution to the y component of the force and field.

16.93

SET UP

The elephant nose fish can detect changes in an electric field as small as 3.0×10^{-6} N/C. We can set this equal to the expression for the electric field due to a point charge, $E = \frac{kQ}{r^2}$, in order to calculate the minimum charge required to create a field of this strength at a distance of $r = 0.75$ m. This charge divided by the magnitude of the charge on one electron (1.602×10^{-19} C) is equal to the number of electrons required to achieve this charge.

SOLVE

Part a)

$$E = \frac{kQ}{r^2}$$

$$Q = \frac{Er^2}{k} = \frac{\left(3.0 \times 10^{-6} \frac{\text{N}}{\text{C}}\right)(0.75\ \text{m})^2}{\left(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)} = \boxed{1.9 \times 10^{-16}\ \text{C}}$$

Part b)

$$1.9 \times 10^{-16}\ \text{C} \times \frac{1\ \text{electron}}{1.602 \times 10^{-19}\ \text{C}} = \boxed{1200\ \text{electrons}}$$

REFLECT

The fish can only detect *changes* in the local electric field of 3.0×10^{-6} N/C, so the additional field can either add to or subtract from the unperturbed field.

Get Help: Picture It – Electric Field
Interactive Example – Zero of E field Two Charges
P'Cast 16.4 – Electric Field Is Zero

16.94

SET UP

A small ball ($m = 1.0 \times 10^{-3}$ kg) possesses a charge of $q = +1$ C and is suspended by a long string in a uniform electric field pointing toward the right. The ball is in equilibrium when the string makes a 9.8-degree angle with the vertical. We can calculate the magnitude of the electric field from Newton's second law. We'll define our coordinate system such that $+x$ points to the right and $+y$ points upward.

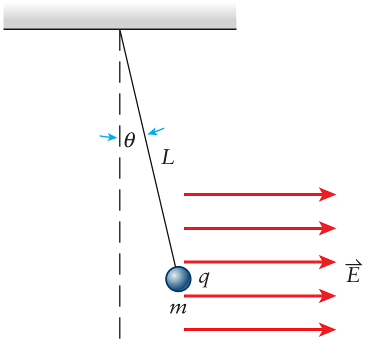


Figure 16-35 Problem 94

SOLVE

Free-body diagram:

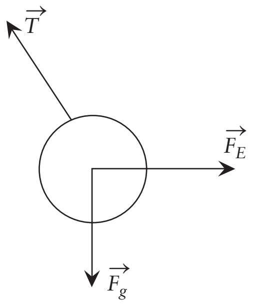


Figure 16-36 Problem 94

Newton's second law, x component:

$$\sum F_x = -T\sin(\theta) + qE = ma_x = 0$$

$$qE = T\sin(\theta)$$

Newton's second law, y component:

$$\sum F_y = T\cos(\theta) - mg = ma_y = 0$$

$$mg = T\cos(\theta)$$

Magnitude of the electric field:

$$\frac{T\sin(\theta)}{T\cos(\theta)} = \tan(\theta) = \frac{qE}{mg}$$

$$E = \frac{mg\tan(\theta)}{q} = \frac{(1.0 \times 10^{-3}\ \text{kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)\tan(9.8°)}{1\ \text{C}} = \boxed{1.7 \times 10^{-3}\frac{\text{N}}{\text{C}}}$$

REFLECT

The length of the string was not necessary in solving this problem.

16.95

SET UP

A small 2.50-kg ball is attached to a thin flexible wire to make a simple pendulum, which is then taken for use in the orbiting space station. Since the space station and, thus, the pendulum are in free fall, the pendulum will not oscillate and its period will be infinite. In order to make the pendulum in space have the same period as the pendulum on Earth, a charge of $q = -8.50 \times 10^{-6}$ C is uniformly distributed over the surface of the ball. The charged pendulum is then placed in a uniform electric field; the force due to the electric field acts as the restoring force for the pendulum. We can use Newton's second law for rotation in order to calculate the magnitude and direction of the electric field that will cause the period of the pendulum in space to equal the period of the pendulum on Earth.

SOLVE

Part a) The pendulum will not swing in orbit since it is in free fall (along with the space station). Therefore, its period would be infinite.

Part b) The electric field should replace gravity, so it should be oriented relative to the pendulum in the same way as gravity is on Earth. You want the force to be downward. Since the ball is negatively charged, the $\boxed{\text{field must point upward}}$ to create a downward force.

Newton's second law for rotation:

$$\sum \tau = I\alpha$$

$$-qEL\sin(\theta) = (mL^2)\left(\frac{d^2\theta}{dt^2}\right)$$

$$\frac{d^2\theta}{dt^2} + \frac{qE}{mL}\sin(\theta) \approx \frac{d^2\theta}{dt^2} + \frac{qE}{mL}\theta = 0$$

$$\omega = \sqrt{\frac{qE}{mL}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{mL}{qE}}$$

$$E = \frac{4\pi^2 mL}{qT^2} = \frac{4\pi^2 mL}{q\left(\frac{4\pi^2 L}{g}\right)} = \frac{mg}{q} = \frac{(2.50\ \text{kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)}{-8.50 \times 10^{-6}\ \text{C}} = \boxed{2.88 \times 10^{6}\frac{\text{N}}{\text{C}}}$$

Part c) The space pendulum depends on the mass of the bob, but the Earth pendulum does not.

REFLECT

Our expression for the electric field, $E = \frac{mg}{q}$, makes sense since it says the electric force must equal the gravitational force on Earth for the periods to be the same.

16.96

SET UP

Three charges—q_A, q_B, and q_C—are placed at the vertices of an equilateral triangle that has sides of length s. Point X is located at the center of the triangle, point Y is located at the midpoint of the side between q_B and q_C, and point Z is located at the midpoint of the side between q_A and q_C. We can use the expression for the electric field due to a point charge, along with geometry and trigonometry, to derive algebraic expressions for the total electric field at X, Y, and Z. After we have an algebraic expression, we can calculate numerical values for the field given $s = 0.10$ m, $q_A = +20 \times 10^{-9}$ C, $q_B = -8 \times 10^{-9}$ C, and $q_C = -10 \times 10^{-9}$ C.

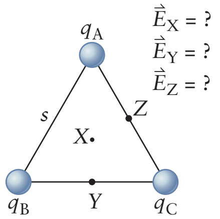


Figure 16-37 Problem 96

SOLVE

Part a)
x component:

$$E_x = E_{A,x} + E_{B,x} + E_{C,x} = 0 + \left(\frac{kq_B}{\left(\frac{s}{\sqrt{3}}\right)^2}\right)\cos(30°) - \left(\frac{kq_C}{\left(\frac{s}{\sqrt{3}}\right)^2}\right)\cos(30°)$$

$$= \left(\frac{3kq_B}{s^2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{3kq_C}{s^2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{3k\sqrt{3}}{2s^2}[q_B - q_C]$$

y component:

$$E_y = E_{A,y} + E_{B,y} + E_{C,y} = -\left(\frac{kq_A}{\left(\frac{s}{\sqrt{3}}\right)^2}\right) + \left(\frac{kq_B}{\left(\frac{s}{\sqrt{3}}\right)^2}\right)\sin(30°) + \left(\frac{kq_C}{\left(\frac{s}{\sqrt{3}}\right)^2}\right)\sin(30°)$$

$$= -\left(\frac{3kq_A}{s^2}\right) + \left(\frac{3kq_B}{s^2}\right)\left(\frac{1}{2}\right) + \left(\frac{3kq_C}{s^2}\right)\left(\frac{1}{2}\right) = \frac{3k}{s^2}\left[-q_A + \frac{q_B}{2} + \frac{q_C}{2}\right]$$

Electric field at X:

$$\boxed{\vec{E}_X = \frac{3k\sqrt{3}}{2s^2}[q_B - q_C]\hat{x} + \frac{3k}{s^2}\left[-q_A + \frac{q_B}{2} + \frac{q_C}{2}\right]\hat{y}}$$

Part b)
x component:

$$E_x = E_{A,x} + E_{B,x} + E_{C,x} = 0 + \left(\frac{kq_B}{\left(\frac{s}{2}\right)^2}\right) - \left(\frac{kq_C}{\left(\frac{s}{2}\right)^2}\right)$$

$$= \left(\frac{4kq_B}{s^2}\right) - \left(\frac{4kq_C}{s^2}\right) = \frac{4k}{s^2}[q_B - q_C]$$

y component:

$$E_y = E_{A,y} + E_{B,y} + E_{C,y} = -\left(\frac{kq_A}{\left(\frac{s\sqrt{3}}{2}\right)^2}\right) + 0 + 0 = -\frac{4kq_A}{3s^2}$$

Electric field at Y:

$$\boxed{\vec{E}_Y = \frac{4k}{s^2}[q_B - q_C]\hat{x} - \frac{4kq_A}{3s^2}\hat{y}}$$

Part c)
x component:

$$E_x = E_{A,x} + E_{B,x} + E_{C,x} = \left(\frac{kq_A}{\left(\frac{s}{2}\right)^2}\right)\cos(60^\circ) + \left(\frac{kq_B}{\left(\frac{s\sqrt{3}}{2}\right)^2}\right)\cos(30^\circ) - \left(\frac{kq_C}{\left(\frac{s}{2}\right)^2}\right)\cos(60^\circ)$$

$$= \left(\frac{4kq_A}{s^2}\right)\left(\frac{1}{2}\right) + \left(\frac{4kq_B}{3s^2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{4kq_C}{s^2}\right)\left(\frac{1}{2}\right) = \frac{2k}{s^2}\left[q_A + \frac{q_B}{\sqrt{3}} - q_C\right]$$

y component:

$$E_y = E_{A,y} + E_{B,y} + E_{C,y} = -\left(\frac{kq_A}{\left(\frac{s}{2}\right)^2}\right)\sin(60^\circ) + \left(\frac{kq_B}{\left(\frac{s\sqrt{3}}{2}\right)^2}\right)\sin(30^\circ) + \left(\frac{kq_C}{\left(\frac{s}{2}\right)^2}\right)\sin(60^\circ)$$

$$= -\left(\frac{4kq_A}{s^2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{4kq_B}{3s^2}\right)\left(\frac{1}{2}\right) + \left(\frac{4kq_C}{s^2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{2k}{s^2}\left[-q_A\sqrt{3} + \frac{q_B}{3} + q_C\sqrt{3}\right]$$

Electric field at Z:

$$\boxed{\vec{E}_Y = \frac{2k}{s^2}\left[q_A + \frac{q_B}{\sqrt{3}} - q_C\right]\hat{x} - \frac{2k}{s^2}\left[-q_A\sqrt{3} + \frac{q_B}{3} + q_C\sqrt{3}\right]\hat{y}}$$

Part d)
Electric field at X, x component:

$$\frac{3k\sqrt{3}}{2s^2}[q_B - q_C] = \frac{3\left(8.99 \times 10^9 \dfrac{C^2}{N \cdot m^2}\right)\sqrt{3}}{2(0.10\text{ m})^2}[(-8 \times 10^{-9}\text{ C}) - (-10 \times 10^{-9}\text{ C})] = 4671\frac{N}{C}$$

Electric field at X, y component:

$$\frac{3k}{s^2}\left[-q_A + \frac{q_B}{2} + \frac{q_C}{2}\right]$$

$$= \frac{3\left(8.99 \times 10^9 \dfrac{C^2}{N \cdot m^2}\right)}{(0.10\text{ m})^2}\left[-(20 \times 10^{-9}\text{ C}) + \frac{(-8 \times 10^{-9}\text{ C})}{2} + \frac{(-10 \times 10^{-9}\text{ C})}{2}\right]$$

$$= -78{,}213\frac{N}{C}$$

Electric field at X:

$$\boxed{\vec{E}_X = \left(4671\frac{N}{C}\right)\hat{x} - \left(78{,}213\frac{N}{C}\right)\hat{y}}$$

Electric field at Y, x component:

$$\frac{4k}{s^2}[q_B - q_C] = \frac{4\left(8.99 \times 10^9 \dfrac{C^2}{N \cdot m^2}\right)}{(0.10\text{ m})^2}[(-8 \times 10^{-9}\text{ C}) - (-10 \times 10^{-9}\text{ C})] = 7192\frac{N}{C}$$

Electric field at Y, y component:

$$-\frac{4kq_A}{3s^2} = -\frac{4\left(8.99 \times 10^9 \dfrac{C^2}{N \cdot m^2}\right)(20 \times 10^{-9}\text{ C})}{3(0.10\text{ m})^2} = -23{,}973\frac{N}{C}$$

Electric field at Y:

$$\boxed{\vec{E}_Y = \left(7192\frac{N}{C}\right)\hat{x} - \left(23{,}973\frac{N}{C}\right)\hat{y}}$$

Electric field at Z, x component:

$$\frac{2k}{s^2}\left[q_A + \frac{q_B}{\sqrt{3}} - q_C\right]$$

$$= \frac{2\left(8.99 \times 10^9 \dfrac{C^2}{N \cdot m^2}\right)}{(0.10\text{ m})^2}\left[(20 \times 10^{-9}\text{ C}) + \frac{(-8 \times 10^{-9}\text{ C})}{\sqrt{3}} - (-10 \times 10^{-9}\text{ C})\right]$$

$$= 45{,}635\frac{N}{C}$$

Electric field at Z, y component:

$$\frac{2k}{s^2}\left[-q_A\sqrt{3}+\frac{q_B}{3}+q_C\sqrt{3}\right]$$

$$=\frac{2\left(8.99\times10^9\frac{C^2}{N\cdot m^2}\right)}{(0.10\text{ m})^2}\left[-(20\times10^{-9}\text{ C})\sqrt{3}+\frac{(-8\times10^{-9}\text{ C})}{3}+(-10\times10^{-9}\text{ C})\sqrt{3}\right]$$

$$=-98{,}221\frac{N}{C}$$

Electric field at Z:

$$\boxed{\vec{E}_Z=\left(45{,}635\frac{N}{C}\right)\hat{x}-\left(98{,}221\frac{N}{C}\right)\hat{y}}$$

REFLECT

If the charges were all equal, many of the components we calculated would be zero due to symmetry.

16.97

SET UP

Six positive charges ($q_1 = 1\times10^{-3}$ C, $q_2 = 2\times10^{-3}$ C, $q_3 = 3\times10^{-3}$ C, $q_4 = 4\times10^{-3}$ C, $q_5 = 5\times10^{-3}$ C, $q_6 = 6\times10^{-3}$ C) are arranged in a regular hexagon with 5-cm-long sides (see figure). The electric field at the center of the hexagon is equal to the vector sum of the electric fields due to each point charge. The magnitude of the electric field due to a point charge q at a distance r away is $E=\frac{kq}{r^2}$. Since all of the charges are positive, the electric field will point away from each charge towards the center of the hexagon.

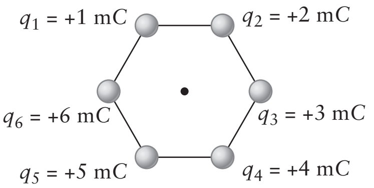


Figure 16-38 Problem 97

SOLVE

Magnitudes of the field due to each point charge at the center:

$$E_1=\frac{kq_1}{r_1^2}=\frac{\left(8.99\times10^9\frac{N\cdot m^2}{C^2}\right)(1\times10^{-3}\text{ C})}{(5\times10^{-2}\text{ m})^2}=3.596\times10^9\frac{N}{C}$$

$$E_2=\frac{kq_2}{r_2^2}=\frac{k(2q_1)}{r_1^2}=2E_1$$

$$E_3=\frac{kq_3}{r_3^2}=\frac{k(3q_1)}{r_1^2}=3E_1$$

$$E_4=\frac{kq_4}{r_4^2}=\frac{k(4q_1)}{r_1^2}=4E_1$$

$$E_5 = \frac{kq_5}{r_5^2} = \frac{k(5q_1)}{r_1^2} = 5E_1$$

$$E_6 = \frac{kq_6}{r_6^2} = \frac{k(6q_1)}{r_1^2} = 6E_1$$

x component of the field at the center:

$$E_x = E_1\cos(60°) - E_2\cos(60°) - E_3 - E_4\cos(60°) + E_5\cos(60°) + E_6$$

$$= [E_1 - E_2 - E_4 + E_5]\cos(60°) + E_6 - E_3 = [E_1 - 2E_1 - 4E_1 + 5E_1]\cos(60°) + 6E_1 - 3E_1$$

$$= [1 - 2 - 4 + 5]E_1\cos(60°) + 6E_1 - 3E_1 = 3E_1 = 3\left(3.596 \times 10^9\frac{\text{N}}{\text{C}}\right) = 1.079 \times 10^{10}\frac{\text{N}}{\text{C}}$$

y component of the field at the center:

$$E_y = -E_1\sin(60°) - E_2\sin(60°) + E_4\sin(60°) + E_5\sin(60°)$$

$$= [-E_1 - E_2 + E_4 + E_5]\sin(60°) = [-E_1 - 2E_1 + 4E_1 + 5E_1]\sin(60°)$$

$$= [-1 - 2 + 4 + 5]E_1\sin(60°) = \frac{6\sqrt{3}}{2}E_1 = 3\sqrt{3}\left(3.596 \times 10^9\frac{\text{N}}{\text{C}}\right) = 1.869 \times 10^{10}\frac{\text{N}}{\text{C}}$$

Electric field at the center has a $\boxed{\text{magnitude of } 2.16 \times 10^{10}\frac{\text{N}}{\text{C}} \text{ with components of}}$

$\boxed{1.08 \times 10^{10}\frac{\text{N}}{\text{C}} \text{ to the right and } 1.87 \times 10^{10}\frac{\text{N}}{\text{C}} \text{ above the horizontal}}$.

REFLECT

There is more positive charge to the left of the center than the right and below than above, so the electric field at the center should point up and to the right. The field must point at an angle of 60 degrees above the horizontal due to symmetry.

16.98

SET UP

A nonconducting flexible ring has a radius of $R = 2.25 \times 10^{-2}$ m. At a point located at $y = 1.50 \times 10^{-2}$ m above the axis of the ring, the electric field has a magnitude of 6850 N/C and points towards the ring. We can use the expression for the magnitude of the electric field, $E = \frac{kQy}{(R^2 + y^2)^{3/2}}$, to calculate the charge that is uniformly distributed on the ring; the charge is negative because the electric field points toward the ring. The ring is cut and straightened into a thin rod without disturbing the charge. The magnitude of the electric field produced by the rod along the axis of the rod at a distance $d = 1.50 \times 10^{-2}$ m from one of its ends is given by $E = \frac{kQ}{d(d + L)}$. If we're situated at a point $d = 1.50 \times 10^{-2}$ m from the center of the rod,

we can treat the rod as being an infinite line that produces a field of magnitude $E = \dfrac{\lambda}{2\pi\epsilon_0 r}$. In either case, the field points toward the rod because the charge is negative.

SOLVE

Charge on the ring:

$$E = \frac{kQy}{(R^2 + y^2)^{3/2}}$$

$$Q = \frac{E(R^2+y^2)^{3/2}}{ky} = \frac{\left(6850\frac{\text{N}}{\text{C}}\right)((2.25 \times 10^{-2}\text{ m})^2 + (1.50 \times 10^{-2})^2)^{3/2}}{\left(8.99 \times 10^{12}\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(1.50 \times 10^{-2})} = 1.00 \times 10^{-9}\text{ C}$$

Because the electric field is pointing toward the ring, the charge must be negative. Therefore $Q = -1.00 \times 10^{-9}$ C.

Length of the ring:

$$L = 2\pi R = 2\pi(2.25 \times 10^{-2}\text{ m}) = 0.141\text{ m}$$

Part a)

$$E = \frac{kQ}{d(d+L)} = \frac{\left(8.99 \times 10^{9}\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(1.00 \times 10^{-9}\text{ C})}{(1.50 \times 10^{-2}\text{ m})((1.50 \times 10^{-2}\text{ m}) + (0.141\text{ m}))} = 3850\frac{\text{N}}{\text{C}}$$

The electric field along the axis of the rod has a magnitude of $\boxed{3850\frac{\text{N}}{\text{C}}\text{ and points toward the rod}}$.

Part b)

$$E = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{\left(\frac{Q}{L}\right)}{2\pi\epsilon_0 r} = \frac{Q}{2\pi\epsilon_0 L r} = \frac{1.00 \times 10^{-9}\text{ C}}{2\pi\left(8.85 \times 10^{-12}\frac{\text{N}\cdot\text{m}}{\text{C}^2}\right)(0.141\text{ m})(1.50 \times 10^{-2}\text{ m})} = 8510\frac{\text{N}}{\text{C}}$$

The electric field along an axis perpendicular to middle of the rod has a magnitude of $\boxed{8510\frac{\text{N}}{\text{C}}\text{ and points toward the center of the rod}}$.

REFLECT

The rod looks "infinite" in part (b) because the length of the rod (0.141 m) is about an order of magnitude larger than the distance from the center of the rod (0.0150 m).

16.99

SET UP

A circular arc of radius r subtends an angle of θ and has a linear charge density of $+\lambda$. Point C is located at the center of the circle. The electric field at C is equal to the integral of the differential field $dE = \frac{kdq}{r^2}\cos(\alpha)$ around the ring from $\alpha = -\frac{\theta}{2}$ to $\frac{\theta}{2}$. The charge is uniformly distributed around the ring, which means we can convert the integral from dq to $d\phi$.

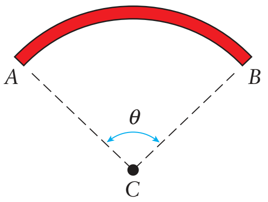


Figure 16-39 Problem 99

SOLVE

$$E_C = \int dE_C = \int \frac{kdq}{r^2}\cos(\alpha) = \frac{k}{r^2}\int(\lambda ds)\cos(\alpha) = \frac{k\lambda}{r^2}\int_{-\theta/2}^{\theta/2}\cos(\alpha)(rd\alpha)$$

$$= \frac{k\lambda}{r}\int_{-\theta/2}^{\theta/2}\cos(\alpha)d\alpha = \frac{k\lambda}{r}[\sin(\alpha)]_{-\theta 2}^{\theta/2} = \frac{k\lambda}{r}\left[\sin\left(\frac{\theta}{2}\right) - \sin\left(-\frac{\theta}{2}\right)\right]$$

$$= \frac{k\lambda}{r}\left[\sin\left(\frac{\theta}{2}\right) + \sin\left(\frac{\theta}{2}\right)\right] = \frac{2k\lambda}{r}\sin\left(\frac{\theta}{2}\right)$$

The electric field at point C has a magnitude of $\boxed{\frac{2k\lambda}{r}\sin\left(\frac{\theta}{2}\right) \text{ and points downward}}$.

REFLECT

The electric field at point C must point straight downward due to symmetry.

16.100

SET UP

A ring of charge lies in the $x-y$ plane centered about the z-axis and has a total uniformly distributed charge of $Q = -50 \times 10^{-6}$ C and a radius of $R = 0.15$ m. Point P is located on the z-axis at $z_P = 0.10$ m. Due to symmetry, the electric field due to the ring at P will only point along the z-axis. The electric field at P can be calculated by integrating the z component of the differential field $dE_z = \frac{kdq}{r^2}\cos(\theta)$ around the ring from $\phi = 0$ to 2π. The charge is uniformly distributed around the ring, which means we can convert the integral from dq to $d\phi$.

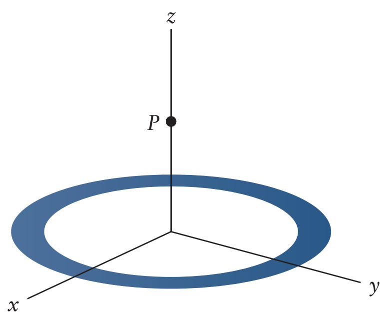


Figure 16-40 Problem 100

SOLVE

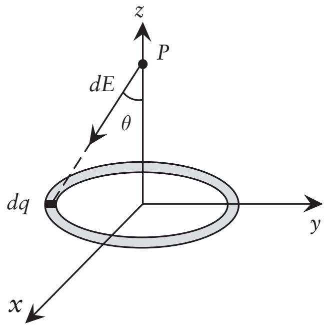


Figure 16-41 Problem 100

$$E_z = \int dE_z = \int \frac{k\,dq}{r^2}\cos(\theta) = k\int_0^{2\pi} \frac{\cos(\theta)}{r^2}\left(\frac{Q}{2\pi}d\phi\right) = \frac{kQ}{2\pi r^2}\cos(\theta)\int_0^{2\pi} d\phi$$

$$= \frac{kQ}{2\pi r^2}\cos(\theta)[\phi]_0^{2\pi} = \frac{kQ}{2\pi r^2}\cos(\theta)[2\pi] = \frac{kQ}{r^2}\cos(\theta)$$

$$= \frac{kQ}{R^2 + z_P^2}\left(\frac{z_P}{\sqrt{R^2 + z_P^2}}\right) = \frac{kQz_P}{(R^2 + z_P^2)^{3/2}}$$

$$\vec{E} = \frac{kQz_P}{(R^2 + z_P^2)^{3/2}}\hat{z} = \frac{\left(8.99 \times 10^9 \dfrac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(-50 \times 10^{-6}\text{ C})(0.10\text{ m})}{((0.15\text{ m})^2 + (0.10\text{ m}))^{3/2}}\hat{z} = \boxed{\left(-7.67 \times 10^6 \frac{\text{N}}{\text{C}}\right)\hat{z}}$$

REFLECT

Thinking about the problem and invoking the symmetry of the charge distribution before performing integrals made the problem much simpler.

16.101

SET UP

In one model, parts of a cloud can be viewed as two round, parallel, oppositely charged sheets with a radius of $R = 2.5 \times 10^3$ m and a distance of 6.0×10^3 m apart. The field midway between the two sheets has a magnitude of 2.0×10^5 N/C. Using the expression for the magnitude of the electric field due to a round disk, $E_{\text{disk}} = \left(\frac{2kQ}{R^2}\right)\left(1 - \frac{y}{\sqrt{R^2 + y^2}}\right)$, we can calculate the magnitude of the charge Q. Another model treats the parts of the cloud as equal but opposite point charges. Again, the field midway between the two charges has a magnitude of 2.0×10^5 N/C. We can use the expression for the magnitude of the electric field due to a point charge, $E_{\text{point charge}} = \frac{kQ}{R^2}$, to calculate Q in this case.

SOLVE

Part a)

$$\vec{E}_{\text{total}} = \vec{E}_1 + \vec{E}_2 = E_{\text{disk}}(-\hat{y}) + E_{\text{disk}}(-\hat{y}) = -2E_{\text{disk}}\hat{y}$$

$$E_{\text{total}} = 2E_{\text{disk}} = 2\left(\frac{2kQ}{R^2}\right)\left(1 - \frac{y}{\sqrt{R^2 + y^2}}\right)$$

$$Q = \frac{E_{\text{total}}R^2}{4k\left(1 - \dfrac{y}{\sqrt{R^2 + y^2}}\right)}$$

$$= \frac{\left(2.0 \times 10^5 \frac{\text{N}}{\text{C}}\right)(2.5 \times 10^3\ \text{m})^2}{4\left(8.99 \times 10^9 \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}\right)\left(1 - \dfrac{3.0 \times 10^3\ \text{m}}{\sqrt{(2.5 \times 10^3\ \text{m})^2 + (3.0 \times 10^3\ \text{m})^2}}\right)} = \boxed{150\ \text{C}}$$

Part b)

$$E_{\text{total}} = 2E_{\text{point charge}} = 2\left(\frac{kQ}{R^2}\right)$$

$$Q = \frac{E_{\text{total}}R^2}{2k} = \frac{\left(2.0 \times 10^5 \frac{\text{N}}{\text{C}}\right)(3.0 \times 10^3\ \text{m})^2}{2\left(8.99 \times 10^9 \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}\right)} = \boxed{100\ \text{C}}$$

REFLECT

Even though the two models are very simplistic, their results agree rather well. Actual measured charges are on the order of a few tens of coulombs.

16.102

SET UP

The maximum electric field measured in clouds during an electrical storm is $E = 2.00 \times 10^5$ N/C. The acceleration of a proton in this electric field can be calculated from Newton's second law. Because the force acting on the proton is constant, we can use the constant acceleration kinematics equations to solve for the distance the proton, which starts at rest, must travel in order to achieve a final speed of $0.10c$. In order to determine whether or not gravity can safely be ignored, we must compare the magnitude of the acceleration due to the field to the acceleration due to gravity.

SOLVE

Part a)

$$\sum F = F_E = m_p a$$

$$eE = m_p a$$

$$a = \frac{eE}{m_p} = \frac{(1.6 \times 10^{-19}\ \mathrm{C})\left(2.00 \times 10^5 \frac{\mathrm{N}}{\mathrm{C}}\right)}{1.67 \times 10^{-27}\ \mathrm{kg}} = \boxed{1.92 \times 10^{13} \frac{\mathrm{m}}{\mathrm{s}^2} = 1.96 \times 10^{12} g}$$

Part b)

$$v^2 = v_0^2 + 2a(\Delta x) = 0 + 2a(\Delta x)$$

$$\Delta x = \frac{v^2}{2a} = \frac{(0.10c)^2}{2a} = \frac{0.010\left(3.0 \times 10^8 \frac{\mathrm{m}}{\mathrm{s}}\right)^2}{2\left(1.92 \times 10^{13} \frac{\mathrm{m}}{\mathrm{s}^2}\right)} = \boxed{23.5\ \mathrm{m}}$$

Part c) Yes, we can neglect the effects of gravity because the acceleration due to the electric field is much, much larger than g.

REFLECT

Electrostatic interactions are usually orders of magnitude larger than the weight of subatomic particles, atoms, ions, and molecules.

16.103

SET UP

An electron with an initial speed of $v_0 = 5.0 \times 10^5$ m/s is traveling in the $+x$ direction when it enters a region with a uniform electric field parallel to the x-axis. The electron travels $\Delta x = 5.0 \times 10^{-2}$ m into the field before coming to rest. Assuming this is the only force acting on the electron, the net force and, thus, the acceleration of the electron are constant. We can calculate the acceleration of the electron from constant acceleration kinematics. The force acting on the electron has a magnitude of eE and must point toward $-x$ because the electron comes to rest. The magnitude of the field E can then be found through Newton's second law. The electric field will point in the opposite direction of the electric force acting on a negative charge.

SOLVE

Acceleration:

$$v^2 = v_0^2 + 2a_x \Delta x$$

$$a_x = \frac{v^2 - v_0^2}{2\Delta x} = \frac{0^2 - \left(5.0 \times 10^5 \frac{\mathrm{m}}{\mathrm{s}}\right)^2}{2(5.0 \times 10^{-2}\ \mathrm{m})} = -2.5 \times 10^{12} \frac{\mathrm{m}}{\mathrm{s}^2}$$

Free-body diagram of the electron:

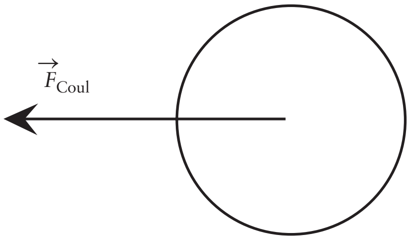


Figure 16-42 Problem 103

Newton's second law:

$$\sum F_x = F_{\text{Coul}} = -|q|E = m_e a_x$$

$$E = -\frac{m_e a_x}{|q|} = -\frac{(9.11 \times 10^{-31}\ \text{kg})\left(-2.5 \times 10^{12}\frac{\text{m}}{\text{s}^2}\right)}{1.6 \times 10^{-19}\ \text{C}} = 14.2\frac{\text{N}}{\text{C}}$$

The electric field has a magnitude of $14.2\frac{\text{N}}{\text{C}}$ and points toward $+x$.

REFLECT

A positive charge would speed up if it entered this field.

16.104

SET UP

An electron, released in a region with a uniform electric field, has an acceleration of $3.00 \times 10^{14}\ \text{m/s}^2$ in the $+x$ direction. The force acting on the electron has a magnitude of eE and must point toward $+x$ because the acceleration points toward $+x$. The magnitude of the field E can then be found through Newton's second law. The electric field will point in the opposite direction of the electric force acting on a negative charge. Since the acceleration is constant, we can use kinematics to calculate the time it takes the electron to reach a final speed of $v = 11{,}200\ \text{m/s}$.

SOLVE

Part a)
Free-body diagram of the electron:

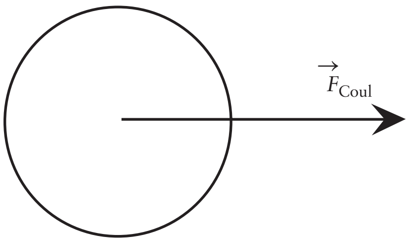


Figure 16-43 Problem 104

Newton's second law:

$$\sum F_x = F_{\text{Coul}} = |q|E = m_e a_x$$

$$E = \frac{m_e a_x}{|q|} = \frac{(9.11 \times 10^{-31}\ \text{kg})\left(3.00 \times 10^{14}\frac{\text{m}}{\text{s}^2}\right)}{1.6 \times 10^{-19}\ \text{C}} = 1708\frac{\text{N}}{\text{C}}$$

The electric field has a magnitude of $\boxed{1708\frac{\text{N}}{\text{C}} \text{ and points toward } -x}$.

Part b)

$$v = v_0 + a_x \Delta t$$

$$\Delta t = \frac{v - v_0}{a_x} = \frac{\left(11{,}200\frac{\text{m}}{\text{s}}\right) - 0}{\left(3.00 \times 10^{14}\frac{\text{m}}{\text{s}^2}\right)} = \boxed{3.73 \times 10^{-11}\ \text{s}}$$

REFLECT
We've assumed that the only force acting on the electron is the force due to the electric field.

16.105

SET UP
The nucleus of an iron atom contains 26 protons and has a radius of $R_{\text{nuc}} = 4.6 \times 10^{-15}$ m. The radius of the atom is 0.50×10^{-10} m. We can treat the nucleus as a positive point charge of magnitude $26e$ for all points outside of the nucleus. The magnitude of the electric field due to a point charge is $E = \frac{kq}{r^2}$; the field will point radially outward because the nucleus is positively charged. We'll assume that the net force acting on the outermost electron is equal to the force due to the electric field of the nucleus at that point. We can calculate that electron's acceleration using Newton's second law.

SOLVE
Part a)

$$E = \frac{kq}{r^2} = \frac{k(26e)}{r_{\text{nuc}}^2} = \frac{\left(8.99 \times 10^9\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(26)(1.6 \times 10^{-19}\ \text{C})}{(4.6 \times 10^{-15}\ \text{m})^2} = 1.8 \times 10^{21}\frac{\text{N}}{\text{C}}$$

The electric field has a magnitude of $\boxed{1.8 \times 10^{21}\frac{\text{N}}{\text{C}} \text{ and points radially outward}}$.

Part b)

$$E = \frac{kq}{r^2} = \frac{k(26e)}{r_{\text{atom}}^2} = \frac{\left(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(26)(1.6 \times 10^{-19}\ \text{C})}{(0.5 \times 10^{-10}\ \text{m})^2} = 1.5 \times 10^{13} \frac{\text{N}}{\text{C}}$$

The electric field has a magnitude of $\boxed{1.5 \times 10^{13} \frac{\text{N}}{\text{C}} \text{ and points radially outward}}$.

Part c)

$$\sum F = F_{\text{Coul}} = |q|E = m_e a$$

$$a = \frac{|q|E}{m_e} = \frac{(1.6 \times 10^{-19}\ \text{C})\left(1.5 \times 10^{13} \frac{\text{N}}{\text{C}}\right)}{9.11 \times 10^{-31}\ \text{kg}} = 2.6 \times 10^{24} \frac{\text{m}}{\text{s}^2}$$

The acceleration of the outermost electron has a magnitude of $\boxed{2.6 \times 10^{24} \frac{\text{m}}{\text{s}^2} \text{ and}}$ $\boxed{\text{points radially inward}}$.

REFLECT

The force acting on the outermost electron will be much less than this due to the effects of the other 25 electrons. These electrons are closer to the nucleus and "screen" the charge of the nucleus so the effective field at the position of the outermost electron is smaller.

16.106

SET UP

An electron with kinetic energy K is traveling to the right along the $+x$-axis through a region with an electric field $\vec{E} = \left(2.00 \times 10^4 \frac{\text{N}}{\text{C}}\right)\hat{y}$. The region is $\Delta x = 0.06$ m long and $\Delta y = 0.02$ m wide. As soon as the electron enters the field, it will experience a force in the $-y$ direction. We can use Newton's second law to find an expression for the acceleration of the electron in the y direction. The force acts only in the y direction, so the velocity in the x direction will remain constant for all time. We can use this to find the time it takes for the particle to travel Δx. The minimum initial kinetic energy the electron can have and still avoid colliding with one of the plates corresponds to the speed at which the electron travels to a final y position of $y_f = -0.01$ m. Since the net force acting on the electron is constant, we can use the constant acceleration kinematics equations to calculate this kinetic energy.

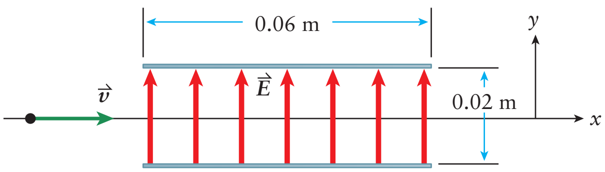


Figure 16-44 Problem 106

SOLVE

Newton's second law:

$$\sum F_y = -F_E = m_e a_y$$

$$-eE = m_e a_y$$

$$a_y = \frac{-eE}{m_e}$$

Time it takes to travel through the plates:

$$v_0 = \frac{\Delta x}{\Delta t}$$

$$\Delta t = \frac{\Delta x}{v_0}$$

Position along the y-axis:

$$y_f = y_0 + v_{0,y}t + \frac{1}{2}a_y(\Delta t)^2 = 0 + 0 + \frac{1}{2}a_y\left(\frac{\Delta x}{v_0}\right)^2 = \frac{a_y(\Delta x)^2}{2v_0^2}$$

$$y_f = \frac{a_y(\Delta x)^2}{2v_0^2} = \frac{\left(-\frac{eE}{m_e}\right)(\Delta x)^2}{2v_0^2} = -\frac{eE(\Delta x)^2}{2m_e v_0^2} = -\frac{eE(\Delta x)^2}{2(2K)} = -\frac{eE(\Delta x)^2}{4K}$$

$$K = -\frac{eE(\Delta x)^2}{4y_f} = -\frac{(1.6 \times 10^{-19}\ \text{C})\left(2.00 \times 10^4 \frac{\text{N}}{\text{C}}\right)(0.06\ \text{m})^2}{4(-0.01\ \text{m})} = \boxed{2.88 \times 10^{-16}\ \text{J}}$$

REFLECT

The speed of the electron will increase when the particle is in between the plates because the electric field is doing work on it.

16.107

SET UP

A spherical oil drop ($\rho_{\text{oil}} = 0.850\ \text{g/cm}^3$) has a diameter of $D = 1.10 \times 10^{-6}$ m and has an excess charge of $q_1 = -4e$. The drop is located in a region with a vertical uniform electric field of magnitude E_1 and remains at rest. Since the net vertical force acting on the drop is zero, the force of the electric field acting on the drop must point upward to oppose the force of gravity acting downward; the electric field will point in the opposite direction of the electric force acting on a negative charge. The magnitudes of these forces must be equal as well. We can find the value of E_1 from this force balance. A second drop with negative charge q_2 is found to remain at rest when $E_2 = 5183$ N/C. Assuming the second drop has the same size as the first drop, the force due to gravity will be the same for both drops. We can then set up the product q_1E_1 to equal q_2E_2, where $q_2 = n_2e$, and solve for the number of excess electrons on the second drop n_2.

SOLVE

Part a) The electric field should point $\boxed{\text{downward}}$.

Part b) Gravity pulls downward on the drop, so the electrical force on it must be upward. Since the drop is negative, the electric field must point downward for the force to point upward.

Part c)

$$\sum F = F_E - F_{\text{grav}} = ma = 0$$

$$F_E = F_{\text{grav}}$$

$$q_1 E_1 = m_{\text{drop}} g$$

$$E = \frac{m_{\text{drop}} g}{q} = \frac{\rho_{\text{oil}} V_{\text{drop}} g}{4e} = \frac{\rho_{\text{oil}} \left(\frac{4}{3}\pi\left(\frac{D}{2}\right)^3\right) g}{4e} = \frac{\pi \rho_{\text{oil}} D^3 g}{24e}$$

$$= \frac{\pi\left(0.850\frac{\text{g}}{\text{cm}^3} \times \frac{1\ \text{kg}}{1000\ \text{g}} \times \left(\frac{100\ \text{cm}}{1\ \text{m}}\right)^3\right)(1.10 \times 10^{-6}\ \text{m})^3 \left(9.8\frac{\text{m}}{\text{s}^2}\right)}{24(1.6 \times 10^{-19}\ \text{C})} = \boxed{9070\frac{\text{N}}{\text{C}}}$$

Part d)

$$q_1 E_1 = q_2 E_2$$

$$(4e)E_1 = (n_2 e)E_2$$

$$n_2 = \frac{4E_1}{E_2} = \frac{4\left(9070\frac{\text{N}}{\text{C}}\right)}{\left(5183\frac{\text{N}}{\text{C}}\right)} = \boxed{7}$$

REFLECT

The more charge on the drop, the weaker the external field needs to be in order to counteract the force due to gravity.

16.108

SET UP

The electric field is zero everywhere except in the region $0 \leq x \leq 3.00 \times 10^{-2}$ m, where there is a uniform electric field $\vec{E} = \left(100\frac{\text{N}}{\text{C}}\right)\hat{y}$. A proton is moving along the $-x$-axis with velocity $\vec{v} = \left(1.00 \times 10^6 \frac{\text{m}}{\text{s}}\right)\hat{x}$. When the proton enters the region $0 \leq x \leq 3.00 \times 10^{-2}$ m, the electric field exerts an upward force on it. The force acts only in the y direction, so the

velocity in the x direction will remain constant for all time. We can use this to calculate the time it takes the proton to travel from $x = 0$ to $x = 3.00 \times 10^{-2}$ m. Next, we can use Newton's second law to calculate the acceleration of the proton in the y direction. The y component of the velocity and the y position at $x = 3.00 \times 10^{-2}$ m can both be calculated using the constant acceleration kinematics equations. After the proton leaves the region with the electric field, it no longer experiences a net force on it, so its velocity will remain constant for all x greater than 3.00×10^{-2} m. We can then find the total change in y from $x = 0$ to $x = 10.0 \times 10^{-2}$ m.

SOLVE

Part a)

Time it takes to arrive to travel from $x = 0$ to $x = 3.00 \times 10^{-2}$ m:

$$v_x = \frac{\Delta x}{\Delta t}$$

$$\Delta t = \frac{\Delta x}{v_x} = \frac{3.00 \times 10^{-2}\ \text{m}}{\left(1.00 \times 10^6 \frac{\text{m}}{\text{s}}\right)} = 3.00 \times 10^{-8}\ \text{s}$$

Newton's second law in the y direction:

$$\sum F_y = F_E = m_\text{p} a_y$$

$$eE = m_\text{p} a_y$$

$$a_y = \frac{eE}{m_\text{p}} = \frac{(1.6 \times 10^{-19}\ \text{C})\left(100 \frac{\text{N}}{\text{C}}\right)}{1.67 \times 10^{-27}\ \text{kg}} = 9.58 \times 10^9 \frac{\text{m}}{\text{s}^2}$$

Velocity at $x = 3.00 \times 10^{-2}$ m:

$$v_y = v_{0,y} + a_y \Delta t = 0 + \left(9.58 \times 10^9 \frac{\text{m}}{\text{s}^2}\right)(3.00 \times 10^{-8}\ \text{s}) = 287 \frac{\text{m}}{\text{s}}$$

$$\boxed{\vec{v} = \left(1.00 \times 10^6 \frac{\text{m}}{\text{s}}\right)\hat{x} + \left(287 \frac{\text{m}}{\text{s}}\right)\hat{y}}$$

Position in the y direction at $x = 3.00 \times 10^{-2}$ m:

$$y_\text{f} = y_0 + v_{0,y} \Delta t + \frac{1}{2} a_y \Delta t = 0 + 0 + \frac{1}{2}\left(9.58 \times 10^9 \frac{\text{m}}{\text{s}^2}\right)(3 \times 10^{-8}\ \text{s}) = \boxed{4.31 \times 10^{-6}\ \text{m}}$$

Part b)

Velocity at $x = 10.0 \times 10^{-2}$ m:

$$\boxed{\vec{v} = \left(1.00 \times 10^{6}\frac{\text{m}}{\text{s}}\right)\hat{x} + \left(287\frac{\text{m}}{\text{s}}\right)\hat{y}}$$

Time it takes to arrive to travel from $x = 3.00 \times 10^{-2}$ m to $x = 10.0 \times 10^{-2}$ m:

$$v_x = \frac{\Delta x}{\Delta t}$$

$$\Delta t = \frac{\Delta x}{v_x} = \frac{7.00 \times 10^{-2}\text{ m}}{\left(1.00 \times 10^{6}\frac{\text{m}}{\text{s}}\right)} = 7.00 \times 10^{-8}\text{ s}$$

Change in y position between $x = 3.00 \times 10^{-2}$ m and $x = 10.0 \times 10^{-2}$ m:

$$v_y = \frac{\Delta y}{\Delta t}$$

$$\Delta y = v_y \Delta t = \left(287\frac{\text{m}}{\text{s}}\right)(7.00 \times 10^{-8}\text{ s}) = 2.01 \times 10^{-5}\text{ m}$$

y position at $x = 10.0 \times 10^{-2}$ m:

$$y_\text{f} = \Delta y_{x=0 \to x=3.00\text{ cm}} + \Delta y_{x=3.00\text{ cm} \to x=10.0\text{ cm}}$$

$$= (4.31 \times 10^{-6}\text{ m}) + (2.01 \times 10^{-5}\text{ m}) = \boxed{2.44 \times 10^{-5}\text{ m}}$$

REFLECT

If we replaced the proton with an electron, all of the numbers would be the same, but the particle would move toward $-y$ rather than $+y$. Therefore, the answers to part (a) would be $\vec{v} = \left(1.00 \times 10^{6}\frac{\text{m}}{\text{s}}\right)\hat{x} - \left(287\frac{\text{m}}{\text{s}}\right)\hat{y}$ and $y_\text{f} = -4.31 \times 10^{-6}$ m.

16.109

SET UP

A sphere of radius R has uniform charge density ρ. A closed Gaussian surface consisting of a circular disk and a hemisphere, each of radius r and concentric with the sphere, surrounds half of the sphere as shown in the figure. The electric field emanates radially outward from the charged sphere as if it were a point charge. Therefore, the flux through the disk is equal to 0. The flux through the hemisphere is equal to EA, where E is the expression for the magnitude of the electric field and A is the surface area of the hemisphere. We can rearrange our expression to show that the total flux through the Gaussian surface is equal to the total enclosed charge divided by ε_0; only the charge located in the top half of the sphere is enclosed by the Gaussian surface.

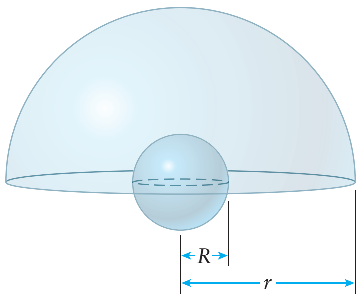


Figure 16-45 Problem 109

SOLVE

Part a)

The $\boxed{\text{flux through the circular disk is zero}}$ because the electric field vectors in that plane are perpendicular to the normal vector of the disk.

Flux through the hemisphere:

$$\Phi = \int \vec{E} \cdot d\vec{A} = EA = \left(\frac{kQ}{r^2}\right)(2\pi r^2) = 2\pi k(\rho V) = 2\pi k\rho\left(\frac{4}{3}\pi R^3\right) = \frac{8\pi^2 k\rho R^3}{3}$$

$$= \frac{8\pi^2\rho R^3}{3}\left(\frac{1}{4\pi\varepsilon_0}\right) = \boxed{\frac{2\pi\rho R^3}{3\varepsilon_0}}$$

Part b)

$$\Phi_{\text{total}} = \frac{2\pi\rho R^3}{3\varepsilon_0} = \frac{\left(\frac{2\pi\rho R^3}{3}\right)}{\varepsilon_0} = \frac{\rho\left(\frac{2\pi R^3}{3}\right)}{\varepsilon_0} = \frac{\rho\left(\frac{V}{2}\right)}{\varepsilon_0} = \frac{q_{\text{encl}}}{\varepsilon_0}$$

REFLECT

Gauss' law holds for any closed Gaussian surface.

16.110

SET UP

A finite line of charge with uniform linear charge density $\lambda = 4.00 \times 10^{-9}$ C/m has a length of $L = 4.00$ m. The line charge lies along the x-axis and is centered about the y-axis. The total charge on the line is equal to the linear charge density multiplied by its length. We can estimate the electric field at a point P located on the y-axis at 120 m by assuming the entire charge is located at the origin. We can then actually calculate the electric field due to the finite line charge by performing an integral. Due to symmetry, the x component of the electric field at point P is equal to zero. Therefore, the electric field strength at P is equal to the magnitude of the y component of the electric field, E_y. We can split the rod up into infinitesimal point charges dq and integrate over the y components of the field due to each point charge, $dE_y = (dE)\cos(\theta)$ where θ is the angle made with the y-axis. The field dE due to the point charge dq is equal to $dE = \frac{kdq}{r^2}$, where r is the straight-line distance between dq and P. We can convert the integral from dq to dx by realizing the charge on the rod is uniformly distributed (that is, the linear charge density is constant). Next, we can estimate the electric field at a point P' located on the y-axis at 2.00×10^{-2} m by assuming the line charge is infinitely long. The electric field for an infinite line charge was derived in Example 16-6 in the text (or can be derived again through Gauss' law), $\vec{E} = \left(\frac{\lambda}{2\pi\epsilon_0 r}\right)\hat{r}$. The derivation of the actual electric field due to the finite line from part (c) was general and will still hold here for $y_{P'} = 2.00 \times 10^{-2}$ m.

SOLVE

Part a)

$$Q_{\text{total}} = \lambda L = \left(4.00 \times 10^{-9}\frac{\text{C}}{\text{m}}\right)(4.00\text{ m}) = \boxed{1.60 \times 10^{-8}\text{ C}}$$

Part b)

$$\vec{E} = \left(\frac{kQ_{\text{total}}}{r^2}\right)\hat{y} = \left(\frac{\left(8.99 \times 10^9\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(1.60 \times 10^{-8}\text{ C})}{(120\text{ m})^2}\right)\hat{y} = \boxed{\left(9.9889 \times 10^{-3}\frac{\text{N}}{\text{C}}\right)\hat{y}}$$

Part c)

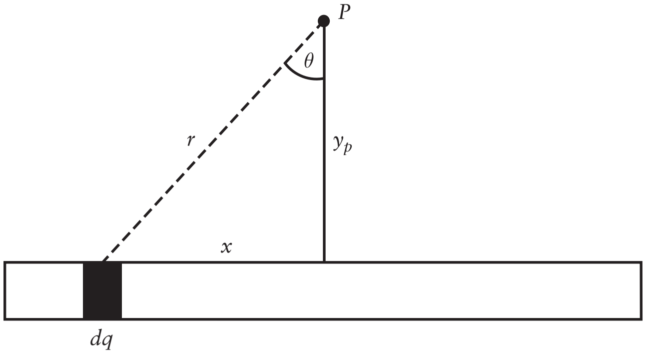


Figure 16-46 Problem 110

$$E_y = \int dE_y = \int (dE)\cos(\theta) = \int\left(\frac{kdq}{r^2}\right)\left(\frac{y_P}{r}\right)$$

Converting the integral from dq to dx using $dq = \lambda dx$:

$$E_y = \int_{-L/2}^{L/2}\left(\frac{k}{r^2}\right)\left(\frac{y_P}{r}\right)(\lambda dx) = k\lambda y_P\int_{-L/2}^{L/2}\frac{dx}{r^3}$$

But $r = \sqrt{x^2 + y_P^2} = (x^2 + y_P^2)^{1/2}$:

$$E_y = k\lambda y_P\int_{-L/2}^{L/2}\frac{dx}{(x^2 + y_P^2)^{3/2}} = k\lambda y_P\left[\frac{x}{y_P^2\sqrt{x^2 + y_P^2}}\right]_{-L/2}^{L/2} = \frac{k\lambda}{y_P}\left[\frac{\left(\frac{L}{2}\right)}{\sqrt{\left(\frac{L}{2}\right)^2 + y_P^2}} - \frac{\left(-\frac{L}{2}\right)}{\sqrt{\left(-\frac{L}{2}\right)^2 + y_P^2}}\right]$$

$$= \frac{k\lambda}{y_P}\left[\frac{L}{\sqrt{\left(\frac{L}{2}\right)^2 + y_P^2}}\right]$$

$$= \frac{\left(8.99 \times 10^9 \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}\right)\left(4.00 \times 10^{-9} \frac{\mathrm{C}}{\mathrm{m}}\right)}{120\ \mathrm{m}} \left[\frac{4.00\ \mathrm{m}}{\sqrt{\left(\frac{4.00\ \mathrm{m}}{2}\right)^2 + (120\ \mathrm{m})^2}}\right] = 9.9875 \times 10^{-3} \frac{\mathrm{N}}{\mathrm{C}}$$

So, $\boxed{\vec{E} = \left(9.9875 \times 10^{-3} \frac{\mathrm{N}}{\mathrm{C}}\right)\hat{y}}$.

Part d)

$$\vec{E} = \left(\frac{\lambda}{2\pi\epsilon_0 r}\right)\hat{y} = \left(\frac{\left(4.00 \times 10^{-9} \frac{\mathrm{C}}{\mathrm{m}}\right)}{2\pi\left(8.85 \times 10^{-12} \frac{\mathrm{C^2}}{\mathrm{N \cdot m^2}}\right)(2.00 \times 10^{-2}\ \mathrm{m})}\right)\hat{y} = \boxed{\left(3596.7 \frac{\mathrm{N}}{\mathrm{C}}\right)\hat{y}}$$

Part e)

$$E_y = \frac{k\lambda}{y_{P'}}\left[\frac{L}{\sqrt{\left(\frac{L}{2}\right)^2 + y_{P'}^2}}\right]$$

$$= \frac{\left(8.99 \times 10^9 \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}\right)\left(4.00 \times 10^{-9} \frac{\mathrm{C}}{\mathrm{m}}\right)}{2.00 \times 10^{-2}\ \mathrm{m}} \left[\frac{4.00\ \mathrm{m}}{\sqrt{\left(\frac{4.00\ \mathrm{m}}{2}\right)^2 + (2.00 \times 10^{-2}\ \mathrm{m})^2}}\right] = 3595.8 \frac{\mathrm{N}}{\mathrm{C}}$$

So, $\boxed{\vec{E} = \left(3595.8 \frac{\mathrm{N}}{\mathrm{C}}\right)\hat{y}}$.

REFLECT

We can treat a charged object of finite size as a point charge when we are very far away from it. We can treat a charged object of finite size as either an infinite line or plane when we are very close to it. The phrases "very far" and "very close" are relative to the actual size of the object.

16.111

SET UP

A semicircular ring of charge has a radius $R = 3.0 \times 10^{-2}$ m and is centered about a point P located at the origin. The charge is not uniformly distributed about the ring but has a linear charge density that depends on the angle θ: $\lambda = -\left(\frac{3.0 \times 10^{-6}\ \mathrm{C}}{\mathrm{m}}\right)\cos(\theta)$, where θ is measured from the $+y$-axis. In order to calculate the

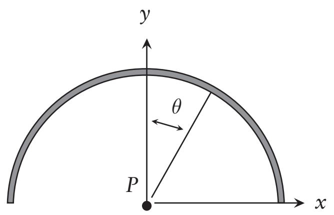


Figure 16-47 Problem 111

electric field at P, we can split the ring up into infinitesimal point charges dq and integrate over the x and y components of the field due to each point charge, $dE_x = (dE)\sin(\theta)$ and $dE_y = (dE)\cos(\theta)$, where θ is the angle made with the y-axis. The field dE due to the point charge dq is equal to $dE = \frac{kdq}{R^2}$. We can convert the integral from dq to ds via the linear charge density, and then from ds to $d\theta$ from the definition of arc length. If the semicircle were extended to be a complete ring, we would expect the electric field at P to be zero due to symmetry.

SOLVE

Part a)
x component:

$$E_x = \int \frac{k}{R^2}\sin(\theta)dq = \frac{k}{R^2}\int \lambda\sin(\theta)ds = \frac{k}{R^2}\int_{-\pi/2}^{\pi/2}\lambda\sin(\theta)Rd\theta$$

$$= \frac{k}{R}\int_{-\pi/2}^{\pi/2}\left(3.0\times10^{-6}\frac{\mathrm{C}}{\mathrm{m}}\right)\cos(\theta)\sin(\theta)d\theta = \frac{k}{R}\left(3.0\times10^{-\pm}\frac{\mathrm{C}}{\mathrm{m}}\right)\left[-\frac{1}{2}(\cos^2(\theta))\right]_{-\pi/2}^{\pi/2}$$

$$= -\frac{k}{2R}\left(3.0\times10^{-6}\frac{\mathrm{C}}{\mathrm{m}}\right)\left[\cos^2\left(\frac{\pi}{2}\right) - \cos^2\left(-\frac{\pi}{2}\right)\right] = 0$$

y component:

$$E_y = -\int \frac{k}{R^2}\cos(\theta)dq = -\frac{k}{R^2}\int \lambda\cos(\theta)ds = -\frac{k}{R^2}\int_{-\pi/2}^{\pi/2}\lambda\cos(\theta)Rd\theta$$

$$= -\frac{k}{R}\int_{-\pi/2}^{\pi/2}\left(-3.0\times10^{-6}\frac{\mathrm{C}}{\mathrm{m}}\right)\cos^2(\theta)d\theta = \frac{k}{R}\left(3.0\times10^{-6}\frac{\mathrm{C}}{\mathrm{m}}\right)\left[\frac{1}{2}(\theta + \sin(\theta)\cos(\theta))\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{k}{2R}\left(3\times .010^{-6}\frac{\mathrm{C}}{\mathrm{m}}\right)\left[\left(\frac{\pi}{2}\right) + \sin\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi}{2}\right) - \left(-\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right)\cos\left(-\frac{\pi}{2}\right)\right]$$

$$= \frac{k}{2R}\left(3.0\times10^{-6}\,\frac{\mathrm{C}}{\mathrm{m}}\right)\left[\left(\frac{\pi}{2}\right) + \left(\frac{\pi}{2}\right)\right] = \frac{\left(8.99\times10^9\,\frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)\left(3.0\times10^{-6}\,\frac{\mathrm{C}}{\mathrm{m}}\right)\pi}{2(3.0\times10^{-2}\ \mathrm{m})}$$

$$= 1\times10^6\,\frac{\mathrm{N}}{\mathrm{C}}$$

The electric field at point P is $\boxed{\vec{E} = \left(1.4\times10^6\frac{\mathrm{N}}{\mathrm{C}}\right)\hat{y}}$.

Part b) The charge distribution is still symmetric about the origin in this case. The field at point P would be equal to zero.

REFLECT

Even though the charge is not uniformly distributed, it is symmetrically distributed about the y-axis; therefore, the x component of the electric field at P should be equal to zero.

16.112

SET UP

A solid sphere of radius R_i and uniform charge density ρ_I is surrounded by a thick, spherical shell of inner radius R_M, outer radius R_o, and uniform charge density ρ_o. We can use Gauss' law to derive an expression for the electric field in the region of point P ($r_P < R_I$), point Q ($R_I \leq r_Q < R_M$), and point R ($r_R \geq R_o$).

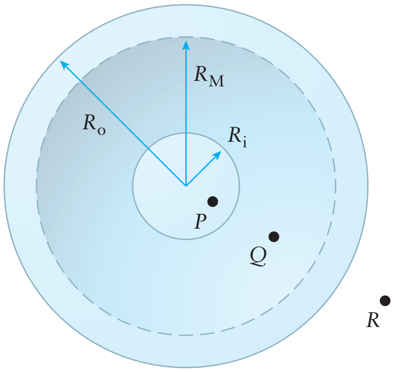


Figure 16-48 Problem 112

SOLVE

At point P ($r_P < R_I$):

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\epsilon_0}$$

$$E(4\pi r_P^2) = \frac{\rho_i V_{\text{encl}}}{\epsilon_0} = \frac{\rho_i\left(\frac{4}{3}\pi r_P^3\right)}{\epsilon_0}$$

$$E = \frac{\rho_i r_P}{3\epsilon_0}$$

The electric field in this region has a magnitude of $\boxed{E = \frac{\rho_i r_P}{3\epsilon_0} \text{ and points radially outward}}$.

At point Q ($R_I \leq r_Q < R_M$):

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\epsilon_0}$$

$$E(4\pi r_Q^2) = \frac{\rho_I V_{\text{sphere}}}{\epsilon_0} = \frac{\rho_I\left(\frac{4}{3}\pi R_I^3\right)}{\epsilon_0}$$

$$E = \frac{\rho_I R_I^3}{3\epsilon_0 r_Q^2}$$

The electric field in this region has a magnitude of $\boxed{E = \frac{\rho_{\mathrm{I}} R_{\mathrm{I}}^3}{3\epsilon_0 r^2} \text{ and points radially outward}}$.

At point R ($r_R \geq R_{\mathrm{o}}$):

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\mathrm{encl}}}{\epsilon_0}$$

$$E(4\pi r_R^2) = \frac{\rho_{\mathrm{I}} V_{\mathrm{sphere}} + \rho_{\mathrm{o}} V_{\mathrm{shell}}}{\epsilon_0} = \frac{\rho_{\mathrm{I}}\left(\frac{4}{3}\pi R_{\mathrm{I}}^3\right) + \rho_{\mathrm{o}}\left(\frac{4}{3}\pi R_{\mathrm{o}}^3 - \frac{4}{3}\pi R_{\mathrm{M}}^3\right)}{\epsilon_0}$$

$$E = \frac{\rho_{\mathrm{I}} R_{\mathrm{I}}^3 + \rho_{\mathrm{o}}(R_{\mathrm{o}}^3 - R_{\mathrm{M}}^3)}{3\epsilon_0 r_R^2}$$

The electric field in this region has a magnitude of $\boxed{E = \frac{\rho_{\mathrm{I}} R_{\mathrm{I}}^3 + \rho_{\mathrm{o}}(R_{\mathrm{o}}^3 - R_{\mathrm{M}}^3)}{3\epsilon_0 r_R^2} \text{ and points radially outward}}$.

REFLECT

It makes sense that the field should increase in magnitude with r within the sphere and decrease in the gap between the two objects as well as outside the object.

16.113

SET UP

Two hollow, concentric, spherical shells are covered with charge. The inner sphere has a radius R_{i} and a surface charge density of $+\sigma_{\mathrm{i}}$; the outer sphere has a radius R_{o} and a surface charge density of $-\sigma_{\mathrm{o}}$. We can use Gauss' law to derive an expression for the electric field everywhere in space. We will use a spherical Gaussian surface of radius r because of the symmetry of the charge distribution. Since the spheres are hollow, the charge only exists on the surface of each sphere. Once we have an algebraic expression for the electric field, we can plug in values $R_{\mathrm{i}} = 5$ cm, $R_{\mathrm{o}} = 8$ cm, $\sigma_{\mathrm{i}} = 20 \times 10^{-6}\ \frac{\mathrm{C}}{\mathrm{cm}^2}$, and $\sigma_{\mathrm{o}} = 14 \times 10^{-6}\ \frac{\mathrm{C}}{\mathrm{cm}^2}$ to calculate the magnitude of the field in each region.

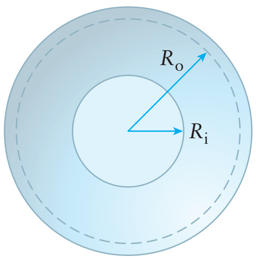
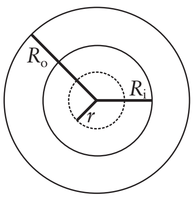


Figure 16-49 Problem 113

SOLVE

Part a)

$r < R_{\mathrm{i}}$

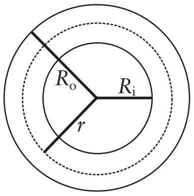


$$\int \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\varepsilon_0} = \frac{0}{\varepsilon_0}$$

$$\boxed{\vec{E} = 0}$$

Part b)

$R_{\text{i}} < r < R_{\text{o}}$



$$\int \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\varepsilon_0}$$

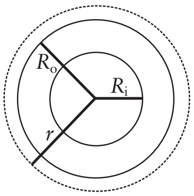
$$EA = \frac{q_{\text{encl}}}{\varepsilon_0}$$

$$E = \frac{q_{\text{encl}}}{A\varepsilon_0} = \frac{4\pi R_{\text{i}}^2 \sigma_{\text{i}}}{(4\pi r^2)\varepsilon_0} = \frac{R_{\text{i}}^2 \sigma_{\text{i}}}{r^2 \varepsilon_0}$$

The electric field has a magnitude of $\boxed{\dfrac{R_{\text{i}}^2 \sigma_{\text{i}}}{r^2 \varepsilon_0} \text{ and points radially outward}}$.

Part c)

$r > R_{\text{o}}$



$$\int \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\varepsilon_0}$$

$$EA = \frac{q_{\text{encl}}}{\varepsilon_0}$$

$$E = \frac{q_{\text{encl}}}{A\varepsilon_0} = \frac{4\pi R_{\text{i}}^2 \sigma_{\text{i}} - 4\pi R_{\text{o}}^2 \sigma_{\text{o}}}{(4\pi r^2)\varepsilon_0} = \frac{R_{\text{i}}^2 \sigma_{\text{i}} - R_{\text{o}}^2 \sigma_{\text{o}}}{r^2 \varepsilon_0}$$

The magnitude of the electric field is $\boxed{\dfrac{R_{\text{i}}^2 \sigma_{\text{i}} - R_{\text{o}}^2 \sigma_{\text{o}}}{r^2 \varepsilon_0} \text{ and points radially outward if } R_{\text{i}}^2 \sigma_{\text{i}} > R_{\text{o}}^2 \sigma_{\text{o}}}$ $\boxed{\text{or inward if } R_{\text{i}}^2 \sigma_{\text{i}} < R_{\text{o}}^2 \sigma_{\text{o}}}$.

Part d)

For $r < R_i$, $\boxed{E = 0}$.

$$\text{For } R_i < r < R_o,\ E = \frac{R_i^2\sigma_i}{r^2\varepsilon_0} = \frac{(5\text{ cm})^2\left(20 \times 10^{-6}\,\dfrac{\text{C}}{\text{cm}^2}\right)}{r^2\left(8.85 \times 10^{-12}\,\dfrac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)} = \boxed{\frac{5.65 \times 10^7}{r^2}\,\frac{\text{N}\cdot\text{m}^2}{\text{C}}}$$

$$\text{For } r > R_o,\ E = \frac{R_i^2\sigma_i - R_o^2\sigma_o}{r^2\varepsilon_0}$$

$$= \frac{(5\text{ cm})^2\left(20 \times 10^{-6}\,\dfrac{\text{C}}{\text{cm}^2}\right) - (8\text{ cm})^2\left(14 \times 10^{-6}\,\dfrac{\text{C}}{\text{cm}^2}\right)}{r^2\left(8.85 \times 10^{-12}\,\dfrac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)}$$

$$= \boxed{\frac{4.47 \times 10^7}{r^2}\,\frac{\text{N}\cdot\text{m}^2}{\text{C}}}$$

REFLECT

The negative value inside the absolute value symbols in part (d) means the electric field points radially inward (*i.e.*, in the negative $\hat{r}$ direction).

16.114

SET UP

A charge $q_1 = +2q$ is at the origin and a charge $q_2 = -q$ is on the x-axis at $x = a$. The total electric field on the x-axis is equal to the vector sum of the field due to q_1 and the field due to q_2. Since we're only interested in the field along the x-axis, we can just look at the x component of the field due to each point charge. We can use superposition to find the algebraic expression for the x component of the electric field due to the two charges for $x < 0$, $0 < x < a$, and $x > a$. The electric field will be equal to zero only in the region $x > a$ because the x component of the electric field due to q_1 points to the right while the x component of the electric field due to q_2 points to the left. Since q_1 is farther from q_2 in this region, there will be a location on the x-axis where their magnitudes are equal. We can then plot E_x versus x to see how the electric field varies along the x-axis.

SOLVE

Part a)

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{k|q_1|}{x^2}(-\hat{x}) + \frac{k|q_2|}{(a+x)^2}\hat{x} = k\left[-\frac{|q_1|}{x^2} + \frac{|q_2|}{(a+x)^2}\right]\hat{x} = \boxed{k\left[-\frac{2q}{x^2} + \frac{q}{(a+x)^2}\right]\hat{x}}$$

Part b)

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{k|q_1|}{x^2}\hat{x} + \frac{k|q_2|}{(a-x)^2}\hat{x} = k\left[\frac{|q_1|}{x^2} + \frac{|q_2|}{(a-x)^2}\right]\hat{x} = \boxed{k\left[\frac{2q}{x^2} + \frac{q}{(a-x)^2}\right]\hat{x}}$$

Part c)

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{k|q_1|}{x^2}\hat{x} + \frac{k|q_2|}{(x-a)^2}(-\hat{x}) = k\left[\frac{|q_1|}{x^2} - \frac{|q_2|}{(x-a)^2}\right]\hat{x} = \boxed{k\left[\frac{2q}{x^2} - \frac{q}{(x-a)^2}\right]\hat{x}}$$

Part d)

$$x = (2 + \sqrt{2})a$$

Part e)

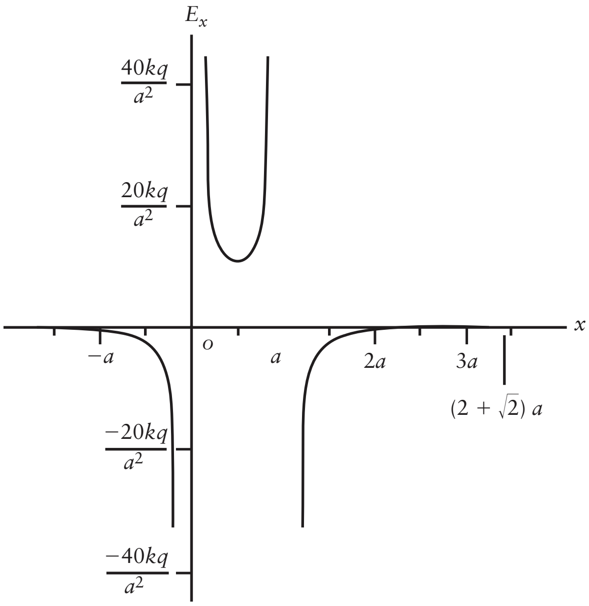


Figure 16-50 Problem 114

Part f) To the left of both charges, the x component of the electric field due to q_1 points to the left and is larger in magnitude than the x component of the electric field due to q_2, which points to the right in that region. Therefore, the x component of the total electric field for $x < 0$ should be negative. In between the two charges, the x component of the electric field due to q_1 and the x component of the electric field due to q_2 both point to the right in that region. Therefore, the x component of the total electric field for $x > 0$ should be positive. Finally, to the right of both charges, the x component of the electric field due to q_1 points to the right while the x component of the electric field due to q_2 points to the left. Since q_1 is farther from q_2 in this region, there will be a location on the x-axis where their magnitudes are equal. This is what we found in part (d). Therefore, the x component of the total electric field for $x > 0$ should start negative and then cross the x-axis and become positive.

REFLECT

The magnitude of the electric field is proportional to the charge but inversely proportional to the *square* of the distance, so this will have a larger effect on the field strength.

16.115

SET UP

A nonconducting, thick spherical shell has an inner radius of R_i and an outer radius of R_o. There is no charge in the cavity ($r < R_i$), but there is a nonuniform volume charge density in the shell itself given by $\rho(r) = \rho_0\left(\frac{R_i}{r}\right)$. We can draw a spherical Gaussian surface of radius r and use Gauss' law to calculate the electric field in the region $r > R_o$. The Gaussian surface encloses the entire shell and all of its charge, so we first need to perform an integral to determine the total charge in the shell. The total charge in the shell q_{shell} is equal to $\int \rho dV$; due to the symmetry of the shell, we'll use spherical coordinates, which means $dV = r^2 dr \sin(\theta) d\theta d\phi$.

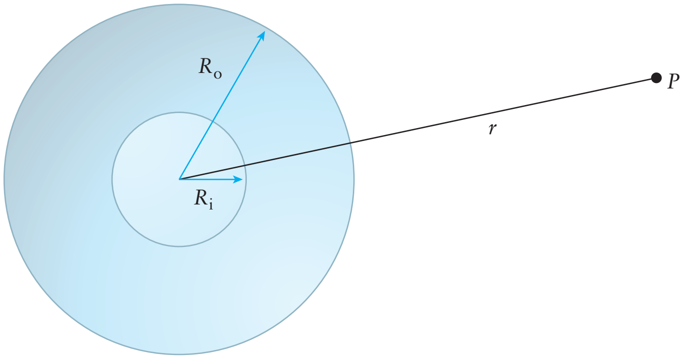


Figure 16-51 Problem 115

SOLVE

Total charge on the shell:

$$q_{\text{shell}} = \int \rho dV = \int_0^{2\pi}\int_0^{\pi}\int_{R_i}^{R_o}\left(\rho_0\frac{R_i}{r}\right)(r^2 dr \sin(\theta) d\theta d\phi) = \rho_0 R_i \int_{R^i}^{R_o} r dr \int_0^{\pi} \sin(\theta) d\theta \int_0^{2\pi} d\phi$$

$$= \rho_0 R_i\left[\frac{1}{2}r^2\right]_{R_i}^{R_o}[-\cos(\theta)]_0^{\pi}[\phi]_0^{2\pi} = \frac{\rho_0 R_i}{2}[R_o^2 - R_i^2][2][2\pi] = 2\pi\rho_0 R_i(R_o^2 - R_i^2)$$

Electric field at P:

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{\text{encl}}}{\epsilon_0}$$

$$EA = \frac{q_{\text{shell}}}{\varepsilon_0}$$

$$E = \frac{q_{\text{shell}}}{A\varepsilon_0} = \frac{2\pi\rho_0 R_i(R_o^2 - R_i^2)}{(4\pi r^2)\varepsilon_0} = \frac{\rho_0 R_i(R_o^2 - R_i^2)}{2\varepsilon_0 r^2}$$

The electric field at point P has a magnitude of $\boxed{\dfrac{\rho_0 R_i(R_o^2 - R_i^2)}{2\varepsilon_0 r^2} \text{ and points radially outward}}$.

REFLECT

In the region $r > R_o$, the shell looks like a point charge with a charge $q = 2\pi\rho_0 R_i(R_o^2 - R_i^2)$. With this in mind our final expression has the general form of the field due to a point charge $E = \dfrac{kq}{r^2}$, as we would expect.

16.116

SET UP

A very long, solid cylinder has a radius R and a uniform volume charge density ρ. We can draw a cylindrical Gaussian surface of radius r and length L that is concentric with the cylinder and apply Gauss' law in order to derive an expression for the electric field for $r < R$ and $r > R$. Due to the symmetry, we expect the electric field to point radially along the axis of the cylinder.

SOLVE

Part a)

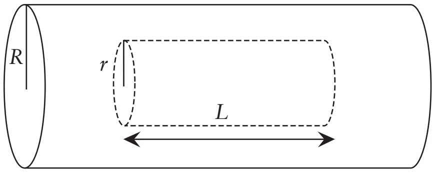


Figure 16-52 Problem 116

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\varepsilon_0}$$

$$EA_{\text{side}} = \frac{\rho(\pi r^2 L)}{\varepsilon_0}$$

$$E = \frac{\pi\rho r^2 L}{A_{\text{side}}\varepsilon_0} = \frac{\pi\rho r^2 L}{(2\pi r L)\varepsilon_0} = \frac{\rho r}{2\varepsilon_0}$$

The electric field for $r < R$ has a magnitude of $\boxed{\dfrac{\rho r}{2\varepsilon_0} \text{ and points radially outward}}$.

Part b)

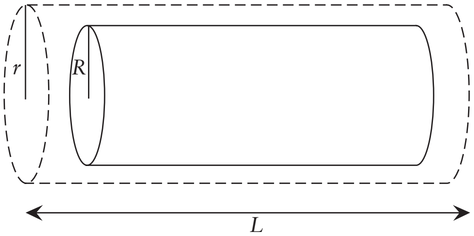


Figure 16-53 Problem 116

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\varepsilon_0}$$

$$EA_{\text{side}} = \frac{\rho(\pi R^2 L)}{\varepsilon_0}$$

$$E_{\text{side}} = \frac{\pi \rho R^2 L}{A\varepsilon_0} = \frac{\pi \rho R^2 L}{(2\pi r L)\varepsilon_0} = \frac{\rho R^2}{2\varepsilon_0 r}$$

The electric field for $r > R$ has a magnitude of $\boxed{\frac{\rho R^2}{2\varepsilon_0 r} \text{ and points radially outward}}$.

REFLECT

The flux through the ends of the Gaussian surface will be zero because their area vectors are perpendicular to the electric field; the only nonzero flux will be through the side of the cylinder, where the electric field and area vectors are parallel.

Chapter 17
Electrostatics II

Conceptual Questions

17.1 The electric potential is the electric potential energy per charge and is a scalar. The electric field is the electric force per unit charge and is a vector. The electric potential depends both on the electric field and also on the region over which the field extends.

17.2 Electric potential is the electric potential energy per unit charge. The electric potential is related to the strength of the electric field itself, independent of any test charge.

17.3 Both electric field and electric potential result from the existence of a charge. If a charge exists, it will create both of these physical quantities. In order to have a nonzero potential energy, a force must be able to do work on an object as it is moved. If only one charge exists, then there is no electric force acting on the charge. If there is no force, then that force can do no work, so there is no potential energy.

17.4 Any negatively charged particle, including an electron, will move in a direction opposite to that of the electric field. The electric field always points in the direction of decreasing potential, so the electron will move in the direction of increasing potential. It will also always move in the direction of decreasing potential energy.

17.5 This statement makes sense only if the zero point of the electric potential has been previously defined.

Get Help: Picture It – Electric Potential

17.6 In isolation an object will always accelerate toward a region of lower potential energy as required by the work-energy theorem. If ΔU is negative, and the charge q is negative, then ΔV must be positive. Another way to think about this question is to recall that electric field lines point in the direction of decreasing electric potential. Since negative charges accelerate in a direction opposite to those field lines, they must be accelerating toward a region of higher electric potential.

17.7 A topographical map details the lines surrounding a mountain, for example, where the elevation is the same. These plots let a hiker know how the terrain changes in elevation as you move from point A to point B. If the lines of elevation are very close together, that will indicate that the mountain is very steep. Walking along these lines of constant elevation is much easier as there is no change in gravitational potential energy. With an equipotential line, the quantity that stays the same is the voltage. With no change in voltage, there is no change in electric potential energy; hence, no work is done when an electric charge moves along an equipotential line. Likewise, these lines will never cross; they would be multivalued and that is not possible. The closer together they are, the greater the electric field is in that region.

17.8 Zero. The electric field is perpendicular to an equipotential; therefore, the work done in moving along an equipotential is zero.

17.9 Part a) Yes, a region of constant potential must have zero electric field.

Part b) No, if the electric field is zero, the potential need only be constant.

17.10 The potential difference depends on the electric field between the conductors. This electric field is proportional to the charge on each of the plates, so the potential difference is proportional to the charge. Since Q and V appear as a ratio in the definition of capacitance, the value of Q will cancel out, leaving only geometric and material properties.

17.11 Increase plate area, decrease separation, and increase the dielectric constant.

17.12 The potential energy stored in a capacitor is proportional to the square of the voltage. If the voltage doubles, the potential energy stored in the capacitor increases by a factor of four.

17.13 Connecting capacitors in series allows the total potential difference to be split among the individual capacitors. Every capacitor has a maximum allowable voltage that cannot be exceeded. If the maximum voltage across a capacitor is exceeded, the resulting electric field will cause dielectric breakdown, destroying the capacitor. By connecting the capacitors in series, each capacitor is kept below its maximum voltage.

17.14 The total capacitance is the sum of the individual capacitances. This means that for a given voltage capacitors in parallel store more charge than any one of the individual capacitors.

17.15 The charge on the capacitor remains constant, while the capacitance decreases. Therefore, the energy stored in the capacitor increases.

Get Help: P'Cast 17.5 – Defibrillator

17.16 Parallel. The equivalent capacitance of the three identical capacitors in parallel will be greater than that of the same three capacitors in series. When connected to a given potential difference, they will store more energy if connected in parallel.

17.17 For a given potential across them, the capacitors in parallel have a greater area and so can store a greater amount of charge, thus a larger capacitance.

17.18 It depends. Inserting a dielectric always increases the capacitance of a capacitor. If the capacitor is isolated, we can use $U = \dfrac{Q^2}{2C}$, from which we see inserting the dielectric will cause the stored energy to decrease. However, if the capacitor is attached to a battery, the potential is constant and we can use $U = \dfrac{CV^2}{2}$, and in this case the stored energy increases.

17.19 The larger dielectric strength increases the maximum voltage that can be applied across the electrodes. The larger dielectric constant decreases the voltage required to store a

given amount of charge, thus increasing the capacitance. The dielectric material can also be used to maintain the separation of the electrodes.

17.20 The electric field in the region between the plates is the same for both capacitors. The potential difference V across a capacitor and the electric field $\vec{E}$ in the region between the plates are related by the equation $V = \left| -\int \vec{E} \cdot d\vec{\ell} \right| = Ed$. If the potential difference across each capacitor is the same, which must be the case for the capacitors in parallel, then the electric fields in the regions between the plates must also be the same. However, A stores more charge.

Multiple-Choice Questions

17.21 **D** (its potential energy decreases and its electric potential decreases). The electric field points in the direction of the force a positive charge would experience. A positive charge moving in this direction would lower its potential energy. The electric field also points towards regions of lower potential.

17.22 **C** (from low potential to high potential). The electric field always points in the direction of decreasing potential, so the electron will move in the direction of increasing potential.

17.23 **C** (perpendicular to the electric field at every point). Electric field lines point perpendicularly to the equipotential surface.

17.24 **C** (is zero). With no change in voltage, there is no change in electric potential energy; hence, no work is done when an electric charge moves along an equipotential line.

17.25 **C** (equal to zero). The potential near the positive charge is large and positive and the potential near the negative charge is equal in magnitude to the positive charge's potential but negative. Therefore, the point exactly in the middle must have a potential of zero due to symmetry.

17.26 **D** ($E \neq 0$; $V = 0$). The electric field points from positive charges to negative charges. The up-and-down line exactly in the middle must have a potential of zero due to symmetry.

17.27 **B** (doubled).

$$\frac{U_2}{U_1} = \frac{\left(\frac{1}{2}\frac{Q_2^2}{C_2}\right)}{\left(\frac{1}{2}\frac{Q_1^2}{C_1}\right)} = \frac{\left(\frac{Q^2}{C_2}\right)}{\left(\frac{Q^2}{C_1}\right)} = \frac{C_1}{C_2} = \frac{\left(\frac{\varepsilon_0 A}{d_1}\right)}{\left(\frac{\varepsilon_0 A}{d_2}\right)} = \frac{d_2}{d_1} = \frac{2d_1}{d_1} = \boxed{2}$$

Get Help: Interactive Example – Parallel Plate Capacitor
P'Cast 17.5 – Defibrillator

17.28 **A** (halved).

$$\frac{U_2}{U_1} = \frac{\left(\frac{1}{2}C_2 V^2\right)}{\left(\frac{1}{2}C_1 V^2\right)} = \frac{C_2}{C_1} = \frac{\left(\frac{\varepsilon_0 A}{d_2}\right)}{\left(\frac{\varepsilon_0 A}{d_1}\right)} = \frac{d_1}{d_2} = \frac{d_1}{2d_1} = \frac{1}{2}$$

17.29 **A** (charge). The charge must be the same due to conservation of charge.

17.30 **B** (voltage). Circuit elements in parallel have the same voltage.

Estimation/Numerical Questions

17.31 The expression for the potential due to a point charge is $V = \frac{kq}{r}$. We would need to be 1 million meters away from a microcoulomb charge in order to experience a potential of a millivolt.

17.32 The voltage of a Van de Graaff generator is usually around 10^5 V.

17.33 The electric potential of an electron located a distance of approximately 5×10^{-11} m from the nucleus of an atom is around 25 V.

17.34 The car would need on the order of 30 million capacitors.

Problems

17.35

SET UP

A uniform electric field has a magnitude $E = 28$ V/m. The field makes an angle of 30 degrees with a path of length $d = 10$ m. The magnitude of the electric field is equal to the potential difference divided by the change in position along the axis of the field. Since the path is located at an angle relative to the field, the difference in position is related to the cosine.

SOLVE

$$E = \frac{\Delta V}{\Delta x}$$

$$\Delta V = E\Delta x = E(d\cos(30^\circ)) = \left(28\frac{\text{V}}{\text{m}}\right)(10\text{ m})\cos(30^\circ) = \boxed{242\text{ V}}$$

REFLECT

An equipotential line makes an angle of 90 degrees with the field.

17.36

SET UP

A charge $q = 1 \times 10^{-6}$ C is placed at a location where the potential is $V = 800$ V. The potential energy of the system is given by $U = qV$.

SOLVE

$$U = qV = (1 \times 10^{-6}\ \text{V})(800\ \text{V}) = \boxed{8 \times 10^{-4}\ \text{J}}$$

REFLECT

The potential energy of the system will decrease (that is, become more stable) if a positive charge is placed at a position where the potential is negative.

17.37

SET UP

A charge ($q_0 = +2.0$ C) is moved through a potential difference of $V = +9.0$ V. The work done by the electric field is equal to $W_{\text{by}\,\vec{E}} = -Vq_0$. Assuming the charge starts and ends at rest, the work required to move the charge is $W_{\text{required}} = -W_{\text{by}\,\vec{E}}$.

SOLVE

$$V = \frac{-W_{\text{by}\,\vec{E}}}{q_0} = \frac{W_{\text{required}}}{q_0}$$

$$W_{\text{required}} = Vq_0 = (9.0\ \text{V})(2.0\ \text{C}) = \boxed{18\ \text{J}}$$

REFLECT

It should require energy to move a positive charge to an area of higher potential.

17.38

SET UP

A potential difference V exists between the inner and outer surfaces of the cell. The inner surface is negatively charged, while the outer surface is positively charged. It requires $W_{\text{required}} = 1.5 \times 10^{-20}$ J of work to move a sodium ion with charge $q_0 = +e$ from the interior of the cell. The potential difference is equal to the work done divided by the charge.

SOLVE

$$V = \frac{-W_{\text{by}\,\vec{E}}}{q_0} = \frac{W_{\text{required}}}{q_0} = \frac{1.5 \times 10^{-20}\ \text{J}}{1.6 \times 10^{-19}\ \text{C}} = 0.094\ \text{V}$$

REFLECT

The exterior of the cell is higher in potential, which makes sense since it is positively charged and it requires positive work to eject a sodium ion from the interior.

17.39

SET UP

The electric field near a charged object is given by $E(r) = E_0\left(\frac{r}{r_0}\right)^3 + E_1\left(\frac{r}{r_0}\right)^2$, where E_0, E_1, and r_0 are constants. The potential difference between an arbitrary point r and $r = 0$ is given by the negative integral of $E(r)$ from $r = 0$ to r. We will assume that $V(0) = 0$.

SOLVE

$$V(r) - V(0) = -\int_0^r E(r)dr = -\int_0^r \left[E_0\left(\frac{r}{r_0}\right)^3 + E_1\left(\frac{r}{r_0}\right)^2 \right] dr = -\left[\frac{E_0}{4r_0^3}r^4 + \frac{E_1}{3r_0^2}r^3 \right]_0^r$$

$$\boxed{V(r) = -\left[\frac{E_0}{4r_0^3}r^4 + \frac{E_1}{3r_0^2}r^3 \right]}$$

REFLECT
In order to be dimensionally correct, E_0 and E_1 must have dimensions of electric field and r_0 must have dimensions of length.

17.40

SET UP
Some of the electric field lines are shown for a system of two charges. The equipotential curves for the system must always be perpendicular to the electric field lines. We can start by drawing dashed lines that are perpendicular to each field line and then connect them together to show the equipotential curves.

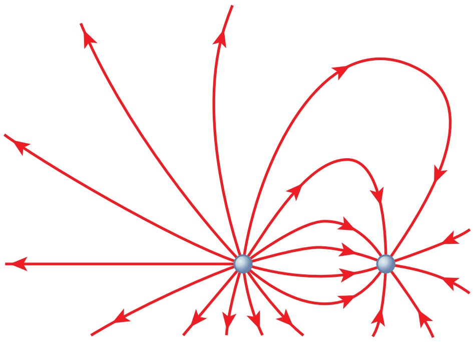


Figure 17-1 Problem 40

SOLVE

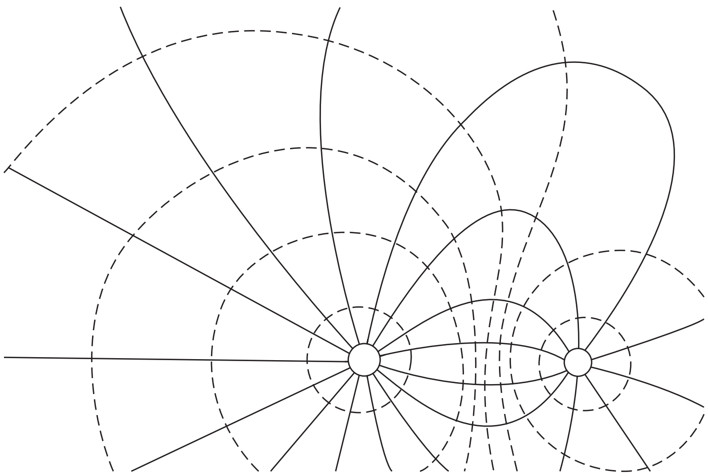


Figure 17-2 Problem 40

REFLECT
The left charge must be positive and the right charge must be negative because the field lines emanate from the left charge and terminate on the right one.

17.41

SET UP
Equipotential lines are shown at 1-m intervals in a two-dimensional region in space. The magnitude of the electric field is equal to the change in the potential divided by the change in position between the equipotential curves. The electric field is perpendicular to the equipotential curves and points toward regions of lower potential.

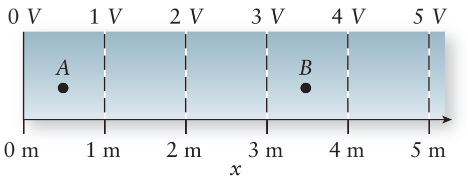


Figure 17-3 Problem 41

SOLVE
Part a)

$$E_A = \left|\frac{\Delta V}{\Delta x}\right| = \left|\frac{(1\text{ V}) - (0\text{ V})}{(1\text{ m}) - (0\text{ m})}\right| = 1\frac{\text{V}}{\text{m}}$$

The electric field at point A has a magnitude of $\boxed{\text{1 V/m and points to the left}}$.

Part b)

$$E_B = \left|\frac{\Delta V}{\Delta x}\right| = \left|\frac{(4\text{ V}) - (3\text{ V})}{(4\text{ m}) - (3\text{ m})}\right| = 1\frac{\text{V}}{\text{m}}$$

The electric field at point B has a magnitude of $\boxed{\text{1 V/m and points to the left}}$.

REFLECT
The electric field must be perpendicular to the equipotential curves, so it must point along the x-axis.

17.42

SET UP
Equipotential lines are shown for a two-dimensional region in space. The magnitude of the electric field is equal to the change in the potential divided by the change in position between the equipotential curves. The electric field is perpendicular to the equipotential curves and points toward regions of lower potential.

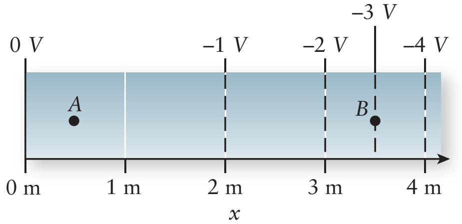


Figure 17-4 Problem 42

SOLVE
Part a)

$$E_A = \left|\frac{\Delta V}{\Delta x}\right| = \left|\frac{(-1\text{ V}) - (0\text{ V})}{(2\text{ m}) - (0\text{ m})}\right| = 0.5\frac{\text{V}}{\text{m}}$$

The electric field at point A has a magnitude of $\boxed{\text{0.5 V/m and points to the right}}$.

Part b)

$$E_B = \left|\frac{\Delta V}{\Delta x}\right| = \left|\frac{(-4\text{ V}) - (-2\text{ V})}{(4\text{ m}) - (3\text{ m})}\right| = 2\frac{\text{V}}{\text{m}}$$

The electric field at point B has a magnitude of $\boxed{2\text{ V/m and points to the right}}$.

REFLECT

The electric field must be perpendicular to the equipotential curves, so it must point along the x-axis.

17.43

SET UP

Two charged, parallel plates are 20 cm apart. The potential of the left plate is $V_{0\text{ cm}} = 100$ V, and the potential of the right plate is $V_{20\text{ cm}} = 0$ V. The electric field is uniform in between the plates and points toward the right. The magnitude of the field is constant between the plates and equal to the total potential difference divided by the distance between the plates. Once we know the magnitude of the field, we can calculate the potential at $x_A = 8$ cm, $x_B = 10$ cm, and $x_C = 18$ cm. The equipotential lines at these three locations are perpendicular to the electric field, which means they are parallel to the plates.

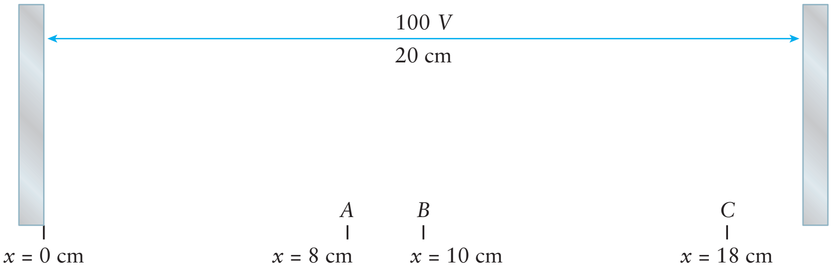


Figure 17-5 Problem 43

SOLVE

Magnitude of the electric field between the plates:

$$E = \left|\frac{\Delta V}{\Delta x}\right| = \left|\frac{(0\text{ V}) - (100\text{ V})}{(20\text{ cm}) - (0\text{ cm})}\right| = 5\frac{\text{V}}{\text{cm}}$$

Potential at point A:

$$E = \left|\frac{\Delta V}{\Delta x}\right| = \left|\frac{V_{20\text{ cm}} - V_{8\text{ cm}}}{(20\text{ cm}) - (8\text{ cm})}\right| = \left|\frac{(0\text{ V}) - V_{8\text{ cm}}}{12\text{ cm}}\right| = 5\frac{\text{V}}{\text{cm}}$$

$$\boxed{V_{8\text{ cm}} = 60\text{ V}}$$

Potential at point B:

$$E = \left|\frac{\Delta V}{\Delta x}\right| = \left|\frac{V_{20\text{ cm}} - V_{10\text{ cm}}}{(20\text{ cm}) - (10\text{ cm})}\right| = \left|\frac{(0\text{ V}) - V_{10\text{ cm}}}{10\text{ cm}}\right| = 5\frac{\text{V}}{\text{cm}}$$

$$\boxed{V_{10\text{ cm}} = 50\text{ V}}$$

Potential at point C:

$$E = \left|\frac{\Delta V}{\Delta x}\right| = \left|\frac{V_{20\text{ cm}} - V_{18\text{ cm}}}{(20\text{ cm}) - (18\text{ cm})}\right| = \left|\frac{(0\text{ V}) - V_{18\text{ cm}}}{2\text{ cm}}\right| = 5\frac{\text{V}}{\text{cm}}$$

$$\boxed{V_{18\text{ cm}} = 10\text{ V}}$$

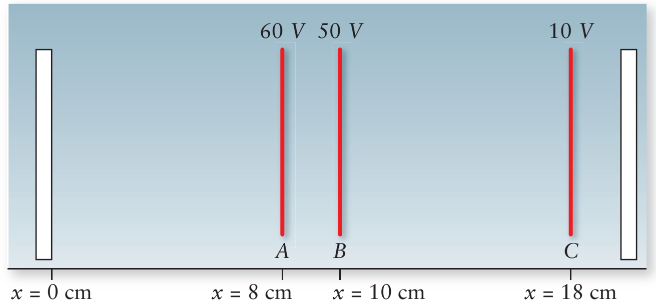


Figure 17-6 Problem 43

REFLECT

All of the equipotential curves in this region will be parallel to the plates and equally spaced.

17.44

SET UP

We are asked to draw the equipotential lines and electric field lines for a pairs of charges—$+q$ and $-q$. The equipotential lines will be the closest together near the charges and spread out as we move away. Electric field lines start on positive charges, end on negative charges, and cannot cross one another.

SOLVE

Parts a) (equipotential lines) and b) (electric fields)

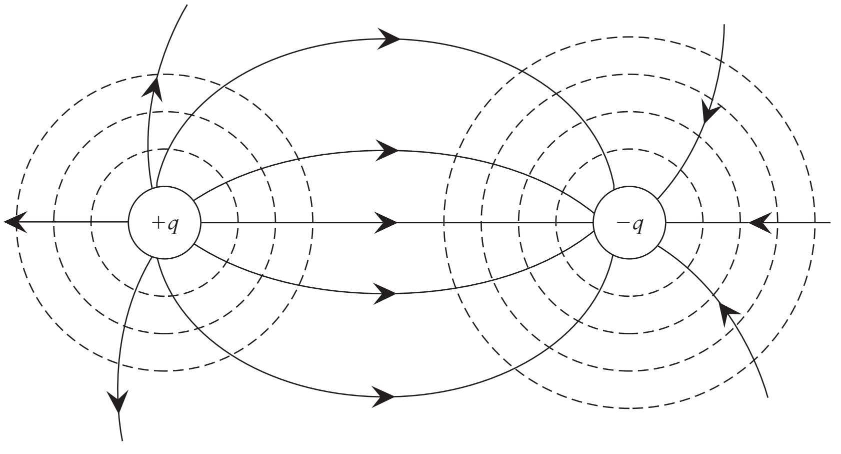


Figure 17-7 Problem 44

REFLECT

This charge configuration is referred to as an electric dipole.

17.45

SET UP

We are asked to draw the equipotential lines and electric field lines for two pairs of charges—two $+q$ and two $-q$. The equipotential lines will be the closest together near the charges and spread out as we move away. Electric field lines start on positive charges, end on negative charges, and cannot cross one another.

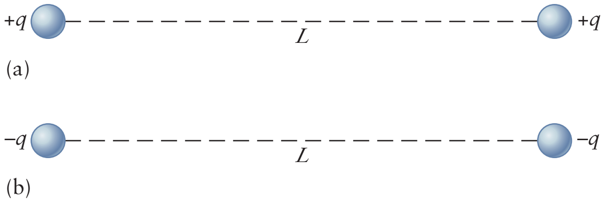


Figure 17-8 Problem 45

SOLVE

Parts a) (Equipotential lines) and b) (Electric fields)

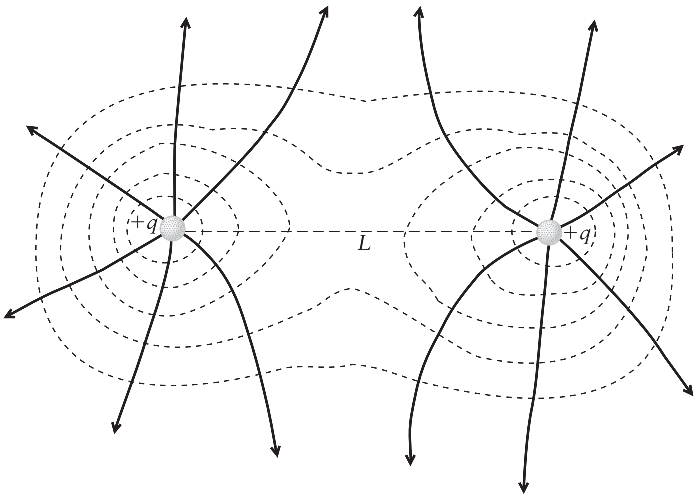


Figure 17-9 Problem 45

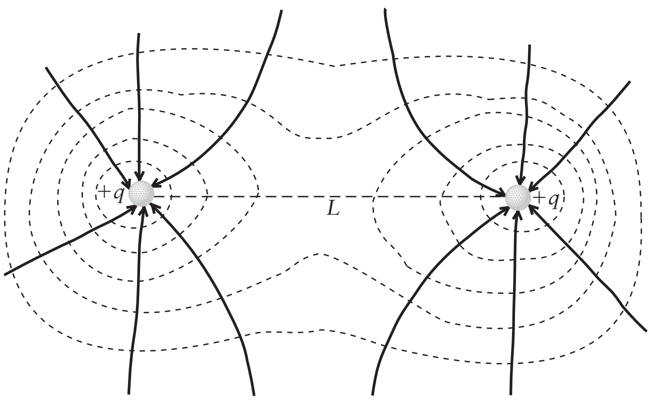


Figure 17-10 Problem 45

REFLECT

Although the shape of the equipotential curves are the same in both setups, the potentials those lines correspond to would not be the same.

17.46

SET UP

We are asked to draw the equipotential lines and electric field lines for a pair of charges: $+2q$ and $-q$. The equipotential lines will be the closest together near the charges and spread out as we move away. Electric field lines start on positive charges, end on negative charges, and cannot cross one another. The number of electric field lines should be proportional to the magnitude of the charge.

SOLVE

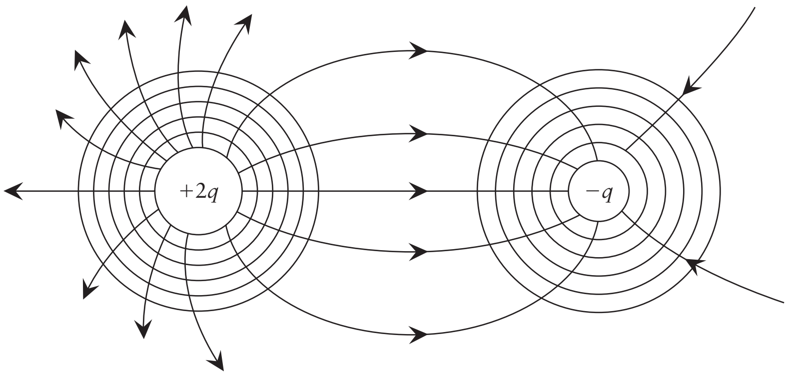


Figure 17-11 Problem 46

REFLECT

The density of the electric field lines is related to the magnitude of the charge through Gauss' law.

17.47

SET UP

A hydrogen nucleus consists of a single proton, so we can model the nucleus as a point charge $q = +1.6 \times 10^{-19}$ C. We can use the expression for the potential due to a point charge as a function of distance, $V = \frac{kq}{r}$, to calculate the potential at $r = 5 \times 10^{-11}$ m.

SOLVE

$$V = \frac{kq}{r} = \frac{\left(8.99 \times 10^{9} \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(1.6 \times 10^{-19}\ \text{C})}{5 \times 10^{-11}\ \text{m}} = \boxed{28.8\ \text{V}}$$

REFLECT

The Bohr radius, which is the most probable distance between the proton and the electron in the ground state of a hydrogen atom, is 5.29×10^{-11} m.

17.48

SET UP

The electric potential due to a point charge $q = +2 \times 10^{-6}$ C at a distance of $r = 0.5 \times 10^{-2}$ cm can be found from the expression for the potential due to a point charge, $V = \frac{kq}{r}$. The sign of the point charge matters, so the potential due to a point charge $q = -2 \times 10^{-6}$ C at a distance $r = 0.5 \times 10^{-2}$ cm will be negative.

SOLVE

Part a)

$$V = \frac{kq}{r} = \frac{\left(8.99 \times 10^{9}\frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}\right)(2 \times 10^{-6}\ \mathrm{C})}{0.5 \times 10^{-2}\ \mathrm{m}} = \boxed{3.6 \times 10^{6}\ \mathrm{V}}$$

Part b) The magnitude of the potential will be the same, but now it is negative: $\boxed{V = -3.6 \times 10^{6}\ \mathrm{V}}$.

REFLECT

Be sure to include the sign of the charge when calculating the electric potential.

17.49

SET UP

The electric potential at a distance of $r = 1.25$ m from a point charge q is $V = -200$ V. We can use the expression for the potential due to a point charge, $V = \frac{kq}{r}$, to calculate the value of q.

SOLVE

$$V = \frac{kq}{r}$$

$$q = \frac{Vr}{k} = \frac{(-200\ \mathrm{V})(1.25\ \mathrm{m})}{\left(8.99 \times 10^{9}\frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}\right)} = \boxed{-2.78 \times 10^{-8}\ \mathrm{C} = -27.8\ \mathrm{nC}}$$

REFLECT

The charge must be negative since the potential is negative.

17.50

SET UP

A point P is located on the same axis as two charges. The potential at point P is zero. For this to be true, the charges must be of opposite sign. Charge q_1 is closer to point P than q_2, which means q_2 must be larger in magnitude. Since the charges are opposite in sign, there will be one more point on the line between the two charges at which the potential is zero.

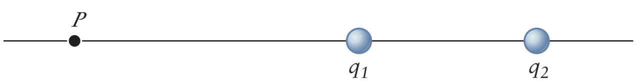


Figure 17-12 Problem 50

SOLVE

Part a) The two charges must be opposite in sign. Charge q_2 must have a greater magnitude than charge q_1.

Part b) In addition to point P and infinity, there will be a point between the two charges where the potential is zero.

REFLECT

It doesn't matter which charge is negative and which is positive, just as long as they are opposite to one another.

17.51

SET UP

Two point charges are placed on the x-axis: $q_1 = 0.5\ \mu\text{C}$ is located at $x = 0$ and $q_2 = -0.2\ \mu\text{C}$ is located at $x = 10$ cm. The electric potential due to a point charge q as a function of distance is $V(r) = \frac{kq}{r}$. We can calculate the position where the electric potential is equal to zero by adding the expressions for the potential due to each charge, setting the sum to zero, and solving for x.

SOLVE

$$V(x) = 0 = \frac{kq_1}{x} + \frac{kq_2}{(x-10)}\ (x \text{ in cm})$$

$$-\frac{q_1}{x} = \frac{q_2}{(x-10)}$$

$$x = \frac{10q_1}{q_1 + q_2} = \frac{10(0.5\ \mu\text{C})}{(0.5\ \mu\text{C}) + (-0.2\ \mu\text{C})} = \boxed{16.7\text{ cm}}$$

REFLECT

It makes sense that the position where $V = 0$ is to the right of both charges since the magnitude of q_1 is larger than the magnitude of q_2, q_1 is positive, and q_2 is negative.

17.52

SET UP

A charge $q_1 = +2.00 \times 10^{-6}$ C is placed at the origin. A second charge $q_2 = -3.00 \times 10^{-6}$ C is placed on the y-axis at $y = 40.0 \times 10^{-2}$ m. The potential at point a, which is located on the x-axis at $x = 40.0 \times 10^{-2}$ m, is equal to the sum of the potential due to q_1 at a and the potential due to q_2 at a. Point b is located at $x = 40.0 \times 10^{-2}$ m, $y = 30.0 \times 10^{-2}$ m. In order to calculate the potential difference between points a and b, we first need to calculate the potential at b, which is equal to the sum of the potential due to q_1 at b and the potential due to q_2 at b. The work required to move an electron at rest from a to rest at point b is equal to the change in the electron's mechanical energy from the work-energy theorem.

SOLVE

Part a)

$$V_a = \frac{kq_1}{r_{1\to a}} + \frac{kq_2}{r_{2\to a}} = k\left[\frac{q_1}{r_{1\to a}} + \frac{q_2}{r_{2\to a}}\right]$$

$$= \left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left[\frac{2.00 \times 10^{-6}\ \text{C}}{40.0 \times 10^{-2}\ \text{m}} + \frac{(-3.00 \times 10^{-6}\ \text{C})}{\sqrt{(40.0 \times 10^{-2}\ \text{m})^2 + (40.0 \times 10^{-2}\ \text{m})^2}}\right]$$

$$= \boxed{-2730\ \text{V}}$$

Part b)
Potential at point b:

$$V_b = \frac{kq_1}{r_{1\to b}} + \frac{kq_2}{r_{2\to b}} = k\left[\frac{q_1}{r_{1\to b}} + \frac{q_2}{r_{2\to b}}\right]$$

$$= \left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left[\frac{2.00 \times 10^{-6}\ \text{C}}{\sqrt{(40.0 \times 10^{-2}\ \text{m})^2 + (30.0 \times 10^{-2}\ \text{m})^2}} + \frac{(-3.00 \times 10^{-6}\ \text{C})}{\sqrt{(40.0 \times 10^{-2}\ \text{m})^2 + (10.0 \times 10^{-2}\ \text{m})^2}}\right] = -29451\ \text{V}$$

Change in potential:

$$V_b - V_a = (-29{,}451\ \text{V}) - (-2730\ \text{V}) = \boxed{-26{,}700\ \text{V}}$$

Part c)

$$W_{\text{required}} = \Delta U + \Delta K = q\Delta V + 0 = (-e)(V_b - V_a)$$

$$= -(1.6 \times 10^{-19}\ \text{C})(-26{,}700\ \text{V}) = \boxed{4.28 \times 10^{-15}\ \text{J}}$$

REFLECT

It should require positive work to move an electron to a point with a more negative potential.

17.53

SET UP

Two equal point charges ($q_1 = q_2 = -0.02 \times 10^{-6}$ C) are placed on the x-axis at the origin and $x = 10 \times 10^{-2}$ m, respectively. The electric potential at point A ($x = 5 \times 10^{-2}$ m, $y = 0$) and at point B ($x = 0$, $y = 5 \times 10^{-2}$ m) is equal to the sum of the potential due to q_1 and the potential due to q_2 at each of those points.

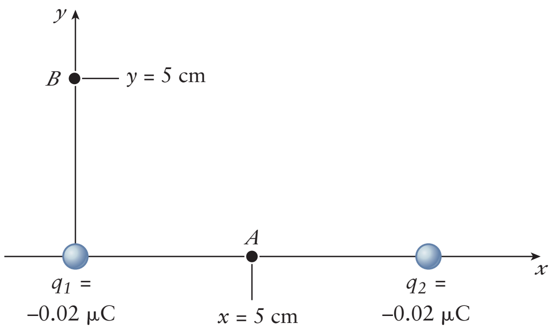


Figure 17-13 Problem 53

SOLVE

Part a)

$$V_A = \frac{kq_1}{r_{1\to A}} + \frac{kq_2}{r_{2\to A}} = k\left[\frac{q_1}{r_{1\to A}} + \frac{q_2}{r_{2\to A}}\right]$$

$$= \left(8.99 \times 10^9 \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}\right)\left[\frac{(-0.02 \times 10^{-6}\ \mathrm{C})}{5 \times 10^{-2}\ \mathrm{m}} + \frac{(-0.02 \times 10^{-6}\ \mathrm{C})}{5 \times 10^{-2}\ \mathrm{m}}\right] = \boxed{-7 \times 10^3\ \mathrm{V}}$$

Part b)

$$V_B = \frac{kq_1}{r_{1\to B}} + \frac{kq_2}{r_{2\to B}} = k\left[\frac{q_1}{r_{1\to B}} + \frac{q_2}{r_{2\to B}}\right]$$

$$= \left(8.99 \times 10^9 \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}\right)\left[\frac{(-0.02 \times 10^{-6}\ \mathrm{C})}{5 \times 10^{-2}\ \mathrm{m}} + \frac{(-0.02 \times 10^{-6}\ \mathrm{C})}{\sqrt{(10 \times 10^{-2}\ \mathrm{m})^2 + (5 \times 10^{-2}\ \mathrm{m})^2}}\right]$$

$$= \boxed{-5 \times 10^3\ \mathrm{V}}$$

REFLECT

The potential at B should be less negative than the potential at A since q_2 is farther away from B.

17.54

SET UP

Four charges are placed in the xy-plane: charge A ($q_A = 10 \times 10^{-6}$ C) is located at $x = -2 \times 10^{-2}$ m, $y = 0$; charge B ($q_B = -20 \times 10^{-6}$ C) is located at $x = +5 \times 10^{-2}$ m, $y = 0$; charge C ($q_C = 18 \times 10^{-6}$ C) is located at $x = -2 \times 10^{-2}$ m, $y = -2 \times 10^{-2}$ m; and charge D ($q_D = -15 \times 10^{-6}$ C) is located at $x = 0$, $y = -3 \times 10^{-2}$ m. The electric potential at the origin is equal to the sum of the potential due to each charge at that point.

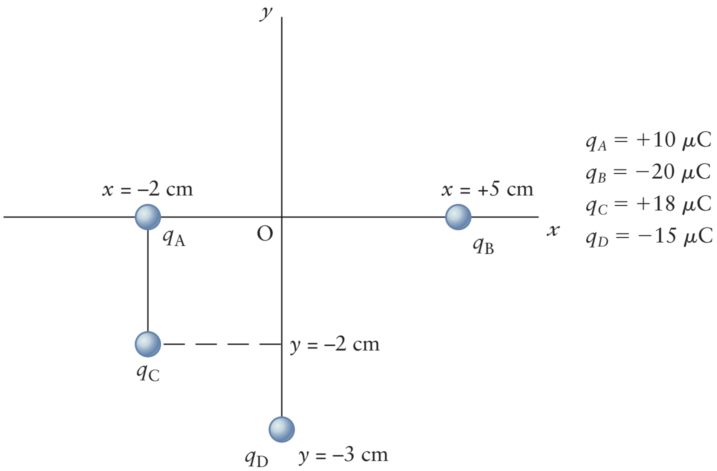


Figure 17-14 Problem 54

SOLVE

$$V = \frac{kq_A}{r_A} + \frac{kq_B}{r_B} + \frac{kq_C}{r_C} + \frac{kq_D}{r_D} = k\left[\frac{q_A}{r_A} + \frac{q_B}{r_B} + \frac{q_C}{r_C} + \frac{q_D}{r_D}\right]$$

$$= \left(8.99 \times 10^9 \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}\right)\left[\frac{10 \times 10^{-6}\ \mathrm{C}}{2 \times 10^{-2}\ \mathrm{m}} + \frac{(-20 \times 10^{-6}\ \mathrm{C})}{5 \times 10^{-2}\ \mathrm{m}}\right.$$

$$\left. + \frac{18 \times 10^{-6}\ \mathrm{C}}{\sqrt{(2 \times 10^{-2}\ \mathrm{m})^2 + (2 \times 10^{-2}\ \mathrm{m})^2}} + \frac{(-15 \times 10^{-6}\ \mathrm{C})}{3 \times 10^{-2}\ \mathrm{m}}\right]$$

$$= \boxed{2.13 \times 10^6\ \mathrm{V}}$$

REFLECT

Remember that we only need to consider the distance from the charge, not the displacement.

17.55

SET UP

A uniformly charged sphere of radius R carries a charge Q. We are asked to derive an expression for the potential at a point $r > R$ from the electric field in that region. After finding the field from Gauss' law, we can relate it to the potential using $V(r) - V(\infty) = -\int_{\infty}^{r} \vec{E} \cdot d\vec{r}$. We'll take the potential at infinity to be 0.

SOLVE

Electric field:

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\mathrm{encl}}}{\varepsilon_0}$$

$$EA = \frac{q_{\mathrm{encl}}}{\varepsilon_0}$$

$$E = \frac{q_{\mathrm{encl}}}{\varepsilon_0 A} = \frac{Q}{\varepsilon_0(4\pi r^2)} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

Potential:

$$V(r) - V(\infty) = -\int_{\infty}^{r} \vec{E} \cdot d\vec{r}$$

$$V(r) - 0 = -\int_{\infty}^{r} E dr$$

$$V(r) = -\int_{\infty}^{r}\left(\frac{Q}{4\pi\varepsilon_0 r^2}\right)dr = -\frac{Q}{4\pi\varepsilon_0}\int_{\infty}^{r}\frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_{\infty}^{r} = \boxed{\frac{Q}{4\pi\varepsilon_0 r} = \frac{kQ}{r}}$$

REFLECT

The potential outside of the sphere has the same algebraic form as the potential due to a point charge, as expected.

17.56

SET UP

A cube with side length $a = 1.00\text{ m}$ has charges ($q = +2.00 \times 10^{-6}\text{ C}$) located at seven of its corners (see figure). The potential at the vacant corner is equal to the sum of the potential due to each of the seven point charges by superposition. We can first use geometry to write all of the distances in terms of a. The work done by an external agent to bring an eighth point charge q from rest at infinity to rest at the vacant corner is equal to the change in this charge's potential energy. If the eighth charge were instead negative, the sign of the work done by the external agent will be opposite.

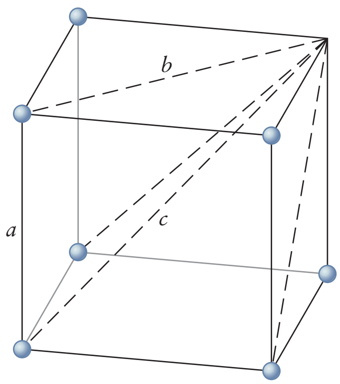


Figure 17-15 Problem 56

SOLVE

Lengths:

$$b = a\sqrt{2}$$

$$c^2 = a^2 + b^2 = a^2 + 2a^2 = 3a^2$$

$$c = a\sqrt{3}$$

Part a)

$$V = 3\left(\frac{kq}{a}\right) + 3\left(\frac{kq}{b}\right) + \left(\frac{kq}{c}\right) = kq\left[\frac{3}{a} + \frac{3}{a\sqrt{2}} + \frac{1}{a\sqrt{3}}\right] = \frac{kq}{a}\left[3 + \frac{3}{\sqrt{2}} + \frac{1}{\sqrt{3}}\right]$$

$$= \frac{\left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(2.00 \times 10^{-6}\text{ C})}{1.00\text{ m}}\left[3 + \frac{3}{\sqrt{2}} + \frac{1}{\sqrt{3}}\right] = \boxed{1.02 \times 10^5\text{ V}}$$

Part b)

$$W_{\text{NC}} = \Delta U + \Delta K = q\Delta V + 0 = (2.00 \times 10^{-6}\text{ C})((1.02 \times 10^5\text{ V}) - 0) = \boxed{2.05\text{ J}}$$

Part c) The potential at the vacant corner remains the same, but the work done by the external agent will be -2.05 J because the electric field does positive work on the negative charge.

REFLECT

The positive charges in the cube will repel the incoming positive charge, so it makes sense that an external agent would need to do positive work.

17.57

SET UP

Three particles, each with charge q, are located at different corners of a rhombus with sides of length a, a short diagonal of length a, and a long diagonal of length b. The relationship between the sides of the rhombus and its diagonals is: $4a^2 = a^2 + b^2$. The total electric potential energy of the charge distribution is equal to the sum of the pairwise potential

energies of the charges, $U_{q_A q_B} = \frac{kq_A q_B}{r}$. The required external work to bring a fourth charge q from infinity to the fourth corner of the rhombus is equal to the change in the particle's mechanical energy. Since the particle starts and ends at rest, the change in the kinetic energy is equal to zero. The potential energy of the particle at infinity is defined to be zero. Finally, we can use our answers to parts a and b to calculate the total electric potential energy of the charge distribution consisting of the four charges.

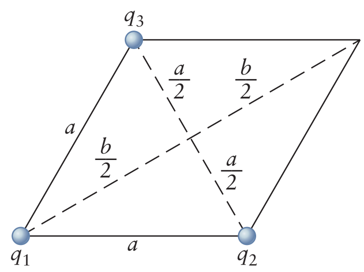


Figure 17-16 Problem 57

SOLVE

Part a)

$$U_{\text{total}} = U_{q_1q_2} + U_{q_2q_3} + U_{q_1q_3} = \frac{kq_1q_2}{a} + \frac{kq_2q_3}{a} + \frac{kq_1q_3}{a} = \frac{k}{a}[q^2 + q^2 + q^2] = \boxed{\frac{3kq^2}{a}}$$

Part b)

$$W = \Delta U = U_{\text{f}} - U_{\text{i}} = U_{\text{f}} - 0 = U_{q_1q_4} + U_{q_2q_4} + U_{q_3q_4} = \frac{kq_1q_4}{b} + \frac{kq_2q_4}{a} + \frac{kq_3q_4}{a}$$

$$= \frac{kq^2}{b} + \frac{kq^2}{a} + \frac{kq^2}{a} = \frac{kq^2}{\sqrt{4a^2 - a^2}} + \frac{2kq^2}{a} = \boxed{\left(2 + \frac{1}{\sqrt{3}}\right)\frac{kq^2}{a}}$$

Part c)

$$U_{\text{total}} = (U_{q_1q_2} + U_{q_2q_3} + U_{q_1q_3}) + (U_{q_1q_4} + U_{q_2q_4} + U_{q_3q_4}) = \left(\frac{3kq^2}{a}\right) + \left(\left(2 + \frac{1}{\sqrt{3}}\right)\frac{kq^2}{a}\right)$$

$$= \boxed{\left(5 + \frac{1}{\sqrt{3}}\right)\frac{kq^2}{a}}$$

REFLECT

We can talk about the electric potential due to one charge, but the electric potential energy due to the interaction between two charges.

17.58

SET UP

The x-axis coincides with the symmetry axis of a uniformly charged thin disk of radius R and uniform surface charge density of $+\sigma$; the disk is centered at the origin. From Section 16-5 of the text, the x component of the electric field due to a uniformly charged disk is given by $E_x = \frac{2kQ}{R^2}\left(1 - \frac{x}{\sqrt{R^2 + x^2}}\right) = 2\pi k\sigma\left(1 - \frac{x}{\sqrt{R^2 + x^2}}\right)$. From Section 17-3 of the text, the electric potential along the axis of a uniformly charged disk is given by $V = \frac{2kQ}{R^2}(\sqrt{R^2 + x^2} - x) = 2\pi k\sigma(\sqrt{R^2 + x^2} - x)$. The potential of the infinite plane has some arbitrary maximum value on the charged plane and then decreases linearly, with the same slope, on either side of the charged plane.

SOLVE

Parts a) (solid lines) and c) (dashed lines):

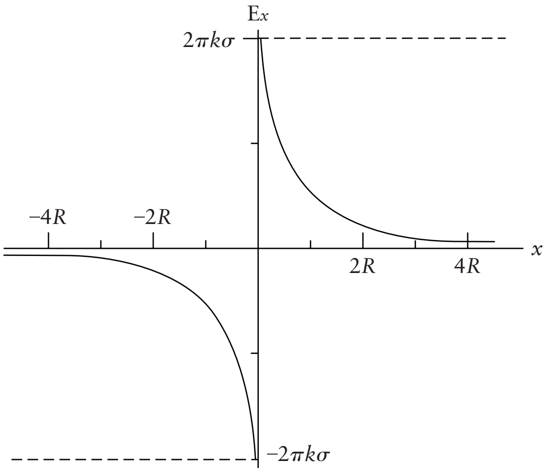


Figure 17-17 Problem 58

Parts b) (solid lines) and c) (dashed lines):

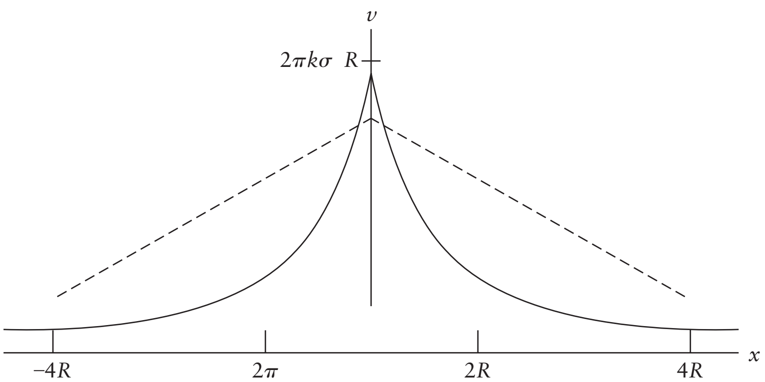


Figure 17-18 Problem 58

REFLECT

As the radius of the disk increases, the plots will change from the solid line ones to the dashed line ones. A charged disk with an infinite radius is equivalent to an infinite sheet of charge.

17.59

SET UP

A capacitor ($C = 2.0 \times 10^{-6}$ F) is connected to a battery ($V = 12$ V). The magnitude of the charge q on each plate of the capacitor is given by $q = CV$.

SOLVE

$$q = CV = (2.0 \times 10^{-6}\text{ F})(12\text{ V}) = \boxed{2.4 \times 10^{-5}\text{ C} = 24\ \mu\text{C}}$$

REFLECT

One of the plates has a charge of $+q$ and the other has a charge of $-q$. The capacitor as a whole has no net charge.

17.60

SET UP

A capacitor stores $q = 10 \times 10^{-6}$ C when it is attached to a battery ($V = 10$ V). The capacitance of the capacitor is given by $q = CV$.

SOLVE

$$q = CV$$

$$C = \frac{q}{V} = \frac{10 \times 10^{-6}\ \text{C}}{10\ \text{V}} = \boxed{1 \times 10^{-6}\ \text{F}}$$

REFLECT

The phrase "store q coulombs of charge" refers to the charge on *each* plate of the capacitor.

17.61

SET UP

A parallel plate capacitor is made of square plates that are 1.00 m on each side. The plates are separated by $d = 1 \times 10^{-3}$ m. The expression for the capacitance of a parallel plate capacitor is $C = \frac{\varepsilon_0 A}{d}$, where A is the surface area of the plates.

SOLVE

$$C = \frac{\varepsilon_0 A}{d} = \frac{\left(8.85 \times 10^{-12} \frac{\text{F}}{\text{m}}\right)(1.00\ \text{m})^2}{1 \times 10^{-3}\ \text{m}} = \boxed{8.85 \times 10^{-9}\ \text{F}}$$

REFLECT

The permittivity of free space ε_0 can be written in terms of $\frac{\text{C}^2}{\text{N} \cdot \text{m}^2}$ or $\frac{\text{F}}{\text{m}}$.

17.62

SET UP

A parallel plate capacitor has a capacitance of $C = 2.0 \times 10^{-12}$ F when the plates are separated by $d = 1 \times 10^{-3}$ m. We can calculate the plate area A from the expression for the capacitance of a parallel plate capacitor, $C = \frac{\varepsilon_0 A}{d}$.

SOLVE

$$C = \frac{\varepsilon_0 A}{d}$$

$$A = \frac{Cd}{\varepsilon_0} = \frac{(2.0 \times 10^{-12}\ \text{F})(1 \times 10^{-3}\ \text{m})}{\left(8.85 \times 10^{-12} \frac{\text{F}}{\text{m}}\right)} = \boxed{2.3 \times 10^{-4}\ \text{m}^2}$$

REFLECT

If the plates were square, they would be 1.5 cm on each side.

17.63

SET UP

A parallel plate capacitor has square plates ($L = 1.0$ m). We can use the expression for the capacitance of a parallel plate capacitor, $C = \frac{\varepsilon_0 A}{d}$, to calculate the separation distance d required to give $C = 8850 \times 10^{-12}$ F.

SOLVE

$$C = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 L^2}{d}$$

$$d = \frac{\varepsilon_0 L^2}{C} = \frac{\left(8.85 \times 10^{-12}\frac{\text{F}}{\text{m}}\right)(1.0\text{ m})^2}{8850 \times 10^{-12}\text{ F}} = 0.0010\text{ m} = \boxed{1.0\text{ mm}}$$

REFLECT

The dielectric constant of air is $\kappa = 1.00058$, which is approximately 1.

Get Help: Picture It – Capacitance
P'Cast 17.3 – Parallel Plates

17.64

SET UP

Parallel plate capacitor 1 has square plates of length L_1 that are separated by $d_1 = 1$ mm. We want to construct a second parallel plate capacitor with square plates of length L_2, where $L_1 = 2L_2$, that has the same capacitance as capacitor 1. We can set the expressions for the two capacitances equal to one another and solve for the required separation distance d_2 for the plates in capacitor 2.

SOLVE

$$C_1 = C_2$$

$$\frac{\varepsilon_0 A_1}{d_1} = \frac{\varepsilon_0 A_2}{d_2}$$

$$\frac{L_1^2}{d_1} = \frac{L_2^2}{d_2}$$

$$d_2 = \frac{L_2^2 d_1}{L_1^2} = \frac{L_2^2 d_1}{(2L_2)^2} = \frac{d_1}{4} = \frac{1.0\text{ mm}}{4} = \boxed{0.25\text{ mm}}$$

REFLECT

We saved ourselves some unnecessary arithmetic by noticing the relationship between the size of the plates of the two capacitors.

17.65

SET UP

A parallel plate capacitor has square plates that have an edge length of 1.00 m and are separated by a distance $d = 1 \times 10^{-3}$ m. It is connected to a 12-V battery. The energy stored in a capacitor is equal to $U = \frac{1}{2}CV^2$, where $C = \frac{\varepsilon_0 A}{d}$ for a parallel plate capacitor.

SOLVE

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)V^2 = \frac{\left(8.85 \times 10^{-12}\frac{\text{F}}{\text{m}}\right)(1.00\text{ m})^2(12\text{ V})^2}{2(1 \times 10^{-3}\text{ m})} = \boxed{6.4 \times 10^{-7}\text{ J}}$$

REFLECT

The expression $U = \frac{1}{2}CV^2$ is the easiest to use since we know the capacitance and the voltage.

17.66

SET UP

When a capacitor is attached to a 10 V battery, it stores 0.0001 J of electrical potential energy. We can solve for the capacitance of the capacitor from the expression for the potential energy stored in a capacitor $U = \frac{1}{2}CV^2$.

SOLVE

$$U = \frac{1}{2}CV^2$$

$$C = \frac{2U}{V^2} = \frac{2(0.0001\text{ J})}{(10\text{ V})^2} = \boxed{2 \times 10^{-6}\text{ F}}$$

REFLECT

A capacitance on the order of microfarads is reasonable for common capacitors.

17.67

SET UP

A capacitor has a capacitance $C = 80 \times 10^{-6}$ F. We can use the expression for the potential energy stored in a capacitor, $U = \frac{1}{2}CV^2$, to calculate the potential difference required to store 160 J of electric energy.

SOLVE

$$U = \frac{1}{2}CV^2$$

$$V = \sqrt{\frac{2U}{C}} = \sqrt{\frac{2(160\text{ J})}{80 \times 10^{-6}\text{ F}}} = \boxed{2000\text{ V}}$$

REFLECT

We can easily calculate this value without a calculator because 2×160 is equal to 320, which is divisible by 80.

17.68

SET UP

A capacitor $C = 2.0 \times 10^{-6}$ F is initially charged to $V_1 = 50$ V. The energy necessary to charge the capacitor to $V_2 = 100$ V is equal to the change in the electric potential energy stored in the capacitor, $\Delta U = \frac{1}{2}CV_2^2 - \frac{1}{2}CV_1^2$.

SOLVE

$$\Delta U = \frac{1}{2}CV_2^2 - \frac{1}{2}CV_1^2 = \frac{C}{2}(V_2^2 - V_1^2) = \frac{2.0 \times 10^{-6}\text{ F}}{2}((100\text{ V})^2 - (50\text{ V})^2) = \boxed{7.5 \times 10^{-3}\text{ J}}$$

REFLECT

Work must be done to change the potential energy stored in the capacitor.

17.69

SET UP

The capacitor ($C = 20 \times 10^{-6}$ F) in a defibrillator has a voltage of 10×10^3 V. The energy released into the patient is equal to the potential energy initially stored by the capacitor, $U = \frac{1}{2}CV^2$.

SOLVE

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(20 \times 10^{-6}\text{ F})(10 \times 10^3\text{ V})^2 = \boxed{1000\text{ J}}$$

REFLECT

We assumed that all of the energy stored in the capacitor is released into the patient.

Get Help: Interactive Example – Parallel Plate Capacitor
P'Cast 17.5 – Defibrillator

17.70

SET UP

A capacitor of unknown capacitance stores 10^{-5} C of charge when connected to a 100 V source. The capacitance is related to the charge and the voltage through $q = CV$. A second unknown capacitor stores 10^{-3} J of energy when connected to a 100 V source. The capacitance is related to the stored energy and the voltage through $U = \frac{1}{2}CV^2$.

SOLVE

Part a)

$$q = CV$$

$$C = \frac{q}{V} = \frac{10^{-5}\ \mathrm{C}}{100\ \mathrm{V}} = \boxed{10^{-7}\ \mathrm{F}}$$

Part b)

$$U = \frac{1}{2}CV^2$$

$$C = \frac{2U}{V^2} = \frac{2(10^{-3}\ \mathrm{J})}{(100\ \mathrm{V})^2} = \boxed{2 \times 10^{-7}\ \mathrm{F}}$$

REFLECT

The charge on the capacitor is linear in the applied voltage, while the energy stored by the capacitor is quadratic in the applied voltage.

17.71

SET UP

Three capacitors—$C_1 = 10\ \mu\mathrm{F}$, $C_2 = 15\ \mu\mathrm{F}$, and $C_3 = 30\ \mu\mathrm{F}$—are connected in parallel and then in series. The equivalent capacitance of the three capacitors in parallel is $C_{\mathrm{equiv}} = C_1 + C_2 + C_3$; the equivalent capacitance of the three capacitors in series is $\frac{1}{C_{\mathrm{equiv}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$.

SOLVE

Part a)

$$C_{\mathrm{equiv}} = C_1 + C_2 + C_3 = (10\ \mu\mathrm{F}) + (15\ \mu\mathrm{F}) + (30\ \mu\mathrm{F}) = \boxed{55\ \mu\mathrm{F}}$$

Part b)

$$\frac{1}{C_{\mathrm{equiv}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{10\ \mu\mathrm{F}} + \frac{1}{15\ \mu\mathrm{F}} + \frac{1}{30\ \mu\mathrm{F}} = \frac{1}{5\ \mu\mathrm{F}}$$

$$\boxed{C_{\mathrm{equiv}} = 5\ \mu\mathrm{F}}$$

REFLECT

Wiring capacitors in parallel increases the capacitance in the circuit. Wiring capacitors in series decreases the capacitance in the circuit.

17.72

SET UP

We are asked to connect four identical capacitors ($C = 1.0$ pF) such that the total capacitance $C_{\mathrm{total}} = 0.75$ pF. We can use the expressions for the equivalent capacitance for capacitors in

parallel $\left(C_{\text{equiv}} = \sum_{i=1}^{N} C_i\right)$ and in series $\left(\frac{1}{C_{\text{equiv}}} = \sum_{i=1}^{N} \frac{1}{C_i}\right)$ to determine which configuration yields the correct equivalent capacitance.

SOLVE

Three of the capacitors should be wired in parallel. This system should then be connected in series to the fourth capacitor.

$$C_1 = C + C + C = (1.0\text{ pF}) + (1.0\text{ pF}) + (1.0\text{ pF}) = 3.0\text{ pF}$$

$$\frac{1}{C_{\text{total}}} = \frac{1}{C_1} + \frac{1}{C} = \frac{1}{3.0\text{ pF}} + \frac{1}{1.0\text{ pF}} = \frac{4}{3.0\text{ pF}}$$

$$C_{\text{total}} = \frac{3.0\text{ pF}}{4} = \boxed{0.75\text{ pF}}$$

REFLECT

We know the capacitors have to be in series because the total equivalent capacitance is less than the capacitance of any individual capacitor.

17.73

SET UP

Two capacitors, C_1 and C_2, are arranged in series and in parallel. The equivalent series capacitance is $C_{\text{series}} = 2.00\ \mu\text{F}$, and the equivalent parallel capacitance is $C_{\text{parallel}} = 8.00\ \mu\text{F}$. We can solve for the values of C_1 and C_2 using the expressions for the equivalent series $\left(\frac{1}{C_{\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2}\right)$ and parallel $(C_{\text{parallel}} = C_1 + C_2)$ capacitances.

SOLVE

Equivalent capacitances:

$$\frac{1}{C_{\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C_{\text{series}} = \frac{C_1 C_2}{C_1 + C_2}$$

$$C_{\text{parallel}} = C_1 + C_2$$

Solving for C_1 and C_2:

$$C_{\text{series}}(C_1 + C_2) = C_1 C_2$$

$$C_{\text{series}} C_{\text{parallel}} = (C_{\text{parallel}} - C_2) C_2$$

$$C_2^2 - C_{\text{parallel}} C_2 + C_{\text{series}} C_{\text{parallel}} = 0$$

$$C_2 = \frac{-(-C_{\text{parallel}}) \pm \sqrt{(-C_{\text{parallel}})^2 - 4(1)(C_{\text{series}}C_{\text{parallel}})}}{2(1)} = \frac{C_{\text{parallel}} \pm \sqrt{C_{\text{parallel}}^2 - 4C_{\text{series}}C_{\text{parallel}}}}{2}$$

$$= \frac{(8.00\ \mu\text{F}) \pm \sqrt{(8.00\ \mu\text{F})^2 - 4(2.00\ \mu\text{F})(8.00\ \mu\text{F})}}{2} = \frac{(8.00\ \mu\text{F}) \pm 0}{2} = \boxed{4.00\ \mu\text{F}}$$

$$C_1 = C_{\text{parallel}} - C_2 = (8.00\ \mu\text{F}) - (4.00\ \mu\text{F}) = \boxed{4.00\ \mu\text{F}}$$

REFLECT
Quickly double-checking our answer, we find that 4 + 4 = 8, and (1/4) + (1/4) = (1/2), which matches the values listed in the problem statement.

Get Help: Interactive Example – Capacitor Network
P'Cast 17.7 – Multiple Capacitors

17.74

SET UP
Two capacitors—$C_1 = 0.50\ \mu\text{F}$ and $C_2 = 0.10\ \mu\text{F}$—are connected in series to a battery ($V = 220$ V). Since the capacitors are wired in series, the final charge will be the same on each capacitor. The final charge is also equal to the charge on a capacitor with the equivalent capacitance as C_1 and C_2 in series. The equivalent capacitance of the two capacitors in series is given by $\frac{1}{C_{\text{equiv}}} = \frac{1}{C_1} + \frac{1}{C_2}$; the charge on the equivalent capacitor is $q_{\text{equiv}} = C_{\text{equiv}}V$.

SOLVE
Equivalent capacitance:

$$\frac{1}{C_{\text{equiv}}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{0.50\ \mu\text{F}} + \frac{1}{0.10\ \mu\text{F}} = \frac{12}{1\ \mu\text{F}}$$

$$C_{\text{equiv}} = \frac{1}{12}\mu\text{F}$$

Charge on each capacitor:

$$q_1 = q_2 = q_{\text{equiv}} = C_{\text{equiv}}V = \left(\frac{1}{12} \times 10^{-6}\ \text{F}\right)(220\ \text{V}) = \boxed{1.8 \times 10^{-5}\ \text{C}}$$

REFLECT
The charge on each capacitor in series must be the same at equilibrium due to the conservation of charge.

17.75

SET UP
Two capacitors—$C_1 = 0.05\ \mu\text{F}$ and $C_2 = 0.10\ \mu\text{F}$—are connected in parallel to a battery ($V = 220$ V). Since the capacitors are wired in parallel, the potential difference across each

one is the same and is equal to the voltage difference across the battery. The final charge on each capacitor can be found through $q = CV$.

SOLVE

$$q_1 = C_1V = (0.05 \times 10^{-6}\text{ F})(220\text{ V}) = \boxed{1.1 \times 10^{-5}\text{ C} = 11\ \mu\text{C}}$$

$$q_2 = C_2V = (0.10 \times 10^{-6}\text{ F})(220\text{ V}) = \boxed{2.2 \times 10^{-5}\text{ C} = 22\ \mu\text{C}}$$

REFLECT

The final charge on capacitors in parallel will only be equal if their capacitances are equal.

17.76

SET UP

A capacitor $C_1 = 2.0 \times 10^{-6}$ F is connected to a battery ($V = 6.0$ V). The battery is disconnected from C_1 and connected to a second capacitor $C_2 = 4.0 \times 10^{-6}$ F. Once the battery is disconnected, no more charge can be added to the system. Therefore, when the two capacitors are connected to one another, the total final charge on the capacitors must equal the total initial charge on the original capacitor due to conservation of charge. Charge will stop flowing between the two capacitors when the final potential difference across C_1 is equal to the final potential difference across C_2.

SOLVE

Initial charge:

$$q_{\text{i}} = C_1V = (2.0 \times 10^{-6}\text{ F})(6.0\text{ V}) = 1.2 \times 10^{-5}\text{ C}$$

Final charges:

$$q_{1,\text{f}} = C_1V_{\text{f}}$$

$$q_{2,\text{f}} = C_2V_{\text{f}}$$

$$\frac{q_{1,\text{f}}}{q_{2,\text{f}}} = \frac{C_1V_{\text{f}}}{C_2V_{\text{f}}} = \frac{C_1}{C_2}$$

$$q_{1,\text{f}} = q_{2,\text{f}}\frac{C_1}{C_2}$$

Applying conservation of charge:

$$q_{\text{i}} = q_{1,\text{f}} + q_{2,\text{f}} = q_{2,\text{f}}\frac{C_1}{C_2} + q_{2,\text{f}} = q_{2,\text{f}}\left(\frac{C_1}{C_2} + 1\right)$$

$$q_{2,\text{f}} = \frac{q_{\text{i}}}{\left(\frac{C_1}{C_2} + 1\right)} = \frac{1.2 \times 10^{-5}\text{ C}}{\left(\left(\frac{2.0 \times 10^{-6}\text{ F}}{4.0 \times 10^{-6}\text{ F}}\right) + 1\right)} = \boxed{8.0 \times 10^{-6}\text{ C}}$$

$$q_{1,\text{f}} = q_{2,\text{f}}\frac{C_1}{C_2} = (8.0 \times 10^{-6}\text{ C})\left(\frac{2.0 \times 10^{-6}\text{ F}}{4.0 \times 10^{-6}\text{ F}}\right) = \boxed{4.0 \times 10^{-6}\text{ C}}$$

REFLECT

We can check our answers by making sure the final charges add up to the total initial charge:

$$q_i \stackrel{?}{=} q_{1,f} + q_{2,f}$$

$$1.2 \times 10^{-5}\ \text{C} \stackrel{?}{=} (8.0 \times 10^{-6}\ \text{C}) + (4.0 \times 10^{-6}\ \text{C}) = 1.2 \times 10^{-5}\ \text{C}$$

17.77

SET UP

Two capacitors ($C_1 = 2.0\ \mu\text{F}$, $C_2 = 6.0\ \mu\text{F}$) that are wired together in parallel are connected in series with a third capacitor ($C_3 = 8.0\ \mu\text{F}$). We can find the total equivalent capacitance between points a and b, C_{ab}, by first finding the equivalent capacitance of the two capacitors in parallel, C_{12}, and then the equivalent capacitance of these in series with the third capacitor.

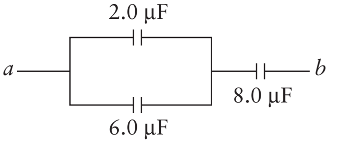


Figure 17-19 Problem 77

SOLVE

Parallel:

$$C_{12} = C_1 + C_2 = (2.0\ \mu\text{F}) + (6.0\ \mu\text{F}) = 8.0\ \mu\text{F}$$

Series:

$$\frac{1}{C_{ab}} = \frac{1}{C_{12}} + \frac{1}{C_3} = \frac{1}{8.0\ \mu\text{F}} + \frac{1}{8.0\ \mu\text{F}} = \frac{2}{8.0\ \mu\text{F}}$$

$$C_{ab} = \frac{8.0\ \mu\text{F}}{2} = \boxed{4.0\ \mu\text{F}}$$

REFLECT

Split the circuit up into smaller parts when calculating equivalent capacitance.

17.78

SET UP

Two capacitors ($C_1 = 0.1\ \mu\text{F}$, $C_2 = 0.3\ \mu\text{F}$) that are wired together in parallel are connected in series with a third capacitor ($C_3 = 0.4\ \mu\text{F}$). We can find the total equivalent capacitance of this setup by first finding the equivalent capacitance of the two capacitors in parallel, C_{12}, and then the equivalent capacitance of these in series with the third capacitor.

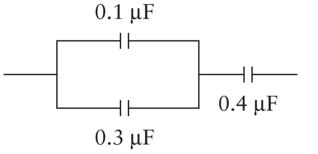


Figure 17-20 Problem 78

SOLVE

Parallel:

$$C_{12} = C_1 + C_2 = (0.1\ \mu\text{F}) + (0.3\ \mu\text{F}) = 0.4\ \mu\text{F}$$

Series:

$$\frac{1}{C_{\text{equiv}}} = \frac{1}{C_{12}} + \frac{1}{C_3} = \frac{1}{0.4\ \mu\text{F}} + \frac{1}{0.4\ \mu\text{F}} = \frac{2}{0.4\ \mu\text{F}}$$

$$C_{\text{equiv}} = \frac{0.4\ \mu\text{F}}{2} = \boxed{0.2\ \mu\text{F}}$$

REFLECT

Split the circuit up into smaller parts when calculating equivalent capacitance.

17.79

SET UP

Three capacitors—$C_1 = 10.0\ \mu\text{F}$, $C_2 = 40.0\ \mu\text{F}$, and $C_3 = 100.0\ \mu\text{F}$—are connected in series across a 12.0 V battery. The equivalent capacitance of three capacitors in series is $\frac{1}{C_{\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$. When they are wired up to the battery, the capacitors will store charge. The capacitors are wired in series, which means the charge on each capacitor will be the same. Because of this, we can use the equivalent capacitance of the capacitor network and the potential difference across the battery to calculate the charge on each capacitor. The potential difference $V = \frac{Q}{C}$ across each capacitor will not be equal, though, since the capacitances are different.

SOLVE

Part a)

$$\frac{1}{C_{\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{10.0\ \mu\text{F}} + \frac{1}{40.0\ \mu\text{F}} + \frac{1}{100.0\ \mu\text{F}} = \frac{27}{200.0\ \mu\text{F}}$$

$$C_{\text{series}} = \frac{200.0\ \mu\text{F}}{27} = \boxed{7.41\ \mu\text{F}}$$

Part b)

$$Q = C_{\text{series}}V = (7.41 \times 10^{-6}\ \text{F})(12.0\ \text{V}) = 8.89 \times 10^{-5}\ \text{C} = \boxed{88.9\ \mu\text{C}}$$

Part c)

$$V_1 = \frac{Q}{C_1} = \frac{88.9 \times 10^{-6}\ \text{C}}{10.0 \times 10^{-6}\ \text{F}} = \boxed{8.89\ \text{V}}$$

$$V_2 = \frac{Q}{C_2} = \frac{88.9 \times 10^{-6}\ \text{C}}{40.0 \times 10^{-6}\ \text{F}} = \boxed{2.22\ \text{V}}$$

$$V_3 = \frac{Q}{C_3} = \frac{88.9 \times 10^{-6}\ \text{C}}{100.0 \times 10^{-6}\ \text{F}} = \boxed{0.889\ \text{V}}$$

REFLECT

The sum of the potential differences across each capacitor should be 12.0 V: (8.89 V) + (2.22 V) + (0.889 V) = 12.0 V. The potential energy stored in the three capacitors in series is equal to the potential energy stored by the equivalent capacitor.

Get Help: Interactive Example – Capacitor Network
P'Cast 17.7 – Multiple Capacitors

17.80

SET UP

Three capacitors—$C_1 = 10.0\ \mu\text{F}$, $C_2 = 40.0\ \mu\text{F}$, and $C_3 = 100.0\ \mu\text{F}$—are connected in parallel to a battery ($V = 12.0$ V). The equivalent capacitance of the three capacitors is equal to $C_{\text{equiv}} = C_1 + C_2 + C_3$. The potential difference across each capacitor will be the same because they are wired in parallel. Therefore, we can calculate the final charge on each capacitor at equilibrium from $q = CV$.

SOLVE

Part a)

$$C_{\text{equiv}} = C_1 + C_2 + C_3 = (10.0\ \mu\text{F}) + (40.0\ \mu\text{F}) + (100.0\ \mu\text{F}) = \boxed{150.0\ \mu\text{F}}$$

Part b)

$$q_1 = C_1 V = (10.0 \times 10^{-6}\ \text{F})(12.0\ \text{V}) = \boxed{1.20 \times 10^{-4}\ \text{C}}$$

$$q_2 = C_2 V = (40.0 \times 10^{-6}\ \text{F})(12.0\ \text{V}) = \boxed{4.80 \times 10^{-4}\ \text{C}}$$

$$q_3 = C_3 V = (100.0 \times 10^{-6}\ \text{F})(12.0\ \text{V}) = \boxed{1.20 \times 10^{-3}\ \text{C}}$$

Part c)

$$V_1 = V_2 = V_3 = V = \boxed{12.0\ \text{V}}$$

REFLECT

Capacitors in parallel have the same potential difference across them; capacitors in series have the same charge on them.

17.81

SET UP

Six capacitors—$C_1 = 10\ \mu\text{F}$, $C_2 = 20\ \mu\text{F}$, $C_3 = 40\ \mu\text{F}$, $C_4 = 50\ \mu\text{F}$, $C_5 = 30\ \mu\text{F}$, and $C_6 = 25\ \mu\text{F}$—are wired as shown in the figure. The potential difference across points a and b is $V_{ab} = 75.0$ V. The charge and energy stored in this system of capacitors are equal to the charge and energy stored in a single capacitor with the equivalent capacitance as the system. After working step-by-step to find the equivalent capacitance of the network of capacitors,

we can find the total charge stored and the total energy stored from $q_{\text{equiv}} = C_{\text{equiv}} V_{ab}$ and $U = \frac{1}{2} C_{\text{equiv}} V_{ab}^2$, respectively.

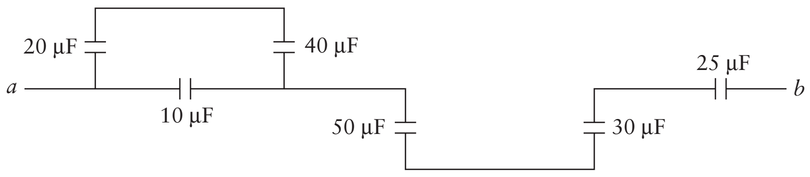


Figure 17-21 Problem 81

SOLVE

Part a)
Capacitors C_2 and C_3 in series:

$$\frac{1}{C_{23}} = \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{20\ \mu\text{F}} + \frac{1}{40\ \mu\text{F}} = \frac{3}{40\ \mu\text{F}}$$

$$C_{23} = \frac{40}{3} \mu\text{F}$$

Capacitors C_1 and C_{23} in parallel:

$$C_{123} = C_1 + C_{23} = (10\ \mu\text{F}) + \left(\frac{40}{3} \mu\text{F}\right) = \frac{70}{3} \mu\text{F}$$

Capacitors C_{123}, C_4, C_5, and C_6 in series:

$$\frac{1}{C_{\text{equiv}}} = \frac{1}{C_{123}} + \frac{1}{C_4} + \frac{1}{C_5} + \frac{1}{C_6} = \frac{3}{70\ \mu\text{F}} + \frac{1}{50\ \mu\text{F}} + \frac{1}{30\ \mu\text{F}} + \frac{1}{25\ \mu\text{F}} = \frac{0.1362}{\mu\text{F}}$$

$$C_{\text{equiv}} = 7.34\ \mu\text{F}$$

Charge:

$$q_{\text{equiv}} = C_{\text{equiv}} V_{ab} = (7.34 \times 10^{-6}\ \text{F})(75.0\ \text{V}) = \boxed{551 \times 10^{-6}\ \text{C} = 551\ \mu\text{C}}$$

Part b)
Energy stored:

$$U = \frac{1}{2} C_{\text{equiv}} V_{ab}^2 = \frac{1}{2}(7.34 \times 10^{-6}\ \text{F})(75.0\ \text{V})^2 = \boxed{2.07 \times 10^{-2}\ \text{J}}$$

REFLECT
It often helps to redraw complicated circuit diagrams such that the individual portions look stereotypically in parallel or in series.

17.82

SET UP

A parallel plate capacitor has plates of 1.0×10^{-2} m by 2.0×10^{-2} m and are separated by a 1.0×10^{-3} m thick sheet of paper. The dielectric constant of paper is $\kappa = 2.7$. The capacitance of this capacitor is given by $C = \frac{\kappa\varepsilon_0 A}{d}$.

SOLVE

$$C = \frac{\kappa\varepsilon_0 A}{d} = \frac{(2.7)\left(8.85 \times 10^{-12}\frac{\text{F}}{\text{m}}\right)(1.0 \times 10^{-2}\text{ m})(2.0 \times 10^{-2}\text{ m})}{1.0 \times 10^{-3}\text{ m}} = \boxed{4.8 \times 10^{-12}\text{ F}}$$

REFLECT

Adding a dielectric to a capacitor increases its capacitance.

17.83

SET UP

A parallel plate capacitor ($A = 20.0\text{ cm}^2$, $d = 1.00 \times 10^{-2}$ m) has a measured capacitance of $C = 0.0142 \times 10^{-6}$ m when filled with a dielectric material κ. We can calculate the value of κ from the expression for the capacitance of a parallel plate capacitor, $C = \frac{\kappa\varepsilon_0 A}{d}$.

SOLVE

$$C = \frac{\kappa\varepsilon_0 A}{d}$$

$$\kappa = \frac{Cd}{\varepsilon_0 A} = \frac{(0.0142 \times 10^{-6}\text{ F})(1.00 \times 10^{-2}\text{ m})}{\left(8.85 \times 10^{-12}\frac{\text{F}}{\text{m}}\right)\left(20.0\text{ cm}^2 \times \left(\frac{1\text{ m}}{100\text{ cm}}\right)^2\right)} = \boxed{8.02 \times 10^3}$$

REFLECT

A dielectric constant on the order of 8000 is very large.

17.84

SET UP

A parallel plate capacitor has square plates of length 0.100 m and are separated by $d = 1 \times 10^{-3}$ m. The area between the plates is filled with a ceramic material ($\kappa = 5.8$). The capacitor is then connected to a battery ($V = 12$ V). The total energy stored in the capacitor is $U = \frac{1}{2}CV^2$, where $C = \frac{\kappa\varepsilon_0 A}{d}$.

SOLVE

Capacitance:

$$C = \frac{\kappa\varepsilon_0 A}{d} = \frac{(5.8)\left(8.85 \times 10^{-12}\frac{\text{F}}{\text{m}}\right)(0.100\text{ m})(0.100\text{ m})}{1 \times 10^{-3}\text{ m}} = 5.13 \times 10^{-10}\text{ F}$$

Energy stored:

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(5.13 \times 10^{-10}\ \text{F})(12\ \text{V})^2 = \boxed{3.7 \times 10^{-8}\ \text{J}}$$

REFLECT

In this case, the dielectric acts to increase the energy stored in the capacitor.

17.85

SET UP

An air-gap capacitor ($C_i = 2800 \times 10^{-12}$ F) accumulates a charge Q_i when it is connected to a 16 V battery. While the battery is still connected, a dielectric material ($\kappa = 5.8$) is placed between the plates of the capacitor, which increases the capacitance to $C_f = \kappa C_i$. The final charge on the capacitor is Q_f. We can use the definition of capacitance to calculate the amount of charge that flowed from the battery to the capacitor upon adding the dielectric.

SOLVE

$$\Delta Q = Q_f - Q_i = C_f V - C_i V = (\kappa C_i)V - C_i V = C_i V(\kappa - 1)$$

$$= (2800 \times 10^{-12}\ \text{F})(16\ \text{V})(5.8 - 1) = \boxed{2.2 \times 10^{-7}\ \text{C}}$$

REFLECT

Inserting a dielectric into a capacitor increases its capacitance, which means it can store more charge for a given potential difference. Therefore, charge will flow from the battery onto the plates of the capacitor. If the battery were not connected when the dielectric was inserted, the charge on the plates would remain constant and the potential difference across the capacitor would decrease.

Get Help: Interactive Example – Parallel Plate Capacitor
P'Cast 17.9 – Capacitance of Membranes

17.86

SET UP

A parallel plate capacitor is made of two square plates that are 1.25×10^{-2} m long and separated by $d = 0.5 \times 10^{-2}$ m. The area between the plates is half-filled with a dielectric ($\kappa = 5$). We can treat this capacitor as two parallel plate capacitors of the same area but half the separation distance in series. One of the capacitors is filled with a dielectric, while the other is filled with air. The capacitance of the original capacitor is equal to the equivalent capacitance of the smaller ones in series.

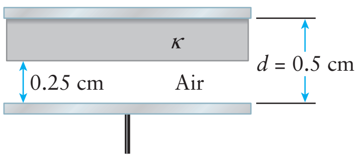


Figure 17-22 Problem 86

SOLVE

$$\frac{1}{C_{\text{equiv}}} = \frac{1}{C_{\kappa=5}} + \frac{1}{C_{\text{air}}} = \frac{\left(\frac{d}{2}\right)}{\kappa\varepsilon_0 A} + \frac{\left(\frac{d}{2}\right)}{\varepsilon_0 A} = \frac{d}{2\varepsilon_0 A}\left(\frac{1}{\kappa} + 1\right)$$

$$C_{\text{equiv}} = \frac{2\varepsilon_0 A}{d}\left(\frac{1}{\kappa}+1\right)^{-1} = \frac{2\left(8.85\times10^{-12}\frac{\text{F}}{\text{m}}\right)(1.25\times10^{-2}\text{ m})(1.25\times10^{-2}\text{ m})}{0.5\times10^{-2}\text{ m}}\left(\frac{1}{5}+1\right)^{-1}$$

$$= \boxed{4.6\times10^{-13}\text{ F}}$$

REFLECT

It makes sense to treat this as a series combination of capacitors because the charge on the "dielectric-filled" capacitor must equal the charge on the "air-filled" capacitor; this charge must equal the charge on the actual capacitor.

17.87

SET UP

A parallel plate capacitor with plate area A and separation distance d is filled with different dielectrics: dielectric with constant κ_1 fills one-quarter of the area and the full separation distance, dielectrics with constants κ_2 and κ_3 fill three-quarters of the area and half of the separation distance (see figure). We can model the capacitor as three separate parallel plate capacitors, one for each dielectric. Capacitors 2 and 3, which are filled with dielectric κ_2 and κ_3, respectively, are in series with each other. Capacitor 1, which is filled with dielectric κ_1, is in parallel with capacitors 2 and 3. We can calculate the capacitance of the original capacitor by applying the equivalent capacitance relationships to our model. We can use our algebraic result from part (a) to determine how replacing dielectric κ_3 with air, which has a dielectric constant of approximately 1, affects the capacitance of the original capacitor.

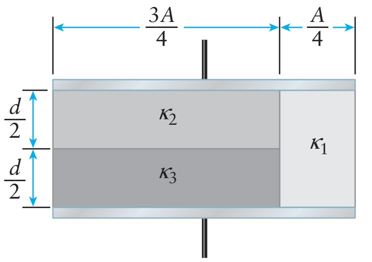


Figure 17-23 Problem 87

SOLVE

Part a)

Capacitance of capacitor 1:

$$C_1 = \frac{\kappa_1\varepsilon_0 A_1}{d_1} = \frac{\kappa_1\varepsilon_0\left(\frac{A}{4}\right)}{d} = \frac{\kappa_1\varepsilon_0 A}{4d}$$

Capacitance of capacitor 2:

$$C_2 = \frac{\kappa_2\varepsilon_0 A_2}{d_2} = \frac{\kappa_2\varepsilon_0\left(\frac{3A}{4}\right)}{\left(\frac{d}{2}\right)} = \frac{3\kappa_2\varepsilon_0 A}{2d}$$

Capacitance of capacitor 3:

$$C_3 = \frac{\kappa_3\varepsilon_0 A_2}{d_2} = \frac{\kappa_3\varepsilon_0\left(\frac{3A}{4}\right)}{\left(\frac{d}{2}\right)} = \frac{3\kappa_3\varepsilon_0 A}{2d}$$

Capacitors 2 and 3 in series:

$$\frac{1}{C_{23}} = \frac{1}{C_2} + \frac{1}{C_3} = \frac{2d}{3\kappa_2\varepsilon_0 A} + \frac{2d}{3\kappa_3\varepsilon_0 A} = \frac{2d}{3\varepsilon_0 A}\left(\frac{1}{\kappa_2} + \frac{1}{\kappa_3}\right)$$

$$C_{23} = \frac{3\varepsilon_0 A}{2d}\left(\frac{1}{\kappa_2} + \frac{1}{\kappa_3}\right)^{-1} = \frac{3\varepsilon_0 A}{2d}\left(\frac{\kappa_2\kappa_3}{\kappa_2 + \kappa_3}\right)$$

Capacitor 23 and 1 in parallel:

$$C_{\text{equiv}} = C_{23} + C_1 = \left[\frac{3\varepsilon_0 A}{2d}\left(\frac{\kappa_2\kappa_3}{\kappa_2 + \kappa_3}\right)\right] + \frac{\kappa_1\varepsilon_0 A}{4d} = \boxed{\frac{\varepsilon_0 A}{2d}\left[\frac{3\kappa_2\kappa_3}{\kappa_2 + \kappa_3} + \frac{\kappa_1}{2}\right]}$$

Part b) The capacitance of that region is reduced since $\kappa_{\text{air}} \approx 1$, so the total capacitance of the parallel plate capacitor is reduced.

REFLECT

Two capacitors are in parallel if the voltages across them are the same. In our model, capacitors 2 and 3 are in parallel with capacitor 1 since the voltage across both 2 and 3 is the same as the voltage across 1.

17.88

SET UP

A lightning bolt transfers $q = 20$ C of charge to the Earth through an average potential difference of $V = 30 \times 10^6$ V. The energy dissipated by the lightning bolt is given by $U = qV$. If all of that energy were absorbed as heat, we can calculate the mass of water at 100 degrees Celsius that would be converted into steam from the latent heat of vaporization of water, $Q = mL_V$, where$L_V = 2260 \times 10^3 \frac{\text{J}}{\text{kg}}$.

SOLVE

Part a)

$$U = qV = (20\text{ C})(30 \times 10^6\text{ V}) = \boxed{6.0 \times 10^8\text{ J}}$$

Part b)

$$Q = mL_V$$

$$m = \frac{Q}{L_V} = \frac{6.0 \times 10^8\text{ J}}{\left(2260 \times 10^3 \frac{\text{J}}{\text{kg}}\right)} = \boxed{265\text{ kg}}$$

REFLECT

This corresponds to about 265 L of water.

17.89

SET UP

A uniform electric field of $E = 2.00 \times 10^3$ N/C points towards $+x$. The potential difference $V_b - V_a$ between $x_a = -0.300$ m and $x_b = 0.500$ m is given by $V_b - V_a = -E(x_b - x_a)$. A positive test charge $q_0 = +2.00 \times 10^{-9}$ C is released from rest at point a. We can use conservation of energy to calculate the kinetic energy of the charge when it passes through point b; the change in the charge's potential energy is equal to $q_0 \Delta V$. If the charge were negative rather than positive, the potential difference between points a and b would be the same since this only depends upon the location of a and b in space, but the negative charge would be accelerated towards $-x$ by the electric field and not pass through point b.

SOLVE

Part a)

$$V_b - V_a = -E(x_b - x_a) = -\left(2.00 \times 10^3 \frac{\text{N}}{\text{C}}\right)((0.500\text{ m}) - (-0.300\text{ m})) = \boxed{-1.60 \times 10^3\text{ V}}$$

Part b)

$$\Delta U + \Delta K = q_0 \Delta V + (K_b - K_a) = q_0 \Delta V + (K_b - 0) = 0$$

$$K_b = -q_0 \Delta V = -(2.00 \times 10^{-9}\text{ C})(-1.60 \times 10^3\text{ V}) = \boxed{3.2 \times 10^{-6}\text{ J}}$$

Part c) The potential difference between the two points would remain the same. However, if a negative charge were placed at rest at point a then it would never reach point b unless an external force acted upon it. The charge would accelerate in the $-x$ direction away from point b.

REFLECT

Use your intuition to help keep your signs straight. The electric field in this problem points towards $+x$, which means point b must be at a lower potential than point a. Therefore, $V_b - V_a$ must be a negative number.

17.90

SET UP

An electric field is described by $\vec{E} = \left(-3000 \frac{\text{V}}{\text{m}^2}\right) x\hat{x}$. The potential difference between $x_a = -0.300$ m and $x_b = +0.500$ m can be found by integrating the electric field over this region. The work required by an external agent to bring a test charge ($q_0 = +2.00 \times 10^{-9}$ C) from rest at x_a to x_b at rest is equal to the charge multiplied by the potential difference. If $x_b = +0.300$ m rather than $+0.500$ m, an external agent would not be required to move the charge since the magnitude of the electric field is symmetric about $x = 0$; a positive charge in an electric field will experience a force pointing in the direction of the electric field. The test charge will initially accelerate from rest toward $x = 0$ and then slow down as it approaches x_b.

SOLVE

Part a)

$$V_b - V_a = -\int_{-0.300}^{+0.500} E_x dx = -\int_{-0.300}^{+0.500} (-3000)x dx = 3000\int_{-0.300}^{+0.500} x dx \text{ (SI units)}$$

$$V_b - V_a = 3000\left[\frac{x^2}{2}\right]_{-0.300}^{+0.500} = 1500[(0.500)^2 - (-0.300)^2] = \boxed{240 \text{ V}}$$

Part b)

$$W = q_0(V_b - V_a) = (+2.00 \times 10^{-9} \text{ C})(240 \text{ V}) = \boxed{4.80 \times 10^{-7} \text{ J}}$$

Part c) The external agent $\boxed{\text{would not need to do work}}$ on the test charge if $x_b = +0.300$ m.

Part d) This is possible because the electric field points in the $+x$ direction for $x < 0$ and in the $-x$ direction for $x > 0$. Furthermore, the magnitude of the electric field is symmetric about $x = 0$. The field does positive work initially, increasing the speed of the test charge until it reaches $x = 0$, at which point the field does negative work, slowing the test charge until it reaches $x = +30.0$ cm.

REFLECT

It makes sense that an external agent would need to do positive work on the charge to move it against the electric field. The external agent would only need to do work on the system from $x = +0.300$ m to $x = +0.500$ m.

17.91

SET UP

A point charge $q_0 = 0.50 \times 10^{-6}$ C is located at the origin. The potential along the x-axis due to a point charge q_0 as a function of x is given by $V(x) = \frac{kq_0}{x}$. A second point charge $q = 1.0 \times 10^{-6}$ C with a mass $m = 0.080 \times 10^{-3}$ kg is placed at $x = 0.80$ m. The potential energy of the system of charges is given by $U = \frac{kq_0 q}{x}$. If the charge q is released from rest at $x = 0.80$ m, we can use conservation of mechanical energy to calculate the speed of the charge at $x = 2.0$ m.

SOLVE

Part a)

$$V(x) = \frac{kq_0}{x}$$

$$V(x = 0.80 \text{ m}) = \frac{\left(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(0.50 \times 10^{-6} \text{ C})}{0.80 \text{ m}} = \boxed{5.6 \times 10^3 \text{ V}}$$

Part b)

$$U = \frac{kq_0q}{x} = q\left(\frac{kq_0}{x}\right) = (1.0 \times 10^{-6}\ \text{C})(5.6 \times 10^{3}\ \text{V}) = \boxed{5.6 \times 10^{-3}\ \text{J}}$$

Part c)

$$U_\text{i} + K_\text{i} = U_\text{f} + K_\text{f}$$

$$U_\text{i} + 0 = \frac{kq_0q}{x_\text{f}} + \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2}{m}\left(U_\text{i} - \frac{kq_0q}{x_\text{f}}\right)}$$

$$= \sqrt{\frac{2}{0.080 \times 10^{-3}\ \text{kg}}\left((5.6 \times 10^{-3}\ \text{J}) - \frac{\left(8.99 \times 10^{9}\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(0.50 \times 10^{-6}\ \text{C})(1.0 \times 10^{-6}\ \text{C})}{2.0\ \text{m}}\right)}$$

$$= \boxed{9.2\frac{\text{m}}{\text{s}}}$$

REFLECT

The potential energy of a point charge placed into a charge distribution is also equal to the product of the point charge and the potential at that location, $U = qV$.

17.92

SET UP

Two electrons that were initially traveling at very high speeds came within 1.0×10^{-15} m of each other. The magnitude of the electric force on each electron at this distance is given by Coulomb's law, $F = \frac{kq_1q_2}{r^2}$, where $q_1 = q_2 = -1.60 \times 10^{-19}$ C. We can compare this force to the weight of a book to try to determine whether or not this force would be able to lift a book off a table. The energy necessary for the electrons to come to rest at a distance of 1.0×10^{-15} m from each other is equal to the potential energy of the electrons at this separation distance, $U = \frac{kq_1q_2}{r}$. Once we know the total potential energy stored in the two-electron system, we can set that equal to the total initial kinetic energy of the system to calculate the initial speed of the two electrons. The initial potential energy of the electrons is equal to zero because we are assuming they begin infinitely far away from each other. Finally, we need to check to make sure our answer is reasonable; for example, we expect it to be large but still must remain less than the speed of light.

SOLVE

Part a)

$$F = \left|\frac{kq_1q_2}{r^2}\right| = \left|\frac{\left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(-1.60 \times 10^{-19}\ \text{C})(-1.60 \times 10^{-19}\ \text{C})}{(1.0 \times 10^{-15}\ \text{m})^2}\right| = \boxed{230\ \text{N}}$$

Part b)

This force is much larger than the force of gravity on a book, so it $\boxed{\text{would be large enough to lift a book}}$.

Part c)

$$U = \frac{kq_1q_2}{r} = \frac{\left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(-1.60 \times 10^{-19}\ \text{C})(-1.60 \times 10^{-19}\ \text{C})}{(1.0 \times 10^{-15}\ \text{m})} = \boxed{2.3 \times 10^{-13}\ \text{J}}$$

Part d)

$$U_\text{i} + K_\text{i} = U_\text{f} + K_\text{f}$$

$$0 + 2\left(\frac{1}{2}m_\text{e}v_\text{i}^2\right) = U_\text{f} + 0$$

$$v_\text{i} = \sqrt{\frac{U_\text{f}}{m_\text{e}}} = \sqrt{\frac{2.3 \times 10^{-13}\ \text{J}}{9.11 \times 10^{-31}\ \text{kg}}} = \boxed{5.0 \times 10^8 \frac{\text{m}}{\text{s}}}$$

Part e) The speed we calculated in part (d) is $\boxed{\text{larger than the speed of light}}$, which is not possible!

REFLECT

An unphysical result means that the model we used to describe the situation was not a good one. In this case we were modeling the electrons as particles using classical mechanics. Special relativity is a much better model when working with particles traveling at high speeds.

17.93

SET UP

Two red blood cells, each of mass $m = 9.0 \times 10^{-14}$ kg, have a negative charge spread uniformly on their surfaces. The first cell carries a charge $q_1 = -2.50 \times 10^{-12}$ C, and the second cell carries a charge of $q_2 = -3.10 \times 10^{-12}$ C. We will model the cells as spheres of diameter $d = 7.5 \times 10^{-6}$ m. Assuming the viscosity of the blood is negligible, we can use conservation of mechanical energy to calculate the speed the red blood cells would need when they are very far such that they just touch; when the cells just touch, their centers will be separated by a distance of d. The magnitude of the net force acting on each red blood cell

is given by Coulomb's law. Plugging this into Newton's second law allows us to calculate the magnitude of the maximum acceleration of the cells.

SOLVE

Part a)

$$U_{\mathrm{i}} + K_{\mathrm{i}} = U_{\mathrm{f}} + K_{\mathrm{f}}$$

$$0 + 2\left(\frac{1}{2}mv^2\right) = \frac{kq_1q_2}{d} + 0$$

$$v = \sqrt{\frac{kq_1q_2}{dm}} = \sqrt{\frac{\left(8.99 \times 10^9 \frac{\mathrm{N} \cdot \mathrm{m}^2}{\mathrm{C}^2}\right)(-2.50 \times 10^{-12}\ \mathrm{C})(-3.10 \times 10^{-12}\ \mathrm{C})}{(7.5 \times 10^{-6}\ \mathrm{m})(9.0 \times 10^{-14}\ \mathrm{kg})}} = \boxed{320\frac{\mathrm{m}}{\mathrm{s}}}$$

Part b)

$$\sum F = F_{\mathrm{Coul}} = \left|\frac{kq_1q_2}{d^2}\right| = ma$$

$$a = \left|\frac{kq_1q_2}{d^2m}\right| = \left|\frac{\left(8.99 \times 10^9 \frac{\mathrm{N} \cdot \mathrm{m}^2}{\mathrm{C}^2}\right)(-2.50 \times 10^{-12}\ \mathrm{C})(-3.10 \times 10^{-12}\ \mathrm{C})}{(7.5 \times 10^{-6}\ \mathrm{m})^2(9.0 \times 10^{-14}\ \mathrm{kg})}\right| = \boxed{1.4 \times 10^{10}\frac{\mathrm{m}}{\mathrm{s}^2}}$$

REFLECT

These values are unrealistic as viscous drag would have a huge effect on blood cells traveling in the bloodstream.

17.94

SET UP

Potassium ions ($q = 1.60 \times 10^{-19}$ C) move across the cell membrane, which has a thickness of $d = 8.0 \times 10^{-9}$ m, from the inside ($V_{\mathrm{i}} = -0.070$ V) to the outside ($V_{\mathrm{out}} = 0$ V). The change in the potential energy of each potassium ion is equal to the charge of the ion multiplied by the potential difference across the membrane. The electrical force exerted on the ions is equal to the charge on the ion multiplied by the electric field. We will assume the electric field is constant throughout the membrane channel, so the electric field has a magnitude of $\frac{\Delta V}{d}$ and points toward lower potential.

SOLVE

Part a)

$$\Delta U = q\Delta V = (1.60 \times 10^{-19}\ \mathrm{C})((0\ \mathrm{V}) - (-0.070\ \mathrm{V})) = \boxed{1.1 \times 10^{-20}\ \mathrm{J}}$$

The potential energy of the potassium ions $\boxed{\text{increases}}$ as they move from the inside to the outside.

Part b)

$$F = qE = q\left(\frac{\Delta V}{d}\right) = (1.60 \times 10^{-19}\ \text{C})\left(\frac{0.070\ \text{V}}{8.0 \times 10^{-9}\ \text{m}}\right) = \boxed{1.4 \times 10^{-12}\ \text{N}}$$

The force on a positive charge points in the same direction as the electric field lines in that region, which point toward areas of lower potential. Therefore, the force points $\boxed{\text{inward}}$.

REFLECT

The electric force will point in the direction that lowers the potential energy of the positive ions. The potential energy was higher outside the cell, so the force should point inward.

17.95

SET UP

An infinitely long line charge $\left(\lambda = 2.00 \times 10^{-9}\frac{\text{C}}{\text{m}}\right)$ lies along the z-axis. We can use Gauss' law to first find the electric field due to this line charge, integrate the field with respect to r to get an expression for the potential as a function of r, and then calculate the potential difference between point b at (40.0 cm, 30.0 cm, 0) and point c at (200 cm, 0, 50.0 cm). Since we're finding the potential difference between two specific points, the reference point where $V = 0$ is irrelevant. The equipotential surfaces are shaped like circular cylinders coaxial with the line charge.

SOLVE

Part a)
Gauss' law:

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\varepsilon_0}$$

$$E(2\pi rL) = \frac{\lambda L}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Potential difference:

$$V(r) = -\int \vec{E} \cdot d\vec{s} = -\int\left(\frac{\lambda}{2\pi\varepsilon_0 r}\right)dr = -\frac{\lambda}{2\pi\varepsilon_0}\ln(r) + C$$

$$V_c - V_b = -\frac{\lambda}{2\pi\varepsilon_0}\ln(r_c) + \frac{\lambda}{2\pi\varepsilon_0}\ln(r_b) = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r_b}{r_c}\right) = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{\sqrt{x_b^2 + y_b^2 + z_b^2}}{\sqrt{x_c^2 + y_c^2 + z_c^2}}\right)$$

$$= \frac{\left(2.00 \times 10^{-9}\frac{\text{C}}{\text{m}}\right)}{2\pi\left(8.85 \times 10^{-12}\frac{\text{C}^2}{\text{N} \cdot \text{m}^2}\right)}\ln\left(\frac{\sqrt{(40.0\ \text{cm})^2 + (30.0\ \text{cm})^2 + (0)^2}}{\sqrt{(200\ \text{cm})^2 + (0)^2 + (50.0\ \text{cm})^2}}\right)$$

$$= \frac{\left(2.00 \times 10^{-9}\frac{\text{C}}{\text{m}}\right)}{2\pi\left(8.85 \times 10^{-12}\frac{\text{C}^2}{\text{N} \cdot \text{m}^2}\right)}\ln\left(\frac{50.0\ \text{cm}}{206.2\ \text{cm}}\right) = \boxed{-51.0\ \text{V}}$$

Part b) Our answer is independent of the location of $V = 0$ since we're interested in the potential difference between two points.

Part c) The equipotential surfaces are the surfaces of cylinders coaxial with the line charge.

REFLECT

We can't use $r = 0$ or $r =$ infinity as the reference point where $V = 0$ for an infinite line of charge because the natural logarithm diverges at both 0 and infinity. Therefore, we usually pick an arbitrary distance R_{ref} to define $V = 0$.

17.96

SET UP

A charged semicircular ring of radius R is centered about the origin and has a linear charge density of $\lambda = \left(4.00 \times 10^{-6} \frac{\text{C}}{\text{m}}\right) \sin(\theta)$, where θ is defined relative to the positive x-axis. In order to calculate the potential at the origin due to this charge distribution, we can split the ring into infinitesimally small point charges of charge dq and perform an integral to add up the contribution due to each part, $V = \int \frac{k}{R} dq$. We can use the definition of the linear charge density, $dq = \lambda ds$, to convert this from an integral with respect to charge to an integral with respect to arc length. Since the linear charge density is a function of θ, we can use the definition of arc length, $ds = R d\theta$, to convert this to an integral with respect to θ from $\theta = 0$ to $\theta = \pi$.

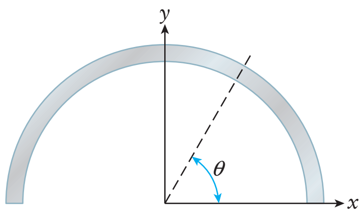


Figure 17-24 Problem 96

SOLVE

$$V = \int \frac{k}{R} dq = \frac{k}{R} \int \lambda ds = \frac{k}{R} \int_0^{\pi} \lambda R d\theta = k \int_0^{\pi} \left(4.00 \times 10^{-6} \frac{\text{C}}{\text{m}}\right) \sin(\theta) d\theta$$

$$= k\left(4.00 \times 10^{-6} \frac{\text{C}}{\text{m}}\right)[-\cos(\theta)]_0^{\pi} = k\left(4.00 \times 10^{-6} \frac{\text{C}}{\text{m}}\right)[-\cos(\pi) + \cos(0)]$$

$$= \left(8.99 \times 10^{9} \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)\left(4.00 \times 10^{-6} \frac{\text{C}}{\text{m}}\right)[2] = \boxed{71{,}900 \text{ V}}$$

REFLECT

We've defined the potential at infinity to be zero. The potential at the center of the ring was independent of the radius of the ring because every point on the ring was equidistant to the origin.

17.97

SET UP

A uniformly charged, insulating, solid sphere has a radius R and a total charge Q. We can use Gauss' law to find an algebraic expression for the electric field everywhere in space. We'll

use a spherical Gaussian surface for the two regions, $r \geq R$ and $r < R$, due to the symmetry of the object. Because the charge is uniformly distributed throughout the sphere, the charge enclosed by the Gaussian sphere in the region $r \geq R$ is proportional to the ratio of the volume of the Gaussian sphere to the volume of the entire sphere. Once we have an algebraic expression for the electric field everywhere in space, we can integrate it with respect to r to find an algebraic expression for the electric potential everywhere in space. We'll use the convention that the potential at infinity is zero. Finally, we can sketch graphs of E_r versus r and V versus r.

SOLVE

Part a)
$r \geq R$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\varepsilon_0}$$

$$E_r(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

$$\boxed{E_r = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{kQ}{r^2}}$$

$r < R$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\varepsilon_0}$$

$$E_r(4\pi r^2) = \frac{Q\left(\dfrac{\left(\dfrac{4}{3}\pi r^3\right)}{\left(\dfrac{4}{3}\pi R^3\right)}\right)}{\varepsilon_0} = \frac{Qr^3}{\varepsilon_0 R^3}$$

$$\boxed{E_r = \frac{Qr}{4\pi\varepsilon_0 R^3} = \frac{kQr}{R^3}}$$

Part b)
$r \geq R$

$$V(\infty) - V(r) = -\int_{\infty}^{r} E_r dr\cos(\pi) = \int_{\infty}^{r}\left(\frac{kQ}{r^2}\right)dr = -kQ\left[\frac{1}{r}\right]_{\infty}^{r} = -\frac{kQ}{r}$$

$$0 - V(r) = -\frac{kQ}{r}$$

$$\boxed{V(r) = \frac{kQ}{r}}$$

$r < R$

$$V(R) - V(r) = -\int_R^r E_r dr \cos(\pi) = \int_R^r \left(\frac{kQr}{R^3}\right) dr = \frac{kQ}{R^3}\left[\frac{r^2}{2}\right]_R^r = \frac{kQ}{2R^3}[r^2 - R^2]$$

$$\frac{kQ}{R} - V(r) = \frac{kQr^2}{2R^3} - \frac{kQ}{2R}$$

$$\boxed{V(r) = \frac{3kQ}{2R} - \frac{kQr^2}{2R^3} = \frac{kQ}{2R}\left[3 - \left(\frac{r}{R}\right)^2\right]}$$

Part c)
E_r versus r:

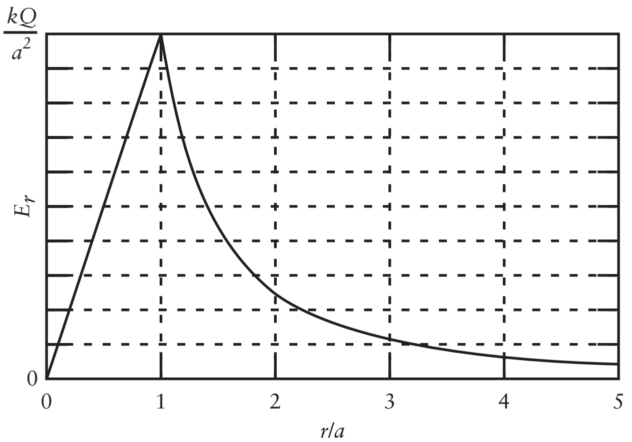


Figure 17-25 Problem 97
V versus r:

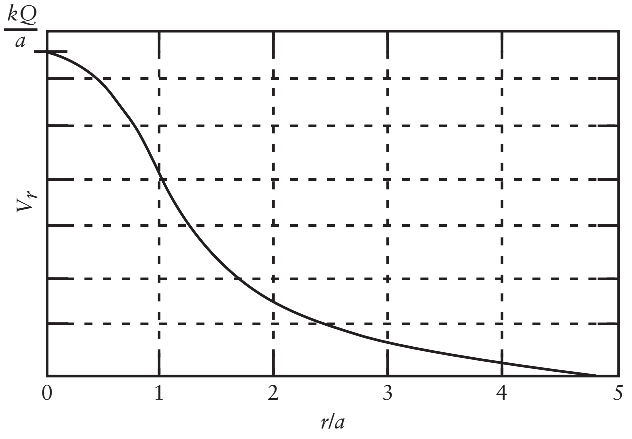


Figure 17-26 Problem 97

REFLECT
Recall that the general expression for the potential difference in terms of the electric field is related to a dot product: $V_a - V_b = -\int_a^b \vec{E} \cdot d\vec{r}$. In this case, the electric field points outward while we integrate inward from infinity toward the center of the sphere, hence, $\cos(\pi)$.

17.98

SET UP

A point P is located a distance $z_P = 0.25$ m above the central axis of a solid disk of radius $R = 0.50$ m. The disk has a total charge of $Q = 0.200$ C, which is uniformly distributed over its surface. We can find the electric potential at point P by splitting the disk into infinitesimal charges dq and summing the contribution of each dq toward the total potential via integration. We will use polar coordinates due to the cylindrical symmetry of the charge distribution. Each infinitesimal charge dq is a variable distance r_{dq} from P, where $r_{dq} = \sqrt{z_P^2 + r^2}$ and r is in the xy-plane.

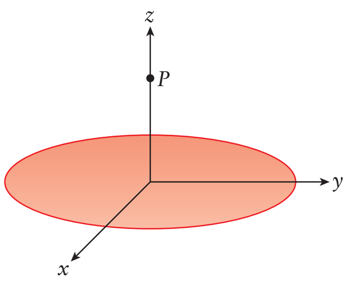


Figure 17-27 Problem 98

SOLVE

$$dV = \frac{kdq}{r_{dq}} = \frac{k(\sigma dA)}{\sqrt{z_P^2 + r^2}} = \frac{k}{\sqrt{z_P^2 + r^2}}\left(\frac{Q}{\pi R^2}\right)(rdrd\theta)$$

$$V_P = \int dV = \int_0^{2\pi}\int_0^R \frac{kQr}{\pi R^2\sqrt{z_P^2 + r^2}}drd\theta = \left(\frac{kQ}{\pi R^2}\right)\int_0^R \frac{rdr}{\sqrt{z_P^2 + r^2}}\int_0^{2\pi} d\theta$$

$$= \left(\frac{kQ}{\pi R^2}\right)\left[\sqrt{z_P^2 + r^2}\right]_0^R \left[\theta\right]_0^{2\pi} = \frac{2kQ}{R^2}\left[\sqrt{z_P^2 + R^2} - z_P\right]$$

$$= \frac{2\left(8.99 \times 10^9 \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}\right)(0.200\ \mathrm{C})}{(0.50\ \mathrm{m})^2}\left[\sqrt{(0.25\ \mathrm{m})^2 + (0.50\ \mathrm{m})^2} - (0.25\ \mathrm{m})\right]$$

$$= \boxed{4.44 \times 10^9\ \mathrm{V}}$$

REFLECT

A charge of 0.200 C is a significant amount, so we would expect the potential at a point 25 cm above the disk to be large.

17.99

SET UP

We can model a neuron as a sphere surrounded by a membrane of thickness $d = 8.0 \times 10^{-9}$ m. The membrane itself carries equal but opposite charges on its inner and outer surfaces. In its resting state, the electric potential everywhere on the inner surface is -70×10^{-3} V. Since the inner and outer surfaces have equal but opposite charges, the potential at the outer surface must be zero because the net charge within the membrane is zero. The potential at the inner surface is negative, so the inner surface must be negative and the outer surface must be positive. The electric field inside the membrane has a magnitude equal to the potential difference across the membrane divided by d and it points radially toward the center of the cell (that is, from a region of high potential to a region of low potential). The potential inside the cell is constant, so the electric field there must be zero.

SOLVE

Part a) The potential of the outer surface must be zero.

Part b) The potential at the inner surface is negative, so the inner surface must be negative and the $\boxed{\text{outer surface must be positive}}$.

Part c)

$$E = \left|\frac{V}{d}\right| = \left|\frac{70 \times 10^{-3}\ \text{V}}{8.0 \times 10^{-9}\ \text{m}}\right| = 8.8 \times 10^{6}\frac{\text{V}}{\text{m}}$$

The electric field inside the membrane has a magnitude of $\boxed{8.8 \times 10^6\ \text{V/m and points}}$ $\boxed{\text{radially toward the center of the cell}}$.

Part d) The electric field inside the cell is $\boxed{\text{zero}}$.

REFLECT

The field due to the neuron outside the cell must also be equal to zero because of Gauss' law.

17.100

SET UP

Twenty-six point charges, each 1.00×10^{-6} C, are evenly spaced between $x = 0$ and $x = 0.250$ m along the x-axis. The total potential at $x = 0.350$ m is equal to the sum of the potential due to each point charge by superposition. A 0.250-m-long rod with a total uniformly distributed charge of 26.0×10^{-6} C stretches from $x = 0$ to $x = 0.250$ m along the x-axis. The potential due to the rod at $x = 0.350$ m can be found through integration by splitting the rod up into infinitesimal point charges dq. Once we calculate the two numerical answers we can compare them with one another and make sense of them.

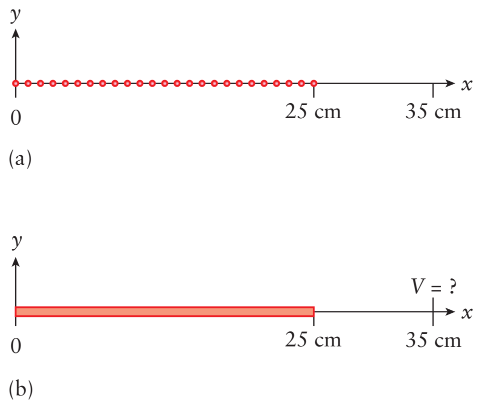


Figure 17-28 Problem 100

SOLVE

Part a)

$$V(x = 0.350\ \text{m}) = \sum_{i=1}^{26} V_i = \sum_{i=1}^{26} \frac{kq_i}{r_i} = kq_i \sum_{i=1}^{26} \frac{1}{r_i}$$

$$= \left(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(1.00 \times 10^{-6}\ \text{C})\left[\frac{1}{0.350\ \text{m}} + \frac{1}{0.340\ \text{m}} + \cdots + \frac{1}{0.100\ \text{m}}\right]$$

$$= \boxed{1.18 \times 10^6\ \text{V}}$$

Part b)

$$V(x = 0.350\text{ m}) = \int dV = \int \frac{k\,dq}{r} = k\int_0^{0.250} \frac{\lambda\,dx}{0.350 - x} = k\lambda\Big[-\ln(0.350 - x)\Big]_0^{0.250}$$

$$= k\lambda\Big[-\ln(0.100) + \ln(0.350)\Big] = \left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{26.0 \times 10^{-6}\text{ C}}{0.250\text{ m}}\right)\ln\left(\frac{0.350}{0.100}\right)$$

$$= \boxed{1.17 \times 10^6\text{ V}}$$

Part c) The two answers are very close to one another. The discrete case is slightly larger than the continuous rod because the point charges closest to $x = 0.350$ m contribute slightly more than the equivalent locations on the rod.

REFLECT

Electric potential is a function of both charge and distance.

17.101

SET UP

An electron leaves the negatively charged plate of a parallel plate capacitor starting from rest and accelerates toward the positively charged plate. The potential difference across the plates is 600 V. We can use conservation of mechanical energy to calculate the speed of the electron when it hits the positive plate. For simplicity, we'll define the potential at the positive plate to be zero, which means the potential at the negative plate will be $V_\text{i} = -600$ V.

SOLVE

$$U_\text{i} + K_\text{i} = U_\text{f} + K_\text{f}$$

$$qV_\text{i} + 0 = 0 + \frac{1}{2}m_\text{e}v^2$$

$$v = \sqrt{\frac{2qV_\text{i}}{m_\text{e}}} = \sqrt{\frac{2(-1.60 \times 10^{-19}\text{ C})(-600\text{ V})}{9.11 \times 10^{-31}\text{ kg}}} = \boxed{1.45 \times 10^7 \frac{\text{m}}{\text{s}}}$$

REFLECT

The plate separation did not factor into our solution because we only needed to know the potential difference across the plates. The separation distance is important for calculating the strength of the electric field between the plates.

17.102

SET UP

We are supplied with five identical capacitors ($C = 10\ \mu$F). We need to determine all of the unique combinations of these five capacitors in series and parallel and determine the equivalent capacitance of each setup. Recall that the relationships for equivalent capacitance for capacitors in series and parallel are $\frac{1}{C_\text{equiv}} = \sum_i \frac{1}{C_i}$ and $C_\text{equiv} = \sum_i C_i$, respectively.

SOLVE

There are 11 unique combinations.

Combination 1)

Figure 17-29 Problem 102

$$\frac{1}{C_{\text{equiv}}} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{5}{C}$$

$$C_{\text{equiv}} = \frac{C}{5} = \frac{10\ \mu\text{F}}{5} = \boxed{2.0\ \mu\text{F}}$$

Combination 2)

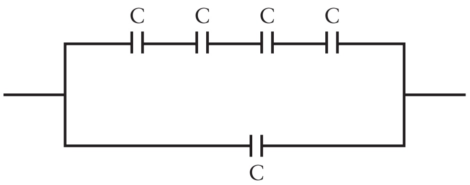


Figure 17-30 Problem 102

$$\frac{1}{C_{\text{top}}} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{4}{C}$$

$$C_{\text{top}} = \frac{C}{4}$$

$$C_{\text{equiv}} = C_{\text{top}} + C = \frac{C}{4} + C = \frac{5C}{4} = \frac{5(10\ \mu\text{F})}{4} = \boxed{13\ \mu\text{F}}$$

Combination 3)

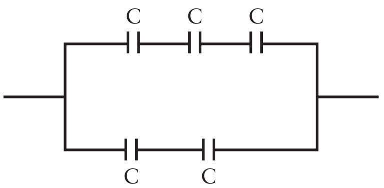


Figure 17-31 Problem 102

$$\frac{1}{C_{\text{top}}} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{3}{C}$$

$$C_{\text{top}} = \frac{C}{3}$$

$$\frac{1}{C_{\text{bottom}}} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C}$$

$$C_{\text{bottom}} = \frac{C}{2}$$

$$C_{\text{equiv}} = C_{\text{top}} + C_{\text{bottom}} = \frac{C}{3} + \frac{C}{2} = \frac{5C}{6} = \frac{5(10\ \mu\text{F})}{6} = \boxed{8.3\ \mu\text{F}}$$

Combination 4)

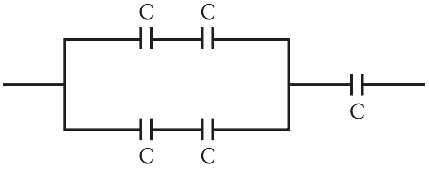


Figure 17-32 Problem 102

$$\frac{1}{C_{\text{top}}} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C}$$

$$C_{\text{top}} = \frac{C}{2}$$

$$\frac{1}{C_{\text{bottom}}} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C}$$

$$C_{\text{bottom}} = \frac{C}{2}$$

$$C_{\text{parallel}} = C_{\text{top}} + C_{\text{bottom}} = \frac{C}{2} + \frac{C}{2} = C$$

$$\frac{1}{C_{\text{equiv}}} = \frac{1}{C_{\text{parallel}}} + \frac{1}{C} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C}$$

$$C_{\text{equiv}} = \frac{C}{2} = \frac{10\ \mu\text{F}}{2} = \boxed{5.0\ \mu\text{F}}$$

Combination 5)

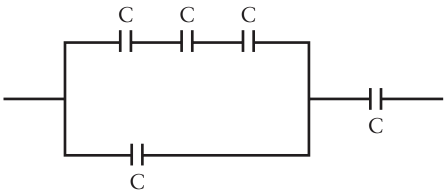


Figure 17-33 Problem 102

$$\frac{1}{C_{\text{top}}} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{3}{C}$$

$$C_{\text{top}} = \frac{C}{3}$$

$$C_{\text{parallel}} = C_{\text{top}} + C = \frac{C}{3} + C = \frac{4C}{3}$$

$$\frac{1}{C_{\text{equiv}}} = \frac{1}{C_{\text{parallel}}} + \frac{1}{C} = \frac{3}{4C} + \frac{1}{C} = \frac{7}{4C}$$

$$C_{\text{equiv}} = \frac{4C}{7} = \frac{4(10\ \mu\text{F})}{7} = \boxed{5.7\ \mu\text{F}}$$

Combination 6)

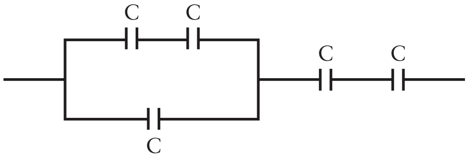


Figure 17-34 Problem 102

$$\frac{1}{C_{\text{top}}} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C}$$

$$C_{\text{top}} = \frac{C}{2}$$

$$C_{\text{parallel}} = C_{\text{top}} + C = \frac{C}{2} + C = \frac{3C}{2}$$

$$\frac{1}{C_{\text{equiv}}} = \frac{1}{C_{\text{parallel}}} + \frac{1}{C} + \frac{1}{C} = \frac{2}{3C} + \frac{1}{C} + \frac{1}{C} = \frac{8}{3C}$$

$$C_{\text{equiv}} = \frac{3C}{8} = \frac{3(10\ \mu\text{F})}{8} = \boxed{3.8\ \mu\text{F}}$$

Combination 7)

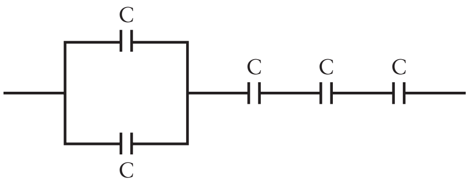


Figure 17-35 Problem 102

$$C_{\text{parallel}} = C + C = 2C$$

$$\frac{1}{C_{\text{equiv}}} = \frac{1}{C_{\text{parallel}}} + \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{1}{2C} + \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{7}{2C}$$

$$C_{\text{equiv}} = \frac{2C}{7} = \frac{2(10\ \mu\text{F})}{7} = \boxed{2.9\ \mu\text{F}}$$

Combination 8)

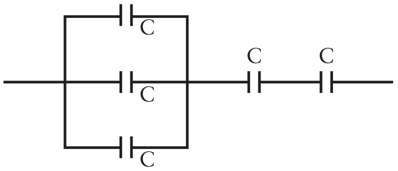


Figure 17-36 Problem 102

$$C_{\text{parallel}} = C + C + C = 3C$$

$$\frac{1}{C_{\text{equiv}}} = \frac{1}{C_{\text{parallel}}} + \frac{1}{C} + \frac{1}{C} = \frac{1}{3C} + \frac{1}{C} + \frac{1}{C} = \frac{7}{3C}$$

$$C_{\text{equiv}} = \frac{3C}{7} = \frac{3(10\ \mu\text{F})}{7} = \boxed{4.3\ \mu\text{F}}$$

Combination 9)

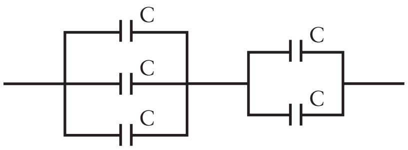


Figure 17-37 Problem 102

$$C_{\text{parallel, 1}} = C + C + C = 3C$$

$$C_{\text{parallel, 2}} = C + C = 2C$$

$$\frac{1}{C_{\text{equiv}}} = \frac{1}{C_{\text{parallel, 1}}} + \frac{1}{C_{\text{parallel, 2}}} = \frac{1}{3C} + \frac{1}{2C} = \frac{5}{6C}$$

$$C_{\text{equiv}} = \frac{6C}{5} = \frac{6(10\ \mu\text{F})}{5} = \boxed{12\ \mu\text{F}}$$

Combination 10)

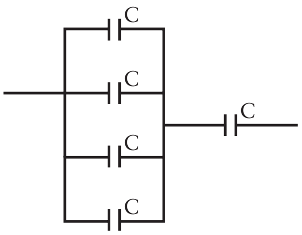


Figure 17-38 Problem 102

$$C_{\text{parallel}} = C + C + C + C = 4C$$

$$\frac{1}{C_{\text{equiv}}} = \frac{1}{C_{\text{parallel}}} + \frac{1}{C} = \frac{1}{4C} + \frac{1}{C} = \frac{5}{4C}$$

$$C_{\text{equiv}} = \frac{4C}{5} = \frac{4(10\ \mu\text{F})}{5} = \boxed{8.0\ \mu\text{F}}$$

Combination 11)

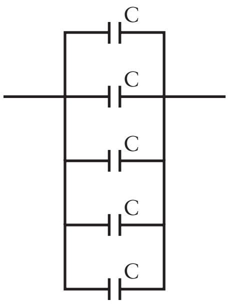


Figure 17-39 Problem 102

$$C_{\text{equiv}} = C + C + C + C + C = 5C = 5(10\ \mu\text{F}) = \boxed{50\ \mu\text{F}}$$

REFLECT

Wiring capacitors in parallel increases the capacitance, whereas wiring them in series decreases the capacitance.

17.103

SET UP

We are asked to find the equivalent capacitance of four capacitors—$C_1 = 3$ pF, $C_2 = 6$ pF, $C_3 = 4$ pF, $C_4 = 6$ pF—wired up as shown (see figure). The potential difference over the entire top branch (made up of C_3 and C_4) is equal to the potential difference over the entire bottom branch (made up of C_1 and C_2), which means these branches are in parallel with one another. The capacitors in each branch (C_1 and C_2, C_3 and C_4) are wired in series since the current through each branch is the same throughout the branch. We can find the overall equivalent capacitance by first finding the equivalent capacitance of each branch and then the equivalent capacitance of the two branches together.

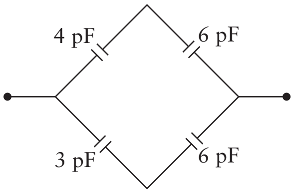


Figure 17-40 Problem 103

The equivalent capacitance of two capacitors in series is $\frac{1}{C_{\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2}$; the equivalent capacitance of two capacitors in parallel is $C_{\text{parallel}} = C_1 + C_2$.

SOLVE

Redrawn diagram:

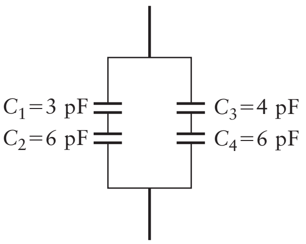


Figure 17-41 Problem 103

Equivalent capacitance:

$$\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{3\text{ pF}} + \frac{1}{6\text{ pF}} = \frac{3}{6\text{ pF}}$$

$$C_{12} = 2\text{ pF}$$

$$\frac{1}{C_{34}} = \frac{1}{C_3} + \frac{1}{C_4} = \frac{1}{4\text{ pF}} + \frac{1}{6\text{ pF}} = \frac{5}{12\text{ pF}}$$

$$C_{34} = \frac{12}{5}\text{pF}$$

$$C_{1234} = C_{12} + C_{34} = (2\text{ pF}) + \left(\frac{12}{5}\text{pF}\right) = \boxed{\frac{22}{5}\text{pF} = 4.4\text{ pF}}$$

REFLECT

Redrawing the circuit diagram using right angles helps when determining which elements are in series and which are in parallel.

Get Help: Interactive Example – Capacitor Network
P'Cast 17.7 – Multiple Capacitors

17.104

SET UP

A capacitor $C_1 = 6\ \mu\text{F}$ is wired in parallel with an unknown capacitor C_x. This setup is then wired in series with $C_2 = 20\ \mu\text{F}$ and $C_3 = 40\ \mu\text{F}$. We are told that the equivalent capacitance of the entire capacitor network is $C_{\text{equiv}} = 8\ \mu\text{F}$. We can apply the relationships for the equivalent capacitance for capacitors in parallel $\left(C_{\text{equiv}} = \sum_{i=1}^{N} C_i\right)$ and in series $\left(\frac{1}{C_{\text{equiv}}} = \sum_{i=1}^{N} \frac{1}{C_i}\right)$ in order to find the value of the unknown capacitor.

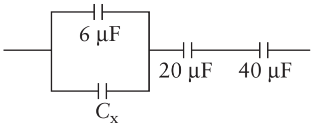


Figure 17-42 Problem 104

SOLVE

Capacitors 1 and x in parallel:

$$C_{1x} = C_1 + C_x$$

Capacitors 2 and 3 in series:

$$\frac{1}{C_{23}} = \frac{1}{C_2} + \frac{1}{C_3} = \frac{C_3 + C_2}{C_2 C_3}$$

Capacitors $1x$ and 23 in series:

$$\frac{1}{C_{\text{equiv}}} = \frac{1}{C_{1x}} + \frac{1}{C_{23}}$$

$$\frac{1}{C_{1x}} = \frac{1}{C_{\text{equiv}}} - \frac{1}{C_{23}} = \frac{1}{C_{\text{equiv}}} - \frac{C_3 + C_2}{C_2 C_3} = \frac{1}{8\ \mu\text{F}} - \frac{(20\ \mu\text{F}) + (40\ \mu\text{F})}{(20\ \mu\text{F})(40\ \mu\text{F})}$$

$$\frac{1}{C_{1x}} = \frac{1}{8\ \mu\text{F}} - \frac{3}{40\ \mu\text{F}} = \frac{1}{20\ \mu\text{F}}$$

Solving for C_x:

$$C_{1x} = C_1 + C_x = 20\ \mu\text{F}$$

$$C_x = (20\ \mu\text{F}) - C_1 = (20\ \mu\text{F}) - (6\ \mu\text{F}) = \boxed{14\ \mu\text{F}}$$

REFLECT

We can quickly check our answer by calculating the equivalent capacitance of the given setup if $C_x = 14\ \mu\text{F}$:

$$\frac{1}{C_{\text{equiv}}} = \frac{1}{C_1 + C_x} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{(6\ \mu\text{F}) + (14\ \mu\text{F})} + \frac{1}{20\ \mu\text{F}} + \frac{1}{40\ \mu\text{F}}$$

$$\frac{1}{C_{\text{equiv}}} = \frac{5}{40\ \mu\text{F}}$$

$$C_{\text{equiv}} = 8\ \mu\text{F}$$

17.105

SET UP

Six capacitors—$C_1 = 7$ pF, $C_2 = 4$ pF, $C_3 = 3$ pF, $C_4 = 6$ pF, $C_5 = 1$ pF, and $C_6 = 2$ pF—are wired together as shown. We can apply the relationships for the equivalent capacitance for capacitors in parallel $\left(C_{\text{equiv}} = \sum_{i=1}^{N} C_i\right)$ and in series $\left(\frac{1}{C_{\text{equiv}}} = \sum_{i=1}^{N} \frac{1}{C_i}\right)$ in order to find the equivalent capacitance of the given capacitor network.

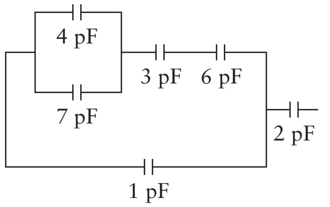


Figure 17-43 Problem 105

SOLVE

Capacitors 1 and 2 in parallel:

$$C_{12} = C_1 + C_2 = (7\ \text{pF}) + (4\ \text{pF}) = 11\ \text{pF}$$

Capacitors 12, 3, and 4 in series:

$$\frac{1}{C_{1234}} = \frac{1}{C_{12}} + \frac{1}{C_3} + \frac{1}{C_4} = \frac{1}{11\ \text{pF}} + \frac{1}{3\ \text{pF}} + \frac{1}{6\ \text{pF}} = \frac{6 + 22 + 11}{66\ \text{pF}}$$

$$C_{1234} = \frac{22}{13}\text{pF}$$

Capacitors 1234 and 5 in parallel:

$$C_{12345} = C_{1234} + C_5 = \left(\frac{22}{13}\text{pF}\right) + (1\ \text{pF}) = \frac{35}{13}\text{pF}$$

Capacitors 12345 and 6 in series:

$$\frac{1}{C_{\text{equiv}}} = \frac{1}{C_{12345}} + \frac{1}{C_6} = \frac{13}{35\ \text{pF}} + \frac{1}{2\ \text{pF}} = \frac{26 + 35}{70\ \text{pF}}$$

$$\boxed{C_{\text{equiv}} = \frac{70}{61}\text{pF} \approx 1.15\ \text{pF}}$$

REFLECT

Whenever calculating equivalent capacitances, start with the smallest unit and work outward. For example, in this problem, we need the equivalent capacitance of C_1 and C_2 together before finding the equivalent capacitance of the upper branch of the network.

17.106

SET UP

Two capacitors—$C_1 = 4\ \mu\text{F}$ and $C_2 = 6\ \mu\text{F}$—are wired in series with a battery ($V = 3$ V). Since the capacitors are in series with each other, the final charge stored on each of them (q_1, q_2) will be the same. The charge on each capacitor will also be equal to the charge on the equivalent capacitor, which we can find by first applying the relationship for the equivalent capacitance for two capacitors in series and then applying the definition of capacitance, $q = CV$.

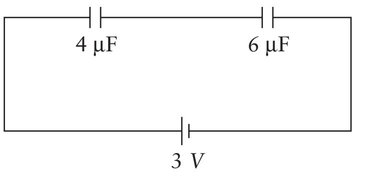


Figure 17-44 Problem 106

SOLVE

Equivalent capacitance:

$$\frac{1}{C_{\text{equiv}}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_1 + C_2}{C_1 C_2}$$

$$C_{\text{equiv}} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(4\ \mu\text{F})(6\ \mu\text{F})}{(4\ \mu\text{F}) + (6\ \mu\text{F})} = 2.4\ \mu\text{F}$$

Charge on the equivalent capacitor:

$$q_{\text{equiv}} = C_{\text{equiv}} V = (2.4\ \mu\text{F})(3\ \text{V}) = 7.2\ \mu\text{C}$$

Charge on each capacitor:

$$\boxed{q_1 = q_2 = q_{\text{equiv}} = 7.2\ \mu\text{C}}$$

REFLECT

The charge stored on each capacitor in series at equilibrium must be the same due to charge conservation.

17.107

SET UP

A cylindrical capacitor consists of an inner cylinder of radius R_1 and charge $+Q$ and an outer thin cylindrical shell of radius R_2 and a charge $-Q$, where $R_2 > R_1$. Both cylinders are a length L and air fills the area between the two cylinders. An algebraic expression for the electric field in the region between the cylinders ($R_1 < r < R_2$) can be found through Gauss' law. Once we have the electric field, we can integrate it with respect to r to find the potential difference between the two cylinders. Finally, the capacitance is equal to the magnitude of the charge on the cylinders divided by the potential difference between them.

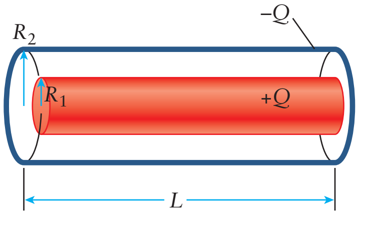


Figure 17-45 Problem 107

SOLVE

Gauss' law:

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\varepsilon_0}$$

$$E(2\pi rL) = \frac{+Q}{\varepsilon_0}$$

$$E = \frac{Q}{2\pi\varepsilon_0 Lr}$$

Potential difference:

$$V_{R_1} - V_{R_2} = -\int_{R_2}^{R_1} E dr = -\int_{R_2}^{R_1}\left(\frac{Q}{2\pi\varepsilon_0 Lr}\right)dr = -\frac{Q}{2\pi\varepsilon_0 L}\left[\ln(r)\right]_{R_2}^{R_1}$$

$$= -\frac{Q}{2\pi\varepsilon_0 L}\left[\ln(R_1) - \ln(R_2)\right] = -\left(\frac{Q}{2\pi\varepsilon_0 L}\right)\ln\left(\frac{R_1}{R_2}\right) = \left(\frac{Q}{2\pi\varepsilon_0 L}\right)\ln\left(\frac{R_2}{R_1}\right)$$

Capacitance:

$$Q = C(V_{R_1} - V_{R_2})$$

$$C = \frac{Q}{V_{R_1} - V_{R_2}} = \frac{Q}{\left(\frac{Q}{2\pi\varepsilon_0 L}\right)\ln\left(\frac{R_2}{R_1}\right)} = \boxed{\frac{2\pi\varepsilon_0 L}{\ln\left(\frac{R_2}{R_1}\right)}}$$

REFLECT

This expression for a cylindrical capacitor follows the same general trends as the expression for a parallel plate capacitor. The capacitance decreases if we increase the distance between the parallel plates. If we increase the difference $R_2 - R_1$ in the cylindrical capacitor, the ratio $\frac{R_2}{R_1}$ and, therefore, the natural log will also increase, which means the capacitance of the cylindrical capacitor will decrease. The capacitance increases with the area of the plates, and we see that the capacitance increases with the length of the cylindrical capacitor.

17.108

SET UP

A Geiger-Müller tube consists of a rigid metal cylindrical wire of radius $a = 0.500 \times 10^{-3}$ m and length $L = 6.50 \times 10^{-2}$ m surrounded by a larger metal cylinder of the same length and radius $b = 15.0 \times 10^{-3}$ m. An algebraic expression for the electric field in terms of the linear charge density λ on the wire in the region between the cylinders ($a < r < b$) can be found through Gauss' law. Once we have the electric field, we can integrate it with respect to r to find an expression for the potential difference between the two cylinders. Setting this equal to the operating voltage of the tube ($V = 925$ V), we can solve for the charge on the surface of the inner wire. We can use our numerical result to find the magnitude of the electric field just outside the surface of the wire (that is, $r = a$) and just inside the surface of the outer cylinder (that is, $r = b$). The capacitance is equal to the magnitude of the charge on the cylinders divided by the potential difference between them. Finally, the energy stored in the Geiger-Müller tube is equal to $U = \frac{1}{2}CV^2$.

SOLVE

Part a)
Gauss' law:

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\varepsilon_0}$$

$$E(2\pi rL) = \frac{\lambda L}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Potential difference:

$$V_a - V_b = V = -\int_b^a Edr = -\int_b^a \left(\frac{\lambda}{2\pi\varepsilon_0 r}\right)dr = -\frac{\lambda}{2\pi\varepsilon_0}\left[\ln(r)\right]_b^a$$

$$= -\frac{\lambda}{2\pi\varepsilon_0}\left[\ln(a) - \ln(b)\right] = -\left(\frac{\lambda}{2\pi\varepsilon_0}\right)\ln\left(\frac{a}{b}\right) = \boxed{\left(\frac{\lambda}{2\pi\varepsilon_0}\right)\ln\left(\frac{b}{a}\right)}$$

Part b)

$$V = \left(\frac{\lambda}{2\pi\varepsilon_0}\right)\ln\left(\frac{b}{a}\right) = \left(\frac{Q}{2\pi\varepsilon_0 L}\right)\ln\left(\frac{b}{a}\right)$$

$$Q = \frac{2\pi\varepsilon_0 VL}{\ln\left(\frac{b}{a}\right)} = \frac{2\pi\left(8.85 \times 10^{-12}\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(925\text{ V})(6.50 \times 10^{-2}\text{ m})}{\ln\left(\frac{15.0 \times 10^{-3}\text{ m}}{0.500 \times 10^{-3}\text{ m}}\right)} = \boxed{9.83 \times 10^{-10}\text{ C}}$$

Part c)
Just outside the surface of the wire:

$$E = \frac{\lambda}{2\pi\varepsilon_0 a} = \frac{Q}{2\pi\varepsilon_0 aL} = \frac{9.83 \times 10^{-10}\text{ C}}{2\pi\left(8.85 \times 10^{-12}\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(0.500 \times 10^{-3}\text{ m})(6.50 \times 10^{-2}\text{ m})}$$

$$= \boxed{5.44 \times 10^5\frac{\text{V}}{\text{m}}}$$

Just inside the surface of the outer cylinder:

$$E = \frac{\lambda}{2\pi\varepsilon_0 b} = \frac{Q}{2\pi\varepsilon_0 bL} = \frac{9.83 \times 10^{-10}\text{ C}}{2\pi\left(8.85 \times 10^{-12}\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(15.0 \times 10^{-3}\text{ m})(6.50 \times 10^{-2}\text{ m})}$$

$$= \boxed{1.81 \times 10^5\frac{\text{V}}{\text{m}}}$$

Part d)

$$C = \frac{Q}{V} = \frac{9.83 \times 10^{-10}\text{ C}}{925\text{ V}} = \boxed{1.06 \times 10^{-12}\text{ F}}$$

Part e)

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(1.06 \times 10^{-12}\text{ F})(925\text{ V})^2 = \boxed{4.55 \times 10^{-7}\text{ J}}$$

REFLECT

This problem is essentially the same as Problem 17.107.

17.109

SET UP

A spherical, air-filled capacitor consists of an inner conductor of radius R_1 and an outer conductor of radius R_2. We can use Gauss' law to find the expression for the electric field in the region $R_1 < r < R_2$ and then integrate it to find the potential difference V between the conductors. Finally, we can rearrange the expression for Q and compare it to $Q = CV$ in order to determine the capacitance. Once we have an expression for C, we can determine the effect that increasing R_1 has on the capacitance.

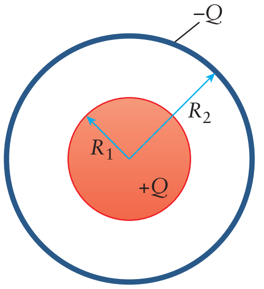


Figure 17-46 Problem 109

SOLVE

Part a)
Gauss' law:

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{+Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

Potential difference:

$$V = -\int_{R_2}^{R_1} \vec{E} \cdot d\vec{s} = -\int_{R_2}^{R_1} E dr = -\int_{R_2}^{R_1} \left(\frac{Q}{4\pi\varepsilon_0 r^2}\right) dr = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_{R_2}^{R_1}$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = \frac{Q}{4\pi\varepsilon_0}\left[\frac{R_2 - R_1}{R_1 R_2}\right]$$

Capacitance:

$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{R_2 - R_1}{R_1 R_2}\right]$$

$$Q = 4\pi\varepsilon_0\left[\frac{R_1 R_2}{R_2 - R_1}\right] V = CV$$

$$\boxed{C = 4\pi\varepsilon_0\left[\frac{R_1 R_2}{R_2 - R_1}\right]}$$

Part b) Since the quantity $R_2 - R_1$ in the denominator will get small, the capacitance will increase, which means the capacitor will store more charge per volt. In addition, the numerator will also get larger, which will also increase the capacitance.

REFLECT

It doesn't matter if the inner conductor is solid or hollow. The charge enclosed by a Gaussian surface with radius $R_1 < r < R_2$ is the same in either case. Also, the expression for the capacitance of a spherical capacitor mimics the capacitance of a parallel plate capacitor—the numerator consists of an area multiplied by ε_0 and the denominator is a measure of the distance between the plates.

17.110

SET UP

A parallel plate capacitor is made by sandwiching sheets of paper ($\kappa = 2.7$, $d = 0.1 \times 10^{-3}$ m) in between three sheets of aluminum foil. The area of the sheets is $A = 10\ \text{m}^2$. Since the aluminum foil sheets are layered, we can treat this capacitor as two smaller parallel plate capacitors in series. After finding the capacitance of each smaller capacitor, we can use the expression for the equivalent capacitance for capacitors in series to find the total capacitance of the object.

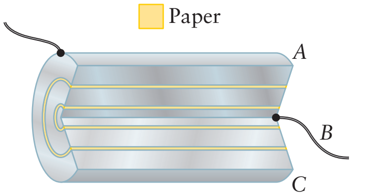


Figure 17-47 Problem 110

SOLVE

Capacitance of each capacitor:

$$C = \frac{\kappa\varepsilon_0 A}{d} = \frac{(2.7)\left(8.85 \times 10^{-12}\frac{\text{F}}{\text{m}}\right)(10\ \text{m}^2)}{0.1 \times 10^{-3}\ \text{m}} = 2.4 \times 10^{-6}\ \text{F}$$

Total capacitance:

$$\frac{1}{C_{\text{equiv}}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C}$$

$$C_{\text{equiv}} = \frac{C}{2} = \frac{2.4 \times 10^{-6}\ \text{F}}{2} = \boxed{1.2 \times 10^{-6}\ \text{F}}$$

REFLECT

Even though the aluminum foil is rolled up, we can still treat this as a parallel plate capacitor.

17.111

SET UP

A parallel plate capacitor ($C = 5 \times 10^{-6}$ F) that has an initial plate separation distance of d_i is initially charged using a battery ($V = 12$ V). The battery is then disconnected and the plates are pulled apart until the separation distance $d_\text{f} = 3d_\text{i}$. The amount of nonconservative work necessary to increase the separation between the plates by a factor of 3 is equal to the change in the potential energy stored in the capacitor.

SOLVE

$$W_{NC} = \Delta U = U_f - U_i = \frac{Q^2}{2C_f} - \frac{Q^2}{2C_i} = \frac{(C_i V)^2}{2}\left(\frac{1}{C_f} - \frac{1}{C_i}\right) = \frac{\left(\frac{\varepsilon_0 A}{d_i}\right)^2 V^2}{2}\left(\frac{d_f}{\varepsilon_0 A} - \frac{d_i}{\varepsilon_0 A}\right)$$

$$= \frac{\varepsilon_0 A V^2}{2d_i^2}(d_f - d_i) = \frac{\varepsilon_0 A V^2}{2d_i^2}((3d_i) - d_i) = \frac{\varepsilon_0 A V^2}{d_i} = C_i V^2$$

$$= (5 \times 10^{-6}\ \text{F})(12\ \text{V})^2 = \boxed{7.2 \times 10^{-4}\ \text{J} = 720\ \mu\text{J}}$$

REFLECT

The easiest way to pick which version of the expression for the potential energy stored in a capacitor $\left(U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV\right)$ to use is to determine what quantity remains constant in the given situation. In this case, the capacitor was initially charged using a 12V battery. The battery was then disconnected, which meant the voltage between the plates could change while the amount of charge on the capacitor plates could not; this is why we used $U = \frac{1}{2}\frac{Q^2}{C}$.

17.112

SET UP

A parallel plate capacitor filled with air has an area A and a plate separation d. The capacitor is attached to a battery that maintains the plates at a potential difference V. The plate separation is then doubled while the battery is still connected. Since V remains constant, we can use $U = \frac{1}{2}CV^2$ to calculate how much the potential energy stored in the capacitor changes. We can use $U = \frac{1}{2}\frac{Q^2}{C}$ to calculate how much the potential energy stored in the capacitor changes if the battery is disconnected before the plate separation is doubled.

SOLVE

Part a)

$$\frac{U_f}{U_i} = \frac{\left(\frac{1}{2}C_f V^2\right)}{\left(\frac{1}{2}C_i V^2\right)} = \frac{C_f}{C_i} = \frac{\left(\frac{\varepsilon_0 A}{d_f}\right)}{\left(\frac{\varepsilon_0 A}{d_i}\right)} = \frac{d_i}{d_f} = \frac{d_i}{2d_i} = \boxed{\frac{1}{2}}$$

Part b)

$$\frac{U_f}{U_i} = \frac{\left(\frac{1}{2}\frac{Q^2}{C_f}\right)}{\left(\frac{1}{2}\frac{Q^2}{C_i}\right)} = \frac{C_i}{C_f} = \frac{\left(\frac{\varepsilon_0 A}{d_i}\right)}{\left(\frac{\varepsilon_0 A}{d_f}\right)} = \frac{d_f}{d_i} = \frac{2d_i}{d_i} = \boxed{2}$$

When the capacitor is disconnected from the battery, it is an isolated conductor, which means the $\boxed{\text{charge remains constant}}$.

REFLECT

The job of a battery is to enforce a known potential difference across its terminals. When the battery is disconnected from the plates of the capacitor, the potential difference between the plates can change.

17.113

SET UP

A parallel plate capacitor filled with air has an area A and a plate separation d. The capacitor is attached to a battery that maintains the plates at a potential difference V. The area of the plates is doubled and the plate separation is halved while the battery is still connected. We can use the expression for the capacitance of a parallel plate capacitor to determine how these changes affect the capacitance of the object. Once we know the ratio of the capacitances, we can relate it to the charge on the plates by $Q = CV$. Since V remains constant while the battery is attached, we can use $U = \frac{1}{2}CV^2$ to calculate how much the potential energy stored in the capacitor changes. If the battery were disconnected before changing the plate area and separation, the capacitance should change in the same fashion since it only depends on the geometry of the object. The charge on the capacitor, rather than V, is now constant because the conductors are isolated. We can use $Q = CV$ and $U = \frac{1}{2}\frac{Q^2}{C}$ to calculate how much the potential difference and potential energy stored in the capacitor change in this case.

SOLVE

Part a)

$$\frac{C_f}{C_i} = \frac{\left(\frac{\varepsilon_0 A_f}{d_f}\right)}{\left(\frac{\varepsilon_0 A_i}{d_i}\right)} = \left(\frac{A_f}{A_i}\right)\left(\frac{d_i}{d_f}\right) = \left(\frac{2A_i}{A_i}\right)\left(\frac{d_i}{\left(\frac{d_i}{2}\right)}\right) = \boxed{4}$$

Part b)

$$\frac{Q_f}{Q_i} = \frac{C_f V}{C_i V} = \frac{C_f}{C_i} = \boxed{4}$$

Part c) The potential difference across the plates remains $\boxed{\text{constant}}$ as long as the battery remains connected.

Part d)

$$\frac{U_f}{U_i} = \frac{\left(\frac{1}{2}C_f V^2\right)}{\left(\frac{1}{2}C_i V^2\right)} = \frac{C_f}{C_i} = \boxed{4}$$

Part e) The capacitance will still increase by a factor of 4, the charge will remain constant, the potential difference across the plates will be reduced by a factor of 4, and the energy will be reduced by a factor of 4.

REFLECT
Think about the problem first and the specific conditions for each part before choosing an equation.

17.114

SET UP
We have a bucketful of identical capacitors, each with a capacitance $C = 1.00\ \mu\text{F}$ and a maximum voltage rating of 250 V. We need to wire together a combination of capacitors such that the total capacitance is 0.75 μF and the maximum voltage rating is 1000 V. Because the desired capacitance is smaller than the individual capacitance, we will have to wire some of the capacitors in series. The total maximum voltage rating for capacitors in series is equal to the sum of the individual maximum voltage ratings. Since 1000 V is 4 times larger than 250 V, we will need 4 capacitors in series. This combination only has a capacitance of $\frac{C}{4}$ or 0.25 μF, but we need a total capacitance of 0.75 μF. Wiring capacitors in parallel increases the equivalent capacitance while having no effect on the maximum voltage rating. We should wire, in parallel, 3 sets of the 4 capacitors in series; therefore, the minimum number of capacitors required to achieve these results is 12.

SOLVE
The minimum number of capacitors required is $\boxed{12}$:

Four capacitors in series:

$$\frac{1}{C_{\text{equiv}}} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} + \frac{1}{C}$$

$$C_{\text{equiv}} = \frac{C}{4}$$

Three sets of these in parallel:

$$C_{\text{equiv}} = \frac{C}{4} + \frac{C}{4} + \frac{C}{4} = \frac{3C}{4} = \frac{3(1.00\ \mu\text{F})}{4} = 0.75\ \mu\text{F}$$

REFLECT
Looking for a pattern is a good way to get started on a problem. The desired maximum voltage rating was 4 times larger than the individual ratings, which was a useful starting point for the solution.

17.115

SET UP
A parallel plate capacitor has an area A and separation d. A conducting slab of thickness d' is inserted between, and parallel to, the plates. This set up is equivalent to two parallel plate capacitors in series. If we define the distances of the gaps between the plates as s_1 and s_2, the separation distances for each "sub-capacitor" are $(d - s_1 - d')$ and $(d - s_2 - d')$, where $d = s_1 + s_2 + d'$. We can use the general expression for the

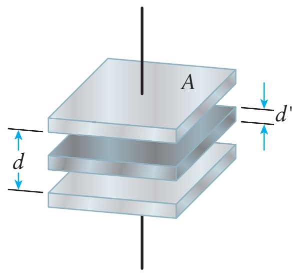


Figure 17-48 Problem 115

capacitance of a parallel plate capacitor, $C = \frac{\varepsilon_0 A}{d}$, and the equivalent capacitance of two capacitors in series, $\frac{1}{C_{\text{equiv}}} = \frac{1}{C_1} + \frac{1}{C_2}$, to calculate the capacitance of the object upon adding the conducting slab.

SOLVE

Part a)

Original capacitance:

$$C_{\text{before}} = \frac{\varepsilon_0 A}{d}$$

Adding the conducting slab:

$$\frac{1}{C_{\text{after}}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{\left(\frac{\varepsilon_0 A}{d - s_1 - d'}\right)} + \frac{1}{\left(\frac{\varepsilon_0 A}{d - s_2 - d'}\right)} = \frac{d - s_1 - d' + d - s_2 - d'}{\varepsilon_0 A}$$

$$= \frac{2d - d' - (s_1 + s_2 + d')}{\varepsilon_0 A} = \frac{2d - d' - (d)}{\varepsilon_0 A} = \frac{d - d'}{\varepsilon_0 A}$$

$$\boxed{C_{\text{after}} = \frac{\varepsilon_0 A}{d - d'}}$$

The capacitance has increased upon adding the conducting slab.

Part b) Since our expression is independent of s_1 and s_2, the effect is independent of the location of the slab.

REFLECT

Even though we treated the "sub-capacitors" as being in series, the capacitance has *increased* upon adding the conducting slab. This may seem counterintuitive—adding identical capacitors in series lowers their equivalent capacitance—but, keep in mind, the "sub-capacitors" are *not identical to the original capacitor*! These "sub-capacitors" have the same cross-sectional area, but a *smaller* separation distance than the original capacitor.

Get Help: Interactive Example – Capacitor Network
P'Cast 17.7 – Multiple Capacitors

17.116

SET UP

Three capacitors ($C = 0.18 \times 10^{-6}$ F) are connected in parallel across a battery ($V_i = 12$ V). We can use $q = CV$ to calculate the initial charge on each capacitor. The battery is disconnected and then the middle capacitor is quickly disconnected and reconnected backward. Since we've reversed one of the capacitors, the total net charge in the circuit will decrease and redistribute equally among the three capacitors.

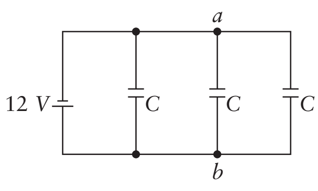


Figure 17-49 Problem 116

After calculating the final charge on each capacitor, we can use $q = CV$ again to find the final potential difference across the capacitors. The energy stored in each capacitor is given by $U = \frac{1}{2}CV^2$; we can set up a ratio to calculate by how much the energy stored in each capacitor changes after reversing the middle capacitor.

SOLVE

Part a)

Initial charge on each of the capacitors:

$$q_i = CV_i = (0.18 \times 10^{-6}\ \text{F})(12\ \text{V}) = 2.16 \times 10^{-6}\ \text{C}$$

Total charge on all three capacitors after reconnecting the middle one:

$$q_{\text{f, total}} = q_i - q_i + q_i = q_i$$

Final potential difference across each capacitor:

$$q_f = CV_f$$

$$V_f = \frac{q_f}{C} = \frac{\left(\frac{q_{\text{f, total}}}{3}\right)}{C} = \frac{q_i}{3C} = \frac{(2.16 \times 10^{-6}\ \text{C})}{3(0.18 \times 10^{-6}\ \text{F})} = \boxed{4.0\ \text{V}}$$

Part b)

$$\frac{U_f}{U_i} = \frac{\left(\frac{1}{2}CV_f^2\right)}{\left(\frac{1}{2}CV_i^2\right)} = \frac{V_f^2}{V_i^2} = \frac{(4.0\ \text{V})^2}{(12\ \text{V})^2} = \boxed{\frac{1}{9}}$$

REFLECT

The potential difference across each capacitor must be the same because they are wired in parallel. Since there is now a lower potential difference across each capacitor, the stored energy must have decreased.

17.117

SET UP

A circuit consists of a battery ($V = 3.0$ V) wired in parallel to two capacitors—$C_1 = 25 \times 10^{-6}$ F, $C_2 = 20 \times 10^{-6}$ F—with a switch S (see figure). At first, the switch is placed at position A, which means C_2 is not in the circuit. The charge on and energy stored by C_1 in this case is given by $q_1 = C_1V$ and $U_1 = \frac{1}{2}C_1V^2$, respectively. The switch is then flipped to position B, which means the completed circuit only consists of the two capacitors. The capacitors form an isolated system, so the sum of the final charges on each capacitor must equal the charge that was initially on C_1 because of charge conservation. Charge will stop flowing in

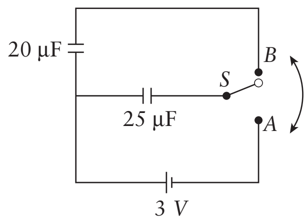


Figure 17-50 Problem 117

this new circuit once the voltage across C_1 is equal to the voltage across C_2. Combining this information allows us to calculate the final charge on each capacitor. We can then use $U = \dfrac{q^2}{2C}$ to calculate the potential energy stored in each capacitor.

SOLVE

Part a)
Circuit diagram:

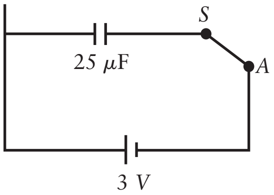


Figure 17-51 Problem 117

Charge:

$$q_1 = C_1V = (25 \times 10^{-6}\text{ F})(3.0\text{ V}) = \boxed{75 \times 10^{-6}\text{ C} = 75\ \mu\text{C}}$$

Energy:

$$U_1 = \frac{1}{2}C_1V^2 = \frac{1}{2}(25 \times 10^{-6}\text{ F})(3.0\text{ V})^2 = \boxed{1.1 \times 10^{-4}\text{ J}}$$

Part b)

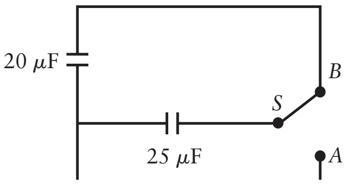


Figure 17-52 Problem 117

Charges:

$$V_{1\text{f}} = V_{2\text{f}}$$

$$\frac{q_{1\text{f}}}{C_1} = \frac{q_{2\text{f}}}{C_2}$$

$$q_{1\text{f}} = q_{2\text{f}}\left(\frac{C_1}{C_2}\right)$$

$$q_1 = q_{1\text{f}} + q_{2\text{f}} = q_{2\text{f}}\left(\frac{C_1}{C_2}\right) + q_{2\text{f}} = q_{2\text{f}}\left(1 + \frac{C_1}{C_2}\right) = C_1V$$

$$q_{2\text{f}} = \frac{C_1V}{\left(1 + \dfrac{C_1}{C_2}\right)} = \frac{C_1C_2V}{C_1 + C_2} = \frac{(25 \times 10^{-6}\text{ F})(20 \times 10^{-6}\text{ F})(3.0\text{ V})}{(25 \times 10^{-6}\text{ F}) + (20 \times 10^{-6}\text{ F})} = \boxed{3.3 \times 10^{-5}\text{ C} = 33\ \mu\text{C}}$$

$$q_{1\text{f}} = q_1 - q_{2\text{f}} = (75\ \mu\text{C}) - (33\ \mu\text{C}) = \boxed{42\ \mu\text{C}}$$

Energies:

$$U_{1f} = \frac{q_{1f}^2}{2C_1} = \frac{(42 \times 10^{-6}\ \text{C})^2}{2(25 \times 10^{-6}\ \text{F})} = \boxed{3.5 \times 10^{-5}\ \text{J} = 35\ \mu\text{J}}$$

$$U_{2f} = \frac{q_{2f}^2}{2C_2} = \frac{(33 \times 10^{-6}\ \text{C})^2}{2(20 \times 10^{-6}\ \text{F})} = \boxed{2.8 \times 10^{-5}\ \text{J} = 28\ \mu\text{J}}$$

REFLECT

In part (a), we used $U = \frac{1}{2}CV^2$ because we knew the final voltage across the capacitor was equal to the battery's voltage. In part (b), we didn't know the numerical value of the final voltage across the capacitors. Rather than perform an unnecessary calculation to find this voltage, we used $U = \frac{q^2}{2C}$ to find the final potential energies. We can also look at the case where the capacitance of one of the capacitors is much larger than the other, say, $C_2 >> C_1$. In this case, $(C_1 + C_2) \approx C_2$ and $q_{2f} = \frac{C_1C_2V}{C_1 + C_2} \approx C_1V$, which means essentially all of the initial charge ends up on the larger capacitor, as we would expect.

17.118

SET UP

The speed of a proton is $v_i = 80 \times 10^3$ m/s in a region where the potential is $V_i = 30$ V. It then moves toward a region of lower potential, where $V_f = -10$ V. We can use conservation of mechanical energy to calculate the speed of the proton at this point.

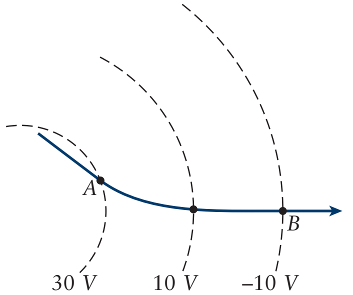


Figure 17-53 Problem 118

SOLVE

$$U_i + K_i = U_f + K_f$$

$$qV_i + \frac{1}{2}m_pv_i^2 = qV_f + \frac{1}{2}m_pv_2^2$$

$$v_2 = \sqrt{\frac{2q(V_i - V_f) + m_pv_i^2}{m_p}}$$

$$= \sqrt{\frac{2(1.60 \times 10^{-19}\ \text{C})((30\ \text{V}) - (-10\ \text{V})) + (1.67 \times 10^{-27}\ \text{kg})\left(80 \times 10^3 \frac{\text{m}}{\text{s}}\right)^2}{1.67 \times 10^{-27}\ \text{kg}}}$$

$$= \boxed{1.2 \times 10^5 \frac{\text{m}}{\text{s}}}$$

REFLECT

The exact path of the proton is not important, only its starting and ending locations are.

17.119

SET UP

A parallel plate capacitor filled with air has a plate separation $d = 0.100 \times 10^{-3}$ m. The potential difference between the plates is $V = 200$ V. The magnitude of the electric field in the region between the plates is equal to the potential difference divided by the plate separation. Assuming the area of the plates is much larger than the separation distance, we can treat the plates as infinite. Applying Gauss' law and superposition, we know the magnitude of the electric field in between the plates is $E = \frac{\sigma}{\varepsilon_0}$. We can rearrange this to solve for the surface charge density on the positive plate. The capacitor is isolated, which means the total amount of charge on it must remain constant. Therefore, if we move the plates of the capacitor closer together, the surface charge density and the magnitude of the electric field must also remain constant. In order for E to remain constant, V must decrease with d.

SOLVE

Part a)

$$E = \left|\frac{V}{d}\right| = \left|\frac{200 \text{ V}}{0.100 \times 10^{-3} \text{ m}}\right| = \boxed{2.00 \times 10^{6} \frac{\text{V}}{\text{m}}}$$

Part b)

$$E = \frac{\sigma}{\varepsilon_0}$$

$$\sigma = E\varepsilon_0 = \left(2.00 \times 10^{6} \frac{\text{V}}{\text{m}}\right)\left(8.85 \times 10^{-12} \frac{\text{F}}{\text{m}}\right) = \boxed{1.77 \times 10^{-5} \frac{\text{C}}{\text{m}^2}}$$

Part c) Since the charge remains constant, the surface charge density also remains constant. This means that the electric field also remains constant. The voltage between the plates, however, is reduced because the quantity d is reduced.

REFLECT

Although we did not use the value for the charge on the plate, we could use it to calculate the area of the plate ($A = 1.1 \text{ m}^2$).

Chapter 18
Moving Charges

Conceptual Questions

18.1 Current is the amount of charge passing a point in the circuit per second. In this sense, it is more like a flux than a vector. It has direction and magnitude but is not really a vector.

18.2 No, when current passes through a resistor, energy is dissipated by it, but current is not used up.

18.3 There is no contradiction. If a conductor is in electrostatic equilibrium, the electric field with it must be zero. However, any conductor that is carrying a current is definitely not in electrostatic equilibrium.

18.4 The equation for the resistance in terms of the resistivity is $R = \frac{\rho L}{A}$. Assuming the geometric factors L and A are the same, if the resistance is twice as large, the resistivity is also twice as large.

18.5 This energy is dissipated as heat in the wire. The amount of energy dissipated is related to the amount of current running through the wire and the resistance of the wire.

18.6 Because free charges are present throughout the volume of a metallic wire, charge begins to flow throughout the entire circuit almost immediately after a switch is closed. The electric field, which is what causes the electrons to move, is set up along the whole length of the wire nearly instantaneously.

18.7 A small resistance will dissipate more power and generate more heat.

18.8 Part a) Since the same current must flow through the ammeter and the circuit element, the two must be in series.

Part b) Resistors in series add algebraically, so the ammeter should have a very small resistance in order not to alter the resistance in the circuit.

18.9 The bird will not be electrocuted because there is no potential difference between the bird's feet. The bird grabs only one high voltage wire and it is not completing a circuit.

18.10 You can wire six lightbulbs in series.

18.11 The voltage drop across each bulb in the old series string was about 1/50 of 110 V, or 2.2 V. The modern parallel connection puts the full 110 V across each bulb. Placing 110 V across one of the old bulbs, designed to operate at 2.2 V, would result in

excessive current in the filament, which would burn out the bulb immediately, perhaps in a spectacular manner.

Get Help: P'Cast 18.7 – Resistors in Combination

18.12 The cell membrane has the ability to alter its permeability properties. At rest, the baseline membrane potential is lower on the inside compared to the outside of a cell (about −70 mV). If the membrane potential depolarizes (that is, becomes a little less negative on the inside surface), sodium ion channels open and sodium ions rush into the cell. The inner surface of the membrane becomes positively charged, and the potential difference quickly swings positive (about +40 mV). This is the peak of the action potential. Just as suddenly, as the potential difference across the cell membrane changes during the initial phase of an action potential, potassium ion channels begin to open. The membrane quickly becomes much more permeable to potassium ions than to sodium ions. Because the concentration of potassium ions is always higher inside cells, positive charge begins to flow out of the cell. This outward positive current drives the membrane potential to its resting level.

18.13 From the peak of the action potential, the membrane potential decays exponentially along the axon, but even a small change in membrane potential causes Na^+ channels further along the axon to open, causing that next piece of membrane to depolarize. In this way action potentials propagate down the axon, amplifying the decaying signal back to its original peak value. This process continues until the action potential reaches the end of the axon.

18.14 A negative current simply means that your initial guess for the direction of positive current was incorrect. The current in that segment is actually in the opposite direction of your initial guess.

18.15 It cannot be changed instantaneously because the resistor in the circuit limits the current (the rate of flow of charge). Because the current is finite, it requires time for the charge to flow on and off of the capacitor plates.

18.16 Part a) The time to charge the capacitor to, say, 5 V, with a 6 V battery is longer than the time to charge it to 5 V with a 12 V battery.

Part b) The time required to charge the capacitor to a given fraction (say, 99%) of its final charge depends only on the time constant $\tau = RC$.

Multiple-Choice Questions

18.17 **A** (increases along length of the wire). The drift speed is inversely proportional to the cross-sectional area of the wire.

Get Help: P'Cast 18.2 – How Fast?

18.18 **C** (both current and voltage). A high enough current may cause cardiac arrest. A high enough voltage may cause dielectric breakdown of the skin.

18.19 **E** (smaller by a factor of ¼).

$$\frac{R_2}{R_1} = \frac{\left(\frac{\rho L}{A_2}\right)}{\left(\frac{\rho L}{A_1}\right)} = \frac{A_1}{A_2} = \frac{\pi R_1^2}{\pi R_2^2} = \frac{\left(\frac{D_1^2}{2}\right)}{\left(\frac{D_2^2}{2}\right)} = \frac{D_1^2}{(2D_1)^2} = \frac{1}{4}$$

18.20 **E** ($4R$).

$$R_\mathrm{f} = \frac{\rho L_\mathrm{f}}{A_\mathrm{f}} = \frac{\rho L_2}{\left(\frac{V}{L_\mathrm{f}}\right)} = \frac{\rho L_\mathrm{f}^2}{V} = \frac{\rho(2L)^2}{AL} = 4\left(\frac{\rho L}{A}\right) = 4R$$

18.21 **D** (2 A). Since we are using the same wire, the resistance remains constant:

$$\frac{V_2}{V_1} = \frac{i_2 R}{i_1 R}$$

$$i_2 = \left(\frac{V_2}{V_1}\right) i_1 = \left(\frac{2\text{ V}}{1\text{ V}}\right)(1\text{ A}) = 2\text{ A}$$

18.22 **D** (80 A).

$$\frac{i_2}{i_1} = \frac{\left(\frac{V}{R_2}\right)}{\left(\frac{V}{R_1}\right)} = \frac{R_1}{R_2} = \frac{\left(\frac{\rho L_1}{A}\right)}{\left(\frac{\rho L_2}{A}\right)} = \frac{L_1}{L_2} = \frac{L}{\left(\frac{L}{2}\right)} = 2$$

$$i_2 = 2i_1 = 2(40\text{ A}) = 80\text{ A}$$

18.23 **C** (0 V). The charge begins and ends at the same location, which has the same potential.

18.24 **A** (increases). Resistors in series add algebraically.

18.25 **B** (decreases). A light bulb acts as a resistor and the equivalent resistance for two resistors in parallel is $\frac{1}{R_\text{equiv}} = \frac{1}{R_1} + \frac{1}{R_2}$.

Get Help: P'Cast 18.7 – Resistors in Combination

18.26 **C** (remain the same). The time constant only depends on the resistance and capacitance of the circuit, not the voltage of the battery.

Estimation/Numerical Questions

18.27 A microwave oven draws about 2 A of current. A hair dryer draws about 10 A of current. A television draws about 0.5 A of current.

18.28 These currents can be in the range of 10^2–10^3 A.

18.29 The resistance of the electrical coils in a toaster is on the order of 10 Ω.

18.30 In the United States, a person uses about 1500 W; in developing countries, a person uses about 100 W.

18.31 About 100 mA of current flows from a cell phone battery.

18.32 About 30 kA of current flows in a lightning bolt.

18.33 The resistor is about 1 kΩ and the capacitor is about 100 μF. This gives a time constant of 0.1 s, which is on the smaller side.

Get Help: Interactive Example – RC III

Problems

18.34

SET UP

A steady current of 35×10^{-3} A exists in a wire. The number of electrons passing through a given point in the wire per second is equal to the current divided by the charge on one electron (1.60×10^{-19} C). Recall that 1 A = 1 C/s.

SOLVE

$$35 \times 10^{-3}\frac{\text{C}}{\text{s}} \times \frac{1 \text{ electron}}{1.60 \times 10^{-19} \text{ C}} = \boxed{2.2 \times 10^{17} \text{ electrons}}$$

REFLECT

A coulomb is a very large amount of charge.

18.35

SET UP

A lightbulb requires a current of 0.50 A to emit a normal amount of light. The number of electrons passing through the bulb in an hour is related to the current divided by the charge on one electron (1.60×10^{-19} C). Recall that 1 A = 1 C/s.

SOLVE

$$1.0 \text{ hr} \times \frac{3600 \text{ s}}{1 \text{ hr}} \times \frac{0.50 \text{ C}}{1 \text{ s}} \times \frac{1 \text{ electron}}{1.60 \times 10^{-19} \text{ C}} = \boxed{1.1 \times 10^{22} \text{ electrons}}$$

REFLECT

This current is typical for 60 W lightbulbs found in your house.

18.36

SET UP

A lightning bolt carries a charge of $\Delta q = 80\ \text{C}$ between a cloud and the ground below in $\Delta t = 0.001\ \text{s}$. The average current is equal to the total charge transported divided by the time interval.

SOLVE

$$i = \frac{\Delta q}{\Delta t} = \frac{80\ \text{C}}{0.001\ \text{s}} = \boxed{8.0 \times 10^4\ \text{A}}$$

REFLECT

Common household currents range from 10–20 A, so 80 kA is a very large current, as one would expect for a lightning bolt.

18.37

SET UP

A synchrotron facility creates an electron beam with a current $i = 0.487\ \text{A}$. We can calculate the number of electrons that pass a given point in an hour from the current and the magnitude of the charge on an electron, $1.6 \times 10^{-19}\ \text{C}$.

SOLVE

$$1\ \text{hr} \times \frac{3600\ \text{s}}{1\ \text{hr}} \times \frac{0.487\ \text{C}}{1\ \text{s}} \times \frac{1\ \text{electron}}{1.6 \times 10^{-19}\ \text{C}} = \boxed{1.09 \times 10^{22}\ \text{electrons}}$$

REFLECT

We were given the current, so we didn't need to use the dimensions of the synchrotron or the speed of the electrons.

18.38

SET UP

A copper wire has a radius of $1.00 \times 10^{-3}\ \text{m}$ and carries a current of $i = 10.0\ \text{A}$. The drift speed of the electrons in the wire is given by $v_{\text{drift}} = \frac{i}{nAe}$, where n is the density of charge carriers, A is the cross-sectional area of the wire, and e is the fundamental charge. The density of charge carriers can be calculated from the density and molar mass of copper, which are $8.95\ \text{g/cm}^3$ and 63.5 g/mol, respectively. We will assume that each copper atom contributes one free electron to the metal.

SOLVE

Density of charge carriers:

$$n = 8.95\frac{\text{g}}{\text{cm}^3} \times \left(\frac{100\ \text{cm}}{1\ \text{m}}\right)^3 \times \frac{1\ \text{mol Cu}}{63.5\ \text{g}} \times \frac{6.02 \times 10^{23}\ \text{atoms Cu}}{1\ \text{mol Cu}} \times \frac{1\ \text{electron}}{1\ \text{atom Cu}}$$

$$= 8.48 \times 10^{28}\frac{\text{electrons}}{\text{m}^3}$$

Drift speed:

$$v_{\text{drift}} = \frac{i}{nAe} = \frac{10.0\text{ A}}{\left(8.48 \times 10^{28}\frac{\text{electrons}}{\text{m}^3}\right)(\pi)(1.00 \times 10^{-3}\text{ m})^2\left(1.60 \times 10^{-19}\frac{\text{C}}{\text{electron}}\right)}$$

$$= \boxed{2.34 \times 10^{-4}\frac{\text{m}}{\text{s}}}$$

REFLECT

The electrons will travel 2.34×10^{-4} m in 1 s. Although a distance on the order of 10^{-4} m/s seems small to us, it's very large compared to the diameter of a copper atom, which is on the order of 10^{-10} m.

18.39

SET UP

There is 0.002 mol of potassium ions K^+ passing through a cell membrane in $\Delta t = 0.04$ s. The current is equal to the total charge passing through the membrane divided by the time interval. We can find the total charge on the ions from the fact that a potassium ion carries a charge of $+1.60 \times 10^{-19}$ C and that there are 6.02×10^{23} ions in a mole. Current is defined as the flow of positive charge. Since potassium ions are positive, the current will flow in the same direction as the flow of the potassium ions.

SOLVE

Part a)

$$i = \frac{\Delta q}{\Delta t} = \frac{\left(0.002\text{ mol K}^+ \times \dfrac{6.02 \times 10^{23}\text{ K}^+}{1\text{ mol K}^+} \times \dfrac{1.60 \times 10^{-19}\text{ C}}{1\text{ K}^+}\right)}{0.04\text{ s}} = \boxed{5 \times 10^3\text{A}}$$

Part b) Potassium ions are positive, so the current flows in the same direction as the flow of ions.

REFLECT

A current is defined generally in terms of charges that can move in a given time frame. In most situations (for example, wires) electrons are these moving charges, but we can also talk about a current due to moving positive (and negative) ions in a solution.

18.40

SET UP

The current in a wire as a function of time is given by the expression $i(t) = (6.00)\sin(3.00t)e^{-\frac{t}{2.00}}$. (All quantities are listed in SI units.) The charge that passes through a point in the wire from $t_1 = 0.500$ s to $t_2 = 1.75$ s is given by $\Delta q = \int_{t_1}^{t_2} i(t)dt$.

SOLVE

Part a)

$$\Delta q = \int_{t_1}^{t_2} i dt = \int_{0.500}^{1.75} [(6.00)\sin(3.00t)e^{-\frac{t}{2.00}}]dt \ \text{(SI units)}$$

$$\Delta q = (6.00)\left[\frac{-2}{37}e^{-\frac{t}{2.00}}(\sin(3.00t) + 6\cos(3.00t))\right]_{0.500}^{1.75}$$

$$= \frac{12.0}{37}[-e^{-\frac{1.75}{2.00}}(\sin(3.00(1.75)) + 6\cos(3.00(1.75)))$$

$$+ e^{-\frac{0.500}{2.00}}(\sin(3.00(0.500)) + 6\cos(3.00(0.500)))]_{0.500}^{1.75}$$

$$= \boxed{5.99 \times 10^{-2}\ \text{C}}$$

Part b)

$$5.99 \times 10^{-2}\ \text{C} \times \frac{1\ \text{electron}}{1.60 \times 10^{-19}\ \text{C}} = \boxed{3.74 \times 10^{17}\ \text{electrons}}$$

REFLECT

The indefinite integral of a function in the form of $e^{\alpha t}\sin(\beta t)$ is $\frac{e^{\alpha t}}{\alpha^2 + \beta^2}(\alpha\sin(\beta t) - \beta\cos(\beta t))$.

18.41

SET UP

The current in an electronic device as a function of time is given by the expression $i(t) = i_0 e^{-2t}$, where $i_0 = 0.544 \times 10^{-3}$. (All quantities are listed in SI units.) The amount of charge that passes through the device from $t_1 = 0$ to $t_2 = 24$ s is given by $\Delta q = \int_{t_1}^{t_2} i(t)dt$.

SOLVE

$$\Delta q = \int_{t_1}^{t_2} i(t)dt = \int_0^{24}[i_0 e^{-2t}]dt = i_0\left[-\frac{1}{2}e^{-2t}\right]_0^{24} = -\frac{0.544 \times 10^{-3}}{2}[e^{-2(24)} - e^{-2(0)}]$$

$$= \boxed{2.72 \times 10^{-4}\ \text{C} = 0.272\ \text{mC}}$$

REFLECT

The current starts at its maximum value of i_0 and exponentially decays to zero as t approaches infinity.

18.42

SET UP

The electric charge flowing from a source moves at a constant speed $v = 20$ m/s. The charge distributed along the x-axis is described by $q(x) = (25 \times 10^{-9})x^2$. (All quantities are in

SI units.) We can calculate the current at $x = 0.01$ m by taking the time derivative of $q(x)$ and evaluating it at $x = 0.01$ m; we will need to invoke the chain rule to accomplish this.

SOLVE

$$i(x) = \frac{dq(x)}{dt} = \frac{dq}{dx}\frac{dx}{dt} = \left(\frac{d}{dx}[25 \times 10^{-9}x^2]\right)(v) \text{ (SI units)}$$

$$i(x) = v(25 \times 10^{-9})(2)[x] = (50 \times 10^{-9})vx$$

$$i(x = 0.01) = (50 \times 10^{-9})(20)(0.01) = \boxed{1.0 \times 10^{-8}\text{ A}}$$

REFLECT

It seems reasonable that the current should be larger in regions where the amount of charge is larger.

18.43

SET UP

A copper wire has a length of $L = 1.0$ m and a diameter of $d = 1.0 \times 10^{-3}$ m. The resistance of this piece of wire is given by $R = \frac{\rho L}{A}$. Copper has a resistivity of $\rho = 1.725 \times 10^{-8}\ \Omega \cdot \text{m}$.

SOLVE

$$R = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2} = \frac{\rho L}{\pi\left(\frac{d}{2}\right)^2} = \frac{4\rho L}{\pi d^2} = \frac{4(1.725 \times 10^{-8}\ \Omega \cdot \text{m})(1.0\text{ m})}{\pi(1.0 \times 10^{-3}\text{ m})^2} = \boxed{0.022\ \Omega}$$

REFLECT

A material has a resistivity; an object has a resistance.

18.44

SET UP

A piece of copper wire has a resistance of $R = 2\ \Omega$ and a diameter of $d = 1 \times 10^{-3}$ m. We can use the expression for the resistance of this piece of wire, $R = \frac{\rho L}{A}$, to calculate its length. Copper has a resistivity of $\rho = 1.725 \times 10^{-8}\ \Omega \cdot \text{m}$.

SOLVE

$$R = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2} = \frac{\rho L}{\pi\left(\frac{d}{2}\right)^2} = \frac{4\rho L}{\pi d^2}$$

$$L = \frac{\pi R d^2}{4\rho} = \frac{\pi(2\ \Omega)(1 \times 10^{-3}\text{ m})^2}{4(1.725 \times 10^{-8}\ \Omega \cdot \text{m})} = \boxed{90\text{ m}}$$

REFLECT

If we compare this result to our answer from Problem 18.43, we see that the resistance in this problem is about 100 times larger, so the length should also be about 100 times larger.

18.45

SET UP

A length of wire has an initial length $L_1 = 8\text{ m}$ and resistance $R_1 = 4\ \Omega$. The wire is then stretched uniformly to a final length $L_2 = 16\text{ m}$. Since the mass and density of the wire must be constant, the volume of the wire must also remain constant. Therefore, the cross-sectional area will decrease upon stretching. We can then calculate the new resistance of the wire R_2 using the definition of R in terms of the resistivity, $R = \frac{\rho L}{A}$.

SOLVE

New cross-sectional area:

$$A_1 L_1 = A_2 L_2$$

$$A_1(8\text{ m}) = A_2(16\text{ m})$$

$$A_2 = \frac{A_1}{2}$$

New resistance:

$$\frac{R_2}{R_1} = \frac{\left(\frac{\rho L_2}{A_2}\right)}{\left(\frac{\rho L_1}{A_1}\right)} = \frac{L_2 A_1}{A_2 L_1} = \frac{L_2 A_1}{\left(\frac{A_1}{2}\right) L_1} = \frac{2L_2}{L_1} = \frac{2(16\text{ m})}{8\text{ m}} = 4$$

$$R_2 = 4R_1 = 4(4\ \Omega) = \boxed{16\ \Omega}$$

REFLECT

Looking at the expression for the resistance in terms of the resistivity, $R = \frac{\rho L}{A}$, increasing the length of the wire and decreasing the cross-sectional area both cause the resistance to increase.

Get Help: P'Cast 18.3 – Stretched Wire

18.46

SET UP

The ratio of the resistances of two objects made out of different conductors is 1:3. The two objects have the same cross-sectional area and length. We can use this information along with the expression for the resistance in terms of the resistivity, $R = \frac{\rho L}{A}$, to calculate the ratio of the resistivities of the two conductors.

SOLVE

$$\frac{R_2}{R_1} = \frac{\left(\frac{\rho_2 L_2}{A_2}\right)}{\left(\frac{\rho_1 L_1}{A_1}\right)} = \frac{\left(\frac{\rho_2 L_1}{A_1}\right)}{\left(\frac{\rho_1 L_1}{A_1}\right)} = \frac{\rho_2}{\rho_1}$$

The ratio of the resistivities should be the same as the ratio of the resistances, that is, $\boxed{1:3}$.

REFLECT

The resistance is directly proportional to the resistivity.

18.47

SET UP

A copper transmission line has a length of $L = 1.0$ mi and a diameter of $d = 1.8 \times 10^{-2}$ m. The resistance of this piece of line is given by $R = \frac{\rho L}{A}$. Copper has a resistivity of $\rho = 1.725 \times 10^{-8}\ \Omega \cdot \text{m}$.

SOLVE

$$R = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2} = \frac{\rho L}{\pi\left(\frac{d}{2}\right)^2} = \frac{4\rho L}{\pi d^2}$$

$$= \frac{4(1.7 \times 10^{-8}\ \Omega \cdot \text{m})\left(1.0 \text{ mi} \times \frac{1.609 \text{ km}}{1 \text{ mi}} \times \frac{1000 \text{ m}}{1 \text{ km}}\right)}{\pi(1.8 \times 10^{-2} \text{ m})^2} = \boxed{0.11\ \Omega}$$

REFLECT

We would expect a power transmission line to have a small resistance.

18.48

SET UP

When $V_1 = 120$ V is applied to a filament of a lightbulb, the current drawn is $i_1 = 0.63$ A. When $V_2 = 3$ V is applied to the same filament, the current drawn is $i_2 = 0.086$ A. We can use Ohm's law to calculate the resistance of the filament in both cases. If the material is ohmic, these resistances should be the same as long as the temperature remains approximately constant.

SOLVE

Resistance at $V_1 = 120$ V:

$$R_1 = \frac{V_1}{i_1} = \frac{120 \text{ V}}{0.63 \text{ A}} = 190\ \Omega$$

Resistance at $V_2 = 3$ V:

$$R_2 = \frac{V_2}{i_2} = \frac{3 \text{ V}}{0.086 \text{ A}} = 35\ \Omega$$

At first glance, seeing that these two calculated values are different may lead you to think the filament is made out of a nonohmic material, but that's not necessarily the case. The temperature of the filament with 120 V across it is much greater than its temperature with only 3 V across it. Most lamp filaments are made from tungsten, which is ohmic with a resistivity that increases with temperature, like most metals. The operating temperature of a typical lightbulb filament is about 2800 K, so we expect the resistance of the filament to be higher when 120 V are across it even though tungsten is ohmic.

REFLECT

If the temperature of the filament were kept constant, its resistance would be independent of current.

18.49

SET UP

A piece of copper wire has a diameter $d = 0.163 \times 10^{-2}$ m and a length $L = 14.0$ m. The resistance of this piece of wire is given by $R = \frac{\rho L}{A}$, where the resistivity of copper is $\rho = 1.725 \times 10^{-8}\ \Omega \cdot \text{m}$. Once we know the resistance, we can use Ohm's law to calculate the current generated in the wire when a potential difference of 3.00 V is applied across it. Finally, the magnitude of the electric field in the wire is given by the potential difference divided by the length of the wire.

SOLVE

Part a)

$$R = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2} = \frac{\rho L}{\pi\left(\frac{d}{2}\right)^2} = \frac{4\rho L}{\pi d^2}$$

$$= \frac{4(1.725 \times 10^{-8}\ \Omega \cdot \text{m})(14.0\ \text{m})}{\pi(0.163 \times 10^{-2}\ \text{m})^2} = \boxed{0.116\ \Omega}$$

Part b)

$$V = iR$$

$$i = \frac{V}{R} = \frac{3.00\ \text{V}}{0.116\ \Omega} = \boxed{25.9\ \text{A}}$$

Part c)

$$E = \left|\frac{\Delta V}{L}\right| = \left|\frac{3.00\ \text{V}}{14.0\ \text{m}}\right| = \boxed{0.214 \frac{\text{V}}{\text{m}}}$$

REFLECT

It only takes a very small field to produce a significant current in a wire.

18.50

SET UP

A thick, hollow, conducting, cylindrical shell has an inner radius r_{i} and an outer radius r_{o}. The conductor has a resistivity ρ. If we split the shell into thin concentric cylindrical shells of thickness dr, the total resistance of the shell is the sum of the resistances due to each one, $\frac{\rho dr}{A}$, where the surface area of a cylinder is $A = 2\pi rL$. We can accomplish this sum through integration, $R = \int_{r_{\mathrm{i}}}^{r_{\mathrm{o}}} \frac{\rho dr}{A}$.

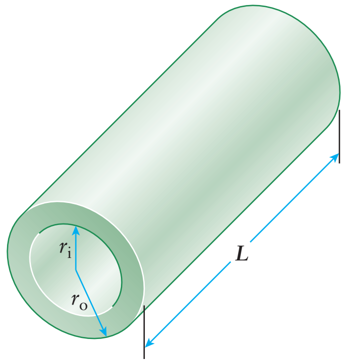


Figure 18-1 Problem 50

SOLVE

$$R = \int_{r_{\mathrm{i}}}^{r_{\mathrm{o}}} \frac{\rho dr}{A} = \rho \int_{r_{\mathrm{i}}}^{r_{\mathrm{o}}} \frac{dr}{2\pi rL} = \frac{\rho}{2\pi L}[\ln(r)]_{r_{\mathrm{i}}}^{r_{\mathrm{o}}} = \frac{\rho}{2\pi L}[\ln(r_{\mathrm{o}}) - \ln(r_i)]_{r_{\mathrm{i}}}^{r_{\mathrm{o}}} = \boxed{\frac{\rho}{2\pi L}\ln\left(\frac{r_o}{r_i}\right)}$$

REFLECT

The resistance of the shell increases as the thickness of the shell (that is, $r_{\mathrm{o}} - r_{\mathrm{i}}$) increases, which makes sense since there is now more material.

18.51

SET UP

A thin, conducting, spherical shell has an inner radius r_{i} and an outer radius r_{o}. The conductor has a resistivity ρ. If we split the shell into thin concentric spheres of thickness dr, the total resistance of the shell is the sum of the resistances due to each one, $\frac{\rho dr}{A}$, where the surface area of a sphere $A = 4\pi r^2$. We can accomplish this sum through integration, $R = \int_{r_{\mathrm{i}}}^{r_{\mathrm{o}}} \frac{\rho dr}{A}$.

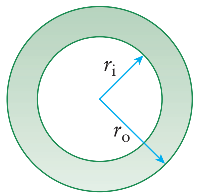


Figure 18-2 Problem 51

SOLVE

$$R = \int_{r_{\mathrm{i}}}^{r_{\mathrm{o}}} \frac{\rho dr}{A} = \rho \int_{r_{\mathrm{i}}}^{r_{\mathrm{o}}} \frac{dr}{4\pi r^2} = \frac{\rho}{4\pi}\left[-\frac{1}{r}\right]_{r_{\mathrm{i}}}^{r_{\mathrm{o}}} = \frac{\rho}{4\pi}\left[-\frac{1}{r_{\mathrm{o}}} + \frac{1}{r_{\mathrm{i}}}\right] = \boxed{\frac{\rho}{4\pi}\left[\frac{1}{r_{\mathrm{i}}} - \frac{1}{r_{\mathrm{o}}}\right]}$$

REFLECT

Resistivity is associated with a material (*e.g.*, copper) while resistance is associated with an object (*e.g.*, a 1-m-long copper wire).

18.52

SET UP

Resistors are marked with four colored bands that designate its resistance. The first two bands correspond to the two significant figures in the resistances; the third band corresponds to the power of 10 by which the significant figures are multiplied; and

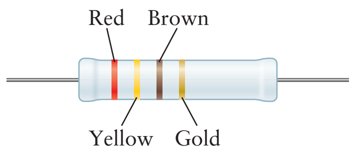


Figure 18-3 Problem 52

the fourth band corresponds to the tolerance (or uncertainty) in the stated value. We can do an Internet search for "resistor color code" to find what the different colors mean.

SOLVE

Red = 2
Yellow = 4
Brown = 1
Gold = ±5%

The resistor is $(24 \times 10^1)\ \Omega \pm 5\%$ or $\boxed{(240 \pm 12)\ \Omega}$.

REFLECT

The bands will usually start all the way on one end of the resistor; that's where you start reading from left to right. If there are only three bands, the tolerance is assumed to be ±20%.

18.53

SET UP

A current of $i_1 = 112$ pA flows through a resistor R_1 when a certain potential difference is applied across it. When that same potential difference is applied to a second resistor R_2, a current of $i_2 = 0.044$ pA is measured. The resistor R_2 is made of the same material as R_1, but the length of R_2 is 25 times larger than the length of R_1, or $L_2 = 25L_1$. We can take a ratio of the two resistances and then use the definition of the resistance in terms of the resistivity for each resistor as well as Ohm's law to find a relationship between the diameters of the two resistors.

SOLVE

$$\frac{R_1}{R_2} = \frac{\left(\frac{\rho L_1}{A_1}\right)}{\left(\frac{\rho L_2}{A_2}\right)} = \frac{L_1 A_2}{L_2 A_1} = \frac{L_1(\pi r_1^2)}{(25L_1)(\pi r_2^2)} = \frac{\left(\frac{d_1}{2}\right)^2}{25\left(\frac{d_2}{2}\right)^2} = \frac{d_1^2}{25 d_2^2}$$

$$\frac{R_1}{R_2} = \frac{\left(\frac{V}{i_1}\right)}{\left(\frac{V}{i_2}\right)} = \frac{i_2}{i_1} = \frac{0.044\ \text{pA}}{112\ \text{pA}} = 3.93 \times 10^{-4}$$

$$\frac{R_1}{R_2} = \frac{\left(\frac{\rho L_1}{A_1}\right)}{\left(\frac{\rho L_2}{A_2}\right)} = \frac{L_1 A_2}{L_2 A_1} = \frac{L_1(\pi r_2^2)}{(25L_1)(\pi r_1^2)} = \frac{\left(\frac{d_2}{2}\right)^2}{25\left(\frac{d_1}{2}\right)^2} = \frac{d_2^2}{25 d_1^2}$$

$$\frac{d_2^2}{25 d_1^2} = \frac{R_1}{R_2} = \frac{i_2}{i_1} = 3.93 \times 10^{-4}$$

$$d_2 = d_1\sqrt{(25)(3.93 \times 10^{-4})} = 0.099 d_1$$

REFLECT

We would expect a smaller current in the narrower, longer resistor for a given potential difference.

18.54

SET UP

We are asked to determine the colors of the first three bands of a 20 G Ω resistor. Resistors are marked with four colored bands that designate their resistance. The first two bands correspond to the two significant figures in the resistances, and the third band corresponds to the power of 10 by which the significant figures are multiplied. We can do an Internet search for "resistor color code" to find what colors correspond to 2, 0, and 9 (for *giga*-, 10^9).

SOLVE

Red = 2
Black = 0
White = 9

The first three bands will be $\boxed{\text{red, black, and then white}}$.

REFLECT

The fourth band corresponds to the tolerance or uncertainty in the stated value.

18.55

SET UP

Electrical cables are made up of a large number of strands of conducting wire. Each cable can have anywhere between 850 to 950 strands, each with a diameter of 0.72 ± 0.07 mm. The resistance of the cable is equal to the sum of the resistances of each component strand. We can determine an expression for this total resistance from combining the expression for the resistance in terms of the resistivity with Ohm's law. Plugging in the provided values will allow us to calculate the ratio of the lowest possible resistance to the average resistance and the ratio of the highest possible resistance to the average resistance of one of these cables.

SOLVE

$$i_{\text{total}} = ni_{\text{strand}} = n\left(\frac{V}{R_{\text{strand}}}\right) = \frac{nV}{\left(\frac{\rho L}{A}\right)} = \frac{nV(\pi r^2)}{\rho L} = \frac{n\pi V\left(\frac{d}{2}\right)^2}{\rho L} = \frac{n\pi V d^2}{4\rho L}$$

$$i_{\text{total}} = \frac{V}{R_{\text{total}}} = \frac{\pi n V d^2}{4\rho L}$$

$$R_{\text{total}} = \frac{4\rho L}{n\pi d^2}$$

Part a)

$$\frac{R_{\text{low}}}{R_{\text{avg}}} = \frac{\left(\frac{4\rho L}{n_{\text{low}}\pi d_{\text{low}}^2}\right)}{\left(\frac{4\rho L}{n_{\text{avg}}\pi d_{\text{avg}}^2}\right)} = \frac{n_{\text{avg}} d_{\text{avg}}^2}{n_{\text{low}} d_{\text{low}}^2} = \frac{(900)(0.72\text{ mm})^2}{(950)(0.79\text{ mm})^2} = \boxed{0.79}$$

Part b)

$$\frac{R_{\text{high}}}{R_{\text{avg}}} = \frac{\left(\frac{4\rho L}{n_{\text{high}}\pi d_{\text{high}}^2}\right)}{\left(\frac{4\rho L}{n_{\text{avg}}\pi d_{\text{avg}}^2}\right)} = \frac{n_{\text{avg}} d_{\text{avg}}^2}{n_{\text{high}} d_{\text{high}}^2} = \frac{(900)(0.72\text{ mm})^2}{(850)(0.65\text{ mm})^2} = \boxed{1.3}$$

REFLECT

The larger the diameter is for a wire, the smaller its resistance will be.

18.56

SET UP

As the temperature changes, the diameter of a given wire increases by 25% and its length decreases by 12%. We can use the expression for the resistance in terms of the resistivity to calculate the ratio of the final resistance to the initial resistance.

SOLVE

$$\frac{R_2}{R_1}=\frac{\left(\frac{\rho L_2}{A_2}\right)}{\left(\frac{\rho L_1}{A_1}\right)}=\frac{L_2A_1}{L_1A_2}=\frac{(0.88L_1)(\pi r_1^2)}{L_1(\pi r_2^2)}=\frac{0.88\left(\frac{d_1}{2}\right)^2}{\left(\frac{d_2}{2}\right)^2}=\frac{0.88d_1^2}{(1.25d_1)^2}=\frac{0.88}{(1.25)^2}=\boxed{0.56}$$

REFLECT

A decrease of 12% in the wire's length means the length of the wire is 88% of its initial value.

18.57

SET UP

A light bulb draws a current of $i = 1.0$ A when connected to a voltage of $V = 12$ V. The resistance of the filament is given by $V = iR$.

SOLVE

$$V = iR$$

$$R=\frac{V}{i}=\frac{12\text{ V}}{1.0\text{ A}}=\boxed{12\ \Omega}$$

REFLECT

A material with a constant resistance over a range of applied potential is said to be ohmic.

18.58

SET UP

A lightbulb that has a resistance of $R = 12.0\ \Omega$ is connected to a battery ($V = 6.0$ V). The current drawn by the bulb is given by $V = iR$.

SOLVE

$$V = iR$$

$$i=\frac{V}{R}=\frac{6.0\text{ V}}{12.0\ \Omega}=\boxed{0.50\text{ A}}$$

REFLECT

Assuming the resistance of the lightbulb remains constant, the current will increase with the potential difference.

18.59

SET UP

The current through a lightbulb ($R = 8.0\ \Omega$) is measured to be 0.50 A. The voltage across the bulb is given by $V = iR$.

SOLVE

$$V = iR = (0.50\text{ A})(8.0\ \Omega) = \boxed{4.0\text{ V}}$$

REFLECT

When working with circuits, it is usually easier to measure the voltage across a circuit element than to measure the current through the element.

18.60

SET UP

We are asked to find the potential difference between points A and B, $V_A - V_B$, for different configurations of resistors and batteries. We can use Ohm's law to find the magnitude of the potential difference across the resistors. The potential drops over a resistor in the direction of the current flow. In the circuit symbol for a battery, the longer vertical line is at a higher potential than the shorter vertical line.

SOLVE

Part a)

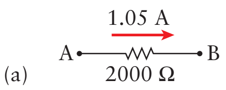


Figure 18-4 Problem 60

$$V_A - V_B = +iR = (1.05\text{ A})(2000\ \Omega) = \boxed{2100\text{ V}}$$

Part b)

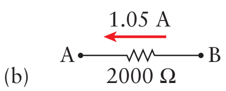


Figure 18-5 Problem 60

$$V_A - V_B = -iR = -(1.05\text{ A})(2000\ \Omega) = \boxed{-2100\text{ V}}$$

Part c)

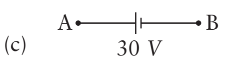


Figure 18-6 Problem 60

$$V_A - V_B = \boxed{+30\text{ V}}$$

Part d)

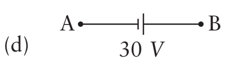


Figure 18-7 Problem 60

$$V_A - V_B = \boxed{-30\text{ V}}$$

REFLECT

We can use the fact that current flows from high potential to low potential to check our signs in parts (a) and (b). In part (a), point A must be at a higher potential than point B, whereas the opposite is true in part (b).

18.61

SET UP

Two resistors—$R_1 = 18\ \Omega$ and $R_2 = 6.0\ \Omega$—are wired in series. The equivalent capacitance of two resistors in series is given by $R_{\text{equiv}} = R_1 + R_2$.

SOLVE

$$R_{\text{equiv}} = R_1 + R_2 = (18\ \Omega) + (6.0\ \Omega) = \boxed{24\ \Omega}$$

REFLECT

The equivalent resistance of resistors in series is larger than the individual resistances.

18.62

SET UP

Two resistors—$R_1 = 18\ \Omega$ and $R_2 = 6.0\ \Omega$—are wired in parallel. The equivalent capacitance of two resistors in series is given by $\frac{1}{R_{\text{equiv}}} = \frac{1}{R_1} + \frac{1}{R_2}$.

SOLVE

$$\frac{1}{R_{\text{equiv}}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$$

$$R_{\text{equiv}} = \frac{R_1 R_2}{R_1 + R_2} = \frac{(18\ \Omega)(6.0\ \Omega)}{(18\ \Omega) + (6.0\ \Omega)} = \boxed{4.5\ \Omega}$$

REFLECT

The equivalent resistance of resistors in parallel is smaller than the individual resistances.

18.63

SET UP

Two resistors, $R_9 = 9.0\ \Omega$ and $R_3 = 3.0\ \Omega$, are wired in series across a battery ($V = 9.0$ V). Since the resistors are connected together in series, the current through each of them is the same. We can calculate the current from the battery voltage and the equivalent resistance of the two resistors in series. The equivalent resistance for the resistors in series is $R_{\text{equiv}} = R_9 + R_3$. Even though the current is the same through both resistors, the voltage drop across each one will be different. We can calculate the voltage drops V_9 and V_3 from the current found in part a and the individual resistances.

SOLVE

Part a)

Equivalent resistance:

$$R_{\text{equiv}} = R_9 + R_3 = (9.0\ \Omega) + (3.0\ \Omega) = 12.0\ \Omega$$

Current:

$$V = iR_{\text{equiv}}$$

$$i = \frac{V}{R_{\text{equiv}}} = \frac{9.0\ \text{V}}{12.0\ \Omega} = \boxed{0.75\ \text{A}}$$

Part b)

$$V_9 = iR_9 = (0.75\ \text{A})(9.0\ \Omega) = \boxed{6.75\ \text{V}}$$

$$V_3 = iR_3 = (0.75\ \text{A})(3.0\ \Omega) = \boxed{2.25\ \text{V}}$$

REFLECT

The sum of the voltages across the two resistors must equal the voltage of the battery:

$$V_9 + V_3 \stackrel{?}{=} V$$

$$(6.75\ \text{V}) + (2.25\ \text{V}) = 9.0\ \text{V}$$

18.64

SET UP

Two resistors, $R_9 = 9.0\ \Omega$ and $R_3 = 3.0\ \Omega$, are wired in parallel across a battery ($V = 9.0$ V). Since the resistors are connected together in parallel and their resistances are different, the current through each of them is different. We can calculate the current through each resistor from Ohm's law. Even though the current is different through each resistor, the voltage drop across each one will be the same since they are wired in parallel.

SOLVE

Part a)
Current through R_1:

$$i_1 = \frac{V}{R_1} = \frac{9.0\ \text{V}}{9.0\ \Omega} = \boxed{1.0\ \text{A}}$$

Current through R_2:

$$i_2 = \frac{V}{R_2} = \frac{9.0\ \text{V}}{3.0\ \Omega} = \boxed{3.0\ \text{A}}$$

Part b)
The voltage drop across the two resistors is the same, $\boxed{V = 9.0\ \text{V}}$.

REFLECT

The sum of the currents through both resistors must equal the current calculated from the equivalent resistance of the circuit:

$$i_1 + i_2 = (1.0\ \Omega) + (3.0\ \Omega) = 4.0\ \Omega$$

$$i_{\text{equiv}} = \frac{V}{R_{\text{equiv}}} = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = (9.0\ \text{V})\left(\frac{1}{9.0\ \Omega} + \frac{1}{3.0\ \Omega}\right) = 4.0\ \text{A}$$

18.65

SET UP

The potential difference across three resistors—$R_1 = 6\ \Omega$, $R_2 = 12\ \Omega$, and $R_3 = 6\ \Omega$—wired in parallel is $V = 3.6$ V. Since the resistors are connected together in parallel, the potential difference across each one will be the same. The current through each can be calculated using Ohm's law. The total current drawn from the power source is equal to the sum of the currents through each resistor.

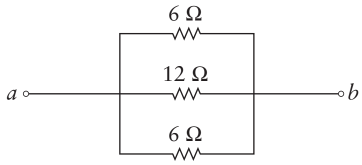


Figure 18-8 Problem 65

SOLVE

Part a)

Current through R_1:

$$i_1 = \frac{V}{R_1} = \frac{3.6\ \text{V}}{6\ \Omega} = \boxed{0.6\ \text{A}}$$

Current through R_2:

$$i_2 = \frac{V}{R_2} = \frac{3.6\ \text{V}}{12\ \Omega} = \boxed{0.3\ \text{A}}$$

Current through R_3:

$$i_3 = \frac{V}{R_3} = \frac{3.6\ \text{V}}{6\ \Omega} = \boxed{0.6\ \text{A}}$$

Part b)

$$i_{\text{total}} = i_1 + i_2 + i_3 = (0.6\ \text{A}) + (1.2\ \text{A}) + (0.6\ \text{A}) = \boxed{1.5\ \text{A}}$$

REFLECT

The total current the three resistors draw from the power source must also equal the current calculated from the equivalent resistance of the circuit:

$$i_{\text{total}} = \frac{V}{R_{\text{equiv}}} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = (3.6\ \text{V})\left(\frac{1}{6\ \Omega} + \frac{1}{12\ \Omega} + \frac{1}{6\ \Omega}\right) = 1.5\ \text{A}$$

18.66

SET UP

Four resistors—$R_1 = 6\ \Omega$, $R_2 = R_x$, $R_3 = 2\ \Omega$, and $R_4 = 4\ \Omega$—are wired together as shown in the figure. The equivalent resistance of this setup is $R_{\text{equiv}} = 8\ \Omega$. We can use the expressions for the equivalent resistance for resistors in series and parallel $\left(R_{\text{equiv}} = \sum_{i=1}^{N} R_i \text{ and } \frac{1}{R_{\text{equiv}}} = \sum_{i=1}^{N} \frac{1}{R_i}\text{, respectively}\right)$ to determine the value of R_x.

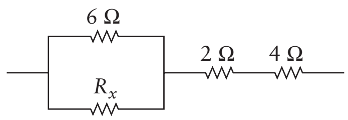


Figure 18-9 Problem 66

SOLVE

$$R_{\text{equiv}} = R_{12} + R_3 + R_4 = R_{12} + (2\ \Omega) + (4\ \Omega) = 8\ \Omega$$

$$R_{12} = 2\ \Omega$$

$$\frac{1}{R_{12}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{6\ \Omega} + \frac{1}{R_x} = \frac{1}{2\ \Omega}$$

$$\boxed{R_x = 3\ \Omega}$$

REFLECT

We can use our answer to explicitly calculate the equivalent resistance of the circuit in order to check our answer:

$$\frac{1}{R_{12}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{6\ \Omega} + \frac{1}{3\ \Omega}$$

$$R_{12} = 2\ \Omega$$

$$R_{\text{equiv}} = R_{12} + R_3 + R_4 = (2\ \Omega) + (2\ \Omega) + (4\ \Omega) = 8\ \Omega$$

18.67

SET UP

Six resistors—$R_1 = 40\ \Omega$, $R_2 = 70\ \Omega$, $R_3 = 30\ \Omega$, $R_4 = 60\ \Omega$, $R_5 = 10\ \Omega$, and $R_6 = 20\ \Omega$—are wired together as shown in the figure. The equivalent resistance of this setup can be found from the expressions for the equivalent resistance for resistors in series and parallel, $R_{\text{equiv}} = \sum_{i=1}^{N} R_i$ and $\frac{1}{R_{\text{equiv}}} = \sum_{i=1}^{N} \frac{1}{R_i}$, respectively.

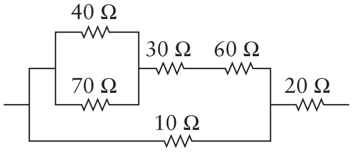


Figure 18-10 Problem 67

SOLVE

$$\frac{1}{R_{12}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{40\ \Omega} + \frac{1}{70\ \Omega} = \frac{11}{280\ \Omega}$$

$$R_{12} = \frac{280}{11}\Omega$$

$$R_{1234} = R_{12} + R_3 + R_4 = \left(\frac{280}{11}\Omega\right) + (30\ \Omega) + (60\ \Omega) = \frac{1270}{11}\Omega$$

$$\frac{1}{R_{12345}} = \frac{1}{R_{1234}} + \frac{1}{R_5} = \frac{11}{1270\ \Omega} + \frac{1}{10\ \Omega} = \frac{138}{1270\ \Omega}$$

$$R_{12345} = \frac{1270}{138}\Omega$$

$$R_{\text{equiv}} = R_{12345} + R_6 = \left(\frac{1270}{138}\Omega\right) + (20\ \Omega) = \boxed{\frac{4030}{138}\Omega \approx 29\ \Omega}$$

REFLECT

When calculating the equivalent resistance of a resistor network, go step-by-step and start with the smallest unit.

18.68

SET UP

A metal wire of length L_1 and resistance $R_1 = 48\ \Omega$ is cut into four equal parts of length $L_2 = \frac{L_1}{4}$. The resistance of each small wire can be found from the expression for the resistance in terms of the resistivity. These small wires are then wired in parallel to create a new wire. The resistance of this new wire is equal to the equivalent resistance of the four smaller wires wired in parallel.

SOLVE

Resistance of each small wire:

$$\frac{R_2}{R_1} = \frac{\left(\frac{\rho L_2}{A}\right)}{\left(\frac{\rho L_1}{A}\right)} = \frac{L_2}{L_1} = \frac{\left(\frac{L_1}{4}\right)}{L_1} = \frac{1}{4}$$

$$R_2 = \frac{R_1}{4}$$

Resistance of new wire:

$$\frac{1}{R_{\text{equiv}}} = \frac{1}{R_2} + \frac{1}{R_2} + \frac{1}{R_2} + \frac{1}{R_2} = \frac{4}{R_2}$$

$$R_{\text{equiv}} = \frac{R_2}{4} = \frac{\left(\frac{R_1}{4}\right)}{4} = \frac{R_1}{16} = \frac{48\ \Omega}{16} = \boxed{3\ \Omega}$$

REFLECT

The resistance of the new wire should be much smaller than the original wire due to two effects: shortening its length and connecting many wires in parallel.

18.69

SET UP

Two resistors A and B (resistances R_A and R_B) are connected in series to a battery ($V = 6.0$ V) and the voltage across resistor A is $V_{A,s} = 4.0$ V. The circuit is disconnected and rewired such that A and B are in parallel with the battery. The current through resistor B in this setup is $i_{B,p} = 2.0$ A. We can determine R_B from the information related to the parallel circuit. In this setup, the voltage across each resistor must be equal to the voltage across the battery. Since we know the voltage across and the current passing through resistor B we can calculate R_B. Now that we know the value for R_B, we can calculate R_A from the information related to the series circuit. First, the sum of the voltages across A and B must equal the voltage across the battery; this will provide us with the voltage across B in the series circuit. We already know R_B, so we can calculate the current in that circuit, which is the same through each element because they are all in series. Finally, by dividing the voltage across A by the series current, we will get R_A.

SOLVE

Resistors in parallel:

$$V_{B,\,p} = V_{A,\,p} = V$$

$$V_{B,\,p} = i_{B,\,p}R_B$$

$$R_B = \frac{V_{B,\,p}}{i_{B,\,p}} = \frac{V}{i_{B,\,p}} = \frac{6.0\text{ V}}{2.0\text{ A}} = \boxed{3.0\ \Omega}$$

Resistors in series:

$$V = V_{A,\,s} + V_{B,\,s}$$

$$V_{B,\,s} = V - V_{A,\,s} = (6.0\text{ V}) - (4.0\text{ V}) = 2.0\text{ V}$$

$$V_{B,\,s} = i_s R_B$$

$$i_s = \frac{V_{B,\,s}}{R_B} = \frac{2.0\text{ V}}{3.0\ \Omega} = 0.667\text{ A}$$

$$V_{A,\,s} = i_s R_A$$

$$R_A = \frac{V_{A,\,s}}{i_s} = \frac{4.0\text{ V}}{0.667\ \Omega} = \boxed{6.0\text{ A}}$$

REFLECT

For problems dealing with networks of resistors and capacitors, it's easiest to split the problem up into smaller steps rather than trying tackle it all at once. Also, realizing what remains constant in a given circuit—voltage for elements in parallel, current for elements in series—will usually help start you off.

18.70

SET UP

Two resistors—$R_1 = 35\ \Omega$ and $R_2 = 65\ \Omega$—are wired in series. A third resistor ($R_3 = 60\ \Omega$) is in parallel with the combination of R_1 and R_2. The potential difference applied between points *a* and *c* is $V = 7.50$ V. We can calculate the current through the top branch of the circuit from its equivalent resistance and Ohm's law. Because R_1 and R_2 are in series, the current is the same through each. The potential difference across R_2 alone is equal to the product of the current and R_2. The current through R_3, which we can find through Ohm's law, will be different than the current through the top branch even though the potential difference across each branch is the same because their resistances are different.

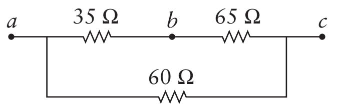


Figure 18-11 Problem 70

SOLVE

Part a)

Current through top branch:

$$i_{top} = \frac{V}{R_{equiv}} = \frac{V}{R_1 + R_2} = \frac{7.50\text{ V}}{(35\ \Omega) + (65\ \Omega)} = 0.075\text{ A}$$

Potential difference across R_2:

$$V_{R_2} = i_{\text{top}}R_2 = (0.075\ \text{A})(65\ \Omega) = \boxed{4.9\ \text{V}}$$

Part b)
Current through bottom branch:

$$i_{\text{bottom}} = \frac{V}{R_3} = \frac{7.50\ \text{V}}{60\ \Omega} = 0.13\ \text{A}$$

The current through the 60 Ω resistor is $\boxed{\text{larger}}$ than the current through the 35 Ω resistor because the potential difference across the two branches is equal and total resistance of the top branch is larger than the resistance of the bottom branch.

REFLECT
The sum of the potential differences across R_1 and R_2 must equal the total potential difference across the circuit:

$$V_{R_1} + V_{R_2} = i_{\text{top}}R_1 + (4.9\ \text{V}) = (0.075\ \text{A})(35\ \Omega) + (4.9\ \text{V}) = (2.6\ \text{V}) + (4.9\ \text{V}) = 7.5\ \text{V}$$

18.71

SET UP
A heater with a power rating of $P = 1500$ W is connected to a voltage source ($V = 120$ V). The current drawn by the heater can be found using $P = Vi$.

SOLVE

$$P = Vi$$

$$i = \frac{P}{V} = \frac{1500\ \text{W}}{120\ \text{V}} = \boxed{12.5\ \text{A}}$$

REFLECT
Common household voltage in the United States is 120 V. Power ratings are calculated assuming this voltage.

18.72

SET UP
A resistor ($R = 4.0\ \Omega$) is connected to a voltage source ($V = 12$ V). The power dissipated by the resistor is equal to $P = \frac{V^2}{R}$.

SOLVE

$$P = \frac{V^2}{R} = \frac{(12\ \text{V})^2}{4.0\ \Omega} = \boxed{36\ \text{W}}$$

REFLECT

The equation $P = \frac{V^2}{R}$ is equivalent to $P = Vi$ through Ohm's law.

18.73

SET UP

The resistance of a coffeemaker is $R = 12\ \Omega$, and it draws a current of $i = 15$ A. The power used by the coffeemaker is given by $P = i^2R$.

SOLVE

$$P = i^2R = (15\text{ A})^2(12\ \Omega) = \boxed{2700\text{ W} = 2.7\text{ kW}}$$

REFLECT

The equation $P = i^2R$ is equivalent to $P = Vi$ through Ohm's law.

18.74

SET UP

A 60 W bulb and a 120 W bulb consume 60 W and 120 W, respectively, when wired in parallel across a 120 V source. The resistance of the bulb, not its power rating, is the inherent feature of the bulb. (In the United States, the power rating for light bulbs is calculated assuming a voltage of 120 V.) We can use this information, along with $P = \frac{V^2}{R}$ to calculate the resistance of each bulb. The bulbs are then wired in series across the 120 V source. Since they are now wired in series, the current through each bulb is the same and can be found using the equivalent resistance and Ohm's law. Finally, the power consumed by each bulb in this configuration is related to this current and the resistance of each bulb.

SOLVE

Resistance of 60 W bulb:

$$P_{60} = \frac{V^2}{R_{60}}$$

$$R_{60} = \frac{V^2}{P_{60}} = \frac{(120\text{ V})^2}{60\text{ W}} = 240\ \Omega$$

Resistance of 120 W bulb:

$$P_{120} = \frac{V^2}{R_{120}}$$

$$R_{120} = \frac{V^2}{P_{120}} = \frac{(120\text{ V})^2}{120\text{ W}} = 120\ \Omega$$

Current through the bulbs in series:

$$V = iR_{\text{equiv}} = i(R_{60} + R_{120})$$

$$i = \frac{V}{R_{60} + R_{120}} = \frac{120\text{ V}}{(240\ \Omega) + (120\ \Omega)} = 0.333\text{ A}$$

Power consumed by 60 W bulb:

$$P_{60,\text{ series}} = i^2R_{60} = (0.333\text{ A})(240\ \Omega) = \boxed{80.0\text{ W}}$$

Power consumed by 60 W bulb:

$$P_{120,\text{ series}} = i^2 R_{120} = (0.333\text{ A})(120\ \Omega) = \boxed{40.0\text{ W}}$$

REFLECT

When purchasing light bulbs, we "know" that a bulb with a higher power rating will be brighter when we use it in a lamp, say, compared with one with a lower power rating. This is because our houses are wired in parallel, not in series. If they were wired in series, the bulb with the lower power rating would be brighter. Houses are not wired in series since removing an appliance from the circuit would break the circuit and cause everything to stop working. Christmas tree lights are a good example of this; if one bulb burns out, they all stop working.

18.75

SET UP

A transmission line with resistance $R = 40\ \Omega$ carries a current $i = 1200$ A. The amount of energy dissipated as heat per second is given by $P = i^2R$.

SOLVE

$$P = i^2R = (1200\text{ A})^2(40\ \Omega) = \boxed{5.8 \times 10^7\text{ W} = 58\text{ MW}}$$

REFLECT

The amount of heat generated per second in a resistive element is proportional to its resistance. Accordingly, appliances whose main job is to provide heat (for example, space heaters, hair dryers, toasters) have high resistances.

18.76

SET UP

A speaker, which has a power output of $P = 40$ W, has a resistance $R = 8\ \Omega$. The current drawn by the speaker can by calculated using $P = i^2R$.

SOLVE

$$P = i^2R$$

$$i = \sqrt{\frac{P}{R}} = \sqrt{\frac{40\text{ W}}{8\ \Omega}} = \boxed{2\text{ A}}$$

REFLECT

We are only allowed one significant figure in our solutions because the resistance only has one significant figure.

18.77

SET UP

The total energy required to run a 1500 W heater for 8 hours is equal to the product of the power and the time interval. Since we are given the cost the power company charges you in kWh ($0.11/kWh), we can multiply the numbers directly and then convert into kWh to find the total cost.

SOLVE

$$(1500\text{ W})(8\text{ h}) \times \frac{1\text{ kW}\cdot\text{h}}{1000\text{ W}\cdot\text{h}} \times \frac{\$0.11}{1\text{ kW}\cdot\text{h}} = \boxed{\$1.32}$$

REFLECT

A kilowatt-hour is a unit of energy commonly used in electricity bills.

Get Help: P'Cast 18.11 – Drying Your Hair

18.78

SET UP

A house is heated by an electric furnace with a power rating of $P = 24$ kW. The electric company charges the consumer $\frac{\$0.10}{\text{kW}\cdot\text{hr}}$. The total utility bill for January, which has 31 days, is \$218. Dividing the total bill amount by the price rate gives the total energy consumed in January. If we divide this amount by the power rating on the furnace, we get the total amount of time the furnace was operating. To find the average time per day the furnace was running we can divide the total time by the number of days in January.

SOLVE

Total energy consumed:

$$\$218 \times \frac{\text{kW}\cdot\text{hr}}{\$0.10} = 2180\text{ kW}\cdot\text{hr}$$

Total time:

$$P = \frac{\Delta E}{\Delta t}$$

$$\Delta t = \frac{\Delta E}{P} = \frac{2180\text{ kW}\cdot\text{hr}}{24\text{ kW}} = 90.8\text{ hr}$$

Time per January day:

$$\frac{90.8\text{ hr}}{31\text{ days}} = \boxed{2.93\frac{\text{hr}}{\text{day}}}$$

REFLECT

We assumed the entire electric bill is due to the furnace, which is unlikely. The actual average time per day the furnace was running should be less than our calculated value.

18.79

SET UP

A resistor $R = 4.00 \times 10^6\ \Omega$ and a capacitor $C = 3.00 \times 10^{-6}$ F are wired in series with a power supply. The time constant τ for this circuit is equal to the product RC.

SOLVE

$$\tau = RC = (4.00 \times 10^6\ \Omega)(3.00 \times 10^{-6}\text{ F}) = \boxed{12.0\text{ s}}$$

REFLECT

The prefix *mega-* corresponds to 10^6 and *micro-* corresponds to 10^{-6}, so if we multiply them together the factors of 10 will cancel.

18.80

SET UP

A capacitor ($C = 20 \times 10^{-6}$ F) and a resistor ($R = 100\ \Omega$) are quickly wired in series to a battery ($V = 6.0$ V). The charge on the capacitor at $t = 0.0010$ s (where $t = 0$ is the time the connection is made) is given by $q(t) = CV\left(1 - e^{-\frac{t}{RC}}\right)$.

SOLVE

$$q(t = 0.0010\text{ s}) = (20 \times 10^{-6}\text{ F})(6.0\text{ V})\left(1 - e^{\frac{0.0010\text{ s}}{(100\ \Omega)(20\times 10^{-6}\text{ F})}}\right) = \boxed{4.7 \times 10^{-5}\text{ C}}$$

REFLECT

The time constant for this circuit is 0.002 s, so $t = 0.0010$ s is half of a time constant in duration.

18.81

SET UP

A capacitor ($C = 10.0 \times 10^{-6}$ F) has an initial charge $q = 100.0 \times 10^{-6}$ C. If a resistor ($R = 20.0\ \Omega$) is connected across the capacitor, the initial current at $t = 0$ is given by $i(t) = -\frac{V}{R}e^{-\frac{t}{RC}}$, where $V = \frac{q}{C}$.

SOLVE

$$i(t = 0) = -\frac{V}{R}e^{-\frac{0}{RC}} = -\frac{V}{R} = -\frac{\left(\frac{q}{C}\right)}{R} = -\frac{q}{RC} = -\frac{100.0 \times 10^{-6}\text{ C}}{(20.0\ \Omega)(10.0 \times 10^{-6}\text{ F})} = -0.500\text{ A}$$

The initial current is $\boxed{0.500\text{ A}}$.

REFLECT

The negative sign in front of the current means charge is leaving the capacitor.

18.82

SET UP

A capacitor ($C = 10 \times 10^{-6}$ F) has an initial charge $q = 80 \times 10^{-6}$ C. If a resistor ($R = 25\ \Omega$) is connected across the capacitor, the initial current at $t = 0$ is given by $i(t) = -\frac{V}{R}e^{-\frac{t}{RC}}$, where $V = \frac{q}{C}$. The time constant for the circuit is equal to RC.

SOLVE

Part a)

$$i(t = 0) = -\frac{V}{R}e^{-\frac{0}{RC}} = -\frac{V}{R} = -\frac{\left(\frac{q}{C}\right)}{R} = -\frac{q}{RC} = -\frac{80 \times 10^{-6}\text{ C}}{(25\ \Omega)(10 \times 10^{-6}\text{ F})} = -0.32\text{ A}$$

The initial current is $\boxed{0.32\text{ A}}$.

Part b)

$$\tau = RC = (25\ \Omega)(10 \times 10^{-6}\ \text{F}) = \boxed{2.5 \times 10^{-4}\ \text{s}}$$

REFLECT

After $t = \tau$ the amount of charge on the capacitor is about 37% of its initial value.

18.83

SET UP

A capacitor ($C = 12.5 \times 10^{-6}$ F) is first charged to a potential $V = 50.0$ V and then discharged through a resistor ($R = 75.0\ \Omega$). The charge on a discharging capacitor as a function of time is described by $q(t) = q_{\text{max}}e^{-\frac{t}{RC}}$. The energy stored in a capacitor is $U = \frac{q^2}{2C}$. We can use these two expressions to calculate the time at which the charge or energy, respectively, reach 10% of their initial value. The current at those times can be calculated using the expression for the current in a discharging series RC circuit as a function of time, $i(t) = -\frac{V}{R}e^{-\frac{t}{RC}}$, where V is the initial voltage across the capacitor.

SOLVE

Part a)

i) Charge:

$$q(t) = q_{\text{max}}e^{-\frac{t}{RC}} = 0.100q_{\text{max}}$$

$$t = -RC\ln(0.100) = -(75.0\ \Omega)(12.5 \times 10^{-6}\ \text{F})\ln(0.100) = \boxed{2.2 \times 10^{-3}\ \text{s}}$$

ii) Energy:

$$U(t) = \frac{1}{2C}(q(t))^2 = \frac{1}{2C}\left(q_{\text{max}}e^{-\frac{t}{RC}}\right)^2 = \frac{q_{\text{max}}^2}{2C}\left(e^{-\frac{2t}{RC}}\right) = U_{\text{max}}\left(e^{-\frac{2t}{RC}}\right)$$

$$U(t) = 0.100U_{\text{max}} = U_{\text{max}}\left(e^{-\frac{2t}{RC}}\right)$$

$$t = -\frac{RC}{2}\ln(0.100) = -\frac{(75.0\ \Omega)(12.5 \times 10^{-6}\ \text{F})}{2}\ln(0.100) = \boxed{1.1 \times 10^{-3}\ \text{s}}$$

Part b)

Current at $t = 2.2 \times 10^{-3}$ s:

$$|i(2.2 \times 10^{-3}\ \text{s})| = \left|-\frac{50.0\ \text{V}}{75.0\ \Omega}e^{-\frac{2.2\times10^{-3}\ \text{s}}{(75.0\ \Omega)(12.5\times10^{-6}\ \text{F})}}\right| = \boxed{0.067\ \text{A}}$$

Current at $t = 1.1 \times 10^{-3}$ s:

$$|i(1.1 \times 10^{-3}\ \text{s})| = \left|-\frac{50.0\ \text{V}}{75.0\ \Omega}e^{-\frac{1.1\times10^{-3}\ \text{s}}{(75.0\ \Omega)(12.5\times10^{-6}\ \text{F})}}\right| = \boxed{0.21\ \text{A}}$$

REFLECT

The time constant $\tau = RC$ allows us to compare two RC circuits and make judgments, for example, on which one charges “faster” or “slower.” A discharging capacitor loses about 37% of its initial charge by $t = \tau$.

Get Help: Interactive Example – RC III

18.84

SET UP

A single ion channel is selectively permeable to potassium ions ($q = +e$) and has a resistance of $R = 1.0 \times 10^9\ \Omega$. The channel is open for $\Delta t = 1.0 \times 10^{-3}$ s while the voltage across the channel is maintained at $V = 80 \times 10^{-3}$ V. The current is equal to the total amount of charge passing through the channel divided by the time interval. By inserting this into Ohm’s law, we can calculate the total number of potassium ions traveling through the channel.

SOLVE

$$V = iR = \left(\frac{\Delta q}{\Delta t}\right)R = \frac{(ne)R}{\Delta t}$$

$$n = \frac{V\Delta t}{eR} = \frac{(80 \times 10^{-3}\ \text{V})(1.0 \times 10^{-3}\ \text{s})}{(1.60 \times 10^{-19}\ \text{C})(1.0 \times 10^{9}\ \Omega)} = \boxed{5.0 \times 10^{5}\ \text{ions}}$$

REFLECT

The ions spontaneously flow through the channel when it is open due to the potential difference across the membrane.

18.85

SET UP

A potassium ion channel carries a current of $i = 1.9 \times 10^{-12}$ A. We can calculate the number of potassium ions ($q = +e$) that pass through the channel in $\Delta t = 1.0 \times 10^{-3}$ s using the definition of current.

SOLVE

$$i = \frac{\Delta q}{\Delta t} = \frac{ne}{\Delta t}$$

$$n = \frac{i\Delta t}{e} = \frac{(1.9 \times 10^{-12}\ \text{A})(1.0 \times 10^{-3}\ \text{s})}{1.60 \times 10^{-19}\ \text{C}} = \boxed{1.2 \times 10^{4}\ \text{ions}}$$

REFLECT

A total charge of 1.9×10^{-15} C passes through the channel during 1 ms.

18.86

SET UP

A certain potassium channel has a diameter $d = 1.0 \times 10^{-9}$ m, a length of $L = 10 \times 10^{-9}$ m, and a resistance of $R = 18 \times 10^{9}\ \Omega$. The resistivity of the solution in the channel is given by $R = \frac{\rho L}{A}$.

SOLVE

$$R = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2} = \frac{\rho L}{\pi\left(\frac{d}{2}\right)^2} = \frac{4\rho L}{\pi d^2}$$

$$\rho = \frac{\pi R d^2}{4L} = \frac{\pi(18 \times 10^9\ \Omega)(1.0 \times 10^{-9}\ \text{m})^2}{4(10 \times 10^{-9}\ \text{m})} = \boxed{1.4\ \Omega \cdot \text{m}}$$

REFLECT

We would expect the resistivity of a biological ionic solution to be somewhere between that of a metal and that of a pure insulator.

18.87

SET UP

Myelination decreases the capacitance of an axon by a factor of ¼, which also decreases the time constant of the effective RC circuit by a factor of ½. We can use a ratio and the definition of the time constant, $\tau = RC$, to determine how the resistance must change.

SOLVE

$$\frac{\tau_2}{\tau_1} = \frac{R_2 C_2}{R_1 C_1}$$

$$\frac{\left(\frac{\tau_1}{2}\right)}{\tau_1} = \frac{R_2\left(\frac{C_1}{4}\right)}{R_1 C_1}$$

$$\frac{1}{2} = \frac{R_2}{4R_1}$$

$$\boxed{\frac{R_2}{R_1} = 2}$$

REFLECT

Myelin is an insulating material that surrounds the axon of a neuron; it helps electric signals to propagate down the neuron quickly and without much distortion.

18.88

SET UP

An action potential with a maximum voltage of $V = 75 \times 10^{-3}$ V propagates at a speed of $v = 64\frac{\text{m}}{\text{s}}$ down an axon of length $L = 0.30$ m. The resistance along the axon is $R = 20{,}000\ \Omega$. The rate at which energy is transmitted down the axon (that is, power) is equal to $P = \frac{V^2}{R}$. The total energy transmitted is equal to the power multiplied by the time interval, which we can find from the definition of speed.

SOLVE

$$P = \frac{V^2}{R} = \frac{\Delta E}{\Delta t}$$

$$\Delta E = \frac{V^2(\Delta t)}{R} = \frac{V^2\left(\frac{L}{v}\right)}{R} = \frac{V^2 L}{Rv} = \frac{(75 \times 10^{-3}\ \text{V})^2(0.30\ \text{m})}{(20{,}000\ \Omega)\left(64\frac{\text{m}}{\text{s}}\right)} = \boxed{1.3 \times 10^{-9}\ \text{J}}$$

REFLECT

The actual total energy will be smaller than our calculated value since we assumed the action potential was constant at its maximum value.

18.89

SET UP

The electric charge on a wayward satellite as a function of charge is described by $q(t) = \left(\frac{100\ \mu\text{C}}{\text{s}^2}\right)t^2 + \left(\frac{150\ \mu\text{C}}{\text{s}}\right)t + (28\ \mu\text{C})$. We can evaluate this function at $t = 0$ and $t = 2.5$ s in order to find the amount of charge the satellite possesses at those times. The rate at which charge flows onto the satellite is equal to the derivative of the charge with respect to time. Evaluating this derivative at $t = 0$ and $t = 12$ s will give the rates at those times.

SOLVE

Part a)

$$q(0) = \left(\frac{100\ \mu\text{C}}{\text{s}^2}\right)(0)^2 + \left(\frac{150\ \mu\text{C}}{\text{s}}\right)(0) + (28\ \mu\text{C}) = \boxed{28\ \mu\text{C}}$$

Part b)

$$q(2.5\ \text{s}) = \left(\frac{100\ \mu\text{C}}{\text{s}^2}\right)(2.5\ \text{s})^2 + \left(\frac{150\ \mu\text{C}}{\text{s}}\right)(2.5\ \text{s}) + (28\ \mu\text{C}) = \boxed{1028\ \mu\text{C}}$$

Part c)

Rate at which charge flows as a function of time:

$$\frac{dq}{dt} = \frac{d}{dt}\left[\left(\frac{100\ \mu\text{C}}{\text{s}^2}\right)t^2 + \left(\frac{150\ \mu\text{C}}{\text{s}}\right)t + (28\ \mu\text{C})\right] = 2\left(\frac{100\ \mu\text{C}}{\text{s}^2}\right)t + \left(\frac{150\ \mu\text{C}}{\text{s}}\right)$$

Rate of charge flow at $t = 0$:

$$\left.\frac{dq}{dt}\right|_{t=0} = 2\left(\frac{100\ \mu\text{C}}{\text{s}^2}\right)(0) + \left(\frac{150\ \mu\text{C}}{\text{s}}\right) = \boxed{150\ \frac{\mu\text{C}}{\text{s}}}$$

Part d)

$$\left.\frac{dq}{dt}\right|_{t=12\text{ s}} = 2\left(\frac{100\ \mu\text{C}}{\text{s}^2}\right)(12\text{ s}) + \left(\frac{150\ \mu\text{C}}{\text{s}}\right) = \boxed{2550\ \frac{\mu\text{C}}{\text{s}}}$$

REFLECT

The functional form for $q(t)$ is an upward-facing parabola, which means the amount of charge at $t = 0$ will be a minimum for $t \geq 0$. The slope of the parabola increases linearly with t, so the rate of charge flow at $t = 0$ will also be a minimum.

Get Help: P'Cast 18.1 – Charging Sphere

18.90

SET UP

An electric eel can generate voltages and currents up to $V = 500$ V and $I = 1.0$ A, respectively. The maximum power that an electric eel can deliver is equal to the product of these two quantities. The resistance of a snorkeler's body while she is swimming in salt water is around $R = 600\ \Omega$. The current that passes through her body is given by Ohm's law. We can compare this value to a current of 0.500 A, which can cause heart fibrillation and death, to see if the eel will harm the snorkeler or not. Finally, we can use $P = iV$ with the current and the eel's voltage to calculate the power received by the snorkeler.

SOLVE

Part a)

$$P = iV = (1.0\text{ A})(500\text{ V}) = \boxed{500\text{ W}}$$

Part b)

$$V = iR$$

$$i = \frac{V}{R} = \frac{500\text{ V}}{600\ \Omega} = \boxed{0.833\text{ A}}$$

$\boxed{\text{Yes}}$, this is large enough to be harmful.

Part c)

$$P = iV = (0.833\text{ A})(500\text{ V}) = \boxed{417\text{ W}}$$

REFLECT

The power received by the snorkeler is less than the maximum power delivered by the eel due to the resistance of the snorkeler's body.

18.91

SET UP

An electric heater consists of a single resistor connected across a power source ($V = 110$ V). The heater is used to heat $m = 0.2000$ kg of water $\left(c = 4186\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)$ from 20 degrees Celsius to 90 degrees Celsius ($\Delta T = 70$ K) in $\Delta t = 162$ s. We will assume that only 90% of the heat dissipated by the resistor goes into heating the water. The heat required to raise the

temperature of an object is equal to $Q = mc\Delta T$. Setting this value equal to 90% of the energy dissipated by the resistor in 162 s will allow us to calculate the resistance of the heater. We can rearrange our algebraic expression to solve for the time required to heat the water using a 12.0 V car battery, assuming everything else about the problem remains the same.

SOLVE

Part a)
Heat dissipated by the resistor:

$$\Delta E = P\Delta t = \left(\frac{V^2}{R}\right)\Delta t$$

Heat gained by the water:

$$Q = mc\Delta T = (0.90)\Delta E$$

Resistance of the heater:

$$mc\Delta T = (0.90)\left(\frac{V^2}{R}\right)\Delta t$$

$$R = (0.90)\left(\frac{V^2}{mc\Delta T}\right)\Delta t = (0.90)\left(\frac{(110\text{ V})^2}{(0.2000\text{ kg})\left(4186\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(70\text{ K})}\right)(162\text{ s}) = \boxed{30\ \Omega}$$

Part b)

$$mc\Delta T = (0.90)\left(\frac{V^2}{R}\right)\Delta t$$

$$\Delta t = \frac{mc(\Delta T)R}{0.90V^2} = \frac{(0.2000\text{ kg})\left(4186\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(70\text{ K})(30.1\ \Omega)}{0.90(12.0\text{ V})^2} = 13{,}612\text{ s}$$

$$(13{,}612\text{ s}) - (162\text{ s}) = 13{,}450\text{ s} \times \frac{1\text{ hr}}{3600\text{ s}} = \boxed{3.7\text{ hr}}$$

REFLECT

It makes sense that it would take a reasonably long time to heat about 200 mL of water using a heater and a car battery.

18.92

SET UP

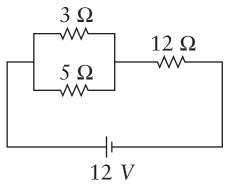
We are shown three circuits made up of a battery and a network of resistors and asked to calculate the current through each resistor. To do so, we will need to invoke Ohm's law along with the equivalent resistance relationships for resistors wired in series and in parallel: $R_{\text{equiv}} = \sum_{i=1}^{N} R_i$ and $\frac{1}{R_{\text{equiv}}} = \sum_{i=1}^{N} \frac{1}{R_i}$, respectively. We will also need to remember that circuit

elements in parallel with one another have the same voltage across them, while circuit elements in series with one another have the same current passing through them. Hint: You will save yourself a lot of time and effort if you solve these problems completely algebraically before plugging in any numbers.

SOLVE

Part a)

$R_1 = 3\ \Omega$, $R_2 = 5\ \Omega$, $R_3 = 12\ \Omega$, $V = 12$ V



(a)

Figure 18-12 Problem 92

Equivalent resistance:

$$\frac{1}{R_{12}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$$

$$R_{12} = \frac{R_1 R_2}{R_1 + R_2}$$

$$R_{\text{equiv}} = R_{12} + R_3 = \frac{R_1 R_2}{R_1 + R_2} + R_3 = \frac{(3\ \Omega)(5\ \Omega)}{(3\ \Omega) + (5\ \Omega)} + (12\ \Omega) = 13.9\ \Omega$$

Current through R_3:

$$V = i_{\text{equiv}} R_{\text{equiv}}$$

$$i_{\text{equiv}} = \frac{V}{R_{\text{equiv}}} = \frac{12\ \text{V}}{13.9\ \Omega} = 0.86\ \text{A}$$

$$\boxed{i_3 = i_{\text{equiv}} = 0.86\ \text{A}}$$

Voltage across R_3:

$$V_3 = i_3 R_3 = (0.86\ \text{A})(12\ \Omega) = 10.4\ \text{V}$$

Voltage across R_1 and R_2:

$$V_1 = V_2 = V - V_3 = (12\ \text{V}) - (10.4\ \text{V}) = 1.6\ \text{V}$$

Current through R_1:

$$V_1 = i_1 R_1$$

$$i_1 = \frac{V_1}{R_1} = \frac{1.6\ \text{V}}{3\ \Omega} = \boxed{0.54\ \text{A}}$$

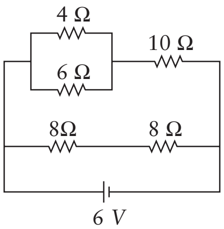
Current through R_2:

$$V_2 = i_2 R_2$$

$$i_2 = \frac{V_2}{R_2} = \frac{1.6\ \text{V}}{5\ \Omega} = \boxed{0.32\ \text{A}}$$

Part b)

$R_1 = 4\ \Omega$, $R_2 = 6\ \Omega$, $R_3 = 10\ \Omega$, $R_4 = 8\ \Omega$, $R_5 = 8\ \Omega$, $V = 6$ V



(b)

Figure 18-13 Problem 92

Equivalent resistance of top branch:

$$\frac{1}{R_{12}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$$

$$R_{12} = \frac{R_1 R_2}{R_1 + R_2}$$

$$R_{\text{top}} = R_{12} + R_3 = \frac{R_1 R_2}{R_1 + R_2} + R_3 = \frac{(4\ \Omega)(6\ \Omega)}{(4\ \Omega) + (6\ \Omega)} + (10\ \Omega) = 12.4\ \Omega$$

Current through R_3:

$$V = i_{\text{top}} R_{\text{top}}$$

$$i_{\text{top}} = \frac{V}{R_{\text{top}}} = \frac{6\ \text{V}}{12.4\ \Omega} = 0.48\ \text{A}$$

$$\boxed{i_3 = i_{\text{top}} = 0.48\ \text{A}}$$

Voltage across R_3:

$$V_3 = i_3 R_3 = (0.48\ \text{A})(10\ \Omega) = 4.8\ \text{V}$$

Voltage across R_1 and R_2:

$$V_1 = V_2 = V - V_3 = (6\ \text{V}) - (4.8\ \text{V}) = 1.2\ \text{V}$$

Current through R_1:

$$V_1 = i_1 R_1$$

$$i_1 = \frac{V_1}{R_1} = \frac{1.2\ \text{V}}{4\ \Omega} = \boxed{0.29\ \text{A}}$$

Current through R_2:

$$V_2 = i_2 R_2$$

$$i_2 = \frac{V_2}{R_2} = \frac{1.2\ \text{V}}{6\ \Omega} = \boxed{0.19\ \text{A}}$$

Equivalent resistance of middle branch:

$$R_{\text{middle}} = R_4 + R_5 = (8\ \Omega) + (8\ \Omega) = 16\ \Omega$$

Current through R_4 and R_5:

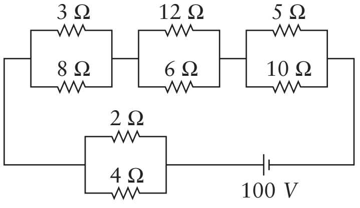
$$V = i_{\text{middle}} R_{\text{middle}}$$

$$i_{\text{middle}} = \frac{V}{R_{\text{middle}}} = \frac{6\ \text{V}}{16\ \Omega} = 0.38\ \text{A}$$

$$\boxed{i_4 = i_5 = i_{\text{middle}} = 0.38\ \text{A}}$$

Part c)

$R_1 = 3\ \Omega$, $R_2 = 8\ \Omega$, $R_3 = 12\ \Omega$, $R_4 = 6\ \Omega$, $R_5 = 5\ \Omega$, $R_6 = 10\ \Omega$, $R_7 = 2\ \Omega$, $R_8 = 4\ \Omega$, $V = 100\ \text{V}$



(c)

Figure 18-14 Problem 92

Equivalent resistance:

$$\frac{1}{R_{12}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$$

$$R_{12} = \frac{R_1 R_2}{R_1 + R_2}$$

$$\frac{1}{R_{34}} = \frac{1}{R_3} + \frac{1}{R_4} = \frac{R_3 + R_4}{R_3 R_4}$$

$$R_{34} = \frac{R_3 R_4}{R_3 + R_4}$$

$$\frac{1}{R_{56}} = \frac{1}{R_5} + \frac{1}{R_6} = \frac{R_5 + R_6}{R_5 R_6}$$

$$R_{56} = \frac{R_5 R_6}{R_5 + R_6}$$

$$\frac{1}{R_{78}} = \frac{1}{R_7} + \frac{1}{R_8} = \frac{R_7 + R_8}{R_7 R_8}$$

$$R_{78} = \frac{R_7 R_8}{R_7 + R_8}$$

$$R_{\text{equiv}} = R_{12} + R_{34} + R_{56} + R_{78} = \frac{R_1 R_2}{R_1 + R_2} + \frac{R_3 R_4}{R_3 + R_4} + \frac{R_5 R_6}{R_5 + R_6} + \frac{R_7 R_8}{R_7 + R_8}$$

$$= \frac{(3\ \Omega)(8\ \Omega)}{(3\ \Omega) + (8\ \Omega)} + \frac{(12\ \Omega)(6\ \Omega)}{(12\ \Omega) + (6\ \Omega)} + \frac{(5\ \Omega)(10\ \Omega)}{(5\ \Omega) + (10\ \Omega)} + \frac{(2\ \Omega)(4\ \Omega)}{(2\ \Omega) + (4\ \Omega)}$$

$$= (2.18\ \Omega) + (4.00\ \Omega) + (3.33\ \Omega) + (1.33\ \Omega) = 10.85\ \Omega$$

Current through equivalent circuit:

$$V = i_{\text{equiv}} R_{\text{equiv}}$$

$$i_{\text{equiv}} = \frac{V}{R_{\text{equiv}}} = \frac{100\ \text{V}}{10.85\ \Omega} = 9.22\ \text{A}$$

Current through R_1 and R_2:

$$V_1 = V_2$$

$$i_1 R_1 = i_2 R_2$$

$$i_{12} = i_1 + i_2 = i_1 + \left(\frac{R_1}{R_2}\right) i_1 = i_1\left(1 + \frac{R_1}{R_2}\right) = i_{\text{equiv}}$$

$$i_1 = \frac{i_{\text{equiv}}}{\left(1 + \frac{R_1}{R_2}\right)} = \frac{9.22\ \text{A}}{\left(1 + \frac{3\ \Omega}{8\ \Omega}\right)} = \boxed{6.70\ \text{A}}$$

$$i_2 = i_{\text{eqiuv}} - i_1 = (9.22\ \text{A}) - (6.70\ \text{A}) = \boxed{2.51\ \text{A}}$$

Current through R_3 and R_4:

$$V_3 = V_4$$

$$i_3 R_3 = i_4 R_4$$

$$i_{34} = i_3 + i_4 = i_3 + \left(\frac{R_3}{R_4}\right) i_3 = i_3\left(1 + \frac{R_3}{R_4}\right) = i_{\text{equiv}}$$

$$i_3 = \frac{i_{\text{equiv}}}{\left(1 + \frac{R_3}{R_4}\right)} = \frac{9.22\ \text{A}}{\left(1 + \frac{12\ \Omega}{6\ \Omega}\right)} = \boxed{3.07\ \text{A}}$$

$$i_4 = i_{\text{eqiuv}} - i_3 = (9.22\ \text{A}) - (3.07\ \text{A}) = \boxed{6.15\ \text{A}}$$

Current through R_5 and R_6:

$$V_5 = V_6$$

$$i_5 R_5 = i_6 R_6$$

$$i_{56} = i_5 + i_6 = i_5 + \left(\frac{R_5}{R_6}\right) i_5 = i_5\left(1 + \frac{R_5}{R_6}\right) = i_{\text{equiv}}$$

$$i_5 = \frac{i_{\text{equiv}}}{\left(1 + \frac{R_5}{R_6}\right)} = \frac{9.22\text{ A}}{\left(1 + \frac{5\ \Omega}{10\ \Omega}\right)} = \boxed{6.15\text{ A}}$$

$$i_6 = i_{\text{eqiuv}} - i_5 = (9.22\text{ A}) - (6.15\text{ A}) = \boxed{3.07\text{ A}}$$

Current through R_7 and R_8:

$$V_7 = V_8$$

$$i_7 R_7 = i_8 R_8$$

$$i_{78} = i_7 + i_8 = i_7 + \left(\frac{R_7}{R_8}\right) i_7 = i_7\left(1 + \frac{R_7}{R_8}\right) = i_{\text{equiv}}$$

$$i_7 = \frac{i_{\text{equiv}}}{\left(1 + \frac{R_7}{R_8}\right)} = \frac{9.22\text{ A}}{\left(1 + \frac{2\ \Omega}{4\ \Omega}\right)} = \boxed{6.15\text{ A}}$$

$$i_8 = i_{\text{eqiuv}} - i_7 = (9.22\text{ A}) - (6.15\text{ A}) = \boxed{3.07\text{ A}}$$

REFLECT

Be careful not to round during intermediate steps of your calculations. This problem is a perfect example of why we solve problems algebraically first and then plug in numbers. We saved ourselves a lot of algebra by noticing patterns in the circuit and the general algebraic expressions.

18.93

SET UP

Four resistors—$R_1 = 5.0\ \Omega$, $R_2 = 10\ \Omega$, $R_3 = 8.0\ \Omega$, and $R_4 = 16\ \Omega$—are connected as shown to a battery ($V = 45$ V). In order to calculate the power dissipated by each resistor, we first need to calculate the current through each resistor. Resistors R_1 and R_2 are wired in series to one another, and resistors R_3 and R_4 are wired in series to one another. This means the current through the top branch of the circuit must be constant and the current through the bottom branch must also be constant. After finding the currents, we can use $P = i^2R$ to calculate the power dissipated by each resistor.

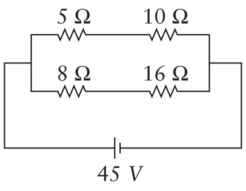


Figure 18-15 Problem 93

SOLVE

Current in the top branch:

$$V = i_{\text{top}} R_{\text{top}} = i_{\text{top}}(R_1 + R_2)$$

$$i_{\text{top}} = \frac{V}{R_1 + R_2} = \frac{45\text{ V}}{(5.0\ \Omega) + (10\ \Omega)} = 3.0\text{ A}$$

Power dissipated by R_1:

$$P_{R_1} = i_{\text{top}}^2 R_1 = (3.0\text{ A})^2(5.0\ \Omega) = \boxed{45\text{ W}}$$

Power dissipated by R_2:

$$P_{R_2} = i_{\text{top}}^2 R_2 = (3.0\text{ A})^2(10\ \Omega) = \boxed{90\text{ W}}$$

Current in the bottom branch:

$$V = i_{\text{bottom}} R_{\text{bottom}} = i_{\text{bottom}}(R_3 + R_4)$$

$$i_{\text{bottom}} = \frac{V}{R_3 + R_4} = \frac{45\text{ V}}{(8.0\ \Omega) + (16\ \Omega)} = 1.9\text{ A}$$

Power dissipated by R_3:

$$P_{R_3} = i_{\text{bottom}}^2 R_3 = (1.9\text{ A})^2(8.0\ \Omega) = \boxed{28\text{ W}}$$

Power dissipated by R_4:

$$P_{R_4} = i_{\text{bottom}}^2 R_4 = (1.9\text{ A})^2(16\ \Omega) = \boxed{56\text{ W}}$$

REFLECT

It makes sense that the current through the top branch should be larger than the current through the bottom branch because the total resistance of the branch is smaller than the total resistance of the bottom branch.

18.94

SET UP

Lightning bolts can carry a maximum charge of $q = 30$ C and can travel between a cloud and the ground in time intervals of $\Delta t = 100 \times 10^{-6}$ s. Potential differences between the cloud and ground were measured to be as high as $V = 400 \times 10^6$ V. The current in a lightning strike is equal to the total charge in the bolt divided by the time it takes to travel between the cloud and ground. The resistance of the air can be found from this current and the potential difference using Ohm's law. The energy transferred in the lightning strike is equal to the power multiplied by the time interval, where $P = iV$. Assuming all of this energy is transferred to water at 100 degrees Celsius as heat, the mass of water that the lightning bolt could evaporate is given by $Q = mL_F$, where $L_F = 2260 \times 10^3$ J/kg.

SOLVE

Part a)

$$i = \frac{\Delta q}{\Delta t} = \frac{30\ \text{C}}{100 \times 10^{-6}\ \text{s}} = \boxed{3.0 \times 10^5\ \text{A}}$$

Household currents are on the order of 10 A, so $\boxed{\text{lightning currents can be about 30,000 times larger}}$.

Part b)

$$V = iR$$

$$R = \frac{V}{i} = \frac{400 \times 10^6\ \text{V}}{3.0 \times 10^5\ \text{A}} = \boxed{1.3 \times 10^3\ \Omega}$$

Part c)

$$P = \frac{\Delta E}{\Delta t} = iV$$

$$\Delta E = iV\Delta t = (3.0 \times 10^5\ \text{A})(400 \times 10^6\ \text{V})(100 \times 10^{-6}\ \text{s}) = \boxed{1.2 \times 10^{10}\ \text{J}}$$

Part d)

$$Q = mL_\text{F}$$

$$m = \frac{Q}{L_\text{F}} = \frac{1.2 \times 10^{10}\ \text{J}}{\left(2260 \times 10^3 \dfrac{\text{J}}{\text{kg}}\right)} = \boxed{5.3 \times 10^3\ \text{kg}}$$

REFLECT

We would expect the amount of charge and energy in a lightning bolt to be very large.

18.95

SET UP

A voltmeter of resistance R_V and an ammeter of resistance R_A are connected to a circuit in order to measure the resistance R_X of an unknown resistor. The actual resistance is $R_\text{X} = \dfrac{V_\text{VM}}{i_\text{X}}$, where V_VM is the reading on the voltmeter and i_X is the actual current through the unknown resistor. The measured resistance is $(R_\text{X})_\text{measured} = \dfrac{V_\text{VM}}{i}$, where i is the reading on the ammeter. The current through the ammeter must equal the sum of the currents through the voltmeter and the unknown resistor due to conservation of charge. Using this and the above relationships, we can derive an expression for the measured unknown resistance in terms of the actual unknown resistance and the resistance of the voltmeter. Assuming $V_\text{VM} = 10.00$ V,

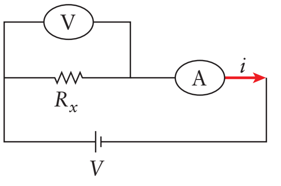


Figure 18-16 Problem 95

$i = 0.01000$ A, and $R_V = 1.000 \times 10^6$ Ω, we can calculate the difference between the measured unknown resistance and the actual unknown resistance using our derived expression.

SOLVE

$$i = i_V + i_X$$

$$\frac{V_{VM}}{(R_X)_{\text{measured}}} = \frac{V_{VM}}{R_V} + \frac{V_{VM}}{R_X}$$

$$\boxed{(R_X)_{\text{measured}} = \left(\frac{1}{R_V} + \frac{1}{R_X}\right)^{-1} = \frac{R_V R_X}{R_V + R_X}}$$

Measured resistance:

$$(R_X)_{\text{measured}} = \frac{V_{VM}}{i} = \frac{10.00 \text{ V}}{0.01000 \text{ A}} = 1000 \ \Omega$$

Actual resistance:

$$\frac{1}{(R_X)_{\text{measured}}} = \frac{1}{R_V} + \frac{1}{R_X}$$

$$R_X = \left(\frac{1}{(R_X)_{\text{measured}}} - \frac{1}{R_V}\right)^{-1} = \left(\frac{1}{1000 \ \Omega} - \frac{1}{1.000 \times 10^6 \ \Omega}\right)^{-1} = 1001 \ \Omega$$

The $\boxed{\text{actual resistance differs from the measured resistance by 1 } \Omega}$.

REFLECT

Voltmeters and ammeters are designed to have high and low resistances, respectively, so as to minimize their effect on the circuit being measured.

18.96

SET UP

A voltmeter of resistance R_V and an ammeter of resistance R_A are connected to a circuit in order to measure the resistance R_X of an unknown resistor. In this circuit the unknown resistor and ammeter are wired in series, and the voltmeter is connected across both of them. The current through the unknown resistor and the ammeter must be the same because they are connected in series. The total potential difference across both the unknown resistor and the ammeter must be equal to the potential difference of the battery. Using Ohm's law, we can relate this potential difference to the measured resistance $(R_X)_{\text{measured}}$, R_X, and R_A, which will allow us to determine the difference between the measured unknown resistance and the actual unknown resistance when $R_A = 10$ Ω.

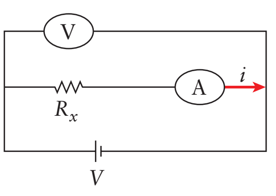


Figure 18-17 Problem 96

SOLVE

$$V = V_{R_X} + V_A$$

$$i_X (R_X)_{\text{measured}} = i_X R_X + i_X R_A$$

$$(R_X)_{\text{measured}} = R_X + R_A$$

$$(R_X)_{\text{measured}} - R_X = R_A = \boxed{10\ \Omega}$$

REFLECT

Comparing our results from Problems 18.95 and 18.96, we see that we get a more accurate result if we attach the voltmeter across only the unknown resistor.

18.97

SET UP

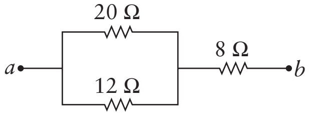


Figure 18-18 Problem 97

Three resistors—$R_1 = 20\ \Omega$, $R_2 = 12\ \Omega$, $R_3 = 8\ \Omega$—are wired together: resistors R_1 and R_2 are parallel to one another and this setup is connected in series to R_3. The voltage across the entire resistor network is $V = 5.00$ V. We can calculate the equivalent total resistance by first calculating the equivalent resistance R_{12} for R_1 and R_2 in parallel and then the total equivalent resistance R_{123} for R_{12} and R_3 in series. Since R_3 is in series with R_{12}, the current in R_3 will be equal to the current in R_{123}, which is equal to $\frac{V}{R_{123}}$. We can then use the current through R_3 and V to calculate the voltage across R_1 and R_2 in parallel. Once we know the voltage across each resistor R_1 and R_2, we can calculate the current through each one. Finally, the power dissipated by each resistor can be calculated directly from the current and voltage, $P = iV$.

SOLVE

Part a)

$$\frac{1}{R_{12}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{20\ \Omega} + \frac{1}{12\ \Omega} = \frac{8}{60\ \Omega}$$

$$R_{12} = \frac{60}{8}\Omega = 7.5\ \Omega$$

$$R_{123} = R_{12} + R_3 = (7.5\ \Omega) + (8\ \Omega) = \boxed{15.5\ \Omega}$$

Part b)

Current through R_3:

$$V = i_3 R_{123}$$

$$i_3 = \frac{V}{R_{123}} = \frac{5.00\ \text{V}}{15.5\ \Omega} = \boxed{0.32\ \text{A}}$$

Voltage across R_1 and R_2:

$$V = V_{12} + V_3$$

$$V_{12} = V - V_3 = V - i_3R_3 = (5.00\text{ V}) - (0.32\text{ A})(8\ \Omega) = 2.44\text{ V}$$

Current through R_1:

$$V_{12} = i_1R_1$$

$$i_1 = \frac{V_{12}}{R_1} = \frac{2.44\text{ V}}{20\ \Omega} = \boxed{0.12\text{ A}}$$

Current through R_2:

$$V_{12} = i_2R_2$$

$$i_2 = \frac{V_{12}}{R_2} = \frac{2.44\text{ V}}{12\ \Omega} = \boxed{0.20\text{ A}}$$

Part c)

Power dissipated by R_1:

$$P_1 = i_1V_{12} = (0.12\text{ A})(2.44\text{ V}) = \boxed{0.29\text{ W}}$$

Power dissipated by R_2:

$$P_2 = i_2V_{12} = (0.20\text{ A})(2.44\text{ V}) = \boxed{0.49\text{ W}}$$

Power dissipated by R_3:

$$P_3 = i_3V_3 = i_3(i_3R_3) = i_3^2R_3 = (0.32\text{ A})^2(8\ \Omega) = \boxed{0.83\text{ W}}$$

Part d) The power dissipated by R_1 is less than the power dissipated by the other two resistors because the current through and voltage across R_1 are lower than the other resistors, and $P = iV$.

REFLECT

Remember that two circuit elements in parallel have the same voltage, and two circuit elements in series have the same current.

Get Help: P'Cast 18.9 – A Flashlight

18.98

SET UP

We are shown two circuits (A and B) made up of many resistors wired to a battery ($V = 10.0$ V). To determine which circuit draws the most power, we first need to determine the equivalent resistance of each circuit. In circuit A, the 3.00 Ω resistors are in series with one another, and the two of them together are in parallel with the 6.00 Ω resistor. Circuit B is just a more complex version of circuit A. The circuit with the smaller equivalent resistance will dissipate more power. The amount of power dissipated is equal to $P = \dfrac{V^2}{R_{\text{equiv}}}$.

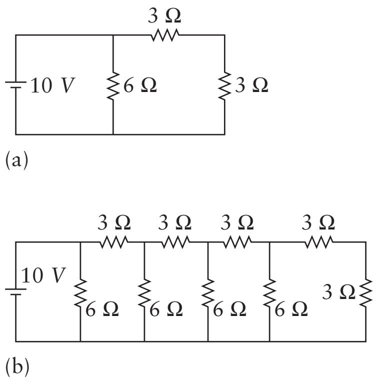


Figure 18-19 Problem 98

SOLVE

Part a)
Equivalent resistance of circuit A:

$$\frac{1}{R_{\text{equiv, A}}} = \frac{1}{6.00\ \Omega} + \frac{1}{(3.00\ \Omega) + (3.00\ \Omega)} = \frac{1}{3.00\ \Omega}$$

$$R_{\text{equiv, A}} = 3.00\ \Omega$$

Equivalent resistance of circuit B: Looking at our calculation for the equivalent resistance of circuit A, we see that the equivalent resistance of the rightmost loop of circuit B is equal to 3 Ω. Redrawing the circuit and replacing the rightmost loop with a 3 Ω resistor, we note that the new rightmost loop is again identical to the resistor loop from circuit A. If we keep reducing the circuit, we see that circuits A and B will have the same equivalent resistance, so the $\boxed{\text{two circuits will draw the same power}}$.

Part b)

$$P = \frac{V^2}{R_{\text{equiv}}} = \frac{(10.0\ \text{V})^2}{3.00\ \Omega} = \boxed{33.3\ \text{W}}$$

REFLECT

If a portion of a circuit diagram doesn't look obviously in series or obviously in parallel, it helps to redraw the diagram such that it does.

18.99

SET UP

The internal resistance of the human body, as measured between someone's hands, is around $R = 500\ \Omega$. We can model the body as a cylinder of length $L = 1.6$ m and diameter $d = 0.14$ m. The effective resistivity of the body can be found from $R = \frac{\rho L}{A}$. The power delivered to the body of a person who touches an electrical outlet at $V = 110$ V is $P = \frac{V^2}{R}$.

SOLVE

Part a)

$$R = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2} = \frac{\rho L}{\pi\left(\frac{d}{2}\right)^2} = \frac{4\rho L}{\pi d^2}$$

$$\rho = \frac{\pi R d^2}{4L} = \frac{\pi(500\ \Omega)(0.14\ \text{m})^2}{4(1.6\ \text{m})} = \boxed{4.8\ \Omega \cdot \text{m}}$$

Part b)

$$P = \frac{V^2}{R} = \frac{(110\ \text{V})^2}{500\ \Omega} = \boxed{24\ \text{W}}$$

REFLECT

The body is mainly comprised of water and ions, so its resistivity should be on the same order as water.

18.100

SET UP

The resistance of the skin, as measured between someone's hands, is around $R = 500 \times 10^3\ \Omega$. We can model the body as a cylinder of length $L = 1.6$ m and radius $r = 0.070$ m. The skin has a thickness of $T = 2.0 \times 10^{-3}$ m. The effective resistivity of the skin can be found from $R = \frac{\rho L}{A}$, where $A = 2\pi r T$ since $T << r$. The resistance depends on not only the resistivity but also the cross-sectional area of the object. The cross-sectional area of the skin is much smaller than that of the interior of the body, which explains the discrepancy between the resistances and resistivities.

SOLVE

Part a)

$$R = \frac{\rho L}{A} = \frac{\rho L}{2\pi r T}$$

$$\rho = \frac{2\pi R r T}{L} = \frac{2\pi(500 \times 10^3\ \Omega)(0.070\ \text{m})(2.0 \times 10^{-3}\ \text{m})}{1.6\ \text{m}} = \boxed{270\ \Omega \cdot \text{m}}$$

Part b) The internal resistivity from Problem 18.99 was 4.8 $\Omega \cdot$ m. The resistivity of the skin is about 60 times larger than the resistivity of the interior of the body.

Part c) The resistance is inversely proportional to the cross-sectional area, which is small for the skin because it is very thin, while the interior of the body has a much larger cross-sectional area and, thus, a smaller resistance.

REFLECT

The relationship $A = 2\pi r T$ comes from the infinitesimal area unit dA, $dA = 2\pi r dr$, where $dr = T$.

18.101

SET UP

The capacitor in an RC has a capacitance of $C = 160 \times 10^{-6}$ F. It takes 10 s for the charge on the capacitor to be 80% of its maximum value. The general expression for the charge as a function of time for a charging RC circuit is $q(t) = q_{max}(1 - e^{-\frac{t}{RC}})$. Rearranging this equation, we can solve for the value of the resistance in the circuit.

SOLVE

$$q(t) = q_{max}\left(1 - e^{-\frac{t}{RC}}\right)$$

$$-\ln\left(1 - \frac{q(t)}{q_{max}}\right) = \frac{t}{RC}$$

$$R = -\frac{t}{C\ln\left(1 - \frac{q(t)}{q_{max}}\right)} = -\frac{10\text{ s}}{(160 \times 10^{-6}\text{ F})\ln\left(1 - \frac{0.80q_{max}}{q_{max}}\right)}$$

$$= -\frac{10\text{ s}}{(160 \times 10^{-6}\text{ F})\ln(0.20)} = \boxed{3.9 \times 10^4\ \Omega = 39\text{ k}\Omega}$$

REFLECT

After $t = \tau$, the charging capacitor will have reached about 63% of its maximum charge. After $t = 2\ \tau$, the charging capacitor will have reached about 86% of its maximum charge. Therefore, we expect τ to be less than 10 s and $2\ \tau$ to be greater than 10 s:

$$\tau = RC = (3.9 \times 10^4\ \Omega)(160 \times 10^{-6}\text{ F}) = 6.2\text{ s}$$

18.102

SET UP

The general expression for the charge on a discharging capacitor as a function of time is $q(t) = q_{max}e^{-\frac{t}{\tau}}$. We can rearrange this expression to find the time (in terms of τ) required for the capacitor to hold 50% of its maximum charge, as well as the percentage of the maximum charge the capacitor will hold after $t = 3\ \tau$.

SOLVE

Part a)

$$q(t) = q_{max}e^{-\frac{t}{\tau}}$$

$$t = -\tau\ln\left(\frac{q(t)}{q_{max}}\right) = -\tau\ln\left(\frac{0.50q_{max}}{q_{max}}\right) = -\tau\ln(0.50) = \boxed{\tau\ln(2) = 0.693\tau}$$

Part b)

$$\frac{q(t = 3\tau)}{q_{\max}} = e^{-\frac{3\tau}{\tau}} = e^{-3} = \boxed{0.0498 = 4.98\%}$$

REFLECT

When in the laboratory, people commonly wait at least 5 τ when saying a capacitor is discharged. At this point, less than 1% (~0.67%) of the initial charge remains on the capacitor.

18.103

SET UP

The energy stored in a capacitor is $U = \frac{q^2}{2C}$. The charge on a charging capacitor as a function of time in a series RC circuit is described by $q(t) = CV\left(1 - e^{-\frac{t}{RC}}\right)$. Combining these two expressions will give us an expression describing the time dependence of the energy stored in a charging capacitor in an RC circuit.

SOLVE

$$U(t) = \frac{1}{2C}(q(t))^2 = \frac{1}{2C}\left(CV\left(1 - e^{-\frac{t}{RC}}\right)\right)^2 = \boxed{\frac{CV^2}{2}\left(1 - e^{-\frac{t}{RC}}\right)^2}$$

REFLECT

At $t = 0$, there is no charge on the capacitor, and the energy stored in it will be 0, as expected. As t approaches infinity, the capacitor should be fully charged and attain its maximum stored energy of $U_{\max} = \frac{q_{\max}^2}{2C} = \frac{CV^2}{2}$.

Get Help: Interactive Example – RC III

18.104

SET UP

A capacitor ($C = 3.0 \times 10^{-6}$ F) is initially charged up using a battery ($V = 12$ V). The charge on the capacitor at this point is $q = CV$. The capacitor is then disconnected from the battery and connected across a resistor ($R = 200\ \Omega$). The current through the resistor as a function of time is given by $i(t) = \frac{V}{R}e^{-\frac{t}{RC}}$; using this we can calculate the initial current through the resistor as well as the time when the current reaches 37% of its initial value.

SOLVE

Part a)

$$q = CV = (3.0 \times 10^{-6}\ \text{F})(12\ \text{V}) = \boxed{3.6 \times 10^{-5}\ \text{C}}$$

Part b)

$$i(t) = \frac{V}{R}e^{-\frac{t}{RC}}$$

$$i(0) = \frac{V}{R}e^{-\frac{0}{RC}} = \frac{V}{R} = \frac{12\text{ V}}{200\ \Omega} = \boxed{0.060\text{ A}}$$

Part c)

$$i(t) = -\frac{V}{R}e^{-\frac{t}{RC}} = i(0)e^{-\frac{t}{RC}}$$

$$t = -RC\ln\left(\frac{i(t)}{i(0)}\right) = -(200\ \Omega)(3.0 \times 10^{-6}\text{ F})\ln\left(\frac{0.37i(0)}{i(0)}\right)$$

$$= -(200\ \Omega)(3.0 \times 10^{-6}\text{ F})\ln(0.37) = \boxed{6.0 \times 10^{-4}\text{ s}}$$

REFLECT

There is no negative sign in our expression for the current as a function of time because we are looking at the current through the resistor rather than the rate of charge leaving the capacitor. After $t = \tau$, the current will be $e^{-1} \approx 0.37$ of its maximum value. In this case, $\tau = 6.0 \times 10^{-4}$ s.

18.105

SET UP

You have a box of 50 Ω resistors and need to wire them up such that the equivalent resistance is either 75 Ω or 60 Ω. The equivalent resistance of resistors in series is larger than the individual resistances, so we will need to put one 50 Ω resistor in series with some combination of resistors in parallel. The equivalent resistances of these combinations need to be 25 Ω and 10 Ω, respectively. We also have a box of 50 μF capacitors and need to wire them up such that the equivalent capacitance is either 75 μF or 60 μF. The equivalent capacitance of capacitors in parallel is larger than the individual capacitances, so we will need to put one 50 μF capacitor in parallel with some combination of capacitors in series. The equivalent capacitances of these combinations need to be 25 μF and 10 μF, respectively.

SOLVE

Part a) Connect two 50 Ω resistors in parallel to make a 25 Ω equivalent resistance, and then connect that combination in series with another 50 Ω resistor to make a total equivalent resistance of 75 Ω.

Part b) Connect five 50 Ω resistors in parallel to make a 10 Ω equivalent resistance, and then connect that combination in series with another 50 Ω resistor to make a total equivalent resistance of 60 Ω.

Part c) Connect two 50 μF capacitors in series to make a 25 μF equivalent capacitance, and then connect that combination in parallel with another 50 μF capacitor to make a total equivalent capacitance of 75 μF.

Part d) Connect five 50 μF capacitors in series to make a 10 μF equivalent capacitance, and then connect that combination in parallel with another 50 μF capacitor to make a total equivalent capacitance of 60 μF.

REFLECT

Remember that the rules for equivalent resistance and equivalent capacitance are opposite from one another.

18.106

SET UP

A circuit probe is a device that gives information about the circuit. Upon connecting it to the circuit element, the probe is now a part of the circuit. Ideally, the probe itself should not affect the quantities you are trying to measure in the circuit. An oscilloscope and a voltmeter, which are two types of probes, are designed to measure the voltage across a circuit element. To correctly measure the voltage across the element, the voltages across the probe should be the same; therefore, the probe should be wired in parallel with the circuit element. So as to not affect the measurement, the added resistance and capacitance of the probe should not affect the resistance or capacitance of the circuit. We can use the expressions for equivalent resistance and capacitance to determine what the relative resistance and capacitance of the probe should be to provide the most accurate measurement.

SOLVE

Part a) The probe is in $\boxed{\text{parallel}}$ with the circuit element.

Part b) (i) The probe should have a $\boxed{\text{very large resistance}}$ because adding a very large resistance in parallel does not appreciably affect the equivalent parallel resistance.

(ii) The probe should have a $\boxed{\text{very small capacitance}}$ because adding a very small capacitance in parallel does not appreciably affect the equivalent parallel capacitance.

REFLECT

Typical resistances for oscilloscopes are on the order of 1 MΩ.

18.107

SET UP

A new 1.5 V battery has a "capacity" of 1250 mA · hr; this quantity has dimensions of current multiplied by time, which makes it a charge. We can use this quantity and the voltage of the battery to determine the total energy stored in the battery. Assuming the battery provides a steady current of 0.400 A, the capacity of the battery divided by this current is equal to the life span of the battery. Finally, we can compare the energy stored in this battery to the energy stored in a 10 μF capacitor charged up to a potential difference of 1.50 V using $U = \frac{1}{2}CV^2$.

SOLVE

Part a)

$$1250\ \text{mA}\cdot\text{hr} \times \frac{1\ \text{A}}{1000\ \text{mA}} \times \frac{3600\ \text{s}}{1\ \text{hr}} = \boxed{4500\ \text{C}}$$

The "capacity" is a $\boxed{\text{measure of charge stored in the battery}}$.

Part b)

$$\Delta E = P\Delta t = (Vi)\Delta t = V(i\Delta t) = V(\text{capacity}) = (1.5\text{ V})(4500\text{ C}) = \boxed{6750\text{ J}}$$

Part c)

$$i = \frac{\Delta q}{\Delta t} = \frac{\text{capacity}}{\Delta t}$$

$$\Delta t = \frac{\text{capacity}}{i} = \frac{4500\text{ C}}{0.400\text{ A}} = 11{,}250\text{ s} \times \frac{1\text{ hr}}{3600\text{ s}} = \boxed{3.13\text{ hr}}$$

Part d)

$$U_{\text{capacitor}} = \frac{1}{2}CV^2 = \frac{1}{2}(10 \times 10^{-6}\text{ F})(1.50\text{ V})^2 = \boxed{1.13 \times 10^{-5}\text{ J}}$$

$$\frac{U_{\text{battery}}}{U_{\text{capacitor}}} = \frac{6750\text{ J}}{1.1 \times 10^{-5}\text{ J}} = 6.0 \times 10^8$$

The $\boxed{\text{battery stores 600 million times as much energy as the capacitor}}$ charged to the same potential.

REFLECT

In order to store 6750 J of energy, the capacitor needs to be either 6 kF or the potential difference across it needs to be about 37 kV.

18.108

SET UP

An ultracapacitor is a high-capacitance device designed to store much more energy than an ordinary capacitor. The surface area of an ultracapacitor is increased by a factor of 100,000, while its plate separation distance is decreased by a factor of 1000. We are told that a 10 μF capacitor is converted into an ultracapacitor; by taking a ratio between the capacitance before and after, we can calculate the capacitance of the ultracapacitor due to these geometric changes. The energy stored by the ultracapacitor is equal to $U = \frac{1}{2}CV^2$, where $V = 1.5$ V. We can use our results from Problem 18.107 to compare the energy stored in the ultracapacitor to the energy stored in the 10 μF capacitor and the 1.5 V battery The time constant for the ultracapacitor is $\tau = 60$ s; using the fact that $\tau = RC$, we can calculate R from the capacitance of the ultracapacitor.

SOLVE

Part a)

$$\frac{C_2}{C_1} = \frac{\left(\dfrac{\varepsilon_0 A_2}{d_2}\right)}{\left(\dfrac{\varepsilon_0 A_1}{d_1}\right)} = \frac{A_2 d_1}{A_1 d_2} = \frac{(100{,}000A_1)d_1}{A_1(0.001d_1)} = 1 \times 10^8$$

$$C_2 = (1 \times 10^8)C_1 = (1 \times 10^8)(10 \times 10^{-6}\text{ F}) = \boxed{1000\text{ F}}$$

Part b)

$$U_{\text{ultracapacitor}} = \frac{1}{2}CV^2 = \frac{1}{2}(1000\text{ F})(1.5\text{ V})^2 = \boxed{1.1 \times 10^3\text{ J}}$$

$$U_{\text{capacitor}} = \frac{1}{2}CV^2 = \frac{1}{2}(10 \times 10^{-6}\text{ F})(1.5\text{ V})^2 = 1.1 \times 10^{-5}\text{ J}$$

$$\frac{U_{\text{ultracapacitor}}}{U_{\text{capacitor}}} = \frac{1.1 \times 10^3\text{ J}}{1.1 \times 10^{-5}\text{ J}} = \boxed{1 \times 10^8}$$

Part c)

$$\frac{U_{\text{ultracapacitor}}}{U_{\text{battery}}} = \frac{1.1 \times 10^3\text{ J}}{6750\text{ J}} = \boxed{0.17}$$

Part d)

$$\tau = RC$$

$$R = \frac{\tau}{C} = \frac{60\text{ s}}{1.1 \times 10^3\text{ F}} = \boxed{0.053\ \Omega}$$

REFLECT

Even though the battery stores more energy than the ultracapacitor, the two energies are much closer than in an ordinary capacitor, which makes an ultracapacitor feasible for energy storage.

Chapter 19
Magnetism

Conceptual Questions

19.1 Use both ends of one iron rod to approach the other iron rods. If both ends of the rod you are holding attract both ends of the other two rods, then the one you are holding is not magnetized iron.

19.2 No, the magnetic field cannot do work on the particles because it is always perpendicular to their velocity.

19.3 The wire is aligned with the magnetic field.

19.4 If the forces do not act at the same point, then the forces can combine to exert a net torque, causing the object to rotate even though the center of mass does not accelerate.

19.5 Part a) The electric field points into the page.

Part b) The beam is deflected into the page.

Part c) The electron beam is not deflected.

19.6 Applying Newton's second law to a charged particle whose velocity is at right angles to a uniform magnetic field $\vec{B}$ results in the equation $qvB = \frac{mv^2}{r}\left(\text{or } r = \frac{mv}{qB}\right)$. As long as the particles have the same charge, the radius of their orbits depends on their momentum, not their velocity alone.

19.7 Yes, because the width of the field determines the length of the wire in the magnetic field. The larger the length of wire in the magnetic field, the larger the force exerted on the wire will be.

19.8 The coil will compress. Each current-carrying loop can be imagined as a dipole that attracts one another, thus making the coil compress.

19.9 Similarities: Both depend on a physical constant associated with the field and the charge element that gives rise to the field. Both laws are inversely proportional to r^2. Differences: The direction of the field in Coulomb's law points either directly toward or away from the charge while the direction of the field in the Biot–Savart law is mutually perpendicular to the r vector and the direction of the current element.

19.10 According to the right-hand rule, because Earth's magnetic field is in a northward direction, the negative charge is moving downward; therefore, the lightning strike would be deflected toward the west.

19.11 Since the directions of the two currents are opposite, the magnetic field from one wire is opposite to the other. This means the magnetic fields will cancel each other to some extent.

19.12 Yes. Two wires repel each other when carrying current in opposite directions—the force on each wire is equal in magnitude and opposite in direction.

19.13 There will be no net force on the wires, but there will be a torque. When two wires are perpendicular, each wire will experience a force on one end and another force, equal in magnitude but opposite in direction, on the other end. This results in zero net force. However, the forces will produce a nonzero torque that will make the wires want to align with each other such that the current in each wire is traveling in the same direction as the current in the neighboring wire.

Multiple-Choice Questions

19.14 **E** (is described by all of the above options, A through D). The magnetic force on a moving charged particle is given by the Lorentz force, $\vec{F} = q\vec{v} \times \vec{B}$.

19.15 **C** (deflected toward the top of the page). The right hand rule and the Lorentz force law give the direction of the force acting on a moving charge in a magnetic field.

19.16 **E** (This situation cannot exist because of the orientation of the velocity and force vectors). The velocity and force vectors must be perpendicular to one another.

19.17 **D** ($-z$ direction). The magnetic force points toward $+z$.

19.18 **A** (zero). The magnetic field due to the coil is parallel to the uniform magnetic field.

19.19 **A** (zero). The force on each infinitesimal portion of the ring points radially, so the torque on the loop is equal to zero.

19.20 **C** (d^{-1}). The magnitude of the magnetic field due to a long, straight wire carrying a current i_0 is $\dfrac{\mu_0 i_0}{2\pi d}$.

19.21 **A** (would remain the same). The magnitude of the magnetic field due to a solenoid is independent of the radius of the solenoid.

19.22 **B** (The force on the i_2 wire is downward and the force on the i_1 wire is upward). Wires carrying currents in opposite directions repel one another.

19.23 **E** (zero force). You can prove this to yourself either through the right hand rule and applying $\vec{F} = i\vec{l} \times \vec{B}$ or through symmetry.

Estimation/Numerical Questions

19.24 The magnitude of the geomagnetic field at the North and South Poles is around 0.6 gauss; at the equator, the field is about 0.3 gauss.

19.25 Assuming that 200 A of current is traveling a distance of 10 m above ground level, the magnitude of the magnetic field at ground level is around 4×10^{-6} T.

19.26 Assuming the wires are about 10 m apart and each carry 1000 A of current, the force per unit length is about 0.02 N/m.

19.27 The beampipe has a radius around 0.25 m, so a current of 25 MA is required to create a field of about 8–10 T.

19.28 The radius is proportional to the mass. Assuming a speed of about 100,000 m/s and a charge of e, the magnetic field required is around 0.3 T.

19.29 The time is equal to the distance traveled divided by the speed:

$$t = \frac{2\pi R}{v} = \frac{2\pi(0.1\text{ m})}{\left(10^5 \frac{\text{m}}{\text{s}}\right)} \approx 6 \times 10^{-6}\text{ s} = 6\ \mu\text{s}$$

19.30 The maximum torque on a coil in a typical drill motor is around 5 N · m.

19.31

r (m)	B (T)
0.001	0.00050
0.002	0.00100
0.003	0.00152
0.004	0.00200
0.005	0.00252
0.006	0.00300
0.007	0.00350
0.008	0.00401
0.009	0.00453
0.010	0.00500
0.015	0.00353
0.020	0.00250
0.025	0.00200
0.030	0.00180
0.035	0.00143
0.040	0.00125
0.045	0.00110
0.050	0.00103
0.100	0.000502

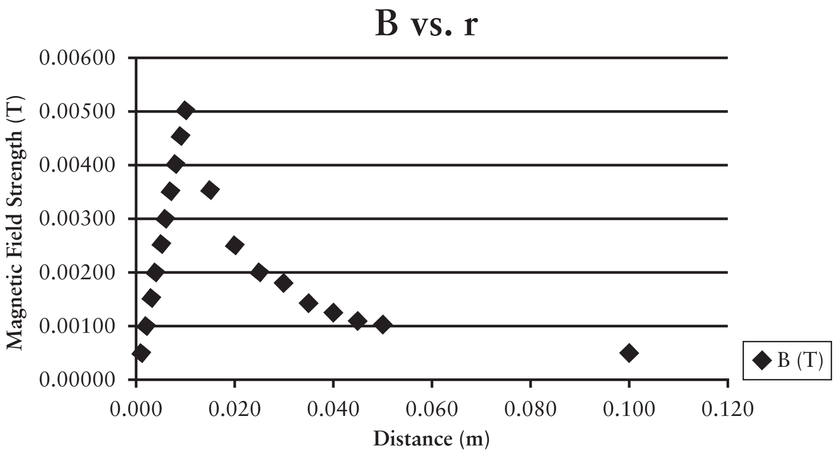


Figure 19-1 Problem 31

Plot of B versus r for $r < 0.01$ m:

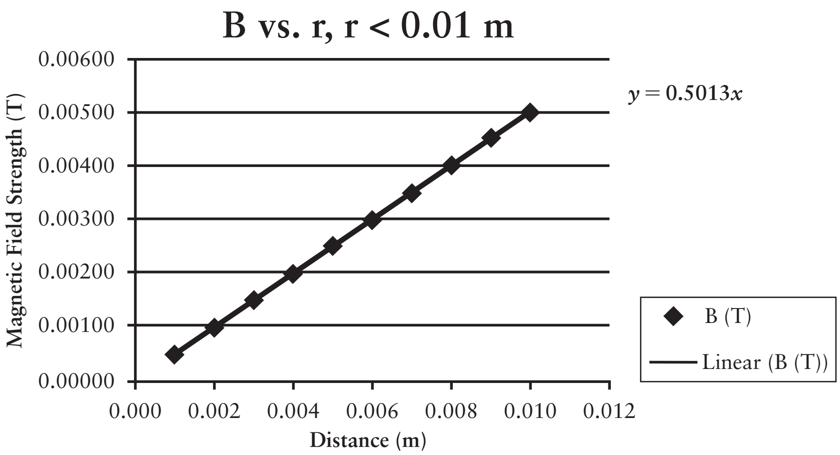


Figure 19-2 Problem 31

Functional form for $B(r)$, $r < 0.01$ m:

$$B = \left(0.5013\frac{\text{T}}{\text{m}}\right)r$$

Plot of B versus r for $r > 0.01$ m:

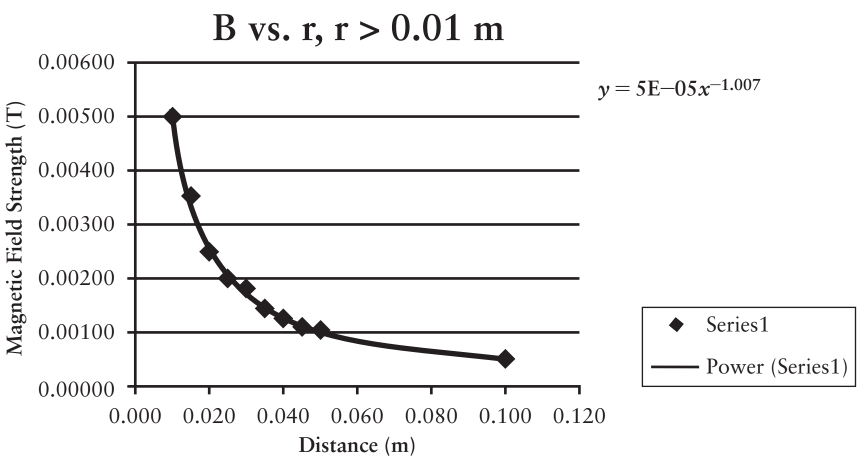


Figure 19-3 Problem 31

Functional form for $B(r)$, $r > 0.01$ m:

$$B \approx (5 \times 10^{-5}\ \mathrm{T \cdot m})r^{-1}$$

Problems

19.32

SET UP

We are given four magnetic field magnitudes and asked to convert between gauss and tesla. The conversion factor we'll use is $1\ \mathrm{G} = 10^{-4}\ \mathrm{T}$.

SOLVE

Part a)

$$5.00\ \mathrm{T} \times \frac{1\ \mathrm{G}}{10^{-4}\ \mathrm{T}} = \boxed{5.00 \times 10^{4}\ \mathrm{G}}$$

Part b)

$$25{,}000\ \mathrm{G} \times \frac{10^{-4}\ \mathrm{T}}{1\ \mathrm{G}} = \boxed{2.5000\ \mathrm{T}}$$

Part c)

$$7.43\ \mathrm{mG} \times \frac{1\ \mathrm{G}}{10^{3}\ \mathrm{mG}} \times \frac{10^{-4}\ \mathrm{T}}{1\ \mathrm{G}} \times \frac{10^{6}\ \mu\mathrm{T}}{1\ \mathrm{T}} = \boxed{0.743\ \mu\mathrm{T}}$$

Part d)

$$1.88\ \mathrm{mT} \times \frac{1\ \mathrm{T}}{10^{3}\ \mathrm{mT}} \times \frac{1\ \mathrm{G}}{10^{-4}\ \mathrm{T}} = \boxed{18.8\ \mathrm{G}}$$

REFLECT

The gauss is a useful unit because Earth's magnetic field is on the order of 0.5 G. It's much easier to work with numbers on the order of 1 than 10^{-4}.

19.33

SET UP

We are shown six scenarios of a positive charge moving with a velocity $\vec{v}$ in a magnetic field $\vec{B}$. We can use the Lorentz force law, $\vec{F} = q\vec{v} \times \vec{B}$, and the right hand rule to determine the direction of the magnetic force acting on the charge.

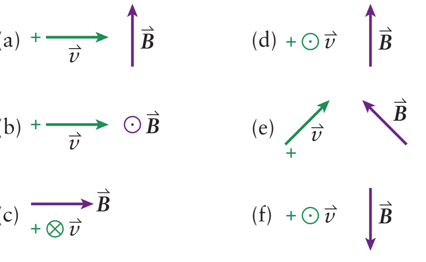


Figure 19-4 Problem 33

SOLVE

Part a) The force points out of the page.

Part b) The force points down.

Part c) The force points down.

Part d) The force points to the left.

Part e) The force points out of the page.

Part f) The force points to the right.

REFLECT

All of the answers would be reversed if the moving charge were negative.

19.34

SET UP

We are shown six scenarios related to a positive charge moving with a velocity $\vec{v}$ in a magnetic field $\vec{B}$. In each case we are given two out of three of the following: $\vec{v}$, $\vec{B}$, or the force $\vec{F}$. We can use the Lorentz force law, $\vec{F} = q\vec{v} \times \vec{B}$, and the right-hand rule to determine the direction of the missing vector.

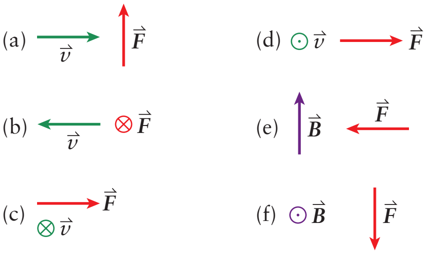


Figure 19-5 Problem 34

SOLVE

Part a) The magnetic field points into the page.

Part b) The magnetic field points up.

Part c) The magnetic field points up.

Part d) The magnetic field points down.

Part e) The velocity points out of the page.

Part f) The velocity points to the right.

REFLECT

All of the answers would be reversed if the moving charge were negative.

19.35

SET UP

A charge $q = 1$ C is moving at a speed $v = 1$ m/s. It is moving at an angle $\varphi = 45°$ relative to a magnetic field of magnitude $B = 1$ T. The magnitude of the Lorentz force on this charge is given by $F = qvB\sin(\theta)$.

SOLVE

$$F = qvB\sin(\varphi) = (1\text{ C})\left(1\frac{\text{m}}{\text{s}}\right)(1\text{ T})\sin(45°) = \boxed{0.7\text{ N}}$$

REFLECT

The fact that the charge is positive does not affect the magnitude of the force, only its direction.

19.36

SET UP

An electron is moving at a velocity $\vec{v}$ parallel to a magnetic field $\vec{B}$. The magnitude and direction of the force are given by the Lorentz force law, $\vec{F} = q\vec{v} \times \vec{B}$. Since the velocity and field are parallel, the cross product will be equal to zero, which means the force will be equal to zero.

SOLVE

The force is equal to $\boxed{\text{zero}}$ because the velocity is parallel to the magnetic field.

REFLECT

Determining the direction of the magnetic force first, rather than its magnitude, will save you a calculation if the cross product is zero.

19.37

SET UP

A proton ($q = 1.6 \times 10^{-19}$ C) travels with a speed of 18 m/s toward the top of the page through a uniform magnetic field of 2.0 T directed into the page. The magnitude and direction of the magnetic force acting on the proton is given by the Lorentz force law, $\vec{F} = q\vec{v} \times \vec{B}$, and the right hand rule.

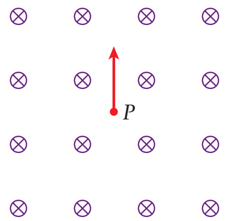


Figure 19-6 Problem 37

SOLVE

Magnitude:

$$F = qvB\sin(\varphi) = (1.6 \times 10^{-19}\ \text{C})\left(18\frac{\text{m}}{\text{s}}\right)(2.0\ \text{T})\sin(90°) = \boxed{5.8 \times 10^{-18}\ \text{N}}$$

Direction: The force points to the $\boxed{\text{left}}$.

REFLECT

This is the maximum value of the force since $\vec{v}$ and $\vec{B}$ are perpendicular to one another.

Get Help: Interactive Example – Motion in a Magnetic Field
P'Cast 19.1 – Potassium Ion

19.38

SET UP

A proton ($q = 1.60 \times 10^{-19}$ C) moving at a speed $v = 2.0 \times 10^{6}$ m/s perpendicularly to a uniform magnetic field $\vec{B}$ experiences a force of magnitude $F = 5.8 \times 10^{-13}$ N. The magnitude of the force acting on the proton is given by the Lorentz force law, $F = qvB\sin(\varphi)$. Rearranging this equation, we can solve for magnitude of the magnetic field B.

SOLVE

$$F = qvB\sin(\varphi)$$

$$B = \frac{F}{qv\sin(\varphi)} = \frac{5.8 \times 10^{-13}\ \text{N}}{(1.60 \times 10^{-19}\ \text{C})(2.0 \times 10^{6})\sin(90°)} = \boxed{1.8\ \text{T}}$$

REFLECT

The velocity, magnetic field, and magnetic force are all mutually perpendicular in this example.

19.39

SET UP

An electron ($q = -1.60 \times 10^{-19}$ C) moves with a velocity $\vec{v} = (1000\hat{x} - 4000\hat{y})$ in a magnetic field $\vec{B} = (-0.10\hat{x} + 0.20\hat{y})$, where all values are in SI units. The magnitude and direction of the magnetic force on the electron are given by $\vec{F} = q(\vec{v} \times \vec{B})$. We can accomplish both tasks at once by taking the determinant to calculate the cross product first and then multiplying it by q.

SOLVE

Cross product:

$$\vec{v} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1000 & -4000 & 0 \\ -0.10 & 0.20 & 0 \end{vmatrix} = \begin{vmatrix} -4000 & 0 \\ 0.20 & 0 \end{vmatrix}\hat{x} - \begin{vmatrix} 1000 & 0 \\ -0.10 & 0 \end{vmatrix}\hat{y} + \begin{vmatrix} 1000 & -4000 \\ -0.10 & 0.20 \end{vmatrix}\hat{z}$$

$$= 0 - 0 + ((1000)(0.20) - (-4000)(-0.10))\hat{z} = (-200\text{ T})\hat{z}$$

Lorentz force:

$$\vec{F} = q(\vec{v} \times \vec{B}) = (-1.60 \times 10^{-19}\text{ C})((-200\text{ T})\hat{z}) = \boxed{(3.20 \times 10^{-17}\text{ N})\hat{z}}$$

REFLECT

We can use the right-hand rule to check our answer. The velocity points toward the fourth quadrant of the *xy*-plane and the magnetic field points toward the second quadrant. The cross product $\vec{v} \times \vec{B}$ will point downward toward $-z$, as we saw, but we need to multiply by q, which is negative, to get the force. This means the force will point toward $+z$.

19.40

SET UP

A proton ($q = 1.60 \times 10^{-19}$ C, $m = 1.67 \times 10^{-27}$ kg) moves in a circle with a speed $v = 280$ m/s in a magnetic field of magnitude $B = 2.0$ T. The circular path of the proton is in a plane perpendicular to the magnetic field. The magnitude of the force due to the magnetic field acting on the proton is given by $F = qvB\sin(\varphi)$, where $\varphi = 90°$ in this case. Assuming that this is the only force acting on the proton, it will be equal to ma through Newton's second law. The acceleration of the proton will be $a_c = \frac{v^2}{R}$, where R is the orbital radius, because the proton is undergoing centripetal motion. Combining all of this information will allow us to calculate R.

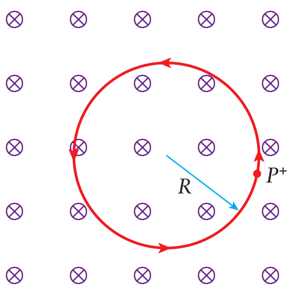


Figure 19-7 Problem 40

SOLVE

$$F = qvB\sin(\varphi) = \frac{mv^2}{R}$$

$$R = \frac{mv}{qB\sin(\varphi)} = \frac{(1.67 \times 10^{-27}\text{ kg})\left(280\frac{\text{m}}{\text{s}}\right)}{(1.60 \times 10^{-19}\text{ C})(2.0\text{ T})\sin(90°)} = \boxed{1.5 \times 10^{-6}\text{ m}}$$

REFLECT

The orbital radius will increase if the particle's mass or speed increases but will decrease if its charge or the magnitude of the external magnetic field decreases.

19.41

SET UP

An electron ($q = -1.6 \times 10^{-19}$ C) travels with a speed of 10^7 m/s in the xy-plane at an angle of 45 degrees above the $+x$-axis through a uniform magnetic field of 3.0 T directed towards $+y$. The magnitude and direction of the magnetic force acting on the proton is given by the Lorentz force law, $\vec{F} = q\vec{v} \times \vec{B}$, and the right hand rule.

SOLVE

Magnitude:

$$F = qvB\sin(\varphi) = (1.6 \times 10^{-19}\text{ C})\left(10^7\frac{\text{m}}{\text{s}}\right)(3.0\text{ T})\sin(45°) = \boxed{3.4 \times 10^{-12}\text{ N}}$$

Direction: The force points in the $\boxed{-z\text{ direction}}$.

REFLECT

Remember to take the sign of the charge into account when assigning the direction of the magnetic force.

Get Help: Interactive Example – Motion in a Magnetic Field
P'Cast 19.1 – Potassium Ion

19.42

SET UP

A uniform magnetic field of magnitude $B = 2.2$ T points along the $+z$-axis. Two scenarios are presented where a charged particle ($q = -1.2 \times 10^{-9}$ C) moves at a speed of $v = 1.0 \times 10^3$ m/s in the field. In the first scenario, the particle is moving in the yz-plane at an angle of 40 degrees relative to the z-axis. In the second case, the particle is moving in the xy-plane at an angle of 40 degrees relative to the x-axis. The magnitude of the force due to the magnetic field acting on the proton is given by $F = qvB\sin(\varphi)$, where φ is the angle between the velocity and the magnetic field vector. The direction of the force is given by the right-hand rule.

SOLVE

Part a)
Magnitude:

$$F = |q|vB\sin(\varphi) = |-1.2 \times 10^{-9}\text{ C}|\left(1.0 \times 10^3\frac{\text{m}}{\text{s}}\right)(2.2\text{ T})\sin(40°) = 1.7 \times 10^{-6}\text{ N}$$

The force on the particle has a magnitude of $\boxed{1.7 \times 10^{-6}\text{ N and points toward } -x}$.

Part b)
Magnitude:

$$F = |q|vB\sin(\varphi) = |-1.2 \times 10^{-9}\text{ C}|\left(1.0 \times 10^3\frac{\text{m}}{\text{s}}\right)(2.2\text{ T})\sin(90°) = 2.6 \times 10^{-6}\text{ N}$$

The force on the particle has a magnitude of $\boxed{2.6 \times 10^6 \text{ N and points at an angle of 130 degrees relative to the } +x\text{-axis}}$.

REFLECT

Remember that the angle φ in the magnitude of the Lorentz force is the angle between $\vec{v}$ and $\vec{B}$, which is not necessarily the angle that one of these vectors makes with an axis.

19.43

SET UP

A beam of protons is moving at a speed of 10^5 m/s in the $+z$ direction through a region with both an electric field and a magnetic field. The electric field has a magnitude of $E = 500$ V/m and points along the $-y$-axis. The force due to the electric field and the force due to the magnetic field acting on the protons must be equal in magnitude and opposite in direction if the protons are to continue traveling in a straight line. We can use the right-hand rule to determine the direction the magnetic field must point such that this is true.

SOLVE

Direction: The $\boxed{\text{magnetic field must point along the } +x\text{-axis}}$.
Magnitude:

$$F_E = F_B$$

$$qE = qvB\sin(\varphi)$$

$$B = \frac{E}{v\sin(\varphi)} = \frac{\left(500\frac{\text{V}}{\text{m}}\right)}{\left(10^5\frac{\text{m}}{\text{s}}\right)\sin(90°)} = \boxed{5 \times 10^{-3}\text{ T}}$$

REFLECT

In the absence of the magnetic field, the protons would have a constant speed in the z direction and be accelerating in the $-y$ direction. In the absence of the electric field, the protons would follow a circular path in the yz-plane.

19.44

SET UP

A beam of particles is deflected by a magnetic field ($B = 1.50$ T). The particles in the beam each have a charge of $q = -2$e and a kinetic energy of $K = 4.00 \times 10^{-13}$ J. The radius of curvature of the deflected beam is $R = 0.200$ m. The magnitude of the force due to the magnetic field acting on the particles is given by $F = qvB\sin(\varphi)$, where $\varphi = 90°$ in this case. Assuming that this is the only force acting on the particles, it will be equal to ma through Newton's second law. The acceleration of the particles will be $a_c = \frac{v^2}{R}$, where R is the orbital radius, because the particles are undergoing centripetal motion. We can use the definition of kinetic energy to rewrite the speed of the particles in terms of their kinetic

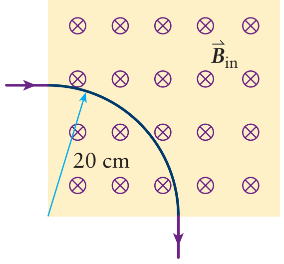


Figure 19-8 Problem 44

energy and mass m. Combining all of this information, we can solve for m. The radius of curvature is proportional to the particle's mass, so a more massive particle will sweep out a larger circle. A beam of positively charged particles would curve in the opposite direction as a beam of negatively charged particles.

SOLVE

Part a)

$$F = |q|vB\sin(\varphi) = \frac{mv^2}{R}$$

$$m = \frac{R|q|B\sin(\varphi)}{v} = \frac{R|q|B\sin(\varphi)}{\sqrt{\frac{2K}{m}}} = \frac{R|q|B\sin(\varphi)\sqrt{m}}{\sqrt{2K}}$$

$$\sqrt{m} = \frac{R|q|B\sin(\varphi)}{\sqrt{2K}}$$

$$m = \frac{[R|q|B\sin(\varphi)]^2}{2K} = \frac{[(0.200\text{ m})(2)(1.60 \times 10^{-19}\text{ C})(1.50\text{ T})\sin(90°)]^2}{2(4.00 \times 10^{-13}\text{ J})}$$

$$= \boxed{1.15 \times 10^{-26}\text{ kg}}$$

Part b)

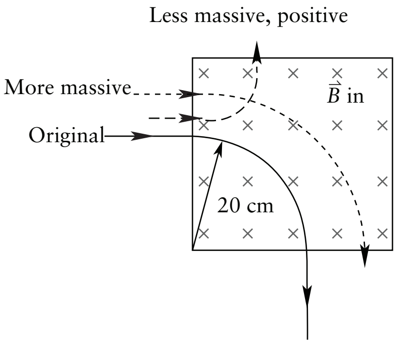


Figure 19-9 Problem 44

REFLECT

A mass of 1.15×10^{-26} kg is 6.94 u; the particles in the beam are most likely Li^{2-} ions.

19.45

SET UP

A straight segment of wire that is 0.350 m long carries a current of 1.40 A in a uniform magnetic field. The segment makes an angle of 53 degrees with $\vec{B}$. The magnitude of the force acting on the segment is 0.200 N. We can calculate the magnitude of the magnetic field from the expression for the magnetic force on a current-carrying wire, $F = ILB\sin(\varphi)$.

SOLVE

$$F = ILB\sin(\varphi)$$

$$B = \frac{F}{IL\sin(\varphi)} = \frac{0.200\text{ N}}{(1.40\text{ A})(0.350\text{ m})\sin(53°)} = \boxed{0.511\text{ T}}$$

REFLECT

The force acts in a direction perpendicular to both the current flow and the magnetic field.

19.46

SET UP

A straight segment of wire that is 0.5 m long carries a current of 2 A in a uniform magnetic field (B = 3 T). The segment makes an angle of 30 degrees with $\vec{B}$. We can calculate the magnitude of the magnetic force on the current-carrying wire from $F = iLB\sin(\varphi)$.

SOLVE

$$F = iLB\sin(\varphi) = (2\text{ A})(0.5\text{ m})(3\text{ T})\sin(30°) = \boxed{2\text{ N}}$$

REFLECT

We're only allowed one significant figure in our result.

19.47

SET UP

A current is flowing through a wire that is parallel to a magnetic field $\vec{B}$. The magnitude and direction of the force acting on the wire due to the magnetic field are given by $\vec{F} = i\vec{l} \times \vec{B}$. Since the direction of the current and the magnetic field are parallel, the cross product will be equal to zero, which means the force will be equal to zero.

SOLVE

The force is equal to $\boxed{\text{zero}}$ because the current is parallel to the magnetic field.

REFLECT

Checking the direction of the magnetic force first, rather than its magnitude, will save you a calculation if the cross product is zero.

19.48

SET UP

A wire of length $l = 0.50$ m carries a current $i = 8.0$ A toward the top of the page. The wire is situated in a uniform magnetic field of magnitude $B = 4.0$ T directed into the page. The magnitude and direction of the force acting on the wire due to the magnetic field are given by $\vec{F} = i\vec{l} \times \vec{B}$; we will need to use the right-hand rule to assign a direction for the cross product.

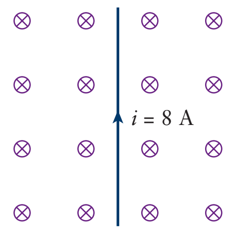


Figure 19-10 Problem 48

SOLVE

Direction: The $\boxed{\text{force will point to the left}}$.
Magnitude:

$$F = iLB\sin(\varphi) = (8.0\text{ A})(0.50\text{ m})(4.0\text{ T})\sin(90°) = \boxed{16\text{ N}}$$

REFLECT

The angle φ is the angle between the direction of the current and the magnetic field vector.

19.49

SET UP

A straight wire has a length $l = 0.80$ m and carries a current of $i = 2.0$ A. It is situated in a magnetic field; the maximum force acting on the wire due to the field is $F = 4.0$ N. The magnitude of the force is given by $F = iLB\sin(\varphi)$, where $\varphi = 90°$ for the maximum force. Rearranging this relationship, we can solve for the magnitude of the magnetic field.

SOLVE

$$F = iLB\sin(\varphi)$$

$$B = \frac{F}{iL\sin(\varphi)} = \frac{4.0\text{ N}}{(2.0\text{ A})(0.80\text{ m})\sin(90°)} = \boxed{2.5\text{ T}}$$

REFLECT

Without knowing the direction of the force, the magnetic field can point in any direction that is perpendicular to the direction of the current.

19.50

SET UP

A straight wire has a length $l = 1.5$ m and carries a current of i. It is situated in a magnetic field of magnitude $B = 1.8$ T. The maximum force acting on the wire due to the field is $F = 2.0$ N. The magnitude of the force is given by $F = iLB\sin(\varphi)$, where $\varphi = 90°$ for the maximum force. Rearranging this relationship, we can solve for the current in the wire.

SOLVE

$$F = iLB\sin(\varphi)$$

$$i = \frac{F}{BL\sin(\varphi)} = \frac{2.0\text{ N}}{(1.8\text{ T})(1.5\text{ m})\sin(90°)} = \boxed{0.74\text{ A}}$$

REFLECT

Without knowing the direction of the force, the current can point in any direction that is perpendicular to the magnetic field.

19.51

SET UP

A long wire of length l stretches along the y-axis and carries a current of $i = 1.0$ A in the $+y$ direction; we can represent the vector $i\vec{l}$ as $il\hat{y}$. This wire is in a uniform magnetic field $\vec{B} = (0.10\text{ T})\hat{x} - (0.20\text{ T})\hat{y} + (0.30\text{ T})\hat{z}$. The magnetic force on a current-carrying wire is $\vec{F} = i\vec{l} \times \vec{B}$. We can use the determinant form of the cross product to calculate the force on the wire per unit length, $\frac{\vec{F}}{l}$.

SOLVE

$$\vec{F} = i\vec{l} \times \vec{B} = (il\hat{y}) \times ((0.10\text{ T})\hat{x} - (0.20\text{ T})\hat{y} + (0.30\text{ T})\hat{z}) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & il & 0 \\ 0.10\text{ T} & -0.20\text{ T} & 0.30\text{ T} \end{vmatrix}$$

$$= (il(0.30\text{ T}))\hat{x} - 0\hat{y} + (il(-0.10\text{ T}))\hat{z}$$

$$\frac{\vec{F}}{l} = (i(0.30\text{ T}))\hat{x} + (i(-0.10\text{ T}))\hat{z} = ((1.0\text{ A})(0.30\text{ T}))\hat{x} + ((1.0\text{ A})(-0.10\text{ T}))\hat{z}$$

$$= \boxed{\left(0.30\frac{\text{N}}{\text{m}}\right)\hat{x} + \left(-0.10\frac{\text{N}}{\text{m}}\right)\hat{z}}$$

REFLECT

The force must be perpendicular to both $\vec{l}$ and $\vec{B}$; this means the dot products $\vec{l} \cdot \vec{F}$ and $\vec{B} \cdot \vec{F}$ must equal zero:

$$\vec{l} \cdot \vec{F} = (l\hat{y}) \cdot \left(\left(0.30\frac{\text{N}}{\text{m}}\right)l\hat{x} + \left(-0.10\frac{\text{N}}{\text{m}}\right)l\hat{z}\right) = 0$$

$$\vec{B} \cdot \vec{F} = ((0.10\text{ T})\hat{x} - (0.20\text{ T})\hat{y} + (0.30\text{ T})\hat{z}) \cdot \left(\left(0.30\frac{\text{N}}{\text{m}}\right)l\hat{x} + \left(-0.10\frac{\text{N}}{\text{m}}\right)l\hat{z}\right)$$

$$= \left(0.03l\frac{\text{T}\cdot\text{N}}{\text{m}}\right) + \left(-0.03l\frac{\text{T}\cdot\text{N}}{\text{m}}\right) = 0$$

19.52

SET UP

A segment of wire of length L makes an angle θ with the $-x$-axis located in the second quadrant. The wire carries a current i pointing toward $+x$ and $-y$. The wire is in the presence of a nonuniform magnetic field $\vec{B} = -\frac{B_0}{L}x\hat{z}$. The force on the wire due to the magnetic field is $\vec{F} = \int i\,d\vec{l} \times \vec{B}$, where $d\vec{l} = dx\hat{x} + dy\hat{y}$. Since the magnetic field is a function of x, we need to use trigonometry to write dy in terms of dx before integrating in the direction of the current. Recall that cross products obey the distributive property over addition and that $\hat{x} \times \hat{z} = -\hat{y}$ and $\hat{y} \times \hat{z} = \hat{x}$.

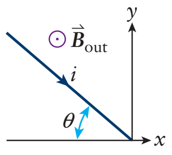


Figure 19-11 Problem 52

SOLVE

Part a)
Writing dy in terms of dx and θ:

$$y = -x\tan(\theta)$$

$$dy = -dx\tan(\theta)$$

Force:

$$\vec{F} = \int i d\vec{l} \times \vec{B} = i\int (dx\hat{x} + dy\hat{y}) \times \left(-\frac{B_0}{L}x\hat{z}\right) = \frac{-iB_0}{L}\int (dx\hat{x} + dy\hat{y}) \times (x\hat{z})$$

$$= \frac{-iB_0}{L}\left[\int xdx(\hat{x} \times \hat{z}) + \int xdy(\hat{y} \times \hat{z})\right]$$

$$= \frac{-iB_0}{L}\left[\int_{-L\cos(\theta)}^{0} xdx(-\hat{y}) + \int_{-L\cos(\theta)}^{0} x(-dx\tan(\theta))(\hat{x})\right]$$

$$= \frac{-iB_0}{L}\left[(-\hat{y})\left[\frac{1}{2}x^2\right]_{-L\cos(\theta)}^{0} + (\hat{x})(-\tan(\theta))\left[\frac{1}{2}x^2\right]_{-L\cos(\theta)}^{0}\right]$$

$$= \frac{-iB_0}{L}\left[(-\hat{y})\left[-\frac{1}{2}(-L\cos(\theta))^2\right] + (\hat{x})(-\tan(\theta))\left[-\frac{1}{2}(-L\cos(\theta))^2\right]\right]$$

$$= \frac{-iB_0}{L}\left[\left[\frac{1}{2}L^2\cos^2(\theta)\right](\hat{y}) + \left[\frac{1}{2}L^2\cos^2(\theta)\tan(\theta)\right](\hat{x})\right]$$

$$= \boxed{-\frac{iB_0L}{2}\left[[\sin(\theta)\cos(\theta)](\hat{x}) + [\cos^2(\theta)](\hat{y})\right]}$$

Part b) If the magnetic field were a function of y instead of x, then y would be the integration variable. This means you would have to find y as a function of x instead of x as a function of y. Because you must integrate in the direction of the current, the lower limit of integration would be $L\sin(\theta)$ and the upper limit of integration would be zero.

REFLECT

We can use the right-hand rule to double check the direction of our answer in part (a). The current points toward $+x$ and $-y$. In the second quadrant, the direction of the magnetic field always points toward $+z$ even though the magnitude is constantly changing. Invoking the right-hand rule, the force should point toward $-x$ and $-y$, as we calculated above. The direction of the force in part (b) would be the opposite of the direction from part (a) because the direction of the magnetic field changed from $+z$ to $-z$.

19.53

SET UP

A round loop of wire ($R = 0.10$ m) carries a current of $i_0 = 100$ A. The loop makes an angle of $\varphi = 30°$ with a magnetic field of $B = 0.244$ T. The angle is then changed to $\varphi = 10°$ and $\varphi = 50°$. The magnitude of the torque on the loop in each case is $\tau = i_0 AB\sin(\varphi)$.

SOLVE

$\varphi = 30°$:

$$\tau = i_0 AB\sin(\varphi) = (100\text{ A})(\pi(0.10\text{ m})^2)(0.244\text{ T})\sin(30°) = \boxed{0.383\text{ N}\cdot\text{m}}$$

$\varphi = 10°$:

$$\tau = i_0 AB\sin(\varphi) = (100\text{ A})(\pi(0.10\text{ m})^2)(0.244\text{ T})\sin(10°) = \boxed{0.133\text{ N}\cdot\text{m}}$$

$\varphi = 50°$:

$$\tau = i_0 AB\sin(\varphi) = (100\text{ A})(\pi(0.10\text{ m})^2)(0.244\text{ T})\sin(50°) = \boxed{0.587\text{ N}\cdot\text{m}}$$

REFLECT

The torque is a maximum when $\varphi = 90°$.

19.54

SET UP

A square loop of wire made up of $N = 100$ turns is 0.10 m on each side and is in the presence of a magnetic field. The wire carries a current of $i_0 = 2.82$ A. The maximum torque acting on the loop is $\tau_{\text{max}} = 0.045\text{ N}\cdot\text{m}$. The magnitude of the maximum torque on the square loop of wire is $\tau = Ni_0 AB\sin(\varphi)$, where $\varphi = 90°$. Rearranging this expression, we can calculate the magnitude of the magnetic field.

SOLVE

$$\tau_{\text{max}} = Ni_0 AB\sin(90°)$$

$$B = \frac{\tau_{\text{max}}}{Ni_0 A} = \frac{0.045\text{ N}\cdot\text{m}}{(100)(2.82\text{ A})[(0.10\text{ m})(0.10\text{ m})]} = \boxed{0.016\text{ T}}$$

REFLECT

The fact that the loop experiences a net torque means it will begin to rotate.

19.55

SET UP

A long, straight wire carries a current in the $+x$ direction. The direction of the magnetic field anywhere in space is given by the cross product term of the Biot–Savart law, $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{i d\vec{l} \times \hat{r}}{r^2}$, where $\hat{r}$ is the unit vector for a given point in space. We can use the right-hand rule to determine the direction of the magnetic field at points O, P, Q, and R.

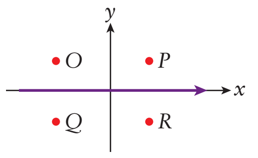


Figure 19-12 Problem 55

SOLVE

The $\boxed{\text{magnetic field at point } O \text{ points toward } +z}$.

The $\boxed{\text{magnetic field at point } P \text{ points toward } +z}$.

The $\boxed{\text{magnetic field at point } Q \text{ points toward } -z}$.

The $\boxed{\text{magnetic field at point } R \text{ points toward } -z}$.

REFLECT

The magnetic field at points O and P should have the same direction, as should the field at points Q and R, since they are located on the same side of the wire.

19.56

SET UP

A long, straight wire carries a current in the $+z$ direction out of the page. The direction of the magnetic field anywhere in space is given by the cross product term of the Biot–Savart law, $d\vec{B} = \frac{\mu_0}{4\pi}\frac{i d\vec{l} \times \hat{r}}{r^2}$, where $\hat{r}$ is the unit vector for a given point in space. We can use the right-hand rule to determine the direction of the magnetic field at points O, P, Q, and R.

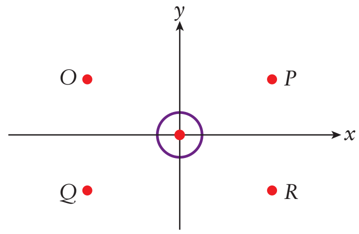


Figure 19-13 Problem 56

SOLVE

The $\boxed{\text{magnetic field at point } O \text{ points down and to the left}}$.

The $\boxed{\text{magnetic field at point } P \text{ points down and to the left}}$.

The $\boxed{\text{magnetic field at point } Q \text{ points down and to the right}}$.

The $\boxed{\text{magnetic field at point } R \text{ points down and to the right}}$.

REFLECT

The magnetic field circulates in a counterclockwise direction around the current-carrying wire.

19.57

SET UP

Two finite wires carry antiparallel currents: wire 1 has a length $L_1 = 0.10$ m with a current $i_1 = 4$ A directed up, wire 2 has a length $L_2 = 0.08$ m with a current $i_2 = 3$ A directed down (see figure). Point P is located $d_1 = 0.07$ m to the right of wire 1 and $d_2 = 0.03$ m to the left of wire 2 along an axis cutting through the center of each wire. The magnetic field at P is the vector sum of the magnetic fields due to wires 1 and 2 at P. The field due to each wire points into the page at P, so the magnitude of the field at P is simply the sum of the magnitudes of the magnetic fields due to wires 1 and 2. We can use the Biot–Savart law to find an expression for the magnitude of the magnetic field due to a finite wire of length L and a distance d from the P in general before plugging in our values.

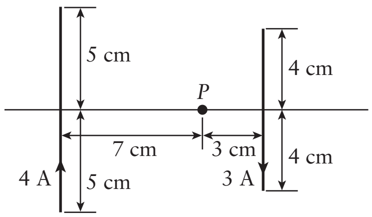


Figure 19-14 Problem 57

SOLVE

Magnetic field due to a finite wire:

$$B = \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{\mu_0}{4\pi}\frac{idl\sin(\varphi)}{r^2} = \frac{\mu_0 i}{4\pi}\int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{dl\left(\frac{d}{r}\right)}{r^2} = \frac{\mu_0 id}{4\pi}\int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{dl}{r^3} = \frac{\mu_0 id}{4\pi}\int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{dl}{(d^2 + l^2)^{\frac{3}{2}}}$$

$$= \frac{\mu_0 id}{4\pi}\left[\frac{l}{d^2\sqrt{d^2 + l^2}}\right]_{-\frac{L}{2}}^{\frac{L}{2}} = \frac{\mu_0 id}{4\pi}\left[\frac{\left(\frac{L}{2}\right)}{d^2\sqrt{d^2 + \left(\frac{L}{2}\right)^2}} - \frac{\left(-\frac{L}{2}\right)}{d^2\sqrt{d^2 + \left(-\frac{L}{2}\right)^2}}\right]$$

$$= \frac{\mu_0 i}{4\pi d}\left[\frac{L}{\sqrt{d^2 + \left(\frac{L}{2}\right)^2}}\right]$$

Magnitude of the magnetic field at point P:

$$B_P = B_1 + B_2 = \frac{\mu_0 i_1}{4\pi d_1}\left[\frac{L_1}{\sqrt{d_1^2 + \left(\frac{L_1}{2}\right)^2}}\right] + \frac{\mu_0 i_2}{4\pi d_2}\left[\frac{L_1}{\sqrt{d_2^2 + \left(\frac{L_2}{2}\right)^2}}\right]$$

$$= \frac{\mu_0}{4\pi}\left[\frac{L_1 i_1}{d_1\sqrt{d_1^2 + \left(\frac{L_1}{2}\right)^2}} + \frac{L_2 i_2}{d_2\sqrt{d_2^2 + \left(\frac{L_2}{2}\right)^2}}\right]$$

$$= \frac{\left(4\pi \times 10^{-7}\frac{\text{T}\cdot\text{m}}{\text{A}}\right)}{4\pi}\left[\frac{(0.10\text{ m})(4\text{ A})}{(0.07\text{ m})\sqrt{(0.07\text{ m})^2 + (0.05\text{ m})^2}} + \frac{(0.08\text{ m})(3\text{ A})}{(0.03\text{ m})\sqrt{(0.03\text{ m})^2 + (0.04\text{ m})^2}}\right]$$

$$= 2.26 \times 10^{-5}\text{ T}$$

The net magnetic field at point P has a magnitude of $\boxed{2.26 \times 10^{-5}\text{ T and points into the page}}$.

REFLECT

This is the same method outlined in Example 19-3 in the textbook.

19.58

SET UP

A long, straight wire is lies along the y-axis and carries a current i toward $+y$ (see figure). Point P is located on the $+x$-axis at a distance x to the right of the wire. We can use the Biot–Savart law, $d\vec{B} = \frac{\mu_0 i d\vec{y} \times \hat{r}}{4\pi r^2}$, to find an expression for the magnetic field due to this long, straight wire by integrating from $y = -\infty$ to $y = +\infty$.

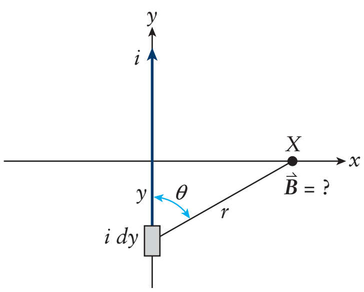


Figure 19-15 Problem 58

SOLVE

$$d\vec{B} = \frac{\mu_0 i d\vec{y} \times \hat{r}}{4\pi r^2} = \frac{\mu_0 i dy \sin(\theta)}{4\pi r^2}(-\hat{z})$$

$$\vec{B} = \int d\vec{B} = \int\left[-\frac{\mu_0 i \sin(\theta) dy}{4\pi r^2}\right]\hat{z} = -\left(\frac{\mu_0 i}{4\pi}\right)\int\left[\frac{\left(\frac{x}{r}\right)}{r^2}dy\right]\hat{z} = -\left(\frac{\mu_0 i x}{4\pi}\right)\int\left[\frac{1}{r^3}dy\right]\hat{z}$$

$$= -\left(\frac{\mu_0 i x}{4\pi}\right)\int_{-\infty}^{+\infty}\left[\frac{1}{(y^2+x^2)^{\frac{3}{2}}}dy\right]\hat{z} = -\left(\frac{\mu_0 i x}{4\pi}\right)\left[\frac{y}{x^2\sqrt{y^2+X^2}}\right]_{-\infty}^{+\infty}\hat{z}$$

$$= -\left(\frac{\mu_0 i}{4\pi x}\right)(2)\left[\frac{1}{\sqrt{1+\left(\frac{x}{y}\right)^2}}\right]_0^{+\infty}\hat{z} = -\left(\frac{\mu_0 i}{2\pi x}\right)[1-0]\hat{z} = \boxed{-\left(\frac{\mu_0 i}{2\pi x}\right)\hat{z}}$$

REFLECT

The next section (19-4: Magnetic Field and Current—Ampère's Law) will introduce a much easier method for arriving at this same answer.

19.59

SET UP

A long, straight wire lies along the y-axis and carries a current i toward $+y$ (see figure). Point P is located on the $+x$-axis at a distance x to the right of the wire. We can use the Biot–Savart law, $d\vec{B} = \dfrac{\mu_0 i d\vec{y} \times \hat{r}}{4\pi r^2}$, to find an expression for the magnetic field due to this long, straight wire by integrating from $\theta = 0$ (which corresponds to $y = -\infty$) to $\theta = \pi$ (which corresponds to $y = +\infty$). Before performing the integral, we first need to rewrite dy and r in terms of θ using trigonometry.

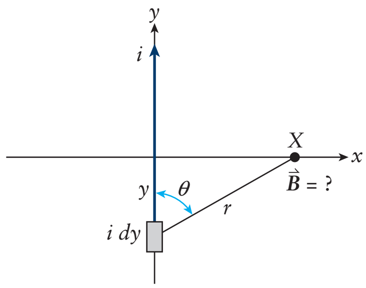


Figure 19-16 Problem 59

SOLVE

Writing dy and r in terms of θ:

$$\tan(\theta) = \frac{x}{-y}$$

$$y = -\frac{x}{\tan(\theta)} = -x\cot(\theta)$$

$$dy = x\csc^2(\theta)d\theta$$

$$\sin(\theta) = \frac{x}{r}$$

$$r = \frac{x}{\sin(\theta)} = x\csc(\theta)$$

Magnetic field at X:

$$d\vec{B} = \frac{\mu_0 i d\vec{y} \times \hat{r}}{4\pi r^2} = \frac{\mu_0 i dy \sin(\theta)}{4\pi r^2}(-\hat{z})$$

$$\vec{B} = \int d\vec{B} = \int\left[-\frac{\mu_0 i \sin(\theta)dy}{4\pi r^2}\right]\hat{z} = -\left(\frac{\mu_0 i}{4\pi}\right)\int_0^{\pi}\left[\frac{\sin(\theta)}{(x\csc(\theta))^2}(x\csc^2(\theta)d\theta)\right]\hat{z}$$

$$= -\left(\frac{\mu_0 i}{4\pi}\right)\int_0^{\pi}\left[\frac{\sin(\theta)}{x}d\theta\right]\hat{z} = -\left(\frac{\mu_0 i}{4\pi x}\right)[-\cos(\theta)]_0^{\pi}\,\hat{z}$$

$$= -\left(\frac{\mu_0 i}{4\pi x}\right)[-\cos(\pi) + \cos(\theta)]\hat{z} = -\left(\frac{\mu_0 i}{4\pi x}\right)[1+1]\hat{z} = \boxed{-\left(\frac{\mu_0 i}{2\pi x}\right)\hat{z}}$$

REFLECT

The answers to Problems 19.58 and 19.59 should be identical since it shouldn't matter whether we integrate with respect to y or θ.

19.60

SET UP

A wire lies along the y-axis and has a length $L = 0.20$ m and a current $i = 2$ A directed toward $+y$. Point P is located $d = 0.050$ m to the right of the wire along the x-axis, which cuts through the center of the wire. The magnetic field at P due to the wire points into the page at P. We can use the Biot–Savart law to find an expression for the magnitude of the magnetic field due to a finite wire of length L and a distance d from the center of the wire in general before plugging in our values.

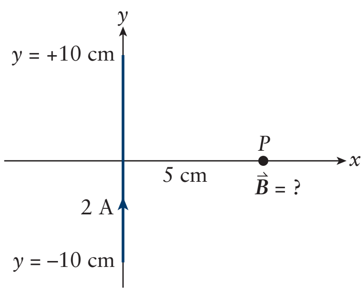


Figure 19-17 Problem 60

SOLVE

Magnitude of the magnetic field due to a finite wire:

$$B = \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{\mu_0}{4\pi}\frac{idl\sin(\varphi)}{r^2} = \frac{\mu_0 i}{4\pi}\int_{-\frac{L}{2}}^{\frac{L}{2}}\frac{dl\left(\frac{d}{r}\right)}{r^2} = \frac{\mu_0 id}{4\pi}\int_{-\frac{L}{2}}^{\frac{L}{2}}\frac{dl}{r^3} = \frac{\mu_0 id}{4\pi}\int_{-\frac{L}{2}}^{\frac{L}{2}}\frac{dl}{(d^2+l^2)^{\frac{3}{2}}}$$

$$= \frac{\mu_0 id}{4\pi}\left[\frac{l}{d^2\sqrt{d^2+l^2}}\right]_{-\frac{L}{2}}^{\frac{L}{2}} = \frac{\mu_0 id}{4\pi}\left[\frac{\left(\frac{L}{2}\right)}{d^2\sqrt{d^2+\left(\frac{L}{2}\right)^2}} - \frac{\left(-\frac{L}{2}\right)}{d^2\sqrt{d^2+\left(-\frac{L}{2}\right)^2}}\right]$$

$$= \frac{\mu_0 i}{4\pi d}\left[\frac{L}{\sqrt{d^2+\left(\frac{L}{2}\right)^2}}\right]$$

Magnetic field at point P:

$$\vec{B} = \frac{\left(4\pi \times 10^{-7}\,\frac{\mathrm{T}\cdot\mathrm{m}}{\mathrm{A}}\right)(2\ \mathrm{A})}{4\pi(0.05\ \mathrm{m})}\left[\frac{0.20\ \mathrm{m}}{\sqrt{(0.05\ \mathrm{m})^2 + \left(\frac{0.20\ \mathrm{m}}{2}\right)^2}}\right](-\hat{z}) = \boxed{-(7 \times 10^{-6}\ \mathrm{T})\hat{z}}$$

REFLECT

Solving the problem algebraically first gives us a general solution to this type of problem, which is much more useful and applicable than a strictly numerical solution.

19.61

SET UP

A wire carrying a current i is bent into two semicircular parts and two flat parts (see figure). The magnetic field at point C is the vector sum of the magnetic fields due to each of the four segments. Since the flat parts lie along the same axis as C, they do not contribute to the field at C. We can use the Biot–Savart law to find an expression for the magnitude of the magnetic field at the center of a semicircular wire for a general radius r; the radius will be constant for a given semicircular segment. The magnetic field at point C due to the closer, smaller semicircle of radius r points into the page (which we'll call $-z$), whereas the magnetic field at point C due to the farther, larger semicircle of radius R points out of the page (or $+z$).

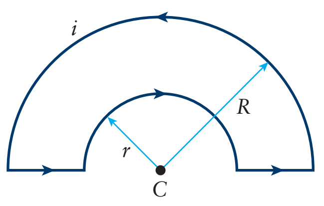


Figure 19-18 Problem 61

SOLVE

Magnitude of magnetic field due to a semicircular wire at its center:

$$B = \int_{\text{semicircle}} \frac{\mu_0}{4\pi}\frac{i|d\vec{l} \times \hat{r}|}{r^2} = \frac{\mu_0 i}{4\pi r^2}\int_{\text{semicircle}} dl = \frac{\mu_0 i}{4\pi r^2}(\pi r) = \frac{\mu_0 i}{2r}$$

Magnetic field at point C:

$$\vec{B} = -\left(\frac{\mu_0 i}{2r}\right)\hat{z} + \left(\frac{\mu_0 i}{2R}\right)\hat{z} = \frac{\mu_0 i}{2}\left(\frac{1}{R} - \frac{1}{r}\right)\hat{z} = \frac{\mu_0 i}{2}\left(\frac{r - R}{rR}\right)\hat{z} = \boxed{-\frac{\mu_0 i}{2}\left(\frac{R - r}{rR}\right)\hat{z}}$$

REFLECT

The magnetic field at point C points into the page because $R > r$. A tangent to a circle is always perpendicular to its radius so $|d\vec{l} \times \hat{r}| = |d\vec{l}\,||\hat{r}| = dl$. The integral of dl around the entire circle is equal to its circumference.

19.62

SET UP

A wire carrying a current i is bent into two circular parts and two flat parts (see figure). The magnetic field at point C is the vector sum of the magnetic fields due to each of the four segments. Since the flat parts lie along the same axis as C, they do not contribute to the field at C. We can use the Biot–Savart law to find an expression for the magnitude of the magnetic field at the center of a quarter-circular wire for a general radius r; the radius will be

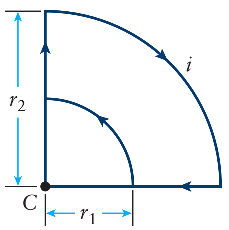


Figure 19-19 Problem 62

constant for a given circular segment. The magnetic field at point C due to the closer, smaller arc of radius r_1 points out of the page (which we'll call $+z$), whereas the magnetic field at point C due to the farther, larger arc of radius r_2 points into the page (or $-z$).

SOLVE

Magnitude of magnetic field due to a quarter-circular wire at its center:

$$B = \int_{\text{quarter-circle}} \frac{\mu_0}{4\pi} \frac{i|d\vec{l} \times \hat{r}|}{r^2} = \frac{\mu_0 i}{4\pi r^2}\int_{\text{quarter-circle}} dl = \frac{\mu_0 i}{4\pi r^2}\left(\frac{\pi r}{2}\right) = \frac{\mu_0 i}{8r}$$

Magnetic field at point C:

$$\vec{B} = \left(\frac{\mu_0 i}{8r_1}\right)\hat{z} - \left(\frac{\mu_0 i}{8r_2}\right)\hat{z} = \frac{\mu_0 i}{8}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)\hat{z} = \boxed{\frac{\mu_0 i}{8}\left(\frac{r_2 - r_1}{r_1 r_2}\right)\hat{z}}$$

REFLECT

The magnetic field at point C points out of the page because $r_2 > r_1$, which means $(r_2 - r_1)$ is positive. A tangent to a circle is always perpendicular to its radius so $|d\vec{l} \times \hat{r}| = |d\vec{l}||\hat{r}| = dl$. The integral of dl around the entire circle is equal to its circumference.

19.63

SET UP

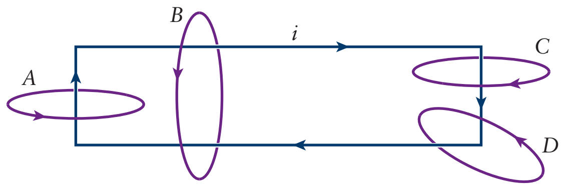


Figure 19-20 Problem 63

A rectangular loop of wire carrying a current i is shown. We can evaluate the integral $\oint \vec{B} \cdot d\vec{l}$ using Ampère's law, $\oint \vec{B} \cdot d\vec{l} = \mu_0 i_{\text{through}}$, where i_{through} is the net current flowing through the loop.

SOLVE

Part a)

$$\oint \vec{B} \cdot d\vec{l} = \boxed{\mu_0 i}$$

Part b)

$$\oint \vec{B} \cdot d\vec{l} = \mu_0(0) = \boxed{0}$$

Part c)

$$\oint \vec{B} \cdot d\vec{l} = \boxed{\mu_0 i}$$

Part d)

$$\oint \vec{B} \cdot d\vec{l} = \boxed{\mu_0 i}$$

REFLECT

The net current through loop B is zero because it encloses the top wire, where the current is moving to the right, and the bottom wire, where the current is moving to the left.

19.64

SET UP

We can use Ampère's law, $\oint \vec{B} \cdot d\vec{l} = \mu_0 i_{\text{through}}$, to derive an expression for the magnitude and direction of the magnetic field at a point that is a radial distance R from a long, straight, current-carrying wire. We will assume the wire carries a current i that points out of the page.

SOLVE

Magnitude:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i_{\text{through}}$$

$$B\oint dl = B(2\pi R) = \mu_0 i$$

$$\boxed{B = \frac{\mu_0 i}{2\pi R}}$$

Direction: The field is $\boxed{\text{counterclockwise}}$ when the current is coming out of the page straight toward you.

REFLECT

This is the expected expression for the magnetic field due to a long, straight, current-carrying wire.

19.65

SET UP

A current of 100 A passes through a wire that is 5 m from a window. We can draw an Ampèrian loop with a radius $r = 5$ m around the wire and apply Ampère's law to find the magnitude of the magnetic field at the window. Once we know the magnitude of the field due to the power line, we can compare it to the magnitude of Earth's magnetic field ($B_{\text{Earth}} = 0.5 \times 10^{-4}$ T).

SOLVE

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i_{\text{through}}$$

$$B\oint dl = B(2\pi r) = \mu_0 i$$

$$B = \frac{\mu_0 i}{2\pi r} = \frac{\left(4\pi \times 10^{-7} \frac{\text{T} \cdot \text{m}}{\text{A}}\right)(100\ \text{A})}{2\pi(5\ \text{m})} = \boxed{4 \times 10^{-6}\ \text{T}}$$

$$\frac{B}{B_{\text{Earth}}} = \frac{4 \times 10^{-6}\ \text{T}}{5 \times 10^{-5}\ \text{T}} = 0.08$$

$$\boxed{B = 0.08 B_{\text{Earth}}}$$

REFLECT

Learning the algebraic expression for the magnitude of a magnetic field due to a straight, current-carrying wire will prove handy.

19.66

SET UP

A solenoid with a winding density $n = 25$ turns/cm carries a current $i_0 = 0.025$ A. The magnitude of the magnetic field inside the solenoid is given by $B = \mu_0 n i_0$.

SOLVE

$$B = \mu_0 n i_0 = \left(4\pi \times 10^{-7}\ \frac{\text{T} \cdot \text{m}}{\text{A}}\right)\left(25 \frac{\text{turns}}{\text{cm}} \times \frac{100\ \text{cm}}{1\ \text{m}}\right)(0.025\ \text{A}) = \boxed{7.9 \times 10^{-5}\ \text{T}}$$

REFLECT

The magnetic field inside the solenoid points along the axis of the solenoid.

19.67

SET UP

We want to make a solenoid that has a diameter $D = 0.0350$ m and length $L = 0.160$ m. When the current through the wire is $i_0 = 3.00$ A, the magnetic field inside the solenoid should be $B = 0.0250$ T. In order to determine how much wire we need to create this solenoid, we first need to calculate the length of wire necessary to make one turn of the solenoid; this is equal to the circumference of the circle created. We can calculate the total number of turns from the expression for the magnitude of the magnetic field within the solenoid, $B = \mu_0 n i_0 = \mu_0 \left(\frac{N}{L}\right) i_0$, where N is the total number of turns. The total number of turns multiplied by the circumference of one turn will give the total amount of wire necessary to form the solenoid.

SOLVE

Circumference of one turn:

$$C = \pi D = \pi(0.0350\ \text{m}) = 0.110\ \text{m}$$

Total length of wire:

$$B = \mu_0 n i_0 = \mu_0\left(\frac{N}{L}\right)i_0$$

$$N = \frac{BL}{\mu_0 i_0} = \frac{(0.0250\text{ T})(0.160\text{ m})}{\left(4\pi \times 10^{-7}\frac{\text{T}\cdot\text{m}}{\text{A}}\right)(3.00\text{ A})} = 1061\text{ turns} \times \frac{0.110\text{ m}}{1\text{ turn}} = \boxed{117\text{ m}}$$

REFLECT

Our algebraic expression for the total number of turns seems reasonable as an increase in the required magnetic field at a constant current or an increase in the length of the solenoid would require an increase in the number of turns. A larger current would require fewer turns to create the same magnetic field.

19.68

SET UP

We can use Ampère's law to derive an expression for the magnetic field inside and outside of a toroid of radius $R = 25$ cm carrying a current $i = 1.25$ A. When calculating the magnetic field, we will use a circular Ampèrian loop of radius r. The total current through this loop is equal to Ni, where N is the total number of turns of wire, inside the toroid. The loop density n for the toroid is equal to $n = 2\frac{\text{turns}}{\text{cm}}$ and is related to N by $n = \frac{N}{2\pi R}$.

SOLVE

Number of loops:

$$n = \frac{N}{2\pi R}$$

$$N = 2\pi n R = 2\pi\left(2\frac{\text{turns}}{\text{cm}}\right)(25\text{ cm}) = 100\pi\text{ turns}$$

Inside the toroid:

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 i_{\text{through}}$$

$$B\oint dl = B(2\pi r) = \mu_0(Ni)$$

$$B = \frac{\mu_0 Ni}{2\pi r} = \frac{\left(4\pi \times 10^{-7}\frac{\text{T}\cdot\text{m}}{\text{A}}\right)(100\pi)(1.25\text{ A})}{2\pi r} = \frac{7.9\text{ T}\cdot\text{m}}{r}$$

The magnetic field inside the toroid has a magnitude of $\boxed{\frac{7.9\text{ T}\cdot\text{m}}{r}\text{, where } r \text{ is the}}$ $\boxed{\text{distance from the center of the toroid, and circulates around the axis of the toroid}}$.

Outside the toroid:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i_{\text{through}} = \mu_0(0)$$

$$\boxed{\vec{B} = 0}$$

REFLECT

The magnetic field outside the toroid should equal zero because the net current through the Ampèrian loop is equal to zero since each turn of wire pierces the plane of the loop twice—once going into the page and once coming out of the page.

19.69

SET UP

A coaxial cable is made of a solid inner conductor of radius R_i and a concentric outer conducting shell of radius R_o. Looking down the cable, the inner conductor carries a current of i coming out of the page, which we'll define as positive current, and the outer conductor carries a current of i going into the page, which we'll define as negative current. There is an insulator in between these two conductors. In order to find the magnetic field in all space, we will use an Amperian loop of radius r and split the problem into three regions—1) $r < R_\text{i}$, 2) $R_\text{i} \leq r < R_\text{o}$, and 3) $r \geq R_\text{o}$—and apply Ampere's law. The Amperian loop in region 1 encloses a fraction of the current distributed throughout the solid inner conductor; assuming the current is uniformly distributed, the enclosed current is equal to the ratio of the cross-sectional area of the Amperian loop to the cross-sectional area of the inner conductor multiplied by the total current. The Amperian loop in region 2 encloses the entire inner conductor. Finally, the net current enclosed by the loop in region 3 is zero, which means the magnetic field in that region is also zero.

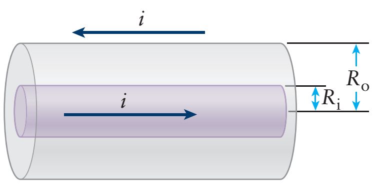


Figure 19-21 Problem 69

SOLVE

Looking at the cable head on:

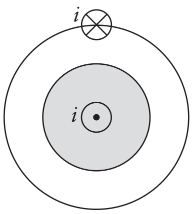


Figure 19-22 Problem 69

$r < R_\text{i}$:

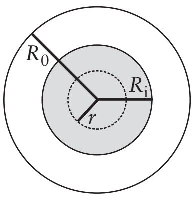


Figure 19-23 Problem 69

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i_{\text{through}}$$

$$B\oint dl = B(2\pi r) = \mu_0\left(\frac{\pi r^2}{\pi R_{\text{i}}^2}\right)i$$

$$\boxed{B = \frac{\mu_0 i r}{2\pi R_{\text{i}}^2}}$$

$R_{\text{i}} \le r < R_{\text{o}}$:

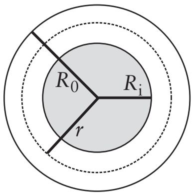


Figure 19-24 Problem 69

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i_{\text{through}}$$

$$B\oint dl = B(2\pi r) = \mu_0 i$$

$$\boxed{B = \frac{\mu_0 i}{2\pi r}}$$

$r \ge R_{\text{o}}$:

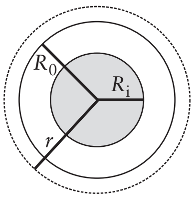


Figure 19-25 Problem 69

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i_{\text{through}} = \mu_0(i - i) = 0$$

$$\boxed{B = 0}$$

REFLECT

The field circulates counterclockwise in regions 1 and 2. As the Amperian loop increases in size in the region $r < R_{\text{i}}$, it encloses more and more current; therefore, the magnetic field should increase with r in this region. In the region $R_{\text{i}} \le r < R_{\text{o}}$, we've fully enclosed the inner current i, so the magnetic field should decrease as r increases.

Get Help: Interactive Example – Coaxial Cylindrical Conductors
P'Cast 19.4 – Magnetic Field Due to a Coaxial Cable

19.70

SET UP

A hollow cylinder carries a total tangential current i on its surface. We can use Ampère's law to find the magnetic field inside and outside the cylinder. The magnetic field outside the cylinder will be equal to zero because the current outside the cylinder is equal to zero. In order to derive an expression for the magnitude of the magnetic field inside the loop, we should draw a rectangular Ampèrian loop of length l and width w that encloses a portion of the current. The current passing through this loop is proportional to w, assuming the current is uniformly distributed over the surface of the cylinder. The direction of the magnetic field inside the cylinder is given by the right-hand rule.

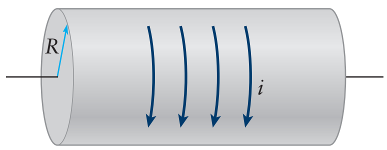


Figure 19-26 Problem 70

SOLVE

Part a)

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i_{\text{through}}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0(0)$$

$$\boxed{B = 0}$$

Part b)

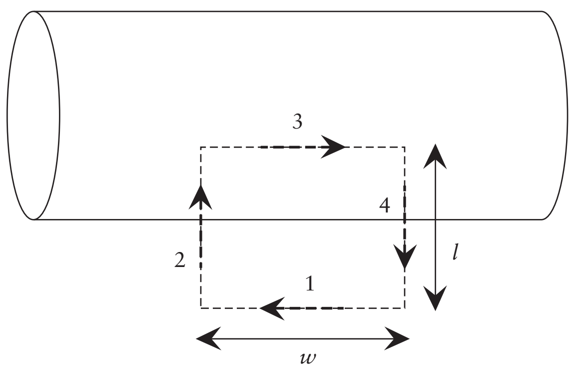


Figure 19-27 Problem 70

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i_{\text{through}}$$

$$\int_1 \vec{B} \cdot d\vec{l} + \int_2 \vec{B} \cdot d\vec{l} + \int_3 \vec{B} \cdot d\vec{l} + \int_4 \vec{B} \cdot d\vec{l} = \mu_0\left(\frac{i}{L}\right)(w)$$

$$0 + \int_2 Bdl\cos(90°) + \int_3 Bdl\cos(0°) + \int_4 Bdl\cos(90°) = \frac{\mu_0 iw}{L}$$

$$B\int_3 dl = \frac{\mu_0 iw}{L}$$

$$Bw = \frac{\mu_0 iw}{L}$$

$$B = \frac{\mu_0 i}{L}$$

The magnetic field inside the cylinder has a magnitude of $B = \frac{\mu_0 i}{L}$ $\boxed{\text{and points toward the right}}$.

REFLECT

Our expression for the magnetic field inside the cylinder has the correct dimensions for a magnetic field. The field is not only independent of the radius of the cylinder but is also constant for a given cylinder.

19.71

SET UP

A long, copper pipe that is 2.0 cm in diameter carries a current $i = 20$ A. We can calculate the magnitude of the magnetic field a distance $R = 2.2$ m away from the central axis of the pipe through Ampère's law. We will use a circular Ampèrian loop of radius R centered about the pipe.

SOLVE

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i_{\text{through}}$$

$$B(2\pi R) = \mu_0 i$$

$$B = \frac{\mu_0 i}{2\pi R} = \frac{\left(4\pi \times 10^{-7} \frac{\text{T} \cdot \text{m}}{\text{A}}\right)(20\text{ A})}{2\pi(2.2\text{ m})} = \boxed{1.8 \times 10^{-6}\text{ T}}$$

REFLECT

In the region outside the copper pipe, the pipe can be modeled as a current-carrying wire.

19.72

SET UP

We need to use the right-hand rule to show that parallel currents attract and opposite currents repel. In either case, we first need to find the direction of the magnetic field due to one wire at the location of the other wire. Once we know this, the direction of the force of the first wire on the second wire is given by the right-hand rule according to $\vec{F} = i\vec{l} \times \vec{B}$.

SOLVE

Parallel currents:

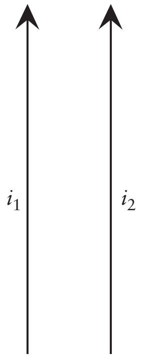


Figure 19-28 Problem 72

The direction of the magnetic field due to wire 1 located at the position of wire 2 is into the page. The force of wire 1 on wire 2 will be to the left. The direction of the magnetic field due to wire 2 at the position of wire 1 is out of the page. The force of wire 2 on wire 1 will be to the right. These forces will cause the wires to move toward one another, so they are attractive.
Opposite currents:

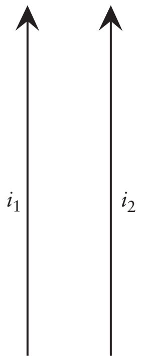


Figure 19-29 Problem 72

The direction of the magnetic field due to wire 1 located at the position of wire 2 is into the page. The force of wire 1 on wire 2 will be to the right. The direction of the magnetic field due to wire 2 at the position of wire 1 is into the page. The force of wire 2 on wire 1 will be to the left. These forces will cause the wires to move away from one another, so they are repulsive.

REFLECT

Remember that it is the magnetic field due to wire 1 (that is, the *external* field) that causes a force on wire 2, not the magnetic field due to wire 2.

19.73

SET UP

Wire 1 ($L_1 = 0.01$ m) carries a current of 2.00 A pointing north. Wire 2 carries a current of 3.60 A pointing south; wire 2 is $d = 1.40$ m to the right of wire 1. The magnitude of the force due to wire 2 on wire 1 is equal to $|i_1\vec{L}_1 \times \vec{B}_2|$. As a reminder, the magnitude of a magnetic field a distance d away from a straight

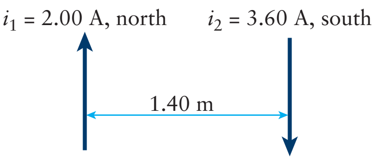


Figure 19-30 Problem 73

current-carrying wire is $\frac{\mu_0 i}{2\pi d}$. The direction of the force on wire 1 due to wire 2 is given by the right hand rule between the direction of i_1 and the direction of B_2. The magnetic field due to wire 2 points directly into the page at the location of wire 1, so the force will point to the left.

SOLVE

Magnitude:

$$F_{2\rightarrow 1} = |i_1\vec{L}_1 \times \vec{B}_2| = i_1 L_1 B_2 = i_1 L_1\left(\frac{\mu_0 i_2}{2\pi d}\right) = \frac{\mu_0 i_1 i_2 L_1}{2\pi d}$$

$$= \frac{\left(4\pi \times 10^{-7}\frac{\text{T}\cdot\text{m}}{\text{A}}\right)(2.00\text{ A})(3.60\ A)(0.01\text{ m})}{2\pi(1.40\text{ m})} = 1.03 \times 10^{-8}\text{ N}$$

The force on wire 1 due to wire 2 has a magnitude of $\boxed{1.03 \times 10^{-8}\text{ N and points to the left}}$.

REFLECT

In general, wires with antiparallel currents repel one another, while wires with parallel currents attract one another. You can prove this to yourself either by performing the similar calculations or by using Newton's third law.

Get Help: P'Cast 19.5 – Wires in a Computer

19.74

SET UP

A long, straight wire that carries a current $i_{\text{wire}} = 40$ A is to the left of a square loop of wire of length $l = 0.060$ m that is carrying a current of $i_{\text{loop}} = 20$ A. The left side of the loop is a distance $r_{\text{left}} = 0.020$ m from the wire; the right side of the loop is a distance $r_{\text{right}} = 0.040$ m from the wire. The net force due to the long, straight wire on the loop is equal to the vector sum of the forces acting on each segment of the loop due to the magnetic field created by the straight wire. The force on each segment of the wire can be calculated using $\vec{F} = i\vec{l} \times \vec{B}$ and the right-hand rule, where i is the current in the loop and $\vec{B}$ is the magnetic field due to the long wire at the location of the loop segment.

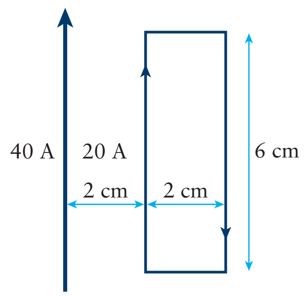


Figure 19-31 Problem 74

SOLVE

Forces on the top and bottom edges:
Since the top and bottom segments of the loop are equidistant from the long wire, the forces on these edges of the loop will cancel out from symmetry.

Force on left edge:

$$\vec{F}_{\text{left}} = i_{\text{loop}}\vec{l}_{\text{left}} \times \vec{B}$$

$$F_{\text{left}} = i_{\text{loop}} l_{\text{left}} B\sin(90°) = i_{\text{loop}} l\left(\frac{\mu_0 i_{\text{wire}}}{2\pi r_{\text{left}}}\right) = \frac{\mu_0 i_{\text{loop}} i_{\text{wire}} l}{2\pi r_{\text{left}}} = \frac{\left(4\pi \times 10^{-7}\frac{\text{T}\cdot\text{m}}{\text{A}}\right)(20\text{ A})(40\text{ A})(0.060\text{ m})}{2\pi(0.020\text{ m})}$$

$$= 4.8 \times 10^{-4}\text{ N}$$

The force on the left edge of the loop has a magnitude of 4.8×10^{-4} N and points to the left.
Force on the right edge:

$$\vec{F}_{\text{right}} = i_{\text{loop}} \vec{l}_{\text{right}} \times \vec{B}$$

$$F_{\text{right}} = i_{\text{loop}} l_{\text{right}} B \sin(90°) = i_{\text{loop}} l \left(\frac{\mu_0 i_{\text{wire}}}{2\pi r_{\text{right}}}\right) = \frac{\mu_0 i_{\text{loop}} i_{\text{wire}} l}{2\pi r_{\text{right}}}$$

$$= \frac{\left(4\pi \times 10^{-7} \frac{\text{T} \cdot \text{m}}{\text{A}}\right)(20\text{ A})(40\text{ A})(0.060\text{ m})}{2\pi(0.040\text{ m})} = 2.4 \times 10^{-4}\text{ N}$$

The force on the right edge of the loop has a magnitude of 2.4×10^{-4} N and points to the right.
Net force on the loop:

$$\sum \vec{F} = \vec{F}_{\text{left}} + \vec{F}_{\text{right}} + (\vec{F}_{\text{top}} + \vec{F}_{\text{bottom}})$$

$$= (4.8 \times 10^{-4}\text{ N})(-\hat{x}) + (2.4 \times 10^{-4}\text{ N})\hat{x} + (0) = -(2.4 \times 10^{-4}\text{ N})\hat{x}$$

The net force acting on the loop is $\boxed{2.4 \times 10^{-4}\text{ N to the left}}$.

REFLECT

Since the long, straight wire is closer to the left edge of the loop, the magnetic field and, therefore, the force on this piece of wire will be larger than on the right edge of the loop.

19.75

SET UP

The fasteners on overhead power lines have a length $l = 0.50$ m and run along the length of the power lines. There are two parallel high voltage wires a distance $d = 2.0$ m apart, each carrying a current of $i = 2500$ A. The magnitude of the force the fastener must withstand is given by $F = ilB\sin(90°)$, where B is the magnitude of the magnetic field due to one wire at the location of the other wire.

SOLVE

$$F = ilB\sin(90°) = il\left(\frac{\mu_0 i}{2\pi d}\right) = \frac{\mu_0 i^2 l}{2\pi d}$$

$$= \frac{\mu_0 i^2 l}{2\pi d} = \frac{\left(4\pi \times 10^{-7} \frac{\text{T} \cdot \text{m}}{\text{A}}\right)(2500\text{ A})^2(0.50\text{ m})}{2\pi(2.0\text{ m})} = \boxed{0.31\text{ N}}$$

REFLECT

The fastener should be able to withstand a force with the same magnitude as the force of gravity on the wire, which is most likely much larger than the magnetic force on each wire.

19.76

SET UP

A high-voltage power line of length $l = 32$ m carries a current $i = 95$ A. We are asked to find the force on the wire due to Earth's magnetic field, which has a magnitude $B = 0.5 \times 10^{-4}$ T and points from south to north at our location. We'll consider three different scenarios for the direction of the current in the wire—from north to south, from east to west, and from southeast to northwest at an angle of 30 degrees north of east. The magnitude of the force is given by $F = ilB\sin(\varphi)$, where φ is the angle between the current and the magnetic field, and the direction of the force is given by the right-hand rule. In order to determine whether these forces will have an appreciable effect on the power lines, we should compare their magnitudes with the magnitude of the force of gravity on the wire.

SOLVE

Part a)

$$F = ilB\,\sin(180°) = \boxed{0}$$

Part b)

$$F = ilB\,\sin(90°) = (95\text{ A})(32\text{ m})(0.50 \times 10^{-4}\text{ T}) = 0.15\text{ N}$$

The force on the wire in this case has a magnitude of $\boxed{\text{0.15 N and points vertically downward}}$.

Part c)

$$F = ilB\,\sin(60°) = (95\text{ A})(32\text{ m})(0.50 \times 10^{-4}\text{ T})\sin(60°) = 0.13\text{ N}$$

The force on the wire in this case has a magnitude of $\boxed{\text{0.13 N and points vertically upward}}$.

Part d) All these forces are negligible compared to the weight of high-voltage cables, so they $\boxed{\text{will not have an appreciable effect}}$.

REFLECT

The maximum force due to Earth's magnetic field occurs when the current is perpendicular to the magnetic field.

19.77

SET UP

A levitating train has a length of $l = 180$ m and a mass of $m = 1.00 \times 10^5$ kg. The current running along the length of the train is $i = 500 \times 10^3$ A; we will assume this current is being carried by a 180-m-long straight wire. Since the train is levitating, the net force in the vertical direction must be equal to zero, which means the magnitude of the force on the train due to the magnetic field is equal to the magnitude of the force of gravity on the train. Setting these equal to one another will allow us to calculate the magnitude of the magnetic field.

SOLVE

$$F_B = F_g$$

$$ilB\sin(90°) = mg$$

$$B = \frac{mg}{il} = \frac{(1.00 \times 10^5\text{ kg})\left(9.81\frac{\text{m}}{\text{s}^2}\right)}{(500 \times 10^3\text{ A})(180\text{ m})} = \boxed{0.0109\text{ T}}$$

REFLECT

We assumed the current was perpendicular to the magnetic field, which means the force due to the magnetic field was a maximum. If this were not the case, the magnetic field would need to be larger than this value.

19.78

SET UP

An electron ($m_e = 9.11 \times 10^{-31}$ kg) and a proton ($m_p = 1.67 \times 10^{-27}$ kg) have the same kinetic energy K upon entering a region of constant magnetic field of magnitude B. The velocity vectors associated with the electron and proton are perpendicular to the field. Assuming the force due to the field is the only force acting on the particles, they will each undergo uniform circular motion and sweep out circular paths of r_e and r_p. We can take a ratio of the expressions for Newton's second law for each particle and rearrange it to solve for the ratio $\frac{r_p}{r_e}$.

SOLVE

$$\frac{F_e}{F_p} = \frac{\left(\frac{m_e v_e^2}{r_e}\right)}{\left(\frac{m_p v_p^2}{r_p}\right)}$$

$$\frac{|-q|v_e B\sin(90°)}{|q|v_p B\sin(90°)} = \frac{v_e}{v_p} = \frac{\left(\frac{m_e v_e^2}{r_e}\right)}{\left(\frac{m_p v_p^2}{r_p}\right)}$$

$$\frac{r_e}{r_p} = \frac{m_p v_p}{m_e v_e}$$

$$\left(\frac{r_e}{r_p}\right)^2 = \left(\frac{m_p v_p}{m_e v_e}\right)^2 = \frac{m_p K}{m_e K} = \frac{m_p}{m_e}$$

$$\frac{r_p}{r_e} = \sqrt{\frac{m_p}{m_e}} = \sqrt{\frac{1.67 \times 10^{-27}\text{ kg}}{9.11 \times 10^{-31}\text{ kg}}} = \boxed{43}$$

REFLECT

The more massive particle should sweep out a larger circle.

19.79

SET UP

An electron enters a region of crossed electric and magnetic fields known as a velocity selector. The electric field has a magnitude of $E_{VS} = 90{,}000$ V/m and points down (towards $-y$), and the magnetic field has a magnitude of $B_{VS} = 0.0053$ T and points into the page. In the velocity selector, the net force on the electron in the y direction is zero since the electron travels in a straight line. Therefore, the magnitude of the force due to the electric field must equal the magnitude of the force due to the magnetic field. Setting these equal gives us an expression for the speed of the electron as it exits the selector. Immediately after the selector, the electron enters a region with a magnetic field $B_0 = 0.00242$ T. The electron then undergoes uniform circular motion in a radius $R = 0.04$ m due to the magnetic force acting on it. We can solve for the mass of the electron by applying Newton's second law once again.

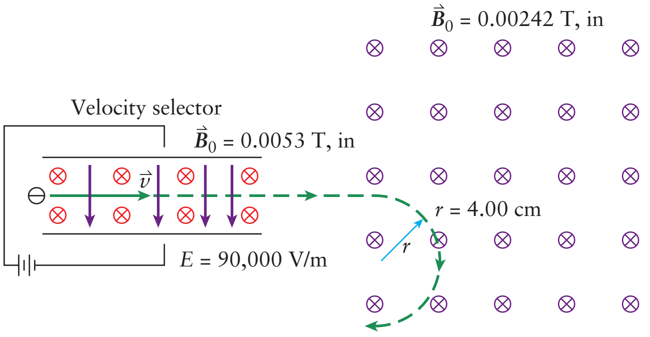


Figure 19-32 Problem 79

SOLVE

Free-body diagram of the electron in the velocity selector:

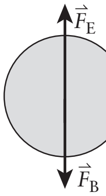


Figure 19-33 Problem 79

Speed of the electron after the velocity selector:

$$\sum F_y = F_E - F_B = ma_y = 0$$

$$qE_{VS} - qvB_{VS} = 0$$

$$v = \frac{E_{VS}}{B_{VS}} = \frac{\left(90{,}000\frac{\text{V}}{\text{m}}\right)}{0.0053\text{ T}} = 1.698 \times 10^7 \frac{\text{m}}{\text{s}}$$

Free-body diagram of the electron as soon as it enters the spectrometer:

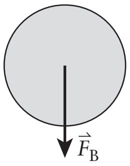


Figure 19-34 Problem 79

Mass of the electron:

$$\sum F_y = -F_B = ma_y = m\left(-\frac{v^2}{R}\right)$$

$$qvB_0 = \frac{mv^2}{R}$$

$$m = \frac{RqB_0}{v} = \frac{RqB_0B_{VS}}{E_{VS}}$$

$$= \frac{(0.040 \text{ m})(1.6 \times 10^{-19} \text{ C})(0.00242 \text{ T})(0.0053 \text{ T})}{\left(90{,}000\frac{\text{V}}{\text{m}}\right)} = \boxed{9.12 \times 10^{-31} \text{ kg}}$$

REFLECT

The accepted mass of the electron (to three significant figures) is 9.11×10^{-31} kg, so our value is reasonable.

Get Help: Interactive Example – Motion in a Magnetic Field
P'Cast 19.1 – Potassium Ion

19.80

SET UP

The largest magnetic field produced was $B = 60$ T. We can calculate the maximum acceleration this field could produce on a sodium ion ($m = 3.8 \times 10^{-26}$ kg) traveling in the aorta at a speed around $v = 0.50$ m/s from Newton's second law and the Lorentz force law. In order to determine if it would be dangerous to expose workers to this magnetic field, we can compare our calculated acceleration to the acceleration due to gravity.

SOLVE

$$F = |q|vB\sin(90°) = ma$$

$$a = \frac{|q|vB}{m} = \frac{(1.60 \times 10^{-19} \text{ C})\left(0.50\frac{\text{m}}{\text{s}}\right)(60 \text{ T})}{3.8 \times 10^{-26} \text{ kg}} = \boxed{1.3 \times 10^{8}\frac{\text{m}}{\text{s}^2}}$$

Yes, this acceleration would be $\boxed{\text{highly dangerous}}$ for workers.

REFLECT

This acceleration is nearly 13 million *g*'s!

19.81

SET UP

A bolt of lightning can transfer $\Delta q = 10\text{ C}$ in $\Delta t = 2.0 \times 10^{-6}\text{ s}$. We will model the bolt of lightning as a long, straight, current-carrying wire. The expression for the magnitude of the magnetic field in this case is $B = \frac{\mu_0 i}{2\pi r}$. We can use this expression to calculate the magnitude of the field due to the lightning bolt at distances of 1.0 m and 1.0×10^3 m from the bolt and compare them to the field created by a current of 10 A in a long, straight wire at the same distances. Finally, we can rearrange the expression to solve for the distance away from the wire that yields the same magnitude field as the field that is 1.0×10^3 m from the lightning bolt.

SOLVE

Part a)
Magnetic field 1.0 m away from the lightning strike:

$$B = \frac{\mu_0 i}{2\pi r} = \frac{\mu_0\left(\frac{\Delta q}{\Delta t}\right)}{2\pi r} = \frac{\mu_0 \Delta q}{2\pi r \Delta t} = \frac{\left(4\pi \times 10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}\right)(10\text{ C})}{2\pi(1.0\text{ m})(2.0 \times 10^{-6}\text{ s})} = \boxed{1.0\text{ T}}$$

This is $\boxed{\text{20,000 times larger than Earth's magnetic field}}$.

Magnetic field 1.0×10^3 m away from the lightning strike:

$$B = \frac{\mu_0 i}{2\pi r} = \frac{\mu_0\left(\frac{\Delta q}{\Delta t}\right)}{2\pi r} = \frac{\mu_0 \Delta q}{2\pi r \Delta t} = \frac{\left(4\pi \times 10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}\right)(10\text{ C})}{2\pi(1.0 \times 10^3\text{ m})(2.0 \times 10^{-6}\text{ s})} = \boxed{1.0 \times 10^{-3}\text{ T}}$$

This is $\boxed{\text{20 times larger than Earth's magnetic field}}$.

Part b)
Magnetic field 1.0 m away from the wire:

$$B = \frac{\mu_0 i}{2\pi r} = \frac{\left(4\pi \times 10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}\right)(10\text{ A})}{2\pi(1.0\text{ m})} = \boxed{2.0 \times 10^{-6}\text{ T}}$$

The $\boxed{\text{field due to the lightning bolt at the same distance is 500,000 times larger}}$ than this field.

Magnetic field 1.0×10^3 m away from the wire:

$$B = \frac{\mu_0 i}{2\pi r} = \frac{\left(4\pi \times 10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}\right)(10\text{ A})}{2\pi(1.0 \times 10^3\text{ m})} = \boxed{2.0 \times 10^{-9}\text{ T}}$$

The $\boxed{\text{field due to the lightning bolt at the same distance is 500,000 times larger}}$ than this field.

Part c)

$$B = \frac{\mu_0 i}{2\pi r}$$

$$r = \frac{\mu_0 i}{2\pi B} = \frac{\left(4\pi \times 10^{-7}\,\frac{\mathrm{T \cdot m}}{\mathrm{A}}\right)(10\ \mathrm{A})}{2\pi(1.0 \times 10^{-3}\ \mathrm{T})} = \boxed{2.0 \times 10^{-3}\ \mathrm{m} = 2.0\ \mathrm{mm}}$$

REFLECT

We would expect the magnetic field due to a lightning bolt to be much larger than the field due to a common wire.

19.82

SET UP

A long, straight wire carries a current $i = 8$ A toward the top of the page. The magnitude of the magnetic field due to this wire at a distance $R = 8 \times 10^{-2}$ m from the wire is given by $B = \frac{\mu_0 i}{2\pi R}$; the direction of the field at this point is given by the right-hand rule.

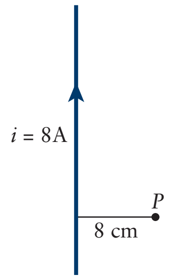


Figure 19-35 Problem 82

SOLVE

$$B = \frac{\mu_0 i}{2\pi R} = \frac{\left(4\pi \times 10^{-7}\,\frac{\mathrm{T \cdot m}}{\mathrm{A}}\right)(8\ \mathrm{A})}{2\pi(8 \times 10^{-2}\ \mathrm{m})} = 2 \times 10^{-5}\ \mathrm{T}$$

The magnetic field at P has a magnitude of $\boxed{2 \times 10^{-5}\ \text{T and points into the page}}$.

REFLECT

The magnetic field for a long, straight wire carrying a uniform current has the same magnitude everywhere in space for a given value of R.

19.83

SET UP

A long, straight wire carries a current i toward the top of the page. A charged particle traveling at a speed v parallel to the wire experiences a force of magnitude $F_1 = 0.8$ N when it is a distance $r_1 = d$ from the wire. The magnitude of the force on the moving charge due to the magnetic field is $F = qvB$ since the velocity is perpendicular to the magnetic field. We can set up a ratio of the force on the particle at $r_2 = 2d$ to the force at $r_1 = d$ in order to calculate the force at r_2, F_2. The magnitude of the magnetic field as a function of distance from the wire is $B = \frac{\mu_0 i}{2\pi r}$.

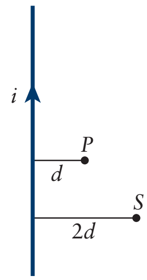


Figure 19-36 Problem 83

SOLVE

$$\frac{F_2}{F_1} = \frac{qvB_2}{qvB_1} = \frac{\left(\dfrac{\mu_0 i}{2\pi(2d)}\right)}{\left(\dfrac{\mu_0 i}{2\pi d}\right)} = \frac{1}{2}$$

$$F_2 = \frac{F_1}{2} = \frac{(0.8\text{ N})}{2} = \boxed{0.4\text{ N}}$$

REFLECT

The force should be smaller the farther the charged particle is from the wire.

19.84

SET UP

Two long, straight wires are parallel to the x-axis at $y = 2.5$ cm and $y = -2.5$ cm. Each wire carries a current of $i = 16$ A. We can use superposition to calculate the magnetic field at $y = 0$, $y = 1$ cm, and $y = 4$ cm. The magnetic field due to a long wire as a function of distance from the wire is given by $B = \dfrac{\mu_0 i}{2\pi r}$ and the right-hand rule.

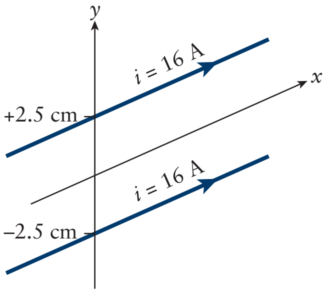


Figure 19-37 Problem 84

SOLVE

Part a)

$$\vec{B} = \vec{B}_{\text{top}} + \vec{B}_{\text{bottom}} = \left(\frac{\mu_0 i}{2\pi r_{\text{top}}}\right)(-\hat{z}) + \left(\frac{\mu_0 i}{2\pi r_{\text{bottom}}}\right)\hat{z}$$

$$= \left(\frac{\mu_0 i}{2\pi(2.5 \times 10^{-2}\text{ m})}\right)(-\hat{z}) + \left(\frac{\mu_0 i}{2\pi(2.5 \times 10^{-2}\text{ m})}\right)\hat{z} = \boxed{0}$$

Part b)

$$\vec{B} = \vec{B}_{\text{top}} + \vec{B}_{\text{bottom}} = \left(\frac{\mu_0 i}{2\pi r_{\text{top}}}\right)(-\hat{z}) + \left(\frac{\mu_0 i}{2\pi r_{\text{bottom}}}\right)\hat{z} = \left(\frac{\mu_0 i}{2\pi}\right)\left[-\frac{1}{r_{\text{top}}} + \frac{1}{r_{\text{bottom}}}\right]\hat{z}$$

$$= \left(\frac{\left(4\pi \times 10^{-7}\,\frac{\text{T}\cdot\text{m}}{\text{A}}\right)(16\text{ A})}{2\pi}\right)\left[-\frac{1}{(1.5 \times 10^{-2}\text{ m})} + \frac{1}{(3.5 \times 10^{-2}\text{ m})}\right]\hat{z} = \boxed{-(1.2 \times 10^{-4}\text{ T})\hat{z}}$$

Part c)

$$\vec{B} = \vec{B}_{\text{top}} + \vec{B}_{\text{bottom}} = \left(\frac{\mu_0 i}{2\pi r_{\text{top}}}\right)\hat{z} + \left(\frac{\mu_0 i}{2\pi r_{\text{bottom}}}\right)\hat{z} = \left(\frac{\mu_0 i}{2\pi}\right)\left[\frac{1}{r_{\text{top}}} + \frac{1}{r_{\text{bottom}}}\right]\hat{z}$$

$$= \left(\frac{\left(4\pi \times 10^{-7}\,\frac{\text{T}\cdot\text{m}}{\text{A}}\right)(16\text{ A})}{2\pi}\right)\left[\frac{1}{(1.5 \times 10^{-2}\text{ m})} + \frac{1}{(6.5 \times 10^{-2}\text{ m})}\right]\hat{z} = \boxed{(2.6 \times 10^{-4}\text{ T})\hat{z}}$$

REFLECT

The magnetic field at $y = 0$ must be zero due to symmetry since the top and bottom wires are equidistant to $y = 0$. The field at $y = 1$ cm should point toward $-z$ because it is closer to the top wire than the bottom. The field at $y = 4$ cm should point toward $+z$ since the magnetic field due to both wires points in that direction at that location.

19.85

SET UP

A thin, nonconducting ring of radius R and total charge Q lies with its center at the origin of xy-plane. The ring is spinning at an angular speed of ω about its center. This spinning charge will generate a magnetic field at point P, which is a vertical distance Z above the xy-plane and a distance $r = \sqrt{Z^2 + R^2}$ from the ring, that we can determine through the Biot–Savart law. Since the ring is centered about the z-axis and is a constant distance R from the axis, we can convert the integral from one over dl to one over $d\theta$ through the definition of the arc length. The current is equal to the change in charge divided by the change in time. For the spinning ring, it completes one full revolution in $\frac{2\pi}{\omega}$. In this time period the entire charge of Q has gone around, which means the current is $i = \frac{Q}{\left(\frac{2\pi}{\omega}\right)}$. The magnetic field at point P can only point along the $+z$-axis due to symmetry, so we need to multiply our result by $\cos(\phi)$, where ϕ is the angle between the magnetic field and the z-axis.

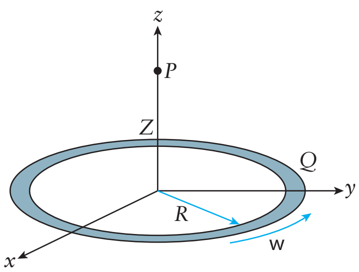


Figure 19-38 Problem 85

SOLVE

Current:

$$i = \frac{\Delta Q}{\Delta T} = \frac{Q}{\left(\frac{2\pi}{\omega}\right)} = \frac{Q\omega}{2\pi}$$

Magnitude of magnetic field at P:

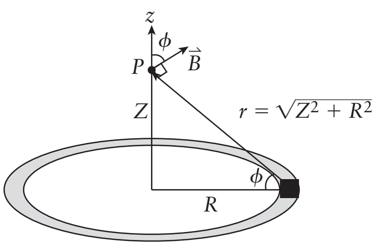


Figure 19-39 Problem 85

$$B_P = \int \frac{\mu_0 i |d\vec{l} \times \hat{r}|}{4\pi r^2} = \frac{\mu_0 i}{4\pi}\int \frac{dl}{r^2} = \frac{\mu_0 i}{4\pi}\int_0^{2\pi} \frac{R\,d\theta}{(Z^2 + R^2)} = \frac{\mu_0 i R}{4\pi(Z^2 + R^2)}\int_0^{2\pi} d\theta = \frac{\mu_0 i R}{4\pi(Z^2 + R^2)}[\theta]_0^{2\pi}$$

$$= \frac{\mu_0 i R}{4\pi(Z^2 + R^2)}[2\pi] = \frac{\mu_0\left(\frac{Q\omega}{2\pi}\right)R}{2(Z^2 + R^2)} = \frac{\mu_0 Q\omega R}{4\pi(Z^2 + R^2)}$$

z component of magnetic field at P:

$$B_{P,z} = B_P \cos(\phi) = \left(\frac{\mu_0 Q\omega R}{4\pi(Z^2 + R^2)}\right)\left(\frac{R}{\sqrt{Z^2 + R^2}}\right) = \frac{\mu_0 Q\omega R^2}{4\pi(Z^2 + R^2)^{\frac{3}{2}}}$$

The magnetic field at point P has a magnitude of $\boxed{\dfrac{\mu_0 Q\omega R^2}{4\pi(Z^2 + R^2)^{\frac{3}{2}}} \text{ and points towards } +z}$.

REFLECT

The right-hand rule for a loop of current shows that the magnetic field generated by the current points towards $+z$ within the loop, which is consistent with our answer.

19.86

SET UP

A thin disk of radius $R = 0.20$ m that has a total charge $Q = 200 \times 10^{-6}$ C lies with its center at the origin of the xy-plane. The ring is spinning at an angular speed of $\omega = 2.5$ rev/s about its center. This spinning charge will generate a magnetic field at point P, which is a vertical distance $z = 0.30$ m above the xy-plane. The current is equal to the change in charge divided by the change in time. For the spinning disk, it completes one full revolution in $\frac{2\pi}{\omega}$. In this time period the entire charge Q has gone around, which means the current is $i = \frac{Q}{\left(\frac{2\pi}{\omega}\right)}$. We can split the disk up into concentric thin rings and perform an integral over r from 0 to R to calculate the total magnetic field at P. We will use the results from Section 19-3 of the text to help get us started.

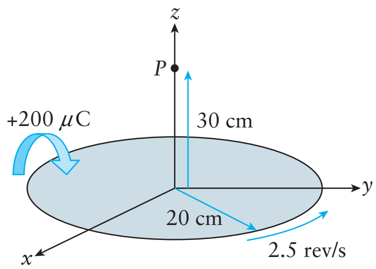


Figure 19-39 Problem 86

SOLVE

Converting ω:

$$\omega = 2.5\frac{\text{rev}}{\text{s}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} = 16\frac{\text{rad}}{\text{s}}$$

Infinitesimal current:

$$di = \frac{dq}{T} = \sigma dA\left(\frac{\omega}{2\pi}\right) = \left(\frac{Q}{\pi R^2}\right)(2\pi r dr)\left(\frac{\omega}{2\pi}\right) = \frac{Q\omega r dr}{\pi R^2}$$

Infinitesimal magnetic field in the z direction:

$$dB_z = \frac{\mu_0}{2}\frac{(di)r^2}{(r^2 + z^2)^{\frac{3}{2}}} = \frac{\mu_0 r^2}{2(r^2 + z^2)^{\frac{3}{2}}}\left(\frac{Q\omega r dr}{\pi R^2}\right)$$

Magnetic field in the z direction:

$$B_z = \int dB_z = \int_0^R \frac{\mu_0 r^2}{2(r^2+z^2)^{\frac{3}{2}}}\left(\frac{Q\omega r dr}{\pi R^2}\right) = \frac{\mu_0 Q\omega}{2\pi R^2}\int_0^R \frac{r^3 dr}{(r^2+z^2)^{\frac{3}{2}}}$$

$$= \frac{\mu_0 Q\omega}{2\pi R^2}\left[2\sqrt{(r^2+z^2)} - \frac{r^2}{\sqrt{r^2+z^2}}\right]_0^R = \frac{\mu_0 Q\omega}{2\pi R^2}\left[2\sqrt{(R^2+z^2)} - \frac{R^2}{\sqrt{R^2+z^2}} - 2z\right]$$

$$= \frac{\mu_0 Q\omega}{2\pi R^2}\left[\frac{2R^2+2z^2-R^2-2z\sqrt{R^2+z^2}}{\sqrt{R^2+z^2}}\right] = \frac{\mu_0 Q\omega}{2\pi R^2}\left[\frac{R^2+2z^2-2z\sqrt{R^2+z^2}}{\sqrt{R^2+z^2}}\right]$$

$$= \frac{\left(4\pi \times 10^{-7}\,\frac{\mathrm{T\cdot m}}{\mathrm{A}}\right)(200\times 10^{-6}\ \mathrm{C})\left(16\frac{\mathrm{rad}}{\mathrm{s}}\right)}{2\pi(0.20\ \mathrm{m})^2}$$

$$\left[\frac{(0.20\ \mathrm{m})^2 + 2(0.30\ \mathrm{m})^2 - 2(0.30\ \mathrm{m})\sqrt{(0.20\ \mathrm{m})^2+(0.30\ \mathrm{m})^2}}{\sqrt{(0.20\ \mathrm{m})^2+(0.30\ \mathrm{m})^2}}\right]$$

$$= \boxed{1.6\times 10^{-10}\ \mathrm{T}}$$

REFLECT

The magnetic field can only point along the z-axis due to symmetry.

19.87

SET UP

A coil of wire has a radius $R = 0.075$ m and consists of $N = 250$ windings. The z component of the magnetic field for a coil of wire with N windings carrying a current i is $B_z = \frac{\mu_0 N i R^2}{2(R^2+z^2)^{\frac{3}{2}}}$, where z is the perpendicular distance measured from the center of the coil. Knowing that the magnitude of the magnetic field at $z = 0.030$ m is $B = 0.50$ T, we can calculate the current in the coil. Once we have the current, we can calculate the magnitude of the field at $z = 0$. One way of increasing the field for a given current is to increase the number of windings in the loop.

SOLVE

Part a)

$$B_z = \frac{\mu_0 N i R^2}{2(R^2+z^2)^{\frac{3}{2}}}$$

$$i = \frac{2(R^2+z^2)^{\frac{3}{2}} B_z}{\mu_0 N R^2} = \frac{2\left((0.075\ \mathrm{m})^2 + (0.030\ \mathrm{m})^2\right)^{\frac{3}{2}}(0.50\ \mathrm{T})}{\left(4\pi\times 10^{-7}\,\frac{\mathrm{T\cdot m}}{\mathrm{A}}\right)(250)(0.075\ \mathrm{m})^2} = 300\ \mathrm{A}$$

Part b)

$$B_z = \frac{\mu_0 N i R^2}{2(R^2 + z^2)^{\frac{3}{2}}} = \frac{\mu_0 N i R^2}{2\left(R^2 + (0)^2\right)^{\frac{3}{2}}} = \frac{\mu_0 N i R^2}{2R^3} = \frac{\mu_0 N i}{2R}$$

$$= \frac{\mu_0 N i}{2R} = \frac{\left(4\pi \times 10^{-7}\,\frac{\text{T}\cdot\text{m}}{\text{A}}\right)(250)(300\text{ A})}{2(0.075\text{ m})} = \boxed{0.62\text{ T}}$$

Part c) The easiest way to achieve the same field with less current is to increase the number of windings, N.

REFLECT

The magnetic field must point along the central axis of the loop.

19.88

SET UP

A steady current of $i = 100$ A passes through a wire ($m = 0.040$ kg, $l = 0.80$ m) that can slide without friction on two parallel, horizontal conducting rails. A uniform magnetic field of $B = 1.2$ T is directed into the page. The magnitude and direction of the force acting on the wire due to the magnetic field are given by $\vec{F}_B = i\vec{l} \times \vec{B}$. Assuming this is the only force acting on the wire in the horizontal direction, the wire will undergo constant acceleration, so we can use the constant acceleration equations to determine the length of the rails necessary for the wire to achieve a final speed of $v_f = 200$ m/s. We can use $\vec{F}_B = i\vec{l} \times \vec{B}$ along with the right-hand rule to determine how our answers would change if $\vec{B}$ were to point out of the page or toward the top of the page.

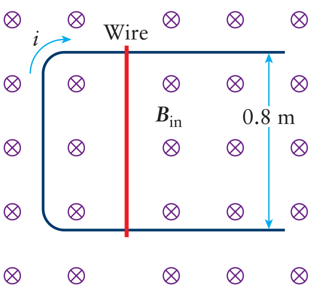


Figure 19-40 Problem 88

SOLVE

Part a) The force on the wire due to the magnetic field will point toward the right, which means the $\boxed{\text{wire will accelerate to the right}}$.

Part b)

$$F_B = ilB\sin(\varphi) = (100\text{ A})(0.80\text{ m})(1.2\text{ T})\sin(90°) = \boxed{96\text{ N}}$$

Part c)

Acceleration of the wire:

$$\sum F_x = F_B = ma_x$$

$$a_x = \frac{F_B}{m}$$

Length of the rails:

$$v_x^2 - v_{0,x}^2 = 2a_x\Delta x$$

$$\Delta x = \frac{v_x^2 - v_{0,x}^2}{2a_x} = \frac{v_x^2 - v_{0,x}^2}{2\left(\frac{F_B}{m}\right)} = \frac{m(v_x^2 - v_{0,x}^2)}{2F_B} = \frac{(0.040\text{ kg})\left(\left(200\frac{\text{m}}{\text{s}}\right)^2 - (0)^2\right)}{2(96\text{ N})} = \boxed{8.3\text{ m}}$$

Part d) If the magnetic field were directed out of the page, the force on the wire due to the magnetic field would point to the left and the wire would accelerate in that direction. The numerical values would remain the same, though.

Part e) If the magnetic field were directed toward the top of the page, the force on the wire due to the magnetic field would be equal to zero because the current and magnetic field would be antiparallel.

REFLECT

If the magnetic field were directed out of the page, we could reverse the current in the loop in order to retain our original answers.

19.89

SET UP

A loop of wire ($r = 2.00 \times 10^{-2}$ m) consisting of $N = 20$ turns is suspended in a region with a magnetic field $B = 0.1000$ T pointing parallel to the plane of the loop. The torque experienced by the loop is $\tau = 4.00 \times 10^{-5}\text{ N}\cdot\text{m}$. The expression for the torque on a current-carrying loop due to a magnetic field is $\tau = Ni_0AB\sin(\varphi)$, where A is the cross-sectional area of the loop. We can rearrange this expression for the current i_0 in the loop. Since there is a net torque on the loop, the loop will rotate in one direction, eventually stop, and then rotate in the opposite direction.

SOLVE

Part a)

$$\tau = Ni_0AB\sin(\varphi)$$

$$i_0 = \frac{\tau}{NAB\sin(\varphi)} = \frac{4.00 \times 10^{-5}\text{ N}\cdot\text{m}}{(20)(\pi)(2.00 \times 10^{-2}\text{ m})^2(0.1000\text{ T})\sin(90°)} = \boxed{0.0159\text{ A} = 15.9\text{ mA}}$$

Part b) The loop will initially rotate to point its magnetic dipole moment in the direction of the magnetic field. At this instant, there is no net torque on the loop, but because of its rotational momentum it will continue to rotate past this point and slow down. After it momentarily comes to rest, it will rotate back toward its initial position. If the system is frictionless, this harmonic oscillation will continue indefinitely.

REFLECT

In order for the loop to continue spinning in the same direction, the current needs to change direction midway through its rotation.

19.90

SET UP

A uniform magnetic field $B = 1.20$ T points toward $+z$. A charged particle ($q = -2.00 \times 10^{-6}$ C) is moving at a speed of $v = 2.20 \times 10^3$ m/s. In the first scenario, the particle is moving at an angle $\varphi = 145°$ with respect to the magnetic field. In the second scenario, the particle is moving in the xy-plane, which means it is moving at an angle of $\varphi = 90°$ with respect to the magnetic field. In either case, the magnitude of the magnetic force on the particle is given by $F = |q|vB\sin(\varphi)$.

SOLVE

Part a)

$$F = |q|vB\sin(\varphi) = |-2.00 \times 10^{-6}\text{ C}|\left(2.20 \times 10^3 \frac{\text{m}}{\text{s}}\right)(1.20\text{ T})\sin(145°) = \boxed{3.03 \times 10^{-3}\text{ N}}$$

Part b)

$$F = |q|vB\sin(\varphi) = |-2.00 \times 10^{-6}\text{ C}|\left(2.20 \times 10^3 \frac{\text{m}}{\text{s}}\right)(1.20\text{ T})\sin(90°) = \boxed{5.28 \times 10^{-3}\text{ N}}$$

REFLECT

We are only interested in the angle the velocity vector makes with respect to the magnetic field, which is located along the z-axis in this example.

19.91

SET UP

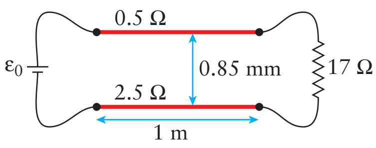


Figure 19-41 Problem 91

Two straight conducting rods—rod 1 has a resistance $R_1 = 0.5\ \Omega$, rod 2 has a resistance $R_2 = 2.5\ \Omega$—that have the same mass $m = 25 \times 10^{-3}$ kg and length $l = 1.0$ m are connected in series by a resistor ($R = 17\ \Omega$) and an external voltage source with potential difference ε_0. We are told that rod 1 "floats" a distance $d = 0.85 \times 10^{-3}$ m above rod 2, which means the net force acting on rod 1 is zero. The forces acting on rod 1 are the force due to the magnetic field generated by rod 2 pointing up and the force due to gravity pointing down. In order to find the magnetic field generated by rod 2, we first need to find the current in the circuit. The two rods and the resistor are in series, which means the current is constant throughout the circuit. The current is equal to the potential difference ε_0 divided by the equivalent resistance of the circuit. The magnitude of the magnetic field due to rod 2 at rod 1 is $B_2 = \dfrac{\mu_0 i}{2\pi d}$ and the magnitude of the magnetic force acting on rod 2 is ilB_2. Setting this expression equal to the force of gravity, we can calculate ε_0.

SOLVE

Equivalent resistance:

$$R_{\text{equiv}} = R_1 + R_2 + R = (0.5\ \Omega) + (2.5\ \Omega) + (17\ \Omega) = 20\ \Omega$$

Current in the circuit:

$$\varepsilon_0 = iR_{\text{equiv}}$$

$$i = \frac{\varepsilon_0}{R_{\text{equiv}}}$$

Magnetic field due to rod 2 at the location of rod 1:

$$B_2 = \frac{\mu_0 i}{2\pi d} = \frac{\mu_0\left(\frac{\varepsilon_0}{R}\right)}{2\pi d} = \frac{\mu_0\varepsilon_0}{2\pi d R_{\text{equiv}}}$$

Free-body diagram for rod 2:

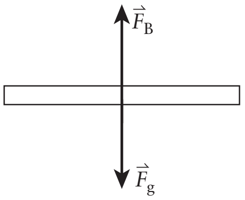


Figure 19-42 Problem 91

Newton's second law for rod 1:

$$\sum F_y = F_B - F_g = ma_y = 0$$

$$F_B = F_g$$

$$ilB_2 = mg$$

$$\left(\frac{\varepsilon_0}{R_{\text{equiv}}}\right)l\left(\frac{\mu_0\varepsilon_0}{2\pi d R_{\text{equiv}}}\right) = mg$$

$$\varepsilon_0 = \sqrt{\frac{2\pi d R_{\text{equiv}}^2 mg}{\mu_0 l}} = \sqrt{\frac{2\pi(0.85 \times 10^{-3}\text{ m})(20\ \Omega)^2(25 \times 10^{-3}\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)}{\left(4\pi \times 10^{-7}\frac{\text{T}\cdot\text{m}}{\text{A}}\right)(1.0\text{ m})}} = \boxed{650\text{ V}}$$

REFLECT

In the space directly above rod 2, its magnetic field points into the page. The current through rod 1 is moving towards the right. The cross product $l_1 \times B_2$ points up as expected since the net force on rod 1 is zero.

Get Help: P'Cast 19.5 – Wires in a Computer

19.92

SET UP

A typical hair dryer draws about $P = 1650$ W when connected to $V = 110$ V. If we model the hair dryer as a long, straight wire in the handle, the current and resistance in the dryer can be found from $P = iV$ and $V = iR$, respectively. The magnitude of the magnetic field due to the

dryer at a distance $R = 0.030$ m from the wire is given by $B = \frac{\mu_0 i}{2\pi R}$. We can then compare this magnitude to the magnitude of Earth's magnetic field to determine if it is dangerous or not.

SOLVE

Part a)

$$P = iV$$

$$i = \frac{P}{V} = \frac{1650\text{ W}}{110\text{ V}} = \boxed{15.0\text{ A}}$$

Part b)

$$P = i^2 R$$

$$R = \frac{P}{i^2} = \frac{1650\text{ W}}{(15.0\text{ A})^2} = \boxed{7.33\ \Omega}$$

Part c)

$$B = \frac{\mu_0 i}{2\pi R} = \frac{\left(4\pi \times 10^{-7}\,\frac{\text{T}\cdot\text{m}}{\text{A}}\right)(15.0\text{ A})}{2\pi(0.030\text{ m})} = \boxed{1.0 \times 10^{-4}\text{ T}}$$

The magnitude of Earth's magnetic field is 0.5×10^{-4} T, so the hair dryer's field is about twice this value, which is very weak. Also, in normal use, the hair dryer is on for only a short amount of time, so its magnetic field is $\boxed{\text{not a health concern}}$.

REFLECT

A current of 15.0 A is a common household current in the United States.

19.93

SET UP

Three long, straight wires—each carrying a current of magnitude i—are located at the corners of a square of side d as shown. Two currents diagonally opposite from one another point into the page, whereas the other current points out of the page. The magnetic field at the fourth corner of the square is equal to the vector sum of the magnetic fields due to each wire at that point through superposition. We'll set up a coordinate system where the current pointing out of the page is positioned at the origin, the positive x-axis points to the right, and the positive y-axis points up. As a reminder, the magnetic field a distance r away from a long, straight wire is $B = \frac{\mu_0 i}{2\pi r}$, while its direction is given by the right-hand rule.

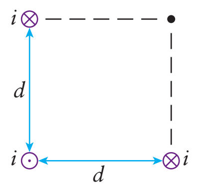


Figure 19-43 Problem 93

SOLVE

Magnetic field due to the top left wire at the fourth corner:

$$\vec{B} = \frac{\mu_0 i}{2\pi d}(-\hat{y}) = -\frac{\mu_0 i}{2\pi d}\hat{y}$$

Magnetic field due to the bottom left wire at the fourth corner:

$$B = \frac{\mu_0 i}{2\pi(d\sqrt{2})}$$

$$B_x = -B\cos(45°) = -\left(\frac{\mu_0 i}{2\sqrt{2}\pi d}\right)\left(\frac{\sqrt{2}}{2}\right) = -\frac{\mu_0 i}{4\pi d}$$

$$B_y = B\sin(45°) = \left(\frac{\mu_0 i}{2\sqrt{2}\pi d}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\mu_0 i}{4\pi d}$$

$$\vec{B} = -\frac{\mu_0 i}{4\pi d}\hat{x} + \frac{\mu_0 i}{4\pi d}\hat{y}$$

Magnetic field due to the bottom right wire at the fourth corner:

$$\vec{B} = \frac{\mu_0 i}{2\pi d}\hat{x}$$

Total magnetic field at the fourth corner:

$$B_{\text{total},\, x} = \frac{\mu_0 i}{2\pi d} - \frac{\mu_0 i}{4\pi d} = \frac{\mu_0 i}{4\pi d}$$

$$B_{\text{total},\, y} = \frac{\mu_0 i}{4\pi d} - \frac{\mu_0 i}{2\pi d} = -\frac{\mu_0 i}{4\pi d}$$

$$\vec{B}_{\text{total}} = \frac{\mu_0 i}{4\pi d}\hat{x} - \frac{\mu_0 i}{4\pi d}\hat{y}$$

Magnitude of the total field:

$$B_{\text{total}} = \sqrt{\left(\frac{\mu_0 i}{4\pi d}\right)^2 + \left(-\frac{\mu_0 i}{4\pi d}\right)^2} = \frac{\mu_0 i}{4\pi d}\sqrt{2} = \frac{\mu_0 i}{2\sqrt{2}\pi d}$$

Angle of the field:

$$\phi = \arctan\left(\frac{\left(-\frac{\mu_0 \mathrm{i}}{4\pi \mathrm{d}}\right)}{\left(\frac{\mu_0 \mathrm{i}}{4\pi \mathrm{d}}\right)}\right) = 315°$$

The magnetic field at the fourth corner has a magnitude of $\boxed{\frac{\mu_0 i}{2\sqrt{2}\pi d} \text{ and makes an angle}}$ $\boxed{\text{of 315 degrees with the } +x\text{-axis}}$.

REFLECT

The currents pointing into the page should have a larger contribution to the total field at the fourth corner because they are closer than the third wire.

19.94

SET UP

A lamp cord, which has a length $l = 2.0$ m, is connected to a light bulb ($P = 75$ W) and an outlet ($V = 110$ V). The cord consists of two insulated parallel wires a distance $d = 4.0 \times 10^{-3}$ m apart; the currents in these wires are antiparallel. The magnitude of the current in the wires is the same and can be calculated using $P = iV$. Once we know the current, we can calculate the magnetic field due to these wires at a point midway between the wires and a distance 2.0×10^{-3} m away from one of the wires and then compare these magnitudes to the magnitude of Earth's magnetic field ($B_{\text{Earth}} = 0.5 \times 10^{-4}$ T). We will need to use the right-hand rule to see if the fields will constructively or destructively add. Finally, the magnitude of the force of one wire on the other is equal to $F = i_1 l B_2 \sin(\varphi)$. This force will be repulsive since the currents in the wires are antiparallel.

SOLVE

Current in the wires:

$$P = iV$$

$$i = \frac{P}{V} = \frac{75\text{ W}}{110\text{ V}} = 0.68\text{ A}$$

Part a)

The magnetic field due to each wire points in the same direction and has the same magnitude.

$$B = \frac{\mu_0 i}{2\pi\left(\frac{d}{2}\right)} + \frac{\mu_0 i}{2\pi\left(\frac{d}{2}\right)} = \frac{2\mu_0 i}{\pi d} = \frac{2\left(4\pi \times 10^{-7}\,\frac{\text{T}\cdot\text{m}}{\text{A}}\right)(0.68\text{ A})}{\pi(4.0 \times 10^{-3}\text{ m})} = \boxed{1.4 \times 10^{-4}\text{ T}}$$

Part b)

The magnetic fields due to each wire point in opposite directions and have different magnitudes.

$$B = \frac{\mu_0 i}{2\pi r_1} - \frac{\mu_0 i}{2\pi r_2} = \frac{\mu_0 i}{2\pi}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$= \frac{\left(4\pi \times 10^{-7}\,\frac{\text{T}\cdot\text{m}}{\text{A}}\right)(0.68\text{ A})}{2\pi}\left[\frac{1}{2.0 \times 10^{-3}\text{ m}} - \frac{1}{6.0 \times 10^{-3}\text{ m}}\right] = \boxed{4.5 \times 10^{-5}\text{ T}}$$

Part c)

Between the wires:

$$\frac{B}{B_{\text{Earth}}} = \frac{1.4 \times 10^{-4}\text{ T}}{0.5 \times 10^{-4}\text{ T}} = \boxed{2.7}$$

Outside the wires:

$$\frac{B}{B_{\text{Earth}}} = \frac{4.5 \times 10^{-5}\ \text{T}}{0.5 \times 10^{-4}\ \text{T}} = \boxed{0.90}$$

Part d)

$$F = i_1 l B_2 \sin(\varphi) = i_1 l\left(\frac{\mu_0 i_2}{2\pi d}\right)\sin(90°) = il\left(\frac{\mu_0 i}{2\pi d}\right)$$

$$= \frac{\mu_0 i^2 l}{2\pi d} = \frac{\left(4\pi \times 10^{-7}\frac{\text{T}\cdot\text{m}}{\text{A}}\right)(0.68\ \text{A})^2(2.0\ \text{m})}{2\pi(4.0 \times 10^{-3}\ \text{m})} = 4.7 \times 10^{-5}\ \text{N}$$

The wires $\boxed{\text{repel one another with a force of } 4.7 \times 10^{-5}\ \text{N}}$.

This force is $\boxed{\text{too small to have an appreciable effect on the insulation}}$.

REFLECT

Even though the magnetic field in between the wires is nearly three times Earth's magnetic field, it is still considered a very small field.

19.95

SET UP

A high voltage power line is located 5.0 m in the air and 12 m horizontally from your house. The wire carries a current of 100 A. We can use the expression $B = \frac{\mu_0 i}{2\pi r}$, where r is the straight-line distance from the wire to your house, to calculate the magnitude of the magnetic field caused by the power line. Once we know the magnitude of the field due to the power line, we can compare it to the magnitude of Earth's magnetic field ($B_{\text{Earth}} = 0.5 \times 10^{-4}$ T) to determine if the power line is likely to affect your health.

SOLVE

$$B = \frac{\mu_0 i}{2\pi r} = \frac{\left(4\pi \times 10^{-7}\frac{\text{T}\cdot\text{m}}{\text{A}}\right)(100\ \text{A})}{2\pi\sqrt{(5.0)^2 + (12\ \text{m})^2}} = \boxed{1.5 \times 10^{-6}\ \text{T}}$$

$$\frac{B}{B_{\text{Earth}}} = \frac{1.5 \times 10^{-6}\ \text{T}}{0.5 \times 10^{-4}\ \text{T}} = 0.031$$

$$\boxed{B = 0.031 B_{\text{Earth}}}$$

Since the magnetic field from the wires is so much smaller than Earth's magnetic field, there should be $\boxed{\text{little or no cause for concern}}$.

REFLECT

Even if we were 1 m from the power line, the magnitude of the magnetic field would still only be 40% of the strength of Earth's field.

19.96

SET UP

Three long, straight wires are positioned on the vertices of an equilateral triangle with sides $s = 2.00$ m. Each wire carries a current $i = 40$ A; the currents of the bottom two wires point out of the page, whereas the current of the top wire points into the page. All three wires are equidistant to a point P located at the center of the triangle. The magnetic field at point P is the vector sum of the magnetic fields due to each of the three wires at that point. Since the wires are equidistant to P, the magnitude of the magnetic field due to each wire at that point will be the same, $B = \frac{\mu_0 i}{2\pi r}$; we can use geometry to rewrite r in terms of s. The direction of each field can be found using the right-hand rule.

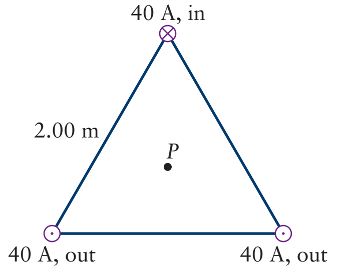


Figure 19-43 Problem 96

SOLVE

Magnetic field vectors and geometry:

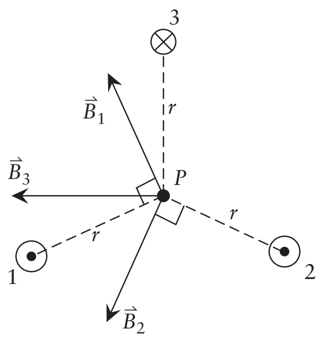


Figure 19-44 Problem 96

Magnitude of the magnetic field of each wire at P:

$$B = \frac{\mu_0 i}{2\pi r} = \frac{\mu_0 i}{2\pi\left(\frac{s}{2\cos(30°)}\right)} = \frac{\mu_0 i\cos(30°)}{\pi s}$$

x component of the field at point P:

$$\sum B_x = B_{1,x} + B_{2,x} + B_{3,x} = -B\cos(60°) - B\cos(60°) - B = -B\left(\frac{1}{2} + \frac{1}{2} + 1\right) = -2B$$

$$= -2\left(\frac{\mu_0 i\cos(30°)}{\pi s}\right) = -\frac{2\left(4\pi \times 10^{-7}\,\frac{\mathrm{T\cdot m}}{\mathrm{A}}\right)(40\ \mathrm{A})\cos(30°)}{\pi(2.00\ \mathrm{m})} = -1.4 \times 10^{-5}\ \mathrm{T}$$

y component of the field at point P:

$$\sum B_y = B_{1,y} + B_{2,y} + B_{3,y} = B\sin(60°) - B\sin(60°) + 0 = 0$$

The magnetic field at point P is $\boxed{\vec{B} = -(1.4 \times 10^{-5}\ \text{T})\hat{x}}$.

REFLECT

The magnetic field due to a long, straight wire must be perpendicular to both the direction of the current and the unit vector pointing from the wire to the point of interest.

19.97

SET UP

Two coils of wire, each with N turns and radius R, are centered about the x-axis—one coil is located at $x = 0$, and the other is located at $x = R$. The magnitude of the magnetic field a distance x along the central axis of a loop of wire made of N turns carrying a current i is $B = \dfrac{\mu_0 N i R^2}{2(R^2 + x^2)^{\frac{3}{2}}}$. From the right-hand rule, the magnetic field of each loop will point toward $+x$. The net magnetic field at $x = \dfrac{R}{2}$ and $x = 2R$ is equal to the sum of the fields at each point due to each loop.

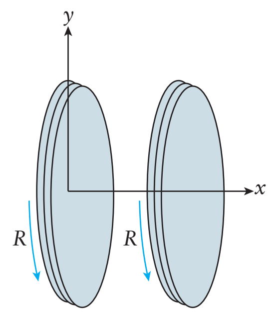


Figure 19-45 Problem 97

SOLVE

Part a)

$$B_x = \frac{\mu_0 N i R^2}{2\left(R^2 + \left(\frac{R}{2}\right)^2\right)^{\frac{3}{2}}} + \frac{\mu_0 N i R^2}{2\left(R^2 + \left(\frac{R}{2}\right)^2\right)^{\frac{3}{2}}} = \frac{8\mu_0 N i R^2}{R^3\sqrt{125}} = \frac{8\mu_0 N i}{R\sqrt{125}} = 0.7155\left(\frac{\mu_0 N i}{R}\right)$$

The total magnetic field at $x = \dfrac{R}{2}$ has a magnitude of $\boxed{0.7155\left(\dfrac{\mu_0 N i}{R}\right) \text{ and points toward } +x}$.

Part b)

$$B_x = \frac{\mu_0 N i R^2}{2(R^2 + (2R)^2)^{\frac{3}{2}}} + \frac{\mu_0 N i R^2}{2(R^2 + R^2)^{\frac{3}{2}}} = \frac{\mu_0 N i R^2}{2\sqrt{125}R^3} + \frac{\mu_0 N i R^2}{4\sqrt{2}R^3} = \frac{\mu_0 N i}{2\sqrt{125}R} + \frac{\mu_0 N i}{4\sqrt{2}R} = 0.2215\left(\frac{\mu_0 N i}{R}\right)$$

The total magnetic field at $x = 2R$ has a magnitude of $\boxed{0.2215\left(\dfrac{\mu_0 N i}{R}\right) \text{ and points toward } +x}$.

REFLECT

The field should be stronger between the loops and weaker outside them.

19.98

SET UP

Earth's magnetic field at the equator has a magnitude of 0.70×10^{-4} T and points directly north. A flat, circular coil that has a radius $R = 0.70$ m and is made up of $N = 10$ turns carries a current i. The magnetic field generated at the center of the coil, which is equal to $B = \frac{\mu_0 N i}{2R}$, completely cancels Earth's field. Rearranging and solving this equation will give us the current necessary to accomplish this. The magnetic field due to the loop must point toward the south in order to cancel out Earth's field; we can use the right-hand rule to determine the direction the current must circulate around the loop.

SOLVE

Part a)

$$B = \frac{\mu_0 N i}{2R}$$

$$i = \frac{2BR}{\mu_0 N} = \frac{2(0.70 \times 10^{-4}\ \mathrm{T})(0.70\ \mathrm{m})}{\left(4\pi \times 10^{-7}\ \frac{\mathrm{T \cdot m}}{\mathrm{A}}\right)(10)} = \boxed{7.8\ \mathrm{A}}$$

Part b) At low latitudes, where Earth's field can be taken as parallel to Earth's surface, the coil should be oriented so the plane of the loop is perpendicular to the surface, with the direction of the current chosen to produce a magnetic field in the direction opposite to Earth's field.

REFLECT

Current should be counterclockwise when looking through the loop toward the north.

19.99

SET UP

A pair of parallel high voltage wires each carrying a current $i = 100$ A are separated by $d = 3.00$ m and lie in the same horizontal plane. We are asked to find the magnetic field at a point P located $z_P = 15.0$ m above the wires, equidistant from both, for a series of configurations of the current. The magnetic field at P is equal to the vector sum of the magnetic fields due to each wire. We can use symmetry and the right-hand rule to determine the magnitude and direction of the field at this point. The magnitude of the magnetic field due to a long, straight wire is given by $B = \frac{\mu_0 i}{2\pi r}$; since the wires are equidistant to P, the magnitude of the field due to each wire will be the same at P. We can compare our results to the magnitude of Earth's magnetic field ($B_{\mathrm{Earth}} = 0.5 \times 10^{-4}$ T) to determine whether the field from the power lines will affect the navigation of migratory birds.

SOLVE

Part a)
Looking north:

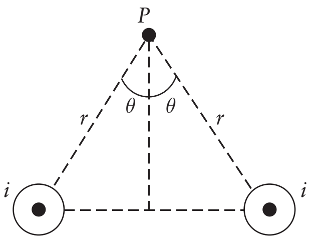


Figure 19-46 Problem 99

Angle:

$$\tan(\theta) = \frac{\left(\frac{d}{2}\right)}{z_P} = \frac{d}{2z_P}$$

$$\theta = \arctan\left(\frac{d}{2z_P}\right) = \arctan\left(\frac{3.00 \text{ m}}{2(15.0 \text{ m})}\right) = 5.71°$$

Magnitude of the field:

$$B_{\text{total}} = 2B_{\text{wire}}\cos(\theta) = 2\left(\frac{\mu_0 i}{2\pi r}\right)\cos(\theta)$$

$$= \frac{\left(4\pi \times 10^{-7}\,\frac{\text{T}\cdot\text{m}}{\text{A}}\right)(100 \text{ A})}{(\pi)\sqrt{(1.50 \text{ m})^2 + (15.0 \text{ m})^2}}\cos(5.71°) = 2.64 \times 10^{-6} \text{ T}$$

The magnetic field at point P has a magnitude of $\boxed{2.64 \times 10^{-6} \text{ T and points due west}}$.

Part b)
Looking north:

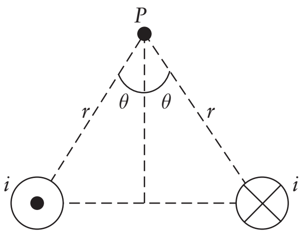


Figure 19-47 Problem 99

Angle:

$$\tan(\theta) = \frac{\left(\frac{d}{2}\right)}{z_P} = \frac{d}{2z_P}$$

$$\theta = \arctan\left(\frac{d}{2z_P}\right) = \arctan\left(\frac{3.00 \text{ m}}{2(15.0 \text{ m})}\right) = 5.71°$$

Magnitude of the field:

$$B_{\text{total}} = 2B_{\text{wire}}\sin(\theta) = 2\left(\frac{\mu_0 i}{2\pi r}\right)\sin(\theta)$$

$$= \frac{\left(4\pi \times 10^{-7}\,\frac{\text{T}\cdot\text{m}}{\text{A}}\right)(100\text{ A})}{(\pi)\sqrt{(1.50\text{ m})^2 + (15.0\text{ m})^2}}\sin(5.71°) = 2.64 \times 10^{-7}\text{ T}$$

The magnetic field at point P has a magnitude of $\boxed{2.64 \times 10^{-7}\text{ T and points vertically upward}}$.

Part c)
Looking west:

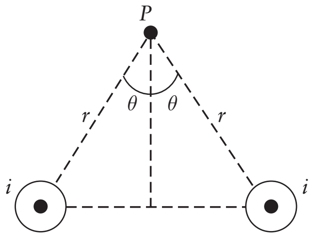


Figure 19-48 Problem 99

Angle:

$$\tan(\theta) = \frac{\left(\frac{d}{2}\right)}{z_P} = \frac{d}{2z_P}$$

$$\theta = \arctan\left(\frac{d}{2z_P}\right) = \arctan\left(\frac{3.00\text{ m}}{2(15.0\text{ m})}\right) = 5.71°$$

Magnitude of the field:

$$B_{\text{total}} = 2B_{\text{wire}}\cos(\theta) = 2\left(\frac{\mu_0 i}{2\pi r}\right)\cos(\theta)$$

$$= \frac{\left(4\pi \times 10^{-7}\,\frac{\text{T}\cdot\text{m}}{\text{A}}\right)(100\text{ A})}{(\pi)\sqrt{(1.50\text{ m})^2 + (15.0\text{ m})^2}}\cos(5.71°) = 2.64 \times 10^{-6}\text{ T}$$

The magnetic field at point P has a magnitude of $\boxed{2.64 \times 10^{-6}\text{ T and points due south}}$.

Part d) The magnetic fields calculated in parts (a) and (c) are about 5% of B_{Earth}. This is fairly small, but it could possibly be enough to interfere with navigation. In part (b), the magnetic field is about 0.5% of B_{Earth}, which seems too small to cause much of a problem.

REFLECT

Determining the direction of the field and invoking symmetry first before plugging in numbers not only brings insight to the problem but also saves us from doing some unnecessary calculations.

19.100

SET UP

We will model the hemisphere of an adult brain ($R_{\text{brain}} = 0.035$ m) as a loop of wire with a radius $R_{\text{loop}} = 0.0325$ m carrying a current i. The magnitude of the magnetic field due to a circular current-carrying coil of radius R is $B = \dfrac{\mu_0 i R^2}{2(z^2 + R^2)^{\frac{3}{2}}}$, where z is the distance away from the center along the axis of the loop. We can plug in $R = R_{\text{brain}}$ and solve for i in order to calculate the current in the brain if the magnetic field has a magnitude $B = 5.0 \times 10^{-15}$ T at a distance of $z = 0.020$ m from the center of the brain. To calculate the magnetic field at the center of the hemisphere of the brain, we should use this calculated current along with $R = R_{\text{loop}}$.

SOLVE

Part a)

$$B = \frac{\mu_0 i R_{\text{brain}}^2}{2(z^2 + R_{\text{brain}}^2)^{\frac{3}{2}}}$$

$$i = \frac{2B\left(z^2 + R_{\text{brain}}^2\right)^{\frac{3}{2}}}{\mu_0 R_{\text{brain}}^2} = \frac{2(5.0 \times 10^{-15}\ \text{T})\left((0.020\ \text{m})^2 + (0.035\ \text{m})^2\right)^{\frac{3}{2}}}{\left(4\pi \times 10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}\right)(0.035\ \text{m})^2} = \boxed{4.3 \times 10^{-10}\ \text{A}}$$

Part b)

$$B = \frac{\mu_0 i R_{\text{loop}}^2}{2\left((0)^2 + R_{\text{loop}}^2\right)^{\frac{3}{2}}} = \frac{\mu_0 i}{2R_{\text{loop}}} = \frac{\left(4\pi \times 10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}\right)(4.3 \times 10^{-10}\ \text{A})}{2(0.0325\ \text{m})} = \boxed{8.3 \times 10^{-15}\ \text{T} = 8.3\ \text{fT}}$$

REFLECT

The magnetic field inside the brain should be larger than the field detected by the sensor outside the brain.

19.101

SET UP

A long, straight wire carries a current of $i_1 = 1.2$ A toward the south. A second long, straight wire is parallel to the first at a distance $R = 0.028$ m and carries a current $i_2 = 3.8$ A toward the north. The magnitude and direction of the force of one wire acting on the other are given by $\vec{F} = i\vec{l} \times \vec{B}$. The force per unit length is found simply by dividing the magnitude of the force F by the length l.

SOLVE

$$F = i_1 l B_2 \sin(90°) = i_1 l\left(\frac{\mu_0 i_2}{2\pi R}\right)$$

$$\frac{F}{l} = \frac{\mu_0 i_1 i_2}{2\pi R} = \frac{\left(4\pi \times 10^{-7} \frac{\text{T} \cdot \text{m}}{\text{A}}\right)(1.2\ \text{A})(3.8\ \text{A})}{2\pi(0.028\ \text{m})} = 3.3 \times 10^{-5}\ \frac{\text{N}}{\text{m}}$$

The force per unit length acting on each wire has a magnitude of $\boxed{3.3 \times 10^{-5}\ \frac{\text{N}}{\text{m}}}$ $\boxed{\text{and is repulsive}}$.

REFLECT

Antiparallel currents repel one another; parallel currents attract one another.

Chapter 20
Magnetic Induction

Conceptual Questions

20.1 The falling bar magnet induces a current in the wall of the pipe. In accordance with Lenz's law, the direction of this induced current is such that it produces a magnetic field that exerts a force on the magnet opposing its motion. The speed of the magnet therefore increases more slowly than in free-fall until it reaches a terminal speed at which the magnetic and gravitational forces are equal and opposite. After this point, the magnet continues to fall, but at a constant speed.

20.2 If the magnetic field is slowly increased, the force on the electron is increased. This results in a centripetal acceleration that causes the electron to move faster and faster in a circular path. Since the mass of the electron increases as it speeds up, the efficiency of the electromagnet is limited at greater speeds.

20.3 Part a) The magnetic coil produces a rapidly varying magnetic field that flows through the plate. From Lenz's law, this induces a current in the plate such that the magnetic field of the plate opposes the field of the magnetic coil, repelling the plate and causing it to levitate.

Part b) The large induced current in the plate dissipates a large amount of power in the plate, heating it up.

Part c) If the plate were an insulating material, this trick would not work because a current could not be induced.

20.4 As someone with a pacemaker or other electronic device walks through regions of spatially varying magnetic fields, the changing magnetic flux through the electronic circuitry will induce an emf in the device. This extra emf could cause unwanted, and possibly fatal, malfunctions of the device.

20.5 A current is induced in loop b while the current in loop a is changing. If the current is in the direction shown and is increasing, the flux of its magnetic field through loop b is upward and increasing. In accordance with Lenz's law, the direction of the induced current in loop b is such that the flux of its magnetic field through loop b is downward, opposing the change in flux that produced it. This means that the current in loop b is in the opposite direction as the current in loop a, and the two loops repel. After the current in loop a stops changing, then the current in loop b becomes zero and there is no force between the loops.

20.6 The induced emf will create an induced current in the clockwise direction, which will exert a force to the left on the sliding wire. This will gradually slow the wire until it comes to a stop.

20.7 Inductance is flux divided by current. Since flux is proportional to current, the current terms cancel, leaving only geometric properties.

20.8 By looking at the expression for the energy stored in an inductor, $U_L = \frac{1}{2}Li^2$, the current must increase by a factor of $\sqrt{2}$ to double the energy stored in the inductor.

20.9 The capacitor has energy $U_E = \frac{q^2}{2C}$ and the inductor has energy $U_L = \frac{1}{2}Li^2$. We see that both energies are proportional to the square of a quantity related to the charge (either i or q). However, the magnetic energy stored is proportional to the inductance, but the electrical energy stored is inversely proportional to the capacitance.

20.10 When the load is increased, the rotating coil is suddenly slowed, perhaps even stopped momentarily. The emf it induces in the primary coil, which produces the magnetic field, decreases drastically and the current drawn increases greatly. The large power drawn is all dissipated as heat in this primary coil, which is what burns out.

20.11 Part a) Assuming the number of coils remains constant, the self-inductance drops by a factor of 2.

Part b) Assuming the length of the solenoid remains constant, the self-inductance is unchanged.

20.12 Part a) The self-inductance of the solenoid will decrease by a factor of 2 if the length is increased by a factor of 2.

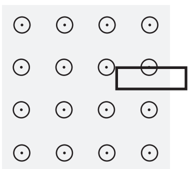
Part b) The self-inductance of the solenoid will increase by a factor of 4 if the diameter is increased by a factor of 2.

20.13 The current drops suddenly toward zero when the switch is opened. This current drop induces a very large emf in the inductor, which produces a large potential difference across the switch gap. This potential is large enough to cause dielectric breakdown of the air in the gap, which causes the spark. The spark allows current to continue to flow for a brief period.

Multiple-Choice Questions

20.14 **D** (all of the above). The magnetic flux is equal to $\Phi_B = \int \vec{B} \cdot d\vec{A} = \int B\cos(\theta)dA$.

20.15 **D** (d). The loop is starting to exit the region with the magnetic field.



(d)

Figure 20-1 Problem 15

20.16 **D** (the battery is connected by closing the switch or the battery is disconnected by opening the switch). Current will be induced in the second coil when the current in the first coil is changing.

20.17 **C** (antiparallel to i_a). The induced current in loop *b* will flow in such a way as to generate a magnetic field to counteract the increase in flux due to loop *a*.

20.18 **E** (There will be no induced current in the loop). The magnetic flux through the loop remains constant as it moves at a constant velocity.

20.19 **A** (clockwise and constant). Lenz's law gives the direction of the induced current (clockwise), while Faraday's law shows that a uniformly decreasing magnetic field will induce a constant potential, which means the current will be constant.

20.20 **D** (1:4).

$$\frac{L_2}{L_1}=\frac{\left(\dfrac{\mu_0 N_2^2 A}{l_2}\right)}{\left(\dfrac{\mu_0 N_1^2 A}{l_1}\right)}=\frac{\left(\dfrac{N_2^2 l_2}{l_2^2}\right)}{\left(\dfrac{N_1^2 l_1}{l_1^2}\right)}=\frac{n_2^2 l}{n_1^2 l}=\frac{n_2^2}{(2n_2)^2}=\frac{1}{4}$$

20.21 **C** (one-quarter of the total energy).

$$U_E=\frac{1}{2C}q_{\max}^2$$

$$\frac{U_{\text{half max}}}{U_{\max}}=\frac{\left(\dfrac{1}{2C}\left(\dfrac{q_{\max}}{2}\right)^2\right)}{\left(\dfrac{1}{2C}q_{\max}^2\right)}=\frac{1}{4}$$

20.22 **D** (quadrupled).

$$\frac{U_{L,2}}{U_{L,1}}=\frac{\left(\dfrac{1}{2}Li_2^2\right)}{\left(\dfrac{1}{2}Li_1^2\right)}=\frac{i_2^2}{i_1^2}=\frac{(2i_1)^2}{i_1^2}=4$$

Estimation/Numerical Questions

20.23 The current in a generator in a large hydroelectric dam is about 16,500 A.

20.24 U.S. power consumption is about 90,000 kWh per citizen. If the Hoover Dam generates about 4 billion kWh of hydroelectric power each year with 17 turbines, then we would need around 134,000 turbines to supply enough energy for the 350,000,000 citizens of the United States. That's over 7500 Hoover Dams!

20.25 The induced voltage created in trans-Atlantic communication cables due to fluctuations in Earth's magnetic field is around 5 V/km, or 15,000 V total. This is such a large variation that it would cause the cable to exceed the maximum current allowed and cease to function. In more modern times, these cables have been changed from coaxial cables to fiber optic cables to help alleviate this concern.

20.26 The change in the geomagnetic field due to solar flares is about 700 nT/min.

20.27 Clearly it was an extremely important achievement for Faraday to conceive of induction (at any time in history). However, it was critical that his accomplishment occur at the approximate time that it did (~1825) because it took several decades for other scientists to fully study and perfect his ideas. Without this fundamental work, it is questionable whether Maxwell would have been able to synthesize electricity and magnetism with optics as he did in 1865. Without Maxwell's equations (and the field theory that followed), the course of scientific history would have been dramatically altered and the entire quantum revolution of Earth's 20th century might never have occurred!

20.28

t (s)	V (V)
0	0
1	2
2	4
3	2
4	0
5	−2
6	−4
7	−2
8	0
9	2
10	4
11	2
12	0
13	−2
14	−4
15	−2
16	0

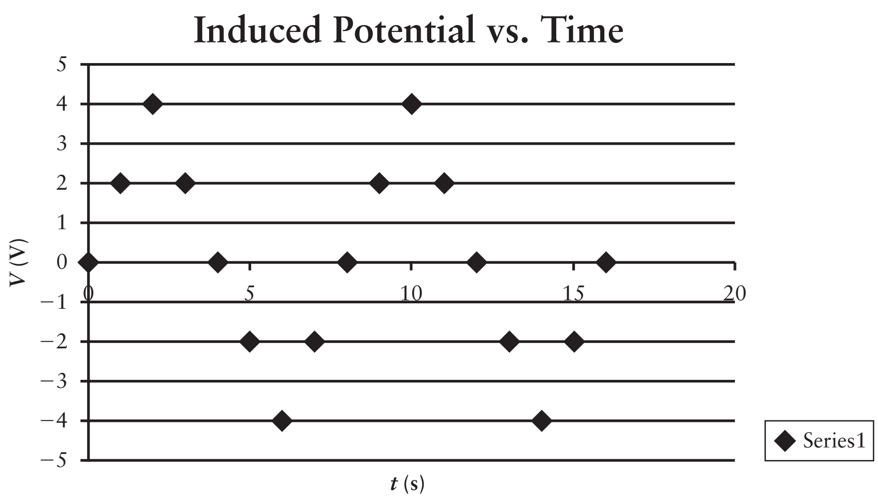


Figure 20-2 Problem 28

The induced potential in the loop as a function of time is described by $V(t) = 4\sin\left(\frac{2\pi t}{8}\right) = 4\sin\left(\frac{\pi t}{4}\right)$ (SI units). This must be equal to $V(t) = -NA\frac{dB}{dt}$ from Faraday's law. Solving for $B(t)$,

$$B(t) = \int dB = \int -\frac{V(t)}{NA} = -\frac{1}{NA}\int 4\sin\left(\frac{\pi t}{4}\right) = \frac{4}{NA}\left[\frac{4}{\pi}\cos\left(\frac{\pi t}{4}\right)\right]$$

$$= \frac{16}{\pi(100)(\pi)(0.25\text{ m})^2}\cos\left(\frac{\pi t}{4}\right) = \boxed{0.26\cos\left(\frac{\pi t}{4}\right)} \quad \text{(SI units)}$$

Problems

20.29

SET UP

A circular coil has $N = 100$ turns and a radius of 0.10 m. There is an external magnetic field $B = 0.0650\text{ T}$ directed perpendicularly through the coil. The magnetic flux through the coil is equal to $\Phi_B = BA\cos(\varphi)$, where A is the cross-sectional area of the coil and φ is the angle between the magnetic field and the area vector of the coil. The external magnetic field is steadily increased from 0.0650 T to 0.100 T over $\Delta t = 0.5\text{ s}$; the induced potential in the coil as a result of this change is $|\varepsilon| = \left|-N\frac{d\Phi_B}{dt}\right|$.

SOLVE

Part a)

$$\Phi_B = BA\cos(\varphi) = (0.0650\text{ T})(\pi)(0.10\text{ m})^2\cos(0°) = \boxed{2.0 \times 10^{-3}\text{ Wb}}$$

Part b)

$$|\varepsilon| = \left|-N\frac{d\Phi_B}{dt}\right| = \left|N\frac{d(BA\cos(0°))}{dt}\right| = \left|NA\frac{\Delta B}{\Delta t}\right|$$

$$= \left|(100)(\pi)(0.10\text{ m})^2\left(\frac{(0.100\text{ T}) - (0.0650\text{ T})}{0.5\text{ s}}\right)\right| = \boxed{0.2\text{ V}}$$

REFLECT

The area vector of the coil points perpendicularly to the plane of the coil.

20.30

SET UP

A rectangular coil consisting of $N = 500$ turns of wire is 0.10 m by 0.25 m. It is rotated about its long axis at an angular frequency ω in a uniform magnetic field $B = 0.58\text{ T}$. The potential induced in the loop is given by $|\varepsilon| = \left|-N\frac{d\Phi_B}{dt}\right|$. In this case, the magnetic field and the area

of the loop remain constant, while the angle between the magnetic field vector and the area vector changes as a function of time, $\varphi = \omega t$. We can apply the definition of the magnetic flux and rearrange this expression to solve for the angular frequency that generates a maximum potential of 110 V.

SOLVE

$$|\varepsilon| = \left|-N\frac{d\Phi_B}{dt}\right| = \left|N\frac{d(BA\cos(\varphi))}{dt}\right| = \left|NBA\frac{d}{dt}\cos(\omega t)\right| = |NBA\omega\sin(\omega t)|$$

$$\varepsilon_{\max} = NBA\omega$$

$$\omega = \frac{\varepsilon_{\max}}{NBA} = \frac{110\text{ V}}{(500)(0.58\text{ T})(0.10\text{ m})} = \boxed{3.8\frac{\text{rad}}{\text{s}}}$$

REFLECT

The maximum of a sine function is equal to its amplitude.

20.31

SET UP

A ring of radius $R = 0.0800$ m is lying in the xy-plane centered about the origin. The ring is in the presence of a magnetic field, $\vec{B} = (6.00\text{ T})\cos(\theta)\hat{z}$, where θ is measured in the xy-plane with respect to the positive x-axis. The magnitude of the magnetic flux through the ring is $\Phi_B = \int \vec{B}\cdot d\vec{A}$, where $d\vec{A}$ is the infinitesimal area vector for a circle. The area vector points normal to the surface, which is towards $+z$ in this case. Since the two vectors are parallel, the dot product $\vec{B}\cdot d\vec{A} = BdA$. The infinitesimal area dA in cylindrical coordinates is $rdrd\theta$. To calculate the flux through the ring, the limits for the integral over dr are $r = 0$ to $r = R$ and for the integral over $d\theta$ are $\theta = 0$ to $\theta = 2\pi$. If we're interested in the flux only through the part of the circle where $x > 0$, the integral over $d\theta$ will be from $\theta = -\pi/2$ to $\theta = +\pi/2$. If we're interested in the flux only through the part of the circle where $y > 0$, the integral over $d\theta$ will be from $\theta = 0$ to $\theta = \pi$.

SOLVE

Part a)

$$\Phi_B = \int \vec{B}\cdot d\vec{A} = \int((6.00\text{ T})\cos(\theta)\hat{z})\cdot(dA\hat{z}) = (6.00\text{ T})\iint\cos(\theta)(rdrd\theta)$$

$$= (6.00\text{ T})\int_0^R rdr\int_0^{2\pi}\cos(\theta)d\theta = (6.00\text{ T})\left[\frac{1}{2}r^2\right]_0^R[\sin(\theta)]_0^{2\pi} = (6.00\text{ T})\left[\frac{1}{2}r^2\right]_0^R[0] = \boxed{0}$$

Part b)

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = (6.00\text{ T})\int_0^R r dr \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(\theta) d\theta = (6.00\text{ T})\left[\frac{1}{2}r^2\right]_0^R [\sin(\theta)]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= (6.00\text{ T})\left[\frac{1}{2}R^2\right]\left[\sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right)\right] = \frac{(6.00\text{ T})}{2}(0.0800\text{ m})^2[2] = \boxed{0.384\text{ Wb}}$$

Part c)

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = (6.00\text{ T})\int_0^R r dr \int_0^{\pi} \cos(\theta) d\theta = (6.00\text{ T})\left[\frac{1}{2}r^2\right]_0^R [\sin(\theta)]_0^{\pi}$$

$$= (6.00\text{ T})\left[\frac{1}{2}R^2\right][\sin(\pi) - \sin(0)] = \boxed{0}$$

REFLECT

We could have also used symmetry to solve parts a and c. Cosine is an even function and positive in the first and fourth quadrants, where $x > 0$. The flux will be positive for values where $x > 0$ and equal in magnitude but negative for values of $x < 0$. Therefore, an integral over the entire ring or over the semicircle where $y > 0$ will be exactly zero.

20.32

SET UP

A coil with a radius of 3.00×10^{-2} m and consisting of $N = 30$ turns of wire is placed in a uniform magnetic field of $B = 1.00$ T directed perpendicularly to the coil. At $t = 0$, the field is uniformly increased until it reaches $B = 1.30$ T at $t = 10.0$ s; the field then remains constant after that. Since the magnetic field is constant before $t = 0$ and after $t = 10.0$ s, the induced potential during these times is equal to zero. The magnitude of the induced potential at $t = 5.00$ s is related to the rate at which the magnetic flux is changing at that time through Faraday's law, $|\varepsilon| = \left|-N\frac{d\Phi_B}{dt}\right|$. We are told that the magnetic field changes uniformly between $t = 0$ and $t = 10.0$ s, so the rate at which the field is changing at $t = 5.00$ s is equal to the change in the field over the entire time interval divided by 10.0 s.

SOLVE

Part a) The magnetic field is constant for $t < 0$ s, so the induced emf in the coil is $\boxed{\text{zero}}$.

Part b)

$$|\varepsilon| = \left|-N\frac{d\Phi_B}{dt}\right| = \left|N\frac{\Delta(BA\cos(0°))}{\Delta t}\right| = \left|NA\frac{\Delta B}{\Delta t}\right|$$

$$= \left|(30)(\pi)(3.00 \times 10^{-2}\text{ m})^2\left(\frac{(1.30\text{ T}) - (1.00\text{ T})}{10.0\text{ s}}\right)\right| = \boxed{2.54 \times 10^{-3}\text{ V} = 2.54\text{ mV}}$$

Part c) The magnetic field is constant for $t > 10.0$ s, so the induced emf in the coil is $\boxed{\text{zero}}$.

Part d)
Magnetic field as a function of time:

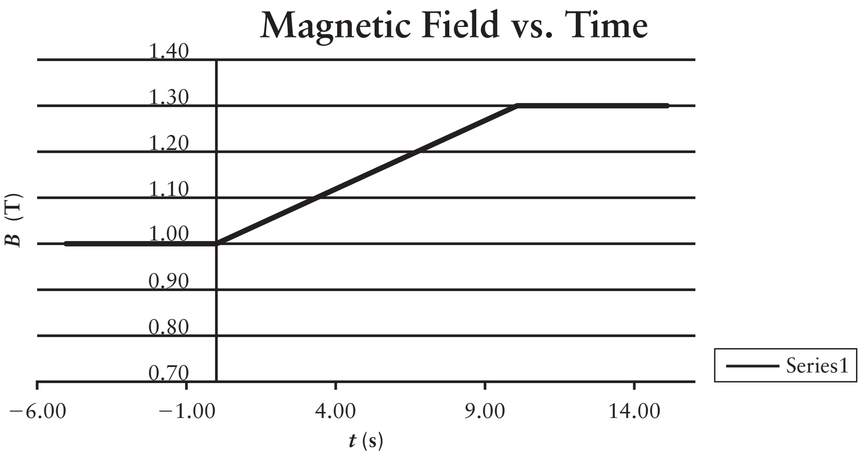


Figure 20-3 Problem 32

Induced potential as a function of time:

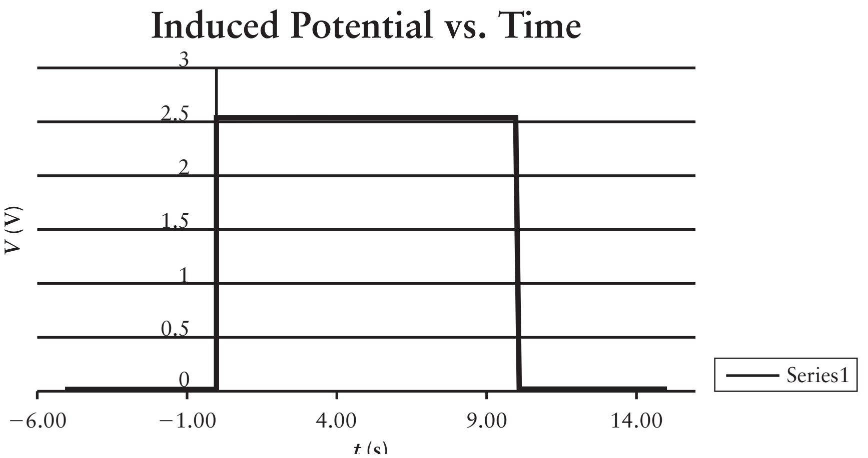


Figure 20-4 Problem 32

REFLECT

There will only be an induced potential in the coil when the magnetic flux through the coil is changing. If the magnetic flux through the coil is constant, then the induced potential in the coil is zero.

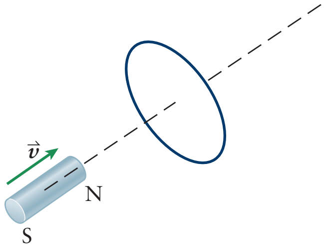
20.33

SET UP

We are shown six different scenarios consisting of a permanent magnet and a loop of wire. We can determine the direction of the induced current in the loop of wire through Lenz's law and the right-hand rule by looking at how the magnetic flux through the loop of wire changes in each case. Remember that magnetic fields emanate from the N pole and terminate on the S pole of a magnet.

SOLVE

Part a)

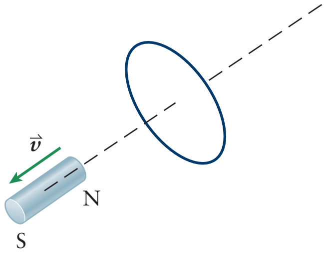


(a)

Figure 20-5 Problem 33

The induced current in the loop will be $\boxed{\text{counterclockwise}}$, as seen from the side with the magnet.

Part b)

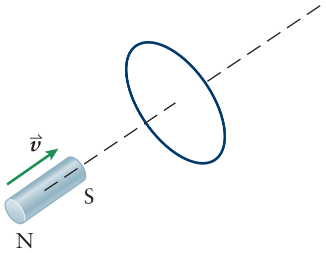


(b)

Figure 20-6 Problem 33

The induced current in the loop will be $\boxed{\text{clockwise}}$, as seen from the side with the magnet.

Part c)

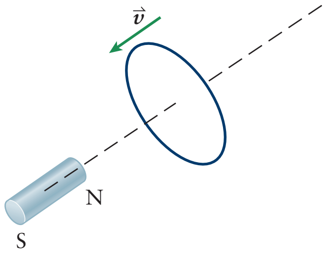


(c)

Figure 20-7 Problem 33

The induced current in the loop will be $\boxed{\text{clockwise}}$, as seen from the side with the magnet.

Part d)

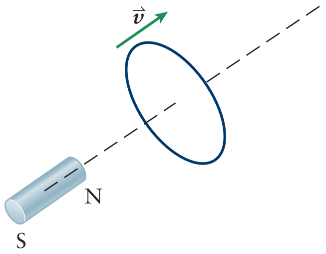


(d)

Figure 20-8 Problem 33

The induced current in the loop will be $\boxed{\text{counterclockwise}}$, as seen from the side with the magnet.

Part e)

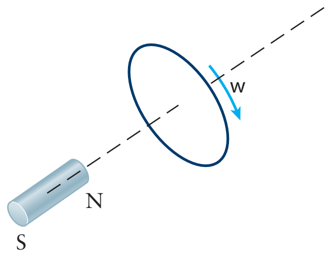


(e)

Figure 20-9 Problem 33

The induced current in the loop will be $\boxed{\text{clockwise}}$, as seen from the side with the magnet.

Part f)



(f)

Figure 20-10 Problem 33

There $\boxed{\text{will not be an induced current}}$ in this case.

REFLECT

There must be a change in the magnetic flux in order for a current to be induced in the wire. In part (f), the magnetic flux through the loop remains constant as the loop spins in its plane.

20.34

SET UP

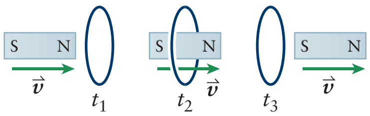


Figure 20-11 Problem 34

A bar magnet, leading with its north pole, is moved at a constant velocity through a wire loop. We'll assume the magnet starts infinitely far to the left of the loop at $t = 0$. As the bar magnet passes through the loop, the magnetic flux is constantly changing, which induces a voltage in the loop according to Faraday's law and Lenz's law. In order to draw a qualitative sketch of the induced voltage in the loop, we will define the area vector of the loop to point toward the initial location of the magnet (to the left in the above figure); this defines our sense of positive induced voltage. The magnetic field pointing to the right is getting stronger as the magnet nears the loop before t_1. Since this corresponds to an increasing negative flux, this will generate an increasingly positive induced voltage until it reaches a maximum. Similarly, after the magnet leaves the loop, the magnetic field pointing to the right is getting weaker. This corresponds to a decreasing negative flux, which results in an increasingly negative induced voltage until it reaches a minimum. The flux is changing the most at t_2, so this corresponds to a maximum in the plot of the magnetic flux versus time. A maximum in the flux corresponds to an induced voltage of zero since the induced current will change direction at that moment.

SOLVE

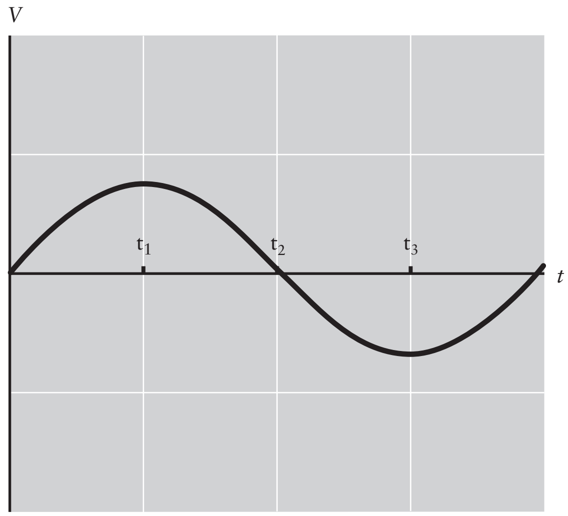


Figure 20-12 Problem 34

REFLECT

The induced voltage is related to the slope of the magnetic flux as a function of time.

20.35

SET UP

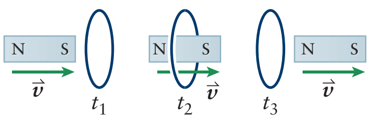


Figure 20-13 Problem 35

A bar magnet, leading with its south pole, is moved at a constant velocity through a wire loop. We'll assume the magnet starts infinitely far to the left of the loop at $t = 0$. As the bar magnet passes through the loop, the magnetic flux is constantly changing, which induces a voltage in the loop according to Faraday's and

Lenz's law. In order to draw a qualitative sketch of the induced voltage in the loop, we will define the area vector of the loop to point towards the initial location of the magnet (to the left in the above figure); this defines our sense of positive induced voltage. The magnetic field pointing to the left is getting stronger as the magnet nears the loop before t_1. Since this corresponds to an increasing positive flux, this will generate an increasingly negative induced voltage until it reaches a minimum. Similarly, after the magnet leaves the loop, the magnetic field pointing to the left is getting weaker. This corresponds to a decreasing positive flux, which results in an increasingly positive induced voltage until it reaches a maximum. The flux is changing the most at t_2, so this corresponds to a maximum in the plot of the magnetic flux versus time. A maximum in the flux corresponds to an induced voltage of zero since the induced current will change direction at that moment.

SOLVE

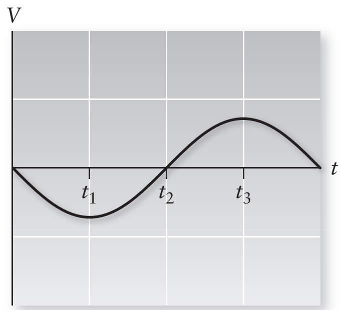


Figure 20-14 Problem 35

REFLECT

The induced voltage is related to the slope of the magnetic flux as a function of time.

20.36

SET UP

A single-turn circular loop of wire that has a radius of 5.0×10^{-2} m lies in a plane perpendicular to a uniform magnetic field. The magnitude of the field changes from $B = 0.20$ T to $B = 0.40$ T within $\Delta t = 0.12$ s. We can use Faraday's law to calculate the induced potential in the loop due to the changing magnetic flux.

SOLVE

$$|\varepsilon| = \left|-\frac{d\Phi_B}{dt}\right| = \left|\frac{\Delta(BA\cos(0°))}{\Delta t}\right| = \left|A\frac{\Delta B}{\Delta t}\right|$$

$$= \left|(\pi)(5.0 \times 10^{-2}\ \text{m})^2\left(\frac{(0.40\ \text{T}) - (0.20\ \text{T})}{0.12\ \text{s}}\right)\right| = \boxed{1.3 \times 10^{-2}\ \text{V}}$$

REFLECT

The current induced in the loop will be in a direction such that a magnetic field is created to oppose this change in flux.

20.37

SET UP

An electromagnetic generator consists of a wound coil ($N = 100$) that has an area of $A = 400\text{ cm}^2$. The coil rotates at $\omega = 60\frac{\text{rev}}{\text{s}}$ in a magnetic field with strength $B = 0.25\text{ T}$. The maximum induced potential is given by $\varepsilon_{\text{max}} = NBA(2\pi\omega)$, where ω is in rev/s.

SOLVE

$$\varepsilon_{\text{max}} = NBA(2\pi\omega) = (100)(0.25\text{ T})\left(400\text{ cm}^2 \times \left(\frac{1\text{ m}}{100\text{ cm}}\right)^2\right)(2\pi)\left(60\frac{\text{rev}}{\text{s}}\right) = \boxed{377\text{ V}}$$

REFLECT

This is the maximum value; the actual potential oscillates between $+377$ V and -377 V.

20.38

SET UP

A rectangular loop of wire with length a and width b lies in the xy-plane. There is a time- and space-dependent magnetic field $\vec{B} = B_0[x\cos(\omega t)\hat{x} + y\cos(\omega t)\hat{z}]$ within the loop. Because the magnetic field changes with time, the magnetic flux through the loop will change in time and there will be an induced potential in the loop of wire according to Faraday's law, $\varepsilon = -\frac{d\Phi_B}{dt}$. We will need to integrate over the area of the loop to calculate the magnetic flux through the loop, $\Phi_B = \int \vec{B}\cdot d\vec{A}$. We will define the area vector of the loop to point toward $+z$. Once we have an algebraic expression for the induced potential in the loop as a function of time, we can take the absolute value of it and plug in the following values to find the magnitude of the induced potential at $t = 2.00$ s: $a = 0.200$ m, $b = 0.100$ m, $B_0 = 0.575$ T/m, and $\omega = 2.22$ rad/s.

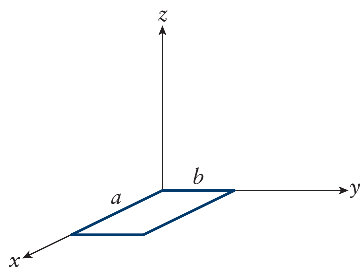


Figure 20-15 Problem 38

SOLVE

Part a)

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left[\int \vec{B}\cdot d\vec{A}\right] = -\frac{d}{dt}\left[\int_0^a\int_0^b (B_0(x\cos(\omega t)\hat{x} + y\sin(\omega t)\hat{z}))\cdot((dx)(dy)\hat{z})\right]$$

$$= -B_0\frac{d}{dt}\left[\int_0^a\int_0^b y\sin(\omega t)dxdy\right] = -B_0\frac{d}{dt}\left[(\sin(\omega t))\int_0^a dx\int_0^b ydy\right]$$

$$= -B_0\frac{d}{dt}\left[(\sin(\omega t))[x]_0^a\left[\frac{y^2}{2}\right]_0^b\right] = -\frac{B_0ab^2}{2}\frac{d}{dt}[\sin(\omega t)] = \boxed{-\frac{B_0ab^2\omega}{2}\cos(\omega t)}$$

Part b)

$$|\varepsilon| = \left|-\frac{Bab^2\omega}{2}\cos(\omega t)\right|$$

$$= \left|-\frac{\left(0.575\frac{\text{T}}{\text{m}}\right)(0.200\text{ m})(0.100\text{ m})^2\left(2.22\frac{\text{rad}}{\text{s}}\right)}{2}\cos\left(\left(2.22\frac{\text{rad}}{\text{s}}\right)(2.00\text{ s})\right)\right|$$

$$= \boxed{3.43\times10^{-4}\text{ V}}$$

REFLECT

Defining the area vector of the loop to point toward $+z$ means a counterclockwise current is considered positive.

20.39

SET UP

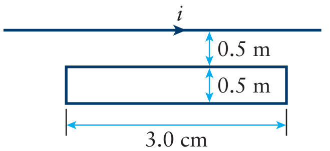


Figure 20-16 Problem 39

A long, straight wire that carries a time-varying current i is parallel to the long side of a rectangular loop of wire and is 0.5 m from the closer side. The loop has a length of 0.030 m and a width of 0.5 m. The magnetic field due to the wire points into the page at the location of the loop and has a magnitude $B = \frac{\mu_0 i}{2\pi y}$, where y is the distance from the wire. Because the magnitude of the magnetic field depends on the distance from the wire, we will need to integrate over the area of the loop in order to calculate the magnetic flux through the loop. We'll define the leftmost side of the loop to be at $x = 0$ and the long, straight wire to be at $y = 0$. Once we have an expression for the magnetic flux through the loop as a function of the current, we can differentiate both sides and rearrange to calculate the time rate of change of the current when the induced voltage in the loop has a magnitude $|\varepsilon| = 2$ V.

SOLVE

Magnetic flux as a function of current:

$$\Phi_B = \int\vec{B}\cdot d\vec{A} = \int_{-0.5}^{-1}\int_0^{0.030}\left(\frac{\mu_0 i}{2\pi y}\right)(\cos(0°))dxdy = \frac{\mu_0 i}{2\pi}\int_0^{0.030}dx\int_{-0.5}^{-1}\frac{dy}{y} = \frac{\mu_0 i}{2\pi}[x]_0^{0.030}[\ln(y)]_{-0.5}^{-1}$$

$$= \frac{0.030\mu_0 i}{2\pi}[\ln(-1)-\ln(-0.5)] = \frac{0.030\mu_0 i}{2\pi}\left[\ln\left(\frac{-1}{-0.5}\right)\right] = \frac{0.030\ln(2)\mu_0 i}{2\pi}$$

Rate of change of the current:

$$|\varepsilon| = \left|-\frac{d\Phi_B}{dt}\right| = \frac{d}{dt}\left[\frac{0.030\ln(2)\mu_0 i}{2\pi}\right] = \left(\frac{0.030\ln(2)\mu_0}{2\pi}\right)\frac{di}{dt}$$

$$\frac{di}{dt} = \frac{|\varepsilon|}{\left(\frac{0.030\ln(2)\mu_0}{2\pi}\right)} = \frac{2\pi|\varepsilon|}{0.030\ln(2)\mu_0} = \frac{2\pi|2\text{ V}|}{0.030\ln(2)\left(4\pi\times10^{-7}\frac{\text{T}\cdot\text{m}}{\text{A}}\right)} = \boxed{4.8\times10^8\frac{\text{A}}{\text{s}}}$$

REFLECT
By choosing the area vector to be parallel to the magnetic field vector and point into the page, we've defined clockwise to be the positive sense of current. Induced voltages are usually much less than 1, so we would expect the time rate of change in this case to be very large.

20.40

SET UP
A 50-turn square coil of wire has a cross-sectional area $A = 5.00$ cm^2 and a resistance $R = 20.0\ \Omega$. The plane of the coil is initially perpendicular to a uniform magnetic field of magnitude $B = 1.00$ T. The coil is suddenly rotated through an angle of 60 degrees in a time period of 0.200 s. Even though the magnetic field and the cross-sectional area remain constant, the angle between the magnetic field vector and the area vector changes. This means a potential and, therefore, a current will be induced in the coil. By combining Faraday's law and Ohm's law, we can calculate the total charge that flows past a point in the coil while the loop is rotated. If the loop is rotated a full 360 degrees, we would expect the net charge passing a given point in the loop to be zero.

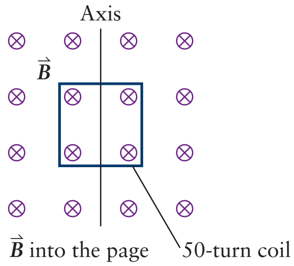


Figure 20-17 Problem 40

SOLVE

Part a)
Change in magnetic flux:

$$\Delta\Phi_B = (BA\cos(\theta_2)) - (BA\cos(\theta_1)) = BA(\cos(\theta_2) - \cos(\theta_1))$$

Faraday's law:

$$|\varepsilon| = \left|-N\frac{\Delta\Phi_B}{\Delta t}\right| = iR = \left(\frac{\Delta q}{\Delta t}\right)R$$

$$\Delta q = \frac{N\Delta\Phi_B}{R} = \frac{NBA(\cos(\theta_2) - \cos(\theta_1))}{R}$$

$$= \frac{(50)(1.00\text{ T})\left(5.00\text{ cm}^2 \times \left(\frac{1\text{ m}}{100\text{ cm}}\right)^2\right)(\cos(60°) - \cos(0°))}{20.0\ \Omega} = \boxed{6.25 \times 10^{-4}\text{ C}}$$

Part b) Zero. Charge travels in one direction for the first half of the rotation, but then the change in flux reverses directions, so the same amount of charge flows in the opposite direction for the second half of the rotation, leading to no net movement of charge for the full rotation.

REFLECT
The time interval is irrelevant since we're only interested in the total amount of charge flowing past a given point.

20.41

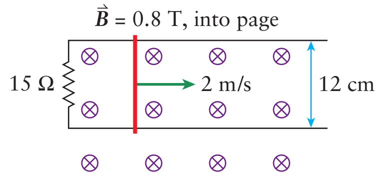


Figure 20-18 Problem 41

SET UP

A pair of parallel conducting rails are a distance $L = 0.12$ m apart and situated at right angles to a uniform magnetic field $B = 0.8$ T pointed into the page. A resistor ($R = 15\ \Omega$) is connected across the rails. A conducting bar is placed on top of the rails and is moved at a constant speed of $v = 2$ m/s to the right. The bar creates a closed loop with a portion of each rail and the resistor. The area of the loop is equal to Lx, where x is the horizontal distance between the resistor and the bar; we'll assume the area vector also points into the page. The area of this closed loop increases as the bar moves to the right, which means the magnetic flux is also increasing. We can use Faraday's law and Ohm's law to calculate the magnitude of the current flowing through the resistor. Since the magnetic flux into the page is increasing, current will be induced in such a way as to counteract this increase according to Lenz's law. From the right hand rule, this corresponds to a counterclockwise current through the loop, so the current through the bar points up. The bar will experience a magnetic force due to the external magnetic field interacting with the current. The current is perpendicular to the field. In this case, the magnitude of the force is given by $F = iLB$, and the direction is given by the right hand rule for cross products.

SOLVE

Part a)

Induced potential:

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}[\vec{B}\cdot\vec{A}] = -\frac{d}{dt}[BA] = -B\frac{d}{dt}[A] = -B\frac{d}{dt}[Lx] = -BL\frac{d}{dt}[x]$$

$$= -BLv = -(0.8\ \text{T})(0.12\ \text{m})\left(2\frac{\text{m}}{\text{s}}\right) = -0.192\ \text{V}$$

Current:

$$\varepsilon = iR$$

$$i = \frac{\varepsilon}{R} = \frac{|-0.192\ \text{V}|}{15\ \Omega} = \boxed{0.0128\ \text{A}}$$

Part b) The current in the bar points $\boxed{\text{up}}$.

Part c)

$$\vec{F} = i\vec{L}\times\vec{B}$$

$$F = iLB = (0.0128\ \text{A})(0.12\ \text{m})(0.8\ \text{T}) = 0.00123\ \text{N}$$

The magnetic force on the bar has a magnitude of $\boxed{\text{0.00123 N and points to the left}}$.

REFLECT

By defining our area vector to point down, we've implicitly chosen a clockwise current to be positive. The negative sign in our induced potential means we have a "negative" current in the loop, *i.e.*, one that flows counterclockwise.

20.42

SET UP

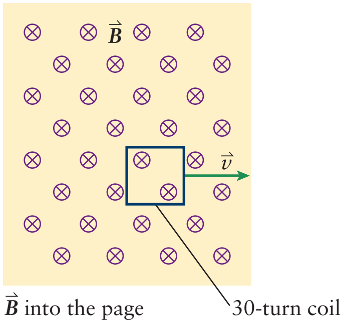


Figure 20-19 Problem 42

A square coil made up of 30 turns of wire with a side length $L = 0.10$ m is moving at a constant speed $v = 0.020$ m/s to the right in a region with a constant, uniform magnetic field $B = 0.60$ T directed perpendicular to the coil. The field drops sharply to zero outside of this region. The induced current in the loop is zero while the coil is traveling completely inside or completely outside the region with the magnetic field because the magnetic flux through the coil is not changing in time. The magnetic flux will change, though, as the coil is leaving the region with the magnetic field. If we define the x-axis to be the horizontal direction and the horizontal length of the coil still in the field to be x, the infinitesimal area element $dA = Ldx$ because the amount of the coil in the vertical direction remains constant. Plugging this into the definition for the magnetic flux and differentiating the expression for the magnetic flux with respect to time, we can relate the magnitude of the time-changing magnetic flux to the current in the coil through Faraday's law and Ohm's law. The total charge that flows past a given point in the coil is equal to the current in the coil multiplied by the total time it takes the coil to leave the field, which we can calculate from the speed of the coil. Finally, the current that flows through the coil will be constant in magnitude and direction for the entire time the coil is transitioning between the regions because it is moving at a constant velocity. We'll define clockwise to be the positive sense of current in the loop.

SOLVE

Part a)

$$\boxed{i = 0}$$

Part b)
Magnitude of the changing magnetic flux:

$$\left|\frac{d\Phi_B}{dt}\right| = \left|\frac{d}{dt}\left[\int \vec{B} \cdot d\vec{A}\right]\right| = \left|B\frac{d}{dt}\left[\int \cos(0°)(Ldx)\right]\right| = \left|BL\frac{dx}{dt}\right| = BLv$$

Current:

$$|\varepsilon| = \left|-N\frac{d\Phi_B}{dt}\right| = iR$$

$$i = \frac{N\left|\frac{d\Phi_B}{dt}\right|}{R} = \frac{NBLv}{R} = \frac{(30)(0.60 \text{ T})(0.10 \text{ m})\left(0.020\frac{\text{m}}{\text{s}}\right)}{0.82\ \Omega} = \boxed{0.044 \text{ A}}$$

Part c)

$$\boxed{i = 0}$$

Part d)

$$i = \frac{\Delta q}{\Delta t} = \frac{\Delta q}{\left(\frac{\Delta x}{v}\right)} = \frac{v\Delta q}{L}$$

$$\Delta q = \frac{iL}{v} = \frac{(0.044\text{ A})(0.10\text{ m})}{\left(0.020\frac{\text{m}}{\text{s}}\right)} = \boxed{0.22\text{ C}}$$

Part e)

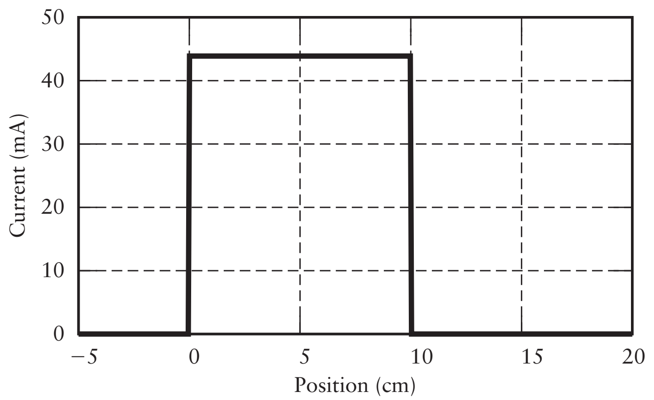


Figure 20-20 Problem 42

REFLECT

It will take the coil 5 s to completely exit the field. A clockwise current is positive because we chose to align the area vector of the coil with the magnetic field.

20.43

SET UP

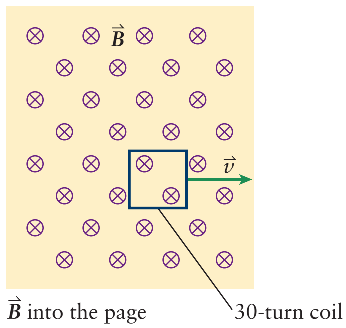


Figure 20-21 Problem 43

A square coil made up of 30 turns of wire with a side length $L = 0.10$ m is moving at a constant speed $v = 0.020$ m/s to the right in a region with a constant, uniform magnetic field $B = 0.60$ T directed perpendicular to the coil. The field drops sharply to zero outside of this region. The induced current in the loop is zero while the coil is traveling completely inside or completely outside the region with the magnetic field because the magnetic flux through the coil is not changing in time. The force acting on each segment of the coil is given by $\vec{F} = i\vec{L} \times \vec{B}$, which means the force on each segment of the coil is equal to zero because the current in the coil is equal to zero. In Problem 20.42, we calculated the induced current in the coil to be $i = 0.044$ A in the clockwise direction as it exits the magnetic field. We can use $\vec{F} = i\vec{L} \times \vec{B}$ to calculate the magnitude and direction of the force acting on each portion of the coil. We'll define the x-axis to be the horizontal direction and the horizontal length of the coil still in the field to be x. If the coil enters the field from the left, we would expect the directions for all of our answers to flip because the induced current would now be counterclockwise according to Lenz's law.

SOLVE

Part a) In parts (a) and (c) of Problem 20.42, the force on each segment of the loop is zero.

In part (b) of Problem 20.42, the force on the right-hand segment is zero. The force on the top segment is $NixB$ in the upward direction. The force on the bottom segment is $NixB$ in the downward direction. The force on the left segment is $NiLB = (30)(0.044\text{ A})(0.100\text{ m})(0.60\text{ T}) = 0.080\text{ N}$ to the left.

Part b) The current will be counterclockwise as viewed. The force on the left-hand segment will be zero. The force on the top and bottom segments will be downward and upward, respectively. The force on the right-hand segment will be to the left.

REFLECT

Because there is a net force acting to the left on the loop as it exits the field, you will need to pull harder to the right in order to keep the loop moving at a constant velocity.

20.44

SET UP

The time rate of change of the current in an inductor ($L = 25 \times 10^{-6}$ H) is $\frac{di}{dt} = 58 \times 10^{-3}\,\frac{\text{A}}{\text{s}}$. The magnitude of the induced potential is given by $|\varepsilon| = \left|-L\frac{di}{dt}\right|$.

SOLVE

$$|\varepsilon| = \left|-L\frac{di}{dt}\right| = \left|-(25 \times 10^{-6}\text{ H})\left(58 \times 10^{-3}\frac{\text{A}}{\text{s}}\right)\right| = \boxed{1.5 \times 10^{-6}\text{ V}}$$

REFLECT

Induced voltages are usually much smaller than 1, so our answer seems reasonable.

20.45

SET UP

A radio antenna is made from a solenoid of length $l = 0.030$ m and cross-sectional area $A = 0.50\text{ cm}^2$. The solenoid, which is filled with air, consists of $N = 300$ turns copper wire. The inductance of a solenoid is $L = \frac{\mu_0 N^2 A}{l}$.

SOLVE

$$L = \frac{\mu_0 N^2 A}{l} = \frac{\left(4\pi \times 10^{-7}\frac{\text{H}}{\text{m}}\right)(300)^2\left(0.50\text{ cm}^2 \times \left(\frac{1\text{ m}}{100\text{ cm}}\right)^2\right)}{(0.030\text{ m})}$$

$$= \boxed{1.88 \times 10^{-4}\text{ H} = 0.188\text{ mH}}$$

REFLECT

The inductance of a solenoid only depends on its dimensions and a physical constant, so the fact that the wire is copper does not affect the inductance.

20.46

SET UP

The current through an inductor ($L = 0.250$ H) is $i = 0.055$ A. The energy stored in the inductor is given by $U_L = \frac{1}{2}Li^2$.

SOLVE

$$U_L = \frac{1}{2}Li^2 = \frac{1}{2}(0.250\ \text{H})(0.055\ \text{A})^2 = \boxed{3.8 \times 10^{-4}\ \text{J}}$$

REFLECT

The energy stored in an inductor is related to the current through the element, whereas the energy stored in a capacitor is related to the charge on the plates.

20.47

SET UP

The current in an inductor ($L = 0.028$ H) varies as a function of time according to $i(t) = i_0 \cos(\omega t)$, where $i_0 = 0.125$ A and $\omega = 25$ rad/s. The induced voltage in the inductor is given by $\varepsilon = -L\frac{di}{dt}$. To find the exact value of the induced voltage at $t = 1.5$ s, we will need to evaluate the derivative at that point.

SOLVE

$$\varepsilon = -L\frac{di}{dt} = -L\frac{d}{dt}[i_0 \cos(\omega t)] = Li_0(\omega \sin(\omega t)) = Li_0\omega \sin(\omega t)$$

$$= (0.028\ \text{H})(0.125\ \text{A})\left(25\frac{\text{rad}}{\text{s}}\right)\sin\left(\left(25\frac{\text{rad}}{\text{s}}\right)(1.5\ \text{s})\right) = \boxed{-0.017\ \text{V}}$$

REFLECT

Just as the current in the inductor oscillates in time, we should expect the induced voltage in the inductor to oscillate in time. As we see here (and will learn more about in Chapter 21), the current and voltage will be out of phase by 90 degrees.

20.48

SET UP

A solenoid that consists of $N = 1600$ turns has a cross-sectional area $A = 6.00 \times 10^{-4}\ \text{m}^2$ and a length $l = 0.200$ m. A current $i = 2.80$ A flows through the solenoid. The inductance of a solenoid is equal to $L = \frac{\mu_0 N^2 A}{l}$, and the energy stored in the solenoid is equal to $U_L = \frac{1}{2}Li^2$. From these expressions, we see that the inductance and the energy stored in the solenoid are both directly proportional to the cross-sectional area. Therefore, if the cross-sectional area is increased, these quantities will increase by the same amount.

SOLVE

Part a)

$$L = \frac{\mu_0 N^2 A}{l} = \frac{\left(4\pi \times 10^{-7}\frac{\text{H}}{\text{m}}\right)(1600)^2(6.00 \times 10^{-4}\text{ m}^2)}{0.200\text{ m}} = \boxed{9.65 \times 10^{-3}\text{ H}}$$

Part b)

$$U_L = \frac{1}{2}Li^2 = \frac{1}{2}(9.65 \times 10^{-3}\text{ H})(2.80\text{ A})^2 = \boxed{3.78 \times 10^{-2}\text{ J}}$$

Part c) The inductance is proportional to the cross-sectional area. Therefore, if the cross-sectional area is doubled, the $\boxed{\text{inductance is doubled}}$. The energy stored in the inductor is proportional to the inductance, so the $\boxed{\text{stored potential energy will also double}}$.

REFLECT

If the number of turns of the solenoid were doubled instead, the inductance and stored potential energy would quadruple.

20.49

SET UP

An LC circuit is made up of an inductor ($L = 14.4 \times 10^{-3}$ H) and a fully charged capacitor ($C = 225 \times 10^{-6}$ F, $Q_0 = 300\ \mu$C). The circuit is completed at $t = 0$. The charge on the capacitor as a function of time is $q(t) = Q_0 \cos(\omega_0 t + \phi)$, where the natural frequency $\omega_0 = \sqrt{\frac{1}{LC}}$. We can determine the phase angle ϕ from the initial conditions of the system. Once we have the expression for the charge as a function of time, we can solve for the charge at $t = 7.5 \times 10^{-3}$ s.

SOLVE

Natural frequency:

$$\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(14.4 \times 10^{-3}\text{ H})(225 \times 10^{-6}\text{ F})}} = 555.6\ \frac{\text{rad}}{\text{s}}$$

Charge as a function of time:

$$q(t) = Q_0 \cos(\omega_0 t + \phi)$$

$$q(0) = Q_0 = Q_0 \cos(\phi)$$

$$\phi = \arccos(1) = 0$$

$$q(t) = Q_0 \cos(\omega_0 t)$$

Charge at $t = 7.5 \times 10^{-3}$ s:

$$q(7.5 \times 10^{-3}\text{ s}) = (300\ \mu\text{C})\cos\left(\left(555.6\ \frac{\text{rad}}{\text{s}}\right)(7.5 \times 10^{-3}\text{ s})\right) = \boxed{-156\ \mu\text{C}}$$

REFLECT

The negative sign in our calculation for the charge at $t = 7.5 \times 10^{-3}$ s means the polarity of the capacitor has flipped relative to its initial setup—the positive plate is now negative and *vice versa*.

20.50

SET UP

An LC circuit is made up of an inductor ($L = 14.4 \times 10^{-3}$ H) and a capacitor ($C = 225 \times 10^{-6}$ F). The circuit is completed at $t = 0$, and the capacitor is fully charged ($Q_0 = 300\ \mu$C) at $t = 2.5 \times 10^{-3}$ s. The charge on the capacitor as a function of time is $q(t) = Q_0 \cos(\omega_0 t + \phi)$, where the natural frequency $\omega_0 = \sqrt{\frac{1}{LC}}$. We can determine the phase angle ϕ using the time at which the charge on the capacitor is a maximum. Once we have the expression for the charge as a function of time, we can solve for the charge at $t = 7.5 \times 10^{-3}$ s.

SOLVE

Natural frequency:

$$\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(14.4 \times 10^{-3}\ \text{H})(225 \times 10^{-6}\ \text{F})}} = 555.6\ \frac{\text{rad}}{\text{s}}$$

Charge as a function of time:

$$q(t) = Q_0 \cos(\omega_0 t + \phi)$$

$$q(2.5 \times 10^{-3}\ \text{s}) = Q_0 = Q_0 \cos(\omega_0(2.5 \times 10^{-3}\ \text{s}) + \phi)$$

$$\omega_0(2.5 \times 10^{-3}\ \text{s}) + \phi = \arccos(1) = 0$$

$$\phi = -\omega_0(2.5 \times 10^{-3}\ \text{s}) = -\left(555.6\ \frac{\text{rad}}{\text{s}}\right)(2.5 \times 10^{-3}\ \text{s}) = -1.39\ \text{rad}$$

$$q(t) = Q_0 \cos(\omega_0 t - 1.39)$$

Charge at $t = 7.5 \times 10^{-3}$ s:

$$q(7.5 \times 10^{-3}\ \text{s}) = (300\ \mu\text{C}) \cos\left(\left(555.6\ \frac{\text{rad}}{\text{s}}\right)(7.5 \times 10^{-3}\ \text{s}) - 1.39\right) = \boxed{-280\ \mu\text{C}}$$

REFLECT

The charge at $t = 7.5 \times 10^{-3}$ s should be larger in this case than in Problem 20.49 because the capacitor doesn't fully charge until $t = 2.5 \times 10^{-3}$ s.

20.51

SET UP

An LC circuit is made up of an inductor ($L = 150 \times 10^{-3}$ H) and a capacitor ($C = 1000 \times 10^{-6}$ F). The natural frequency f_0 of this circuit is equal to $f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$.

SOLVE

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \frac{1}{2\pi}\sqrt{\frac{1}{(150 \times 10^{-3}\ \text{H})(1000 \times 10^{-6}\ \text{F})}} = \boxed{13\ \text{Hz}}$$

REFLECT
Be careful not to confuse frequency and angular frequency.

20.52

SET UP
An LC circuit is made up of an inductor ($L = 1.0 \times 10^{-3}$ H) and a capacitor C. The natural frequency of this circuit is $f_0 = 980 \times 10^3$ Hz. We can use the definition of the natural frequency, $f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$, to calculate the capacitance C.

SOLVE

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

$$C = \frac{1}{L(2\pi f_0)^2} = \frac{1}{4\pi^2 f_0^2 L} = \frac{1}{4\pi^2(980 \times 10^3\ \text{Hz})(1.0 \times 10^{-3}\ \text{H})} = \boxed{2.6 \times 10^{-11}\ \text{F}}$$

REFLECT
A capacitance of 26 pF is reasonable.

20.53

SET UP
An LC circuit is made up of an inductor ($L = 400 \times 10^{-3}$ H) and a capacitor ($C = 100 \times 10^{-6}$ F). In an LC circuit, both the current and charge are time dependent with the current 90 degrees out of phase with the charge. The maximum current in this particular circuit occurs at $t = 0$, which means the charge on the capacitor is 0. Since the charge and current differ in phase by 90 degrees, it will take one-quarter of a period for the charge to reach its maximum value. The period of the oscillation is equal to $\frac{2\pi}{\omega_0}$, where $\omega_0 = \sqrt{\frac{1}{LC}}$.

SOLVE
Natural frequency:

$$\omega = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(400 \times 10^{-3}\ \text{H})(100 \times 10^{-6}\ \text{F})}} = 158\frac{\text{rad}}{\text{s}}$$

Period of oscillation:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\left(158\frac{\text{rad}}{\text{s}}\right)} = 0.0398\ \text{s}$$

One-quarter period:

$$\frac{T}{4} = \frac{0.0398\ \text{s}}{4} = \boxed{0.01\ \text{s}}$$

REFLECT

We could have also solved this using calculus. The extrema of a function are found by setting the derivative equal to zero and solving. We're interested in the maximum charge; the first derivative of the charge with respect to time is the current. The current as a function of time in this LC circuit is $i(t) = i_{\text{max}}\cos(\omega_0 t)$. Setting this equal to zero and solving for t, we find that $t = \dfrac{\arccos(0)}{\omega_0} = \dfrac{\left(\dfrac{\pi}{2}\right)}{\omega_0} = \dfrac{2\pi}{4\omega_0} = \dfrac{T}{4}$.

20.54

SET UP

A capacitor ($C = 200 \times 10^{-12}$ F) is charged to $V = 120$ V and then immediately connected to an inductor. The maximum energy stored in the inductor is equal to the total initial energy stored in the capacitor, $U_{E,\text{i}} = \dfrac{1}{2}CV^2$, through conservation of energy.

SOLVE

$$U_{L,\text{max}} = U_{E,\text{i}} = \frac{1}{2}CV^2 = \frac{1}{2}(200 \times 10^{-12}\ \text{F})(120\ \text{V})^2 = \boxed{1.44 \times 10^{-6}\ \text{J}}$$

REFLECT

The stored energy in the circuit will oscillate between energy stored in the electric field and energy stored in the magnetic field.

20.55

SET UP

An LC circuit is made up of an inductor ($L = 0.250$ H) and a capacitor ($C = 875 \times 10^{-6}$ F). The selectivity of an LC circuit is given by the quotient $\dfrac{L}{C}$.

SOLVE

$$\text{selectivity} = \frac{L}{C} = \frac{0.250\ \text{H}}{875 \times 10^{-6}\ \text{F}} = \boxed{286\frac{\text{H}}{\text{F}}}$$

REFLECT

A large selectivity corresponds to a narrow filter bandwidth.

20.56

SET UP

An LC circuit is made up of an inductor ($L = 150 \times 10^{-6}$ H) and a capacitor ($C = 2.20 \times 10^{-12}$ F). Assuming the capacitor is fully charged at $t = 0$, the charge on the capacitor as

a function of time is given by $q(t) = Q_0 \cos(\omega_0 t)$, where $\omega_0 = \sqrt{\frac{1}{LC}}$. Rearranging this expression, we can solve for the time at which $q(t) = 0.75Q_0$.

SOLVE

$$q(t) = Q_0 \cos(\omega_0 t) = Q_0 \cos\left(\left(\sqrt{\frac{1}{LC}}\right)t\right)$$

$$t = (\sqrt{LC}) \arccos\left(\frac{q(t)}{Q_0}\right)$$

$$= (\sqrt{(150 \times 10^{-6}\ \text{H})(2.20 \times 10^{-12}\ \text{F})}) \arccos\left(\frac{0.75Q_0}{Q_0}\right) = \boxed{1.31 \times 10^{-8}\ \text{s}}$$

REFLECT

Given the small values for L and C, we would expect the time to reach 75% of the maximum value to be on the order of 10^{-8} s.

20.57

SET UP

We can use the expression for the energy stored in a capacitor, $U_E = \frac{q^2}{2C}$, to determine the ratio between the energy stored when the charge on the capacitor is one-half the maximum $\left(\text{that is, } q = \frac{Q_0}{2}\right)$ and the total energy stored in both the inductor and the capacitor in a given LC circuit. The sum of the energy stored in both the capacitor and the inductor at any given time is equal to the energy stored in the capacitor when it has its maximum charge (that is, $q = Q_0$).

SOLVE

$$\frac{U_E}{U_{\text{total}}} = \frac{\left(\frac{\left(\frac{Q_0}{2}\right)^2}{2C}\right)}{\left(\frac{Q_0^2}{2C}\right)} = \frac{\left(\frac{Q_0^2}{4}\right)}{Q_0^2} = \boxed{\frac{1}{4}}$$

REFLECT

When the charge on the capacitor is one-half the maximum value, three-quarters of the energy is stored in the inductor.

20.58

SET UP

The ratio between the energy stored in the capacitor and the total energy stored in both the inductor and the capacitor in a given LC circuit is equal to $\frac{1}{2}$. We can use the expression for

the energy stored in a capacitor, $U_E = \frac{q^2}{2C}$, to determine the charge stored on the capacitor in terms of the maximum charge Q_0 in this case. The sum of the energy stored in both the capacitor and the inductor at any given time is equal to the energy stored in the capacitor when it has its maximum charge (that is, $q = Q_0$).

SOLVE

$$\frac{U_E}{U_{\text{total}}} = \frac{\left(\frac{q^2}{2C}\right)}{\left(\frac{Q_0^2}{2C}\right)} = \frac{q^2}{Q_0^2} = \frac{1}{2}$$

$$\boxed{q = \frac{Q_0}{\sqrt{2}}}$$

REFLECT

When $q = 0.707Q_0$, the energy is equally distributed between the capacitor and inductor in an LC circuit.

20.59

SET UP

An LR circuit is made up of an inductor ($L = 22 \times 10^{-3}$ H) and a resistor ($R = 360\ \Omega$). The time constant for an LR circuit is $\tau = \frac{L}{R}$.

SOLVE

$$\tau = \frac{L}{R} = \frac{22 \times 10^{-3}\text{ H}}{360\ \Omega} = \boxed{6.11 \times 10^{-5}\text{ s}}$$

REFLECT

The current in an LR circuit exponentially decays or grows, whereas the current in an LC circuit oscillates in time.

20.60

SET UP

The time constant for an LR circuit made up of an inductor L and resistor R_1 is $\tau_1 = 256$ ms. We can use the expression for the time constant of an LR circuit, $\tau = \frac{L}{R}$, and set up a ratio to calculate the time constant of the circuit if the resistance is increased by a factor of 6 (that is, $R_2 = 6R_1$) while the inductance remains constant.

SOLVE

$$\frac{\tau_2}{\tau_1} = \frac{\left(\frac{L_2}{R_2}\right)}{\left(\frac{L_1}{R_1}\right)} = \frac{\left(\frac{L}{6R_1}\right)}{\left(\frac{L}{R_1}\right)} = \frac{1}{6}$$

$$\tau_2 = \frac{\tau_1}{6} = \frac{256\text{ ms}}{6} = \boxed{42.7\text{ ms}}$$

REFLECT

The time constant is inversely proportional to the resistance, so an increase in the resistance will result in a decrease in the time constant.

20.61

SET UP

An LR circuit consists of an inductor ($L = 10$ H) and a resistor ($R = 100\ \Omega$). The time constant for an LR circuit is $\tau = \frac{L}{R}$.

SOLVE

$$\tau = \frac{L}{R} = \frac{10\text{ H}}{100\ \Omega} = \boxed{0.10\text{ s}}$$

REFLECT

The time constant of a circuit allows us to compare the time dependence of different circuits.

20.62

SET UP

An LR circuit is made up of an inductor ($L = 12$ H), a resistor ($R = 3.0\ \Omega$), and a battery ($V = 12$ V). The current at $t = 2$ s can be found from the expression for the current as a function of time in an LR circuit, $i(t) = \frac{V}{R}\left(1 - e^{-\frac{t}{\tau}}\right)$, where the time constant $\tau = \frac{L}{R}$.

SOLVE

$$i(t) = \frac{V}{R}\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$i(2\text{ s}) = \left(\frac{12\text{ V}}{3.0\ \Omega}\right)\left(1 - e^{-2.0\text{ s}/(\text{L/R})}\right) = \left(\frac{12\text{ V}}{3.0\ \Omega}\right)\left(1 - \mathrm{e}^{-2.0\text{ s}/(12\text{ H}/3.0\ \Omega)}\right) = \boxed{1.6\text{ A}}$$

REFLECT

Current starts to flow at $t = 0$ and exponentially approaches a maximum value as t approaches infinity.

20.63

SET UP

An LR circuit is made up of an inductor ($L = 12\text{ H}$) and a resistor ($R = 3.0\ \Omega$). The current as a function of time in an LR circuit is given by $i(t) = \frac{V}{R}(1 - e^{-\frac{t}{\tau}}) = i_{max}(1 - e^{-\frac{t}{\tau}})$, where the time constant $\tau = \frac{L}{R}$. We can rearrange this equation to calculate the time when $i(t) = 0.5i_{\text{max}}$.

SOLVE

$$i(t) = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right) = i_{\text{max}}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$t = -\left(\frac{L}{R}\right)\ln\left(1 - \frac{i(t)}{i_{\text{max}}}\right) = -\left(\frac{L}{R}\right)\ln\left(1 - \frac{0.5i_{\text{max}}}{i_{\text{max}}}\right) = -\left(\frac{12\text{ H}}{3.0\ \Omega}\right)\ln(0.5) = \boxed{2.8\text{ s}}$$

REFLECT

The time constant for this circuit is 4.0 s. After 4.0 s, the current is 63% of its maximum. Since we're interested in the time it takes to reach 50% of its maximum, our answer should be less than one time constant.

20.64

SET UP

An inductor ($L = 0.050\text{ H}$) is placed in series with a battery ($V = 3.0\text{ V}$), a resistor ($R = 5.0\ \Omega$), and a switch. When the switch is closed at $t = 0$, current will flow through the circuit as a function of time according to $i(t) = \frac{V}{R}\left(1 - e^{-\frac{t}{\tau}}\right)$, where $\tau = \frac{L}{R}$. We can use the given values to find the current at $t = 0.040\text{ s}$ and as t approaches infinity.

SOLVE

$$i(t) = \frac{V}{R}\left(1 - e^{-\frac{t}{\tau}}\right) = \frac{V}{R}(1 - e^{-t/(L/R)}) = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

Part a)

$$i(0.04\text{ s}) = \left(\frac{3.0\text{ V}}{5.0\ \Omega}\right)\left(1 - e^{-\left(\frac{5.0\ \Omega}{50\times 10^{-3}\text{ H}}\right)(0.04\text{ s})}\right) = \boxed{0.59\text{ A}}$$

Part b)

$$i(\infty) = \lim_{t\to\infty}\left[\frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)\right] = \frac{V}{R} = \frac{3.0\text{ V}}{5.0\ \Omega} = \boxed{0.60\text{ A}}$$

REFLECT

Although it will take an infinite amount of time to reach the final value, people usually wait on the order of 5–10 time constants before considering the value "final".

20.65

SET UP

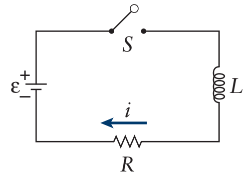


Figure 20-22 Problem 65

An LR circuit is made from a battery ε, an inductor L, a resistor R, and a switch S. At $t = 0$ the switch is closed and current starts to flow. The power dissipated by the resistor as a function is given by $P(t) = i(t)\varepsilon$, where $i(t) = \frac{\varepsilon}{R}\left(1 - e^{-\frac{R}{L}t}\right)$. The maximum power dissipated by the resistor is equal to $P_{\max} = \frac{\varepsilon^2}{R}$. We can combine these relationships to find the time at which one-half of the battery power is dissipated by the resistor. Once we know this time, we can plug it into the expression for the current in the circuit as a function of time to find the current at that instant. Evaluating the derivative of the current at that time will tell us how fast the current is changing at that instant.

SOLVE

Part a)

Maximum power dissipated by the resistor:

$$P_{\max} = \frac{\varepsilon^2}{R}$$

Power dissipated by the resistor as a function of time:

$$P(t) = i(t)\varepsilon = \left(\frac{\varepsilon}{R}\left(1 - e^{-\frac{R}{L}t}\right)\right)\varepsilon = \frac{\varepsilon^2}{R}\left(1 - e^{-\frac{R}{L}t}\right) = P_{\max}\left(1 - e^{-\frac{R}{L}t}\right)$$

Time at which resistor dissipates one-half of the battery's power:

$$P(t) = 0.5P_{\max} = P_{\max}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$0.5 = 1 - e^{-\frac{R}{L}t}$$

$$0.5 = e^{-\frac{R}{L}t}$$

$$t = -\left(\frac{L}{R}\right)\ln(0.5) = \left(\frac{L}{R}\right)\ln(2)$$

Current at this time:

$$i(t) = \frac{\varepsilon}{R}\left(1 - e^{-\frac{R}{L}\left(\left(\frac{L}{R}\right)\ln(2)\right)}\right) = \frac{\varepsilon}{R}\left(1 - e^{-\ln(2)}\right) = \frac{\varepsilon}{R}\left(1 - \left(\frac{1}{2}\right)\right) = \boxed{\frac{\varepsilon}{2R}}$$

Part b)

Rate of change of the current:

$$\frac{di(t)}{dt} = \frac{d}{dt}\left[\frac{\varepsilon}{R}\left(1 - e^{-\frac{R}{L}t}\right)\right] = \frac{\varepsilon}{R}\left(\frac{R}{L}e^{-\frac{R}{L}t}\right) = \frac{\varepsilon}{L}e^{-\frac{R}{L}t}$$

Rate of change of the current at $t = \left(\frac{L}{R}\right)\ln(2)$:

$$\left.\frac{di}{dt}\right|_{t=\left(\frac{L}{R}\right)\ln(2)} = \frac{\varepsilon}{L}e^{-\frac{R}{L}\left(\left(\frac{L}{R}\right)\ln(2)\right)} = \frac{\varepsilon}{L}e^{\ln(0.5)} = \boxed{\frac{\varepsilon}{2L}}$$

REFLECT

We see that the value of the current at a given time is related to the resistance in the circuit and the rate of change of the current is related to the inductance.

20.66

SET UP

A tightly wound solenoid has a length $l = 0.180$ m, diameter $D = 2.00 \times 10^{-2}$ m, and $N =$ 1500 turns. It is made with 22-gauge ($r = 0.322 \times 10^{-3}$ m) copper wire ($\rho = 1.725 \times 10^{-8}\ \Omega \cdot \text{m}$). The inductance of a solenoid is given by $L = \frac{N^2\mu_0 A}{l}$, and the resistance of the solenoid is related to the total length of wire used to make the solenoid and the resistivity of copper. Treating the solenoid as an RL circuit, the time constant is equal to $\tau = \frac{L}{R}$.

SOLVE

Part a)

$$L = \frac{N^2\mu_0 A}{l} = \frac{N^2\mu_0\pi\left(\frac{D}{2}\right)^2}{l} = \frac{(1500)^2\left(4\pi \times 10^{-7}\frac{\text{H}}{\text{m}}\right)(\pi)\left(\frac{2.00 \times 10^{-2}\text{ m}}{2}\right)^2}{0.180\text{ m}}$$

$$= \boxed{4.93 \times 10^{-3}\text{ H}}$$

Part b)

$$R = \frac{\rho(\text{length})}{A} = \frac{\rho(N\pi D)}{\pi r^2} = \frac{(1.725 \times 10^{-8}\ \Omega \cdot \text{m})(1500)(\pi)(2.00 \times 10^{-2}\text{ m})}{\pi(0.322 \times 10^{-3}\text{ m})^2} = \boxed{4.99\ \Omega}$$

Part c)

$$\tau = \frac{L}{R} = \frac{4.93 \times 10^{-3}\text{ H}}{4.99\ \Omega} = \boxed{9.88 \times 10^{-4}\text{ s}}$$

REFLECT

Copper wire is routinely used in building circuits for its low resistivity, so we would expect a copper wire solenoid to have a small resistance.

20.67

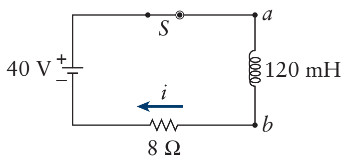


Figure 20-23 Problem 67

SET UP

An LR circuit is made up of a battery ($\varepsilon = 40$ V), an inductor ($L = 0.120$ H), and a resistor ($R = 8\ \Omega$). At some arbitrary moment in time, the current through the resistor is measured to be

$i = 6.00$ A to the left. We can use Kirchhoff's loop rule to calculate the rate of change of the current, the potential difference $V_a - V_b$ at that moment, and the conditions under which the current is increasing. The induced potential difference across an inductor has a magnitude of $L\frac{di}{dt}$; the potential difference across a resistor has a magnitude of Ri.

SOLVE

Part a)

Kirchhoff's loop rule:

$$\varepsilon - V_{\mathrm{L}} - V_{\mathrm{R}} = 0$$

$$\varepsilon - \left(L\frac{di}{dt}\right) - Ri = 0$$

$$\frac{di}{dt} = \frac{\varepsilon - Ri}{L} = \frac{(40\ \mathrm{V}) - (8\ \Omega)(6.00\ \mathrm{A})}{0.120\ \mathrm{H}} = \boxed{-66.7\frac{\mathrm{A}}{\mathrm{s}}}$$

Part b) The current is $\boxed{\text{decreasing}}$ because $\frac{di}{dt} < 0$.

Part c)

$$V_{\mathrm{L}} = \varepsilon - V_{\mathrm{R}} = \varepsilon - Ri = (40\ \mathrm{V}) - (8\ \Omega)(6.00\ \mathrm{A}) = \boxed{-8.00\ \mathrm{V}}$$

Part d) If $i < 5$ A, then $V_b < V_a$, which would mean the potential difference across the inductor is opposing that of the battery, which means the current is increasing.

REFLECT

At $i = 5$ A, the rate of change of the current is equal to zero. For $i < 5$ A the current is increasing, for $i > 5$ A the current is decreasing; this means 5 A is the maximum value of the current in the circuit.

20.68

SET UP

An inductor ($L = 0.0400$ H) is connected to a resistor ($R = 5.00\ \Omega$) and a battery ($V = 36.0$ V). The energy stored in the inductor in the steady state is $U_L = \frac{1}{2}Li^2$, where i is the steady-state current in the circuit given by Ohm's law, $V = iR$.

SOLVE

$$U_L = \frac{1}{2}Li^2 = \frac{1}{2}L\left(\frac{V}{R}\right)^2 = \frac{1}{2}(0.0400\ \mathrm{H})\left(\frac{36.0\ \mathrm{V}}{5.00\ \Omega}\right)^2 = \boxed{1.04\ \mathrm{J}}$$

REFLECT

The term "steady state" refers to the value a very long time after the switch is flipped, that is, as time approaches infinity.

20.69

SET UP

A copper wire of length L is formed into a circular coil with a radius r and N turns. The total length of the wire L is related to the dimensions of the coil through the circumference, so $L = 2\pi rN$. A time-varying magnetic field is directed perpendicular through the coil. We can use Faraday's law and then Ohm's law to find the induced potential and induced current in the coil, respectively, as a function of N. From here, we can explore how i depends on N to find the value of N that maximizes the induced current in the coil.

SOLVE

Induced potential:

$$|\varepsilon| = \left|-N\frac{d\Phi_B}{dt}\right| = \left|N\frac{d}{dt}[\vec{B}\cdot\vec{A}]\right| = N\left|\frac{d}{dt}[B(t)(\pi r^2)\cos(0°)]\right| = N\pi r^2\left|\frac{dB(t)}{dt}\right|$$

Induced current:

$$i = \frac{|\varepsilon|}{R} = \frac{\left(N\pi r^2\left|\frac{dB(t)}{dt}\right|\right)}{\left(\frac{\rho L}{A_{\text{wire}}}\right)} = \frac{N\pi\left(\frac{L}{2\pi N}\right)^2 A_{\text{wire}}}{\rho L}\left|\frac{dB(t)}{dt}\right| = \frac{LA_{\text{wire}}}{4\pi N\rho}\left|\frac{dB(t)}{dt}\right|$$

This function is a maximum when $\boxed{N = 1}$.

REFLECT

Since the current is proportional to $\frac{1}{N}$, the maximum current will result from the smallest possible value of N, which is $N = 1$.

20.70

SET UP

A rectangular loop of wire that is 0.0500 m wide and 0.0800 m long has a resistance of $R = 2.00\ \Omega$ and lies in the xy-plane. The left side of the loop is located 0.0500 m away from a long, straight wire carrying a time-dependent current, $i_{\text{wire}} = i_0\sin(\omega t)$, where $i_0 = 15.0$ A and $\omega = 120\pi\frac{\text{rad}}{\text{s}}$. Because the current and, therefore, the magnetic field created by this wire change with time, the magnetic flux through the loop will change in time and there will be an induced potential in the loop of wire according to Faraday's law, $\varepsilon = -\frac{d\Phi_B}{dt}$. We will need to integrate over the area of the loop to calculate the magnetic flux through the loop, $\Phi_B = \int\vec{B}\cdot d\vec{A}$. We will define the area vector of the loop to point into the page. Once we have an expression for the induced potential in the loop as a function of time, we can use Ohm's law to calculate the induced current in the loop at $t = 0$ and $t = 2.09 \times 10^{-3}$ s.

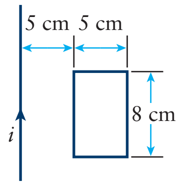


Figure 20-24 Problem 70

SOLVE

Magnetic flux:

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = \iint \left(\frac{\mu_0 i_{\text{wire}}}{2\pi x} \right) (dxdy) \cos(0°) = \left(\frac{\mu_0 i_{\text{wire}}}{2\pi} \right) \int_{0.0500\ m}^{0.100\ m} \frac{dx}{x} \int_0^{0.0800\ m} dy$$

$$= \left(\frac{\mu_0 i_0 \sin(\omega t)}{2\pi} \right) \Big[\ln(x) \Big]_{0.0500\text{ m}}^{0.100\text{ m}} \Big[y \Big]_0^{0.0800\text{ m}}$$

$$= \left(\frac{\left(4\pi \times 10^{-7} \frac{\text{T} \cdot \text{m}}{\text{A}}\right)(15.0\text{ A}) \sin\left(\left(120\pi \frac{\text{rad}}{\text{s}}\right) t\right)}{2\pi} \right) \left[\ln\left(\frac{0.100\text{ m}}{0.0500\text{ m}} \right) \right] [0.0800\text{ m}]$$

$$= (1.66 \times 10^{-7}\text{ T} \cdot \text{m}^2) \sin\left(\left(120\pi \frac{\text{rad}}{\text{s}}\right) t\right)$$

Induced potential:

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left[(1.66 \times 10^{-7}\text{ T} \cdot \text{m}^2) \sin\left(\left(120\pi \frac{\text{rad}}{\text{s}}\right) t\right) \right]$$

$$= -(1.66 \times 10^{-7}\text{ T} \cdot \text{m}^2)\left(120\pi \frac{\text{rad}}{\text{s}}\right)\left[\cos\left(\left(120\pi \frac{\text{rad}}{\text{s}}\right) t\right) \right]$$

$$= -(6.27 \times 10^{-5}\text{ V}) \cos\left(\left(120\pi \frac{\text{rad}}{\text{s}}\right) t\right)$$

Induced current:

$$i_{\text{ind}} = \frac{\varepsilon}{R} = \frac{-(6.27 \times 10^{-5}\text{ V}) \cos\left(\left(120\pi \frac{\text{rad}}{\text{s}}\right) t\right)}{(2.00\ \Omega)} = -(3.14 \times 10^{-5}\text{ A}) \cos\left(\left(120\pi \frac{\text{rad}}{\text{s}}\right) t\right)$$

Induced current at $t = 0$ s:

$$i_{\text{ind}}(0) = -(3.14 \times 10^{-5}\text{ A}) \cos\left(\left(120\pi \frac{\text{rad}}{\text{s}}\right)(0)\right) = \boxed{-3.14 \times 10^{-5}\text{ A}}$$

Induced current at $t = 2.09 \times 10^{-3}$ s:

$$i_{\text{ind}}(2.09 \times 10^{-3}\text{ s}) = -(3.14 \times 10^{-5}\text{ A}) \cos\left(\left(120\pi \frac{\text{rad}}{\text{s}}\right)(2.09 \times 10^{-3}\text{ s})\right)$$

$$= \boxed{-2.21 \times 10^{-5}\text{ A}}$$

REFLECT

The negative sign associated with the current means the current will flow in the counterclockwise direction, which agrees with Lenz's law.

20.71

SET UP

A magnetic field of $B = 0.45 \times 10^{-4}$ T is directed straight down, perpendicular to the plane of a circular coil of wire. The wire is made up of $N = 250$ turns and has an initial radius of $r_1 = 0.20$ m. The radius of the circle is increased to $r_2 = 0.30$ m in a period of $\Delta t = 15 \times 10^{-3}$ s. For simplicity, we'll assume the rate at which the radius changes is constant, the number of coils remains constant, and that the resistance of the coil also remains constant at $R = 25\ \Omega$ throughout the stretch. We can use Faraday's law of induction, $\varepsilon = N\frac{d\Phi_B}{dt}$, to calculate the voltage induced across the coil. Once we have the induced voltage across the coil, we can use Ohm's law to calculate the inducted current in the coil. Finally, the flux directed downward through the coil is increasing when the area of the loop is increasing; the induced magnetic field should point upward, so as to counteract this increase in flux. Therefore, the induced current will flow counterclockwise (when viewed from above), as given by the right-hand rule.

SOLVE

Part a)

$$\varepsilon = N\frac{d\Phi_B}{dt} = N\frac{d}{dt}[\vec{B}\cdot\vec{A}] = NB\frac{dA}{dt} = NB\left(\frac{A_f - A_i}{\Delta t}\right) = NB\pi\left(\frac{r_f^2 - r_i^2}{\Delta t}\right)$$

$$= (250)(0.45 \times 10^{-4}\ \text{T})\pi\left(\frac{(0.30\ \text{m})^2 - (0.20\ \text{m})^2}{15 \times 10^{-3}\ \text{s}}\right) = \boxed{0.118\ \text{V}}$$

Part b)

$$\varepsilon = iR$$

$$i = \frac{\varepsilon}{R} = \frac{0.118\ \text{V}}{25\ \Omega} = \boxed{0.00471\ \text{A}}$$

Part c) The induced current in the loop is $\boxed{\text{counterclockwise}}$ when viewed from above.

REFLECT

Our assumption that the resistance remained constant made the problem much easier to solve, but was an oversimplification. The resistance of the wire would increase linearly as we increase the radius of the coil because we've changed the dimensions of the coil.

20.72

SET UP

A long, rectangular loop of width w, mass m, and resistance R is being pushed to the left with a constant force of magnitude F into a uniform magnetic field of magnitude B pointing into the page. We will define the negative x-axis to point toward the left and x to be the length of the loop that is within the region of the magnetic field at a time t. We can find the induced current in the

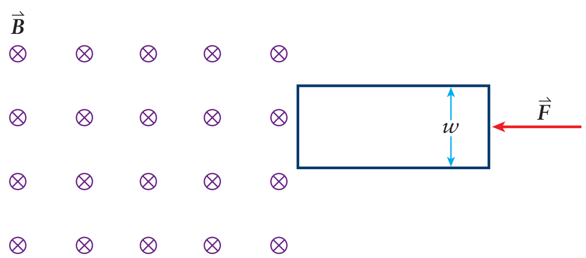


Figure 20-25 Problem 72

loop after a time t from Faraday's laws and Ohm's laws. For our calculation, we will define the area vector of the loop to point into the page, which means a clockwise current is considered positive. Since a current is induced, the loop will experience a force due to the magnetic field; the direction of this force is given by the right-hand rule. The net force acting on the loop in the x direction is equal to the mass of the loop multiplied by the first derivative of the speed of the loop with respect to time through Newton's second law. We can then solve this separable differential equation for the speed of the loop as a function of time, assuming the loop starts from rest and enters the magnetic field at $t = 0$.

SOLVE

Induced potential in the loop after a time t:

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}[\vec{B} \cdot \vec{A}] = -\frac{d}{dt}[B(wx)\cos(0°)] = -Bw\frac{dx}{dt} = -Bwv$$

Induced current in the loop after a time t:

$$i_{\text{induced}} = \frac{\varepsilon}{R} = \frac{-Bwv}{R}$$

Net force on the loop:

$$\sum F_x = -F + F_B = ma$$

$$-F + (|i_{\text{induced}}|wB) = m\left(\frac{dv}{dt}\right)$$

$$-F + \left(\frac{Bwv}{R}\right)wB = m\left(\frac{dv}{dt}\right)$$

$$\left(\frac{B^2w^2}{mR}\right)\left(v - \frac{FR}{B^2w^2}\right) = \frac{dv}{dt}$$

Speed of the loop as a function of time:

$$\int_0^t \frac{B^2w^2}{mR}dt = \int_0^v \frac{dv}{\left(v - \frac{FR}{B^2w^2}\right)}$$

$$\frac{B^2w^2}{mR}[t]_0^t = \left[\ln\left(v - \frac{FR}{B^2w^2}\right)\right]_0^v$$

$$\frac{B^2w^2t}{mR} = \left[\ln\left(v - \frac{FR}{B^2w^2}\right) - \ln\left(-\frac{FR}{B^2w^2}\right)\right]$$

$$\frac{B^2w^2t}{mR} = \left[\ln\left(\frac{\left(v - \frac{FR}{B^2w^2}\right)}{-\frac{FR}{B^2w^2}}\right)\right] = \ln\left(1 - \frac{B^2w^2v}{FR}\right)$$

$$e^{\frac{B^2w^2t}{mR}} = 1 - \frac{B^2w^2v}{FR}$$

$$\boxed{v = \left(\frac{FR}{B^2w^2}\right)\left[1 - e^{\frac{B^2w^2t}{mR}}\right]}$$

REFLECT

As more of the loop enters the magnetic field, the magnitude of the induced current should increase, which will then increase the magnitude of the magnetic force. This increasing magnetic force opposes the constant force from the push; therefore, the speed of the loop should eventually level off.

20.73

SET UP

A square metal loop that is 0.45 m on each side is rotated at an angular speed of ω in Earth's magnetic field ($B = 0.50 \times 10^{-4}$ T). We can find the induced potential as a function of time through Faraday's law. Since the angle between the magnetic field and the area vector changes in time, the induced potential will change in time. Setting the expression for the magnitude of the induced potential in the loop to 120 V, we can calculate the value of ω and determine whether or not it is feasible to rotate the loop at that speed.

SOLVE

Part a)
Induced potential as a function of time:

$$|\varepsilon| = \left|-\frac{d\Phi_B}{dt}\right| = \left|-\frac{d}{dt}[\vec{B}\cdot\vec{A}]\right| = \left|-\frac{d}{dt}[BA\cos(\theta(t))]\right|$$

$$= \left|-BA\frac{d}{dt}[\cos(\omega t)]\right| = BA\omega|\sin(\omega t)|$$

Angular speed:

$$BA\omega = 120\text{ V}$$

$$\omega = \frac{120\text{ V}}{BA} = \frac{120\text{ V}}{(0.50 \times 10^{-4}\text{ T})(0.45\text{ m})(0.45\text{ m})}$$

$$= 1.2 \times 10^{7}\,\frac{\text{rad}}{\text{s}} \times \frac{1\text{ rev}}{2\pi\text{ rad}} = \boxed{1.9 \times 10^{6}\frac{\text{rev}}{\text{s}}}$$

Part b) This is nearly 2 million rev/s, which $\boxed{\text{does not seem feasible}}$.

REFLECT

Increasing the area of the loop would decrease the angular speed required to achieve an induced potential of 120 V, but even a square loop of wire that was 1 m on each side would need to rotate at over 38,000 rad/s!

20.74

SET UP

Magnetic fields pointing along the surface of Earth are created by activity on the Sun. We can model a strand of power lines on Earth's surface as a loop of wire that is 1.0×10^{3} m by 5.0 m. The magnitude of the potential induced in this loop is equal to $|\varepsilon| = 6.0$ V. We can find the rate at which the magnetic field must change to induce this potential in the power grid through Faraday's law. We will assume the magnetic field along Earth's surface is perpendicular to the plane of the loop formed by the power lines.

SOLVE

$$|\varepsilon| = \left|-\frac{d\Phi_B}{dt}\right| = \left|-\frac{d}{dt}[\vec{B}\cdot\vec{A}]\right| = \left|-\frac{d}{dt}[BA\cos(0°)]\right| = A\left|\frac{dB}{dt}\right|$$

$$\left|\frac{dB}{dt}\right| = \frac{|\varepsilon|}{A} = \frac{6.0\text{ V}}{(5.0\text{ m})(1.0\times10^{3}\text{ m})} = \boxed{1.2\times10^{-3}\frac{\text{T}}{\text{s}}}$$

REFLECT

A change on the order of 1×10^{-3} T/s doesn't seem like much, but this corresponds to a change by a factor of 25 larger than Earth's magnetic field each second.

20.75

SET UP

The current in a loop of wire rises from zero to its maximum value over a time interval of $\Delta t = 75 \times 10^{-6}$ s. The magnetic field created from this current is uniform over a circular area of diameter $D = 2.0 \times 10^{-2}$ m and has a maximum magnitude of $B = 0.50$ T. We can use Faraday's law to find the magnitude of the average potential induced over this circular area from the current.

SOLVE

$$|\varepsilon| = \left|-\frac{\Delta\Phi_B}{\Delta t}\right| = \frac{\Delta[\vec{B}\cdot\vec{A}]}{\Delta t} = \frac{\Delta\left[B\left(\pi\left(\frac{D}{2}\right)^2\right)\cos(0°)\right]}{\Delta t} = \frac{\pi D^2}{4}\left(\frac{\Delta B}{\Delta t}\right)$$

$$= \frac{\pi D^2}{4}\left(\frac{B_{\text{f}} - B_{\text{i}}}{\Delta t}\right) = \frac{\pi(2.0\times10^{-2}\text{ m})^2}{4}\left(\frac{(0.50\text{ T}) - 0}{75\times10^{-6}\text{ s}}\right) = \boxed{2.1\text{ V}}$$

REFLECT

We are calculating the average induced potential because we are only using the initial and final values for the magnetic field.

20.76

SET UP

A conducting disk of radius R_{disk} is rotated about its axis at an angular speed ω in the presence of a uniform magnetic field of magnitude B directed perpendicularly to the surface of the disk. As it rotates, a differential segment dr of a radial line of the disk sweeps out a differential area dA in a time dt. This differential area is a rectangle that has a length dr on one side and ds on the other side. We can use the definitions of arc length and angular speed to rewrite the area in terms of dt. Next, we can use Faraday's law to calculate the potential induced in this differential segment of the disk; integrating this expression over the entire radius of the disk will give us the induced potential along the disk. Finally, we can divide this potential by the resistance R across the radius of the disk to find an expression for the induced current in the circuit. To determine whether the center of the disk or the outside edge is at a higher potential, we need to determine which direction the Lorentz force would point on a positive charge that is rotating with the disk. The location on the disk with a higher concentration of positive charge would have a higher relative potential.

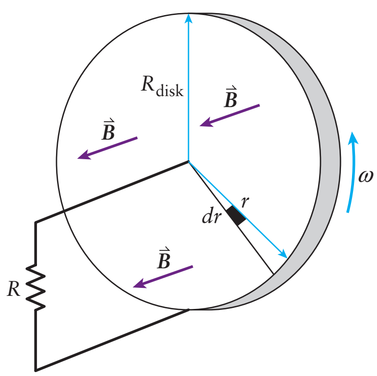


Figure 20-26 Problem 76

SOLVE

Part a)

Differential area swept out in a time dt:

$$dA = (ds)(dr) = (rd\theta)dr = r(\omega dt)dr$$

Induced potential along a differential segment:

$$|\varepsilon_{\text{segment}}| = \left|-\frac{d\Phi_B}{dt}\right| = \left|-\frac{d}{dt}[\vec{B}\cdot\vec{A}]\right| = \left|-\frac{d}{dt}[BA\cos(0°)]\right| = \left|-B\frac{dA}{dt}\right| = \left|-B\frac{(r\omega dtdr)}{dt}\right| = B\omega rdr$$

Induced potential along the radius of the disk:

$$|\varepsilon_{\text{total}}| = \int_0^{R_{\text{disk}}} B\omega rdr = B\omega\left[\frac{1}{2}r^2\right]_0^{R_{\text{disk}}} = \frac{B\omega R_{\text{disk}}^2}{2}$$

Induced current:

$$i_{\text{induced}} = \frac{|\varepsilon_{\text{total}}|}{R} = \frac{\left(\dfrac{B\omega R_{\text{disk}}^2}{2}\right)}{R} = \boxed{\frac{B\omega R_{\text{disk}}^2}{2R}}$$

Part b) The outside edge is at a higher potential because a positive charge moving in the counterclockwise direction would experience a force toward the outside edge, and negative charges would feel a force toward the center.

REFLECT

Even though the mobile charge carriers in the metal disk would be the electrons, current is defined in terms of positive charges.

20.77

SET UP

A pot contains a round metal coil that is 0.120 m in diameter and has a resistance $R = 22.5 \times 10^{-3}\ \Omega$. The surface of the stove produces a uniform vertical magnetic field that oscillates sinusoidally in time with amplitude $B_0 = 0.850$ T and frequency 60.0 Hz. The time-changing magnetic field induces a time-changing potential in the metal coil according to Faraday's law. Since the magnetic field and the area vector are parallel to one another, the dot product is equal to the product of their magnitudes. The average rate of heat produced in a resistive wire is equal to the time average of the potential squared divided by the resistance, $\langle P\rangle = \frac{\langle\varepsilon^2\rangle}{R}$. Assuming all of this heat generated by the coil is absorbed by $m = 0.500$ kg of water $\left(c = 4186\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)$, we can calculate the time required to increase the temperature of the water by 30 K.

SOLVE

Part a)

Induced potential:

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}[\vec{B}\cdot\vec{A}] = -\frac{d}{dt}[(B_0\cos(\omega t))A] = -B_0A\frac{d}{dt}[\cos(\omega t)] = B_0A\omega\ \sin(\omega t)$$

Average rate of heat produced:

$$\langle P\rangle = \frac{\langle \varepsilon^2\rangle}{R} = \frac{\langle (B_0 A\omega\ \sin(\omega t))^2\rangle}{R} = \frac{(B_0 A\omega)^2\langle \sin^2(\omega t)\rangle}{R} = \frac{(B_0 A\omega)^2\left(\frac{1}{2}\right)}{R}$$

$$= \frac{\left((0.850\text{ T})\left(\pi\left(\frac{0.120\text{ m}}{2}\right)^2\right)(2\pi(60.0\text{ Hz}))\right)^2}{2(22.5\times 10^{-3}\ \Omega)} = \boxed{292\text{ W}}$$

Part b)

$$Q = Pt = mc\Delta T$$

$$t = \frac{mc\Delta T}{P} = \frac{(0.500\text{ kg})\left(4186\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(30\text{ K})}{292\text{ W}} = 215\text{ s}\times\frac{1\text{ min}}{60\text{ s}} = \boxed{3.59\text{ min}}$$

REFLECT

It would take about 9.5 min for the water to start boiling, which seems on the long side of reasonable. When calculating the time averages, recall that the average of a constant is equal to the constant and the average over one period of $\sin^2(\omega t) = \frac{1}{2}$.

20.78

SET UP

A permanent bar magnet with the north pole pointing downward is dropped into a solenoid with an ammeter wired in series with the coils. We can use Lenz's law to determine the direction of the current through the ammeter when the magnet is dropped into the solenoid and then pulled upward through the solenoid.

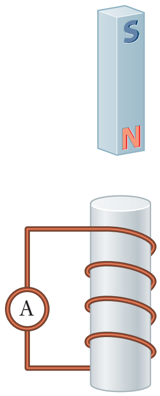


Figure 20-27 Problem 78

SOLVE

Part a) The magnetic flux is increasing in the downward direction, which means the magnetic field induced in the solenoid must point upward. Therefore, the current around the solenoid will be counterclockwise when viewed from above, which means the current will $\boxed{\text{pass up through the ammeter}}$.

Part b) If the magnet were pulled upward through the solenoid, the magnetic flux would be decreasing in the downward direction, so the current would $\boxed{\text{pass down through the ammeter}}$.

REFLECT

We would expect the direction of the current to change if we reverse the motion of the permanent magnet.

20.79

SET UP

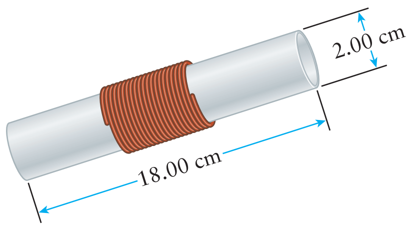


Figure 20-28 Problem 79

A tightly wound solenoid ($N_{\text{solenoid}} = 1500$, $l_{\text{solenoid}} = 0.180$ m, $d_{\text{solenoid}} = 0.0200$ m) carrying a current i_{solenoid} is surrounded by a circular coil ($N_{\text{coil}} = 20$, $d_{\text{coil}} = 0.0300$ m). The circular coil is connected across a resistor of very high resistance, which means the current through the coil is essentially zero for all time. The current in the solenoid is changing at a rate of $\frac{di_{\text{solenoid}}}{dt} = 100\frac{\text{A}}{\text{s}}$. The induced potential in the coil is equal to the sum of the induced potential due to the self-inductance of the coil and the induced potential due to the mutual inductance from the solenoid. Since the current in the coil remains essentially constant, the self-inductance term goes to zero. The mutual inductance for the system is given by $M = \frac{N_{\text{coil}}\Phi_{B,\text{ coil}}}{i_{\text{solenoid}}}$. The magnetic flux through the coil arises from the magnetic field produced by the solenoid, $B_{\text{solenoid}} = \mu_0\frac{N_{\text{solenoid}}}{l_{\text{solenoid}}}i_{\text{solenoid}}$. This magnetic field is only produced in the region inside the solenoid, which is smaller than the cross-sectional area of the coil; therefore, when calculating the flux, the effective area is the cross-sectional area of the solenoid, not the coil. The problem would be a bit more difficult if we had to calculate the magnetic flux through the solenoid due to the coil since the magnitude and direction of the magnetic field due to the coil changes along the radius and length of the solenoid. The easiest method would involve invoking the mutual inductance of the system and using our result from part a.

SOLVE

Part a)

$$\varepsilon_{\text{coil}} = -L_{\text{coil}}\frac{di_{\text{coil}}}{dt} - M\frac{di_{\text{solenoid}}}{dt} \approx 0 - M\frac{di_{\text{solenoid}}}{dt} = -\left(\frac{N_{\text{coil}}\Phi_{B,\text{ coil}}}{i_{\text{solenoid}}}\right)\frac{di_{\text{solenoid}}}{dt}$$

$$= -\left(\frac{N_{\text{coil}}}{i_{\text{solenoid}}}\right)(B_{\text{solenoid}}A_{\text{solenoid}})\frac{di_{\text{solenoid}}}{dt}$$

$$= -\left(\frac{N_{\text{coil}}}{i_{\text{solenoid}}}\right)\left(\mu_0\frac{N_{\text{solenoid}}}{l_{\text{solenoid}}}i_{\text{solenoid}}\right)\left(\pi\left(\frac{d_{\text{solenoid}}}{2}\right)^2\right)\frac{di_{\text{solenoid}}}{dt}$$

$$= -\frac{\mu_0\pi}{4}\left(\frac{N_{\text{solenoid}}N_{\text{coil}}d^2_{\text{solenoid}}}{l_{\text{solenoid}}}\right)\left(\frac{di_{\text{solenoid}}}{dt}\right)$$

$$|\varepsilon_{\text{coil}}| = \frac{\left(4\pi \times 10^{-7}\frac{\text{H}}{\text{m}}\right)\pi}{4}\left(\frac{(1500)(20)(0.0200\text{ m})^2}{0.180\text{ m}}\right)\left(100\frac{\text{A}}{\text{s}}\right)$$

$$= \boxed{0.00658\text{ V} = 6.58\text{ mV}}$$

Part b) In the new situation, it is much more difficult to calculate the flux through the solenoid from the coil because the magnitude and direction of the magnetic field change along the

length and radius of the solenoid, respectively. The problem is not impossible, though. From the existing problem, we can calculate the mutual inductance, M. Once we have that, the problem is reduced to $\varepsilon_{\text{solenoid}} = M\frac{di_{\text{coil}}}{dt}$.

REFLECT

A few mV seems reasonable for an induced potential. In general, if you need a quantity that was not provided in the problem statement, rather than panicking and quitting, just go ahead and give it a name and variable and carry it though your calculation. You could report your final answer in terms of this variable or, better yet, the quantity may cancel out during the calculation. For example, we did not know the current in the solenoid, but needed to carry it through our calculation. Upon defining it to be i_{solenoid} and carrying it through, the current term from the mutual inductance ended up cancelling with the current term from the magnetic flux.

20.80

SET UP

A circular coil has $N = 20$ turns and a radius of $R_{\text{coil}} = 0.800 \times 10^{-2}$ m is inside and coaxial to a tightly wound solenoid that is 0.180 m long and is made up of 1500 turns of wire. The circular coil is connected across a resistor of very high resistance. The current inside the solenoid is changing at a rate of $\frac{di}{dt} = 100\frac{\text{A}}{\text{s}}$.

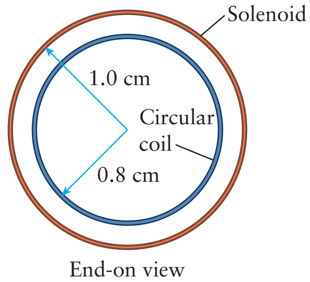


Figure 20-29 Problem 80

The magnitude of the induced potential is given by Faraday's law, $|\varepsilon| = \left|-N\frac{d\Phi_B}{dt}\right|$. The magnetic flux through the coil arises from the magnetic field produced by the solenoid, $B = \mu_0 ni = \mu_0\frac{N}{l}i$.

SOLVE

$$|\varepsilon| = \left|-N\frac{d\Phi_B}{dt}\right| = \left|-N\frac{d}{dt}[\vec{B}\cdot\vec{A}]\right| = \left|-N\frac{d}{dt}[BA\cos(0°)]\right| = NA\left|-\frac{dB}{dt}\right|$$

$$= N(\pi R_{\text{coil}}^2)\left|-\frac{d}{dt}[\mu_0 ni]\right| = \pi N\mu_0 nR_{\text{coil}}^2\left(\frac{di}{dt}\right)$$

$$= \pi(20\text{ turns})\left(4\pi \times 10^{-7}\frac{\text{T}\cdot\text{m}}{\text{A}}\right)\left(\frac{1500\text{ turns}}{0.180\text{ m}}\right)(0.800 \times 10^{-2}\text{ m})^2\left(100\frac{\text{A}}{\text{s}}\right)$$

$$= \boxed{4.21 \times 10^{-3}\text{ V}}$$

REFLECT

This is a reasonable order of magnitude for an induced potential.

20.81

SET UP

A long solenoid that is 1.0×10^{-2} m in radius and $l = 0.20$ m in length consists of $N = 5000$ turns. The magnitude of the magnetic field inside the solenoid when a current $i = 5.0$ A is in

it is equal to $B = \mu_0 ni$. The self-inductance of a solenoid is given by $L = \dfrac{\mu_0 N^2 A}{l}$. Finally, if the current drops to zero in a time interval $\Delta t = 0.10$ s, the magnitude of the induced potential is $|\varepsilon| = \left|-L\dfrac{\Delta i}{\Delta t}\right|$.

SOLVE

Part a)

$$B = \mu_0 ni = \left(4\pi \times 10^{-7}\frac{\mathrm{T\cdot m}}{\mathrm{A}}\right)\left(\frac{5000\ \text{turns}}{0.20\ \mathrm{m}}\right)(5.0\ \mathrm{A}) = \boxed{0.16\ \mathrm{T}}$$

Part b)

$$L = \frac{\mu_0 N^2 A}{l} = \frac{\left(4\pi \times 10^{-7}\frac{\mathrm{H}}{\mathrm{m}}\right)(5000\ \text{turns})^2(\pi)(1.0 \times 10^{-2}\ \mathrm{m})^2}{0.20\ \mathrm{m}} = \boxed{0.049\ \mathrm{H}}$$

Part c)

$$|\varepsilon| = \left|-L\frac{\Delta i}{\Delta t}\right| = \left|-(0.049\ \mathrm{H})\frac{(0-(5.0\ \mathrm{A}))}{(0.10\ \mathrm{s})}\right| = \boxed{2.4\ \mathrm{V}}$$

REFLECT

The more quickly the current changes, the larger the magnitude of the induced potential in the solenoid.

20.82

SET UP

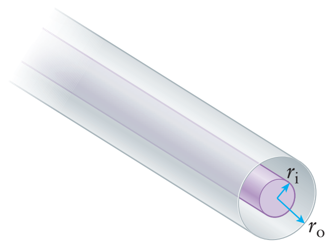


Figure 20-30 Problem 80

A coaxial cable is made from an inner conductor of radius r_i and an outer conductor of radius r_o. The two conductors carry equal currents in opposite directions. From Ampère's law, we know that we only need to consider the magnetic field due to the inner conductor in the region between the conductors. We can find an expression for the magnetic flux by integrating the flux through a small rectangle of length l and width dr from $r = r_i$ to $r = r_o$. The inductance of the coaxial cable is equal to the magnetic flux divided by the current. Finally, we can divide this by the length l to find the inductance per unit length of the cable.

SOLVE

Magnetic flux between the conductors:

$$\Phi_B = \int \vec{B}\cdot d\vec{A} = \int B(ldr)\cos(0°) = l\int_{r_i}^{r_o}\left(\frac{\mu_0 i}{2\pi r}\right)dr = \frac{\mu_0 il}{2\pi}[\ln(r)]_{r_i}^{r_o} = \left(\frac{\mu_0 il}{2\pi}\right)\ln\left(\frac{r_o}{r_i}\right)$$

Inductance:

$$L = \frac{\Phi_B}{i} = \frac{\left(\frac{\mu_0 il}{2\pi}\right)\ln\left(\frac{r_o}{r_i}\right)}{i} = \frac{\mu_0 l}{2\pi}\ln\left(\frac{r_o}{r_i}\right)$$

Inductance per unit length:

$$\boxed{\frac{L}{l} = \frac{\mu_0}{2\pi}\ln\left(\frac{r_o}{r_i}\right)}$$

REFLECT

The wider the gap between the conductors, the larger the inductance per unit length.

20.83

SET UP

In the real world, solenoids are made with wire, which has some resistance associated with it. Let's say a cylindrical solenoid ($l_{\text{solenoid}} = 35.0 \times 10^{-2}$ m, $d_{\text{solenoid}} = 4.50 \times 10^{-2}$ m) is made of copper wire ($\rho_{\text{copper}} = 1.725 \times 10^{-8}\ \Omega \cdot \text{m}$, $d_{\text{wire}} = 0.6438 \times 10^{-3}$ m) that is wound about a central axis. The adjacent loops touch each other but do not overlap, which means the total length of the solenoid is equal to the number of loops multiplied by the diameter of the wire. The resistance of the solenoid is equal to the resistance of the entire length of wire; the length of the wire is equal to the number of loops multiplied by the circumference of the solenoid. The resistance of the copper wire is equal to $R = \frac{\rho_{\text{copper}} l_{\text{wire}}}{A_{\text{wire}}}$. The inductance of a solenoid is $L = \frac{\mu_0 N^2 A_{\text{solenoid}}}{l_{\text{solenoid}}}$.

SOLVE

Part a)

$$l_{\text{solenoid}} = N d_{\text{wire}}$$

$$N = \frac{l_{\text{solenoid}}}{d_{\text{wire}}} = \frac{35.0 \times 10^{-2}\ \text{m}}{0.6438 \times 10^{-3}\ \text{m}} = \boxed{544\ \text{loops}}$$

Part b)

Resistance:

$$R = \frac{\rho_{\text{copper}} l_{\text{wire}}}{A_{\text{wire}}} = \frac{\rho_{\text{copper}}(N(\pi d_{\text{solenoid}}))}{\left(\pi\left(\frac{d_{\text{wire}}}{2}\right)^2\right)} = \frac{4N\rho_{\text{copper}} d_{\text{solenoid}}}{d_{\text{wire}}^2}$$

$$= \frac{4(544)(1.725 \times 10^{-8}\ \Omega \cdot \text{m})(4.50 \times 10^{-2}\ \text{m})}{(0.6438 \times 10^{-3}\ \text{m})^2} = \boxed{4.07\ \Omega}$$

Inductance:

$$L = \frac{\mu_0 N^2 A_{\text{solenoid}}}{l_{\text{solenoid}}} = \frac{\mu_0 N^2\left(\pi\left(\frac{d_{\text{solenoid}}}{2}\right)^2\right)}{l_{\text{solenoid}}} = \frac{\mu_0 \pi N^2 d_{\text{solenoid}}^2}{4 l_{\text{solenoid}}}$$

$$= \frac{\left(4\pi \times 10^{-7}\frac{\text{H}}{\text{m}}\right)\pi(544)^2(4.50 \times 10^{-2}\ \text{m})^2}{4(35.0 \times 10^{-2}\ \text{m})} = \boxed{1.69 \times 10^{-3}\ \text{H} = 1.69\ \text{mH}}$$

REFLECT

Copper wire is known to be a good conductor so we would expect the resistance of the solenoid to be small.

20.84

SET UP

An object ($m = 5.0$ kg) is attached to a spring ($k = 1.34 \times 10^3$ N/m) and set into harmonic motion. We want to model this system using an LC circuit with $L = 4.0$ H. In order to accurately model the spring–mass system, the circuit should have the same resonant frequency. The resonant frequency of an LC circuit is $\omega_0 = \sqrt{\frac{1}{LC}}$, and the resonant frequency of a spring–mass system is $\omega_0 = \sqrt{\frac{k}{m}}$. Setting these equal to one another will allow us to solve for the required value of C.

SOLVE

$$\sqrt{\frac{1}{LC}} = \sqrt{\frac{k}{m}}$$

$$\frac{1}{LC} = \frac{k}{m}$$

$$C = \frac{m}{Lk} = \frac{5.0 \text{ kg}}{(4.0 \text{ H})\left(1.34 \times 10^3 \frac{\text{N}}{\text{m}}\right)} = \boxed{9.3 \times 10^{-4} \text{ F}}$$

REFLECT

Both the circuit and the spring–mass system will oscillate with a period of $T = 0.38$ s.

20.85

SET UP

A capacitor ($C = 25.0 \times 10^{-6}$ F) and an inductor ($L = 10.0 \times 10^{-3}$ H) are wired in series with an open switch between them. The initial energy stored in the capacitor is $U_{E,\text{i}} = \frac{Q_0^2}{2C}$, where Q_0 is the maximum charge on the capacitor. When the switch is closed, the charge on the capacitor will oscillate in time and can be described by $q(t) = Q_0 \cos(\omega t)$, where $\omega = \frac{1}{\sqrt{LC}}$. We can plug this into the expression for the energy stored in a capacitor to solve for the times when the energy in the capacitor is one-half of its initial value (that is, the energy is equally shared between the capacitor and inductor) and when the energy stored in the capacitor is zero (that is, when the inductor stores all of the energy in the circuit).

SOLVE

Part a)

$$U_E = \frac{U_{E,\,\mathrm{i}}}{2} = \frac{\left(\frac{Q_0^2}{2C}\right)}{2} = \frac{Q_0^2}{4C}$$

$$\frac{[q(t)]^2}{2C} = \frac{Q_0^2 \cos^2(\omega t)}{2C} = \frac{Q_0^2 \cos^2\left(\frac{t}{\sqrt{LC}}\right)}{2C} = \frac{Q_0^2}{4C}$$

$$\cos\left(\frac{t}{\sqrt{LC}}\right) = \sqrt{\frac{1}{2}}$$

$$\frac{t}{\sqrt{LC}} = \arccos\left(\sqrt{\frac{1}{2}}\right) = \frac{\pi}{4}$$

$$t = \frac{\pi\sqrt{LC}}{4} = \frac{\pi\sqrt{(10.0 \times 10^{-3}\ \mathrm{H})(25.0 \times 10^{-6}\ \mathrm{F})}}{4} = \boxed{3.93 \times 10^{-4}\ \mathrm{s}}$$

Part b)

$$U_E = \frac{Q_0^2 \cos^2\left(\frac{t}{\sqrt{LC}}\right)}{2C} = 0$$

$$\cos\left(\frac{t}{\sqrt{LC}}\right) = 0$$

$$\frac{t}{\sqrt{LC}} = \arccos(0) = \frac{\pi}{2}$$

$$t = \frac{\pi\sqrt{LC}}{2} = \frac{\pi\sqrt{(10.0 \times 10^{-3}\ \mathrm{H})(25.0 \times 10^{-6}\ \mathrm{F})}}{2} = \boxed{7.85 \times 10^{-4}\ \mathrm{s}}$$

REFLECT

The exact value of the initial energy stored in the capacitor is not necessary since we're only interested in fractions of the initial energy. The first time the energy will be shared between the capacitor and inductor should be one-eighth of a period:

$$\frac{T}{8} = \frac{\left(\frac{2\pi}{\omega}\right)}{8} = \frac{\pi}{4\omega} = \frac{\pi\sqrt{LC}}{4}.$$

20.86

SET UP

The time rate of change of the current in an LR circuit is $\frac{di}{dt} = J_0 \cos(\omega t)$, where $J_0 = 0.035\frac{\mathrm{A}}{\mathrm{s}}$ and $\omega = 2.0\frac{\mathrm{rad}}{\mathrm{s}}$. The resistance of the circuit is $R = 50\ \Omega$. To find the current at $t = 0.40$ s, we

need to integrate the expression for the time rate of change of the current with respect to t and then evaluate the result at $t = 0.40$ s; we'll assume the current at $t = 0$ is 0. The maximum current in the circuit is equal to the amplitude of this function. The voltage across the LR circuit is equal to the maximum current multiplied by the resistance of the circuit from Ohm's law. We can then use Kirchhoff's loop rule to solve for the inductance L of the circuit.

SOLVE

Part a)
Current as a function of time:

$$\frac{di}{dt} = J_0 \cos(\omega t)$$

$$\int_0^{i(t)} di = \int_0^t J_0 \cos(\omega t)dt$$

$$i(t) = J_0\left[\frac{\sin(\omega t)}{\omega}\right]_0^t = \frac{J_0}{\omega}\sin(\omega t)$$

Current at $t = 0.400$ s:

$$i(0.40 \text{ s}) = \frac{\left(0.035\frac{\text{A}}{\text{s}}\right)}{\left(2.0\frac{\text{rad}}{\text{s}}\right)}\sin\left(\left(2.0\frac{\text{rad}}{\text{s}}\right)(0.40 \text{ s})\right) = \boxed{0.013 \text{ A}}$$

Part b)
Maximum current in the circuit:

$$i_{\text{max}} = \frac{J_0}{\omega}$$

Total voltage across the circuit:

$$V = i_{\text{max}}R = \frac{J_0 R}{\omega}$$

Loop rule:

$$V - iR - L\frac{di}{dt} = 0$$

$$i_{\text{max}}R - iR = L\frac{di}{dt}$$

$$L = \frac{(i_{\text{max}} - i)R}{\left(\frac{di}{dt}\right)} = \frac{\left(\left(\frac{J_0}{\omega}\right) - i\right)R}{(J_0 \cos(\omega t))}$$

This expression holds for all time t. Since we've already calculated values at $t = 0.40$ s, we will use those:

$$L = \frac{\left(\frac{\left(0.035\frac{\text{A}}{\text{s}}\right)}{\left(2.0\frac{\text{rad}}{\text{s}}\right)} - (0.013\ \text{A})\right)(50\ \Omega)}{\left(0.035\frac{\text{A}}{\text{s}}\right)\cos\left(\left(2.0\frac{\text{rad}}{\text{s}}\right)(0.40\ \text{s})\right)} = \boxed{10\ \text{H}}$$

REFLECT

The time constant for this circuit is $\tau = 0.20$ s, so $t = 0.40$ s is equal to 2τ.

20.87

SET UP

A circuit consists of a battery ($V_0 = 12$ V), a switch S, three resistors ($R_1 = 4.0\ \Omega$, $R_2 = 8.0\ \Omega$, and $R_3 = 2.0\ \Omega$), and an inductor L wired as shown. Initially the switch has been open for a very long time, which means there is no current flowing in the circuit. At the moment S is closed, the induced voltage across the inductor is a maximum, so the current in the right loop of the circuit will be equal to zero and the circuit consists of only R_1 and R_2 in series. After the switch has been closed for a very long time, the currents will no longer fluctuate and remain constant in time; therefore, the voltage across the inductor will be equal to zero. We can use this fact along with Kirchhoff's rules to calculate the steady-state currents in the circuit. Once the switch is reopened after the steady state is reached, the current through R_1 will immediately drop to zero and the current in through R_3 will reverse. Eventually, a long time after the switch is reopened, each of the currents in the circuit will be equal to zero.

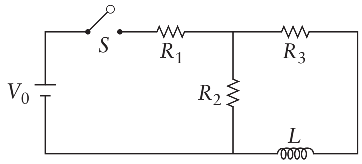


Figure 20-31 Problem 87

SOLVE

Part a)

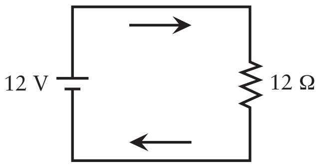


Figure 20-32 Problem 80

Part b)

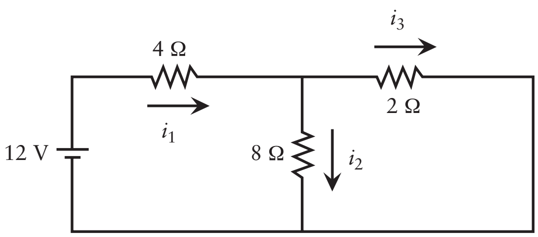


Figure 20-33 Problem 80

Part c)

In the steady state, $\frac{di}{dt} = 0$, so $V_L = -L\left(\frac{di}{dt}\right) = 0$.

Right loop:

$$i_2R_2 - i_3R_3 - V_L = i_2R_2 - i_3R_3 - 0 = 0$$

$$i_2R_2 = i_3R_3$$

$$i_3 = \frac{i_2R_2}{R_3}$$

Junction rule:

$$i_1 = i_2 + i_3 = i_2 + \frac{i_2R_2}{R_3} = i_2\left(1 + \frac{R_2}{R_3}\right)$$

Left loop:

$$V_0 - i_1R_1 - i_2R_2 = 0$$

$$V_0 - i_2\left(1 + \frac{R_2}{R_3}\right)R_1 - i_2R_2 = 0$$

Currents in the steady state:

$$i_2 = \frac{V_0}{\left[R_1 + \left(\frac{R_1R_2}{R_3}\right) + R_2\right]} = \frac{12\text{ V}}{\left[(4.0\ \Omega) + \frac{(4.0\ \Omega)(8.0\ \Omega)}{2.0\ \Omega} + (8.0\ \Omega)\right]} = \frac{12}{28}\text{A} = 0.43\text{ A}$$

$$i_3 = \frac{i_2R_2}{R_3} = \left(\frac{12}{28}\text{A}\right)\left(\frac{8.0\ \Omega}{2.0\ \Omega}\right) = 1.7\text{ A}$$

$$i_1 = i_2\left(1 + \frac{R_2}{R_3}\right) = \left(\frac{12}{28}\text{A}\right)\left(1 + \frac{8.0\ \Omega}{2.0\ \Omega}\right) = 2.1\text{ A}$$

Immediately after the switch is reopened:

$$\boxed{i_1 = 0}$$

$$i_1 = i_2 + i_3 = 0$$

$$i_2 = -i_3$$

$$\boxed{i_3 = -1.7\text{ A}}$$

$$i_2 = -i_3 = -(-1.7\text{ A}) = \boxed{1.7\text{ A}}$$

Part d)

$$\boxed{i_1 = i_2 = i_3 = 0}$$

REFLECT

In part (c), only the right loop of the circuit has current flowing in it, which means resistors R_2 and R_3 are in series and should have the same current flowing through them.

Chapter 21
AC Circuits

Conceptual Questions

21.1 It means that the current peaks one-fourth of a period after the voltage drop peaks. One-fourth of a period corresponds to a phase difference of 90 degrees.

21.2 The statement is false. The average power delivered by an ideal transformer equals the average power it receives. Transformers are not power sources, so they cannot deliver more average power than they receive. In fact, they deliver less average power than they receive because of internal dissipative processes.

21.3 Ohm's law ($V = iR$) is not precisely valid for AC circuits. Instead, the "reactance" and the "impedance" are defined for AC circuits: $V = iX$ and $V = iZ$.

21.4 Many AC voltages are sinusoidal. A sine (or cosine) function has an average value of zero, since equal amounts of the function lie above and below the axis. If you calculate the "root-mean-square" value of the function, all of the values of the function—both negative and positive—are first squared before averaged. Therefore, the negative values do not cancel out the positive values and the general features of the waveform can be explained.

21.5

Country	V_{rms}	Frequency (Hz)
Angola	220	50
Botswana	231	50
Ecuador	120–127	60
Mexico	127	60
Slovenia	220	50

21.6 Since the power that is dissipated in an appliance is proportional to the square of the voltage, even a small increase can lead to a rather large power surge.

21.7 No. These devices usually work over a range of voltages and will not function properly if a transformer is used. They contain "smart" circuitry that senses if the voltage is 120 V or 240 V. (Many people needlessly buy transformers for their laptops. What may be needed, depending on the destination, is an adapter plug.)

21.8 Part a) A transformer is a device that transfers electrical energy from one circuit to another through inductively coupled conductors (the transformer's coils).

Part b) A varying current in the first (or primary) coil creates a varying magnetic flux in the transformer's core and, thus, a varying magnetic field through the secondary coil. This varying magnetic field induces a varying electromotive force (or voltage) in the secondary coil. This effect is called "mutual inductance."

Part c) A transformer requires an AC input so that the changing flux in the primary coil will induce a varying electromotive force according to Faraday's law.

21.9 PCBs were widely used for many applications, especially as dielectric fluids in transformers, capacitors, and coolants. PCBs have low water solubilities (0.0027–0.42 ng/L for Aroclors) and low vapor pressures at room temperature, but they have high solubilities in most organic solvents, oils, and fats. They have high dielectric constants, very high thermal conductivity, high flash points (from 170–380 degrees Celsius), and are fairly chemically inert (being extremely resistant to oxidation, reduction, addition, elimination, and electrophilic substitution).

21.10 It saves copper. The primary power loss in transmission equals i_{rms}^2R, where R is the resistance of the transmission lines, and the rate at which energy is transported equals $V_{\text{rms}}i_{\text{rms}}$. Thus, at high voltage, power can be transmitted with a lower current, and the lower the current, the less the transmission loss. The other way to reduce transmission loss would be to decrease the resistance of the transmission lines. The only practical way to do that is to use thicker wire, which requires more of the metal that the wires are made of. The voltages are stepped down for consumer safety.

21.11 The electrical energy stored in the capacitor is $U_E = \frac{1}{2}\frac{1}{C}q^2$, which varies as $\cos^2(\omega t)$, and the magnetic energy stored in the inductor is $U_L = \frac{1}{2}Li^2$, which varies as $\sin^2(\omega t)$. Therefore, when a maximum amount of energy is stored in the capacitor, no energy is stored in the inductor, and *vice versa*.

21.12 The instantaneous current in each device is the same because they are in series. The voltage drop across the inductor leads the current by 90 degrees, whereas the voltage drop across the capacitor lags the current by 90 degrees; thus, the two voltage drops are 180 degrees out of phase. The equivalent reactance of the combination is the ratio of the rms voltage drop across the combination to the rms current through it. Because the two voltage drops are 180 degrees out of phase, the rms voltage drop across the combination equals the difference of the individual rms voltage drops; so the equivalent reactance equals the difference between the individual reactances. Represented as a phasor, the voltage across the combination $\vec{V} = \vec{V}_L + \vec{V}_C$. Because $\vec{V}_L$ and $\vec{V}_C$ are oppositely directed, the magnitude of $\vec{V}$ equals the difference between their magnitudes.

21.13 The reactance of a capacitor decreases with increased frequency because the capacitor requires time to charge before it can begin to resist current. With a more quickly alternating current, there is even less time to charge, so there is less resistance. With an inductor, the strongest resistance to alternating current happens when the current changes the quickest. This happens at higher frequencies.

21.14 No. This would be true only if the net reactance of the circuit were zero, which would mean (a) that there is negligible inductance and capacitance in the circuit or (b) that the circuit is at resonance.

21.15 An inductor with resistance is equivalent to a series LR combination. The tangent of the phase angle equals the ratio of the inductive resistance to the resistance. The phase angle varies with frequency because the inductive resistance does.

21.16 The property of a capacitor (or inductor) to offer resistance to the flow of charge in an AC circuit is called "reactance." The SI units of reactance are ohms (Ω).

21.17 Impedance describes the overall opposition to the flow of charge in a circuit driven by a time-varying voltage. The SI units of impedance are ohms (Ω).

21.18 The triangle we formed in carrying out the vector sum of the V_L, V_C, and V_R phasors enables us to find the phase angle φ. We defined this as the angle between the voltage phasor V and the current phasor i.

21.19 Inductive reactance is directly proportional to the frequency (or angular frequency) of the voltage signal while capacitive reactance is inversely proportional to the frequency. So while X_C is large for low frequencies, X_L is large for high frequencies. In the LRC circuit, then, the capacitor serves as a simple high-pass filter, suppressing low frequencies from getting to the tweeter, and the inductor functions as a simple low-pass filter, suppressing high frequencies from getting to the woofer.

Multiple-Choice Questions

21.20 **D** (it is relatively straightforward to change the voltage delivered by an AC source). Transformers, which operate using mutual inductance, are used to convert from a high voltage to a low voltage.

21.21 **D** ($2\sqrt{2}V$). The peak-to-peak voltage is twice the peak voltage. The peak voltage is $\sqrt{2}$ times the root-mean-square voltage.

21.22 **B** (the root-mean-square average of the voltage as it varies with time). The root-mean-square average is not influenced by whether the values are positive or negative.

21.23 **A** (1/4 of the previous value). The power delivered is constant, so i is proportional to $\frac{1}{V}$. For $P_{\text{loss}} = i^2R$, as V goes up, P_{loss} decreases by $\frac{1}{V^2}$.

21.24 **A** (use a step-up transformer). The output voltage from a step-up transformer is greater than the input voltage.

21.25 **D** (B leads A by π). The general form of a sinusoidal voltage is $V(t) = V_0 \sin(\omega t - \varphi)$ for $\varphi > 0$.

21.26 **D** (the impedance equals the resistance). When the impedance is equal to the resistance, the term $\left(\omega L - \dfrac{1}{\omega C}\right) = 0$. Therefore, the phase angle $\varphi = 0°$.

21.27 **A** (the phase angle is zero).

$$\tan(\varphi) = \frac{X_L - X_C}{R} = 0$$

$$\varphi = \arctan(0) = 0$$

21.28 **C** (in inductor plus AC circuits). The circuit must only have an AC power source and an inductor.

21.29 **C** (inductive). The reactance for an inductor is $X_L = \omega L$.

Estimation/Numerical Questions

21.30 The power lost through a resistor is $P = i^2R$ for a DC circuit. The power lost through a resistor for an AC circuit is $P_{\text{avg}} = i_{\text{rms}}^2 R = \frac{1}{2}i^2R$. Therefore, the power lost on average is one-half for the AC case versus the DC case.

21.31 Most household appliances draw between 1 and 20 amps. Most households have a main circuit breaker that "trips" at 150 A or so.

21.32

$$\frac{V_{\text{s}}}{V_{\text{p}}} = \frac{N_{\text{s}}}{N_{\text{p}}} = \frac{120\text{ V}}{24\text{ V}} \sim 5$$

21.33
Impedance:

$$Z = \sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2} = \sqrt{\left(2\pi f L - \frac{1}{2\pi f C}\right)^2 + R^2}$$

$$= \sqrt{\left(2\pi(60\text{ Hz})(0.100\text{ H}) - \frac{1}{2\pi(60\text{ Hz})(133 \times 10^{-6}\text{ C})}\right)^2 + (50\ \Omega)^2} = 53\ \Omega$$

Current:

$$i_{\text{max}} = \frac{V_{\text{max}}}{Z} = \frac{12\text{ V}}{53.1\ \Omega} = 0.23\text{ A}$$

Plot of current versus time:

t (s)	V (V)	i (A)
1	12	0.23
2	12	0.23
3	12	0.23
4	12	0.23
5	12	0.23
6	−12	−0.23
7	−12	−0.23
8	−12	−0.23
9	−12	−0.23
10	−12	−0.23
11	12	0.23
12	12	0.23
13	12	0.23
14	12	0.23
15	12	0.23
16	−12	−0.23
17	−12	−0.23
18	−12	−0.23
19	−12	−0.23
20	−12	−0.23
21	12	0.23
22	12	0.23
23	12	0.23
24	12	0.23
25	12	0.23

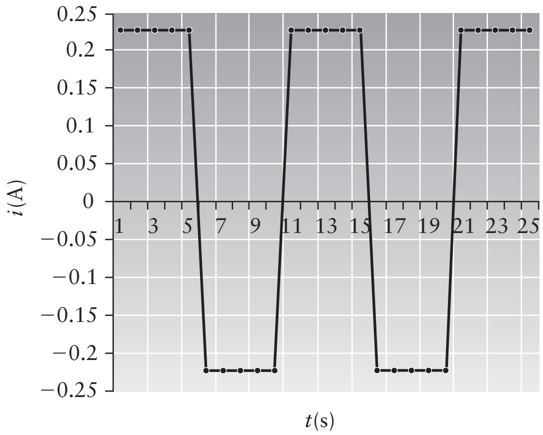


Figure 21-1 Problem 33

Problems

21.34

SET UP

A sinusoidally varying voltage is represented by $V(t) = (75\text{ V})\sin(120\pi t)$. We can compare this equation to the general form of a sinusoidally varying voltage. The general form of such a voltage is $V(t) = V_0 \sin(\omega t)$, where V_0 is the peak voltage and ω is the angular frequency. The frequency is related to the angular frequency by a factor of 2π, $f = \frac{\omega}{2\pi}$.

SOLVE

Peak voltage $= \boxed{75\text{ V}}$.
Frequency:

$$f = \frac{\omega}{2\pi} = \frac{\left(120\pi \frac{\text{rad}}{\text{s}}\right)}{2\pi} = \boxed{60\text{ Hz}}$$

REFLECT
AC voltages in the United States oscillate at 60 Hz.

21.35

SET UP
An AC generator supplies a root-mean-square voltage of $V_{rms} = 120$ V at a frequency of $f = 60$ Hz. We can use the definitions of peak voltage and angular frequency to write an algebraic expression for the instantaneous voltage as a function of time in the form $V(t) = V_0 \sin(\omega t)$.

SOLVE
Peak voltage:

$$V_0 = V_{rms}\sqrt{2} = (120 \text{ V})\sqrt{2} = 120\sqrt{2} \text{ V}$$

Angular frequency:

$$\omega = 2\pi f = 2\pi(60 \text{ Hz}) = 120\pi\frac{\text{rad}}{\text{s}}$$

Voltage expression:

$$V(t) = (120\sqrt{2} \text{ V})\sin(120\pi \text{t})$$

REFLECT
This corresponds to a peak voltage of around 170 V.

21.36

SET UP
A light bulb ($P_{avg} = 60$ W) is plugged into a wall socket ($V_{rms} = 120$ V). The root-mean-square current drawn by the light bulb is given by $P_{avg} = i_{rms}V_{rms}$.

SOLVE

$$P_{avg} = i_{rms}V_{rms}$$

$$i_{rms} = \frac{P_{avg}}{V_{rms}} = \frac{60 \text{ W}}{120 \text{ V}} = \boxed{0.50 \text{ A}}$$

REFLECT
In the United States, light bulbs are rated at a voltage of 120 V.

21.37

SET UP
The peak voltage across the terminals of a sinusoidal AC source ($f = 60$ Hz) is $V_0 = 17$ V. The voltage at $t = 0$ is equal to 0, which means we can model the voltage as a function of time by $V(t) = V_0 \sin(\omega t) = V_0 \sin((2\pi f)t)$. Using the provided information and the algebraic form for $V(t)$, we can solve for the voltage at $t = 2.00 \times 10^{-3}$ s.

SOLVE

$$V(t = 2.00 \times 10^{-3} \text{ s}) = (17 \text{ V})\sin(2\pi(60 \text{ Hz})(2.00 \times 10^{-3} \text{ s})) = \boxed{12 \text{ V}}$$

REFLECT

The maximum voltage occurs at one-quarter cycle or $t = 0.0042$ s, which means $V(t = 0.002 \text{ s})$ should be positive and less than 17 V.

21.38

SET UP

An AC voltage is represented by $V(t) = (200 \text{ V})\sin(120\pi t)$, which means the peak voltage $V_0 = 200$ V. The rms voltage is related to the peak voltage by $V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$.

SOLVE

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}} = \frac{200 \text{ V}}{\sqrt{2}} = \boxed{141 \text{ V}}$$

REFLECT

The general expression for a time-varying voltage is $V(t) = V_0 \sin(\omega t)$.

21.39

SET UP

The root-mean-square current passing through a resistor ($R = 50\ \Omega$) is $i_{\text{rms}} = 12.0$ A. The root-mean-square voltage drop across the resistor is equal to $V_{\text{rms}} = i_{\text{rms}}R$ through Ohm's law. The maximum voltage drop is related to the root-mean-square voltage drop by $V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$.

SOLVE

Root-mean-square voltage:

$$V_{\text{rms}} = i_{\text{rms}}R = (12.0 \text{ A})(50\ \Omega) = 600 \text{ V}$$

Peak voltage:

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$$

$$V_0 = V_{\text{rms}}\sqrt{2} = (600 \text{ V})\sqrt{2} = 850 \text{ V}$$

REFLECT

The maximum voltage drop across the resistor is the same as the peak voltage.

21.40

SET UP

The peak-to-peak current passing through a resistor ($R = 150\ \Omega$) is $i_{\text{peak-to-peak}} = 24.0$ A; this is also equal to twice the maximum current i_0. After solving for the peak current, we can use Ohm's law to find the maximum voltage V_0 across the resistor. The root-mean-square current is equal to $i_{\text{rms}} = \frac{i_0}{\sqrt{2}}$.

SOLVE

Part a)
Maximum current:

$$i_{\text{peak-to-peak}} = 2i_0$$

$$i_0 = \frac{i_{\text{peak-to-peak}}}{2} = \frac{24.0\ \text{A}}{2} = 12.0\ \text{A}$$

Maximum voltage:

$$V_0 = i_0 R = (12.0\ \text{A})(150\ \Omega) = \boxed{1800\ \text{V}}$$

Part b)

$$i_{\text{rms}} = \frac{i_0}{\sqrt{2}} = \frac{12.0\ \text{A}}{\sqrt{2}} = \boxed{8.49\ \text{A}}$$

REFLECT

The peak-to-peak value of an oscillating function is equal to twice its amplitude, as it is the difference between the peak and trough.

21.41

SET UP

We are asked to derive an expression relating the root-mean-square current through a resistor R to the maximum voltage across it. We can relate the root-mean-square current to the root-mean-square voltage using Ohm's law. From there, we can plug in the definition of the peak voltage in terms of the root-mean-square voltage. We can then rearrange this expression and use the provided values to solve for the root-mean-square current, peak voltage, or resistance.

SOLVE

Part a)

$$V_{\text{rms}} = i_{\text{rms}} R$$

$$\frac{V_0}{\sqrt{2}} = i_{\text{rms}} R$$

$$\boxed{V_0 = \sqrt{2}\, i_{\text{rms}} R}$$

Part b)

$$i_{\text{rms}} = \frac{V_0}{R\sqrt{2}} = \frac{50\ \text{V}}{(100\ \Omega)\sqrt{2}} = \boxed{0.35\ \text{A}}$$

Part c)

$$V_0 = \sqrt{2}\, i_{\text{rms}} R = \sqrt{2}(2.5\ \text{A})(200\ \Omega) = \boxed{710\ \text{V}}$$

Part d)

$$R = \frac{V_0}{i_{\text{rms}}\sqrt{2}} = \frac{28\text{ V}}{(0.127\ \Omega)\sqrt{2}} = \boxed{160\ \Omega}$$

REFLECT

We could have derived the first expression using Ohm's law in terms of the peak voltage and current:

$$V_0 = i_0 R$$

$$V_0 = (\sqrt{2}i_{\text{rms}})R$$

21.42

SET UP

We are asked to prove that the integral $\int_0^T \frac{\cos(2\omega t)}{2}dt$ is equal to zero. To help us easily evaluate the integral, we can rewrite the angular frequency ω in terms of the frequency f, and then write f in terms of the period of the oscillation T.

SOLVE

$$\int_0^T \frac{\cos(2\omega t)}{2}dt = \frac{1}{2}\left[\frac{1}{2\omega}\sin(2\omega t)\right]_0^T = \frac{1}{4\omega}[\sin(2\omega T) - \sin(0)] = \frac{1}{4\omega}[\sin(2(2\pi f)T) - 1]$$

$$= \frac{1}{4\omega}\left[\sin\left(4\pi\left(\frac{1}{T}\right)T\right) - 1\right] = \frac{1}{4\omega}[\sin(4\pi) - 1] = \frac{1}{4\omega}[1 - 1] = \boxed{0}$$

REFLECT

The total area under an oscillating function over one complete cycle is equal to zero because there is an equal amount of area above the axis as there is below it.

21.43

SET UP

Each member of a family of four uses the hair dryer for 3.00 min every day. The hair dryer draws a root-mean-square current of $i_{\text{rms}} = 15.6$ A when a voltage of $V_{\text{rms}} = 120$ V is placed across it. The average power dissipated by the hair dryer P_{avg} is equal to $P_{\text{avg}} = i_{\text{rms}}V_{\text{rms}}$. Multiplying the average power in kW by the total time the hair dryer is used in a year in hours will give the total energy in kWh. The local utility company charges \$0.095 per kWh, so we can multiply the total energy by the cost to get the total yearly cost to operate the hair dryer.

SOLVE

Average power:

$$P_{\text{avg}} = i_{\text{rms}}V_{\text{rms}} = (15.6\text{ A})(120\text{ V}) = 1872\text{ W} = 1.872\text{ kW}$$

Total time the hair dryer is used in a year:

$$t = 4(3.00\text{ min})(365\text{ days}) = 4380\text{ min} \times \frac{1\text{ hr}}{60\text{ min}} = 73.0\text{ hr}$$

Total energy used:

$$E = (1.872\ \text{kW})(73.0\ \text{hr}) = 137\ \text{kWh}$$

Total cost:

$$\text{cost} = 137\ \text{kWh} \times \frac{\$0.095}{\text{kWh}} = \boxed{\$12.98}$$

REFLECT

This works out to be a little over \$1 per month, which seems reasonable.

21.44

SET UP

The primary coil ($N_p = 240$ turns) of a transformer is connected to an AC line that has an applied voltage of $V_p = 120$ V. The secondary coil has $N_s = 80$ turns. We can solve for the voltage V_s in the secondary coil using the relationship between the number of turns and the voltages of each coil, which is $V_s = \frac{N_s}{N_p} V_p$.

SOLVE

$$V_s = \frac{N_s}{N_p} V_p = \frac{80\ \text{turns}}{240\ \text{turns}}(120\ \text{V}) = \boxed{40.0\ \text{V}}$$

REFLECT

This is a step-down transformer because the secondary voltage is less than the primary voltage.

21.45

SET UP

The primary coil ($N_p = 400$ turns) of a step-down transformer is connected to an AC line that has an applied voltage of $V_p = 120$ V. The secondary coil voltage is $V_s = 6.50$ V. The relationship between the number of turns and the voltages of each coil is $V_s = \frac{N_s}{N_p} V_p$. We can rearrange this equation to calculate the number of turns of the secondary coil N_s.

SOLVE

$$V_s = \frac{N_s}{N_p} V_p$$

$$N_s = \frac{V_s N_p}{V_p} = \frac{(6.50\ \text{V})(400\ \text{turns})}{120\ \text{V}} = 21.7 = \boxed{22\ \text{turns}}$$

REFLECT

We are told this is a step-down transformer, which means the $N_s < N_p$.

Get Help: P'Cast 21.2 – High Voltage Transformer

21.46

SET UP

The primary coil ($N_p = 240$ turns) of a transformer is connected to an AC line that has an alternating current of $i_p = 3$ A. The secondary coil has $N_s = 80$ turns. The current in the secondary coil is related to the number of turns and the currents through the primary coil by $i_s = \frac{N_p}{N_s} i_p$.

SOLVE

$$i_s = \frac{N_p}{N_s} i_p = \frac{240 \text{ turns}}{80 \text{ turns}}(3 \text{ A}) = \boxed{9 \text{ A}}$$

REFLECT

This is a step-down transformer because the *voltage* across the secondary coil is smaller than the *voltage* across the primary coil.

21.47

SET UP

A transformer produces an output voltage that is 500% larger than the input voltage, that is, $V_s = 5V_p$. The input current is $i_p = 10$ A. We can rearrange $V_p i_p = V_s i_s$ to solve for the output current.

SOLVE

$$V_p i_p = V_s i_s$$

$$i_s = \frac{V_p i_p}{V_s} = \frac{V_p i_p}{(5V_p)} = \frac{i_p}{5} = \frac{10 \text{ A}}{5} = \boxed{2 \text{ A}}$$

REFLECT

The rate of energy transferred between the two coils must be equal, which is how we arrive at the expression $V_p i_p = V_s i_s$.

21.48

SET UP

The primary voltage of a step-up transformer is $V_p = 120$ V. The secondary coil has $N_s =$ 20,000 turns and needs to output a voltage of $V_s = 1.2 \times 10^4$ V. The number of coils in the primary transformer can be found by rearranging $V_s = \frac{N_s}{N_p} V_p$.

SOLVE

$$V_s = \frac{N_s}{N_p} V_p$$

$$N_p = N_s \frac{V_p}{V_s} = (20{,}000 \text{ turns})\left(\frac{120 \text{ V}}{1.2 \times 10^4 \text{ V}}\right) = \boxed{200 \text{ turns}}$$

REFLECT

The output voltage of a step-up transformer is larger than the input voltage.

21.49

SET UP

The primary coil ($N_p = 400$ turns) of a step-down transformer is connected to an AC line that has an applied voltage of $V_p = 120$ V. The secondary coil voltage is $V_s = 6.30$ V. The relationship between the number of turns and the voltages of each coil is $V_s = \frac{N_s}{N_p}V_p$. We can rearrange this equation to calculate the number of turns of the secondary coil N_s. The secondary coil supplies a current of $i_s = 15.0$ A. We can use the relationship between the currents and the voltages of the two coils, $i_s = \frac{V_p}{V_s}i_p$, to calculate the current in the primary coil.

SOLVE

Part a)

$$V_s = \frac{N_s}{N_p}V_p$$

$$N_s = \frac{V_s N_p}{V_p} = \frac{(6.30\text{ V})(400\text{ turns})}{120\text{ V}} = \boxed{21\text{ turns}}$$

Part b)

$$i_s = \frac{V_p}{V_s}i_p$$

$$i_p = \frac{V_s}{V_p}i_s = \left(\frac{6.30\text{ V}}{120\text{ V}}\right)(15.0\ A) = \boxed{0.788\text{ A}}$$

REFLECT

A step-down transformer is designed such that the primary coil has a high voltage but low current and the secondary coil has a low voltage but high current.

Get Help: P'Cast 21.2 – High Voltage Transformer

21.50

SET UP

The ratio of the number of turns in the primary coil to the number of turns in the secondary coil of a step-down transformer is 25:1. The input voltage across the primary coil is $V_p = 750$ V. We can use this information along with $V_s = \frac{N_s}{N_p}V_p$ to calculate the output voltage across the secondary coil of the transformer. The current in the output coil is $i_s = 25$ A; the power delivered to the secondary coil is given by $P = i_s V_s$.

SOLVE

Output voltage:

$$V_s = \frac{N_s}{N_p}V_p = \left(\frac{1}{25}\right)(750\text{ V}) = 30\text{ V}$$

Power:

$$P = i_s V_s = (25\text{ A})(30\text{ V}) = \boxed{750\text{ W}}$$

REFLECT

Our calculated output voltage is less than the input voltage as expected for a step-down transformer.

21.51

SET UP

A high-voltage discharge tube employs a step-up transformer to convert the input voltage of $V_p = 120$ V to an output voltage of $V_s = 5000$ V. The ratio of the number of coils in the secondary to the number of coils in the primary is related to the ratio of these voltages by $V_s = \frac{N_s}{N_p}V_p$. The discharge tube dissipates a power of $P = 75$ W, which is equal to the product of the input current and the input voltage as well as the product of the output current and the output voltage. Rearranging these allows us to calculate the currents in the two coils. Finally, once we have the input voltage and the input current, we can use Ohm's law to find the effective resistance of the input circuit.

SOLVE

Part a)

$$V_s = \frac{N_s}{N_p}V_p$$

$$\frac{N_s}{N_p} = \frac{V_s}{V_p} = \frac{5000\text{ V}}{120\text{ V}} = 42$$

The ratio of the number of coils in the secondary to the number of coils in the primary is $\boxed{42{:}1}$.

Part b)
Primary current:

$$P = i_p V_p$$

$$i_p = \frac{P}{V_p} = \frac{75\text{ W}}{120\text{ V}} = \boxed{0.63\text{ A}}$$

Secondary current:

$$P = i_s V_s$$

$$i_s = \frac{P}{V_s} = \frac{75\text{ W}}{5000\text{ V}} = \boxed{0.015\text{ A}}$$

Part c)

$$V_p = i_p R$$

$$R = \frac{V_p}{i_p} = \frac{120\ \text{V}}{0.63\ \text{A}} = \boxed{190\ \Omega}$$

REFLECT

We could have also solved for the effective resistance in the input circuit using the power:

$$P = i_p^2 R$$

$$R = \frac{P}{i_p^2} = \frac{75\ \text{W}}{(0.63\ \text{A})^2} = 190\ \Omega$$

21.52

SET UP

At $t = 2.0 \times 10^{-3}$ s, a voltage phasor with a peak voltage of $V_0 = 12$ V makes an angle of $\frac{\pi}{4}$rad with the x-axis. This angle is equal to ωt, where ω is the angular frequency of the voltage. The voltage at any time t, $V(t)$, is given by the projection of the phasor on the y-axis, $V(t) = V_0 \sin(\omega t)$.

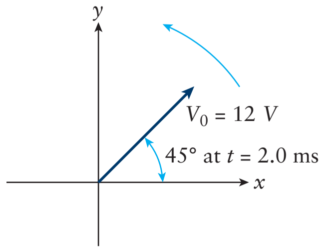


Figure 21-2 Problem 52

SOLVE

Part a)

$$\omega t = \frac{\pi}{4}$$

$$\omega = \frac{\pi}{4t} = \frac{\pi}{4(2.0 \times 10^{-3}\ \text{s})} = \boxed{390 \frac{\text{rad}}{\text{s}}}$$

Part b)

$$V(t) = V_0 \sin(\omega t) = (12\ \text{V})\sin\left(\frac{\pi}{4}\right) = \boxed{8.5\ \text{V}}$$

REFLECT

By assuming this is the first rotation of the phasor, we know the angle must be equal to $\frac{\pi}{4}$rad, and not, say, $\frac{9\pi}{4}$rad.

21.53

SET UP

At $t = 2.0 \times 10^{-3}$ s, a voltage phasor has a value of 50 V and makes an angle of $\frac{\pi}{4}$rad with the x-axis. This angle is equal to ωt,

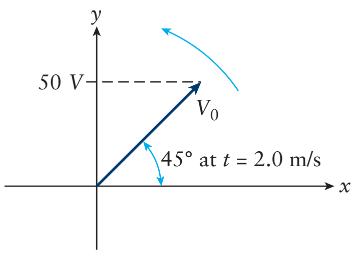


Figure 21-3 Problem 53

where ω is the angular frequency of the voltage. The voltage at any given time $V(t)$ is equal to the projection of the phasor onto the y-axis, $V(t) = V_0 \sin(\omega t)$, where V_0 is the peak voltage.

SOLVE

Angular frequency:

$$\omega t = \frac{\pi}{4}$$

$$\omega = \frac{\left(\frac{\pi}{4}\right)}{t} = \frac{\left(\frac{\pi}{4}\right)}{2.0 \times 10^{-3}\ \text{s}} = \boxed{390\frac{\text{rad}}{\text{s}}}$$

Peak voltage:

$$V(t) = V_0 \sin(\omega t)$$

$$V_0 = \frac{V(t)}{\sin(\omega t)} = \frac{50\ \text{V}}{\sin\left(\frac{\pi}{4}\right)} = \frac{50\ \text{V}}{\left(\frac{1}{\sqrt{2}}\right)} = \boxed{71\ \text{V}}$$

REFLECT

Remember to convert the angle into radians before performing a calculation.

21.54

SET UP

An alternating current source oscillates at a frequency of $f = 60$ Hz and has a peak voltage of $V_0 = 170$ V. The voltage phasor at $t = 3.0 \times 10^{-3}$ s is represented by an arrow of length V_0 that makes an angle ωt with the $+x$-axis, where ω is the angular frequency.

SOLVE

Angle:

$$\omega t = (2\pi f)t = 2\pi(60\ \text{Hz})(3.0 \times 10^{-3}\ \text{s}) = 1.1\ \text{rad} = 65^\circ$$

Phasor:

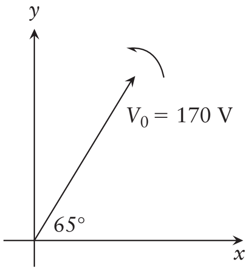


Figure 21-4 Problem 54

REFLECT

The phasor rotates counterclockwise as this corresponds to an increasing angle.

21.55

SET UP

We are given three pairs of phasors and asked to draw phasor diagrams for each case showing both the phase difference and the relative magnitudes of the two phasors. We should draw phasor $A(t)$ first in each case, so that we can use the phase angle between the two phasors to draw $B(t)$.

SOLVE

Part a)

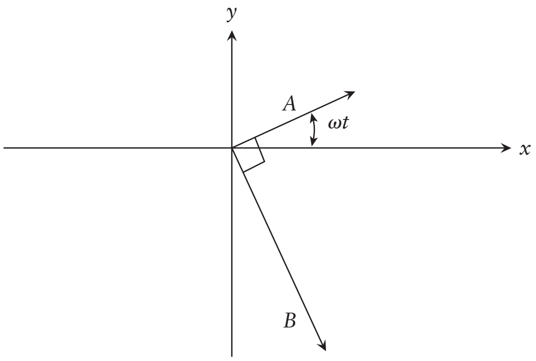


Figure 21-5 Problem 55

Part b)

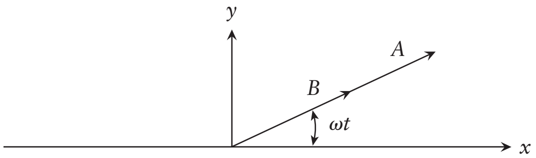


Figure 21-6 Problem 55

Part c)

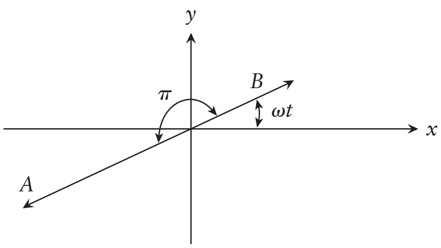


Figure 21-7 Problem 55

REFLECT

A phase angle of $\frac{\pi}{2}$ rad is equal to 90 degrees; a phase angle of π rad is equal to 180 degrees.

21.56

SET UP

An LRC circuit consists of a resistor, an inductor ($L = 5.00 \times 10^{-3}$ H), and a capacitor ($C = 1.00 \times 10^{-6}$ F). We can use the expression $\omega_0 = \sqrt{\frac{1}{LC}}$ and the fact that $\omega_0 = 2\pi f_0$ to calculate the natural frequency of this LRC circuit.

SOLVE

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

$$2\pi f_0 = \sqrt{\frac{1}{LC}}$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{(5.00 \times 10^{-3}\text{ H})(1.00 \times 10^{-6}\text{ F})}} = \boxed{2.25 \times 10^{3}\text{ Hz}}$$

REFLECT

The natural (or resonant) frequency of a circuit is commonly given in Hz, rather than rad/s.

21.57

SET UP

A circuit that contains a resistor, an inductor ($L = 5.00$ H), and a capacitor resonates at $f_0 = 1000$ Hz. Using the expression for the natural frequency of an LRC circuit, $\omega_0 = \sqrt{\frac{1}{LC}}$, we can calculate the value of the capacitor. Recall that $\omega_0 = 2\pi f_0$.

SOLVE

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

$$\text{C} = \frac{1}{L\omega_0^2} = \frac{1}{L(2\pi f_0)^2} = \frac{1}{4\pi^2(5.00\text{ H})(1000\text{ Hz})^2} = \boxed{5.07 \times 10^{-9}\text{ F} = 5.07\text{ nF}}$$

REFLECT

The resistor does not affect the natural frequency of the circuit, only the maximum current.

21.58

SET UP

A circuit that consists of a resistor, an inductor, and a capacitor ($C = 125 \times 10^{-9}$ F) resonates at $f_0 = 250$ Hz. Using the expression for the natural frequency of an LRC circuit, $\omega_0 = \sqrt{\frac{1}{LC}}$, we can calculate the value of the inductor. Recall that $\omega_0 = 2\pi f_0$.

SOLVE

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

$$L = \frac{1}{C\omega_0^2} = \frac{1}{C(2\pi f_0)^2} = \frac{1}{4\pi^2(125 \times 10^{-9}\text{ F})(250\text{ Hz})^2} = \boxed{3.24\text{ H}}$$

REFLECT

The resistance does not affect the natural frequency of the circuit, only the maximum current.

21.59

SET UP

A sinusoidal voltage ($f = 60$ Hz) is applied to an inductor ($L = 0.20$ H). The reactance of the inductor is given by $X_L = \omega L = (2\pi f)L$.

SOLVE

$$X_L = \omega L = (2\pi f)L = 2\pi(60\text{ Hz})(0.20\text{ H}) = \boxed{75\ \Omega}$$

REFLECT

The reactance of an inductor is directly proportional to the frequency of the applied voltage.

21.60

SET UP

A sinusoidal voltage ($f = 60$ Hz, $V_{\text{rms}} = 120$ V) is applied to an inductor ($L = 0.20$ H). The reactance of the inductor is given by $X_L = \omega L = (2\pi f)L$. Once we know the reactance of the inductor, we can use Ohm's law and the relationship between the root-mean-square value and the peak value to calculate the peak current in the circuit.

SOLVE

Reactance:

$$X_L = \omega L = (2\pi f)L = 2\pi(60\text{ Hz})(0.20\text{ H}) = 75\ \Omega$$

Peak current:

$$V_{\text{rms}} = i_{\text{rms}}X_L = \left(\frac{i_0}{\sqrt{2}}\right)X_L$$

$$i_0 = \frac{V_{\text{rms}}\sqrt{2}}{X_L} = \frac{(120\text{ V})\sqrt{2}}{75\ \Omega} = \boxed{2.3\text{ A}}$$

REFLECT

The current will oscillate between +2.3 A and −2.3 A.

21.61

SET UP

A sinusoidal voltage ($f = 60$ Hz) is applied to a capacitor ($C = 50.0 \times 10^{-6}$ F). The reactance of the capacitor is given by $X_C = \frac{1}{\omega C} = \frac{1}{(2\pi f)C}$.

SOLVE

$$X_C = \frac{1}{\omega C} = \frac{1}{(2\pi f)C} = \frac{1}{2\pi(60\text{ Hz})(50.0 \times 10^{-6}\text{ F})} = \boxed{53\ \Omega}$$

REFLECT

The reactance of a capacitor or inductor is equivalent to the resistance of a resistor, but the reactance depends upon the frequency of the applied voltage, while we usually assume the resistance remains constant.

Get Help: P'Cast 21.3 – High-Pass Filter

21.62

SET UP

A capacitor C has a capacitive reactance of $X_{C,1} = 160\ \Omega$ at $f_1 = 60$ Hz. We can set up a ratio using these data and the algebraic expression for the reactance of a capacitor to calculate the reactance of the capacitor at $f_2 = 600$ Hz.

SOLVE

$$\frac{X_{C,2}}{X_{C,1}} = \frac{\left(\frac{1}{\omega_2 C}\right)}{\left(\frac{1}{\omega_1 C}\right)} = \frac{\omega_1}{\omega_2} = \frac{2\pi f_1}{2\pi f_2} = \frac{f_1}{f_2}$$

$$X_{C,2} = \left(\frac{f_1}{f_2}\right)X_{C,1} = \left(\frac{60\text{ Hz}}{600\text{ Hz}}\right)(160\ \Omega) = \boxed{16\ \Omega}$$

REFLECT

The capacitive reactance is inversely proportional to the frequency of the applied voltage, so the reactance should be smaller at $f_2 = 600$ Hz than at $f_1 = 60$ Hz.

21.63

SET UP

A sinusoidal voltage ($f = 60$ Hz, $V_{rms} = 120$ V) is applied to a capacitor ($C = 50.0 \times 10^{-6}$ F). The reactance of the capacitor is given by $X_C = \frac{1}{\omega C} = \frac{1}{(2\pi f)C}$. Once we know the reactance of the capacitor, we can use Ohm's law and the relationship between the root-mean-square value and the peak value to calculate the peak current in the circuit.

SOLVE

Reactance:

$$X_C = \frac{1}{\omega C} = \frac{1}{(2\pi f)C} = \frac{1}{2\pi(60\text{ Hz})(50.0 \times 10^{-6}\text{ F})} = 53\ \Omega$$

Peak current:

$$V_{\text{rms}} = i_{\text{rms}} X_C = \left(\frac{i_0}{\sqrt{2}}\right) X_C$$

$$i_0 = \frac{V_{\text{rms}}\sqrt{2}}{X_C} = \frac{(120\text{ V})\sqrt{2}}{53\ \Omega} = \boxed{3.2\text{ A}}$$

REFLECT

The current in the circuit will oscillate between +3.2 A and −3.2 A.

21.64

SET UP

Voltages oscillating at frequencies of $f = 60.0$ Hz, 6000 Hz, and 6.00×10^6 Hz are applied to a capacitor ($C = 5.0 \times 10^{-6}$ F). The resulting reactance of the capacitor is given by $X_C = \frac{1}{\omega C} = \frac{1}{(2\pi f)C}$.

SOLVE

Part a)

$$X_C = \frac{1}{\omega C} = \frac{1}{(2\pi f)C} = \frac{1}{2\pi(60\text{ Hz})(5.0 \times 10^{-6}\text{ F})} = \boxed{530\ \Omega}$$

Part b)

$$X_C = \frac{1}{\omega C} = \frac{1}{(2\pi f)C} = \frac{1}{2\pi(6000\text{ Hz})(5.0 \times 10^{-6}\text{ F})} = \boxed{5.3\ \Omega}$$

Part c)

$$X_C = \frac{1}{\omega C} = \frac{1}{(2\pi f)C} = \frac{1}{2\pi(6.0 \times 10^6\text{ Hz})(5.0 \times 10^{-6}\text{ F})} = \boxed{5.3 \times 10^{-3}\ \Omega}$$

REFLECT

The reactance of the capacitor is inversely proportional to the frequency of the applied voltage. Therefore, we would expect the reactance to decrease as we increase the frequency.

21.65

SET UP

Voltages oscillating at frequencies of $f = 60$ Hz, 6000 Hz, and 6×10^6 Hz are applied to an inductor ($L = 5.0 \times 10^{-3}$ H). The resulting reactance of the inductor is given by $X_L = \omega L = (2\pi f)L$.

SOLVE

Part a)

$$X_L = 2\pi(60\ \text{Hz})(5.0 \times 10^{-3}\ \text{H}) = \boxed{1.88\ \Omega}$$

Part b)

$$X_L = 2\pi(6000\ \text{Hz})(5.0 \times 10^{-3}\ \text{H}) = \boxed{188\ \Omega}$$

Part c)

$$X_L = 2\pi(6 \times 10^6\ \text{Hz})(5.0 \times 10^{-3}\ \text{H}) = \boxed{1.88 \times 10^5\ \Omega = 188\ \text{k}\Omega}$$

REFLECT

We could have also used proportional reasoning to calculate the answers to parts b and c. The frequency in part b is 100 times larger than the frequency in part a, which means the reactance will also be 100 times larger. The frequency in part c is 1000 times larger than the frequency in part b, which means the reactance will also be 1000 times larger.

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21.66

SET UP

A root-mean-square voltage of $V_{\text{rms}} = 120$ V is connected across a resistor ($R = 100\ \Omega$). The root-mean-square current i_{rms} is given by Ohm's law. It is related to the peak current i_0 by a factor of $\sqrt{2}$, $i_0 = i_{\text{rms}}\sqrt{2}$. Finally, the average power dissipated in the resistor is equal to the product of the root-mean-square voltage and the root-mean-square current.

SOLVE

Part a)

$$V_{\text{rms}} = i_{\text{rms}}R$$

$$i_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{120\ \text{V}}{100\ \Omega} = \boxed{1.20\ \text{A}}$$

Part b)

$$i_0 = i_{\text{rms}}\sqrt{2} = (1.20\ \text{A})\sqrt{2} = \boxed{1.70\ \text{A}}$$

Part c)

$$P_{\text{avg}} = i_{\text{rms}}V_{\text{rms}} = (1.20\ \text{A})(120\ \text{V}) = \boxed{144\ \text{W}}$$

REFLECT

The resistance of a resistor is independent of the frequency of the applied voltage.

21.67

SET UP

A sinusoidal voltage ($V_0 = 50.0$ V, $f = 400$ Hz) is applied to a capacitor C. The root-mean-square current in the circuit is $i_{\text{rms}} = 0.400$ A. We can use Ohm's law and the algebraic expression for the capacitive reactance, $X_C = \frac{1}{\omega C} = \frac{1}{(2\pi f)C}$, to solve for the value of C; recall that $i_0 = i_{\text{rms}}\sqrt{2}$. We can use the algebraic expression from part (a) to determine what will happen to the value of i_{rms} if the frequency of the applied voltage is increased.

SOLVE

Part a)

$$V_0 = i_0 X_C = (i_{\text{rms}}\sqrt{2})\left(\frac{1}{\omega C}\right) = \frac{i_{\text{rms}}\sqrt{2}}{(2\pi f)C}$$

$$C = \frac{i_{\text{rms}}\sqrt{2}}{(2\pi f)V_0} = \frac{(0.400\text{ A})\sqrt{2}}{2\pi(400\text{ Hz})(50.0\text{ V})} = \boxed{4.50 \times 10^{-6}\text{ F} = 4.50\ \mu\text{F}}$$

Part b) The root-mean-square current will increase. As ω is increased, X_C is reduced, so i_{rms} will increase.

REFLECT

It is always best to completely solve a problem algebraically first and plug in the numbers last. This way, it's not only easier to check your answer for mistakes, but you can also gain some intuition through proportional reasoning.

21.68

SET UP

An oscillating voltage ($V_{\text{rms}} = 40$ V, $f = 100$ Hz) is first applied to an inductor ($L = 0.20$ H) and then to a capacitor ($C = 50 \times 10^{-6}$ F). In either case, we can relate the root-mean-square voltage and the reactance of the circuit element to the root-mean-square current through Ohm's law. The peak current can be found from the root-mean-square current from $i_{\text{rms}} = \frac{i_0}{\sqrt{2}}$. The inductive reactance is $X_L = \omega L$, and the capacitive reactance is $X_C = \frac{1}{\omega C}$.

SOLVE

Part a)

$$V_{\text{rms}} = i_{\text{rms}} X_L = \left(\frac{i_0}{\sqrt{2}}\right)(\omega L) = \left(\frac{i_0}{\sqrt{2}}\right)(2\pi f)L$$

$$i_0 = \frac{V_{\text{rms}}\sqrt{2}}{2\pi f L} = \frac{(40\text{ V})\sqrt{2}}{2\pi(100\text{ Hz})(0.20\text{ H})} = \boxed{0.45\text{ A}}$$

Part b)

$$V_{rms} = i_{rms}X_C = \left(\frac{i_0}{\sqrt{2}}\right)\left(\frac{1}{\omega C}\right) = \left(\frac{i_0}{\sqrt{2}}\right)\frac{1}{(2\pi f)C}$$

$$i_0 = 2\pi V_{rms} f C\sqrt{2} = 2\pi(40\text{ V})(100\text{ Hz})(50 \times 10^{-6}\text{ F})\sqrt{2} = \boxed{1.8\text{ A}}$$

REFLECT

The peak current through an inductor is inversely proportional to the frequency of the applied voltage, whereas the peak current through a capacitor is directly proportional to the frequency of the applied voltage.

21.69

SET UP

A sinusoidal voltage ($V_{rms} = 40.0$ V, $f = 100$ Hz) is applied to a resistor ($R = 100\ \Omega$), an inductor ($L = 0.200$ H), and a capacitor ($C = 50.0 \times 10^{-6}$ F). The current as a function of time for a resistor, inductor, and capacitor each attached separately to an AC voltage source is $i_R(t) = \frac{V_0}{R}\sin(\omega t)$, $i_L(t) = \frac{V_0}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$, and $i_C(t) = \omega C V_0 \sin\left(\omega t + \frac{\pi}{2}\right)$, respectively. The peak current is the amplitude of the oscillation in all cases. The average power delivered to the resistor is equal to $P_{R,\text{ avg}} = \frac{1}{2}\frac{V_0^2}{R}$. Since the power is equal to the voltage multiplied by the current, the average power delivered to the inductor and capacitor is zero because the current and voltage are 90 degrees out of phase.

SOLVE

Part a)

Peak current:

$$i_R(t) = \frac{V_0}{R}\sin(\omega t)$$

$$i_{R,\text{ max}} = \frac{V_0}{R} = \frac{V_{rms}\sqrt{2}}{R} = \frac{(40.0\text{ V})\sqrt{2}}{100\ \Omega} = \boxed{0.566\text{ A}}$$

Average power:

$$P_{R,\text{ avg}} = \frac{1}{2}\frac{V_0^2}{R} = \frac{1}{2}\frac{(V_{rms}\sqrt{2})^2}{R} = \frac{V_{rms}^2}{R} = \frac{(40.0\text{ V})^2}{100\ \Omega} = \boxed{16.0\text{ W}}$$

Part b)

Peak current:

$$i_L(t) = \frac{V_0}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$i_{L,\,\max} = \frac{V_0}{\omega L} = \frac{V_{\text{rms}}\sqrt{2}}{(2\pi f)L} = \frac{(40.0\text{ V})\sqrt{2}}{2\pi(100\text{ Hz})(0.200\text{ H})} = \boxed{0.450\text{ A}}$$

Average power:

$$P_{C,\,\text{avg}} = 0$$

Part c)

Peak current:

$$i_C(t) = \omega C V_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$i_{C,\,\max} = \omega C V_0 = (2\pi f)C(V_{\text{rms}}\sqrt{2}) = 2\pi\sqrt{2} f C V_{\text{rms}}$$

$$= 2\pi\sqrt{2}(100\text{ Hz})(50.0\times 10^{-6}\text{ F})(40.0\text{ V}) = \boxed{1.78\text{ A}}$$

Average power:

$$P_{C,\,\text{avg}} = 0$$

REFLECT

The average of a function is $f_{\text{avg}} = \dfrac{1}{b-a}\displaystyle\int_a^b f(x)dx$, so the time average of the power delivered to a circuit element is $P_{\text{avg}} = \dfrac{1}{T}\displaystyle\int_0^T i(t)V(t)dt$. For a capacitor and an inductor, the integrand is proportional to $\sin(\omega t)\cos(\omega t)$, so the integral exactly equals zero.

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21.70

SET UP

An inductor ($L = 0.035$ H) with an intrinsic resistance $R = 0.20\ \Omega$ is connected in series to a capacitor ($C = 200\times 10^{-6}$ F) and an AC voltage source ($V_{\text{rms}} = 45$ V, $f = 60$ Hz). In order to find the root-mean-square current in the circuit, we first need to find the impedance Z of the circuit, where $Z = \sqrt{\left(\omega L - \dfrac{1}{\omega C}\right)^2 + R^2}$. The root-mean-square current is equal to the root-mean-square voltage divided by the impedance, according to Ohm's law. The phase angle between the time-varying current and the time-varying voltage is given by $\tan(\varphi) = \dfrac{\omega L - \left(\dfrac{1}{\omega C}\right)}{R}$, where $\omega = 2\pi f$.

SOLVE

Part a)
Impedance:

$$Z = \sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2} = \sqrt{\left((2\pi f)L - \frac{1}{(2\pi f)C}\right)^2 + R^2}$$

$$= \sqrt{\left(2\pi(60\text{ Hz})(0.035\text{ H}) - \frac{1}{2\pi(60\text{ Hz})(200 \times 10^{-6}\text{ F})}\right)^2 + (0.20\ \Omega)^2} = 0.21\ \Omega$$

Root-mean-square current:

$$V_{\text{rms}} = i_{\text{rms}} Z$$

$$i_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{45\text{ V}}{0.21\ \Omega} = \boxed{210\ \Omega}$$

Part b)

$$\tan(\varphi) = \frac{\omega L - \left(\frac{1}{\omega C}\right)}{R}$$

$$\varphi = \arctan\left(\frac{\omega L - \left(\frac{1}{\omega C}\right)}{R}\right) = \arctan\left(\frac{(2\pi f)\text{L} - \left(\frac{1}{(2\pi f)C}\right)}{R}\right)$$

$$= \arctan\left(\frac{2\pi(60\text{ Hz})(0.035\text{ H}) - \left(\frac{1}{2\pi(60\text{ Hz})(200 \times 10^{-6}\text{ F})}\right)}{0.20\ \Omega}\right) = \boxed{-0.33\text{ rad}}$$

REFLECT

The contribution to the impedance from the inductive and capacitive reactances is about an order of magnitude smaller than the resistance in the circuit. Therefore, the impedance should be very close to the resistance.

21.71

SET UP

An AC voltage, described by $V(t) = (10\text{ V})\sin(12\pi t)$, is applied to an LRC series circuit with $L = 0.250$ H, $R = 20\ \Omega$, and $C = 350 \times 10^{-6}$ F. We want to know the instantaneous voltage across each element at a time $t = 0.04$ s. First we need to find the peak current in the circuit

from the peak voltage and the impedance. Next, the phase angle between the time-varying current and the time-varying voltage is given by $\tan(\varphi) = \dfrac{\omega L - \left(\dfrac{1}{\omega C}\right)}{R}$, where $\omega = 12\pi \dfrac{\text{rad}}{\text{s}}$. Finally, the voltage across the resistor, capacitor, and inductor in the LRC circuit as a function of time are given by $V_R(t) = i_0 R \sin(12\pi t + |\varphi|)$, $V_C(t) = \dfrac{i_0}{\omega C}\sin\left(12\pi t + |\varphi| - \dfrac{\pi}{2}\right)$, and $V_L(t) = i_0 \omega L \sin\left(12\pi t + |\varphi| + \dfrac{\pi}{2}\right)$, respectively.

SOLVE

Peak current:

$$i_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{\left(\omega L - \dfrac{1}{\omega C}\right)^2 + R^2}}$$

$$= \frac{10\text{ V}}{\sqrt{\left(\left(12\pi \dfrac{\text{rad}}{\text{s}}\right)(0.250\text{ H}) - \left(\dfrac{1}{\left(12\pi \dfrac{\text{rad}}{\text{s}}\right)(350 \times 10^{-6}\text{ F})}\right)\right)^2 + (20\ \Omega)^2}}$$

$$= \frac{10\text{ V}}{69.3\ \Omega} = 0.1443\text{ A}$$

Phase angle:

$$\tan(\varphi) = \frac{\omega L - \left(\dfrac{1}{\omega C}\right)}{R}$$

$$\varphi = \arctan\left(\frac{\omega L - \left(\dfrac{1}{\omega C}\right)}{R}\right)$$

$$= \arctan\left(\frac{\left(12\pi \dfrac{\text{rad}}{\text{s}}\right)(0.250\text{ H}) - \left(\dfrac{1}{\left(12\pi \dfrac{\text{rad}}{\text{s}}\right)(350 \times 10^{-6}\text{ F})}\right)}{20\ \Omega}\right) = -1.278\text{ rad}$$

Voltage across the resistor at $t = 0.04$ s:

$$V_R(t) = i_0 R \sin(12\pi t + |\varphi|)\ \text{(SI units)}$$

$$V_R(0.04) = (0.1443)(20)\sin(12\pi(0.04) + 1.278) = \boxed{1.004\text{ V}}$$

Voltage across the capacitor at $t = 0.04$ s:

$$V_C(t) = \frac{i_0}{\omega C}\sin\left(12\pi t + |\varphi| - \frac{\pi}{2}\right) \text{ (SI units)}$$

$$V_C(0.04) = \frac{(0.1443)}{(12\pi)(350 \times 10^{-6})}\sin\left(12\pi(0.04) + 1.278 - \frac{\pi}{2}\right) = \boxed{10.25 \text{ V}}$$

Voltage across the inductor at $t = 0.04$ s:

$$V_L(t) = i_0\omega L\sin\left(12\pi t + |\varphi| + \frac{\pi}{2}\right) \text{ (SI units)}$$

$$V_L(0.04) = (0.1443)(12\pi)(0.250)\sin\left(12\pi(0.04) + 1.278 + \frac{\pi}{2}\right) = \boxed{-1.275 \text{ V}}$$

REFLECT

We can double check our answer to make sure the sum of the voltages across each element at $t = 0.04$ s equals the total voltage at $t = 0.04$ s:

$$V(t) = V_0 \sin(12\pi t)$$

$$V(0.04) = (10)\ \sin(12\pi(0.04)) = 9.98 \text{ V}$$

$$V_R(0.04) + V_C(0.04) + V_L(0.04) = (1.004 \text{ V}) + (10.25 \text{ V}) + (-1.275 \text{ V}) = 9.98 \text{ V}$$

21.72

SET UP

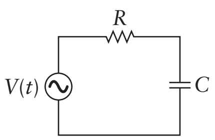

Figure 21-8 Problem 72

An AC circuit consists of a voltage source, which oscillates at $f = 300$ Hz in part (a) and $f = 5000$ Hz in part (b), a capacitor ($C = 1.5 \times 10^{-6}$ F) and a resistor ($R = 100\ \Omega$) in series with negligible inductance. The phase angle between the potential and the current is given by $\tan(\varphi) = \frac{\omega L - \left(\frac{1}{\omega C}\right)}{R}$, where $\omega = 2\pi f$. We can use Ohm's law to find the ratio between the voltage across the capacitor to the voltage across the resistor; this will help us determine the relative sizes for the phasor diagram.

SOLVE

Part a)
Phase angle:

$$\tan(\varphi) = \frac{\omega L - \left(\frac{1}{\omega C}\right)}{R}$$

$$\varphi = \arctan\left(\frac{\omega L - \left(\frac{1}{\omega C}\right)}{R}\right) = \arctan\left(\frac{0 - \left(\frac{1}{(2\pi f)C}\right)}{R}\right) = \arctan\left(-\frac{1}{2\pi f C R}\right)$$

$$= \arctan\left(-\frac{1}{2\pi(300\text{ Hz})(1.5 \times 10^{-6}\text{ F})(100\ \Omega)}\right) = \boxed{-1.3\text{ rad}}$$

Relative potentials:

$$\frac{V_C}{V_R} = \frac{iX_C}{iR} = \frac{\left(\frac{1}{\omega C}\right)}{R} = \frac{\left(\frac{1}{(2\pi f)C}\right)}{R} = \frac{1}{2\pi(300\text{ Hz})(1.5 \times 10^{-6}\text{ F})(100\ \Omega)} = 3.5$$

$$V_C = 3.5 V_R$$

Phasor diagram:

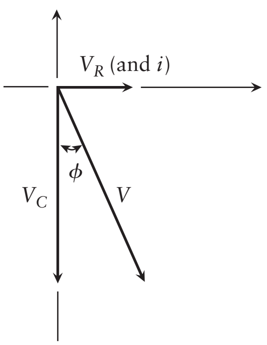

Figure 21-9 Problem 72

Part b)
Phase angle:

$$\tan(\varphi) = \frac{\omega L - \left(\frac{1}{\omega C}\right)}{R}$$

$$\varphi = \arctan\left(\frac{\omega L - \left(\frac{1}{\omega C}\right)}{R}\right) = \arctan\left(\frac{0 - \left(\frac{1}{(2\pi f)C}\right)}{R}\right) = \arctan\left(-\frac{1}{2\pi f C R}\right)$$

$$= \arctan\left(-\frac{1}{2\pi(5000\text{ Hz})(1.5 \times 10^{-6}\text{ F})(100\ \Omega)}\right) = \boxed{-0.21\text{ rad}}$$

Relative potentials:

$$\frac{V_C}{V_R} = \frac{iX_C}{iR} = \frac{\left(\frac{1}{\omega C}\right)}{R} = \frac{\left(\frac{1}{(2\pi f)C}\right)}{R} = \frac{1}{2\pi(5000\ \text{Hz})(1.5 \times 10^{-6}\ \text{F})(100\ \Omega)} = 0.21$$

$$V_C = 0.21 V_R$$

Phasor diagram:

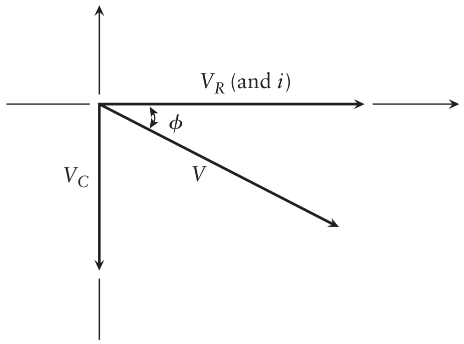

Figure 21-10 Problem 72

REFLECT

The current is in phase with the voltage across the resistor but out of phase by 90 degrees with the voltage across the capacitor.

21.73

SET UP

An LRC circuit has an inductance of $L = 0.60$ H, a capacitance of $C = 3.5 \times 10^{-6}$ F, and a resistance of $R = 250\ \Omega$. The AC voltage is oscillating at a frequency of $f = 60$ Hz. The impedance of the circuit is given by $Z = \sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2}$, where $\omega = 2\pi f$. The tangent of the phase angle between the current and the voltage is equal to $\tan(\varphi) = \frac{\omega L - \left(\frac{1}{\omega C}\right)}{R}$.

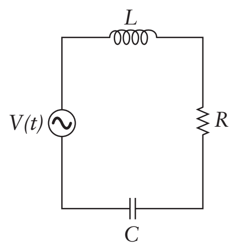

Figure 21-11 Problem 73

SOLVE

Part a)

$$Z = \sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2} = \sqrt{\left((2\pi f)L - \frac{1}{(2\pi f)C}\right)^2 + R^2}$$

$$= \sqrt{\left(2\pi(60\ \text{Hz})(0.60\ \text{H}) - \frac{1}{2\pi(60\ \text{Hz})(3.5 \times 10^{-6}\ \text{F})}\right)^2 + (250\ \Omega)^2} = \boxed{590\ \Omega}$$

Part b)

$$\tan(\varphi) = \frac{\omega L - \left(\frac{1}{\omega C}\right)}{R}$$

$$\varphi = \arctan\left(\frac{\omega L - \left(\frac{1}{\omega C}\right)}{R}\right) = \arctan\left(\frac{(2\pi f)L - \left(\frac{1}{(2\pi f)C}\right)}{R}\right)$$

$$= \arctan\left(\frac{2\pi(60\text{ Hz})(0.60\text{ H}) - \left(\frac{1}{2\pi(60\text{ Hz})(3.5\times 10^{-6}\text{ F})}\right)}{250\ \Omega}\right) = -65°$$

The angle between the current and voltage is $\boxed{65°}$.

REFLECT

We took the absolute value of the phase angle since we were only interested in the magnitude of the angle between the current and the voltage.

21.74

SET UP

An LRC circuit has an inductance of $L = 0.60$ H, a capacitance of $C = 3.5 \times 10^{-6}$ F, and a resistance of $R = 280\ \Omega$. The AC voltage is oscillating at a frequency of $f = 60$ Hz and has a root-mean-square amplitude of $V_{rms} = 150$ V. The impedance of the circuit is given by $Z = \sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2}$, where $\omega = 2\pi f$. Once we know the impedance of the circuit, we can calculate the root-mean-square current from Ohm's law and the root-mean-square voltage.

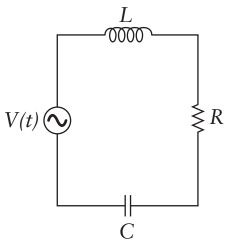

Figure 21-12 Problem 74

SOLVE

Impedance:

$$Z = \sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2} = \sqrt{\left((2\pi f)L - \frac{1}{(2\pi f)C}\right)^2 + R^2}$$

$$= \sqrt{\left(2\pi(60\text{ Hz})(0.60\text{ H}) - \frac{1}{2\pi(60\text{ Hz})(3.5\times 10^{-6}\text{ F})}\right)^2 + (280\ \Omega)^2} = 600\ \Omega$$

Root-mean-square current:

$$V_{\mathrm{rms}} = i_{\mathrm{rms}} Z$$

$$i_{\mathrm{rms}} = \frac{V_{\mathrm{rms}}}{Z} = \frac{150\ \mathrm{V}}{600\ \Omega} = \boxed{0.25\ \mathrm{A}}$$

REFLECT

The capacitive reactance is approximately 760 Ω, and the inductive reactance is around 230 Ω.

21.75

SET UP

In an LRC series circuit, the inductive reactance is $X_L = 2500\ \Omega$, the capacitive reactance is $X_C = 3400\ \Omega$, and the resistance is $R = 1000\ \Omega$. The impedance of the circuit is given by $Z = \sqrt{(X_L - X_C)^2 + R^2}$.

SOLVE

$$Z = \sqrt{(X_L - X_C)^2 + R^2} = \sqrt{((2500\ \Omega) - (3400\ \Omega))^2 + (1000\ \Omega)^2} = \boxed{1345\ \Omega}$$

REFLECT

Since $X_C > X_L$ the capacitor will have a greater effect on the current than the inductor, so this circuit is more “capacitive” than “inductive”.

Get Help: P’Cast 21.4 – A Series LRC Circuit Driven by an AC Voltage – Impedance

21.76

SET UP

In an LRC circuit, the net reactance is $|X_L - X_C| = 1400\ \Omega$ and the resistance is $R = 600\ \Omega$. The impedance of the circuit is given by $Z = \sqrt{(X_L - X_C)^2 + R^2}$.

SOLVE

$$Z = \sqrt{(X_L - X_C)^2 + R^2} = \sqrt{(1400\ \Omega)^2 + (600\ \Omega)^2} = \boxed{1523\ \Omega}$$

REFLECT

Only the magnitude of the difference between the inductive and capacitive reactances is necessary to calculate the impedance. The sign associated with the difference comes into play when calculating the phase angle.

21.77

SET UP

An inductor, which has a reactance $X_L = 10\ \Omega$ and a resistance $R = 8.0\ \Omega$, is connected in series with an AC voltage source ($V_{\mathrm{rms}} = 120\ \mathrm{V}$). After finding the impedance of the circuit, we can use Ohm’s law to calculate the root-mean-square current drawn by the inductor.

SOLVE

Impedance:

$$Z = \sqrt{(X_L - X_C)^2 + R^2} = \sqrt{((10\ \Omega) - 0)^2 + (8.0\ \Omega)^2} = 12.8\ \Omega$$

Root-mean-square current:

$$V_{\text{rms}} = i_{\text{rms}} Z$$

$$i_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{120\ \text{V}}{12.8\ \Omega} = \boxed{9.37\ \text{A}}$$

REFLECT

The peak voltage is 170 V, which leads to a peak current of 13 A.

21.78

SET UP

An LRC circuit consists of an inductor ($L = 0.20$ H), a resistor ($R = 500\ \Omega$), and a capacitor ($C = 2.0 \times 10^{-6}$ F) all wired in series to a sinusoidal voltage source, $V(t) = (100\ \text{V})\sin(1000\pi t)$, which means $V_0 = 100$ V and $f = 500$ Hz. The impedance Z of the circuit is given by $Z = \sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2} = \sqrt{\left((2\pi f)L - \frac{1}{(2\pi f)C}\right)^2 + R^2}$. The peak current can be found through Ohm's law, $V_0 = i_0 Z$. Finally, the general form of the time-varying current in the circuit is $i(t) = i_0 \sin(\omega t - \varphi)$, where the phase angle φ is given by $\tan(\varphi) = \dfrac{\omega L - \left(\frac{1}{\omega C}\right)}{R} = \dfrac{(2\pi f)L - \left(\frac{1}{(2\pi f)C}\right)}{R}$.

SOLVE

Part a)

$$Z = \sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2} = \sqrt{\left((2\pi f)L - \frac{1}{(2\pi f)C}\right)^2 + R^2}$$

$$= \sqrt{\left(2\pi(500\ \text{Hz})(0.20\ \text{H}) - \frac{1}{2\pi(500\ \text{Hz})(2.0 \times 10^{-6}\ \text{F})}\right)^2 + (500\ \Omega)^2} = 686\ \Omega$$

Part b)

$$V_0 = i_0 Z$$

$$i_0 = \frac{V_0}{Z} = \frac{100\ \text{V}}{686\ \Omega} = \boxed{0.146\ \text{A}}$$

Part c)
Phase angle:

$$\tan(\varphi) = \frac{\omega L - \left(\frac{1}{\omega C}\right)}{R}$$

$$\varphi = \arctan\left(\frac{\omega L - \left(\frac{1}{\omega C}\right)}{R}\right) = \arctan\left(\frac{(2\pi f)L - \left(\frac{1}{(2\pi f)C}\right)}{R}\right)$$

$$= \arctan\left(\frac{2\pi(500\text{ Hz})(0.20\text{ H}) - \left(\frac{1}{2\pi(500\text{ Hz})(2.0\times10^{-6}\text{ F})}\right)}{500\ \Omega}\right) = 0.75$$

Current:

$$i(t) = i_0 \sin(\omega t - \varphi) = \boxed{(0.146\text{ A})\sin(1000\pi t - 0.75)}$$

REFLECT

The resonant frequency of the circuit is $f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \frac{1}{2\pi}\sqrt{\frac{1}{(0.20\text{ H})(2.0\times10^{-6}\text{ F})}} =$ 250 Hz. Since the power source is oscillating off resonance, the current and voltage will be out of phase.

21.79

SET UP

A resistor ($R = 250\ \Omega$) and an inductor ($L = 0.04$ H) are wired in series with an AC voltage source ($V_{max} = 100$ V). The impedance of the circuit is given by $Z = \sqrt{(\omega L)^2 + R^2} = \sqrt{(2\pi f L)^2 + R^2}$, since the capacitance in the circuit is negligible. The peak current in the circuit is equal to the peak input voltage divided by the impedance. Once we have the current in terms of V_{max}, we can use the expression for the voltage across the inductor $V_{L,\,max} = i_{0,\,max}X_L$ to solve for the peak voltage across the inductor for $f = 50$ Hz and $f = 5000$ Hz. We can determine what this circuit filters by comparing the voltage across the inductor to the input voltage as a function of frequency and seeing if there is a pattern.

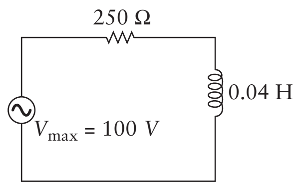

Figure 21-13 Problem 79

SOLVE

Impedance:

$$Z = \sqrt{(\omega L)^2 + R^2} = \sqrt{(2\pi f L)^2 + R^2}$$

Current:

$$i_{0,\,max} = \frac{V_{in,\,max}}{Z} = \frac{V_{max}}{\sqrt{(2\pi f L)^2 + R^2}}$$

Peak voltage across the inductor:

$$V_{\mathrm{L,\,max}} = i_{0,\,\mathrm{max}} X_{\mathrm{L}} = \left(\frac{V_{\max}}{\sqrt{(2\pi fL)^2 + R^2}}\right)(2\pi fL) = \frac{2\pi fL}{\sqrt{(2\pi fL)^2 + R^2}} V_{\max}$$

Part a)

$$V_{\mathrm{L,\,max\ at\ 50\ Hz}} = \frac{2\pi(50\ \mathrm{Hz})(0.04\ \mathrm{H})}{\sqrt{(2\pi(50\ \mathrm{Hz})(0.04\ \mathrm{H}))^2 + (250\ \Omega)^2}}(100\ \mathrm{V}) = \boxed{5.02\ \mathrm{V} = 0.0502 V_{\max}}$$

Part b)

$$V_{\mathrm{L,\,max\ at\ 5000\ Hz}} = \frac{2\pi(5000\ \mathrm{Hz})(0.04\ \mathrm{H})}{\sqrt{(2\pi(5000\ \mathrm{Hz})(0.04\ \mathrm{H}))^2 + (250\ \Omega)^2}}(100\ \mathrm{V}) = \boxed{98.1\ \mathrm{V} = 0.981 V_{\max}}$$

Part c) Looking at our answers for parts (a) and (b), this *LR* circuit $\boxed{\text{attenuates low-frequency signals, while allowing high-frequency signals to pass}}$. (That is, the voltage across the inductor is much smaller than V_{in} for low-frequency signals and is on the order of V_{in} for high-frequency signals.) This circuit acts as a high-pass filter.

REFLECT

A low-pass filter allows low-frequency signals to pass and attenuates high-frequency signals.

21.80

SET UP

A high-pass filter is made using a capacitor $C = 75 \times 10^{-9}$ F and an AC voltage source ($V_{\mathrm{rms}} = 120$ V). We are asked to compare the root-mean-square currents produced in the circuit if the AC source frequency is $f = 60$ Hz and then if $f = 100$ Hz. The root-mean-square current through the capacitor can be found using $V_{\mathrm{rms}} = i_{\mathrm{rms}} X_{\mathrm{C}}$, where $X_{\mathrm{C}} = \frac{1}{\omega C} = \frac{1}{(2\pi f)C}$.

SOLVE

Root-mean-square current at $f = 60$ Hz:

$$V_{\mathrm{rms}} = i_{\mathrm{rms}} X_{\mathrm{C}}$$

$$i_{\mathrm{rms}} = \frac{V_{\mathrm{rms}}}{X_C} = \frac{V_{\mathrm{rms}}}{\left(\frac{1}{\omega C}\right)} = \omega C V_{\mathrm{rms}} = (2\pi f) C V_{\mathrm{rms}}$$

$$= 2\pi(60\ \mathrm{Hz})(75 \times 10^{-9}\ \mathrm{F})(120\ \mathrm{V}) = 3.4 \times 10^{-3}\ \mathrm{A}$$

Root-mean-square current at $f = 100$ Hz:

$$V_{\mathrm{rms}} = i_{\mathrm{rms}} X_{\mathrm{C}}$$

$$i_{\mathrm{rms}} = \frac{V_{\mathrm{rms}}}{X_C} = \frac{V_{\mathrm{rms}}}{\left(\frac{1}{\omega C}\right)} = \omega C V_{\mathrm{rms}} = (2\pi f) C V_{\mathrm{rms}}$$

$$= 2\pi(100\ \mathrm{Hz})(75 \times 10^{-9}\ \mathrm{F})(120\ \mathrm{V}) = 5.7 \times 10^{-3}\ \mathrm{A}$$

The $\boxed{\text{higher the frequency, the greater is the current}}$ output from the capacitor.

REFLECT

This is a high-pass filter because we get an increased response from the circuit when we increase the frequency of the applied voltage.

21.81

SET UP

The variable capacitor of an FM radio tuner is set such that the capacitance is $C = 5.800 \times 10^{-12}$ F. This variable capacitor is in series with an inductor ($L = 0.4000 \times 10^{-6}$ H) and an AC voltage source ($V_0 = 9$ V). The circuit has a net resistance of $R = 1000\ \Omega$. The natural frequency of the circuit is equal to $\omega_0 = 2\pi f_0 = \sqrt{\frac{1}{LC}}$. The peak current through the circuit is equal to the peak voltage divided by the impedance Z, where $Z = \sqrt{\left(\omega_0 L - \frac{1}{\omega_0 C}\right)^2 + R^2}$.

SOLVE

Part a)

$$\omega_0 = 2\pi f_0 = \sqrt{\frac{1}{LC}}$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \frac{1}{2\pi}\sqrt{\frac{1}{(0.4000 \times 10^{-6}\ \text{H})(5.800 \times 10^{-12}\ \text{F})}} = \boxed{1.045 \times 10^{8}\ \text{Hz} = 104.5\ \text{MHz}}$$

Part b)
Impedance:

$$Z = \sqrt{\left(\omega_0 L - \frac{1}{\omega_0 C}\right)^2 + R^2} = \sqrt{\left((2\pi f_0)L - \frac{1}{(2\pi f_0)C}\right)^2 + R^2}$$

$$= \sqrt{\left(2\pi(1.045 \times 10^{8}\ \text{Hz})(0.4000 \times 10^{-6}\ \text{H}) - \frac{1}{2\pi(1.045 \times 10^{8}\ \text{Hz})(5.800 \times 10^{-12}\ \text{F})}\right)^2 + (1000\ \Omega)^2}$$

$$= 1000\ \Omega$$

Peak current:

$$V_0 = i_0 Z$$

$$i_0 = \frac{V_0}{Z} = \frac{9\ \text{V}}{1000\ \Omega} = \boxed{0.009\ \text{A}}$$

REFLECT

Even though we usually refer to FM radio stations solely as numbers, the units associated with these values are MHz. A radio station of 104.5 is a reasonable number on the radio dial.

21.82

SET UP

An LRC series circuit that has a capacitance $C = 18 \times 10^{-12}$ F resonates at $f = 58 \times 10^{6}$ Hz. We can rearrange the expression for the natural frequency of an LRC circuit, $\omega_0 = \sqrt{\frac{1}{LC}}$, to solve for the inductance of the circuit. Recall that $\omega = 2\pi f$.

SOLVE

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

$$L = \frac{1}{\omega_0^2 C} = \frac{1}{(2\pi f)^2 C} = \frac{1}{4\pi^2(58 \times 10^6\ \text{Hz})^2(18 \times 10^{-12}\ \text{F})} = \boxed{4.2 \times 10^{-7}\ \text{H}}$$

REFLECT

The frequency of a television tuner can be varied in order to change the frequency it is attempting to receive.

21.83

SET UP

A circuit contains a resistor, an inductor ($L = 6.0$ H), and a capacitor ($C = 5.0 \times 10^{-9}$ F) wired in series. Using the expression for the natural frequency of an LRC circuit, $\omega_0 = \sqrt{\frac{1}{LC}}$, we can calculate the resonant frequency of the circuit. Recall that $\omega_0 = 2\pi f_0$.

SOLVE

$$\omega_0 = 2\pi f_0 = \sqrt{\frac{1}{LC}}$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \frac{1}{2\pi}\sqrt{\frac{1}{(6.0\ \text{H})(5.0 \times 10^{-9}\ \text{F})}} = \boxed{920\ \text{Hz}}$$

REFLECT

The resistor does not affect the natural frequency of the circuit, only the maximum current.

Get Help: P'Cast 21.9 – Greatest Hits 99.7

21.84

SET UP

An inductor ($L = 2.5$ H) is connected in series with a resistor ($R = 1200\ \Omega$) and an AC voltage source ($V_{\text{rms}} = 50$ V, $f = 60$ Hz). We need to calculate the impedance in order to find the root-mean-square current in the circuit. The voltage drop across the resistor V_R is equal to the root-mean-square current in the circuit multiplied by the resistance R. Since the circuit

only consists of the inductor and the resistor, the voltage drop across the inductor is related to the voltage drop across the resistor V_R and V_{rms} through $V_{rms}^2 = V_R^2 + V_L^2$.

SOLVE

Part a)
Impedance:

$$Z = \sqrt{(\omega L)^2 + R^2} = \sqrt{(2\pi f L)^2 + R^2} = \sqrt{4\pi^2 f^2 L^2 + R^2}$$

$$= \sqrt{4\pi^2(60\ \text{Hz})^2(2.5\ \text{H})^2 + (1200\ \Omega)^2} = 1526\ \Omega$$

Root-mean-square current:

$$V_{rms} = i_{rms} Z$$

$$i_{rms} = \frac{V_{rms}}{Z} = \frac{50\ \text{V}}{1526\ \Omega} = 0.033\ \text{A}$$

Voltage drop across the resistor:

$$V_R = i_{rms} R = (0.033\ \text{A})(1200\ \Omega) = \boxed{39\ \text{V}}$$

Part b)

$$V_{rms}^2 = V_R^2 + V_L^2$$

$$V_L = \sqrt{V_{rms}^2 - V_R^2} = \sqrt{(50\ \text{V})^2 - (39\ \text{V})^2} = \boxed{31\ \text{V}}$$

REFLECT

We could have also calculated the voltage drop across the inductor directly from the root-mean-square current, $V_L = i_{rms} X_L = i_{rms}(\omega L) = i_{rms}(2\pi f)L$.

21.85

SET UP

We are asked to determine an expression for the root-mean-square voltage associated with a specific plot of voltage vs. time (see figure). The signal repeats every T and varies linearly from $+V_0$ to $-V_0$, which means we can represent the voltage as a function of time by $V(t) = \frac{-2V_0}{T}t + V_0$ for $0 < t < T$. The root-mean-square of the voltage involves first squaring the voltage, then finding its average, and finally taking the square root. As a reminder, the average of a function over an interval from $t = a$ to $t = b$ is equal to $f_{avg} = \frac{1}{b-a}\int_a^b f(t)dt$.

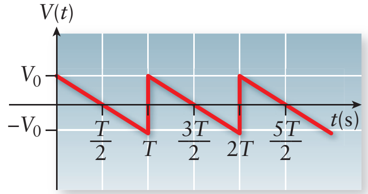

Figure 21-14 Problem 85

SOLVE

Root-mean-square:

$$V_{\text{rms}}^2 = \frac{1}{T}\int (V(t))^2 dt = \frac{1}{T}\int_0^T \left(\frac{-2V_0}{T}t + V_0\right)^2 dt = \frac{1}{T}\int_0^T \left(\frac{4V_0^2}{T^2}t^2 - \frac{4V_0^2}{T}t + V_0^2\right)dt$$

$$= \frac{V_0^2}{T}\int_0^T \left(\frac{4}{T^2}t^2 - \frac{4}{T}t + 1\right)dt = \frac{V_0^2}{T}\left[\frac{4}{3T^2}t^3 - \frac{2}{T}t^2 + t\right]_0^T = \frac{V_0^2}{T}\left[\frac{4}{3T^2}T^3 - \frac{2}{T}T^2 + T\right]$$

$$= \frac{V_0^2}{T}\left[\frac{4}{3}T - 2T + T\right] = \frac{V_0^2}{T}\left[\frac{1}{3}T\right] = \frac{V_0^2}{3}$$

$$V_{\text{rms}} = \sqrt{\frac{V_0^2}{3}} = \boxed{\frac{V_0}{\sqrt{3}}}$$

REFLECT

From dimensional analysis, there is no way the root-mean-square voltage can depend on the period.

21.86

SET UP

A resistor ($R = 150\ \Omega$) is connected across an AC voltage source ($V_0 = 75$ V). We want to replace the AC source with a DC source of voltage V_{DC} that dissipates heat at the same rate, which means the average power in each case will be equal. The average power dissipated through a resistor attached to an AC source is given by $P_{\text{avg, AC}} = \frac{V_{\text{rms}}^2}{R}$, whereas the power dissipated through a resistor attached to a DC power source is equal to $P_{\text{avg, DC}} = \frac{V_{\text{DC}}^2}{R}$. By setting these two expressions equal to one another and invoking the definition of the root-mean-square voltage in terms of the peak voltage, we can solve for the value of V_{DC}.

SOLVE

$$P_{\text{avg, AC}} = P_{\text{avg, DC}}$$

$$\frac{V_{\text{rms}}^2}{R} = \frac{V_{\text{DC}}^2}{R}$$

$$\frac{\left(\frac{V_0}{\sqrt{2}}\right)^2}{R} = \frac{V_{\text{DC}}^2}{R}$$

$$V_{\text{DC}} = \frac{V_0}{\sqrt{2}} = \frac{75\ \text{V}}{\sqrt{2}} = \boxed{53\ \text{V}}$$

REFLECT

We can double check our answer by explicitly calculating the average power dissipated in each case:

$$P_{\text{avg, AC}} = \frac{V_{\text{rms}}^2}{R} = \frac{\left(\frac{V_0}{\sqrt{2}}\right)^2}{R} = \frac{V_0^2}{2R} = \frac{(75\ \text{V})^2}{2(150\ \Omega)} = 19\ \text{W}$$

$$P_{\text{avg, DC}} = \frac{V_{\text{DC}}^2}{R} = \frac{(53\ \text{V})^2}{(150\ \Omega)} = 19\ \text{W}$$

21.87

SET UP

An immersion coil heater has resistance $R = 4.50\ \Omega$ and is connected to a household AC source ($V_{\text{rms}} = 120$ V). The rate at which heat is dissipated by the coil is equal to its power, $P_{\text{avg}} = \frac{V_{\text{rms}}^2}{R}$; this is also equal to the rate at which heat is absorbed by a cup of water ($m =$ 0.250 kg) that is initially at a temperature of 293 K. The heat required to heat the water to 373 K is equal to $Q = mc\Delta T$, where $c = 4186 \frac{\text{J}}{\text{kg} \cdot \text{K}}$ for water. We can solve for the time required to heat the water to 373 K by setting these two expressions for the power equal to each other. The root-mean-square current and, by extension, the peak current through the coil are given by Ohm's law.

SOLVE

Part a)

$$P_{\text{avg}} = \frac{Q}{\Delta t}$$

$$\Delta t = \frac{Q}{P_{\text{avg}}} = \frac{mc\Delta T}{\left(\frac{V_{\text{rms}}^2}{R}\right)} = \frac{Rmc\Delta T}{V_{\text{rms}}^2} = \frac{(4.50\ \Omega)(0.250\ \text{kg})\left(4186\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(80.0\ \text{K})}{(120\ \text{V})^2} = \boxed{26.2\ \text{s}}$$

Part b)

Root-mean-square current:

$$V_{\text{rms}} = i_{\text{rms}} R$$

$$i_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{120\ \text{V}}{4.50\ \Omega} = \boxed{26.7\ \text{A}}$$

Peak current:

$$i_0 = i_{\text{rms}}\sqrt{2} = \boxed{37.7\ \text{A}}$$

REFLECT

An immersion coil is used to quickly heat liquids, so a time of 26.2 s seems reasonable. In order to dissipate a large amount of heat in a short amount of time, the current through the coil needs to be large.

21.88

SET UP

You bring an electric razor designed for the United States with a power rating of $P_{\text{USA}} = 5.00$ W with you on a trip to Rome. The standard voltage in the United States is $V_{\text{rms, USA}} = 120$ V, and the standard voltage in Rome is $V_{\text{rms, Rome}} = 240$ V. The resistance R of the razor is intrinsic to the device, so this remains constant regardless of where you use it. Knowing this, we can set up a proportionality to solve for the power drawn by the razor in Rome. Once we know the power consumed by the razor in the two locations, we can calculate the root-mean-square current drawn by the razor in each case. If the current increases by a lot, then the razor is in danger of being damaged. In order to decrease the current through the razor, we will need to use a step-down transformer to decrease the European standard voltage to the U.S. standard voltage; to determine the ratio of the number of turns in the two coils of the transformer, we can compare the two standard voltages. Finally, we can find a value for R from the power drawn and the standard voltage in the United States.

SOLVE

Part a)

$$R_{\text{USA}} = R_{\text{Rome}}$$

$$\frac{V^2_{\text{rms, USA}}}{P_{\text{USA}}} = \frac{V^2_{\text{rms, Rome}}}{P_{\text{Rome}}}$$

$$P_{\text{Rome}} = \left(\frac{V^2_{\text{rms, Rome}}}{V^2_{\text{rms, USA}}}\right)P_{\text{USA}} = \left(\frac{240\text{ V}}{120\text{ V}}\right)^2(5.00\text{ W}) = \boxed{20.0\text{ W}}$$

Part b)
Root-mean-square current, United States:

$$P_{\text{USA}} = i_{\text{rms, USA}}V_{\text{rms, USA}}$$

$$i_{\text{rms, USA}} = \frac{P_{\text{USA}}}{V_{\text{rms, USA}}} = \frac{5.00\text{ W}}{120\text{ V}} = 0.0417\text{ A}$$

Root-mean-square current, Rome:

$$P_{\text{Rome}} = i_{\text{rms, Rome}}V_{\text{rms, Rome}}$$

$$i_{\text{rms, Rome}} = \frac{P_{\text{Rome}}}{V_{\text{rms, Rome}}} = \frac{20.0\text{ W}}{240\text{ V}} = 0.0833\text{ A}$$

The razor draws twice as much current in Rome as in the United States, so it will $\boxed{\text{most likely be damaged}}$.

Part c)

$$V_s = \frac{N_s}{N_p} V_p$$

$$\frac{N_p}{N_s} = \frac{V_p}{V_s} = \frac{240\text{ V}}{120\text{ V}} = 2$$

To reduce the supplied voltage from 240 V to 120 V, we will need to use a $\boxed{\text{step-down transformer with twice as many windings in the primary coil as the secondary coil}}$.

Part d)

$$P_{\text{USA}} = \frac{V_{\text{rms, USA}}^2}{R}$$

$$R = \frac{V_{\text{rms, USA}}^2}{P_{\text{USA}}} = \frac{(120\text{ V})^2}{5.00\text{ W}} = \boxed{2880\ \Omega}$$

REFLECT

The standard voltage in Rome is larger than the standard voltage in the United States, so we would expect the power drawn by the razor in Rome to be larger than the power drawn in the United States.

21.89

SET UP

A DC current of $i_{\text{DC}} = 0.060$ A or an AC current of $i_{\text{AC}} = 0.015$ A can cause respiratory paralysis. We can use Ohm's law and the definition of the peak AC current in terms of the root-mean-square AC current to solve for the DC voltage, root-mean-square AC voltage, and peak AC voltage that could cause respiratory paralysis, assuming a body resistance of $R = 1000\ \Omega$.

SOLVE

DC voltage:

$$V_{\text{DC}} = i_{\text{DC}}R = (0.060\text{ A})(1000\ \Omega) = \boxed{60\text{ V}}$$

AC voltage:

$$V_{\text{AC, rms}} = i_{\text{AC}}R = (0.015\text{ A})(1000\ \Omega) = \boxed{15\text{ V}}$$

$$V_{\text{AC, 0}} = V_{\text{AC, rms}}\sqrt{2} = \boxed{21\text{ V}}$$

REFLECT

The danger associated with alternating current was one of the reasons Thomas Edison used in his argument for an electric power system based on direct current.

21.90

SET UP

A power cord has a resistance $R = 8.00 \times 10^{-2}\ \Omega$. The power delivered by the power cord and a device is measured to be $P_{\text{delivered}} = 1500$ W. To calculate the power dissipated by the

power cord itself at a root-mean-square voltage of $V_{\text{rms}} = 12.0\text{ V}$ or $V_{\text{rms}} = 120\text{ V}$, we first need to calculate the root-mean-square current drawn in each case from $P_{\text{delivered}} = i_{\text{rms}}V_{\text{rms}}$. The power dissipated by the power cord alone is related to this current and the resistance of the cord, $P_{\text{cord}} = i_{\text{rms}}^2 R$. An ideal power cord would dissipate as little power as possible so that nearly all of it is delivered to the device.

SOLVE

Part a)
Current through the cord:

$$P_{\text{delivered}} = i_{\text{rms}}V_{\text{rms}}$$

$$i_{\text{rms}} = \frac{P_{\text{delivered}}}{V_{\text{rms}}} = \frac{1500\text{ W}}{12.0\text{ V}} = 125\text{ A}$$

Power dissipated by the cord:

$$P_{\text{cord}} = i_{\text{rms}}^2 R = (125\text{ A})^2(8.00 \times 10^{-2}\ \Omega) = \boxed{1250\text{ W}}$$

Part b)
Current through the cord:

$$P_{\text{delivered}} = i_{\text{rms}}V_{\text{rms}}$$

$$i_{\text{rms}} = \frac{P_{\text{delivered}}}{V_{\text{rms}}} = \frac{1500\text{ W}}{120\text{ V}} = 12.5\text{ A}$$

Power dissipated by the cord:

$$P_{\text{cord}} = i_{\text{rms}}^2 R = (12.5\text{ A})^2(8.00 \times 10^{-2}\ \Omega) = \boxed{12.5\text{ W}}$$

Part c) We should use a voltage of 120 V. A much smaller proportion of the total power is dissipated in the cord, which means more is delivered to the device. It is much more efficient than the 12.0-V delivery system.

REFLECT

In both cases, the power dissipated by the cord alone is less than the total power delivered.

21.91

SET UP

An AC generator is connected across a light bulb ($R = 8.50\ \Omega$). The electrical generator is made by rotating a flat coil in a uniform magnetic field ($B = 0.225\text{ T}$). The flat coil has 33 windings and is square, measuring 0.150 m on each side. The coil rotates at $\omega = 745\frac{\text{rev}}{\text{min}}$ about an axis perpendicular to the magnetic field and parallel to its two opposite sides. This orientation means the area vector will rotate around at a frequency of ω. We'll assume the magnetic field vector and area vector begin parallel. The alternating induced potential in the coil can be determined through Faraday's law. The amplitude of the resulting function is also equal to the voltage amplitude V_0 across the light bulb. The current amplitude for the bulb is

equal to $i_0 = \frac{V_0}{R}$ through Ohm's law. The average rate heat is generated is equal to the average power emitted by the light bulb, $P_{avg} = i_{rms}^2 R$, where $i_{rms} = \frac{i_0}{\sqrt{2}}$. The energy consumed by the light bulb in an hour is equal to its power multiplied by 1 hour.

SOLVE

Part a)

Angular frequency:

$$\omega = 745 \frac{\text{rev}}{\text{min}} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} = \frac{745\pi}{30} \frac{\text{rad}}{\text{s}}$$

Induced potential:

$$V = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}[NBA \cos(\omega t)] = -NBA\frac{d}{dt}[\cos(\omega t)] = NBA\omega \sin(\omega t)$$

$$= (33)(0.225 \text{ T})(0.150 \text{ m})^2\left(\frac{745\pi}{30} \frac{\text{rad}}{\text{s}}\right)\sin(\omega t) = (13.0 \text{ V})\sin(\omega t)$$

$$\boxed{V_0 = 13.0 \text{ V}}$$

Current:

$$i_0 = \frac{V_0}{R} = \frac{13.0 \text{ V}}{8.50 \ \Omega} = \boxed{1.53 \text{ A}}$$

Part b)

$$P_{avg} = i_{rms}^2 R = \left(\frac{i_0}{\sqrt{2}}\right)^2 R = \frac{(1.53 \text{ A})^2}{2}(8.50 \ \Omega) = \boxed{9.99 \text{ W}}$$

Part c)

$$E_{consumed} = Pt = (9.99 \text{ W})\left(1 \text{ hr} \times \frac{3600 \text{ s}}{1 \text{ hr}}\right) = \boxed{3.60 \times 10^4 \text{ J}}$$

REFLECT

In the United States, we buy light bulbs based on the power they consume at a voltage of 120 V rather than their resistance. Accordingly, a 60-W light bulb has a resistance of 240 Ω.

21.92

SET UP

A heating coil consists of $N = 750$ turns, has a length $l = 8.50 \times 10^{-2}$ m, a diameter $d = 1.25 \times 10^{-2}$ m, and a resistance $R = 2.15 \ \Omega$. The coil is connected in series to a capacitor ($C = 2240 \times 10^{-6}$ F) and an AC voltage source ($V_{rms} = 120$ V, $f = 60.0$ Hz). To find the impedance of this circuit, we first need to find the inductance of the coil. The self-inductance of a coil of wire is given by $L = \frac{\mu_0 N^2 A}{l}$, where A is the cross-sectional area

of the coil. After finding the value of L, we can find the impedance in the circuit using $Z = \sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2} = \sqrt{\left((2\pi f)L - \frac{1}{(2\pi f)C}\right)^2 + R^2}$. The root-mean-square current drawn by the coil can be found using Ohm's law. The peak current drawn by the coil is equal to $i_0 = i_{\text{rms}}\sqrt{2}$.

SOLVE

Part a)
Inductance:

$$L = \frac{\mu_0 N^2 A}{l} = \frac{\mu_0 N^2(\pi r^2)}{l} = \frac{\pi\mu_0 N^2\left(\frac{d}{2}\right)^2}{l} = \frac{\pi\mu_0 N^2 d^2}{4l}$$

$$= \frac{\pi\left(4\pi \times 10^{-7}\frac{\text{H}}{\text{m}}\right)(750 \text{ turns})^2(1.25 \times 10^{-2} \text{ m})^2}{4(8.50 \times 10^{-2} \text{ m})} = 1.02 \times 10^{-3} \text{ H}$$

Impedance:

$$Z = \sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2} = \sqrt{\left((2\pi f)L - \frac{1}{(2\pi f)C}\right)^2 + R^2}$$

$$= \sqrt{\left(2\pi(60.0 \text{ Hz})(1.02 \times 10^{-3} \text{ H}) - \frac{1}{2\pi(60.0 \text{ Hz})(2240 \times 10^{-6} \text{ F})}\right)^2 + (2.15\ \Omega)^2}$$

$$= \boxed{2.29\ \Omega}$$

Part b)

$$V_{\text{rms}} = i_{\text{rms}} Z$$

$$i_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{120 \text{ V}}{2.29\ \Omega} = \boxed{52.3 \text{ A}}$$

Part c)

$$i_0 = i_{\text{rms}}\sqrt{2} = (52.3 \text{ A})\sqrt{2} = \boxed{74.0 \text{ A}}$$

REFLECT

We would expect a heating coil to draw a large amount of current in order to deliver a large amount of power to quickly heat an object.

21.93

SET UP

An electrician is working with high-voltage AC power lines where $V_0 = 2.5 \times 10^4$ V. The resistance of his body is $R = 1000\ \Omega$ when he is wet and $R = 500 \times 10^3\ \Omega$ when he is dry.

In either case, we can calculate the root-mean-square current through his body from Ohm's law and the peak voltage. Alternating currents around 15 mA can cause respiratory paralysis (see Problem 21.89), so we can compare our calculated currents to this value to determine whether the electrician is in significant danger. The electrician is wearing protective shoes that have soles that are 8.0×10^{-2} m by 20.0×10^{-2} m and are designed to limit the AC root-mean-square current to no more than 1.0×10^{-3} A. In order to maximize the resistance of the worker, the shoes should have a small cross-sectional area. If he does stand on two feet, then his legs and, therefore, his shoes are in parallel. We can use Ohm's law to calculate the total required resistance through the two shoes from the root-mean-square voltage and maximum allowed root-mean-square current. The resistance of one shoe is equal to twice the required resistance since they are in parallel. Finally, we can relate the resistance of one shoe to the resistivity of the material of the sole that is $L = 2.0 \times 10^{-2}$ m thick through $R = \dfrac{\rho L}{A}$.

SOLVE

Part a)
Root-mean-square voltage:

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}} = \frac{2.5 \times 10^4 \text{ V}}{\sqrt{2}} = 1.8 \times 10^4 \text{ V}$$

Root-mean-square current when wet:

$$V_{\text{rms}} = i_{\text{rms}} R$$

$$i_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{1.8 \times 10^4 \text{ V}}{1000\ \Omega} = \boxed{18 \text{ A}}$$

An alternating current of $\boxed{\text{18 A is deadly}}$.
Root-mean-square current when dry:

$$V_{\text{rms}} = i_{\text{rms}} R$$

$$i_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{1.8 \times 10^4 \text{ V}}{500 \times 10^3\ \Omega} = \boxed{0.035 \text{ A} = 35 \text{ mA}}$$

An alternating current of $\boxed{\text{35 mA is not safe}}$.

Part b) The shoes must increase his resistance. Since $R = \dfrac{\rho L}{A}$, a smaller area A gives a larger resistance (and, thus, a smaller current). The electrician should stand on $\boxed{\text{one foot}}$, if possible.

Part c)
Required resistance:

$$R = \frac{V_{\text{rms}}}{i_{\text{rms}}} = \frac{1.8 \times 10^4 \text{ V}}{1.0 \times 10^{-3} \text{ A}} = 1.8 \times 10^7\ \Omega$$

The resistance of the electrician's body is negligible compared with this value, so the soles of the shoes must provide all of the needed resistance. His legs (and, therefore, shoes) are in parallel, so each shoe must provide twice the needed resistance.

Resistivity:

$$R = \frac{\rho L}{A}$$

$$\rho = \frac{AR}{L} = \frac{((8.0 \times 10^{-2}\ \text{m})(20.0 \times 10^{-2}\ \text{m}))(2)(1.8 \times 10^{7}\ \Omega)}{2.0 \times 10^{-2}\ \text{m}} = \boxed{2.8 \times 10^{7}\ \Omega \cdot \text{m}}$$

REFLECT

The resistivity of hard rubber is on the order of $10^{13}\ \Omega \cdot \text{m}$, so a minimum required resistivity on the order of $10^{7}\ \Omega \cdot \text{m}$ seems reasonable.

21.94

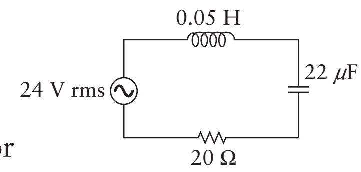

Figure 21-15 Problem 94

SET UP

An LRC circuit consists of an AC voltage source ($V_{\text{rms}} = 24$ V), a resistor ($R = 20\ \Omega$), a capacitor ($C = 22 \times 10^{-6}$ F), and an inductor ($L = 0.050$ H). We are asked to determine the peak voltages across the resistor, capacitor, and inductor when the frequency of the voltage source is $f = 100$ Hz and $f = f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$ (that is, on resonance). In either case, we first need to find the impedance Z of the circuit in order to calculate the maximum current in the circuit. Once we have the peak current in the circuit, the maximum voltage across each circuit element is given by Ohm's law where we use the resistance, capacitive reactance, and inductive reactance for the voltage over the resistor, capacitor, and inductor, respectively. At first glance it may not make sense that the maximum voltages we calculate sum up to a total voltage larger than the applied voltage, but we need to remember that the three voltages are not in phase with one another. At resonance, the maximum voltages across the inductor and capacitor should be equal because they are exactly out of phase with one another and the maximum voltage across the resistor should equal the maximum voltage of the AC source.

SOLVE

Part a)

Impedance at $f = 100$ Hz:

$$Z = \sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2} = \sqrt{\left((2\pi f)L - \frac{1}{(2\pi f)C}\right)^2 + R^2}$$

$$= \sqrt{\left(2\pi(100\ \text{Hz})(0.050\ \text{H}) - \frac{1}{2\pi(100\ \text{Hz})(22 \times 10^{-6}\ \text{F})}\right)^2 + (20\ \Omega)^2} = 46\ \Omega$$

Maximum current in the circuit at $f = 100$ Hz:

$$V_{\text{rms}} = i_{\text{rms}} Z = \left(\frac{i_0}{\sqrt{2}}\right) Z$$

$$i_0 = \frac{V_{\text{rms}}\sqrt{2}}{Z} = \frac{(24\ \text{V})\sqrt{2}}{46\ \Omega} = 0.75\ \text{A}$$

Maximum voltage across the resistor at $f = 100$ Hz:

$$V_{R,0} = i_0 R = (0.75\ \text{A})(20\ \Omega) = \boxed{15\ \text{V}}$$

Maximum voltage across the capacitor at $f = 100$ Hz:

$$V_{C,0} = i_0 X_C = i_0\left(\frac{1}{\omega C}\right) = \frac{i_0}{(2\pi f)C} = \frac{0.75\ \text{A}}{2\pi(100\ \text{Hz})(22 \times 10^{-6}\ \text{F})} = \boxed{54\ \text{V}}$$

Maximum voltage across the inductor at $f = 100$ Hz:

$$V_{L,0} = i_0 X_L = i_0(\omega L) = i_0(2\pi f)L = 2\pi(100\ \text{Hz})(0.75\ \text{A})(0.050\ \text{H}) = \boxed{23\ \text{V}}$$

Part b)
Resonance frequency:

$$\omega_0 = 2\pi f_0 = \sqrt{\frac{1}{LC}}$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \frac{1}{2\pi}\sqrt{\frac{1}{(0.050\ \text{H})(22 \times 10^{-6}\ \text{F})}} = \boxed{150\ \text{Hz}}$$

Maximum current in the circuit at $f = 100$ Hz:

$$V_{\text{rms}} = i_{\text{rms}} Z = \left(\frac{i_0}{\sqrt{2}}\right)R$$

$$i_0 = \frac{V_{\text{rms}}\sqrt{2}}{R} = \frac{(24\ \text{V})\sqrt{2}}{20\ \Omega} = 1.7\ \text{A}$$

Maximum voltage across the resistor at resonance:

$$V_{R,0} = i_0 R = (1.7\ \text{A})(20\ \Omega) = \boxed{34\ \text{V}}$$

Maximum voltage across the capacitor at resonance:

$$V_{C,0} = i_0 X_C = i_0\left(\frac{1}{\omega_0 C}\right) = \frac{i_0}{(2\pi f_0)C} = \frac{1.7\ \text{A}}{2\pi(150\ \text{Hz})(22 \times 10^{-6}\ \text{F})} = \boxed{81\ \text{V}}$$

Maximum voltage across the inductor at resonance:

$$V_{L,0} = i_0 X_L = i_0(\omega_0 L) = i_0(2\pi f_0)L = 2\pi(150\ \text{Hz})(1.7\ \text{A})(0.050\ \text{H}) = \boxed{81\ \text{V}}$$

Part c) The maximum voltages across the capacitor, inductor, and resistor are not in phase with each other, so their sum can be larger than the maximum applied potential.

Part d) At resonance, the voltages across the capacitor and inductor must cancel each other, so their maximum voltages must be the same.

REFLECT

The impedance of the circuit at resonance should be smaller than the impedance off-resonance since we would expect the largest current in the circuit at resonance.

21.95

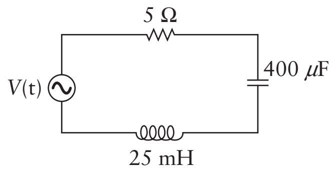

Figure 21-16 Problem 95

SET UP

An AC voltage source is connected to a resistor ($R = 5\ \Omega$), a capacitor ($C = 400 \times 10^{-6}$ F), and an inductor ($L = 0.025$ H), all wired in series. The current in the circuit is $i(t) = (10\text{ A})\sin(120\pi t)$, which means $\omega = 120\pi \frac{\text{rad}}{\text{s}}$. The total impedance of this circuit can be calculated from $Z = \sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2}$. The current and voltage are in phase for a resistor, which means $\varphi_R = 0$. For a capacitor, the voltage lags the current by $\frac{\pi}{2}$ rad, so $\varphi_C = -\frac{\pi}{2}$. Finally, the voltage leads the current by $\frac{\pi}{2}$ rad in an inductor, which means $\varphi_L = \frac{\pi}{2}$.

SOLVE

Part a)

$$Z = \sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2}$$

$$= \sqrt{\left(\left(120\pi \frac{\text{rad}}{\text{s}}\right)(0.025\text{ H}) - \left(\frac{1}{\left(120\pi \frac{\text{rad}}{\text{s}}\right)(400 \times 10^{-6}\text{ F})}\right)\right)^2 + (5\ \Omega)^2} = \boxed{5.73\ \Omega}$$

Part b)

$$\varphi_R = 0$$

$$\varphi_C = -\frac{\pi}{2}$$

$$\varphi_L = \frac{\pi}{2}$$

REFLECT

The current through all three circuit elements is equal since they are wired up in series. This is an inductive circuit since $X_L > X_C$.

Get Help: Interactive Example – AC Circuit 1
P'Cast 21.4 – A Series LRC Circuit Driven by an AC Voltage – Impedance

21.96

SET UP

The maximum charge Q_0 on the capacitor in an LRC circuit is equal to the product of the capacitance and the peak voltage across the capacitor, $Q_0 = CV_{C,0}$. The peak voltage across the capacitor is equal to the capacitive reactance multiplied by the peak current, which we can relate to the peak voltage and the impedance of the circuit. Plugging in all of these

expressions, we can then rearrange the resulting expression to show that the maximum charge is equal to $\frac{V_0}{\sqrt{\left(\omega^2 L - \frac{1}{C}\right)^2 + (\omega R)^2}}$. To determine the angular frequency at which Q_0 is a maximum, we should differentiate our expression for Q_0 with respect to ω, evaluate it at $\omega = \omega_0$, set it equal to zero, and solve for ω_0.

SOLVE

Part a)

$$Q_0 = CV_{C,0} = C(X_C i_0) = C\left(\frac{1}{\omega C}\right)\left(\frac{V_0}{Z}\right) = \frac{V_0}{\omega Z}$$

$$= \frac{V_0}{\omega\sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2}} = \frac{V_0}{\omega\sqrt{\left(\frac{1}{\omega^2}\right)\left(\omega^2 L - \frac{1}{C}\right)^2 + \frac{(\omega R)^2}{\omega^2}}}$$

$$= \boxed{\frac{V_0}{\sqrt{\left(\omega^2 L - \frac{1}{C}\right)^2 + (\omega R)^2}}}$$

Part b)

$$\left.\frac{dQ_0}{d\omega}\right|_{\omega=\omega_0} = 0 = \frac{d}{d\omega}\left[\frac{V_0}{\sqrt{\left(\omega^2 L - \frac{1}{C}\right)^2 + (\omega R)^2}}\right]\Bigg|_{\omega=\omega_0}$$

$$= \left[\frac{V_0}{\left(\left(\omega_0^2 L - \frac{1}{C}\right)^2 + (\omega_0 R)^2\right)^{\frac{3}{2}}}\right]\left(2\left(\omega_0^2 L - \frac{1}{C}\right)(2\omega_0 L) + 2\omega_0 R^2\right)$$

$$\left(\omega_0^2 L - \frac{1}{C}\right)(2\omega_0 L) + \omega_0 R^2 = 0$$

$$R^2 = -\left(\omega_0^2 L - \frac{1}{C}\right)(2L)$$

$$\omega_0^2 L = \frac{1}{C} - \frac{R^2}{2L}$$

$$\boxed{\omega_0 = \sqrt{\frac{1}{CL} - \frac{R^2}{2L^2}}}$$

REFLECT

We can plug in the expression we found for the angular frequency to calculate the maximum charge on the capacitor:

$$Q_{0,\,\max} = \frac{V_0}{\sqrt{\left(\omega_0^2 L - \frac{1}{C}\right)^2 + (\omega_0 R)^2}} = \frac{V_0}{\sqrt{\left(\left(\frac{1}{CL} - \frac{R^2}{2L^2}\right)L - \frac{1}{C}\right)^2 + \left(\frac{1}{CL} - \frac{R^2}{2L^2}\right)R^2}}$$

$$= \frac{V_0}{\sqrt{\left(\frac{1}{C} - \frac{R^2}{2L} - \frac{1}{C}\right)^2 + \left(\frac{R^2}{CL} - \frac{R^4}{2L^2}\right)}} = \frac{V_0}{\sqrt{\frac{R^4}{4L^2} + \left(\frac{R^2}{CL} - \frac{R^4}{2L^2}\right)}} = \frac{V_0}{\sqrt{\frac{R^2}{CL} - \frac{R^4}{4L^2}}}$$

$$= \frac{V_0}{\frac{R}{L}\sqrt{\frac{L}{C} - \frac{R^2}{4}}} = \frac{LV_0}{R\sqrt{\frac{L}{C} - \frac{R^2}{4}}}$$

This expression has the dimensions of charge, as expected.

21.97

SET UP

An LRC AC circuit consists of an AC voltage source, resistor ($R = 125\ \Omega$), an inductor ($L = 12.5 \times 10^{-3}$ H), and a parallel plate capacitor having a plate separation $d = 2.10 \times 10^{-3}$ m with a dielectric material completely filling the region between the plates. The rectangular plates of the capacitor are 4.25×10^{-2} m by 6.20×10^{-2} m. The maximum root-mean-square current occurs when the frequency of the AC voltage source is set to $f_0 = 55.0$ Hz. Since the maximum current occurs at this frequency, this must be the natural frequency of the LRC circuit. We can use the expression for the capacitance of a parallel-plate capacitor along with the expression for the natural frequency of the circuit to calculate the dielectric constant of the material between the capacitor plates. Table 17-2 lists the dielectric constants of some common materials; we can compare the value we calculated for κ to those in the table in order to see if the required dielectric constant is actually feasible.

SOLVE

Part a)

$$\omega_0 = 2\pi f_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{L\left(\frac{\kappa\varepsilon_0 A}{d}\right)}}$$

$$(2\pi f_0)^2 = \frac{d}{L\kappa\varepsilon_0 A}$$

$$\kappa = \frac{d}{4\pi^2\varepsilon_0 f_0^2 LA}$$

$$= \frac{2.10 \times 10^{-3}\ \text{m}}{4\pi^2\left(8.85 \times 10^{-12}\frac{\text{F}}{\text{m}}\right)(55.0\ \text{Hz})^2(12.5 \times 10^{-3}\ \text{H})((4.25 \times 10^{-2}\ \text{m})(6.20 \times 10^{-2}\ \text{m}))}$$

$$= \boxed{6.03 \times 10^{7}}$$

Part b) Table 17-2 tells us that this is not a feasible dielectric constant since it is much greater than that of ordinary materials. However, the required capacitance ($C = 670\ \mu\text{F}$) is easily within the range of ultracapacitors.

REFLECT

Ultracapacitors employ methods that make the effective plate separation negligible, which means the capacitance should be very large.

21.98

SET UP

A circuit consists of a capacitor $C = 15.0 \times 10^{-6}$ F, a resistor R, and an inductor L wired in series with an AC power source ($V_0 = 25.0$ V) of variable frequency. When the voltage source is dialed to the natural frequency of this LRC circuit ($f_0 = 44.5$ Hz), the root-mean-square current reaches its maximum value of $i_{\text{rms}} = 60.0 \times 10^{-3}$ A. Knowing the natural frequency of the circuit allows us to calculate the value of L. Because the circuit is operating at resonance, the impedance is simply equal to the resistance R. The value of R can be calculated through Ohm's law. Once we have numerical values for C, L, and R, we can calculate the impedance and then the root-mean-square current at $f = 60.0$ Hz.

SOLVE

Natural frequency:

$$\omega_0 = 2\pi f_0 = 2\pi(44.5\ \text{Hz}) = 280\frac{\text{rad}}{\text{s}}$$

Inductance:

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

$$L = \frac{1}{\omega_0^2 C} = \frac{1}{\left(280\frac{\text{rad}}{\text{s}}\right)^2 (15.0 \times 10^{-6}\ \text{F})} = 0.853\ \text{H}$$

Impedance at resonance:

$$Z = \sqrt{(X_L - X_C)^2 + R^2} = \sqrt{0 + R^2} = R$$

Resistance:

$$V_{\text{rms}} = i_{\text{rms}} Z = i_{\text{rms}} R$$

$$R = \frac{V_{\text{rms}}}{i_{\text{rms}}} = \frac{\left(\frac{V_0}{\sqrt{2}}\right)}{i_{\text{rms}}} = \frac{V_0}{i_{\text{rms}}\sqrt{2}} = \frac{25.0\ \text{V}}{(65.0 \times 10^{-3}\ \text{A})\sqrt{2}} = 272\ \Omega$$

Current at $f = 60.0$ Hz:

$$i_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{\left(\frac{V_0}{\sqrt{2}}\right)}{\sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2}} = \left(\frac{V_0}{\sqrt{2}}\right)\frac{1}{\sqrt{\left((2\pi f)L - \frac{1}{(2\pi f)C}\right)^2 + R^2}}$$

$$= \left(\frac{25.0\text{ V}}{\sqrt{2}}\right)\frac{1}{\sqrt{\left(2\pi(60.0\text{ Hz})(0.853\text{ H}) - \frac{1}{2\pi(60.0\text{ Hz})(15.0\times 10^{-6}\text{ F})}\right)^2 + (272\ \Omega)^2}}$$

$$= \boxed{0.0574\text{ A}}$$

REFLECT

The root-mean-square current at $f = 60.0$ Hz should be smaller than the root-mean-square current at $f = 44.5$ Hz because we are now off-resonance.

21.99

SET UP

The current in a circuit consisting of an AC power supply and a capacitor is found to be $i(t) = i_0 \sin(\omega t)$. The current in the circuit is equal to the first derivative of the charge on the capacitor with respect to time. Assuming the charge is equal to zero at $t = 0$, we can solve for the charge as a function of time via separation of variables.

SOLVE

$$i(t) = \frac{dq}{dt} = i_0 \sin(\omega t)$$

$$\int_{q(0)}^{q(t)} dq = \int_0^t i_0 \sin(\omega t)dt$$

$$q(t) - q(0) = i_0\left[-\frac{1}{\omega}\cos(\omega t)\right]_0^t$$

$$q(t) - 0 = -\frac{i_0}{\omega}[\cos(\omega t) - \cos(0)]_0^t$$

$$q(t) = -\frac{i_0}{\omega}[\cos(\omega t) - 1] = \boxed{\frac{i_0}{\omega}[1 - \cos(\omega t)]}$$

REFLECT

The charge on the capacitor oscillates in time between a minimum value of 0 and a maximum value of $\frac{2i_0}{\omega}$.

Get Help: P'Cast 21.3 – High-Pass Filter

21.100

SET UP

An AC power supply supplies a voltage of $V(t) = V_0 \sin(\omega t + \varphi)$ and is connected in series with a capacitor C. The charge on the capacitor as a function of time is equal to $q(t) = CV(t)$. The current in the circuit is equal to the time derivative of the charge on the capacitor.

SOLVE

Charge on the capacitor:

$$q(t) = CV(t) = C(V_0 \sin(\omega t + \varphi))$$

Current through the capacitor:

$$i(t) = \frac{dq}{dt} = \frac{d}{dt}[C(V_0 \sin(\omega t + \varphi))] = CV_0(\omega_0 \cos(\omega t + \varphi))$$

$$= (\omega_0 C)V_0 \cos(\omega t + \varphi) = \boxed{\frac{V_0}{X_C}\cos(\omega t + \varphi)}$$

REFLECT

The phase angle between the voltage and the current is $\frac{\pi}{2}$rad as expected.

21.101

SET UP

An inductor L is in series with a resistor R and a sinusoidal voltage source, $V(t) = V_0 \sin(\omega t)$. Kirchhoff's loop rule around the circuit results in the following differential equation, $V_0 \sin(\omega t) = L\frac{di}{dt} + Ri$. We are asked to explicitly show that $i(t) = i_0 \sin(\omega t - \varphi)$ is a solution to this differential equation. In the process of showing this, we can also derive expressions for the phase angle φ and the peak current i_0.

SOLVE

Part a)

$$V_0 \sin(\omega t) = L\frac{di}{dt} + Ri$$

$$V_0 \sin(\omega t) \stackrel{?}{=} L\frac{d}{dt}[i_0 \sin(\omega t - \varphi)] + R[i_0 \sin(\omega t - \varphi)]$$

$$V_0 \sin(\omega t) \stackrel{?}{=} Li_0[\omega\cos(\omega t - \varphi)] + Ri_0 \sin(\omega t - \varphi)$$

$$V_0 \sin(\omega t) \stackrel{?}{=} Li_0\, \omega[\cos(\omega t)\cos(\varphi) + \sin(\omega t)\sin(\varphi)] + Ri_0[\sin(\omega t)\cos(\varphi) - \cos(\omega t)\sin(\varphi)]$$

$$V_0 \sin(\omega t) \stackrel{?}{=} [Li_0\, \omega\cos(\varphi) - Ri_0 \sin(\varphi)]\cos(\omega t) + [Li_0\, \omega\sin(\varphi) + Ri_0 \cos(\varphi)]\sin(\omega t)$$

This is true as long as:

$$Li_0\,\omega\cos(\varphi) - Ri_0\sin(\varphi) = 0 \text{ and } Li_0\,\omega\sin(\varphi) + Ri_0\cos(\varphi) = V_0$$

Part b)

$$Li_0\,\omega\sin(\varphi) + Ri_0\cos(\varphi) = \mathrm{i}_0(R\cos(\varphi) + L\omega\sin(\varphi)) = V_0$$

$$i_0\left(R\left(\frac{R}{\sqrt{(\omega L)^2 + R^2}}\right) + \omega L\left(\frac{\omega L}{\sqrt{(\omega L)^2 + R^2}}\right)\right) = V_0$$

$$V_0 = i_0\left(\frac{R^2 + (\omega L)^2}{\sqrt{(\omega L)^2 + R^2}}\right) = i_0\sqrt{(\omega L)^2 + R^2}$$

$$\boxed{i_0 = \frac{V_0}{\sqrt{(\omega L)^2 + R^2}} = \frac{V_0}{Z}}$$

Part c)

$$Li_0\,\omega\cos(\varphi) = Ri_0\sin(\varphi)$$

$$\boxed{\tan(\varphi) = \frac{\omega L}{R} = \frac{X_L}{R}}\text{, which means } \sin(\varphi) = \frac{\omega L}{\sqrt{(\omega L)^2 + R^2}} \text{ and } \cos(\varphi) = \frac{R}{\sqrt{(\omega L)^2 + R^2}}$$

REFLECT

Our expressions for the phase angle and the peak current match the expected expressions for an *LR* AC circuit.

21.102

SET UP

The circuit seen in the figure is known as a high-pass filter. The input is an AC signal composed of many frequencies. The output detected is the voltage across the resistor. The current in the circuit is equal to the input voltage divided by the impedance of the circuit. Once we have the current in terms of V_i, we can use the expression for the voltage across the resistor $V_\mathrm{o} = i_0 R$ to solve for the ratio between the output and input voltages.

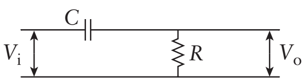

Figure 21-17 Problem 102

SOLVE

Current:

$$i_0 = \frac{V_\mathrm{i}}{Z} = \frac{V_\mathrm{i}}{\sqrt{\left(\frac{1}{\omega C}\right)^2 + R^2}}$$

Voltage across the resistor:

$$V_\mathrm{o} = i_0 R = \frac{V_\mathrm{i} R}{\sqrt{\left(\frac{1}{\omega C}\right)^2 + R^2}}$$

Ratio between V_o and V_i:

$$\frac{V_o}{V_i} = \frac{R}{\sqrt{\left(\frac{1}{\omega C}\right)^2 + R^2}} = \frac{R}{R\sqrt{\left(\frac{1}{\omega CR}\right)^2 + 1}} = \boxed{\frac{1}{\sqrt{1 + \left(\frac{1}{2\pi fRC}\right)^2}}}$$

REFLECT

Looking at our final expression for the ratio of the voltages, we see the $\frac{V_o}{V_i}$ gets small as f gets small, which means the amplitude of the low-frequency components of the input signal is very small. This is consistent with our expectation that a high-pass filter allows high-frequency signals to pass (relatively) unaffected while attenuating the low-frequency ones.

21.103

SET UP

The circuit seen in the figure is known as a low-pass filter. The input is an AC signal composed of many frequencies. The output detected is the voltage across the capacitor. The current in the circuit is equal to the input voltage divided by the impedance of the circuit. Once we have the current in terms of V_i, we can use the expression for the voltage across the capacitor $V_o = i_0 X_C$ to solve for the ratio between the output and input voltages.

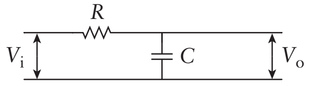

Figure 21-18 Problem 103

SOLVE

Current:

$$i_0 = \frac{V_i}{Z} = \frac{V_i}{\sqrt{\left(\frac{1}{\omega C}\right)^2 + R^2}}$$

Voltage across the capacitor:

$$V_o = i_0 X_C = \left(\frac{V_i}{\sqrt{\left(\frac{1}{\omega C}\right)^2 + R^2}}\right)\left(\frac{1}{\omega C}\right)$$

Ratio between V_o and V_i:

$$\frac{V_o}{V_i} = \frac{1}{\omega C\sqrt{\left(\frac{1}{\omega C}\right)^2 + R^2}} = \frac{1}{\sqrt{(\omega C)^2\left(\frac{1}{\omega C}\right)^2 + (\omega C)^2(R^2)}} = \boxed{\frac{1}{\sqrt{1 + (2\pi fRC)^2}}}$$

REFLECT

Looking at our final expression for the ratio of the voltages, we see the $\frac{V_o}{V_i}$ gets small as f gets large, which means the amplitude of the high-frequency components of the input signal is very small. This is consistent with our expectation that a low-pass filter allows low-frequency signals to pass (relatively) unaffected while attenuating the high-frequency ones.

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21.104

SET UP

The Q factor of an LRC circuit is defined as the ratio of the voltage across the inductor to the voltage across the resistor at resonance. Using Ohm's law and the expression for the natural frequency of an LRC circuit, we can show that the expression for the Q factor is $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$. Using this and the fact that $\omega_0 = 2\pi f_0 = \sqrt{\frac{1}{LC}}$, we can solve for the values of L and R required to produce a factor of $Q = 1000$ at a resonance frequency of $f_0 = 1.0 \times 10^6$ Hz, assuming $C = 0.0010 \times 10^{-6}$ F.

SOLVE

Part a)

$$Q = \frac{V_L}{V_R} = \frac{i_0 X_L}{i_0 R} = \frac{\omega_0 L}{R} = \frac{\left(\sqrt{\frac{1}{LC}}\right)L}{R} = \boxed{\frac{1}{R}\sqrt{\frac{L}{C}}}$$

Part b)
Inductance:

$$\omega_0 = 2\pi f_0 = \sqrt{\frac{1}{LC}}$$

$$L = \frac{1}{(2\pi f_0)^2 C} = \frac{1}{4\pi^2(1.0 \times 10^6\ \text{Hz})^2(0.0010 \times 10^{-6}\ \text{F})} = \boxed{2.5 \times 10^{-5}\ \text{H}}$$

Resistance:

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$R = \frac{1}{Q}\sqrt{\frac{L}{C}} = \frac{1}{1000}\sqrt{\frac{2.5 \times 10^{-5}\ \text{H}}{0.0010 \times 10^{-6}\ \text{F}}} = \boxed{0.16\ \Omega}$$

REFLECT

The square root of a henry divided by a farad is equal to an ohm.

Chapter 22
Electromagnetic Waves

Conceptual Questions

22.1 Part a) E (gamma rays), C (ultraviolet light), B (red light), D (infrared light), A (microwaves).

Part b) Gamma rays are highest energy, and microwaves are lowest energy.

22.2 Without the seminal work of Faraday, Gauss, Coulomb, Ampère, and others, it would not have been possible for Maxwell to complete his theory. Even though physics would have continued to progress with the work of Faraday, Gauss, and Ampère, it was Maxwell who really pushed the forefronts of science by understanding the connections among electricity, magnetism, and optics.

22.3 The electric field is perpendicular to the magnetic field and the velocity of the wave. That means that the electric fields will vibrate, say, along the x-axis, the magnetic field along the y-axis, and the EM radiation will move along the z-axis. These three vectors are mutually perpendicular.

22.4 No, a steady current does not emit an electromagnetic wave; only a time-varying current can emit electromagnetic waves. The radiation is the result of accelerating charges, which can only occur when the current is changing.

22.5 The speed of light is proportional to the frequency of the oscillating electric and magnetic fields ($c \propto f$).

22.6 The energy of the electromagnetic waves is proportional to the frequency of the oscillating electric and magnetic fields ($E \propto f$).

22.7 All EM waves move at the same speed ($c = 3.00 \times 10^8$ m/s). However, the wavelength times the frequency equals this constant ($c = \lambda f$). Therefore, the wavelength can vary (smaller/larger) in proportion to the frequency (larger/smaller).

22.8 (i) $\oint \vec{B} \cdot d\vec{l} = \mu_0 i_{\text{through}}$ = (b) the source of a magnetic field is an electric current

(ii) $\oint \vec{E} \cdot d\vec{l} = -\dfrac{d\Phi_B}{dt}$ = (c) changing magnetic fields induce changing electric fields

(iii) $\oint \vec{B} \cdot d\vec{A} = 0$ = (d) changing electric fields induce changing magnetic fields

(iv) $\oint \vec{E} \cdot d\vec{A} = \dfrac{q_{\text{encl}}}{\varepsilon_0}$ = (a) the source of an electric field is an electric charge

22.9 A partial derivative is the extension of an ordinary derivative that is used when the function of interest has a dependence on more than one variable. In determining $\frac{\partial E}{\partial t}$, take the derivative of $E(x,t)$ with respect to t, treating x as constant. In determining $\frac{\partial E}{\partial x}$, take the derivative of $E(x,t)$ with respect to x, treating t as constant.

22.10

$$\left[\frac{\partial^2 B}{\partial x^2}\right] = \left[\frac{1}{c^2}\frac{\partial^2 B}{dt^2}\right]$$

$$\frac{\mathrm{T}}{\mathrm{m}^2} \stackrel{?}{=} \left(\frac{1}{(\mathrm{m/s})}\right)^2 \left(\frac{\mathrm{T}}{\mathrm{s}^2}\right)$$

$$\frac{\mathrm{T}}{\mathrm{m}^2} \stackrel{?}{=} \left(\frac{\mathrm{s}^2}{\mathrm{m}^2}\right)\left(\frac{\mathrm{T}}{\mathrm{s}^2}\right)$$

$$\frac{\mathrm{T}}{\mathrm{m}^2} = \frac{\mathrm{T}}{\mathrm{m}^2}$$

22.11 The wave equation cannot contain a negative sign because that would predict that the speed of light of EM waves is *imaginary*! There are no complex numbers allowed (especially for the speed of light).

22.12 Possible answers are microwaves, visible light, IR rays, and UV light from the Sun.

Multiple-Choice Questions

22.13 **C** (have same speed). All EM radiation travels at the speed of light.

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22.14 **C** (Photon energy that is higher). X-rays are higher in energy than visible light.

22.15 **D** (sound). EM waves can travel in a vacuum.

22.16 **D** (wavelengths that are shorter). Radio waves are lower in energy than visible light.

22.17 **B** (wavelengths that are longer). X-rays are higher in energy than visible light.

22.18 **C** (there are both electric and magnetic fields). The time-varying electric field generates a magnetic field.

22.19 **E** (to both time-independent and time-dependent electric and magnetic fields). Maxwell's equations are the fundamental relationships underlying all electric, magnetic, and electromagnetic phenomena, regardless of their time dependence.

22.20 C (0°). The electric and magnetic fields are exactly in phase in an electromagnetic wave.

Estimation/Numerical Questions

22.21

$$v = \frac{2\pi R_{\text{Earth}}}{\Delta t}$$

$$\Delta t = \frac{2\pi R_{\text{Earth}}}{v} = \frac{2\pi R_{\text{Earth}}}{0.1c} = \frac{2\pi(6.38 \times 10^6 \text{ m})}{0.1\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)} = 1.3 \text{ s}$$

22.22 Part a) The frequency of AM radio waves is around 1000 kHz, so the wavelength is 300 m.

Part b) The frequency of FM radio waves is around 100 MHz, so the wavelength is 3 m.

22.23 Green light has a wavelength around 550 nm. The energy of a green photon is then

$$E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)}{550 \times 10^{-9} \text{ m}} = 3.62 \times 10^{-19} \text{ J}$$

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22.24 One eV is equal to 1.6×10^{-19} J. A simple rule-of-thumb conversion is 60 eV is equal to 1 aJ. Recall that 1 aJ $= 10^{-18}$ J.

22.25 Ultraviolet light would be potentially classified as "ionizing radiation"; the wavelength of ultraviolet light is around 10 nm.

22.26 Assuming the average wavelength of the light bulb is 550 nm, the energy per photon is $E_{\text{photon}} = \dfrac{hc}{\lambda} = \dfrac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)}{550 \times 10^{-9} \text{ m}} = 3.62 \times 10^{-19}$ J. The number of photons emitted in the 1000-hr lifetime of a 100-W light bulb is $1000 \text{ hr} \times \dfrac{3600 \text{ s}}{1 \text{ hr}} \times \dfrac{100 \text{ J}}{\text{s}} \times \dfrac{1 \text{ photon}}{3.62 \times 10^{-19} \text{ J}} = 1 \times 10^{27}$ photons.

22.27 The people 300 km away receive the news first:

$$t_{\text{radio}} = \frac{\Delta x}{c} = \frac{300 \times 10^3 \text{ m}}{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)} = 1 \times 10^{-3} \text{ s}$$

$$t_{\text{sound}} = \frac{\Delta x}{v} = \frac{3 \text{ m}}{\left(340 \frac{\text{m}}{\text{s}}\right)} = 8.8 \times 10^{-3} \text{ s}$$

22.28

t (s)	E (N/C)	B ($\times 10^{-9}$ T)
0.000	100	333
0.785	70.7	236
1.571	0	0
2.356	−70.7	−236
3.142	−100	−333
3.927	−70.7	−236
4.712	0	0
5.498	70.7	236
6.283	100	333
7.069	70.7	236
7.854	0	0
8.639	−70.7	−236
9.425	−100	−333
10.210	−70.7	−236
10.996	0	0
11.781	70.7	236
12.566	100	333
13.352	70.7	236
14.137	0	0
14.923	−70.7	−236
15.708	−100	−333
16.493	−70.7	−236
17.279	0	0
18.064	70.7	236
18.850	100	333

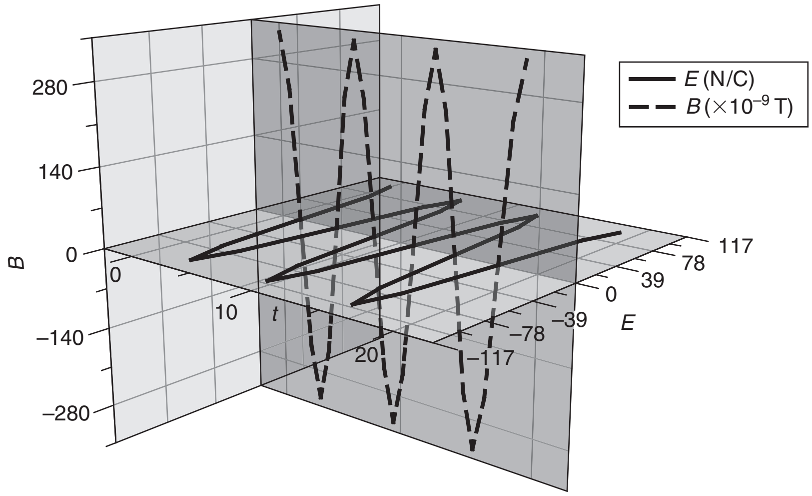


Figure 22-1 Problem 28

The electric and magnetic fields oscillate in time in phase with one another.

Problems

22.29

SET UP

We are given a list of frequencies of some photons and asked to calculate their wavelengths and determine which type of electromagnetic radiation each photon is. The wavelength of a photon is related to its frequency by the speed of light, $\lambda = \frac{c}{f}$. We can use Figure 22-1 from the text to determine the type of radiation.

SOLVE

A) $\lambda = \dfrac{c}{f} = \dfrac{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)}{4.14 \times 10^{15}\ \text{Hz}} = \boxed{7.25 \times 10^{-8}\ \text{m}}$, ultraviolet light

B) $\lambda = \dfrac{c}{f} = \dfrac{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)}{7.00 \times 10^{14}\ \text{Hz}} = \boxed{4.29 \times 10^{-7}\ \text{m}}$, purple visible light

C) $\lambda = \frac{c}{f} = \frac{\left(3.00 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{8.00 \times 10^{16}\ \text{Hz}} = \boxed{3.75 \times 10^{-9}\ \text{m}}$, x-ray

D) $\lambda = \frac{c}{f} = \frac{\left(3.00 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{3.00 \times 10^{13}\ \text{Hz}} = \boxed{1.00 \times 10^{-5}\ \text{m}}$, infrared light

E) $\lambda = \frac{c}{f} = \frac{\left(3.00 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{9.00 \times 10^{12}\ \text{Hz}} = \boxed{3.33 \times 10^{-5}\ \text{m}}$, infrared light

F) $\lambda = \frac{c}{f} = \frac{\left(3.00 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{3.44 \times 10^{17}\ \text{Hz}} = \boxed{8.72 \times 10^{-10}\ \text{m}}$, x-ray

G) $\lambda = \frac{c}{f} = \frac{\left(3.00 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{8.23 \times 10^{15}\ \text{Hz}} = \boxed{3.65 \times 10^{-8}\ \text{m}}$, ultraviolet light

H) $\lambda = \frac{c}{f} = \frac{\left(3.00 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{6.00 \times 10^{15}\ \text{Hz}} = \boxed{5.00 \times 10^{-8}\ \text{m}}$, ultraviolet light

REFLECT

Visible light has wavelengths ranging from 380 nm (violet) to 750 nm (red). Wavelengths smaller than this region correspond to ultraviolet light, and wavelengths larger than this region correspond to infrared light.

22.30

SET UP

We are given a list of the frequencies of some photons and asked to calculate their wavelengths. The wavelength of a photon is related to the frequency by the speed of light, $\lambda = \frac{c}{f}$.

SOLVE

A) $\lambda = \frac{c}{f} = \frac{\left(3.00 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{7.50 \times 10^{15}\ \text{Hz}} = \boxed{4.00 \times 10^{-8}\ \text{m}}$

B) $\lambda = \frac{c}{f} = \frac{\left(3.00 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{6.00 \times 10^{14}\ \text{Hz}} = \boxed{5.00 \times 10^{-7}\ \text{m}}$

$$\text{C) } \lambda = \frac{c}{f} = \frac{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)}{5.00 \times 10^{14}\ \text{Hz}} = \boxed{6.00 \times 10^{-7}\ \text{m}}$$

$$\text{D) } \lambda = \frac{c}{f} = \frac{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)}{4.29 \times 10^{14}\ \text{Hz}} = \boxed{6.99 \times 10^{-7}\ \text{m}}$$

$$\text{E) } \lambda = \frac{c}{f} = \frac{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)}{7.50 \times 10^{16}\ \text{Hz}} = \boxed{4.00 \times 10^{-9}\ \text{m}}$$

$$\text{F) } \lambda = \frac{c}{f} = \frac{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)}{2.66 \times 10^{16}\ \text{Hz}} = \boxed{1.13 \times 10^{-8}\ \text{m}}$$

$$\text{G) } \lambda = \frac{c}{f} = \frac{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)}{8.23 \times 10^{17}\ \text{Hz}} = \boxed{3.65 \times 10^{-10}\ \text{m}}$$

$$\text{H) } \lambda = \frac{c}{f} = \frac{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)}{6.00 \times 10^{18}\ \text{Hz}} = \boxed{5.00 \times 10^{-11}\ \text{m}}$$

REFLECT

The frequency and wavelength are inversely proportional to one another, so a smaller wavelength will have a larger frequency, and vice versa.

22.31

SET UP

We are given a list of the wavelengths of some photons and asked to calculate their frequencies. The frequency of a photon is related to the wavelength by the speed of light, $f = \frac{c}{\lambda}$.

SOLVE

$$\text{A) } f = \frac{c}{\lambda} = \frac{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)}{700 \times 10^{-9}\ \text{m}} = \boxed{4.29 \times 10^{14}\ \text{Hz}}$$

$$\text{B) } f = \frac{c}{\lambda} = \frac{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)}{600 \times 10^{-9}\ \text{m}} = \boxed{5.00 \times 10^{14}\ \text{Hz}}$$

$$\text{C) } f = \frac{c}{\lambda} = \frac{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)}{500 \times 10^{-9}\ \text{m}} = \boxed{6.00 \times 10^{14}\ \text{Hz}}$$

D) $f = \dfrac{c}{\lambda} = \dfrac{\left(3.00 \times 10^{8} \dfrac{\text{m}}{\text{s}}\right)}{400 \times 10^{-9}\ \text{m}} = \boxed{7.50 \times 10^{14}\ \text{Hz}}$

E) $f = \dfrac{c}{\lambda} = \dfrac{\left(3.00 \times 10^{8} \dfrac{\text{m}}{\text{s}}\right)}{100 \times 10^{-9}\ \text{m}} = \boxed{3.00 \times 10^{15}\ \text{Hz}}$

F) $f = \dfrac{c}{\lambda} = \dfrac{\left(3.00 \times 10^{8} \dfrac{\text{m}}{\text{s}}\right)}{0.033 \times 10^{-9}\ \text{m}} = \boxed{9.01 \times 10^{18}\ \text{Hz}}$

G) $f = \dfrac{c}{\lambda} = \dfrac{\left(3.00 \times 10^{8} \dfrac{\text{m}}{\text{s}}\right)}{500 \times 10^{-6}\ \text{m}} = \boxed{6.00 \times 10^{11}\ \text{Hz}}$

H) $f = \dfrac{c}{\lambda} = \dfrac{\left(3.00 \times 10^{8} \dfrac{\text{m}}{\text{s}}\right)}{63.3 \times 10^{-12}\ \text{m}} = \boxed{4.74 \times 10^{18}\ \text{Hz}}$

REFLECT

The frequency and wavelength are inversely proportional to one another, so a smaller wavelength will have a larger frequency, and vice versa.

Get Help: P'Cast 22.1 – Visible Light

22.32

SET UP

We are given a list of the wavelengths of some photons and asked to calculate their frequencies. The frequency of a photon is related to the wavelength by the speed of light, $f = \dfrac{c}{\lambda}$.

SOLVE

A) $f = \dfrac{c}{\lambda} = \dfrac{\left(3.00 \times 10^{8} \dfrac{\text{m}}{\text{s}}\right)}{800 \times 10^{-9}\ \text{m}} = \boxed{3.75 \times 10^{14}\ \text{Hz}}$

B) $f = \dfrac{c}{\lambda} = \dfrac{\left(3.00 \times 10^{8} \dfrac{\text{m}}{\text{s}}\right)}{650 \times 10^{-9}\ \text{m}} = \boxed{4.62 \times 10^{14}\ \text{Hz}}$

C) $f = \dfrac{c}{\lambda} = \dfrac{\left(3.00 \times 10^{8} \dfrac{\text{m}}{\text{s}}\right)}{550 \times 10^{-9}\ \text{m}} = \boxed{5.45 \times 10^{14}\ \text{Hz}}$

D) $f = \dfrac{c}{\lambda} = \dfrac{\left(3.00 \times 10^{8} \dfrac{\text{m}}{\text{s}}\right)}{450 \times 10^{-9}\ \text{m}} = \boxed{6.67 \times 10^{14}\ \text{Hz}}$

E) $f = \dfrac{c}{\lambda} = \dfrac{\left(3.00 \times 10^{8} \dfrac{\text{m}}{\text{s}}\right)}{2.22 \times 10^{-9}\ \text{m}} = \boxed{1.35 \times 10^{17}\ \text{Hz}}$

F) $f = \dfrac{c}{\lambda} = \dfrac{\left(3.00 \times 10^{8} \dfrac{\text{m}}{\text{s}}\right)}{1.10 \times 10^{-8}\ \text{m}} = \boxed{2.73 \times 10^{16}\ \text{Hz}}$

G) $f = \dfrac{c}{\lambda} = \dfrac{\left(3.00 \times 10^{8} \dfrac{\text{m}}{\text{s}}\right)}{50 \times 10^{-6}\ \text{m}} = \boxed{6.0 \times 10^{12}\ \text{Hz}}$

H) $f = \dfrac{c}{\lambda} = \dfrac{\left(3.00 \times 10^{8} \dfrac{\text{m}}{\text{s}}\right)}{33.4 \times 10^{-3}\ \text{m}} = \boxed{8.98 \times 10^{9}\ \text{Hz}}$

REFLECT

The frequency and wavelength are inversely proportional to one another, so a smaller wavelength will have a larger frequency, and vice versa.

22.33

SET UP

We are given a list of energies of some photons and asked to calculate their wavelengths and frequencies. The wavelength λ and frequency f of a photon is related to its energy by $E = \dfrac{hc}{\lambda} = hf$. Recall that $1\ \text{eV} = 1.602 \times 10^{-19}\ \text{J}$.

SOLVE

A)

Wavelength:

$$\lambda = \frac{hc}{E} = \frac{(6.626 \times 10^{-34}\ \text{J} \cdot \text{s})\left(3.00 \times 10^{8} \dfrac{\text{m}}{\text{s}}\right)}{2.33 \times 10^{-19}\ \text{J}} = \boxed{8.53 \times 10^{-7}\ \text{m} = 853\ \text{nm}}$$

Frequency:

$$f = \frac{E}{h} = \frac{2.33 \times 10^{-19}\ \text{J}}{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}} = \boxed{3.52 \times 10^{14}\ \text{Hz}}$$

B)

Wavelength:

$$\lambda = \frac{hc}{E} = \frac{(6.626 \times 10^{-34}\ \text{J} \cdot \text{s})\left(3.00 \times 10^{8} \dfrac{\text{m}}{\text{s}}\right)}{4.50 \times 10^{-19}\ \text{J}} = \boxed{4.42 \times 10^{-7}\ \text{m} = 442\ \text{nm}}$$

Frequency:

$$f = \frac{E}{h} = \frac{4.50 \times 10^{-19}\ \text{J}}{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}} = \boxed{6.79 \times 10^{14}\ \text{Hz}}$$

C)
Wavelength:

$$\lambda = \frac{hc}{E} = \frac{(6.626 \times 10^{-34}\ \text{J} \cdot \text{s})\left(3.00 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{3.20 \times 10^{-19}\ \text{J}} = \boxed{6.21 \times 10^{-7}\ \text{m} = 621\ \text{nm}}$$

Frequency:

$$f = \frac{E}{h} = \frac{3.20 \times 10^{-19}\ \text{J}}{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}} = \boxed{4.83 \times 10^{14}\ \text{Hz}}$$

D)
Wavelength:

$$\lambda = \frac{hc}{E} = \frac{(6.626 \times 10^{-34}\ \text{J} \cdot \text{s})\left(3.00 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{8.55 \times 10^{-19}\ \text{J}} = \boxed{2.32 \times 10^{-7}\ \text{m} = 232\ \text{nm}}$$

Frequency:

$$f = \frac{E}{h} = \frac{8.55 \times 10^{-19}\ \text{J}}{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}} = \boxed{1.29 \times 10^{15}\ \text{Hz}}$$

E)
Energy in joules:

$$E = 63.3\ \text{eV} \times \frac{1.602 \times 10^{-19}\ \text{J}}{1\ \text{eV}} = 1.01 \times 10^{-17}\ \text{J}$$

Wavelength:

$$\lambda = \frac{hc}{E} = \frac{(6.626 \times 10^{-34}\ \text{J} \cdot \text{s})\left(3.00 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{1.01 \times 10^{-17}\ \text{J}} = \boxed{1.96 \times 10^{-8}\ \text{m} = 19.6\ \text{nm}}$$

Frequency:

$$f = \frac{E}{h} = \frac{1.01 \times 10^{-17}\ \text{J}}{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}} = \boxed{1.53 \times 10^{16}\ \text{Hz}}$$

F)
Energy in joules:

$$E = 8.77\ \text{eV} \times \frac{1.602 \times 10^{-19}\ \text{J}}{1\ \text{eV}} = 1.40 \times 10^{-18}\ \text{J}$$

Wavelength:

$$\lambda = \frac{hc}{E} = \frac{(6.626 \times 10^{-34}\ \mathrm{J \cdot s})\left(3.00 \times 10^{8}\frac{\mathrm{m}}{\mathrm{s}}\right)}{1.40 \times 10^{-18}\ \mathrm{J}} = \boxed{1.41 \times 10^{-7}\ \mathrm{m} = 141\ \mathrm{nm}}$$

Frequency:

$$f = \frac{E}{h} = \frac{1.40 \times 10^{-18}\ \mathrm{J}}{6.626 \times 10^{-34}\ \mathrm{J \cdot s}} = \boxed{2.12 \times 10^{15}\ \mathrm{Hz}}$$

G)
Energy in joules:

$$E = 1.98\ \mathrm{eV} \times \frac{1.602 \times 10^{-19}\ \mathrm{J}}{1\ \mathrm{eV}} = 3.17 \times 10^{-19}\ \mathrm{J}$$

Wavelength:

$$\lambda = \frac{hc}{E} = \frac{(6.626 \times 10^{-34}\ \mathrm{J \cdot s})\left(3.00 \times 10^{8}\frac{\mathrm{m}}{\mathrm{s}}\right)}{3.17 \times 10^{-19}\ \mathrm{J}} = \boxed{6.27 \times 10^{-7}\ \mathrm{m} = 627\ \mathrm{nm}}$$

Frequency:

$$f = \frac{E}{h} = \frac{3.17 \times 10^{-19}\ \mathrm{J}}{6.626 \times 10^{-34}\ \mathrm{J \cdot s}} = \boxed{4.79 \times 10^{14}\ \mathrm{Hz}}$$

H)
Energy in joules:

$$E = 4.55\ \mathrm{eV} \times \frac{1.602 \times 10^{-19}\ \mathrm{J}}{1\ \mathrm{eV}} = 7.29 \times 10^{-19}\ \mathrm{J}$$

Wavelength:

$$\lambda = \frac{hc}{E} = \frac{(6.626 \times 10^{-34}\ \mathrm{J \cdot s})\left(3.00 \times 10^{8}\frac{\mathrm{m}}{\mathrm{s}}\right)}{7.29 \times 10^{-19}\ \mathrm{J}} = \boxed{2.73 \times 10^{-7}\ \mathrm{m} = 273\ \mathrm{nm}}$$

Frequency:

$$f = \frac{E}{h} = \frac{7.29 \times 10^{-19}\ \mathrm{J}}{6.626 \times 10^{-34}\ \mathrm{J \cdot s}} = \boxed{1.10 \times 10^{15}\ \mathrm{Hz}}$$

REFLECT

The energy of a photon is directly proportional to its frequency and inversely proportional to its wavelength.

22.34

SET UP

We are given a list of energies of some photons and asked to calculate their wavelengths and frequencies. The wavelength λ and frequency f of a photon are related to its energy by $E = \frac{hc}{\lambda} = hf$. Recall that 1 eV $= 1.602 \times 10^{-19}$ J.

SOLVE

A)
Wavelength:

$$\lambda = \frac{hc}{E} = \frac{(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})\left(3.00 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{3.45 \times 10^{-19}\ \text{J}} = \boxed{5.76 \times 10^{-7}\ \text{m} = 576\ \text{nm}}$$

Frequency:

$$f = \frac{E}{h} = \frac{3.45 \times 10^{-19}\ \text{J}}{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}} = \boxed{5.21 \times 10^{14}\ \text{Hz}}$$

B)
Wavelength:

$$\lambda = \frac{hc}{E} = \frac{(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})\left(3.00 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{4.80 \times 10^{-19}\ \text{J}} = \boxed{4.14 \times 10^{-7}\ \text{m} = 414\ \text{nm}}$$

Frequency:

$$f = \frac{E}{h} = \frac{4.80 \times 10^{-19}\ \text{J}}{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}} = \boxed{7.24 \times 10^{14}\ \text{Hz}}$$

C)
Wavelength:

$$\lambda = \frac{hc}{E} = \frac{(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})\left(3.00 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{1.28 \times 10^{-18}\ \text{J}} = \boxed{1.55 \times 10^{-7}\ \text{m} = 155\ \text{nm}}$$

Frequency:

$$f = \frac{E}{h} = \frac{1.28 \times 10^{-18}\ \text{J}}{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}} = \boxed{1.93 \times 10^{15}\ \text{Hz}}$$

D)
Wavelength:

$$\lambda = \frac{hc}{E} = \frac{(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})\left(3.00 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{4.33 \times 10^{-20}\ \text{J}} = \boxed{4.59 \times 10^{-6}\ \text{m} = 4590\ \text{nm}}$$

Frequency:

$$f = \frac{E}{h} = \frac{4.33 \times 10^{-20}\ \text{J}}{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}} = \boxed{6.53 \times 10^{13}\ \text{Hz}}$$

E)
Energy in joules:

$$E = 931 \times 10^{6}\ \text{eV} \times \frac{1.602 \times 10^{-19}\ \text{J}}{1\ \text{eV}} = 1.49 \times 10^{-10}\ \text{J}$$

Wavelength:

$$\lambda = \frac{hc}{E} = \frac{(6.626 \times 10^{-34}\ \text{J} \cdot \text{s})\left(3.00 \times 10^{8} \frac{\text{m}}{\text{s}}\right)}{1.49 \times 10^{-10}\ \text{J}} = \boxed{1.33 \times 10^{-15}\ \text{m}}$$

Frequency:

$$f = \frac{E}{h} = \frac{1.49 \times 10^{-10}\ \text{J}}{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}} = \boxed{2.25 \times 10^{23}\ \text{Hz}}$$

F)
Energy in joules:

$$E = 2.88 \times 10^{3}\ \text{eV} \times \frac{1.602 \times 10^{-19}\ \text{J}}{1\ \text{eV}} = 4.61 \times 10^{-16}\ \text{J}$$

Wavelength:

$$\lambda = \frac{hc}{E} = \frac{(6.626 \times 10^{-34}\ \text{J} \cdot \text{s})\left(3.00 \times 10^{8} \frac{\text{m}}{\text{s}}\right)}{4.61 \times 10^{-16}\ \text{J}} = \boxed{4.31 \times 10^{-10}\ \text{m} = 0.431\ \text{nm}}$$

Frequency:

$$f = \frac{E}{h} = \frac{4.61 \times 10^{-16}\ \text{J}}{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}} = \boxed{6.96 \times 10^{17}\ \text{Hz}}$$

G)
Energy in joules:

$$E = 7.88\ \text{eV} \times \frac{1.602 \times 10^{-19}\ \text{J}}{1\ \text{eV}} = 1.26 \times 10^{-18}\ \text{J}$$

Wavelength:

$$\lambda = \frac{hc}{E} = \frac{(6.626 \times 10^{-34}\ \text{J} \cdot \text{s})\left(3.00 \times 10^{8} \frac{\text{m}}{\text{s}}\right)}{1.26 \times 10^{-18}\ \text{J}} = \boxed{1.57 \times 10^{-7}\ \text{m} = 157\ \text{nm}}$$

Frequency:

$$f = \frac{E}{h} = \frac{1.26 \times 10^{-18}\ \text{J}}{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}} = \boxed{1.91 \times 10^{15}\ \text{Hz}}$$

H)
Energy in joules:

$$E = 13.6\ \text{eV} \times \frac{1.602 \times 10^{-19}\ \text{J}}{1\ \text{eV}} = 2.18 \times 10^{-18}\ \text{J}$$

Wavelength:

$$\lambda = \frac{hc}{E} = \frac{(6.626 \times 10^{-34}\ \text{J} \cdot \text{s})\left(3.00 \times 10^{8} \frac{\text{m}}{\text{s}}\right)}{2.18 \times 10^{-18}\ \text{J}} = \boxed{9.12 \times 10^{-8}\ \text{m} = 91.2\ \text{nm}}$$

Frequency:

$$f = \frac{E}{h} = \frac{2.18 \times 10^{-18}\ \text{J}}{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}} = \boxed{3.29 \times 10^{15}\ \text{Hz}}$$

REFLECT
An energy of 931 MeV is equivalent to a mass of 1 u through Einstein's famous equation, $E = mc^2$. An energy of 13.6 eV is equal to the ionization energy of a hydrogen atom in its ground state.

22.35

SET UP
The range of typical frequencies for an FM radio spans from $f_{\text{low}} = 88.0$ MHz to $f_{\text{high}} = 108$ MHz. We can calculate the wavelength from these frequencies using $\lambda = \frac{c}{f}$.

SOLVE
Low frequency:

$$\lambda_{\text{low}} = \frac{c}{f_{\text{low}}} = \frac{\left(3.00 \times 10^{8} \frac{\text{m}}{\text{s}}\right)}{88.0 \times 10^{6}\ \text{Hz}} = \boxed{3.41\ \text{m}}$$

High frequency:

$$\lambda_{\text{low}} = \frac{c}{f_{\text{low}}} = \frac{\left(3.00 \times 10^{8} \frac{\text{m}}{\text{s}}\right)}{108 \times 10^{6}\ \text{Hz}} = \boxed{2.78\ \text{m}}$$

REFLECT
Wavelength and frequency are inversely proportional, so the lower end of the frequency range should correspond to a larger wavelength and vice versa.

22.36

SET UP

A radio station's antenna has a height of $h = 75$ m, which corresponds to one-quarter of the wavelength it transmits. Multiplying the height by 4 and plugging it into $f = \frac{c}{\lambda}$ will give us the frequency of the radio wave.

SOLVE

$$f = \frac{c}{\lambda} = \frac{c}{4h} = \frac{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)}{4(75\text{ m})} = \boxed{1.0 \times 10^6\text{ Hz} = 1.0\text{ MHz}}$$

REFLECT

The typical range of AM radio frequencies is 500–1600 kHz.

22.37

SET UP

The speed of light in a vacuum is $c = 3.00 \times 10^8$ m/s. The distance light travels in 10×10^{-9} s can be found by multiplying the speed by the time interval.

SOLVE

$$c = \frac{\Delta x}{\Delta t}$$

$$\Delta x = c(\Delta t) = \left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)(10 \times 10^{-9}\text{ s}) = \boxed{3\text{ m}}$$

REFLECT

One light-year is the distance light travels during one year: 1 ly $= 9.4607 \times 10^{12}$ km.

22.38

SET UP

The speed of light in a vacuum is $c = 3.00 \times 10^8$ m/s. The time it takes light to travel a distance of 300×10^3 m can be found by dividing this distance by the speed of light.

SOLVE

$$c = \frac{\Delta x}{\Delta t}$$

$$\Delta t = \frac{\Delta x}{c} = \frac{300 \times 10^3\text{ m}}{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)} = \boxed{1.00 \times 10^{-3}\text{ s}}$$

REFLECT

Light travels 1 m in 3.3 ns.

22.39

SET UP

The distance between Earth and the Moon is approximately 3.84×10^8 m. The time it takes a radio signal to travel between Earth and the Moon is equal to this distance divided by the speed of light, c.

SOLVE

$$c = \frac{\Delta x}{\Delta t}$$

$$\Delta t = \frac{\Delta x}{c} = \frac{3.84 \times 10^8 \text{ m}}{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)} = \boxed{1.28 \text{ s}}$$

REFLECT

The distance between Earth and the Moon is on the order of 10^8, as is the speed of light. Therefore, we would expect our answer to be on the order of 1.

22.40

SET UP

The formula describing the speed of light in terms of the frequency and wavelength of a photon is $c = f\lambda$. We can rewrite this in terms of the angular frequency and wavenumber using their definitions, $\omega = 2\pi f$ and $k = \frac{2\pi}{\lambda}$, respectively.

SOLVE

$$c = f\lambda = \left(\frac{\omega}{2\pi}\right)\left(\frac{2\pi}{k}\right) = \boxed{\frac{\omega}{k}}$$

REFLECT

This is equivalent to multiplying the numerator and the denominator of the equation by 2π.

22.41

SET UP

We can rewrite the speed of light in terms of the angular frequency ω and the wavenumber k, $c = \frac{\omega}{k}$, in order to solve for the wavenumber of a photon with $\omega = 6.28 \times 10^{15}\frac{\text{rad}}{\text{s}}$ or the angular frequency of a photon with $k = 4\pi \times 10^6\frac{\text{rad}}{\text{m}}$. The frequency is equal to $f = \frac{\omega}{2\pi}$, and the wavenumber is related to the wavelength though its definition, $k = \frac{2\pi}{\lambda}$.

SOLVE

Part a)

$$c = \frac{\omega}{k}$$

$$k = \frac{\omega}{c} = \frac{\left(6.28 \times 10^{15}\frac{\text{rad}}{\text{s}}\right)}{\left(3.00 \times 10^8\frac{\text{m}}{\text{s}}\right)} = \boxed{2.09 \times 10^7\frac{\text{rad}}{\text{m}}}$$

Part b)

Angular frequency:

$$c = \frac{\omega}{k}$$

$$\omega = kc = \left(3.00 \times 10^{8}\frac{\text{m}}{\text{s}}\right)\left(4\pi \times 10^{6}\frac{\text{rad}}{\text{m}}\right) = \boxed{3.77 \times 10^{15}\frac{\text{rad}}{\text{s}}}$$

Frequency:

$$f = \frac{\omega}{2\pi} = \frac{\left(3.77 \times 10^{15}\frac{\text{rad}}{\text{s}}\right)}{2\pi} = \boxed{6.00 \times 10^{14}\ \text{Hz}}$$

Wavelength:

$$k = \frac{2\pi}{\lambda}$$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{\left(4\pi \times 10^{6}\frac{\text{rad}}{\text{m}}\right)} = \boxed{5.00 \times 10^{-7}\ \text{m}}$$

REFLECT

The photon in part (a) is 300-nm ultraviolet light. The photon in part (b) is blue-green visible light.

22.42

SET UP

The magnetic field of an electromagnetic wave is described by

$B(x,t) = (0.7\ \mu\text{T})\sin\left[2\pi\left(\left(4.00 \times 10^{6}\frac{\text{rad}}{\text{m}}\right)x - \left(1.20 \times 10^{15}\frac{\text{rad}}{\text{s}}\right)t\right)\right]$.

The general form of an electromagnetic wave as a function of space and time is

$B(x,t) = A\sin[kx - \omega t] = A\sin\left[2\pi\left(\left(\frac{x}{\lambda}\right) - ft\right)\right]$, where A is the amplitude, k is the wave number, λ is the wavelength, ω is the angular frequency, and f is the frequency. The speed of the wave is equal to $v = \frac{\omega}{k} = \lambda f$. The period of the wave is the reciprocal of the frequency.

SOLVE

Part a)

$$\boxed{A = 0.7\ \mu\text{T}}$$

Part b)

$$v = \lambda f = (4.00 \times 10^6\ \text{m}^{-1})^{-1}(1.20 \times 10^{15}\ \text{Hz}) = \boxed{3.00 \times 10^8 \frac{\text{m}}{\text{s}}}$$

Part c)

$$\boxed{f = 1.20 \times 10^{15}\ \text{Hz}}$$

Part d)

$$T = \frac{1}{f} = \frac{1}{1.20 \times 10^{15}\ \text{Hz}} = \boxed{8.33 \times 10^{-16}\ \text{s}}$$

Part e)

$$\lambda = (4.00 \times 10^6\ \text{m}^{-1})^{-1} = \boxed{2.50 \times 10^{-7}\ \text{m}}$$

REFLECT

All electromagnetic waves travel at a speed c.

22.43

SET UP

The electric field of an electromagnetic wave is described by $E(x,t) = 400\cos((40\pi \times 10^{-2})x - (120\pi \times 10^6)t)$, where all of the constants are in the appropriate SI units. The general form of the electric field as a function of x and t is $E(x,t) = E_0\cos(kx - \omega t)$, where k is the wave number and ω is the angular frequency, which can be read directly from the equation. The wavelength λ is related to the wave number through $\lambda = \frac{2\pi}{k}$; the frequency f is related to the angular frequency through $f = \frac{\omega}{2\pi}$.

SOLVE

Part a)

$$\boxed{k = 40\pi \times 10^{-2} \frac{\text{rad}}{\text{m}}}$$

Part b)

$$\boxed{\omega = 120\pi \times 10^{6} \frac{\text{rad}}{\text{s}}}$$

Part c)

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{\left(40\pi \times 10^{-2} \frac{\text{rad}}{\text{m}}\right)} = \boxed{5.0\ \text{m}}$$

Part d)

$$f = \frac{\omega}{2\pi} = \frac{\left(120\pi \times 10^6 \frac{\text{rad}}{\text{s}}\right)}{2\pi} = \boxed{60 \times 10^6\ \text{Hz} = 60\ \text{MHz}}$$

REFLECT

The speed of the wave is $v = \frac{\omega}{k} = \frac{\left(120\pi \times 10^6 \frac{\text{rad}}{\text{s}}\right)}{\left(40\pi \times 10^{-2} \frac{\text{rad}}{\text{m}}\right)} = 3.0 \times 10^8 \frac{\text{m}}{\text{s}}$, which makes sense given that this is an electromagnetic wave.

22.44

SET UP

The magnitude of the electric field in a region of space as a function of time is given by $E = \left(0.2\frac{\text{N}}{\text{C}}\right)\sin\left(\left(1000\frac{\text{rad}}{\text{s}}\right)t\right)$. The maximum displacement current through a surface ($A = 1\ \text{m}^2$) that is perpendicular to the electric field is equal to the amplitude of $\varepsilon_0 \frac{d\Phi_E}{dt}$, where $\Phi_E = \vec{E} \cdot \vec{A}$.

SOLVE

Displacement current:

$$\varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{d}{dt}[\vec{E} \cdot \vec{A}] = \varepsilon_0 \frac{d}{dt}[EA\cos(90°)] = \varepsilon_0 A \frac{d}{dt}\left[\left(0.2\frac{\text{N}}{\text{C}}\right)\sin\left(\left(1000\frac{\text{rad}}{\text{s}}\right)t\right)\right]$$

$$= \left(8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}\right)(1\ \text{m}^2)\left(0.2\frac{\text{N}}{\text{C}}\right)\left[\left(1000\frac{\text{rad}}{\text{s}}\right)\cos\left(\left(1000\frac{\text{rad}}{\text{s}}\right)t\right)\right]$$

$$= (2 \times 10^{-9}\ \text{A})\cos\left(\left(1000\frac{\text{rad}}{\text{s}}\right)\text{t}\right)$$

Maximum displacement current:

$$\left[\varepsilon_0 \frac{d\Phi_E}{dt}\right]_{\text{max}} = 2 \times 10^{-9}\ \text{A}$$

REFLECT

The displacement current is usually orders of magnitude smaller than currents found in typical wires.

22.45

SET UP

Charge is flowing onto the positive plate and off of the negative plate of a parallel plate capacitor with closely spaced plates at a rate of $\frac{dq}{dt} = 2.8\ \text{A}$. If we treat the parallel plates

as being approximately infinite, the electric field in between the plates has a magnitude of $E = \frac{q}{A\varepsilon_0}$, where A is the cross-sectional area. The displacement current through the capacitor between the plates is equal to $\varepsilon_0 \frac{d\Phi_E}{dt}$, where $\Phi_E = \vec{E} \cdot \vec{A}$.

SOLVE

$$\varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{d}{dt}[\vec{E} \cdot \vec{A}] = \varepsilon_0 \frac{d}{dt}[EA] = \varepsilon_0 \frac{d}{dt}\left[\left(\frac{q}{A\varepsilon_0}\right)A\right] = \frac{dq}{dt} = \boxed{2.8\text{ A}}$$

REFLECT

It makes sense that the displacement current "through" the capacitor should equal the rate of charge flowing onto the positive plate and off of the negative plate.

22.46

SET UP

The electric field between the plates of a parallel-plate capacitor (radius 4×10^{-2} m) is increasing at a rate of $\frac{dE}{dt} = 1.0 \times 10^6 \frac{\text{V}}{\text{m} \cdot \text{s}}$. The magnitude of the magnetic field a distance $r = 5.0 \times 10^{-2}$ m from the axis of the capacitor is related to the displacement current through Ampère's law, $\oint \vec{B} \cdot d\vec{l} = \mu_0\left(\varepsilon_0 \frac{d\Phi_E}{dt}\right)$. We should use the size of the plates of the capacitor when calculating the electric flux and the distance from the capacitor axis when calculating the line integral around dl.

SOLVE

$$\oint \vec{B} \cdot d\vec{l} = \mu_0\left(\varepsilon_0 \frac{d\Phi_E}{dt}\right)$$

$$\oint Bdl\cos(0°) = \mu_0\varepsilon_0\left(\frac{d(\vec{E} \cdot \vec{A})}{dt}\right)$$

$$B\oint dl = \mu_0\varepsilon_0\left(\frac{d(EA\cos(0°))}{dt}\right)$$

$$B(2\pi r) = \mu_0\varepsilon_0 A\left(\frac{dE}{dt}\right)$$

$$\begin{aligned} B &= \frac{\mu_0\varepsilon_0 A}{2\pi r}\left(\frac{dE}{dt}\right) \\ &= \frac{\left(4\pi \times 10^{-7}\frac{\text{T} \cdot \text{m}}{\text{A}}\right)\left(8.85 \times 10^{-12}\frac{\text{C}^2}{\text{N} \cdot \text{m}^2}\right)(\pi(4 \times 10^{-2}\text{ m})^2)}{2\pi(5 \times 10^{-2}\text{ m})}\left(1.0 \times 10^6\frac{\text{V}}{\text{m} \cdot \text{s}}\right) \\ &= 2 \times 10^{-13}\text{ T} \end{aligned}$$

REFLECT

The magnitude of the magnetic field due to the displacement current should be small.

22.47

SET UP

Charge flows onto the positive plate of a parallel-plate capacitor at a rate of $i = 1.5$ A. The magnitude of the magnetic field a distance $r = 3.0 \times 10^{-2}$ m from the axis of the capacitor is related to the current through Ampère's law, $\oint \vec{B} \cdot d\vec{l} = \mu_0 i$.

SOLVE

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i$$

$$\oint B dl \cos(0°) = \mu_0 i$$

$$B \oint dl = \mu_0 i$$

$$B(2\pi r) = \mu_0 i$$

$$B = \frac{\mu_0 i}{2\pi r} = \frac{\left(4\pi \times 10^{-7} \frac{\mathrm{T \cdot m}}{\mathrm{A}}\right)(1.5\ \mathrm{A})}{2\pi(3.0 \times 10^{-2}\ \mathrm{m})} = \boxed{1.0 \times 10^{-5}\ \mathrm{T}}$$

REFLECT

The size of the plates did not explicitly factor into this problem.

22.48

SET UP

We are asked to calculate the partial derivatives of various functions. In order to take the partial derivative with respect to time, we need to hold the position fixed and treat it as a constant. When taking the partial derivative with respect to position, we hold the time fixed and treat it as a constant.

SOLVE

A)

$$\frac{\partial}{\partial x}[A \sin(kx - \omega t)] = \boxed{kA \cos(kx - \omega t)}$$

B)

$$\frac{\partial}{\partial t}[A \cos(kx - \omega t)] = \boxed{\omega A \sin(kx - \omega t)}$$

C)

$$\frac{\partial^2}{\partial t \partial x}[Ae^{i(kx - \omega t)}] = \frac{\partial}{\partial t}[(ik)Ae^{i(kx - \omega t)}] = (Aik)\frac{\partial}{\partial t}[e^{i(kx - \omega t)}]$$

$$= (Aik)(-i\omega)e^{i(kx - \omega t)} = \boxed{\omega k A e^{i(kx - \omega t)}}$$

D)

$$\frac{\partial^2}{\partial x^2}[Ae^{i(kx-\omega t)}] = \frac{\partial}{\partial x}[A(ik)e^{i(kx-\omega t)}] = (Aik)\frac{\partial}{\partial x}[e^{i(kx-\omega t)}]$$
$$= (Aik)(ik)e^{i(kx-\omega t)} = \boxed{-Ak^2e^{i(kx-\omega t)}}$$

E)

$$\frac{\partial^2}{\partial x \partial t}[At^2x^3] = \frac{\partial}{\partial x}[2Atx^3] = \boxed{6Atx^2}$$

F)

$$\frac{\partial^2}{\partial t \partial x}[At^2x^3] = \frac{\partial}{\partial t}[3At^2x^2] = \boxed{6Atx^2}$$

REFLECT

When calculating higher-order partial derivatives, it's easiest to calculate the derivatives step-by-step rather than trying to do it all at once.

22.49

SET UP

We are asked to determine $\frac{\partial^2 f}{\partial t^2}$, where $f(x,t) = Ae^{\alpha t}\sin(kx - \omega t)$. In order to take the partial derivatives with respect to time, we need to hold the position fixed and treat it as a constant.

SOLVE

$$f(x,t) = Ae^{\alpha t}\sin(kx - \omega t)$$

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial t}[Ae^{\alpha t}\sin(kx - \omega t)] = A[\alpha e^{\alpha t}\sin(kx - \omega t) - e^{\alpha t}\omega\cos(kx - \omega t)]$$
$$= Ae^{\alpha t}[\alpha\sin(kx - \omega t) - \omega\cos(kx - \omega t)]$$

$$\frac{\partial^2 f}{\partial t^2} = \frac{\partial}{\partial t}[Ae^{\alpha t}[\alpha\sin(kx - \omega t) - \omega\cos(kx - \omega t)]]$$
$$= A\alpha e^{\alpha t}[\alpha\sin(kx - \omega t) - \omega\cos(kx - \omega t)] + Ae^{\alpha t}[-\alpha\omega\cos(kx - \omega t) - \omega^2\sin(kx - \omega t)]$$
$$= \boxed{\alpha^2Ae^{\alpha t}\sin(kx - \omega t) - 2\alpha\omega Ae^{\alpha t}\cos(kx - \omega t) - \omega^2Ae^{\alpha t}\sin(kx - \omega t)}$$

REFLECT

It's always easiest to first explicitly calculate the first derivative when trying to find the second derivative, rather than trying to do it all in one step.

22.50

SET UP

We are asked to derive the relationship $\frac{\partial B}{\partial x} = -\mu_0\varepsilon_0\frac{\partial E}{\partial t}$ starting with Ampère's law with no conduction current, $\oint \vec{B}\cdot d\vec{l} = \mu_0\varepsilon_0\frac{d\Phi_E}{dt}$. We can consider a portion of a transverse electromagnetic wave traveling toward $+x$ with its electric field oscillating along the y-axis and its magnetic field oscillating along the z-axis. We can draw a rectangular Amperian loop of width Δx and length Δz between two neighboring field lines. We will eventually take Δx to be infinitesimally small, so the difference in magnitude between these neighboring fields at that point will be dB. The oscillating electric field is directed perpendicularly to this Amperian loop, which means the electric flux through the loop will be a maximum. Plugging these results into both sides of Ampère's law and rearranging, we will arrive at $\frac{\partial B}{\partial x} = -\mu_0\varepsilon_0\frac{\partial E}{\partial t}$.

SOLVE

Amperian loop:

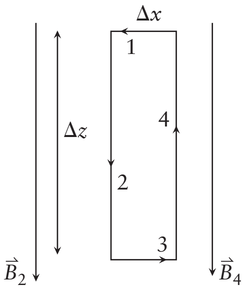


Figure 22-2 Problem 50

Left-hand side of Ampère's law:

$$\oint \vec{B}\cdot d\vec{l} = \int \vec{B}_1\cdot dx + \int \vec{B}_2\cdot dz + \int \vec{B}_3\cdot dx + \int \vec{B}_4\cdot dz$$

$$= \int B_1 dx\cos(90°) + \int B_2 dz\cos(0°) + \int B_3 dx\cos(90°) + \int B_4 dz\cos(180°)$$

$$= 0 + \int B_2 dz + 0 - \int B_4 dz = B_2\Delta z - B_4\Delta z = B_2\Delta z - (B_2 + dB)\Delta z = -dB\Delta z$$

Right-hand side of Ampère's law:

$$\mu_0\varepsilon_0\frac{d\Phi_E}{dt} = \mu_0\varepsilon_0\frac{d}{dt}[\vec{E}\cdot\vec{A}] = \mu_0\varepsilon_0\frac{d}{dt}[E(\Delta x\Delta z)\cos(0°)] = \mu_0\varepsilon_0\Delta x\Delta z\frac{dE}{dt}$$

Setting them equal:

$$\oint \vec{B}\cdot d\vec{l} = \mu_0\varepsilon_0\frac{d\Phi_E}{dt}$$

$$-dB\Delta z = \mu_0\varepsilon_0\Delta x\Delta z\frac{dE}{dt}$$

$$-dB = \mu_0\varepsilon_0\Delta x\frac{dE}{dt}$$

Taking Δx to be infinitesimally small:

$$-dB = \mu_0\varepsilon_0 dx\frac{dE}{dt}$$

$$\frac{dB}{dx} = -\mu_0\varepsilon_0\frac{dE}{dt}$$

Replacing the full derivatives with partial derivatives:

$$\boxed{\frac{\partial B}{\partial x} = -\mu_0\varepsilon_0\frac{\partial E}{\partial t}}$$

REFLECT

This final equation is useful when relating the partial derivatives of the magnetic and electric fields.

22.51

SET UP

Starting with $\frac{\partial B}{\partial x} = -\mu_0\varepsilon_0\frac{\partial E}{\partial t}$ we are asked to show that both the electric and magnetic fields obey the wave formula with a speed equal to the speed of light, $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$. First, by taking the partial time derivative of both sides, we can arrive at the wave equation, $\frac{\partial^2 E}{\partial x^2} = \mu_0\varepsilon_0\frac{\partial^2 E}{\partial t^2}$, after rearranging and invoking that $\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$. If we take the partial derivative of both sides of the starting equation with respect to x, we will arrive at the wave equation for the magnetic field, $\frac{\partial^2 B}{\partial x^2} = \mu_0\varepsilon_0\frac{\partial^2 B}{\partial t^2}$, after rearranging and invoking that $\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$.

SOLVE

Part a)

$$\frac{\partial}{\partial t}\left[\frac{\partial B}{\partial x}\right] = \frac{\partial}{\partial t}\left[-\mu_0\varepsilon_0\frac{\partial E}{\partial t}\right]$$

$$\frac{\partial}{\partial x}\left[\frac{\partial B}{\partial t}\right] = -\mu_0\varepsilon_0\frac{\partial^2 E}{\partial t^2}$$

$$\frac{\partial}{\partial x}\left[-\frac{\partial E}{\partial x}\right] = -\mu_0\varepsilon_0\frac{\partial^2 E}{\partial t^2}$$

$$\frac{\partial^2 E}{\partial x^2} = \mu_0\varepsilon_0\frac{\partial^2 E}{\partial t^2}$$

This is the wave equation where $\frac{1}{c^2} = \mu_0\varepsilon_0$, or $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$.

Part b)

$$\frac{\partial}{\partial x}\left[\frac{\partial B}{\partial x}\right]=\frac{\partial}{\partial x}\left[-\mu_0\varepsilon_0\frac{\partial E}{\partial t}\right]$$

$$\frac{\partial^2 B}{\partial x^2}=-\mu_0\varepsilon_0\frac{\partial}{\partial x}\left[\frac{\partial E}{\partial t}\right]$$

$$\frac{\partial^2 B}{\partial x^2}=-\mu_0\varepsilon_0\frac{\partial}{\partial t}\left[\frac{\partial E}{\partial x}\right]$$

$$\frac{\partial^2 B}{\partial x^2}=-\mu_0\varepsilon_0\frac{\partial}{\partial t}\left[-\frac{\partial B}{\partial t}\right]$$

$$\frac{\partial^2 B}{\partial x^2}=\mu_0\varepsilon_0\frac{\partial^2 B}{\partial t^2}$$

This is the wave equation where $\frac{1}{c^2}=\mu_0\varepsilon_0$, or $c=\frac{1}{\sqrt{\mu_0\varepsilon_0}}$.

REFLECT

We can switch the order of the partial derivatives as long as the function, its first derivatives, and its second derivatives are continuous at any given point.

22.52

SET UP

The wave equations for plane electromagnetic waves are $\frac{\partial^2 E}{\partial x^2}=\mu_0\varepsilon_0\frac{\partial^2 E}{dt^2}$ and $\frac{\partial^2 B}{\partial x^2}=\mu_0\varepsilon_0\frac{\partial^2 B}{dt^2}$. We can perform the requisite partial derivatives to show that $E(x,t)=E_0\sin(kx-\omega t)$ and $B(x,t)=B_0\sin(kx-\omega t)$, respectively, satisfy these partial differential equations. Remember that $c=\frac{1}{\sqrt{\mu_0\varepsilon_0}}$.

SOLVE

Derivatives of the electric field:

$$\frac{\partial^2}{\partial x^2}[E_0\sin(kx-\omega t)]=E_0\frac{\partial}{\partial x}[k\cos(kx-\omega t)]=-k^2E_0\sin(kx-\omega t)$$

$$\frac{\partial^2}{\partial t^2}[E_0\sin(kx-\omega t)]=E_0\frac{\partial}{\partial t}[-\omega\cos(kx-\omega t)]=-\omega^2E_0\sin(kx-\omega t)$$

Electric field partial differential equation:

$$\frac{\partial^2 E}{\partial x^2}\stackrel{?}{=}\mu_0\varepsilon_0\frac{\partial^2 E}{dt^2}$$

$$-k^2E_0\sin(kx-\omega t)\stackrel{?}{=}\mu_0\varepsilon_0(-\omega^2\mathrm{E}_0\sin(kx-\omega t))$$

$$k^2\stackrel{?}{=}\mu_0\varepsilon_0\omega^2$$

$$\frac{k^2}{\omega^2} \stackrel{?}{=} \mu_0\varepsilon_0$$

$$\frac{1}{c^2} = \mu_0\varepsilon_0$$

Derivatives of the magnetic field:

$$\frac{\partial^2}{\partial x^2}[B_0 \sin(kx - \omega t)] = B_0\frac{\partial}{\partial x}[k \cos(kx - \omega t)] = -k^2 B_0 \sin(kx - \omega t)$$

$$\frac{\partial^2}{\partial t^2}[B_0 \sin(kx - \omega t)] = B_0\frac{\partial}{\partial t}[-\omega \cos(kx - \omega t)] = -\omega^2 B_0 \sin(kx - \omega t)$$

Electric field partial differential equation:

$$\frac{\partial^2 B}{\partial x^2} \stackrel{?}{=} \mu_0\varepsilon_0\frac{\partial^2 B}{dt^2}$$

$$-k^2 B_0 \sin(kx - \omega t) \stackrel{?}{=} \mu_0\varepsilon_0(-\omega^2 \mathrm{B}_0 \sin(kx - \omega t))$$

$$k^2 \stackrel{?}{=} \mu_0\varepsilon_0\omega^2$$

$$\frac{k^2}{\omega^2} \stackrel{?}{=} \mu_0\varepsilon_0$$

$$\frac{1}{c^2} = \mu_0\varepsilon_0 \quad ✓$$

REFLECT

Be careful when keeping track of the negative signs when differentiating with respect to time.

22.53

SET UP

We are asked to determine whether the function $E(x,t) = E_0[\sin(kx - \omega t) + \cos(kx - \omega t)]$ satisfies the one-dimensional wave equation, $\frac{\partial^2 E}{\partial x^2} = \mu_0\varepsilon_0\frac{\partial^2 E}{\partial t^2}$, where $\omega = kc$ and $\mu_0\varepsilon_0 = \frac{1}{c^2}$. In order to take the partial derivatives with respect to position or time, we need to hold the time or position fixed, respectively, and treat it as a constant.

SOLVE

Taking derivatives:

$$E(x,t) = E_0[\sin(kx - \omega t) + \cos(kx - \omega t)]$$

$$\frac{\partial E}{\partial x} = \frac{\partial}{\partial x}[E_0[\sin(kx - \omega t) + \cos(kx - \omega t)]] = E_0[k\cos(kx - \omega t) - k\sin(kx - \omega t)]$$

$$\frac{\partial^2 E}{\partial x^2} = \frac{\partial}{\partial x}[E_0[k\cos(kx - \omega t) - k\sin(kx - \omega t)]] = E_0[-k^2\sin(kx - \omega t) - k^2\cos(kx - \omega t)]$$
$$= -k^2E_0[\sin(kx - \omega t) + \cos(kx - \omega t)]$$

$$\frac{\partial E}{\partial t} = \frac{\partial}{\partial t}[E_0[\sin(kx - \omega t) + \cos(kx - \omega t)]] = E_0[-\omega\cos(kx - \omega t) + \omega\sin(kx - \omega t)]$$

$$\frac{\partial^2 E}{\partial t^2} = \frac{\partial}{\partial t}[E_0[-\omega\cos(kx - \omega t) + \omega\sin(kx - \omega t)]] = E_0[-\omega^2\sin(kx - \omega t) - \omega^2\cos(kx - \omega t)]$$
$$= -\omega^2E_0[\sin(kx - \omega t) + \cos(kx - \omega t)]$$

Wave equation:

$$\frac{\partial^2 E}{\partial x^2} = \mu_0\varepsilon_0\frac{\partial^2 E}{\partial t^2}$$

$$-k^2E_0[\sin(kx - \omega t) + \cos(kx - \omega t)] \stackrel{?}{=} \mu_0\varepsilon_0[-\omega^2E_0[\sin(kx - \omega t) + \cos(kx - \omega t)]]$$

$$k^2 \stackrel{?}{=} \mu_0\varepsilon_0\omega^2$$

$$k^2 \stackrel{?}{=} \left(\frac{1}{c^2}\right)(kc)^2 = k^2$$

REFLECT

The functions $E(x,t) = E_0\sin(kx - \omega t)$, $E(x,t) = E_0\cos(kx - \omega t)$, or their linear combination, $E(x,t) = E_0[\sin(kx - \omega t) + \cos(kx - \omega t)]$, are all solutions to the one-dimensional wave equation.

22.54

SET UP

We are asked to determine whether the function $B(x,t) = B_0\sin(kx)\cos(\omega t)$ satisfies the one-dimensional wave equation, $\frac{\partial^2 B}{\partial x^2} = \mu_0\varepsilon_0\frac{\partial^2 B}{\partial t^2}$, where $\omega = kc$ and $\mu_0\varepsilon_0 = \frac{1}{c^2}$. In order to take the partial derivatives with respect to position or time, we need to hold the time or position fixed, respectively, and treat it as a constant.

SOLVE

Derivatives of the magnetic field:

$$\frac{\partial^2}{\partial x^2}[B_0\sin(kx)\cos(\omega t)] = \frac{\partial}{\partial x}[kB_0\cos(kx)\cos(\omega t)] = -k^2B_0\sin(kx)\cos(\omega t)$$

$$\frac{\partial^2}{\partial t^2}[B_0\sin(kx)\cos(\omega t)] = \frac{\partial}{\partial t}[-\omega B_0\sin(kx)\sin(\omega t)] = -\omega^2B_0\sin(kx)\cos(\omega t)$$

Wave equation:

$$\frac{\partial^2 B}{\partial x^2} \stackrel{?}{=} \mu_0\varepsilon_0\frac{\partial^2 B}{\partial t^2}$$

$$-k^2B_0\sin(kx)\cos(\omega t) \stackrel{?}{=} \mu_0\varepsilon_0(-\omega^2 B_0\sin(kx)\cos(\omega t))$$

$$k^2 \stackrel{?}{=} \mu_0\varepsilon_0\omega^2$$

$$k^2 \stackrel{?}{=} \left(\frac{1}{c^2}\right)(kc)^2 = k^2$$

REFLECT

The function $B(x,t) = B_0\sin(kx)\cos(\omega t)$ describes a standing wave.

22.55

SET UP

We are asked to calculate the wavelengths associated with photons with energy $E_{\text{low}} = 3.00\text{ eV}$ and $E_{\text{high}} = 20.0\text{ eV}$ and determine the part of the electromagnetic spectrum in which the light lies. The wavelength λ is equal to $\lambda = \frac{hc}{E}$. Recalling that the visible spectrum spans from around 380–750 nm will be useful when determining the region of the electromagnetic spectrum.

SOLVE

Part a)

$$\lambda_{\text{low energy}} = \frac{hc}{E_{\text{low}}} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})\left(3.00\times10^{8}\frac{\text{m}}{\text{s}}\right)}{\left(3.00\text{ eV}\times\frac{1.602\times10^{-19}\text{ J}}{1\text{ eV}}\right)} = \boxed{4.14\times10^{-7}\text{ m} = 414\text{ nm}}$$

$$\lambda_{\text{high energy}} = \frac{hc}{E_{\text{high}}} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})\left(3.00\times10^{8}\frac{\text{m}}{\text{s}}\right)}{\left(20.0\text{ eV}\times\frac{1.602\times10^{-19}\text{ J}}{1\text{ eV}}\right)} = \boxed{6.20\times10^{-8}\text{ m} = 62.0\text{ nm}}$$

Part b) These wavelengths extend from violet visible light into the ultraviolet region.

REFLECT

It makes sense that ultraviolet light, which is known to cause skin cancer, would cause damage to DNA.

22.56

SET UP

A dental x-ray delivers about $E_{\text{total}} = 4.0\times10^{-6}\text{ J}$ of energy during each scan. The wavelength of the x-rays is $\lambda = 0.025\times10^{-9}\text{ m}$. The energy of one photon of this wavelength is given by

$E = \frac{hc}{\lambda}$. In order to find the number of photons absorbed during each scan, we need to divide the total energy absorbed by the energy of a single photon.

SOLVE

Part a)

$$E_{1\text{ photon}} = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34}\text{ J}\cdot\text{s})\left(3.00 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{0.025 \times 10^{-9}\text{ m}}$$

$$= 7.95 \times 10^{-15}\text{ J} \times \frac{1\text{ eV}}{1.602 \times 10^{-19}\text{ J}} = \boxed{5.0 \times 10^{4}\text{ eV}}$$

Part b)

$$N = \frac{E_{\text{total}}}{E_{1\text{ photon}}} = \frac{4.0 \times 10^{-6}\text{ J}}{7.95 \times 10^{-15}\text{ J}} = \boxed{5.0 \times 10^{8}\text{ photons}}$$

REFLECT

The gray (Gy) is the SI unit describing the amount of radiation required for 1 kg of matter to absorb 1 J of energy.

22.57

SET UP

A laser is made up of a cylindrical beam of diameter $d = 0.750 \times 10^{-2}$ m. The energy is pulsed, lasting $\Delta t = 1.50 \times 10^{-9}$ s. Each pulse contains an energy of $E = 2.00$ J. The length (in meters) of the pulse can be found by multiplying the pulse duration by the speed of light c. Using this length, we can calculate the volume of the cylindrical pulse and then the energy per unit volume for each pulse.

SOLVE

Part a)

$$c = \frac{\Delta x}{\Delta t}$$

$$\Delta x = c\Delta t = \left(3.00 \times 10^{8}\frac{\text{m}}{\text{s}}\right)(1.50 \times 10^{-9}\text{ s}) = \boxed{0.45\text{ m}}$$

Part b)

$$\frac{E}{V} = \frac{E}{\pi R^2(\Delta x)} = \frac{E}{\pi\left(\frac{d}{2}\right)^2(\Delta x)} = \frac{4E}{\pi d^2(\Delta x)} = \frac{4(2.00\text{ J})}{\pi(0.750 \times 10^{-2}\text{ m})^2(0.45\text{ m})} = \boxed{1.01 \times 10^{5}\frac{\text{J}}{\text{m}^3}}$$

REFLECT

As a rule of thumb, light travels a distance of 1 m in 3.3 ns.

22.58

SET UP

The energy of a photon of blue-green light ($\lambda = 525 \times 10^{-9}$ m) is equal to $E = \frac{hc}{\lambda}$. The wave number of the photon is $k = \frac{2\pi}{\lambda}$.

SOLVE

Part a)

$$E = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34}\ \mathrm{J \cdot s})\left(3.00 \times 10^{8}\frac{\mathrm{m}}{\mathrm{s}}\right)}{525 \times 10^{-9}\ \mathrm{m}} = \boxed{3.79 \times 10^{-19}\ \mathrm{J}}$$

$$3.79 \times 10^{-19}\ \mathrm{J} \times \frac{1\ \mathrm{eV}}{1.602 \times 10^{-19}\ \mathrm{J}} = \boxed{2.36\ \mathrm{eV}}$$

Part b)

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{525 \times 10^{-9}\ \mathrm{m}} = \boxed{1.20 \times 10^{7}\frac{\mathrm{rad}}{\mathrm{m}}}$$

REFLECT

The light has a frequency of 5.71×10^{14} Hz.

22.59

SET UP

The quantity $\varepsilon_0 \frac{d\Phi_E}{dt}$ is known as the displacement current. We need to show that it does, in fact, have SI units of amperes, the same units as current. Looking at the units of each term, the SI units of ε_0 are $\frac{\mathrm{C}^2}{\mathrm{N \cdot m^2}}$, and the SI units of $\frac{d\Phi_E}{dt}$ are related to the SI units for electric field $\left(\frac{\mathrm{N}}{\mathrm{C}}\right)$, area ($\mathrm{m}^2$), and time (s). We need to combine these to ensure they equal amperes.

SOLVE

$$\left[\varepsilon_0 \frac{d\Phi_E}{dt}\right] = \left[\varepsilon_0 \frac{d[\vec{E} \cdot \vec{A}]}{dt}\right] = \left(\frac{\mathrm{C}^2}{\mathrm{N \cdot m^2}}\right)\frac{\left(\frac{\mathrm{N}}{\mathrm{C}}\right)(\mathrm{m}^2)}{\mathrm{s}} = \frac{\mathrm{C}}{\mathrm{s}} = \boxed{\mathrm{A}}$$

REFLECT

We could have also represented the SI units of electric field as $\left(\frac{\mathrm{V}}{\mathrm{m}}\right)$ and defined ε_0 in terms of $\frac{\mathrm{C}}{\mathrm{V \cdot m}}$.

Chapter 23
Wave Properties of Light

Conceptual Questions

23.1 Christiaan Huygens suggested that every point along the front of a wave be treated as many separate sources of tiny "wavelets" that move at the speed of the wave. This is important to see how light moves from one medium to a different medium and allows you to predict the bending that occurs in Snell's law.

23.2 The last color should be reddish due to the dispersive qualities of light. The index of refraction for red light is the smallest, and it will be bent through the largest angle according to Snell's law.

23.3 The light coming from the Sun is refracted as it passes through Earth's atmosphere. Because of the wavelength dependence of the index of refraction, red light bends less than orange than yellow than green than blue than violet. In addition, bluer light is scattered more than red light as it passes through the atmosphere. Because of these effects, more red/orange colored light can pass into the shadow of Earth, making the Moon appear red.

23.4 No, the critical angle only depends on the index of refraction of the material that it enters and that it reflects off.

23.5 The swimming pool seems shallower. If you follow a light ray that bounces off the bottom of the swimming pool and ends up entering your eyes, the light exits the water at an angle from the normal larger than the angle at which it arrived at the water−air boundary. This is because the index of refraction of water is greater than that of air. Your brain assumes that light travels in a straight line, so you interpret the origin of the ray on the floor of the swimming pool to be along the line of the light ray as it enters your eyes. The distance you measure to the origin of the light ray is fixed by your stereo vision, so the swimming pool appears to be shallower than it really is.

23.6 Fiber optics used for communication and laparoscopic surgery are two common uses of total internal reflection.

23.7 When light moves from a medium of a larger index of refraction toward a medium with a smaller index of refraction, the angle of refraction will *increase*. If you make the incident angle larger and larger, the refracted angle will ultimately approach 90 degrees. After the light moves past this critical value, it is reflected rather than refracted. In mathematical terms, $\arcsin(x)$ is undefined if $x > 1$. Since $x = \frac{n_2}{n_1}$, this means $n_2 < n_1$.

23.8 Polarizing filters are made of materials that contain long-chain molecules that absorb the electric fields of light waves that impinge upon them. There is a decrease

in the intensity of the waves. The intensity is proportional to the square of the amplitude, so depending on the alignment of the polarizing filter and the plane of the polarized light, there will be less light leaving than entering the filter according to $I = E_0^2\cos^2(\theta) = I_0\cos^2(\theta)$.

23.9 When light enters a material with a negative index of refraction, the light refracts "back" away from the normal to the surface as seen in the picture below:

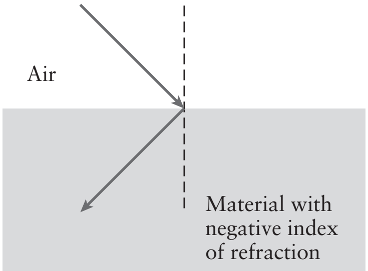


Figure 23-1 Problem 9

Get Help: P'Cast 23.1 – Seeing Under Water

23.10 Polarized sunglasses use polarizing filters to preferentially absorb light that harms your eyes. Since most glare is created by light that is reflected off of horizontal surfaces (for example, a lake, a ski slope), it is advantageous to block out the light that is horizontally polarized. As such, the lenses of polarized sunglasses only allow vertically polarized light to enter. Naturally, the intensity of the light that passes through such lenses is decreased in intensity.

23.11 Soap bubbles reflecting different colors, thin film coatings on photographic lenses ("nonreflective coatings"), and oil floating in a puddle of water are all examples of thin film interference.

23.12 These colors come from thin film interference. Part of each light wave striking the gasoline film reflects from the top surface, and part reflects from the bottom surface. These two reflected waves interfere with one another in a wavelength-dependent manner and make the layer appear brightly colored.

23.13 Yes, diffraction occurs for all wavelike phenomena. Sound waves are diffracted through an open window and water waves are diffracted around an obstacle in the water, for example.

23.14 Since the laboratory demonstration of diffraction involved passing light through a rather small slit (on the order of the wavelength of the light itself), it took scientists a while to perfect techniques that allowed them to make such small openings. For example, when Newton performed the single slit experiment, he observed no interference effects because the opening was so much larger than the wavelength of the light that he was using. About 100 years later, Thomas Young was able to use slits that were much narrower (closer to the size of the wavelength of light that he was using) and the diffraction effects were much more easily observed.

23.15 Sunlight includes all of the colors of the rainbow and one of the delightful aspects of a diamond is its strong dispersion. The diamond bends violet light much more than red light so that sunlight is separated into its different colors as it passes through the diamond's surface.

23.16 Part a) At Brewster's angle, the parallel component is completely refracted. Therefore, no light is reflected in this case.

Part b) Some of the incident light is reflected and some is refracted. Both the reflected and the refracted beams of light will be polarized perpendicularly to the plane of incidence.

Multiple-Choice Questions

23.17 **D** (electromagnetic, sound, and water waves). Refraction and interference are general phenomena for all types of waves.

23.18 **B** (it bends with an angle smaller than θ with respect to the normal to the boundary surface). Glass has a higher index of refraction than air, so the refracted ray will bend toward the normal in the glass.

23.19 **B** (refraction). Refraction refers to the bending of a light ray when it enters a medium with a different index of refraction.

23.20 **A** (red). The index of refraction for red light in crown glass is smaller than the index of refraction for blue light, which means the speed of red light is larger than the speed of blue light in crown glass.

23.21 B $\left(\frac{1}{2}I_0\right)$. The intensity of unpolarized light drops by a factor of two when it passes through a linear polarizer. The second polarizer does not affect the intensity of the light because the two polarizers are aligned (*i.e.*, $\theta = 0$).

Get Help: Interactive Example – Polarization I
Interactive Example – Polarization II

23.22 D $\left(\frac{1}{4}I_0\right)$.

$$I_1 = \frac{1}{2}I_0$$

$$I_2 = I_1\cos^2(45°) = \left(\frac{1}{2}I_0\right)\cos^2(45°) = \left(\frac{1}{2}I_0\right)\left(\frac{1}{2}\right) = \frac{1}{4}I_0$$

23.23 **A** (shrinks with all the fringes getting narrower). The fringe width is directly proportional to the wavelength of the light.

23.24 **A** (shrinks with all the fringes getting narrower). The fringe width is inversely proportional to the slit width.

23.25 **A** ($w_a > w_b$). The width of the slit is inversely proportional to the fringe width.

(a) (b)

Figure 23-2 Problem 25

Estimation/Numerical Questions

23.26 Part a)

$$v_{\text{air}} = \frac{c}{n_{\text{air}}} = \frac{\left(2.998 \times 10^8 \frac{\text{m}}{\text{s}}\right)}{1.0003} = 2.989 \times 10^8 \frac{\text{m}}{\text{s}}$$

Part b)

$$v_{\text{water}} = \frac{c}{n_{\text{water}}} = \frac{\left(2.998 \times 10^8 \frac{\text{m}}{\text{s}}\right)}{1.33} = 2.25 \times 10^8 \frac{\text{m}}{\text{s}}$$

Part c)

$$v_{\text{glass}} = \frac{c}{n_{\text{glass}}} = \frac{\left(2.998 \times 10^8 \frac{\text{m}}{\text{s}}\right)}{1.55} = 1.93 \times 10^8 \frac{\text{m}}{\text{s}}$$

23.27 Common indices of refraction range from ~1.0 for air to 2.4 for diamond.

23.28 Part a) The distance from the Moon to Earth is 3.84×10^8 m, which means it takes about 1 s for light to travel between the Moon and Earth.

Part b) The distance from the Sun to Earth is 1.5×10^{11} m, which means it takes about 500 s, which is a little over 8 min, for light to travel between the Sun and Earth.

Part c) The distance from Alpha Centuri to Earth is 4.2 light-years, which means it takes about 4.2 years for light to travel between the Alpha Centuri and Earth.

23.29 We can use $c = \dfrac{\Delta x}{\Delta t}$ to calculate the distance light travels.

Part a)

$$\Delta x = c(\Delta t) = \left(3.0 \times 10^8 \frac{\text{m}}{\text{s}}\right)(1 \text{ s}) = 3.0 \times 10^8 \text{ m}$$

Part b)

$$\Delta x = c(\Delta t) = \left(3.0 \times 10^8 \frac{\text{m}}{\text{s}}\right)(60\ \text{s}) = 2 \times 10^{10}\ \text{m}$$

Part c)

$$\Delta x = c(\Delta t) = \left(3.0 \times 10^8 \frac{\text{m}}{\text{s}}\right)\left(1\ \text{yr} \times \frac{365.25\ \text{days}}{1\ \text{yr}} \times \frac{24\ \text{hr}}{1\ \text{day}} \times \frac{3600\ \text{s}}{1\ \text{hr}}\right) = 9.5 \times 10^{15}\ \text{m}$$

23.30 In air, the visible spectrum ranges from around 400–700 nm. Using the index of refraction of water (n = 1.33), this range becomes 300–530 nm.

23.31

x (cm)	y (cm)
−4.00	−2.00
−3.00	−1.52
−2.00	−1.02
−1.00	−0.514
0.00	0.00
1.00	0.296
2.00	0.595
3.00	0.901
4.00	1.20

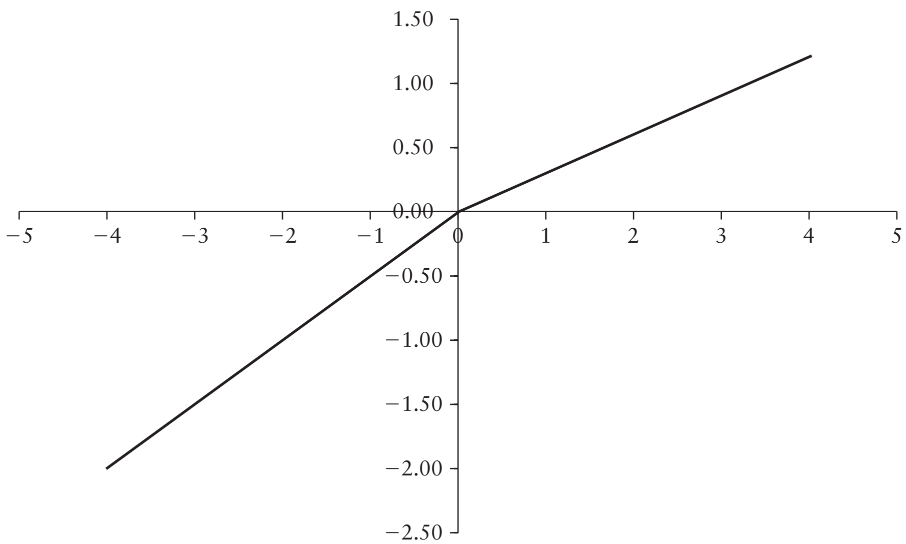


Figure 23-3 Problem 31

Angle of incidence:

$$\theta_1 = \arctan\left(\frac{-2.00}{-4}\right) = 27°$$

Angle of refraction:

$$\theta_2 = \arctan\left(\frac{1.20}{4}\right) = 17°$$

Ratio of indices of refraction:

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

$$\frac{n_2}{n_1} = \frac{\sin(\theta_1)}{\sin(\theta_2)} = \frac{\sin(27°)}{\sin(17°)} = 1.56$$

Problems

23.32

SET UP

The speed of light in a new plastic is $v = 1.97 \times 10^8 \frac{\text{m}}{\text{s}}$. The index of refraction of a material is given by $n = \frac{c}{v}$.

SOLVE

$$n = \frac{c}{v} = \frac{\left(2.998 \times 10^8 \frac{\text{m}}{\text{s}}\right)}{\left(1.97 \times 10^8 \frac{\text{m}}{\text{s}}\right)} = \boxed{1.52}$$

REFLECT

An index of refraction around 1.5 is reasonable for a plastic.

23.33

SET UP

The index of refraction for a vacuum is $n_{\text{vac}} = 1.00000$. The index of refraction for air is $n_{\text{air}} = 1.00029$. We can use the speed of light in each medium, $n = \frac{c}{v}$, to determine the ratio of time required for light to travel through $x = 1000$ m of air to the time required for light to travel through $x = 1000$ m of vacuum.

SOLVE

$$\frac{t_{\text{air}}}{t_{\text{vac}}} = \frac{\left(\frac{x}{v_{\text{air}}}\right)}{\left(\frac{x}{v_{\text{vac}}}\right)} = \frac{v_{\text{vac}}}{v_{\text{air}}} = \frac{\left(\frac{c}{n_{\text{vac}}}\right)}{\left(\frac{c}{n_{\text{air}}}\right)} = \frac{n_{\text{air}}}{n_{\text{vac}}} = \frac{1.00029}{1.00000} = \boxed{1.00029}$$

REFLECT

It should take more time for light to travel through air than through vacuum.

23.34

SET UP

We are given a list of indices of refraction for various common media. The speed of light in a medium is related to its index of refraction through $n = \frac{c}{v}$.

SOLVE

$$\text{A) } v = \frac{c}{n} = \frac{\left(2.998 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{1.31} = \boxed{2.29 \times 10^{8}\frac{\text{m}}{\text{s}}}$$

$$\text{B) } v = \frac{c}{n} = \frac{\left(2.998 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{1.36} = \boxed{2.20 \times 10^{8}\frac{\text{m}}{\text{s}}}$$

$$\text{C) } v = \frac{c}{n} = \frac{\left(2.998 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{1.49} = \boxed{2.01 \times 10^{8}\frac{\text{m}}{\text{s}}}$$

$$\text{D) } v = \frac{c}{n} = \frac{\left(2.998 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{1.54} = \boxed{1.95 \times 10^{8}\frac{\text{m}}{\text{s}}}$$

$$\text{E) } v = \frac{c}{n} = \frac{\left(2.998 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{1.77} = \boxed{1.69 \times 10^{8}\frac{\text{m}}{\text{s}}}$$

$$\text{F) } v = \frac{c}{n} = \frac{\left(2.998 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{2.42} = \boxed{1.24 \times 10^{8}\frac{\text{m}}{\text{s}}}$$

$$\text{G) } v = \frac{c}{n} = \frac{\left(2.998 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{1.47} = \boxed{2.04 \times 10^{8}\frac{\text{m}}{\text{s}}}$$

$$\text{H) } v = \frac{c}{n} = \frac{\left(2.998 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{1.65} = \boxed{1.82 \times 10^{8}\frac{\text{m}}{\text{s}}}$$

REFLECT

The speed of light in a medium will always be slower than the speed of light in a vacuum.

23.35

SET UP

The speed of light in methylene iodide is $v_m = 1.72 \times 10^8 \frac{m}{s}$. The index of refraction of water is $n_w = 1.33$, which means the speed of light in water is $v_w = \frac{c}{n_w}$. We can use these data along with the definition of speed, $v = \frac{\Delta x}{\Delta t}$, to calculate the distance of methylene iodide that light must travel through such that it takes the same amount of time as light traveling through 1000×10^3 m of water.

SOLVE

$$v = \frac{\Delta x}{\Delta t}$$

$$\Delta t = \frac{\Delta x}{v}$$

$$\frac{(\Delta x)_m}{v_m} = \frac{(\Delta x)_w}{v_w}$$

$$(\Delta x)_m = \frac{(\Delta x)_w v_m}{v_w} = \frac{(\Delta x)_w v_m}{\left(\frac{c}{n_w}\right)} = \frac{(\Delta x)_w v_m n_w}{c}$$

$$= \frac{(1000 \times 10^3 \text{ m})\left(1.72 \times 10^8 \frac{\text{m}}{\text{s}}\right)(1.33)}{\left(3.0 \times 10^8 \frac{\text{m}}{\text{s}}\right)} = \boxed{7.63 \times 10^5 \text{ m} = 763 \text{ km}}$$

REFLECT

The index of refraction of methylene iodide is $n_m = \frac{c}{v_m} = \frac{\left(3.0 \times 10^8 \frac{\text{m}}{\text{s}}\right)}{\left(1.72 \times 10^8 \frac{\text{m}}{\text{s}}\right)} = 1.74$, which means light will travel more slowly in methylene iodide compared to water. Therefore, we expect the distance of methylene iodide should be smaller than the distance of water.

Get Help: Interactive Example – Refraction
P'Cast 23.1 – Seeing Under Water

23.36

SET UP

Light travels from air ($n_{air} = 1.00029$) toward water ($n_{water} = 1.33$) at an angle of 27 degrees with respect to the normal. We can use Snell's law to calculate the angle of refraction after the light enters the water.

SOLVE

$$n_{\text{air}} \sin(\theta_{\text{air}}) = n_{\text{water}} \sin(\theta_{\text{water}})$$

$$\sin(\theta_{\text{water}}) = \left(\frac{n_{\text{air}}}{n_{\text{water}}}\right)\sin(\theta_{\text{air}})$$

$$\theta_{\text{water}} = \arcsin\left[\left(\frac{n_{\text{air}}}{n_{\text{water}}}\right)\sin(\theta_{\text{air}})\right] = \arcsin\left[\left(\frac{1.00029}{1.33}\right)\sin(27°)\right] = \boxed{20°}$$

REFLECT

The index of refraction of water is larger than the index of refraction for air, so we expect the ray to bend toward the normal resulting in a smaller angle.

23.37

SET UP

We are shown four different scenarios in which light is traveling from a medium of lower index of refraction into a medium of higher index of refraction. We can use Snell's law to determine the missing angle in each case.

SOLVE

Part a)

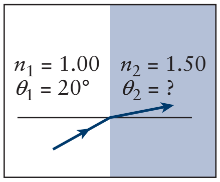


Figure 23-4 Problem 37

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

$$\sin(\theta_2) = \left(\frac{n_1}{n_2}\right)\sin(\theta_1)$$

$$\theta_2 = \arcsin\left[\left(\frac{n_1}{n_2}\right)\sin(\theta_1)\right] = \arcsin\left[\left(\frac{1.00}{1.50}\right)\sin(20°)\right] = \boxed{13.2°}$$

Part b)

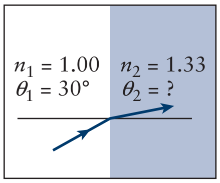


Figure 23-5 Problem 37

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

$$\sin(\theta_2) = \left(\frac{n_1}{n_2}\right)\sin(\theta_1)$$

$$\theta_2 = \arcsin\left[\left(\frac{n_1}{n_2}\right)\sin(\theta_1)\right] = \arcsin\left[\left(\frac{1.00}{1.33}\right)\sin(30°)\right] = \boxed{22.1°}$$

Part c)

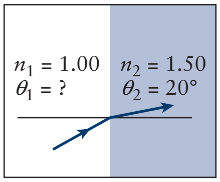


Figure 23-6 Problem 37

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

$$\sin(\theta_1) = \left(\frac{n_2}{n_1}\right)\sin(\theta_2)$$

$$\theta_1 = \arcsin\left[\left(\frac{n_2}{n_1}\right)\sin(\theta_2)\right] = \arcsin\left[\left(\frac{1.50}{1.00}\right)\sin(20°)\right] = \boxed{30.9°}$$

Part d)

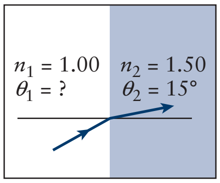


Figure 23-7 Problem 37

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

$$\sin(\theta_1) = \left(\frac{n_2}{n_1}\right)\sin(\theta_2)$$

$$\theta_1 = \arcsin\left[\left(\frac{n_2}{n_1}\right)\sin(\theta_2)\right] = \arcsin\left[\left(\frac{1.50}{1.00}\right)\sin(15°)\right] = \boxed{22.8°}$$

REFLECT

The refracted angle in the second medium should be smaller than the incident angle in all four cases because the second medium has a larger index of refraction in each case.

23.38

SET UP

We are shown four different scenarios in which light is traveling from one medium into a second medium. We can use Snell's law to determine the missing indices of refraction for each case.

SOLVE

Part a)

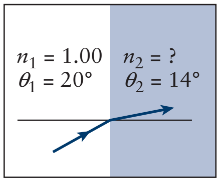


Figure 23-8 Problem 38

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

$$n_2 = n_1\left(\frac{\sin(\theta_1)}{\sin(\theta_2)}\right) = (1.00)\left(\frac{\sin(20°)}{\sin(14°)}\right) = \boxed{1.4}$$

Part b)

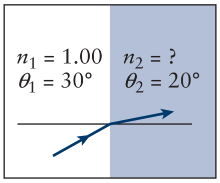


Figure 23-9 Problem 38

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

$$n_2 = n_1\left(\frac{\sin(\theta_1)}{\sin(\theta_2)}\right) = (1.00)\left(\frac{\sin(30°)}{\sin(20°)}\right) = \boxed{1.5}$$

Part c)

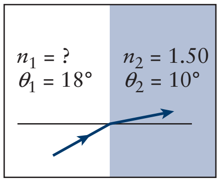


Figure 23-10 Problem 38

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

$$n_1 = n_2\left(\frac{\sin(\theta_2)}{\sin(\theta_1)}\right) = (1.50)\left(\frac{\sin(14°)}{\sin(18°)}\right) = \boxed{1.2}$$

Part d)

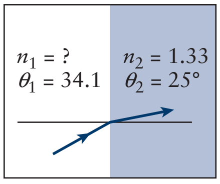


Figure 23-11 Problem 38

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

$$n_1 = n_2\left(\frac{\sin(\theta_2)}{\sin(\theta_1)}\right) = (1.33)\left(\frac{\sin(34°)}{\sin(48°)}\right) = \boxed{1.0}$$

REFLECT

The index of refraction of the second medium should be larger than the index of refraction in the first medium in all four cases because the angle of refraction is smaller than the angle of incidence.

23.39

SET UP

Total internal reflection occurs when the incident angle of light traveling from a medium with index of refraction n_1 into a medium with index of refraction n_2 is greater than the critical angle $\theta_c = \arcsin\left(\frac{n_2}{n_1}\right)$. It can only occur when $n_1 > n_2$. We can use the provided indices of refraction to calculate the critical angle in each case.

SOLVE

A) $\theta_c = \arcsin\left(\frac{1.00}{1.50}\right) = \boxed{41.8°}$

B) $\theta_c = \arcsin\left(\frac{1.00}{1.33}\right) = \boxed{48.8°}$

C) $\theta_c = \arcsin\left(\frac{1.33}{1.56}\right) = \boxed{58.5°}$

D) $\boxed{\text{no critical angle}}$

REFLECT

Because $-1 \leq \sin(\theta) \leq 1$, $\arcsin\left(\frac{1.55}{1.00}\right) = \arcsin(1.55)$ is undefined.

Get Help: Picture It – Reflection and Refraction
P'Cast 23.2 – Critical Angle for a Glass–Air Boundary
P'Cast 23.3 – Critical Angles

23.40

SET UP

Total internal reflection occurs when the incident angle of light traveling from a medium with index of refraction n_1 into a medium with index of refraction n_2 is greater than the critical angle $\sin(\theta_c) = \frac{n_2}{n_1}$. It can only occur when $n_1 > n_2$. We can use the provided critical angles to calculate the initial indices of refraction in each case.

SOLVE

$$\sin(\theta_c) = \frac{n_2}{n_1}$$

$$n_1 = \frac{n_2}{\sin(\theta_c)}$$

A) $n_1 = \dfrac{n_2}{\sin(\theta_c)} = \dfrac{1.00}{\sin(48.5°)} = \boxed{1.34}$

B) $n_1 = \dfrac{n_2}{\sin(\theta_c)} = \dfrac{1.00}{\sin(47.0°)} = \boxed{1.37}$

C) $n_1 = \dfrac{n_2}{\sin(\theta_c)} = \dfrac{1.00}{\sin(42.6°)} = \boxed{1.48}$

D) $n_1 = \dfrac{n_2}{\sin(\theta_c)} = \dfrac{1.00}{\sin(35.0°)} = \boxed{1.74}$

E) $n_1 = \dfrac{n_2}{\sin(\theta_c)} = \dfrac{1.00}{\sin(55.7°)} = \boxed{1.21}$

F) $n_1 = \dfrac{n_2}{\sin(\theta_c)} = \dfrac{1.00}{\sin(38.5°)} = \boxed{1.61}$

G) $n_1 = \dfrac{n_2}{\sin(\theta_c)} = \dfrac{1.00}{\sin(22.2°)} = \boxed{2.65}$

H) $n_1 = \dfrac{n_2}{\sin(\theta_c)} = \dfrac{1.00}{\sin(75.0°)} = \boxed{1.04}$

REFLECT

As the critical angle approaches 90 degrees, the index of refraction of the initial medium approaches 1.

23.41

SET UP

Light is traveling from sapphire ($n_1 = 1.77$) into air ($n_2 = 1.00$). Total internal reflection occurs when the incident angle of light traveling from a medium with index of refraction n_1 into a medium with index of refraction n_2 is greater than the critical angle $\theta_c = \arcsin\left(\frac{n_2}{n_1}\right)$. We can use the provided indices of refraction to calculate the critical angle in this case.

SOLVE

$$\theta_c = \arcsin\left(\frac{n_2}{n_1}\right) = \arcsin\left(\frac{1.00}{1.77}\right) = \boxed{34.4°}$$

REFLECT
This is a reasonable value for the critical angle.

23.42

SET UP
A fiber optic cable is made of acrylic ($n_{acrylic} = 1.50$). Light is incident on the left side of the cable at an angle θ_1 (see figure). The light will refract at the interface according to Snell's law and head off at an angle θ_2 toward the top wall of the cable. The angle at which this ray hits the top wall, relative to that normal, is $\theta_3 = (90° - \theta_2)$. As long as θ_3 is larger than the critical angle, the light will be totally internally reflected. We can look at the extremes of θ_1 and determine for which range of incoming angles this occurs.

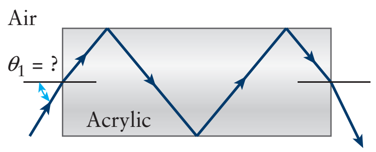


Figure 23-12 Problem 42

SOLVE
Ray entering the cable:

$$n_{air}\sin(\theta_1) = n_{acrylic}\sin(\theta_2)$$

$$\sin(\theta_2) = \frac{n_{air}}{n_{acrylic}}\sin(\theta_1)$$

Angle of ray bouncing off the side of the cable:

$$\theta_3 = 90° - \theta_2$$

Critical angle:

$$\theta_c = \arcsin\left(\frac{1}{1.50}\right) = 41.8°$$

As long as $\theta_3 > \theta_c$, the light will be totally internally reflected.

Limits of θ_1:
$\theta_1 = 90°$:

$$(1.00)\sin(90°) = (1.50)\sin(\theta_2)$$

$$\theta_2 = 41.8°$$

$$\theta_3 = 90° - 41.8° = 48.2°$$

$\theta_1 \to 0°$:

$$(1.00)\sin(\theta_1) = (1.50)\sin(\theta_2)$$

$$\theta_2 \to 0°$$

$$\theta_3 = 90° - \theta_2 \to 90°$$

Since θ_3 is always larger than θ_c for any value of θ_1, the $\boxed{\text{light will always be totally internally reflected}}$.

REFLECT

Fiber optic cables employ total internal reflection to ensure light signals propagate long distances with minimal losses.

23.43

SET UP

A scuba diver wants to look up from inside the water ($n_{\text{water}} = 1.33$) to see her friend standing on a very distant shore ($n_{\text{air}} = 1.00$). Since her friend is located so far away, the angle of the light ray is essentially 90 degrees. We can use Snell's law to calculate the angle the scuba diver must look.

SOLVE

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

$$(1.33)\sin(\theta_1) = (1.00)\sin(90°)$$

$$\theta_1 = \arcsin\left(\frac{1.00}{1.33}\right) = \boxed{48.8°}$$

REFLECT

This is equivalent to finding the critical angle for the system.

Get Help: Picture It – Reflection and Refraction
P'Cast 23.2 – Critical Angle for a Glass–Air Boundary
P'Cast 23.3 – Critical Angles

23.44

SET UP

A point source of light is a distance $L = 2.50$ m below the surface of a pool of water ($n = 1.33$). The volume of the "cone of light" formed in the water is related to the critical angle for total internal reflection between the water and the air. We can use this angle and trigonometry to calculate the radius r of the cone. The volume of a cone is $V = \frac{1}{3}\pi r^2 L$.

SOLVE

Critical angle:

$$\theta_c = \arcsin\left(\frac{n_2}{n_1}\right) = \arcsin\left(\frac{1.00}{1.33}\right) = 48.75°$$

Radius of cone:

$$\tan(\theta_c) = \frac{r}{L}$$

$$r = L\tan(\theta_c) = (2.50 \text{ m})\tan(48.75°) = 2.85 \text{ m}$$

Volume of cone:

$$V = \frac{1}{3}\pi r^2 L = \frac{1}{3}\pi(2.85 \text{ m})^2(2.50 \text{ m}) = \boxed{21.3 \text{ m}^3}$$

REFLECT

This question is similar to Got the Concept 23-3.

23.45

SET UP

A block of glass ($n_1 = 1.55$) is completely submerged in water ($n_2 = 1.33$). The critical angle for light traveling from the glass to the water is given by $\theta_c = \arcsin\left(\frac{n_2}{n_1}\right)$.

SOLVE

$$\theta_c = \arcsin\left(\frac{n_2}{n_1}\right) = \arcsin\left(\frac{1.33}{1.55}\right) = \boxed{59.1°}$$

REFLECT

Total internal reflection is a general phenomenon for light traveling from a medium of higher index of refraction toward a medium of lower index of refraction; one of the media does not need to be air.

23.46

SET UP

For a certain optical medium, the speed of light varies from $v_{\text{violet}} = 1.90 \times 10^8 \frac{\text{m}}{\text{s}}$ for violet light to $v_{\text{violet}} = 2.00 \times 10^8 \frac{\text{m}}{\text{s}}$ for red light. We can calculate the indices of refraction for violet light and red light using $n = \frac{c}{v}$. Once we have the indices of refraction, we can calculate the angle of refraction for both colors of light when white light makes an incident angle of 30 degrees and 60 degrees.

SOLVE

Part a)
Violet light:

$$n_{\text{violet}} = \frac{c}{v_{\text{violet}}} = \frac{\left(2.998 \times 10^8 \frac{\text{m}}{\text{s}}\right)}{\left(1.90 \times 10^8 \frac{\text{m}}{\text{s}}\right)} = \boxed{1.58}$$

Red light:

$$n_{\text{red}} = \frac{c}{v_{\text{red}}} = \frac{\left(2.998 \times 10^8 \frac{\text{m}}{\text{s}}\right)}{\left(2.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)} = \boxed{1.50}$$

Part b)
Violet light:

$$n_{\text{air}}\sin(\theta_{\text{air}}) = n_{\text{violet}}\sin(\theta_{\text{violet}})$$

$$\sin(\theta_{\text{violet}}) = \left(\frac{n_{\text{air}}}{n_{\text{violet}}}\right)\sin(\theta_{\text{air}})$$

$$\theta_{\text{violet}} = \arcsin\left[\left(\frac{n_{\text{air}}}{n_{\text{violet}}}\right)\sin(\theta_{\text{air}})\right] = \arcsin\left[\left(\frac{1.00}{1.58}\right)\sin(30°)\right] = \boxed{18.4°}$$

Red light:

$$n_{\text{air}}\sin(\theta_{\text{air}}) = n_{\text{red}}\sin(\theta_{\text{red}})$$

$$\sin(\theta_{\text{red}}) = \left(\frac{n_{\text{air}}}{n_{\text{red}}}\right)\sin(\theta_{\text{air}})$$

$$\theta_{\text{red}} = \arcsin\left[\left(\frac{n_{\text{air}}}{n_{\text{red}}}\right)\sin(\theta_{\text{air}})\right] = \arcsin\left[\left(\frac{1.00}{1.50}\right)\sin(30°)\right] = \boxed{19.5°}$$

Part c)
Violet light:

$$n_{\text{air}}\sin(\theta_{\text{air}}) = n_{\text{violet}}\sin(\theta_{\text{violet}})$$

$$\sin(\theta_{\text{violet}}) = \left(\frac{n_{\text{air}}}{n_{\text{violet}}}\right)\sin(\theta_{\text{air}})$$

$$\theta_{\text{violet}} = \arcsin\left[\left(\frac{n_{\text{air}}}{n_{\text{violet}}}\right)\sin(\theta_{\text{air}})\right] = \arcsin\left[\left(\frac{1.00}{1.58}\right)\sin(60°)\right] = \boxed{33.2°}$$

Red light:

$$n_{\text{air}}\sin(\theta_{\text{air}}) = n_{\text{red}}\sin(\theta_{\text{red}})$$

$$\sin(\theta_{\text{red}}) = \left(\frac{n_{\text{air}}}{n_{\text{red}}}\right)\sin(\theta_{\text{air}})$$

$$\theta_{\text{red}} = \arcsin\left[\left(\frac{n_{\text{air}}}{n_{\text{red}}}\right)\sin(\theta_{\text{air}})\right] = \arcsin\left[\left(\frac{1.00}{1.50}\right)\sin(60°)\right] = \boxed{35.3°}$$

REFLECT

We expect violet light to bend more than red light, so the refracted angles should be smaller for violet light than for red light.

23.47

SET UP

A beam of light shines at an angle of 45 degrees on an equilateral glass prism ($n_2 = 1.57$) that is 10 cm on each side; the incident ray hits the midpoint of the left face. To determine the angle at which the light emerges from the opposite face, we need to apply both Snell's law and geometry. Recall that we measure the incoming and refracted angles relative to the normal

of the interface. In part (b), we need to consider the effects of dispersion on the light ray. We will assume the light in the ray spans the visible spectrum from red ($n_{\text{red}} = 1.568$) to violet ($n_{\text{violet}} = 1.572$). Accordingly, we can apply Snell's law and geometry to determine the distance between the exit points of the red and violet rays.

SOLVE

Part a)

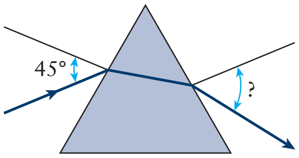


Figure 23-13 Problem 47

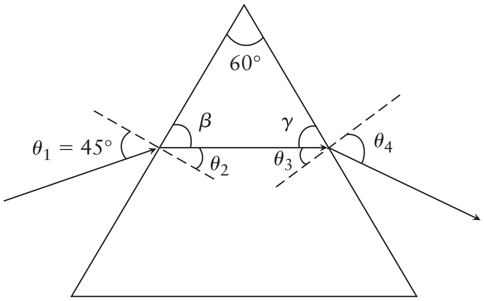


Figure 23-14 Problem 47

Refraction at air–glass interface:

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

$$\sin(\theta_2) = \left(\frac{n_1}{n_2}\right)\sin(\theta_1)$$

$$\theta_2 = \arcsin\left[\left(\frac{n_1}{n_2}\right)\sin(\theta_1)\right] = \arcsin\left[\left(\frac{1.00}{1.57}\right)\sin(45°)\right] = 26.8°$$

Geometry:

$$\beta = 90° - 26.8° = 63.2°$$

$$\gamma = 180° - 60° - \beta = 180° - 60° - 63.2° = 56.8°$$

$$\theta_3 = 90° - 56.8° = 33.2°$$

Refraction at glass–air interface:

$$n_3 \sin(\theta_3) = n_4 \sin(\theta_4)$$

$$\sin(\theta_4) = \frac{n_3}{n_4}\sin(\theta_3)$$

$$\theta_4 = \arcsin\left[\frac{n_3}{n_4}\sin(\theta_3)\right] = \arcsin\left[\left(\frac{1.57}{1.00}\right)\sin(33.2°)\right] = \boxed{59.4°}$$

Part b)

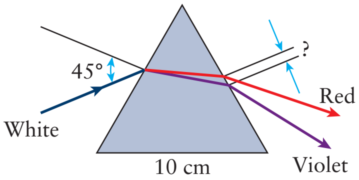


Figure 23-15 Problem 47

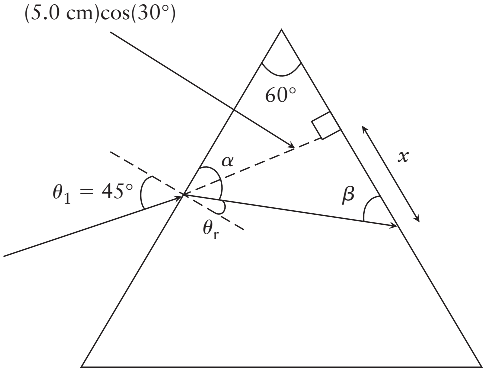


Figure 23-16 Problem 47

Red light:

$$n_{\text{air}}\sin(\theta_1) = n_{\text{red}}\sin(\theta_{\text{red}})$$

$$\sin(\theta_{\text{red}}) = \frac{n_{\text{air}}}{n_{\text{red}}}\sin(\theta_1)$$

$$\theta_{\text{red}} = \arcsin\left[\frac{n_{\text{air}}}{n_{\text{red}}}\sin(\theta_1)\right] = \arcsin\left[\frac{1.00}{1.568}\sin(45°)\right] = 26.81°$$

$$\alpha_{\text{red}} = 90° - \theta_{\text{red}} = 90° - 26.81° = 63.19°$$

$$\beta_{\text{red}} = 180° - 60° - \alpha_{\text{red}} = 120° - 63.19° = 56.81°$$

$$x_{\text{red}} = \frac{(5.0\text{ cm})\cos(30°)}{\tan(\beta_{\text{red}})} = \frac{(5.0\text{ cm})\cos(30°)}{\tan(56.81°)} = 2.833\text{ cm}$$

Violet light:

$$n_{\text{air}}\sin(\theta_1) = n_{\text{violet}}\sin(\theta_{\text{violet}})$$

$$\sin(\theta_{\text{violet}}) = \frac{n_{\text{air}}}{n_{\text{violet}}}\sin(\theta_1)$$

$$\theta_{\text{violet}} = \arcsin\left[\frac{n_{\text{air}}}{n_{\text{violet}}}\sin(\theta_1)\right] = \arcsin\left[\frac{1.00}{1.572}\sin(45°)\right] = 26.73°$$

$$\alpha_{\text{violet}} = 90° - \theta_{\text{violet}} = 90° - 26.73° = 63.27°$$

$$\beta_{\text{violet}} = 180° - 60° - \alpha_{\text{violet}} = 120° - 63.27° = 56.73°$$

$$x_{\text{violet}} = \frac{(5.0\text{ cm})\cos(30°)}{\tan(\beta_{\text{violet}})} = \frac{(5.0\text{ cm})\cos(30°)}{\tan(56.73°)} = 2.841\text{ cm}$$

Difference between exit points:

$$\Delta x = x_{\text{violet}} - x_{\text{red}} = (2.841\text{ cm}) - (2.833\text{ cm}) = \boxed{0.00793\text{ cm}}$$

REFLECT

We would expect dispersion effects to be small for a small prism.

23.48

SET UP

A light beam containing two colors of light—450 nm and an unknown wavelength—strikes a piece of glass with incident angle $\theta_1 = 45.0°$. The index of refraction for the 450-nm light is $n_{450} = 1.48$. We can use Snell's law to calculate the angle of refraction θ_{450} for the 450-nm light. We are told the refracted ray for the unknown light is 0.275° away from the other refracted ray. Using this information along with Snell's law, we can calculate the possible indices of refraction for the unknown light.

SOLVE

Angle of refraction, $\lambda = 450$ nm:

$$n_1 \sin(\theta_1) = n_{450} \sin(\theta_{450})$$

$$\sin(\theta_{450}) = \frac{n_1}{n_{450}} \sin(\theta_1)$$

$$\theta_{450} = \arcsin\left[\frac{n_1}{n_{450}} \sin(\theta_1)\right] = \arcsin\left[\frac{1.00}{1.48} \sin(45.0°)\right] = 28.5°$$

Angle of refraction for unknown light:

$$\theta_{\text{unk}} = 28.3° \text{ or } 28.8°$$

Index of refraction for unknown light:

$$n_1\sin(\theta_1) = n_{\text{unk}} \sin(\theta_{\text{unk}})$$

$$n_{\text{unk}} = \frac{n_1\sin(\theta_1)}{sin(\theta_{\text{unk}})} = \frac{(1.00)\sin(45.0°)}{\sin(28.3°)} = \boxed{1.49}$$

or

$$n_{\text{unk}} = \frac{n_1\sin(\theta_1)}{\sin(\theta_{\text{unk}})} = \frac{(1.00)\sin(45.0°)}{\sin(28.8°)} = \boxed{1.47}$$

REFLECT

We are not told if the unknown refracted ray is above or below the 450-nm refracted ray, so we need to take into account both possibilities. It makes sense that the index of refraction for the unknown light should be similar to the index of refraction for the 450-nm light because an angle of 0.275° is not very large.

23.49

SET UP

Blue and yellow light are incident on a glass slab ($t = 12$ cm) at an angle of 25 degrees relative to the normal. The index of refraction for the blue light in the glass is $n_{\text{glass, b}} = 1.545$, and the index of refraction for the yellow light in the glass is $n_{\text{glass, y}} = 1.523$. We are interested in the distance separating the two rays when they emerge from the other side of the slab. In order to calculate this, we first need to find the vertical displacement of each ray (either y_{b} or y_{y} for blue or yellow light, respectively) once it exits the slab. Applying Snell's law at the first interface will give the angle of the refracted light (either θ_{b} or θ_{y} for blue or yellow light, respectively) in the glass slab. The ray will travel at this angle until it reaches the other side. We can draw a triangle and relate the thickness of the slab t and the tangent of θ_{b} (or θ_{y}) to the vertical displacement y_{b} (or y_{y}). The distance between the rays is equal to the difference between the two vertical displacements.

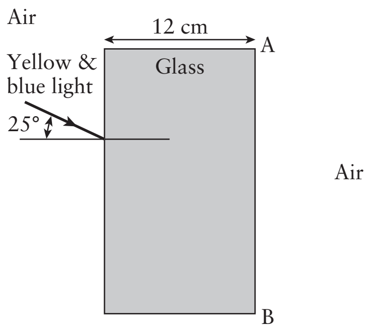


Figure 23-17 Problem 49

SOLVE

Blue light, angle of refraction:

$$n_{\text{air}} \sin(\theta_1) = n_{\text{glass, b}} \sin(\theta_{\text{b}})$$

$$\theta_{\text{b}} = \arcsin\left(\frac{n_{\text{air}}}{n_{\text{glass, b}}} \sin(\theta_1)\right) = \arcsin\left(\left(\frac{1.000}{1.545}\right)\sin(25°)\right) = 15.87°$$

Blue light, vertical displacement:

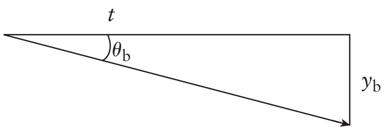


Figure 23-18 Problem 49

$$\tan(\theta_b) = \frac{y_{\text{b}}}{t}$$

$$y_{\text{b}} = t\tan(\theta_b) = (12 \text{ cm})\tan(15.87°) = 3.4126 \text{ cm}$$

Yellow light, angle of refraction:

$$n_{\text{air}} \sin(\theta_1) = n_{\text{glass, y}} \sin(\theta_{\text{y}})$$

$$\theta_{\text{y}} = \arcsin\left(\frac{n_{\text{air}}}{n_{\text{glass, y}}} \sin(\theta_1)\right) = \arcsin\left(\left(\frac{1.000}{1.523}\right)\sin(25°)\right) = 16.11°$$

Yellow light, vertical displacement:

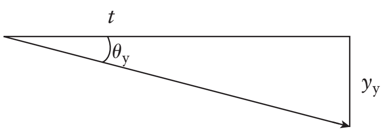


Figure 23-19 Problem 49

$$\tan(\theta_y) = \frac{y_y}{t}$$

$$y_y = t\tan(\theta_y) = (12\text{ cm})\tan(16.11°) = 3.4660\text{ cm}$$

Vertical difference between the blue and yellow rays:

$$\Delta y = y_y - y_b = (3.4660\text{ cm}) - (3.4126\text{ cm}) = \boxed{0.0534\text{ cm}}$$

REFLECT

The thicker the piece of glass, the farther apart the blue and yellow rays will emerge from one another, which makes sense. The shallower the angle of the incoming light is, the closer together the rays will emerge, which also makes sense.

23.50

SET UP

Unpolarized light of intensity $I_0 = 78\frac{\text{W}}{\text{m}^2}$ is passed through an optical filter that is oriented in the vertical direction. The light that emerges from the filter will be polarized along the polarization axis of the filter. The intensity of the light will decrease by a factor of 2.

SOLVE

$$I_1 = \frac{1}{2}I_0 = \frac{1}{2}\left(78\frac{\text{W}}{\text{m}^2}\right) = 39\frac{\text{W}}{\text{m}^2}$$

The resulting light is $\boxed{\text{vertically polarized and has an intensity of } 39\frac{\text{W}}{\text{m}^2}}$.

REFLECT

Whenever unpolarized light is passed through a linear polarizer, the intensity of the emerging light is one-half the initial intensity.

23.51

SET UP

Vertically polarized light $\left(I_0 = 400\frac{\text{W}}{\text{m}^2}\right)$ is passed through two polarizing filters. The first is oriented at an angle of 30 degrees relative to the vertical, and the second is oriented at an angle of 75 degrees relative to the vertical. We can use $I_1 = I_0\cos^2(\theta)$ in succession to determine the final intensity and polarization of the light after passing through the two filters. Remember that θ is the angle between the polarization axis of the light and the polarization axis of the filter, which is not necessarily the same as the angle the filter makes with the vertical.

SOLVE

$$I_1 = I_0 \cos^2(30°)$$

$$I_2 = I_1 \cos^2(\theta) = (I_0 \cos^2(30°))\cos^2((75°) - (30°)) = (I_0)\left(\frac{\sqrt{3}}{2}\right)^2 \cos^2(45°)$$

$$= \left(400\frac{\text{W}}{\text{m}^2}\right)\left(\frac{3}{4}\right)\left(\frac{1}{\sqrt{2}}\right)^2 = 150\frac{\text{W}}{\text{m}^2}$$

The resulting light is $\boxed{\text{polarized 75 degrees from the vertical and has an intensity of } 150\frac{\text{W}}{\text{m}^2}}$.

REFLECT

The intensity after a polarizer drops by a factor of 2 *only* if the initial light was unpolarized.

23.52

SET UP

We are asked to prove that unpolarized light emerging from a linear polarizer has an intensity equal to one-half of the initial intensity. We can do this by finding the average intensity over all angles by evaluating the integral $I_{\text{average}} = \frac{1}{2\pi}\int_0^{2\pi} I_0 \cos^2(\theta)d\theta$.

SOLVE

$$I_{\text{average}} = \frac{1}{2\pi}\int_0^{2\pi} I_0 \cos^2(\theta)d\theta = \frac{I_0}{2\pi}\int_0^{2\pi} \cos^2(\theta)d\theta = \frac{I_0}{2\pi}\left[\frac{1}{2}(\theta + \sin(\theta)\cos(\theta))\right]_0^{2\pi}$$

$$= \frac{I_0}{4\pi}[(2\pi + \sin(2\pi)\cos(2\pi)) - (0 + \sin(0)\cos(0))] = \frac{I_0}{4\pi}[2\pi] = \boxed{\frac{1}{2}I_0}$$

REFLECT

We can integrate $\cos^2(\theta)$ by invoking the identities: $\cos^2(\theta) = \frac{1}{2}(1 + \cos(2\theta))$ and $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$.

23.53

SET UP

Vertically polarized light with an initial intensity of I_0 is sent through a polarizing filter that makes a relative angle of θ to the filter. The intensity of the light after the polarizer is reduced by 25%, *i.e.*, $I = 0.75I_0$. The intensity of polarized light after it passes through a linear polarizer that makes a relative angle of θ is equal to $I = I_0\cos^2(\theta)$.

SOLVE

$$I = 0.75I_0 = I_0\cos^2(\theta)$$

$$\theta = \arccos(\pm\sqrt{0.75}) = \boxed{30°, 150°}$$

REFLECT

It makes sense that there should be two angles due to the symmetry of the system.

Get Help: Interactive Example – Polarization I
Interactive Example – Polarization II

23.54

SET UP

Unpolarized light passes through three consecutive polarizers, each 25 degrees relative to the previous one. The intensity of the light emerging after the set of polarizers is $I_3 = 250\frac{\text{W}}{\text{m}^2}$. We can use $I = I_0\cos^2(\theta)$ in succession to determine the initial intensity of the light. Recall that the intensity of initially unpolarized light drops by a factor of 2 upon passing through a polarizer.

SOLVE

$$I_1 = \frac{1}{2}I_0$$

$$I_2 = I_1\cos^2(25°) = \left(\frac{1}{2}I_0\right)\cos^2(25°)$$

$$I_3 = I_2\cos^2(25°) = \left(\left(\frac{1}{2}I_0\right)\cos^2(25°)\right)\cos^2(25°) = \left(\frac{1}{2}I_0\right)\cos^4(25°)$$

$$I_0 = \frac{2I_3}{\cos^4(25°)} = \frac{2\left(250\frac{\text{W}}{\text{m}^2}\right)}{\cos^4(25°)} = \boxed{741\frac{\text{W}}{\text{m}^2}}$$

REFLECT

If the resulting light is horizontally polarized, polarizer 1 makes an angle of 40 degrees relative to the vertical.

23.55

SET UP

The critical angle between two optical media is $\theta_c = 60°$. The critical angle is equal to $\theta_c = \arcsin\left(\frac{n_2}{n_1}\right)$. We can rearrange this equation to solve for the ratio of the indices of refraction in order to calculate Brewster's angle, $\theta_B = \arctan\left(\frac{n_2}{n_1}\right)$, for these media.

SOLVE

Critical angle:

$$\theta_c = \arcsin\left(\frac{n_2}{n_1}\right)$$

$$\sin(\theta_c) = \frac{n_2}{n_1}$$

Brewster's angle:

$$\theta_{\mathrm{B}} = \arctan\left(\frac{n_2}{n_1}\right) = \arctan(\sin(\theta_{\mathrm{c}})) = \arctan(\sin(60°)) = \boxed{41°}$$

REFLECT

Brewster's angle occurs when the sum of the incident angle and the refracted angle is equal to 90 degrees.

23.56

SET UP

We can use $\theta_{\mathrm{B}} = \arctan\left(\frac{n_2}{n_1}\right)$ and $\theta_{\mathrm{c}} = \arcsin\left(\frac{n_2}{n_1}\right)$ to calculate Brewster's angle and the critical angle, respectively, for light traveling from water ($n_1 = 1.33$) into air ($n_2 = 1.00$).

SOLVE

Part a)

$$\theta_{\mathrm{B}} = \arctan\left(\frac{n_2}{n_1}\right) = \arctan\left(\frac{1.00}{1.33}\right) = \boxed{36.9°}$$

Part b)

$$\theta_{\mathrm{c}} = \arcsin\left(\frac{n_2}{n_1}\right) = \arcsin\left(\frac{1.00}{1.33}\right) = \boxed{48.8°}$$

REFLECT

The critical angle is larger.

23.57

SET UP

We can use $\theta_{\mathrm{B}} = \arctan\left(\frac{n_2}{n_1}\right)$ to calculate Brewster's angle for light traveling from air ($n_1 = 1.00$) into plastic ($n_2 = 1.49$).

SOLVE

$$\theta_{\mathrm{B}} = \arctan\left(\frac{n_2}{n_1}\right) = \arctan\left(\frac{1.49}{1.00}\right) = \boxed{56.1°}$$

REFLECT

The critical angle is undefined in this case because the light is traveling into a medium with a higher index of refraction.

23.58

SET UP

We can use $\theta_{\mathrm{B}} = \arctan\left(\frac{n_2}{n_1}\right)$ to calculate Brewster's angle for light traveling from water ($n_1 = 1.33$) into glass ($n_2 = 1.55$).

SOLVE

$$\theta_B = \arctan\left(\frac{n_2}{n_1}\right) = \arctan\left(\frac{1.55}{1.33}\right) = \boxed{49.4°}$$

REFLECT

Brewster's angle exists in general for any two media; air does not need to be one of them.

23.59

SET UP

A person observes that the light rays from the Sun that bounce off the air–water surface are linearly polarized along the horizontal. When incoming light strikes the air–water boundary at Brewster's angle, $\theta_B = \arctan\left(\frac{n_{water}}{n_{air}}\right)$, the reflected light is completely polarized parallel to the surface. Brewster's angle is measured relative to the vertical, so the angle relative to the horizontal is equal to $\theta = 90° - \theta_B$.

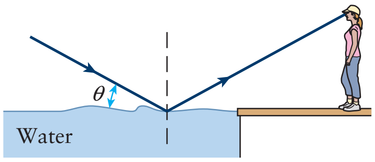


Figure 23-20 Problem 59

SOLVE

Brewster's angle:

$$\theta_B = \arctan\left(\frac{n_{water}}{n_{air}}\right) = \arctan\left(\frac{1.33}{1.00}\right) = 53.1°$$

Angle above the horizontal:

$$\theta = 90° - \theta_B = 90° - 53.1° = \boxed{36.9°}$$

REFLECT

If the Sun is close to, but not exactly at, this angle, the reflected light will be strongly, not completely, polarized.

Get Help: P'Cast 23.6 – Brewster's Angle for Air to Water

23.60

SET UP

A thin film of index n_{film} coats a glass slab of index n_{glass}. We will assume that $n_{film} < n_{glass}$. Light is reflected from the front surface and the back surface of the film back into air. If the light reflects off of a medium with a higher index of refraction than the medium in which it begins, the reflected light will experience a shift of half a wavelength. If the light reflects off of a medium of lower index, there will be no phase shift.

SOLVE

Part a) Light will be $\boxed{\text{shifted by half a wavelength}}$ when it reflects off the air–film interface because $n_{air} < n_{film}$.

Part b) Light will be $\boxed{\text{shifted by half a wavelength}}$ when it reflects off the film–glass interface because $n_{film} < n_{glass}$.

REFLECT

When solving thin film interference problems, we need to consider not only phase changes due to reflections but also phase changes due to the light traveling more slowly in the film.

23.61

SET UP

The wavelength of red light ($\lambda_{vac} = 700$ nm) inside a glass slab ($n = 1.55$) is equal to the wavelength of the light in the vacuum divided by the index of refraction of the medium, $\lambda_n = \frac{\lambda_{vac}}{n}$.

SOLVE

$$\lambda_n = \frac{\lambda_{vac}}{n} = \frac{700\text{ nm}}{1.55} = \boxed{452\text{ nm}}$$

REFLECT

Although the wavelength changes, the frequency of the light remains constant.

23.62

SET UP

White light illuminates a thin film ($n_{film} = 1.28$) normal to the surface, and we observe that both indigo light ($\lambda_i = 450$ nm) and yellow light ($\lambda_y = 600$ nm) are strongly reflected. The film has a thickness t and is sitting on top of glass ($n_{water} = 1.50$). The condition on the path difference for constructive interference in the case when the film has a smaller index of refraction than the medium under it is $(m + 1)\frac{\lambda_0}{n_{film}}$. Note that the wavelength of light changes when the index of refraction changes. We are interested in the wavelength of light in the film, so we need to divide the given wavelength by the index of refraction of the film. Since the thickness of the film is t, this path difference should be equal to $2t$. This same path difference strongly reflects *two* wavelengths of light, so we will have two expressions with different integers, m_i for the indigo light and m_y for the yellow light. Setting the thicknesses equal to one another, we can solve for the smallest integers that satisfy the resulting relationship. The minimum thickness of the film can be found by plugging the integer back into the constructive interference relationship.

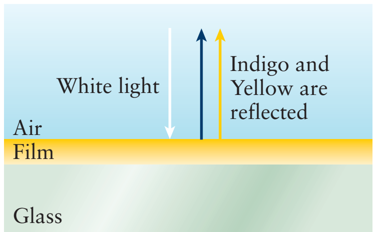


Figure 23-21 Problem 62

SOLVE

Indigo light:

$$2t = (m_i + 1)\frac{\lambda_i}{n_{film}}$$

Yellow light:

$$2t = (m_y + 1)\frac{\lambda_y}{n_{film}}$$

Solving for m's:

$$(m_{\text{i}} + 1)\frac{\lambda_{\text{i}}}{n_{\text{film}}} = (m_{\text{y}} + 1)\frac{\lambda_{\text{y}}}{n_{\text{film}}}$$

$$(m_{\text{i}} + 1)(450 \text{ nm}) = (m_{\text{y}} + 1)(600 \text{ nm})$$

$$3m_{\text{i}} + 3 = 4m_{\text{y}} + 4$$

$$3m_{\text{i}} - 4m_{\text{y}} = 1$$

This is true if $m_{\text{i}} = 3$ and $m_{\text{y}} = 2$.
Thickness:

$$t = (m_{\text{i}} + 1)\frac{\lambda_{\text{i}}}{2n_{\text{film}}} = (3 + 1)\frac{450 \text{ nm}}{2(1.28)} = \boxed{703 \text{ nm}}$$

REFLECT

We could have also used the expression for the thickness t in terms of the yellow light:

$$t = (m_{\text{y}} + 1)\frac{\lambda_{\text{y}}}{2n_{\text{film}}} = (2 + 1)\frac{600 \text{ nm}}{2(1.28)} = \boxed{703 \text{ nm}}$$

23.63

SET UP

White light illuminates a thin film ($n_{\text{film}} = 1.35$) normal to the surface, and we observe that both blue light ($\lambda_{\text{b}} = 500$ nm) and red light ($\lambda_{\text{r}} = 700$ nm) are strongly reflected. The film has a thickness t and is floating on top of water ($n_{\text{water}} = 1.33$). The condition on the path difference for constructive interference in the case where the film has a higher index of refraction than the medium under it is $\left(m + \frac{1}{2}\right)\frac{\lambda_0}{n_{\text{film}}}$. Note that the wavelength of light changes when the index of refraction changes. We are interested in the wavelength of light in the film, so we need to divide the given wavelength by the index of refraction of the film. Since the thickness of the film is t, this path difference should be equal to $2t$. This same path difference strongly reflects *two* wavelengths of light, so we will have two expressions with different integers, m_{b} for the blue light and m_{r} for the red light. Setting the thicknesses equal to one another, we can solve the smallest integers that satisfy the resulting relationship. The minimum thickness of the film can be found by plugging the integer back into the constructive interference relationship.

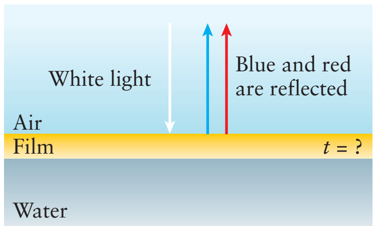


Figure 23-22 Problem 63

SOLVE

Blue light:

$$2t = \left(m_{\text{b}} + \frac{1}{2}\right)\frac{\lambda_{\text{b}}}{n_{\text{film}}}$$

Red light:

$$2t = \left(m_{\text{r}} + \frac{1}{2}\right)\frac{\lambda_{\text{r}}}{n_{\text{film}}}$$

Solving for m's:

$$\left(m_b + \frac{1}{2}\right)\frac{\lambda_b}{n_{film}} = \left(m_r + \frac{1}{2}\right)\frac{\lambda_r}{n_{film}}$$

$$\left(m_b + \frac{1}{2}\right)(500\text{ nm}) = \left(m_r + \frac{1}{2}\right)(700\text{ nm})$$

$$5m_b + \frac{5}{2} = 7m_r + \frac{7}{2}$$

$$5m_b - 7m_r = 1$$

This is true if $m_b = 3$ and $m_r = 2$.

Thickness:

$$t = \left(m_b + \frac{1}{2}\right)\frac{\lambda_b}{2n_{film}} = \left(3 + \frac{1}{2}\right)\frac{500\text{ nm}}{2(1.35)} = \boxed{648\text{ nm}}$$

REFLECT

Since we were looking for the minimum thickness of the film, we chose the smallest values of m_b and m_r that satisfied the equation $5m_b - 7m_r = 1$. The next set of integers that satisfies that relationship is $m_b = 10$ and $m_r = 7$, which corresponds to $t = 1944$ nm.

Get Help: Interactive Example – Film on Water
P'Cast 23.8 – Reducing the Reflection

23.64

SET UP

White light illuminates a soap film (index of refraction n_{film}) normal to the surface, and we observe that both blue light ($\lambda_b = 420$ nm) and green light ($\lambda_r = 560$ nm) are not reflected. The film has a thickness $t = 625$ nm and is sandwiched by air ($n_{air} = 1.00$). The condition on the path difference for destructive interference in the case when the film has a higher index of refraction than the medium under it is $(m + 1)\frac{\lambda_0}{n_{film}}$. Note that the wavelength of light changes when the index of refraction changes. We are interested in the wavelength of light in the film, so we need to divide the given wavelength by the index of refraction of the film. Since the thickness of the film is t, this path difference should be equal to $2t$. This same path difference causes *two* wavelengths of light to be destroyed, so we will have two expressions with different integers, m_b for the blue light and m_g for the green light. Setting the thicknesses equal to one another, we can solve for the smallest integers that satisfy the resulting relationship. The index of refraction of the film can be found by plugging the integer back into the destructive interference relationship.

SOLVE

Blue light:

$$2t = (m_b + 1)\frac{\lambda_b}{n_{film}}$$

Green light:

$$2t = (m_g + 1)\frac{\lambda_r}{n_{film}}$$

Solving for m's:

$$(m_b + 1)\frac{\lambda_b}{n_{film}} = (m_g + 1)\frac{\lambda_r}{n_{film}}$$

$$(m_b + 1)(420\text{ nm}) = (m_g + 1)(560\text{ nm})$$

$$4.2m_b + 4.2 = 5.6m_g + 5.6$$

$$4.2m_b - 5.6m_g = 1.4$$

This is true if $m_b = 3$ and $m_g = 2$.
Index of refraction:

$$n_{film} = (m_b + 1)\frac{\lambda_b}{2t} = (3 + 1)\frac{420\text{ nm}}{2(625\text{ nm})} = \boxed{1.34}$$

REFLECT

The net phase change due to reflections in the film is equal to one-half wavelength. This is why the phase change due to the path difference must be equal to one full wavelength for destructive interference to occur.

23.65

SET UP

White light illuminates a thin film of cooking oil ($n_{film} = 1.38$) normal to the surface, and we want to know the four smallest thicknesses of the film that strongly reflect blue light ($\lambda_b =$ 518 nm). We'll call the thickness of the film t, and it is floating on top of water ($n_{water} = 1.33$). The condition on the path difference for constructive interference in the case when the film has a higher index of refraction than the medium under it is $\left(m + \frac{1}{2}\right)\frac{\lambda_0}{n_{film}}$. Note that the wavelength of light changes when the index of refraction changes. We are interested in the wavelength of light in the film, so we need to divide the given wavelength by the index of refraction of the film. Since the thickness of the film is t, this path difference should be equal to $2t$. The four smallest thicknesses of the film can be found by plugging $m = 0$, 1, 2, and 3 into the constructive interference relationship.

SOLVE

Constructive interference:

$$2t = \left(m + \frac{1}{2}\right)\frac{\lambda_b}{n_{film}}$$

$$t = \left(m + \frac{1}{2}\right)\frac{\lambda_b}{2n_{film}}$$

Thicknesses:

$m = 0$:

$$t = \left(0 + \frac{1}{2}\right)\frac{518\text{ nm}}{2(1.38)} = \boxed{93.8\text{ nm}}$$

$m = 1$:

$$t = \left(1 + \frac{1}{2}\right)\frac{518\text{ nm}}{2(1.38)} = \boxed{282\text{ nm}}$$

$m = 2$:

$$t = \left(2 + \frac{1}{2}\right)\frac{518\text{ nm}}{2(1.38)} = \boxed{469\text{ nm}}$$

$m = 3$:

$$t = \left(3 + \frac{1}{2}\right)\frac{518\text{ nm}}{2(1.38)} = \boxed{657\text{ nm}}$$

REFLECT

These are reasonable thicknesses for a thin oil film.

23.66

SET UP

White light illuminates a glass lens ($n_{\text{lens}} = 1.56$) that is coated with a thin antireflective film ($n_{\text{film}} = 1.39$) normal to the surface. We want to calculate the minimum thickness t of the film that causes light of wavelength $\lambda_0 = 550$ nm to not be reflected. The condition on the path difference for destructive interference in the case when the film has a lower index of refraction than the medium under it is $\left(m + \frac{1}{2}\right)\frac{\lambda_0}{n_{\text{film}}}$. Note that the wavelength of light changes when the index of refraction changes. We are interested in the wavelength of light in the film, so we need to divide the given wavelength by the index of refraction of the film. Since the thickness of the film is t, this path difference should be equal to $2t$. The smallest thickness of the film can be found by plugging $m = 0$ into the destructive interference relationship.

SOLVE

Destructive interference:

$$2t = \left(m + \frac{1}{2}\right)\frac{\lambda_0}{n_{\text{film}}}$$

$$t = \left(m + \frac{1}{2}\right)\frac{\lambda_0}{2n_{\text{film}}}$$

Minimum thickness:

$$t = \left(0 + \frac{1}{2}\right)\frac{550\text{ nm}}{2(1.39)} = \boxed{98.9\text{ nm}}$$

REFLECT

The net phase change due to reflections in the film is equal to a full wavelength. This is why the phase change due to the path difference must be equal to half a wavelength for destructive interference to occur.

23.67

SET UP

A pool of water ($n_{water} = 1.33$) covered with a thin film of oil ($t = 450$ nm, $n_{oil} = 1.45$) is illuminated with white light and viewed from straight above. We want to know which visible wavelengths are *not* present in the reflected light, which means these wavelengths undergo total destructive interference. The condition on the path difference for destructive interference in the case where the film has a higher index of refraction than the medium under it is $(m + 1)\dfrac{\lambda}{n_{oil}}$. Note that the wavelength of light changes when the index of refraction changes. We are interested in the wavelength of light in the oil, so we need to divide the given wavelength by the index of refraction of the oil. Since the thickness of the oil film is t, this path difference should be equal to $2t$. We can solve for an expression for the wavelength in terms of the integer m. This will allow us to plug in consecutive values for m (starting from $m = 0$) and determine which wavelengths lie in the visible region (around 380 nm–750 nm).

SOLVE

$$2t = (m + 1)\frac{\lambda}{n_{oil}}$$

$$\lambda = \frac{2tn_{oil}}{m + 1} = \frac{2(450 \text{ nm})(1.45)}{m + 1} = \frac{1305 \text{ nm}}{m + 1}$$

For $m = 0$, $\lambda = 1305$ nm, which is in the infrared.

For $m = 1$, $\boxed{\lambda = 653 \text{ nm}}$, which is red/orange.

For $m = 2$, $\boxed{\lambda = 435 \text{ nm}}$, which is violet/indigo.

For $m = 3$, $\lambda = 326$ nm, which is ultraviolet.

REFLECT

The visible wavelength that is strongly reflected is 522 nm (green).

Get Help: Interactive Example – Film on Water
P'Cast 23.8 – Reducing the Reflection

23.68

SET UP

Light ($\lambda = 550 \times 10^{-9}$ m) is incident on a single slit ($w = 10 \times 10^{-6}$ m). The angular location θ of the first three dark fringes in the diffraction pattern can be found using $\sin(\theta) = m\dfrac{\lambda}{w}$, where $m = 1, 2,$ and 3.

SOLVE

$$\sin(\theta) = m\frac{\lambda}{w}$$

$$\theta = \arcsin\left(m\frac{\lambda}{w}\right)$$

$m = 1$:

$$\theta = \arcsin\left((1)\left(\frac{550 \times 10^{-9}\text{ m}}{10 \times 10^{-6}\text{ m}}\right)\right) = \boxed{3.2°}$$

$m = 2$:

$$\theta = \arcsin\left((2)\left(\frac{550 \times 10^{-9}\text{ m}}{10 \times 10^{-6}\text{ m}}\right)\right) = \boxed{6.3°}$$

$m = 3$:

$$\theta = \arcsin\left((3)\left(\frac{550 \times 10^{-9}\text{ m}}{10 \times 10^{-6}\text{ m}}\right)\right) = \boxed{9.5°}$$

REFLECT

The diffraction pattern is symmetric about its central maximum, so dark fringes are also located at $\theta = -3.2°$, $-6.3°$, and $-9.5°$.

23.69

SET UP

Light ($\lambda = 475$ nm) is incident on a single slit ($w = 800$ nm). The angular location θ of the first three dark fringes in the diffraction pattern can be found using $\sin(\theta) = m\frac{\lambda}{w}$, where $m = 1, 2,$ and 3.

SOLVE

$$\sin(\theta) = m\frac{\lambda}{w}$$

$$\theta = \arcsin\left(m\frac{\lambda}{w}\right)$$

$m = 1$:

$$\theta = \arcsin\left((1)\left(\frac{475\text{ nm}}{800\text{ nm}}\right)\right) = \boxed{36.4°}$$

$m = 2$:

$$\theta = \arcsin\left((2)\left(\frac{475\text{ nm}}{800\text{ nm}}\right)\right) = \text{undefined}$$

$m = 3$:

$$\theta = \arcsin\left((3)\left(\frac{475\text{ nm}}{800\text{ nm}}\right)\right) = \text{undefined}$$

Only one dark fringe at $\theta = 36.4°$ exists in the diffraction pattern.

REFLECT

As the slit width decreases, the width of the central maximum increases.

23.70

SET UP

Light ($\lambda = 633$ nm) is incident on a single slit ($w = 1500$ nm). The angular location θ of the dark fringes in the diffraction pattern can be found using $\sin(\theta) = m\frac{\lambda}{w}$, where m is an integer. In order to find the highest-order dark fringe found in the diffraction pattern, we need to determine the largest value of m for which the right-hand side of that equation is still less than or equal to 1.

SOLVE

$$\sin(\theta) = m\frac{\lambda}{w}$$

$$\theta = \arcsin\left(m\frac{\lambda}{w}\right) = \arcsin\left(m\left(\frac{633 \text{ nm}}{1500 \text{ nm}}\right)\right)$$

A value of $m = 3$ will cause the argument of the arcsine to be larger than 1. Therefore, the highest-order dark fringe seen in the diffraction pattern is $\boxed{m = 2}$.

REFLECT

The sine function oscillates between $+1$ and -1, so the argument of the arcsine must be between those values.

23.71

SET UP

The highest-order dark fringe found in a diffraction pattern corresponds to $m = 6$. The slit used in the experiments has a width $w = 3500$ nm. Assuming the sixth-order fringes are at the extremes of the pattern (that is, $\sin(\theta) = 1$), we can use $\sin(\theta) = m\frac{\lambda}{w}$ to calculate the wavelength λ of the light.

SOLVE

$$\sin(\theta) = m\frac{\lambda}{w}$$

$$\lambda = \frac{w\sin(\theta)}{m} = \frac{(3500 \text{ nm})(1)}{6} = \boxed{583 \text{ nm}}$$

REFLECT

There is actually a range of wavelengths that would yield a maximum dark fringe corresponding to $m = 6$ since m must be an integer. The next-order dark fringe ($m = 7$) appears when $\lambda = 500$ nm.

23.72

SET UP

Blue light ($\lambda = 500 \times 10^{-9}$ m) shines on a single slit of width w. The central bright spot of the resulting diffraction pattern on a screen a distance $L = 3.55$ m behind the single slit has a width $(2y_1) = 8.75 \times 10^{-2}$ m. The relationship $\sin(\theta) = \frac{m\lambda}{w}$ gives the angular position of the dark fringes. Using trigonometry and the small angle approximation ($\sin(\theta) \approx \tan(\theta)$), we can rewrite this in terms of the position of the mth dark fringe on the screen, y_m. Plugging in $m = 1$ and rearranging the expression for w will allow us to calculate the slit width.

SOLVE

Small angle approximation:

$$\sin(\theta) \approx \tan(\theta) = \frac{y_m}{L}$$

Location of first dark fringe:

$$\sin(\theta) = \frac{m\lambda}{w} = \frac{(1)\lambda}{w} = \frac{\lambda}{w}$$

$$\frac{\lambda}{w} \approx \frac{y_1}{L}$$

$$y_1 \approx \frac{\lambda L}{w}$$

Width of central maximum:

$$2y_1 \approx \frac{2\lambda L}{w}$$

Slit width:

$$w \approx \frac{2\lambda L}{(2y_1)} = \frac{2(500 \times 10^{-9}\text{ m})(3.55\text{ m})}{(8.75 \times 10^{-2}\text{ m})} = \boxed{4.06 \times 10^{-5}\text{ m} = 40.6\ \mu\text{m}}$$

REFLECT

It is easier to measure the position of the dark fringes experimentally rather than their angular position.

23.73

SET UP

The beam from a He-Ne laser illuminates a single slit of width $w = 1850$ nm. In the resulting diffraction pattern, the first dark fringe appears at an angle of 20.0 degrees from the central maximum. We can use the expression for the angular position of the first dark fringe,
$\sin(\theta) = \frac{\lambda}{w}$, to calculate the wavelength of the laser light.

SOLVE

$$\sin(\theta) = \frac{\lambda}{w}$$

$$\lambda = w\sin(\theta) = (1850 \text{ nm})\sin(20.0°) = \boxed{633 \text{ nm}}$$

REFLECT

A typical He-Ne laser appears red, and a wavelength of 633 nm is well within the range of red visible light.

23.74

SET UP

Yellow light ($\lambda = 625 \times 10^{-9}$ m) shines on a single slit of width w. The central bright spot of the resulting diffraction pattern on a screen a distance $L = 1.58$ m behind the single slit has a width $(2y_1) = 0.24$ m. The relationship $\sin(\theta) = \frac{m\lambda}{w}$ gives the angular position of the dark fringes. Using trigonometry and the small angle approximation ($\sin(\theta) \approx \tan(\theta)$), we can rewrite this in terms of the position of the mth dark fringe on the screen, y_{m}. Plugging in $m = 1$ and rearranging the expression for w will allow us to calculate the slit width.

SOLVE

Small angle approximation:

$$\sin(\theta) \approx \tan(\theta) = \frac{y_m}{L}$$

Location of first dark fringe:

$$\sin(\theta) = \frac{m\lambda}{w} = \frac{(1)\lambda}{w} = \frac{\lambda}{w}$$

$$\frac{\lambda}{w} \approx \frac{y_1}{L}$$

$$y_1 \approx \frac{\lambda L}{w}$$

Width of central maximum:

$$2y_1 \approx \frac{2\lambda L}{w}$$

Slit width:

$$w \approx \frac{2\lambda L}{(2y_1)} = \frac{2(625 \times 10^{-9} \text{ m})(1.58 \text{ m})}{(0.24 \text{ m})} = \boxed{8.2 \times 10^{-6} \text{ m} = 8.2\ \mu\text{m}}$$

REFLECT

For a given wavelength of light, the central maximum gets wider as the slit width gets smaller.

23.75

SET UP

Red light ($\lambda = 633 \times 10^{-9}$ m) shines on a circular aperture of diameter $D = 0.18 \times 10^{-3}$ m. A diffraction pattern is formed on a screen $L = 2.0$ m behind the aperture. The relationship $\sin(\theta) = 1.22\frac{\lambda}{w}$ gives the angular position of the first dark ring. Using trigonometry and the small angle approximation ($\sin(\theta) \approx \tan(\theta)$), we can rewrite this in terms of the radius of the central maximum, R. The diameter of the central maximum is equal to $2R$.

SOLVE

Small angle approximation:

$$\sin(\theta) \approx \tan(\theta) = \frac{R}{L}$$

Location of first dark ring:

$$\sin(\theta) = 1.22\frac{\lambda}{D}$$

$$1.22\frac{\lambda}{D} \approx \frac{R}{L}$$

Diameter of central maximum:

$$D_{\text{central max}} = 2R \approx 2\left(1.22\frac{\lambda L}{D}\right) = 2.44\left(\frac{(633 \times 10^{-9}\text{ m})(2.0\text{ m})}{0.18 \times 10^{-3}\text{ m}}\right) = \boxed{1.7 \times 10^{-2}\text{ m} = 17\text{ mm}}$$

REFLECT

It makes sense that the diameter of the central maximum will increase if the screen is moved farther away from the aperture.

23.76

SET UP

The average person's pupil has a diameter $D = 5.0 \times 10^{-3}$ m, and the average eye is most sensitive to light of wavelength $\lambda = 555 \times 10^{-9}$ m. The angular resolution of the eye θ_R is given by $\sin(\theta_R) = 1.22\frac{\lambda}{D}$.

SOLVE

$$\sin(\theta_R) = 1.22\frac{\lambda}{D}$$

$$\theta_R = \arcsin\left(1.22\frac{\lambda}{D}\right) = \arcsin\left(1.22\left(\frac{555 \times 10^{-9}\text{ m}}{5.0 \times 10^{-3}\text{ m}}\right)\right) = \boxed{1.4 \times 10^{-4}\text{ rad}}$$

REFLECT

From a height of 2 m, this corresponds to a resolution of 0.1–0.5 mm.

23.77

SET UP

The objective mirror of a telescope has a diameter $D = 5.08$ m. We can use $\sin(\theta_R) = 1.22\frac{\lambda}{D}$ to find the angular resolution of the telescope θ_R at $\lambda = 560 \times 10^{-9}$ m in both radians and degrees.

SOLVE

$$\sin(\theta_R) = 1.22\frac{\lambda}{D}$$

$$\theta_R = \arcsin\left(1.22\frac{\lambda}{D}\right) = \arcsin\left(1.22\left(\frac{560 \times 10^{-9}\text{ m}}{5.08\text{ m}}\right)\right)$$

$$= \boxed{1.34 \times 10^{-7}\text{ rad} = 7.71 \times 10^{-6}\text{ degrees}}$$

REFLECT

We would expect the resolution of a telescope to be very small in order to distinguish objects that are very far from Earth.

23.78

SET UP

The Hubble Space Telescope has a diameter $D = 2.4$ m. We can use $\sin(\theta_R) = 1.22\frac{\lambda}{D}$ to find the angular resolution of the telescope θ_R at $\lambda = 540 \times 10^{-9}$ m.

SOLVE

$$\sin(\theta_R) = 1.22\frac{\lambda}{D}$$

$$\theta_R = \arcsin\left(1.22\frac{\lambda}{D}\right) = \arcsin\left(1.22\left(\frac{540 \times 10^{-9}\text{ m}}{2.4\text{ m}}\right)\right) = \boxed{2.7 \times 10^{-7}\text{ rad}}$$

REFLECT

We would expect the resolution of a telescope to be very small in order to distinguish objects that are very far from Earth.

23.79

SET UP

Light of wavelength λ is sent through a circular aperture of diameter D. The diffraction pattern is projected on a screen located a distance $L = 0.85$ m from the aperture. The first dark ring is $15{,}000\lambda$ from the center of the central maximum. The angular position of the first dark ring is given by $\sin(\theta) = 1.22\frac{\lambda}{D}$. Using the small angle approximation, we can relate this

to $\tan(\theta)$ by geometry. Putting all of this together will allow us to calculate the diameter of the aperture D.

SOLVE

Geometry:

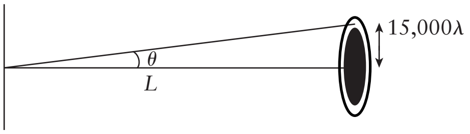


Figure 23-23 Problem 79

$$\tan(\theta) = \frac{15{,}000\lambda}{L}$$

Diameter of aperture:

$$\sin(\theta) = 1.22\frac{\lambda}{D}$$

Using the small angle approximation, $\sin(\theta) \approx \tan(\theta) = \dfrac{15{,}000\lambda}{L}$:

$$\frac{15{,}000\lambda}{L} = 1.22\frac{\lambda}{D}$$

$$D = \frac{1.22L}{15{,}000} = \frac{1.22(0.85\text{ m})}{15{,}000} = \boxed{6.91 \times 10^{-5}\text{ m}}$$

REFLECT

The wavelength of visible light is on the order of hundreds of nanometers. If we assume 5×10^{-7} m as a typical value, the first dark ring is 7.5×10^{-3} m from the center of the central maximum. This is approximately 2 orders of magnitude smaller than the distance between the aperture and the screen, so our use of the small angle approximation is justified.

Get Help: P'Cast 23.9 – The Hubble Space Telescope

23.80

SET UP

The eye has an aperture diameter of $D = 3.00 \times 10^{-3}$ m when bright headlights are pointed at it. We can use trigonometry and the small angle approximation to calculate the maximum distance L at which a person can still distinguish two headlights separated by $x = 1.50$ m as distinct; the diffraction-limited angular resolution is given by $\sin(\theta_R) = 1.22\frac{\lambda}{D}$.

SOLVE

Angular resolution:

$$\sin(\theta_R) = 1.22\frac{\lambda}{D}$$

Small angle approximation:

$$\sin(\theta_R) \approx \tan(\theta_R)$$

$$1.22\frac{\lambda}{D} \approx \frac{x}{L}$$

$$L \approx \frac{xD}{1.22\lambda} = \frac{(1.50\text{ m})(3.00 \times 10^{-3}\text{ m})}{1.22(550 \times 10^{-9}\text{ m})} = \boxed{6.71 \times 10^{3}\text{ m} = 6.71\text{ km}}$$

REFLECT

A distance of 6.71 km is a little over 4 mi, which seems on the larger side of reasonable. Keep in mind, we assumed that only diffraction affected the ability of a person to distinguish the headlights and that everything else—the person's vision, the weather, and so on—had no effect.

23.81

SET UP

One way of describing the speed of light is by characterizing materials by the increase in time required for light to travel a given distance through the material as compared to traveling the same distance in a vacuum. In this case, the speed of light in the medium is equal to the speed of light in the vacuum divided by the factor by which the time increased. The index of refraction of the medium is equal to the speed of light in the vacuum divided by the speed of light in the medium.

SOLVE

Speed of light:

$$v = \frac{c}{\text{time increase}}$$

$$v_{100\%} = \frac{\left(2.998 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{1.00} = \boxed{3.00 \times 10^{8}\frac{\text{m}}{\text{s}}}$$

$$v_{125\%} = \frac{\left(2.998 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{1.25} = \boxed{2.40 \times 10^{8}\frac{\text{m}}{\text{s}}}$$

$$v_{150\%} = \frac{\left(2.998 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{1.50} = \boxed{2.00 \times 10^{8}\frac{\text{m}}{\text{s}}}$$

$$v_{200\%} = \frac{\left(2.998 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{2.00} = \boxed{1.50 \times 10^{8}\frac{\text{m}}{\text{s}}}$$

$$v_{500\%} = \frac{\left(2.998 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{5.00} = \boxed{6.00 \times 10^{7}\frac{\text{m}}{\text{s}}}$$

$$v_{1000\%} = \frac{\left(2.998 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{10.00} = \boxed{3.00 \times 10^{7}\frac{\text{m}}{\text{s}}}$$

Index of refraction:

$$n = \frac{c}{v}$$

$$v = \frac{c}{n}$$

This is the same relationship we used to calculate the speeds of light in the first part. Accordingly, the index of refraction is equal to the time increase in each case.
Table:

Optical material with percentage of time required for light to pass through compared to an equal length of vaccum	Speed of light	Index of refraction
100%	3.00×10^8m/s	1.00
125%	2.40×10^8m/s	1.25
150%	2.00×10^8m/s	1.50
200%	1.50×10^8m/s	2.00
500%	6.00×10^7m/s	5.00
1000%	3.00×10^7m/s	10.00

REFLECT
Setting up a table like this helps build intuition regarding indices of refraction.

23.82

SET UP
A light ray is traveling from one medium ($n_1 = 1.00$) toward a second medium (n_2). The ray makes an incident angle of 45.0 degrees with respect to the vertical. In part (a), we are told that the refracted ray hits a point C that is 0.750 cm to the right of the vertical at a depth of 2.00 cm (see figure). In part (b), we are told that $n_2 = 1.55$. We can use trigonometry and Snell's law to calculate the n_2 for part (a) and the distance BC for part (b).

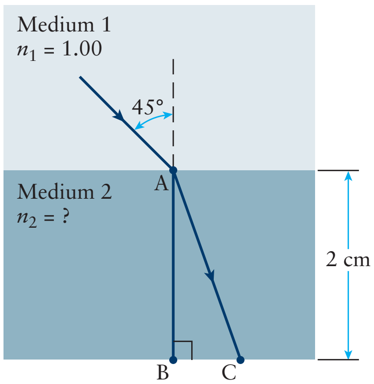


Figure 23-24 Problem 82

SOLVE

Part a)
Angle of refraction:

$$\tan(\theta_2) = \frac{\text{BC}}{\text{AB}}$$

$$\theta_2 = \arctan\left(\frac{\text{BC}}{\text{AB}}\right) = \arctan\left(\frac{0.750\text{ cm}}{2.00\text{ cm}}\right) = 20.6°$$

Index of refraction of medium 2:

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

$$n_2 = n_1 \frac{\sin(\theta_1)}{\sin(\theta_2)} = (1.00)\left(\frac{\sin(45.0°)}{\sin(20.6°)}\right) = \boxed{2.01}$$

Part b)
Angle of refraction:

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

$$\sin(\theta_2) = \frac{n_1}{n_2} \sin(\theta_1)$$

$$\theta_2 = \arcsin\left[\frac{n_1}{n_2} \sin(\theta_1)\right] = \arcsin\left[\left(\frac{1.00}{1.55}\right)\sin(45.0°)\right] = 27.1°$$

Distance BC:

$$\tan(\theta_2) = \frac{\text{BC}}{\text{AB}}$$

$$\text{BC} = (\text{AB})\tan(\theta_2) = (2.00\text{ cm})\tan(27.1°) = \boxed{1.03\text{ cm}}$$

REFLECT
The larger the refracted angle, the larger the distance BC.

23.83

SET UP
Many slabs of materials of different indices of refraction are placed side-by-side and sandwiched by air (see figure). We are asked to prove that the final refracted angle only depends on the indices of refraction of the initial and final media and the incident angle. To do so, we can apply Snell's law at each interface and see how it holds in general.

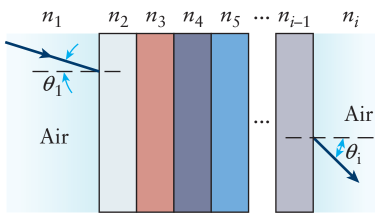


Figure 23-25 Problem 83

SOLVE
Snell's law at the first interface:

$$n_A \sin(\theta_A) = n_1 \sin(\theta_1)$$

Snell's law at the second interface:

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

$$\vdots$$

Snell's law at the $(j-1)$th interface:

$$n_{j-1} \sin(\theta_{j-1}) = n_j \sin(\theta_j)$$

$$\vdots$$

Snell's law at the last interface:

$$n_i \sin(\theta_i) = n_B \sin(\theta_B)$$

If we successively apply Snell's law to each interface and invoke the transitive property, then we see that $\boxed{n_A \sin(\theta_A) = n_B \sin(\theta_B)}$,

REFLECT

Since the first and last media are both air, the refracted angle will be equal to the incident angle. Assuming the other media have nonnegligible thicknesses, the outgoing light ray will be shifted vertically relative to the incoming ray.

23.84

SET UP

Fermat's principle states that light travels between two points in the path that minimizes the time of flight. We can use this principle to derive the law of reflection ($\theta_i = \theta_r$) and Snell's law ($n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$). The total time the light travels between two points is equal to the distance traveled divided by the speed of the light. We can write an algebraic expression describing this in terms of the physical parameters and geometry of the system. Then we can minimize the total time with respect to the physical parameters; mathematically, this consists of taking the derivative and setting it equal to zero. We can then rearrange our results to arrive at the law of reflection and Snell's law.

SOLVE

Part a)

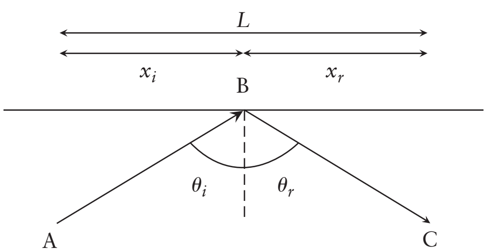


Figure 23-26 Problem 84

Total time of flight:

$$t = t_{AB} + t_{BC} = \frac{AB}{c} + \frac{BC}{c} = \frac{\left(\frac{x_i}{\sin(\theta_i)}\right)}{c} + \frac{\left(\frac{x_r}{\sin(\theta_r)}\right)}{c}$$

Minimization:

$$\frac{dt}{dx_i} = 0 = \frac{d}{dx_i}\left[\frac{1}{c}\left(\frac{x_i}{\sin(\theta_i)} + \frac{x_r}{\sin(\theta_r)}\right)\right] = \frac{1}{c}\left[\frac{1}{\sin(\theta_i)} + \left(\frac{dx_r}{dx_i}\right)\left(\frac{1}{\sin(\theta_r)}\right)\right]$$

But $L = x_i + x_r$

$$\frac{dL}{dx_i} = 0 = 1 + \frac{dx_r}{dx_i}$$

$$\frac{dx_r}{dx_i} = -1$$

Plugging this into our minimization:

$$0 = \frac{1}{c}\left[\frac{1}{\sin(\theta_i)} + (-1)\left(\frac{1}{\sin(\theta_r)}\right)\right]$$

$$0 = \frac{1}{\sin(\theta_i)} - \frac{1}{\sin(\theta_r)}$$

$$\sin(\theta_i) = sin(\theta_r)$$

$$\boxed{\theta_i = \theta_r}$$

Part b)

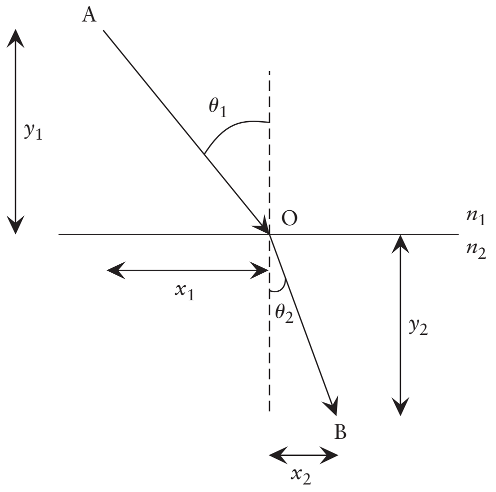


Figure 23-27 Problem 84

Total time:

$$t = t_{\text{AO}} + t_{\text{OB}} = \frac{\text{AO}}{v_1} + \frac{\text{OB}}{v_2} = \frac{y_1 \sec(\theta_1)}{v_1} + \frac{y_2 \sec(\theta_2)}{v_2}$$

Minimization:

$$\frac{dt}{d\theta_1} = 0 = \frac{d}{d\theta_1}\left[\frac{y_1 \sec(\theta_1)}{v_1} + \frac{y_2 \sec(\theta_2)}{v_2}\right] = \frac{y_1}{v_1}\sec(\theta_1)\tan(\theta_1) + \frac{y_2}{v_2}\sec(\theta_2)\tan(\theta_2)\left(\frac{d\theta_2}{d\theta_1}\right)$$

But $x_1 + x_2 =$ constant,

$$x_1 + x_2 = y_1 \tan(\theta_1) + y_2 \tan(\theta_2) = \text{constant}$$

$$\frac{d}{d\theta_1}[y_1 \tan(\theta_1) + y_2 \tan(\theta_2)] = 0$$

$$y_1 \sec^2(\theta_1) + y_2 \sec^2(\theta_2)\left(\frac{d\theta_2}{d\theta_{1s}}\right) = 0$$

$$\frac{d\theta_2}{d\theta_{1s}} = \frac{-y_1 \sec^2(\theta_1)}{y_2 \sec^2(\theta_2)}$$

Plugging this into our minimization:

$$0 = \frac{y_1}{v_1}\sec(\theta_1)\tan(\theta_1) + \frac{y_2}{v_2}\sec(\theta_2)\tan(\theta_2)\left(\frac{-y_1 \sec^2(\theta_1)}{y_2 \sec^2(\theta_2)}\right)$$

$$0 = \frac{1}{v_1}\sec(\theta_1)\tan(\theta_1) - \frac{1}{v_2}\sec(\theta_2)\tan(\theta_2)\left(\frac{\sec^2(\theta_1)}{\sec^2(\theta_2)}\right)$$

$$0 = \frac{1}{v_1}\tan(\theta_1) - \frac{1}{v_2}\tan(\theta_2)\left(\frac{\sec(\theta_1)}{\sec(\theta_2)}\right)$$

$$\frac{1}{v_1}\frac{\tan(\theta_1)}{\sec(\theta_1)} = \frac{1}{v_2}\frac{\tan(\theta_2)}{\sec(\theta_2)}$$

$$\frac{1}{v_1}\sin(\theta_1) = \frac{1}{v_2}\sin(\theta_2)$$

$$\left(\frac{n_1}{c}\right)\sin(\theta_1) = \left(\frac{n_2}{c}\right)\sin(\theta_2)$$

$$\boxed{n_1\sin(\theta_1) = n_2\sin(\theta_2)}$$

REFLECT

We arbitrarily chose to minimize the time with respect to x_i and θ_1. The derivation also works if we minimize with respect to x_r and θ_2.

23.85

SET UP

An object is located $d = 0.850$ m below the surface of a pond of clear water ($n_2 = 1.33$) directly under a dock. The end of the dock is located a horizontal distance D from the object. If we cannot see the object from the end of the dock, then all of the light from the object is totally internally reflected. Using the indices of refraction of water and air ($n_1 = 1.00$), we can find the critical angle. The distance D can be calculated from the critical angle and trigonometry. If the object is seen at any distance from the dock, then the light from the object will never be totally internally reflected. Therefore, the index of refraction of the water must be lower than the index of refraction of air.

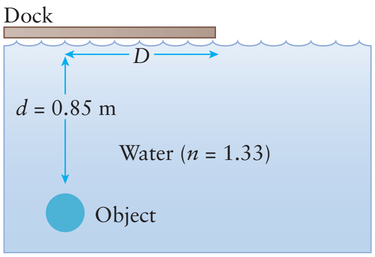


Figure 23-28 Problem 85

SOLVE

Part a)
Critical angle:

$$\theta_c = \arcsin\left(\frac{n_2}{n_1}\right) = \arcsin\left(\frac{1.00}{1.33}\right) = 48.8°$$

Distance of the dock:

$$\tan(\theta_c) = \frac{D}{d}$$

$$D = d\tan(\theta_c) = (0.850\ \text{m})\tan(48.8°) = \boxed{0.969\ \text{m}}$$

Part b) You would have to change the index of refraction of water so that it is smaller than the index of refraction of air. Under that condition totally internal reflection cannot occur.

REFLECT

Totally internal reflection is only possible when traveling from a medium with a higher index into a medium with a lower index.

23.86

SET UP

A light ray travels from air ($n_1 = 1.00$) through a layer of acrylic ($n_2 = 1.50$) and into seawater ($n_3 = 1.37$). The light ray impinges the air–acrylic interface at an angle of $\theta_1 = 40°$ with respect to the vertical. We will need to apply Snell's law at each interface to determine the angles of refraction in both the acrylic and the water.

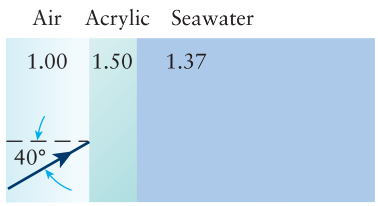


Figure 23-29 Problem 86

SOLVE

Part a)

$$n_1\sin(\theta_1) = n_2\sin(\theta_2)$$

$$\sin(\theta_2) = \frac{n_1}{n_2}\sin(\theta_1)$$

$$\theta_2 = \arcsin\left[\frac{n_1}{n_2}\sin(\theta_1)\right] = \arcsin\left[\left(\frac{1.00}{1.50}\right)\sin(40°)\right] = \boxed{25°}$$

Part b)

$$n_2\sin(\theta_2) = n_3\sin(\theta_3)$$

$$\sin(\theta_3) = \frac{n_2}{n_3}\sin(\theta_2)$$

$$\theta_3 = \arcsin\left[\frac{n_2}{n_3}\sin(\theta_2)\right] = \arcsin\left[\left(\frac{1.50}{1.37}\right)\sin(25°)\right] = \boxed{28°}$$

REFLECT

The refracted angle should be larger than the incident angle when traveling from a region of higher index of refraction into a region of lower index of refraction.

23.87

SET UP

A flat glass surface ($n_{\text{glass}} = 1.54$) has a uniform layer of water ($n_{\text{water}} = 1.33$) on top of the glass. We want to know the minimum angle of incidence that light coming from the glass must strike the glass–water interface such that the light is totally internally reflected by the water–air interface. The easiest way of tackling this problem is by working backwards. First, we can calculate the critical angle for the water–air interfaces from $\theta_c = \arcsin\left(\frac{n_{\text{air}}}{n_{\text{water}}}\right)$. This angle is also equal to the angle at which the light is refracted by the glass–water interface, which means we can apply Snell's law and calculate the incident angle of the light in the glass.

SOLVE

Critical angle for water–air interface:

$$\theta_c = \arcsin\left(\frac{n_{\text{air}}}{n_{\text{water}}}\right) = \arcsin\left(\frac{1.00}{1.33}\right) = 48.75°$$

Angle of incidence for the glass–water interface:

$$n_{\text{glass}} \sin(\theta_1) = n_{\text{water}} \sin(\theta_c)$$

$$\theta_1 = \arcsin\left(\frac{n_{\text{water}}}{n_{\text{glass}}} \sin(\theta_c)\right) = \arcsin\left(\frac{1.33}{1.54} \sin(48.75°)\right) = \boxed{40.5°}$$

REFLECT

We could have saved ourselves an intermediate step by realizing the water layer effectively does not enter into the calculation. Applying Snell's law at the first interface, we find $n_{\text{glass}} \sin(\theta_1) = n_{\text{water}} \sin(\theta_2)$. Applying it again at the second interface we get $n_{\text{water}} \sin(\theta_2) = n_{\text{air}} \sin(90°)$. Combining these two expressions, we can eliminate the term related to the water and see that $n_{\text{glass}} \sin(\theta_1) = n_{\text{air}}$, or $\theta_1 = \arcsin\left(\frac{n_{\text{air}}}{n_{\text{glass}}}\right) = \arcsin\left(\frac{1.00}{1.54}\right) = 40.5°$.

23.88

SET UP

A small source of light is located in water ($n_{\text{water}} = 1.33$) at a depth of $d = 3.00$ m. The radius of the circle of light seen at the surface ($n_{\text{air}} = 1.00$) is related to the critical angle for total internal reflection and the depth d by the tangent.

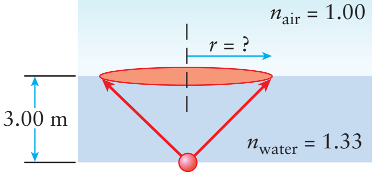


Figure 23-30 Problem 88

SOLVE

Critical angle:

$$\theta_c = \arcsin\left(\frac{n_{\text{air}}}{n_{\text{water}}}\right) = \arcsin\left(\frac{1.00}{1.33}\right) = 48.8°$$

Radius:

$$\tan(\theta_c) = \frac{r}{d}$$

$$r = d\tan(\theta_c) = (3.00 \text{ m})\tan(48.8°) = \boxed{3.42 \text{ m}}$$

REFLECT

The deeper the source of light is in the water, the larger the circle of light it makes.

23.89

SET UP

A baseball is located directly beneath a lily pad of diameter d in a pool of water ($n_2 = 1.33$) that has a depth $D = 4.00$ m. If we cannot see the baseball from anywhere, then that means the lily pad is larger than the circle of light coming from the ball due to totally internal reflection. Using the indices of refraction of water and air ($n_1 = 1.00$), we can find the critical angle. The radius, and then diameter, of the lily pad can be calculated from the critical angle and trigonometry.

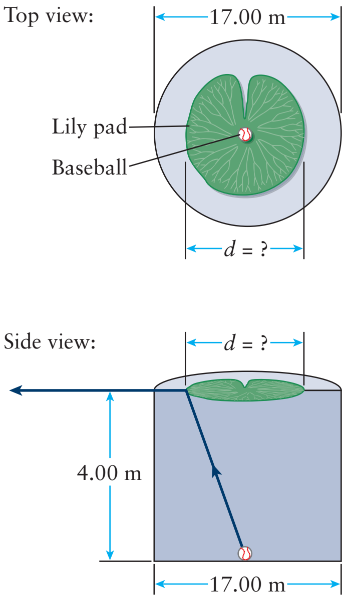


Figure 23-31 Problem 89

SOLVE

Critical angle:

$$\theta_c = \arcsin\left(\frac{n_2}{n_1}\right) = \arcsin\left(\frac{1.00}{1.33}\right) = 48.8°$$

Radius of the lily pad:

$$\tan(\theta_c) = \frac{r}{D}$$

$$r = D\tan(\theta_c) = (4.00\text{ m})\tan(48.8°) = 4.56\text{ m}$$

Diameter of the lily pad:

$$d = 2r = 2(4.56\text{ m}) = \boxed{9.12\text{ m}}$$

REFLECT

The diameter of the pool was not used since the diameter of the lily pad is what affects whether we can see the baseball.

23.90

SET UP

Light rays fall normally on the vertical surface of a glass prism ($n_{\text{glass}} = 1.55$). We want to know the largest value of θ (see figure) such that the ray is totally internally reflected at the slanted face. From geometry, the sum of θ and the critical angle is equal to 90 degrees. Using this condition and the definition of the critical angle, we can solve for θ when the prism is surrounded by air ($n_{\text{air}} = 1.00$) and when it is surrounded by water ($n_{\text{water}} = 1.33$).

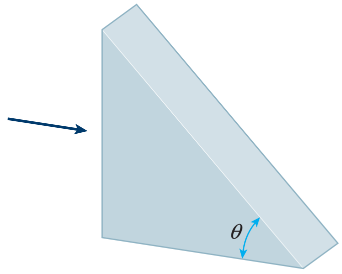


Figure 23-32 Problem 90

SOLVE

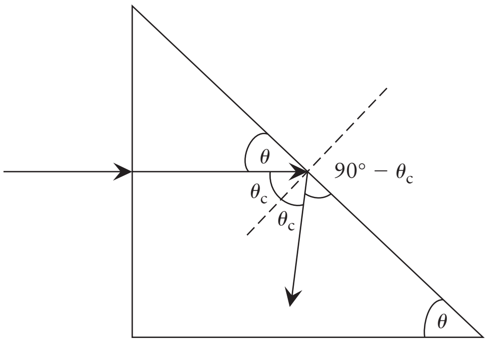


Figure 23-33 Problem 90

Part a)

$$\theta + \theta_c = 90°$$

$$\theta = 90° - \theta_c = 90° - \arcsin\left(\frac{n_{\text{air}}}{n_{\text{glass}}}\right) = 90° - \arcsin\left(\frac{1.00}{1.55}\right) = \boxed{49.8°}$$

Part b)

$$\theta + \theta_c = 90°$$

$$\theta = 90° - \theta_c = 90° - \arcsin\left(\frac{n_{\text{water}}}{n_{\text{glass}}}\right) = 90° - \arcsin\left(\frac{1.33}{1.55}\right) = \boxed{30.9°}$$

REFLECT

The effects of refraction are larger when there is a larger difference between the indices of refraction of the two media.

23.91

SET UP

Blue light and red light are incident on a glass slab at an angle of 35.0 degrees (see figure). The index of refraction for the blue light in the slab is $n_B = 1.521$, and the index of refraction for the red light in the slab is $n_R = 1.514$. We want to know the distance between the exit points of the red light and blue light on the opposite side of the slab. Since the index of refraction

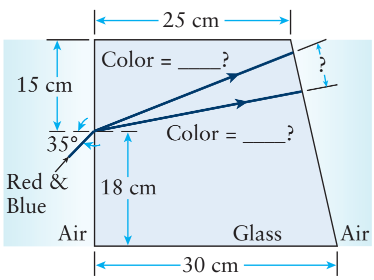


Figure 23-34 Problem 91

for the red light is smaller than that for the blue, the red light will bend less than the blue; therefore, the top ray exiting the right side of the slab corresponds to the red light. We can use Snell's law to determine the angles of refraction for the two rays. In order to determine the locations of the exit points on the slanted face, we need to treat the rays as lines and write their mathematical equations. We will also want to describe the slanted face with the equation of its line. The intersection of each ray's line with the line of the slanted face is equal to the exit point for each light ray. Once we determine the exact positions in space for the exit points, we can determine the distance between these two points.

SOLVE

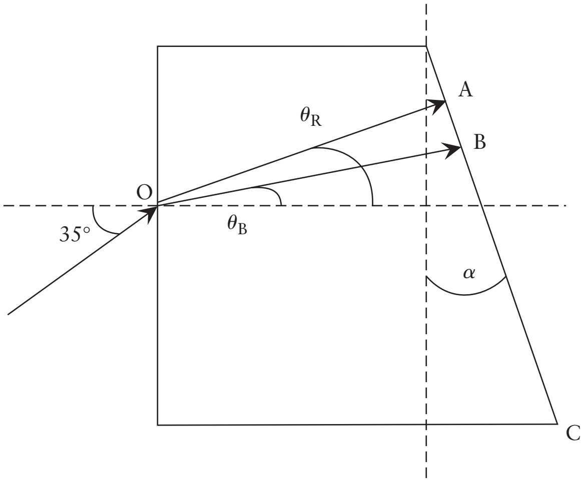


Figure 23-35 Problem 91

Angle of refraction, red light:

$$n_1 \sin(\theta_1) = n_R \sin(\theta_R)$$

$$\sin(\theta_R) = \frac{n_1}{n_R}\sin(\theta_1)$$

$$\theta_R = \arcsin\left[\frac{n_1}{n_R}\sin(\theta_1)\right] = \arcsin\left[\frac{1.000}{1.514}\sin(35.0°)\right] = 22.26°$$

Angle of refraction, blue light:

$$n_1 \sin(\theta_1) = n_B \sin(\theta_B)$$

$$\sin(\theta_B) = \frac{n_1}{n_B}\sin(\theta_1)$$

$$\theta_B = \arcsin\left[\frac{n_1}{n_B}\sin(\theta_1)\right] = \arcsin\left[\frac{1.000}{1.521}\sin(35.0°)\right] = 22.15°$$

Equation of line $\overline{\text{OA}}$:

$$y_R = x\tan(22.26°) = 0.4094x$$

Equation of line $\overline{\text{OB}}$:

$$y_R = x\tan(22.15°) = 0.4072x$$

Slope of line $\overline{\text{CBA}}$:

$$m_{CBA} = \frac{\Delta y}{\Delta x} = \frac{-33.0\text{ cm}}{5.00\text{ cm}} = -\frac{33.0}{5.00}$$

Intercept of line $\overline{\text{CBA}}$:

$$y = 0 \text{ at } x = (25.0\text{ cm}) + (15.0\text{ cm})\left(\frac{5.00}{33.0}\right) = 27.27\text{ cm}$$

$$0 = \left(-\frac{33.0}{5.00}\right)(27.27\text{ cm}) + b$$

$$b = 180\text{ cm}$$

Equation of line $\overline{\text{CBA}}$:

$$y_{CBA} = -\frac{33.0}{5.00}x + (180\text{ cm})$$

Intersection of red light and line $\overline{\text{CBA}}$:

$$y_R = y_{CBA}$$

$$0.4094x = -\frac{33.0}{5.00}x + 180$$

$$7.01x = 180$$

$$x = 25.68\text{ cm}$$

$$y = 0.4094(25.68\text{ cm}) = 10.51\text{ cm}$$

Intersection of blue light and line $\overline{\text{CBA}}$:

$$y_B = y_{CBA}$$

$$0.4072x = -\frac{33.0}{5.00}x + 180$$

$$7.007x = 180$$

$$x = 25.69\text{ cm}$$

$$y = 0.4094(25.68\text{ cm}) = 10.46\text{ cm}$$

Distance between the red and blue intersections:

$$AB = \sqrt{((25.69\text{ cm}) - (25.68\text{ cm})^2) + ((10.46\text{ cm}) - (10.51\text{ cm})^2)}$$

$$= \boxed{0.0538\text{ cm}}$$

REFLECT

We would not expect dispersion effects to be large in this case, given how small the glass slab is.

23.92

SET UP

Brewster's angle for light that passes from water ($n_{\text{water}} = 1.33$) into a certain plastic of index n_{plastic} is $\theta_B = 61.4°$. We can use the definition of Brewster's angle to calculate n_{plastic}. From this, we can use the definition of the critical angle for totally internal reflection to find the critical angle for the light traveling from the plastic into air.

SOLVE

Index of refraction of the plastic:

$$\tan(\theta_B) = \frac{n_{\text{plastic}}}{n_{\text{water}}}$$

$$n_{\text{plastic}} = n_{\text{water}} \tan(\theta_B) = (1.33)\tan(61.4°) = 2.44$$

Critical angle:

$$\theta_c = \arcsin\left(\frac{n_{\text{air}}}{n_{\text{plastic}}}\right) = \arcsin\left(\frac{1.00}{2.44}\right) = \boxed{24.2°}$$

REFLECT

The critical angle for the light traveling from the plastic back into water is around 33 degrees. This value should be larger than the plastic–air interface because the indices of refraction for the plastic and water are closer to each other than the plastic and air.

23.93

SET UP

Unpolarized light with an intensity of $I_0 = 100\frac{\text{W}}{\text{m}^2}$ is incident on three polarizers. Two of them are placed with their transmission axes perpendicular to each other. A third polarizer is placed in between the two crossed polarizers such that the transmission axis of the second polarizer is oriented 30 degrees relative to that of the first. This means the transmission axis of the third polarizer is oriented 60 degrees relative to that of the second. In general, the intensity of unpolarized light drops by a factor of two when it passes through a linear polarizer. The intensity of the now linearly polarized light after it passes through a polarizer depends on the previous intensity and the square of the cosine of the angle between the polarization axis of the light and the transmission axis of the polarizer. The orientation of the middle polarizer that maximizes the transmitted intensity can be found by differentiating the general form for the transmitted intensity with respect to θ, setting it equal to zero, and solving for θ. Since the first and third polarizers are perpendicular to one another, we can call the angle between polarizers 1 and 2 θ and the angle between polarizers 2 and 3 $(90° - \theta)$.

SOLVE

Part a)

$$I_1 = \frac{I_0}{2}$$

$$I_2 = I_1 \cos^2(30°) = \left(\frac{I_0}{2}\right)\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3I_0}{8}$$

$$I_3 = I_2 \cos^2(60°) = \left(\frac{3I_0}{8}\right)\left(\frac{1}{2}\right)^2 = \frac{3I_0}{32} = \frac{3}{32}\left(100\frac{\mathrm{W}}{\mathrm{m}^2}\right) = \boxed{9.375\frac{\mathrm{W}}{\mathrm{m}^2}}$$

Part b)

Finding I_3 in terms of the general angle θ:

$$I_2 = I_1 \cos^2(\theta) = \frac{I_0}{2}\cos^2(\theta)$$

$$I_3 = I_2 \cos^2(90° - \theta) = \left(\frac{I_0}{2}\cos^2(\theta)\right)(\sin^2(\theta)) = \frac{I_0}{2}(\sin(\theta)\,cos(\theta))^2$$

$$= \frac{I_0}{2}\left(\frac{\sin(2\theta)}{2}\right)^2 = \frac{I_0}{8}\sin^2(2\theta)$$

Finding the maximum:

$$\frac{dI_3}{d\theta} = 0$$

$$\frac{d}{d\theta}\left[\frac{I_0}{8}\sin^2(2\theta)\right] = \frac{I_0}{8}(4\sin(2\theta)\cos(2\theta)) = \frac{I_0}{4}(\sin(4\theta)) = 0$$

$$4\theta = \arcsin(0) = 0,\ \pi,\ 2\pi\ldots$$

The solution $\theta = 0$ yields a minimum intensity, which means the maximum occurs at

$$4\theta = \pi$$

$$\theta = \frac{\pi}{4} = \boxed{45°}$$

REFLECT

After the fact, it makes sense that the middle sheet should be at an angle of 45 degrees relative to each polarizer due to symmetry in order to maximize the transmitted intensity.

Get Help: Interactive Example – Polarization I
Interactive Example – Polarization II

23.94

SET UP

White light illuminates a thin film ($n_{\mathrm{film}} = 2.10$) normal to the surface, and we observe that both blue light ($\lambda_b = 495$ nm) and red light ($\lambda_r = 660$ nm) are not reflected. The film has a

thickness t and is coating a glass lens ($n_{\text{glass}} = 1.57$). The condition on the path difference for destructive interference in the case when the film has a higher index of refraction than the medium under it is $(m+1)\frac{\lambda_0}{n_{\text{film}}}$. Note that the wavelength of light changes when the index of refraction changes. We are interested in the wavelength of light in the film, so we need to divide the given wavelength by the index of refraction of the film. Since the thickness of the film is t, this path difference should be equal to $2t$. This same path difference does not reflect *two* wavelengths of light, so we will have two expressions with different integers, m_{b} for the blue light and m_{r} for the red light. Setting the thicknesses equal to one another, we can solve for the smallest integers that satisfy the resulting relationship. The minimum thickness of the film can be found by plugging the integer back into the constructive interference relationship.

SOLVE

Blue light ($\lambda_{\text{b}} = 495\text{ nm}$):

$$2t = (m_{\text{b}}+1)\frac{\lambda_{\text{b}}}{n_{\text{film}}}$$

Red light ($\lambda_{\text{r}} = 660\text{ nm}$):

$$2t = (m_{\text{r}}+1)\frac{\lambda_{\text{r}}}{n_{\text{film}}}$$

Solving for m's:

$$(m_{\text{b}}+1)\frac{\lambda_{\text{b}}}{n_{\text{film}}} = (m_{\text{r}}+1)\frac{\lambda_{\text{r}}}{n_{\text{film}}}$$

$$(m_{\text{b}}+1)(495\text{ nm}) = (m_{\text{r}}+1)(660\text{ nm})$$

$$3m_{\text{b}} + 3 = 4m_{\text{r}} + 4$$

$$3m_{\text{b}} - 4m_{\text{r}} = 1$$

This is true if $m_{\text{b}} = 3$ and $m_{\text{r}} = 2$.

Thickness:

$$t = (m_{\text{b}}+1)\frac{\lambda_{\text{b}}}{2n_{\text{film}}} = (3+1)\frac{495\text{ nm}}{2(2.10)} = \boxed{471\text{ nm}}$$

REFLECT

The net phase change due to reflections in the film is equal to one-half wavelength. This is why the phase change due to the path difference must be equal to one full wavelength for destructive interference to occur.

23.95

SET UP

Unpolarized light ($I_0 = 850\frac{\text{W}}{\text{m}^2}$) is incident on a series of polarizers. The axis of the first polarizer is horizontal. The axis of

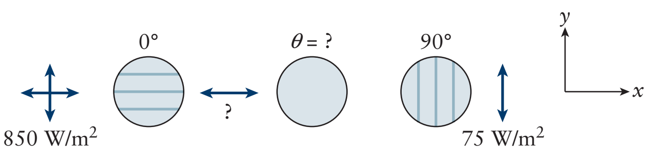


Figure 23-36 Problem 95

the second polarizer makes an angle θ with the horizontal. The axis of the third polarizer is vertical, or $(90° - \theta)$ with respect to the second polarizer. The intensity after the third polarizer is $I_3 = 75\frac{\text{W}}{\text{m}^2}$. The intensity after the first polarizer drops by a factor of 2 because the light is initially unpolarized. After each successive polarizer, the intensity is related to the angle between the polarization axis of the light and the polarization axis of the polarizer by $I_{\text{after}} = I_{\text{before}} \cos^2(\theta)$. We can use this to express the final intensity I_3 in terms of the initial intensity I_0 and the unknown angle in order to solve for θ.

SOLVE

After the first polarizer:

$$I_1 = \frac{1}{2}I_0$$

After the second polarizer:

$$I_2 = I_1 \cos^2(\theta) = \left(\frac{1}{2}I_0\right)\cos^2(\theta)$$

After the third polarizer:

$$I_3 = I_2 \cos^2(90° - \theta) = \left(\frac{1}{2}I_0 \cos^2(\theta)\right)\cos^2(90° - \theta) = \frac{I_0}{2}\cos^2(\theta)\sin^2(\theta)$$

$$= \frac{I_0}{2}\left(\frac{1}{2}\sin(2\theta)\right)^2 = \frac{I_0}{8}\sin^2(2\theta)$$

Angle of the second polarizer:

$$\sin^2(2\theta) = \frac{8I_3}{I_0}$$

$$\sin(2\theta) = \sqrt{\frac{8I_3}{I_0}}$$

$$2\theta = \arcsin\left(\sqrt{\frac{8I_3}{I_0}}\right)$$

$$\theta = \frac{\arcsin\left(\sqrt{\frac{8I_3}{I_0}}\right)}{2} = \frac{\arcsin\left(\sqrt{\frac{8\left(75\frac{\text{W}}{\text{m}^2}\right)}{\left(850\frac{\text{W}}{\text{m}^2}\right)}}\right)}{2} = \boxed{29° \text{ or } 61°}$$

REFLECT

There are two answers due to the symmetry of the polarizer.

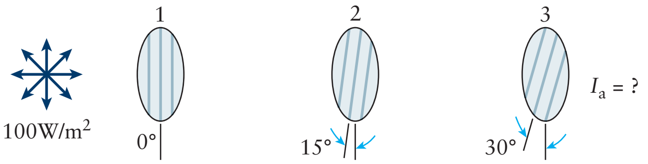
23.96

SET UP

Unpolarized light $\left(I_0 = 100\dfrac{\text{W}}{\text{m}^2}\right)$ shines on two sets of polarizers. The first set ("situation a") consists of 3 polarizers, each rotated 15 degrees relative to the previous one. The second set ("situation b") consists of 31 polarizers, each rotated 1 degree relative to the previous one. In either case, the intensity after the first polarizer is equal to one-half the initial intensity because the light is initially unpolarized. We can calculate the final intensities in the two situations by applying $I_{\text{after}} = I_{\text{before}} \cos^2(\theta)$ for each polarizer.

SOLVE

Intensity in situation (a):



(a) The vertical polarizer (1) is followed by a second filter that is tilted at 15° from the vertical, and then by a third filter tilted at 30° from the vertical.

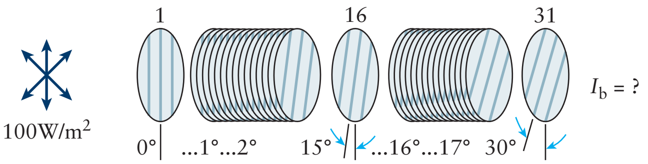
Figure 23-37 Problem 96

$$I_1 = \frac{1}{2}I_0$$

$$I_2 = I_1 \cos^2(15°) = \left(\frac{1}{2}I_0\right)\cos^2(15°)$$

$$I_a = I_2 \cos^2(15°) = \left(\frac{1}{2}I_0 \cos^2(15°)\right)\cos^2(15°) = \frac{1}{2}I_0 \cos^4(15°)$$

Intensity in situation (b):



(b) The vertical polarizer (1) is followed by 30 polarizing filters, each one tilted 1° more than the previous one...the figure is meant to be illustrative only.

Figure 23-38 Problem 96

$$I_1 = \frac{1}{2}I_0$$

$$I_2 = I_1 \cos^2(1°) = \left(\frac{1}{2}I_0\right)\cos^2(1°)$$

$$I_3 = I_2 \cos^2(1°) = \left(\frac{1}{2}I_0 \cos^2(1°)\right)\cos^2(1°) = \frac{1}{2}I_0 \cos^4(1°)$$

$$\vdots$$

$$I_b = I_{30} \cos^2(1°) = \frac{1}{2}I_0 \cos^{60}(1°)$$

Ratio of final intensities:

$$\frac{I_a}{I_b} = \frac{\left(\frac{1}{2}I_0 \cos^4(15°)\right)}{\left(\frac{1}{2}I_0 \cos^{60}(1°)\right)} = \frac{\cos^4(15°)}{\cos^{60}(1°)} = \boxed{0.88}$$

REFLECT

Even though there are more polarizers in situation (b), the intensity decreases more quickly in situation (a) due to the larger angles between neighboring polarizers.

23.97

SET UP

White light illuminates a soap film (index of refraction $n_{\text{film}} = 1.33$) normal to the surface, and we observe that light with wavelengths $\lambda_{400} = 400$ nm, $\lambda_{480} = 480$ nm, and $\lambda_{600} = 600$ nm is not reflected. The film has a thickness t and is sandwiched by air ($n_{\text{air}} = 1.00$). The condition on the path difference for destructive interference in the case when the film has a higher index of refraction than the medium under it is $(m + 1)\dfrac{\lambda_0}{n_{\text{film}}}$. Note that the wavelength of light changes when the index of refraction changes. We are interested in the wavelength of light in the film, so we need to divide the given wavelength by the index of refraction of the film. Since the thickness of the film is t, this path difference should be equal to $2t$. This same path difference causes *three* wavelengths of light to be destroyed, so we will have three expressions with different integers (m_{400}, m_{480}, m_{600}). Setting the thicknesses equal to one another, we can solve for the smallest integers that satisfy the resulting relationship. The thickness of the film can be found by plugging the integers back into the destructive interference relationship.

SOLVE

400-nm light:

$$2t = (m_{400} + 1)\frac{\lambda_{400}}{n_{\text{film}}}$$

480-nm light:

$$2t = (m_{480} + 1)\frac{\lambda_{480}}{n_{\text{film}}}$$

600-nm light:

$$2t = (m_{600} + 1)\frac{\lambda_{600}}{n_{\text{film}}}$$

Solving for m's:

$$(m_{400}+1)\frac{\lambda_{400}}{n_{\text{film}}}=(m_{480}+1)\frac{\lambda_{480}}{n_{\text{film}}}=(m_{600}+1)\frac{\lambda_{600}}{n_{\text{film}}}$$

$$(m_{400}+1)(400\text{ nm})=(m_{480}+1)(480\text{ nm})=(m_{600}+1)(600\text{ nm})$$

$$10m_{400}+10=12m_{480}+12=15m_{600}+15$$

This is true if $m_{400}=5$, $m_{480}=4$, and $m_{600}=3$.
Minimum thickness:

$$t=(m_{400}+1)\frac{\lambda_{400}}{2n_{\text{film}}}=(5+1)\frac{400\text{ nm}}{2(1.33)}=\boxed{902\text{ nm}}$$

REFLECT

We could have also used the expression for the destructive interference of the other wavelengths:

$$t=(m_{480}+1)\frac{\lambda_{480}}{2n_{\text{film}}}=(4+1)\frac{480\text{ nm}}{2(1.33)}=902\text{ nm}$$

$$t=(m_{600}+1)\frac{\lambda_{600}}{2n_{\text{film}}}=(3+1)\frac{600\text{ nm}}{2(1.33)}=902\text{ nm}$$

23.98

SET UP

A thin brass sheet ($\alpha=18.7\times10^{-6}\text{ K}^{-1}$) has a thin slit of width w_1 scratched into it. At room temperature ($T_1=295$ K), a laser beam of wavelength λ is shined on the slit and the first diffraction minimum occurs at $\theta_1=25°$. The brass sheet is then immersed in liquid nitrogen ($T_2=77$ K) until it reaches thermal equilibrium. This will cause the brass to contract along each dimension, which means the slit width will also change according to $\Delta w=w_2-w_1=w_1\alpha\Delta T$, where w_2 is the slit width at T_2. Using this new slit width, we can calculate the angular position of the first diffraction minimum when the laser is shined on the slit when the brass is at T_2.

SOLVE

Room temperature:

$$\sin(\theta_1)=\frac{\lambda}{w_1}$$

$$w_1\sin(\theta_1)=\lambda$$

Liquid nitrogen temperature:

$$\sin(\theta_2)=\frac{\lambda}{w_2}$$

$$w_2\sin(\theta_2)=\lambda$$

Thermal contraction:

$$\Delta w = w_2 - w_1 = w_1\alpha(T_2 - T_1) = w_1\alpha\Delta T$$

$$w_2 = w_1 + w_1\alpha\Delta T = w_1(1 + \alpha\Delta T)$$

Solving for θ_2:

$$w_1 \sin(\theta_1) = w_2 \sin(\theta_2)$$

$$w_1 \sin(\theta_1) = (w_1(1 + \alpha\Delta T))\sin(\theta_2)$$

$$\sin(\theta_1) = (1 + \alpha\Delta T)\sin(\theta_2)$$

$$\sin(\theta_2) = \frac{\sin(\theta_1)}{1 + \alpha\Delta T}$$

$$\theta_2 = \arcsin\left(\frac{\sin(\theta_1)}{1 + \alpha\Delta \mathrm{T}}\right) = \arcsin\left(\frac{\sin(25.0°)}{1 + (18.7 \times 10^{-6}\ \mathrm{K}^{-1})((77\ \mathrm{K}) - (295\ \mathrm{K}))}\right) = \boxed{25.1°}$$

REFLECT

Since the slit width decreases, we expect the central maximum to widen.

23.99

SET UP

A slip of paper of thickness T is placed between the edges of two thin plates of glass that have a length of $L = 0.125$ m. This causes the top plate to make an angle θ with respect to the bottom plate. When light ($\lambda = 600 \times 10^{-9}$ m) is shone normally on the glass plates, interference fringes due to destructive interference are observed. The spacing along the plate between neighboring fringes is $x = 0.200 \times 10^{-3}$ m. The thickness of the air film changes along the length of the plates. We can use the relationship for destructive interference in order to calculate the difference in the thickness of the film at successive fringes; this distance is related to x by $\sin(\theta)$. Since we expect the thickness of one sheet of paper to be very small, the angle between the glass plates will also be very small, which means we can invoke the small angle approximation: $\tan(\theta) \approx \sin(\theta) \approx \theta$. This angle is also related to the thickness of the paper and the length of the bottom plate by $\tan(\theta)$, which is approximately equal to θ. By setting the two expressions for θ equal to one another, we can solve for T.

SOLVE

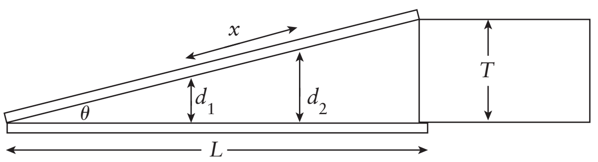


Figure 23-39 Problem 99

Destructive interference at successive fringes:

$$2d_1 = \left(m_1 + \frac{1}{2}\right)\lambda$$

$$2d_2 = \left(m_2 + \frac{1}{2}\right)\lambda = \left((m_1 + 1) + \frac{1}{2}\right)\lambda = \left(m_1 + \frac{3}{2}\right)\lambda$$

Horizontal distance between successive fringes:

$$2d_2 - 2d_1 = \left(m_1 + \frac{3}{2}\right)\lambda - \left(m_1 + \frac{1}{2}\right)\lambda = \lambda$$

$$d_2 - d_1 = \frac{\lambda}{2}$$

Angle in terms of the distance along the plate between successive fringes:

$$\sin(\theta) = \frac{d_2 - d_1}{x} = \frac{\left(\frac{\lambda}{2}\right)}{x} = \frac{\lambda}{2x}$$

$$\sin(\theta) \approx \theta \approx \frac{\lambda}{2x}$$

Angle in terms of the thickness of the paper:

$$\tan(\theta) = \frac{T}{L}$$

$$\tan(\theta) \approx \theta \approx \frac{T}{L}$$

Thickness of the paper:

$$\frac{T}{L} = \frac{\lambda}{2x}$$

$$T = \frac{\lambda L}{2x} = \frac{(600 \times 10^{-9}\text{ m})(0.125\text{ m})}{2(0.200 \times 10^{-3}\text{ m})} = \boxed{1.88 \times 10^{-4}\text{ m} = 188\ \mu\text{m}}$$

REFLECT

A thickness of about 0.2 mm seems reasonable for a single sheet of paper.

23.100

SET UP

A thin layer of SiO ($n_{\text{film}} = 1.45$) is used to coat a solar cell ($n_{\text{cell}} = 3.5$). In part (a), we want to calculate the minimum thickness t of the film that causes light of wavelength $\lambda_{400} = 400$ nm to not be reflected. The condition on the path difference for destructive interference in the case when the film has a lower index of refraction than the medium under it is $\left(m + \frac{1}{2}\right)\frac{\lambda_0}{n_{\text{film}}}$. Note that the wavelength of light changes when the index of refraction changes. We are interested in the wavelength of light in the film, so we need to divide the given wavelength by the index of refraction of the film. Since the thickness of the film is t, this path difference should be equal to $2t$. The smallest thickness of the film can be found by plugging $m = 0$ into the destructive interference relationship. Once we know the thickness, we can rearrange the expression for destructive interference to see if any other visible wavelengths are canceled as well. In part (b), the thickness of the SiO film is tripled relative to the thickness in part

(a) ($t_{new} = 3t$). We can apply the condition for destructive interference and the condition for constructive interference when the film has a lower index of refraction than the medium under it, $(m + 1)\frac{\lambda_0}{n_{film}}$, to calculate the visible wavelengths that are canceled or reinforced, respectively, in this new film.

SOLVE

Part a)
Minimum thickness:

$$2t = \left(m_{400} + \frac{1}{2}\right)\frac{\lambda_{400}}{n_{film}}$$

$$t = \left(m_{400} + \frac{1}{2}\right)\frac{\lambda_{400}}{2n_{film}} = \left(0 + \frac{1}{2}\right)\frac{(400\text{ nm})}{2(1.45)} = \boxed{69.0\text{ nm}}$$

Other canceled visible wavelengths:

$$2t = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_{film}}$$

$$\lambda = \frac{2tn_{film}}{\left(m + \frac{1}{2}\right)} = \frac{2(69.0\text{ nm})(1.45)}{\left(m + \frac{1}{2}\right)} = \frac{200\text{ nm}}{\left(m + \frac{1}{2}\right)}$$

This will always be less than 200 nm for any positive integer m; therefore, $\boxed{\text{no additional wavelengths are canceled}}$.

Part b)
Canceled wavelengths:

$$2t_{new} = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_{film}}$$

$$\lambda = \frac{2t_{new}n_{film}}{\left(m + \frac{1}{2}\right)} = \frac{2((3)(69.0\text{ nm}))(1.45)}{\left(m + \frac{1}{2}\right)} = \frac{600\text{ nm}}{\left(m + \frac{1}{2}\right)}$$

The only visible wavelength canceled occurs for $m = 1$, which corresponds to $\boxed{\lambda = 400\text{ nm}}$.
Reinforced wavelengths:

$$2t_{new} = (m + 1)\frac{\lambda}{n_{film}}$$

$$\lambda = \frac{2t_{new}n_{film}}{(m + 1)} = \frac{2((3)(69.0\text{ nm}))(1.45)}{(m + 1)} = \frac{600\text{ nm}}{(m + 1)}$$

The only visible wavelength reinforced occurs for $m = 0$, which corresponds to $\boxed{\lambda = 600\text{ nm}}$.

REFLECT

There are no visible wavelengths strongly reflected in the first case.

23.101

SET UP

A pane of glass ($n_{glass} = 1.54$) is coated with a thin film of higher index and thickness $t = 155$ nm. The thin film should have an index of refraction n_{film} such that 550-nm light is not reflected. The condition on the path difference for destructive interference in the case when the film has a higher index of refraction than the medium under it is $(m + 1)\frac{\lambda_0}{n_{film}}$. Note that the wavelength of light changes when the index of refraction changes. We are interested in the wavelength of light in the film, so we need to divide the given wavelength by the index of refraction of the film. Since the thickness of the film is t, this path difference should be equal to $2t$. The index of refraction of the film can be found by plugging $m = 0$ into the destructive interference relationship. Once we know n_{film}, we can then calculate the next three thinnest films that will cancel 550-nm light; these thicknesses correspond to $m = 1, 2$, and 3, respectively.

SOLVE

Part a)

$$2t = (m + 1)\frac{\lambda}{n_{film}}$$

$$n_{film} = (m + 1)\frac{\lambda}{2t} = (0 + 1)\frac{550 \text{ nm}}{2(155 \text{ nm})} = \boxed{1.77}$$

Part b)

$$2t = (m + 1)\frac{\lambda}{n_{film}}$$

$$t = (m + 1)\frac{\lambda}{2n_{film}}$$

Thickness for $m = 1$:

$$t = (1 + 1)\frac{550 \text{ nm}}{2(1.77)} = \boxed{310 \text{ nm}}$$

Thickness for $m = 2$:

$$t = (2 + 1)\frac{550 \text{ nm}}{2(1.77)} = \boxed{465 \text{ nm}}$$

Thickness for $m = 3$:

$$t = (3 + 1)\frac{550 \text{ nm}}{2(1.77)} = \boxed{620 \text{ nm}}$$

REFLECT

The net phase change due to reflections in the film is equal to one-half wavelength. This is why the phase change due to the path difference must be equal to one full wavelength for destructive interference to occur.

23.102

SET UP

A thin wire of thickness $T = 5 \times 10^{-6}$ m is placed between the edges of two thin plates of glass that have a length of $L = 0.0800$ m. This causes the top plate to make an angle θ with respect to the bottom plate. When light ($\lambda = 500 \times 10^{-9}$ m) is shined normally on the glass plates, interference fringes due to destructive interference are observed. The spacing along the plate between neighboring fringes is x. The thickness of the air film changes along the length of the plates. We can use the relationship for destructive interference in order to calculate the difference in the thickness of the film at successive fringes; this distance is related to x by $\sin(\theta)$. Since we expect the thickness of one sheet of paper to be very small, the angle between the glass plates will also be very small, which means we can invoke the small angle approximation: $\tan(\theta) \approx \sin(\theta) \approx \theta$. This angle is also related to the thickness of the paper and the length of the bottom plate by $\tan(\theta)$, which is approximately equal to θ. By setting the two expressions for θ equal to one another, we can solve for x.

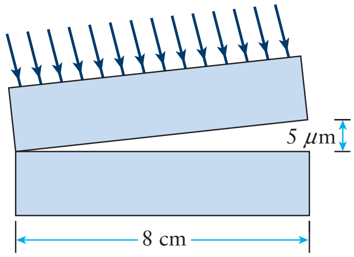


Figure 23-40 Problem 102

SOLVE

Destructive interference at successive fringes:

$$2d_1 = \left(m_1 + \frac{1}{2}\right)\lambda$$

$$2d_2 = \left(m_2 + \frac{1}{2}\right)\lambda = \left((m_1 + 1) + \frac{1}{2}\right)\lambda = \left(m_1 + \frac{3}{2}\right)\lambda$$

Horizontal distance between successive fringes:

$$2d_2 - 2d_1 = \left(m_1 + \frac{3}{2}\right)\lambda - \left(m_1 + \frac{1}{2}\right)\lambda = \lambda$$

$$d_2 - d_1 = \frac{\lambda}{2}$$

Angle in terms of the distance along the plate between successive fringes:

$$\sin(\theta) = \frac{d_2 - d_1}{x} = \frac{\left(\frac{\lambda}{2}\right)}{x} = \frac{\lambda}{2x}$$

$$\sin(\theta) \approx \theta \approx \frac{\lambda}{2x}$$

Angle in terms of the thickness of the paper:

$$\tan(\theta) = \frac{T}{L}$$

$$\tan(\theta) \approx \theta \approx \frac{T}{L}$$

Spacing of the dark fringes:

$$\frac{T}{L} = \frac{\lambda}{2x}$$

$$x = \frac{\lambda L}{2T} = \frac{(500 \times 10^{-9}\text{ m})(0.0800\text{ m})}{2(5 \times 10^{-6}\text{ m})} = \boxed{4 \times 10^{-3}\text{ m} = 4\text{ mm}}$$

REFLECT

The thicker the wire, the closer together are the dark fringes along the length of the plate.

23.103

SET UP

The distance between the crest and adjacent trough of a tsunami wave is 250 mi, which means the wavelength will be twice this distance. The period of the tsunami wave is $T = 1$ hr. The speed of the wave is equal to the wavelength divided by the period. The time it takes the tsunami to travel a distance of $\Delta x = 600$ mi is equal to that distance divided by the wave speed. These waves travel between a 100-mile-wide opening. To determine whether we can use the diffraction relationship $w\sin(\theta) = m\lambda$, we need to compare the slit width w to the wavelength λ. The slit needs to be "narrow," which means w must be approximately the same size or smaller than the wavelength.

SOLVE

Part a)

$$v = \lambda f = \frac{\lambda}{T} = \frac{2(250\text{ mi})}{1\text{ hr}} = \boxed{500\text{ mph}}$$

Part b)

$$v = \frac{\Delta x}{\Delta t}$$

$$\Delta t = \frac{\Delta x}{v} = \frac{600\text{ mi}}{500\text{ mph}} = \boxed{1.2\text{ hr}}$$

Part c) The formula $w\sin(\theta) = m\lambda$ applies only if the wavelength is smaller than the slit width w. In this case $w = 100$ mi and $\lambda = 500$ mi, so the formula $\boxed{\text{would } \textit{not} \text{ apply}}$.

REFLECT

If we were to apply $w\sin(\theta) = m\lambda$ to find the first diffraction minimum, we would find that $\sin(\theta) = \frac{\lambda}{w} = \frac{500\text{ mi}}{100\text{ mi}} = 5$. The sine of an angle can only yield a result between -1 and 1, so there is no solution to this equation. Physically, this means the first diffraction minimum does not exist, and, thus, diffraction does not occur.

23.104

SET UP

Two point sources that emit light at $\lambda = 500 \times 10^{-9}$ m are located $L = 100$ m from a photographer. The point sources are photographed using a camera with an aperture of diameter $D = 0.0105$ m. We can use trigonometry and the small angle approximation to calculate the minimum separation distance x at which the point sources are resolved; the diffraction-limited angular resolution is given by $\sin(\theta_R) = 1.22\frac{\lambda}{D}$.

SOLVE

Angular resolution:

$$\sin(\theta_R) = 1.22\frac{\lambda}{D}$$

Small angle approximation:

$$\sin(\theta_R) \approx \tan(\theta_R)$$

$$1.22\frac{\lambda}{D} \approx \frac{x}{L}$$

$$x \approx 1.22\frac{\lambda L}{D} = \frac{1.22(500 \times 10^{-9}\text{ m})(100\text{ m})}{(0.0105\text{ m})} = \boxed{5.81 \times 10^{-3}\text{ m} = 5.81\text{ mm}}$$

REFLECT

A larger aperture not only lets in more light, but also it resolves objects that are very close together.

23.105

SET UP

The angular resolution of light ($\lambda = 575 \times 10^{-9}$ m) when looking through an aperture of diameter $D = 0.75 \times 10^{-3}$ m is given by $\sin(\theta_R) = 1.22\frac{\lambda}{D}$. We can compare this to the angular resolution through a 4-mm pupil, $\theta_R = 1.75 \times 10^{-4}$ rad; we expect the resolution through the pupil to be smaller than the resolution through the aperture.

SOLVE

$$\sin(\theta_R) = 1.22\frac{\lambda}{D}$$

$$\theta_R = \arcsin\left(1.22\frac{\lambda}{\mathrm{D}}\right) = \arcsin\left(1.22\left(\frac{575 \times 10^{-9}\text{ m}}{0.75 \times 10^{-3}\text{ m}}\right)\right) = \boxed{9.4 \times 10^{-4}\text{ rad}}$$

This is larger than the resolution for the pupil, which makes sense because the pupil is larger in diameter than the hole.

REFLECT

A smaller resolution means the two objects can be closer together and still be resolved as being separate objects.

23.106

SET UP

The pupil of the eye is a circular aperture with a diameter that varies from $D = 2.0 \times 10^{-3}$ m in bright light to $D = 8.0 \times 10^{-3}$ m in dim light. We can use $\sin(\theta_R) = 1.22\frac{\lambda}{D}$ to calculate the angular resolution of the pupil for light of wavelength $\lambda = 550 \times 10^{-9}$ m in both bright and dim light. From these results, we can try to make sense of the observation that squinting helps us see things more clearly.

SOLVE

Part a)

Bright light:

$$\sin(\theta_R) = 1.22\frac{\lambda}{D}$$

$$\theta_R = \arcsin\left(1.22\frac{\lambda}{\mathrm{D}}\right) = \arcsin\left(1.22\left(\frac{550 \times 10^{-9}\ \mathrm{m}}{2.0 \times 10^{-3}\ \mathrm{m}}\right)\right) = \boxed{3.4 \times 10^{-4}\ \mathrm{rad}}$$

Dim light:

$$\sin(\theta_R) = 1.22\frac{\lambda}{D}$$

$$\theta_R = \arcsin\left(1.22\frac{\lambda}{\mathrm{D}}\right) = \arcsin\left(1.22\left(\frac{550 \times 10^{-9}\ \mathrm{m}}{8.0 \times 10^{-3}\ \mathrm{m}}\right)\right) = \boxed{8.4 \times 10^{-5}\ \mathrm{rad}}$$

The angular resolution is $\boxed{\text{better in dim light}}$ than in bright light because the angular resolution is smaller for dim light.

Part b) By squinting we partially close our eye, thereby limiting the light entering it. This causes the pupil to enlarge, which improves our angular resolution.

REFLECT

Our pupils get larger ("dilate") in dim light so that more light can enter.

23.107

SET UP

In bright light the pupil of the eye has a diameter $D = 2.0 \times 10^{-3}$ m. Blue-green light ($\lambda = 500$ nm) undergoes diffraction as it passes through the circular pupil. The angular position of the first dark ring is given by $\sin(\theta_1) = 1.22\frac{\lambda}{D}$. The diameter of the eyeball is $d = 25$ mm. Ignoring the curvature of the eyeball, we can relate the radius of the central bright spot x_1 to the angle associated with the first dark ring through geometry and the small

angle approximation, $x_1 = \theta_1 d$. In normal vision, we don't see alternating bright and dark rings, which must mean the dark rings are extremely narrow.

SOLVE

Part a)

$$\sin(\theta_1) = 1.22\frac{\lambda}{D}$$

$$\theta_1 = \arcsin\left[1.22\left(\frac{500 \times 10^{-9}\text{ m}}{2.0 \times 10^{-3}\text{ m}}\right)\right] = \boxed{3.1 \times 10^{-4}\text{ rad} = 0.017°}$$

Part b)

$$x_1 = \theta_1 d = (3.1 \times 10^{-4}\text{ rad})(25\text{ mm}) = \boxed{7.6 \times 10^{-3}\text{ mm}}$$

Part c) The dark ring and the surrounding bright rings are so close together that the bright rings are essentially right next to one another and mask the dark ring in between. This eliminates the diffraction effect and gives us only bright light.

REFLECT

Since the angle of the first diffraction minimum is so small, ignoring the curvature of the eyeball is a reasonable assumption.

23.108

SET UP

The pupil of a house cat's eye narrows to a thin vertical slit of width $w = 0.500 \times 10^{-3}$ m in very bright light of wavelength $\lambda = 550 \times 10^{-9}$ m. We can use $\sin(\theta) = \frac{m\lambda}{w}$ where $m =$ 1, 2, and 3 to calculate the three smallest angles corresponding to diffraction minima. If we consider that the index of refraction in the eye is $n = 1.4$, we need to use the wavelength of the light in the eye, $\lambda_{\text{eye}} = \frac{\lambda}{n}$, when calculating the angular positions of the three lowest-order maxima. To determine if the cat would see alternating fringes, we need to consider how close the dark fringes are to one another; if they are very closely spaced, the cat will not perceive them.

SOLVE

Part a)

$$\sin(\theta) = \frac{m\lambda}{w}$$

$$\theta = \arcsin\left(\frac{m\lambda}{w}\right)$$

$m = 1$:

$$\theta = \arcsin\left(\frac{(1)(550 \times 10^{-9}\text{ m})}{0.500 \times 10^{-3}\text{ m}}\right) = \boxed{1.10 \times 10^{-3}\text{ rad}}$$

$m = 2$:

$$\theta = \arcsin\left(\frac{(2)(550 \times 10^{-9}\text{ m})}{0.500 \times 10^{-3}\text{ m}}\right) = \boxed{2.20 \times 10^{-3}\text{ rad}}$$

$m = 3$:

$$\theta = \arcsin\left(\frac{(3)(550 \times 10^{-9}\text{ m})}{0.500 \times 10^{-3}\text{ m}}\right) = \boxed{3.30 \times 10^{-3}\text{ rad}}$$

Part b)
Wavelength in the eye:

$$\lambda_{\text{eye}} = \frac{\lambda}{n} = \frac{550 \times 10^{-9}\text{ m}}{1.4} = 393 \times 10^{-9}\text{ m}$$

$m = 1$:

$$\theta = \arcsin\left(\frac{(1)(393 \times 10^{-9}\text{ m})}{0.500 \times 10^{-3}\text{ m}}\right) = \boxed{7.9 \times 10^{-4}\text{ rad}}$$

$m = 2$:

$$\theta = \arcsin\left(\frac{(2)(393 \times 10^{-9}\text{ m})}{0.500 \times 10^{-3}\text{ m}}\right) = \boxed{1.6 \times 10^{-3}\text{ rad}}$$

$m = 3$:

$$\theta = \arcsin\left(\frac{(3)(393 \times 10^{-9}\text{ m})}{0.500 \times 10^{-3}\text{ m}}\right) = \boxed{2.4 \times 10^{-3}\text{ rad}}$$

Part c) The bright and dark fringes are too closely spaced for the cat to see them, so it would just see continuous bright light that decreased in intensity away from the center.

REFLECT

This is the same reason why we do not perceive alternating light and dark fringes in our vision.

23.109

SET UP

An optical telescope can achieve an angular resolution of 0.25 arcsec $\left(1\text{ arcsec} = \left(\frac{1}{3600}\right)^{\circ} = 4.85 \times 10^{-6}\text{ rad}\right)$. Rayleigh's criterion for the angular resolution through a circular aperture is $\sin(\theta_R) = 1.22\frac{\lambda}{D}$, where we're interested in the wavelengths of visible light. We will use a wavelength of 550×10^{-9} m to represent visible light. We can use this, along with the Rayleigh criterion and the desired resolution of 0.25 arcsec, to calculate the minimum value of D.

SOLVE

Part a)

$$\sin(\theta_R) = 1.22\frac{\lambda}{D}$$

$$D = 1.22\frac{\lambda}{\sin(\theta_R)} = 1.22\frac{\lambda}{\sin\left(\frac{1}{4}\text{arcsec} \times \frac{4.85 \times 10^{-6}\text{ rad}}{1\text{ arcsec}}\right)} = (1.01 \times 10^6)\lambda$$

$$= (1.01 \times 10^6)(550 \times 10^{-9}\text{ m}) = 0.55\text{ m} = 55\text{ cm}$$

The minimum diameter aperture should be around $\boxed{55\text{ cm}}$ to achieve a resolution of 0.25 arcsec.

Part b) Building a telescope with a diameter much larger than this won't improve the resolution significantly as long as you have to look through Earth's atmosphere. Telescopes are built larger than the diffraction-limited diameter for greater light collecting power, which allows you to see dimmer objects.

REFLECT

The Rayleigh criterion is just one rule of thumb used to determine whether or not two objects can be resolved through a circular aperture.

Get Help: P'Cast 23.9 – The Hubble Space Telescope

23.110

SET UP

The primary mirror of the Herschel infrared telescope has a diameter $D = 3.5$ m and focuses light in the range of 55×10^{-6} m to 672×10^{-6} m. The angular resolution of the telescope is given by $\sin(\theta_R) = 1.22\frac{\lambda}{D}$. Since the angular resolution is directly proportional to the wavelength, the smallest wavelength of the range will give the maximum angular resolution. We can use the same expression to determine the diameter of the mirror required to achieve the same resolution at a wavelength of 550×10^{-9} m. Finally, the size x of the smallest infrared source that the telescope can resolve at a distance $d = 150$ light-years can be found through trigonometry and the small angle approximation; the size x is related to the distance d through the tangent.

SOLVE

Part a)

$$\sin(\theta_R) = 1.22\frac{\lambda}{D}$$

$$\theta_R = \arcsin\left(1.22\frac{\lambda}{\mathrm{D}}\right)$$

Maximum angular resolution:

$$\theta_{\mathrm{R}} = \arcsin\left(1.22\left(\frac{55 \times 10^{-6}\ \mathrm{m}}{3.5\ \mathrm{m}}\right)\right) = \boxed{1.9 \times 10^{-5}\ \mathrm{rad} = 4.0''}$$

Part b)

$$D = 1.22\frac{\lambda}{\sin(\theta_{\mathrm{R}})} = 1.22\left(\frac{550 \times 10^{-9}\ \mathrm{m}}{\sin(1.9 \times 10^{-5}\ \mathrm{rad})}\right) = \boxed{0.035\ \mathrm{m}}$$

Part c)

$$\sin(\theta_{\mathrm{R}}) \approx \theta_{\mathrm{R}} \approx \tan(\theta_{\mathrm{R}}) = \frac{x}{d}$$

$$x \approx d\theta_{\mathrm{R}} = \left(150\ \mathrm{ly} \times \frac{9.461 \times 10^{15}\ \mathrm{m}}{1\ \mathrm{ly}}\right)(1.9 \times 10^{-5}\ \mathrm{rad}) = \boxed{2.7 \times 10^{13}\ \mathrm{m}}$$

REFLECT

A lens diameter of a few centimeters seems reasonable for a visible light telescope.

23.111

SET UP

The objective of a telescope that is located a distance $L = 200 \times 10^3$ m above Earth has a diameter $D = 1.02$ m. The minimum separation of two objects on the ground emitting visible light ($\lambda = 550 \times 10^{-9}$ m) can be found through trigonometry and the small angle approximation; the separation distance x is related to the distance L through the tangent. Taking atmospheric turbulence into account, two objects on Earth's surface can be distinguished only if their angular separation is $1.00''$ (where $3600'' = 1°$); in the small angle approximation, the separation distance x is equal to the arclength of the subtended angle.

SOLVE

Part a)

$$\sin(\theta_{\mathrm{R}}) \approx \tan(\theta_{\mathrm{R}})$$

$$1.22\frac{\lambda}{D} \approx \frac{x}{L}$$

$$x \approx 1.22\frac{\lambda L}{D} = 1.22\left(\frac{(550 \times 10^{-9}\ \mathrm{m})(200 \times 10^3\ \mathrm{m})}{1.02\ \mathrm{m}}\right) = \boxed{0.132\ \mathrm{m} = 13.2\ \mathrm{cm}}$$

Part b)

Conversion into radians:

$$1.00'' \times \frac{1°}{3600''} \times \frac{\pi\ \mathrm{rad}}{180°} = 4.85 \times 10^{-6}\ \mathrm{rad}$$

Separation distance:

$$x = L\theta = (200 \times 10^3\ \mathrm{m})(4.85 \times 10^{-6}\ \mathrm{rad}) = \boxed{0.970\ \mathrm{m}}$$

Since the resolution with atmospheric turbulence is less than the diffraction-limited resolution, the atmospheric resolution is the limiting factor, so the telescope can resolve only objects separated by about 1 m.

REFLECT

For a small angle, there is very little curvature in the arc associated with the subtended angle, so we can approximate it as a straight line rather than a curve.

23.112

SET UP

The primary mirror of a telescope has a diameter $D = 42$ m. The farthest Jupiter-sized planet (diameter $x = 1.43 \times 10^8$ m) the telescope could resolve can be found through trigonometry and the small angle approximation; the size x is related to the distance L through the tangent. We will assume the light the telescope focuses has a wavelength $\lambda = 550 \times 10^{-9}$ m. We can use the same algebraic expression to calculate the minimum diameter of the mirror needed to resolve a Jupiter-sized planet at a distance $L = 20$ light-years away.

SOLVE

Part a)

$$\sin(\theta_{\mathrm{R}}) \approx \tan(\theta_{\mathrm{R}})$$

$$1.22\frac{\lambda}{D} \approx \frac{x}{L}$$

$$L \approx \frac{xD}{1.22\lambda} = \frac{(1.43 \times 10^8 \text{ m})(42 \text{ m})}{1.22(550 \times 10^{-9} \text{ m})} = \boxed{9.0 \times 10^{15} \text{ m} = 0.95 \text{ ly}}$$

Part b)

$$1.22\frac{\lambda}{D} \approx \frac{x}{L}$$

$$D \approx \frac{1.22\lambda L}{x} = \frac{1.22(550 \times 10^{-9} \text{ m})\left(20 \text{ ly} \times \dfrac{9.461 \times 10^{15} \text{ m}}{1 \text{ ly}}\right)}{1.43 \times 10^8 \text{ m}} = \boxed{890 \text{ m}}$$

REFLECT

In order to resolve a Jupiter-sized planet, the mirror would need to be almost a kilometer in diameter!

Chapter 24
Geometrical Optics

Conceptual Questions

24.1 Actually, a plane mirror does neither, but instead inverts objects back to front. If the mirror inverted right and left, then the object's right hand that points east would appear on the images as a right hand pointing toward the west. Because the image is inverted back to front, the object facing north is transformed into an image that faces south. Also, the object's right hand is transformed into a left hand in the image.

24.2 Reflected light is usually diffuse or scattered because it reflects from an object's surface in many random directions. This occurs mainly for two reasons: Light shining on a surface generally comes from various directions and most surfaces are uneven.

24.3 A real image forms where light rays come together, whereas no light rays actually meet where a virtual image forms.

24.4 The mirror must be convex because only convex mirrors produce images that are upright and smaller than the object.

24.5 This phrase refers to the fact that the images in curved mirrors are not located at the same point as images in plane mirrors. In fact, the mirrors are designed so that if you see a vehicle in your rearview mirror, you should not change lanes because the vehicle is too close.

24.6 The radius of curvature of a plane mirror is equal to infinity. In terms of the mirror equation, if $d_i = -d_o$, then $\frac{1}{f}$ must go to zero, which means $f = \infty$.

24.7 Additional information is needed. In accordance with the lensmaker's equation, if the radius of curvature of the front surface is larger, it is a diverging lens; if the radius of curvature of the front surface is smaller, it is a converging lens.

Get Help: Picture It: Converging Lens

24.8 The focal length of the glass lens will increase. At the water–glass interface, the rays are not refracted as strongly as at the air–glass interface; therefore, the focal length increases.

24.9 The full image forms, but it is dimmer than before because it is formed with half as much light.

24.10 Our brain interprets an inverted image on the retina as a physically upright object. The brain "flips" the signal from the optic nerve.

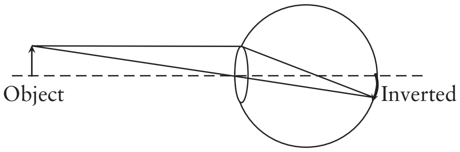


Figure 24-1 Problem 10

24.11 Since the brain can be "trained" to interpret all the nerve inputs that it receives, it is feasible that upright and inverted could be "redefined." It must be very disconcerting, however, for those several days when "up is down and down is up!"

24.12 A nearsighted eye focuses the image in front of the retinal plane, and a farsighted eye focuses the image in back of the retinal plane. We can draw ray diagrams to see why converging lenses are used to correct farsightedness, whereas diverging lenses are used to correct nearsightedness.

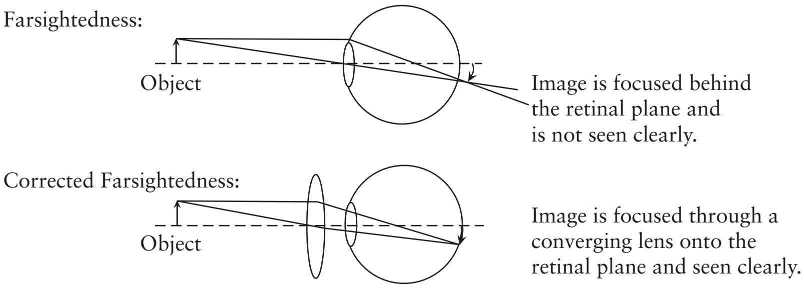


Figure 24-2 Problem 12

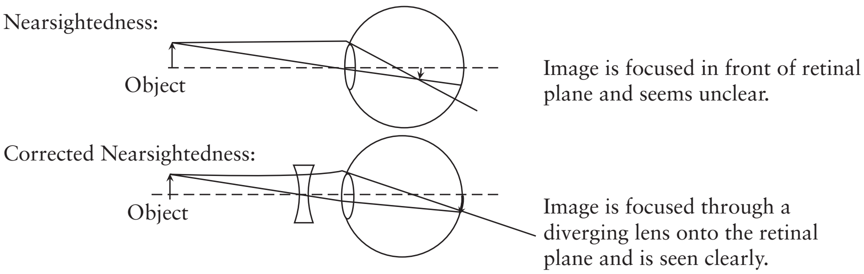


Figure 24-3 Problem 12

24.13 The condition known as "presbyopia" is associated with a weakening of the muscles around the eye and inflexibility in the crystalline lens system as a person ages. This loss in adaptive amplitude leads to the inability to focus on objects that are close up. Basically, it is entropy at work in the physiological system. Myopia, on the other hand, is a disorder that affects as many as 50% of the world's population, but it is not brought on by the aging process.

24.14 Pinhole glasses are not made of glass at all but of an opaque substance such as metal or plastic. The user looks through any of the many small holes in the material. These holes have the effect of reducing the width of the bundle of diverging rays (called a "pencil of light") coming from each point on the viewed object. Normally, the full opening of the pupil admits light. It is the improper bending of the outermost rays in that pencil of light that causes refractive errors such as myopia, hyperopia (farsightedness), presbyopia (diminished focusing range with age), and astigmatism to be noticeable. Pinholes can bring about clearer vision in all these conditions. By blocking these peripheral rays and only letting into the eye those rays that pass through the central portion of the pupil, any refractive error in the lens or cornea is not noticed as much. The pupil may be wide open, but only the central portion is receiving light. The improvement in visual acuity can be striking.

Multiple-Choice Questions

24.15 **B** (The height of the image stays the same and the image distance increases). For a plane mirror, the image distance equals the object distance, so the image distance will increase as the object distance increases. Also, the image height is equal to the object height; since the height of the object doesn't change, neither will the height of the image.

24.16 **B** (a concave mirror). A concave mirror can converge light rays.

24.17 **A** (magnified and real).

$$\frac{1}{d_{\mathrm{O}}}+\frac{1}{d_{\mathrm{I}}}=\frac{1}{f}$$

$$\frac{1}{d_{\mathrm{I}}}=\frac{1}{f}-\frac{1}{d_{\mathrm{O}}}$$

$d_{\mathrm{O}} > f$, which means $\dfrac{1}{f} > \dfrac{1}{d_{\mathrm{O}}}$; therefore, $d_{\mathrm{I}} > 0$ and $m > 1$.

24.18 **B** (a concave mirror). A concave mirror can focus light rays to a point.

24.19 **B** (real and inverted).

$$\frac{1}{d_{\mathrm{O}}}+\frac{1}{d_{\mathrm{I}}}=\frac{1}{f}$$

$$\frac{1}{d_{\mathrm{I}}}=\frac{1}{f}-\frac{1}{d_{\mathrm{O}}}=\frac{1}{\left(\dfrac{r}{2}\right)}-\frac{1}{r}=\frac{2}{r}-\frac{1}{r}=\frac{1}{r}$$

$$d_{\mathrm{I}}=r$$

$$m = -\frac{d_{\mathrm{I}}}{d_{\mathrm{O}}} = -\frac{r}{r} = -1$$

A positive image distance means the image is real. A negative magnification indicates the image is inverted.

24.20 **D** (smaller and virtual).

$$\frac{1}{d_{\mathrm{O}}} + \frac{1}{d_{\mathrm{I}}} = \frac{1}{f}$$

$$\frac{1}{d_{\mathrm{I}}} = \frac{1}{f} - \frac{1}{d_{\mathrm{O}}}$$

$f < 0$, which means $d_{\mathrm{O}} < 0$ and $|d_{\mathrm{I}}| < |d_{\mathrm{O}}|$, so $m < 1$.

24.21 **B** (a concave mirror). The image distance needs to be larger in magnitude than the object distance, and the final image will be virtual.

24.22 **A** (short focal length (<<1 m) convex lens). The closer the object is to the lens, the larger the image will be.

24.23 **B** (The objective lens is a short focal length, convex lens and the eyepiece functions as a simple magnifier). The real image from the objective lens of a compound microscope converges just inside the focal point of the eyepiece, which means the angular size of the overall image will be much larger than the angular size of the object.

Get Help: Interactive Example – Two Lens System
P'Cast 24.6 – Magnification

Estimation/Numerical Questions

24.24 Average corrective lenses are about ±3 diopters.

24.25 The blind spot mirror has a focal length of about 3 m.

24.26 The spherical mirror has a focal length of about 1 m.

24.27 A shiny spoon (f about 5 cm), a shiny salad bowl (f about 15 cm), and a reflector in a flashlight (f about 2 cm).

Get Help: Interactive Example – Concave Mirror

24.28 A shiny door knob (f about 1 cm), a shiny backside of a spoon (f about 1 cm), and a glass ornament (f about 2 cm).

24.29

d_O (cm)	d_I (cm)	$1/d_O$ (cm^{-1})	$1/d_I$ (cm^{-1})
30	98	0.033	0.010
35	67	0.029	0.015
40	53	0.025	0.019
45	47	0.022	0.021
50	42	0.020	0.024
55	38	0.018	0.026
60	37	0.017	0.027
65	35	0.015	0.029
70	34	0.014	0.029
75	33	0.013	0.030
80	32	0.013	0.031

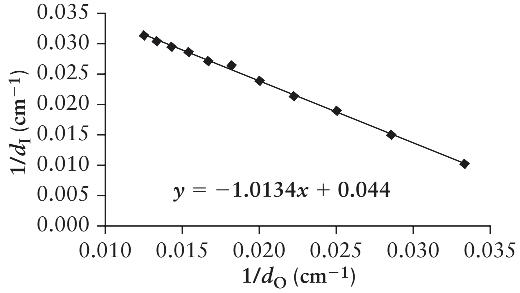


Figure 24-4 Problem 29

$$\frac{1}{d_I} = -\frac{1}{d_O} + \frac{1}{f}$$

$$f = \frac{1}{0.044\ \text{cm}^{-1}} \approx 23\ \text{cm}$$

Problems

24.30

SET UP

The angle of incidence on a flat mirror is 0°. The angle of reflection is equal to the angle of incidence.

SOLVE

$$\boxed{\theta_r = \theta_i = 0°}$$

REFLECT

An angle of 0° means the light comes in normal to the surface of the mirror.

24.31

SET UP

Two flat mirrors are perpendicular to each other. An incoming beam of light makes an angle of 30 degrees with respect to the first mirror. The beam will reflect off the mirror at an angle of 30 degrees according to the law of reflection. The beam then makes an angle of 60 degrees with respect to the second mirror, or 30 degrees with respect to the normal of the second mirror. Again, applying the law of reflection, we see that the outgoing beam will make an angle of 30 degrees with respect to the normal of the second mirror.

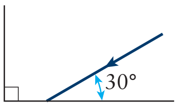


Figure 24-5 Problem 31

SOLVE

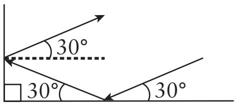


Figure 24-6 Problem 31

The outgoing beam will make an angle of $\boxed{30 \text{ degrees}}$ with respect to the normal of the second mirror.

REFLECT

Recognizing common geometric patterns will go a long way in solving optics problems. For example, the path of the ray leaving the first mirror and hitting the second mirror makes a 30-60-90 triangle with the two mirrors.

24.32

SET UP

A man ($h_O = 1.8$ m) stands in front of a plane mirror. The image height is equal to the object height for a plane mirror.

SOLVE

$$h_I = h_O = \boxed{1.8 \text{ m}}$$

REFLECT

This will always be the case for a plane mirror.

24.33

SET UP

A 1.80-m-tall person wants to see a full image of himself in a plane mirror. We can use geometry to prove that the length of the mirror must be at least one-half the height of the person.

SOLVE

The mirror must be at least $\boxed{0.900 \text{ m}}$ tall.

REFLECT

This result is independent of how far (or close) you stand to the mirror. Try it out yourself!

24.34

SET UP

A plane mirror is 10 m away from and parallel to a second plane mirror. An object is placed exactly between the two mirrors. The original object creates an image in both the left and the right mirrors. These images then act as objects for the other mirror and are thus reflected again. This process repeats and there will be an infinite number of images. In every case, the image distance is equal to the object distance for the mirror.

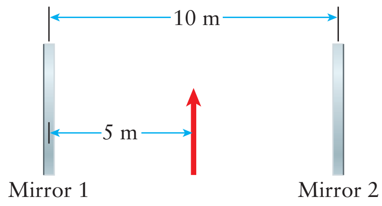


Figure 24-7 Problem 34

SOLVE

Illustration of the resulting images:

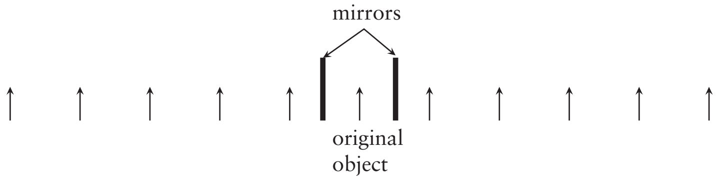


Figure 24-8 Problem 34

The first images formed in the left mirror are $\boxed{\text{5 m, 15 m, 25 m, 35 m, and 45 m to the left of the left mirror}}$.

The first images formed in the right mirror are $\boxed{\text{5 m, 15 m, 25 m, 35 m, and 45 m to the left of the right mirror}}$.

REFLECT

Since the system is symmetric, the resulting images must also be symmetric.

24.35

SET UP

A plane mirror is 10 m away from and parallel to a second plane mirror. An object is placed 3 m to the right of the left mirror, which means it is 7 m to the left of the right mirror. The original object creates an image in both the left and the right mirror. These images then act as objects for the other mirror and are thus reflected again. This process repeats and there will be an infinite number of images. In every case, the image distance is equal to the object distance for the mirror.

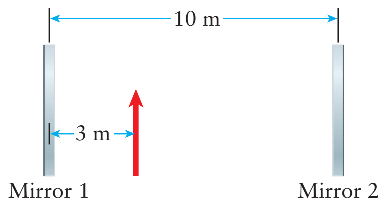


Figure 24-9 Problem 35

SOLVE

Illustration of the resulting images:

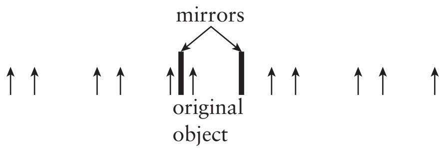


Figure 24-10 Problem 35

The first images formed in the left mirror are $\boxed{\text{3 m, 17 m, 23 m, 37 m, and 43 m to the left of the left mirror}}$.

The first images formed in the right mirror are $\boxed{\text{7 m, 13 m, 27 m, 47 m, and 53 m to the left of the left mirror}}$.

REFLECT

You can see this effect for yourself at the hair salon or in a mirror maze or even if you have two mirrors in your bathroom.

24.36

SET UP

Your friend is standing a distance $x = \frac{L}{2}$ in front of a plane mirror of length L. He is standing exactly at the midpoint of the mirror. You stand at points A–E, which are separated by a distance of $\frac{3L}{4}$, to see if you can see the reflection of your friend's face at each point. Since the angle of incidence must equal the angle of reflection, the largest possible angle of incidence is given by $\tan(\theta_\text{i}) = \frac{\left(\frac{L}{2}\right)}{x} = \frac{\left(\frac{L}{2}\right)}{\left(\frac{L}{2}\right)} = 1$, or $\theta_\text{i} = 45°$. Given the location of the points, this means the reflection can only be seen at points $\boxed{B, C, \text{ and } D}$.

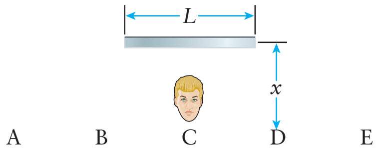


Figure 24-11 Problem 36

SOLVE

The image of your friend's face will be visible in the mirror at points $\boxed{B, C, \text{ and } D}$.

REFLECT

At points A and E, you will see a reflection of what is to the right and left of your friend, respectively.

24.37

SET UP

A person stands a distance d_O in front of a plane mirror. We can use a protractor and a ruler to draw two rays from the object that will reflect off of the mirror to determine where the image is produced.

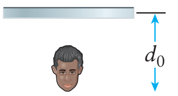


Figure 24-12 Problem 37

SOLVE

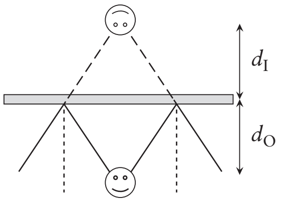


Figure 24-13 Problem 37

REFLECT

The image produced by a plane mirror is virtual and the magnitude of the image distance is equal to the magnitude of the object distance.

24.38

SET UP

Two people stand a distance d_O in front of a plane mirror. We can use a protractor and a ruler to draw a ray from the second person that reflects off of the mirror and into the eyes of the first person to determine where the image of the second person is seen by the first person.

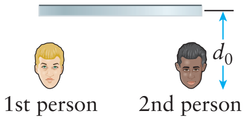


Figure 24-14 Problem 38

SOLVE

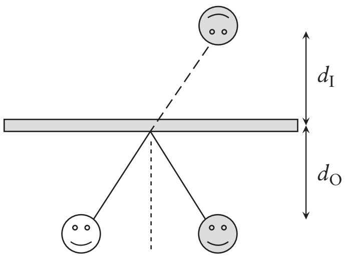


Figure 24-15 Problem 38

REFLECT

The image produced by a plane mirror is virtual and the magnitude of the image distance is equal to the magnitude of the object distance.

24.39

SET UP

An autofocusing camera sends infrared waves from a transmitter and receives the reflected waves that bounce off of the objects in front of the camera to determine at which distance to focus the lens. The camera is placed 2 m in front of a plane mirror in hopes of photographing the image. Since the infrared waves will bounce off the surface of the mirror, the camera will focus there rather than the location of the image.

SOLVE

The camera will focus the lens on the surface of the mirror, which is 2 m away from the virtual image. The photograph will be out of focus a little bit.

REFLECT

The closer you stand to the mirror, the better the autofocusing feature on the camera should work.

24.40

SET UP

We can use the ray trace diagrams from section 24-2 of the text to help determine how the images seen in a spherical, concave mirror will differ when $d_O < f$ and $d_O > f$.

SOLVE

When your eye is close to a spherical concave mirror (that is, $d_O < f$), the image is upright and virtual. When your eye is far away (that is, $d_O > f$), the image is inverted and real.

REFLECT
You can prove this to yourself using a shiny spoon.

24.41

SET UP
We can use ray tracing in order to determine if there are any situations where a real image is formed by a spherical, concave mirror.

SOLVE
An arrow is placed in front of a spherical, concave mirror. We will trace two light rays from the tip of the arrow to the mirror and as they reflect from it.

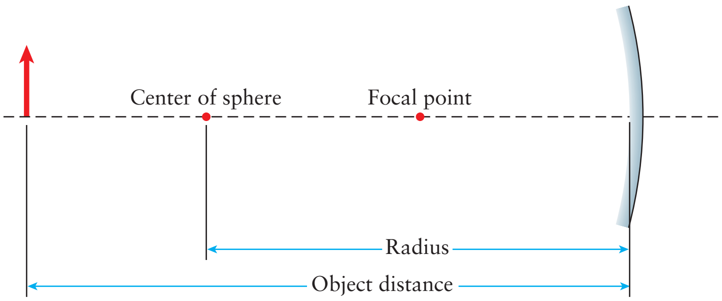


Figure 24-16 Problem 41

The image of the point of the arrow forms where the two rays cross. The image is inverted relative to the object.

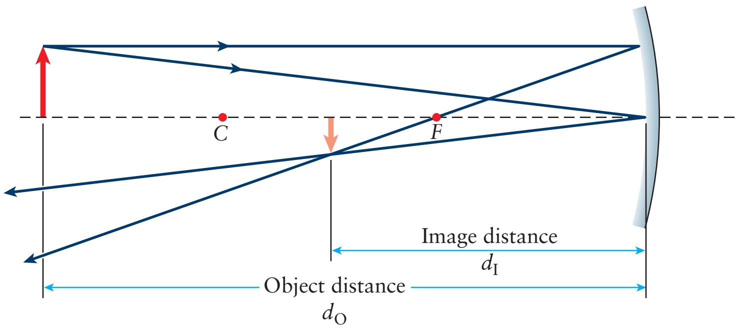


Figure 24-17 Problem 41

Two light rays are traced from the tip of the arrow in order to find the location of the image.

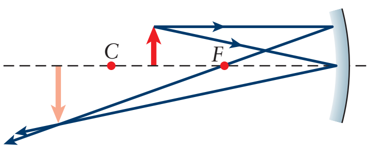


Figure 24-18 Problem 41

As long as $d_O > f$, the image will be real in a concave, spherical mirror. In this case, the focal point is located between the mirror and the object.

REFLECT
An object placed outside the center of curvature of a concave, spherical mirror will result in an image smaller than the object; an object placed in between the center of curvature

and the focal point of a concave, spherical mirror will result in an image larger than the object.

Get Help: Picture It – Spherical Mirror

24.42

SET UP

A simple mnemonic device to remember the difference in shape between a concave mirror and a convex mirror is that a concave mirror makes a shape like a cave.

SOLVE

A concave mirror goes "in like a cave."

REFLECT

A convex mirror bumps outward.

24.43

SET UP

An object is placed in front of a concave mirror that has a radius of curvature of $r = 10$ cm. The object distance is $d_O = 8.0$ cm, and the focal length is $f = r/2 = +5.0$ cm. First, we can draw a ray trace diagram of the setup to determine the image distance and magnification of the image. A parallel ray from the tip of the object will reflect off of the mirror through the focal point. A ray from the tip of the object striking the center of the mirror is also easy to reflect by applying the law of reflection. The image distance is the distance from the center of the mirror to the position of the image; the magnification is the ratio of the image height to the object height. An image located on the reflective side of the mirror is considered real. If the image is upside down relative to the object, it is said to be inverted. Secondly, we can apply the mirror equation, $\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f}$, and its sign conventions to confirm the results from the ray trace diagram. A positive image distance is considered real, and a negative magnification corresponds to an inverted image.

SOLVE

Part a)

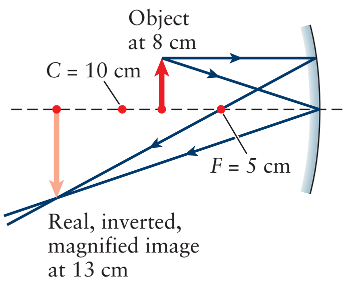


Figure 24-19 Problem 43

Part b)

Image distance:

$$\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f}$$

$$\frac{1}{d_I} = \frac{1}{f} - \frac{1}{d_O} = \frac{1}{5.0\text{ cm}} - \frac{1}{8.0\text{ cm}} = \frac{3.0}{40\text{ cm}}$$

$$d_I = \frac{40\text{ cm}}{3.0} = \boxed{13\text{ cm}}$$

Magnification:

$$m = -\frac{d_I}{d_O} = -\frac{13\text{ cm}}{8.0\text{ cm}} = \boxed{-1.7}$$

The image is $\boxed{\text{real}}$ because the image distance is positive. The image is $\boxed{\text{inverted}}$ because $m < 0$.

REFLECT

A ray trace diagram must be drawn to scale if you want to quantitatively measure distances and heights.

Get Help: Interactive Example – Concave Mirror
P'Cast 24.4 – An Object Far from a Concave Mirror
P'Cast 24.5 – An Object Close to a Concave Mirror

24.44

SET UP

An object ($h_O = 1.0$ cm) is placed in front of a spherical concave mirror that has a radius of curvature $R = 10$ cm, which means the focal length is $f = +5.0$ cm. The object distance is $d_O = 3.0$ cm. First, we can draw a ray trace diagram of the setup to determine the image distance and magnification of the image. A parallel ray from the tip of the object will reflect off of the mirror through the focal point. A ray from the tip of the object striking the center of the mirror is also easy to reflect by applying the law of reflection. The image distance is the distance from the center of the mirror to the position of the image; the magnification is the ratio of the image height to the object height. An image located on the reflective side of the mirror is considered real. An inverted image is upside-down relative to the object. We can also apply the mirror equation, $\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f}$, and its sign conventions to confirm the results from the ray trace diagram. A positive image distance is considered real, and a negative magnification corresponds to an inverted image.

SOLVE

Part a)

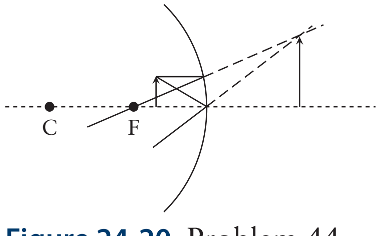


Figure 24-20 Problem 44

Part b)
Image distance:

$$\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f}$$

$$d_I = \frac{d_O f}{d_O - f} = \frac{(3.0\text{ cm})(5.0\text{ cm})}{(3.0\text{ cm}) - (5.0\text{ cm})} = \boxed{-7.5\text{ cm}}$$

Image height:

$$h_I = -\frac{d_I}{d_O} h_O = -\left(\frac{-7.5\text{ cm}}{3.0\text{ cm}}\right)(1.0\text{ cm}) = \boxed{2.5\text{ cm}}$$

REFLECT

The object is virtual and upright, as we would expect for an object placed within the focal point of a spherical concave mirror.

24.45

SET UP

An object ($h_O = 1.0$ cm) is placed at a distance $d_O = 6.0$ cm in front of a spherical concave mirror ($R = 10.0$ cm). The focal length f of a spherical mirror is equal to $f = \frac{R}{2}$, which is $f = 5.0$ cm in this case. Two simple rays to draw in the ray trace diagram are the ray from the top of the object that hits the mirror parallel to the optical axis and the ray that hits the point of the mirror on the optical axis. The first ray passes through the focal point after it reflects off the mirror, while the second ray is symmetric about the optical axis. The numerical value for the image distance can be calculated using the mirror equation, $\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f}$; the height of the image is found using $h_I = -\frac{d_I}{d_O} h_O$.

SOLVE

Part a)

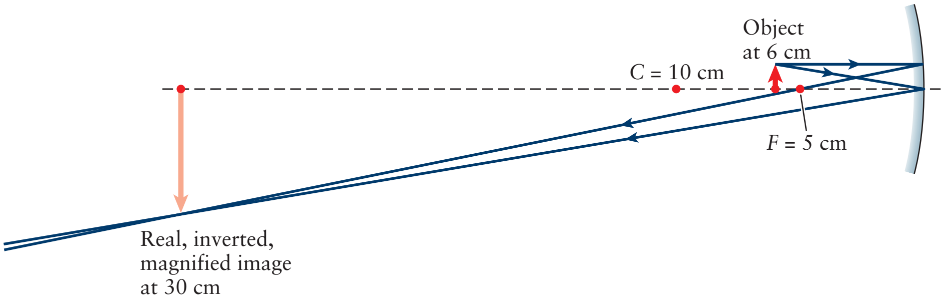


Figure 24-21 Problem 45

Part b)
Image distance:

$$\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f}$$

$$d_I = \frac{d_O f}{d_O - f} = \frac{(6.0\text{ cm})(5.0\text{ cm})}{(6.0\text{ cm}) - (5.0\text{ cm})} = \boxed{30\text{ cm}}$$

Image height:

$$h_I = -\frac{d_I}{d_O}h_O = -\left(\frac{30\text{ cm}}{6.0\text{ cm}}\right)(1.0\text{ cm}) = \boxed{-5.0\text{ cm}}$$

REFLECT

The image will be real and inverted.

24.46

SET UP

An object ($h_O = 10$ cm) is placed at various distances ($d_O = 5.0$ cm, 20 cm, 50 cm, 100 cm) in front of a spherical concave mirror ($R = 20$ cm). The focal length f of a spherical mirror is equal to $f = \frac{R}{2}$, which is $f = 10$ cm in this case. The numerical value for the image distance can be calculated using the mirror equation, $\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f}$; the height of the image is found using $h_I = -\frac{d_I}{d_O}h_O$. If d_I is negative, the image is virtual; if d_I is positive, the image is real. The object will be upright if h_I is positive; it will be inverted if h_I is negative.

SOLVE

Part a)
Image distance:

$$\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f}$$

$$d_I = \frac{d_O f}{d_O - f} = \frac{(5.0\text{ cm})(10\text{ cm})}{(5.0\text{ cm}) - (10\text{ cm})} = \boxed{-10\text{ cm}}$$

Image height:

$$h_I = -\frac{d_I}{d_O}h_O = -\left(\frac{-10\text{ cm}}{5.0\text{ cm}}\right)(10\text{ cm}) = \boxed{20\text{ cm}}$$

The image is $\boxed{\text{virtual and upright}}$.

Part b)
Image distance:

$$\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f}$$

$$d_I = \frac{d_O f}{d_O - f} = \frac{(20\text{ cm})(10\text{ cm})}{(20\text{ cm}) - (10\text{ cm})} = \boxed{20\text{ cm}}$$

Image height:

$$h_I = -\frac{d_I}{d_O}h_O = -\left(\frac{20\text{ cm}}{20\text{ cm}}\right)(10\text{ cm}) = \boxed{-10\text{ cm}}$$

The image is $\boxed{\text{real and inverted}}$.

Part c)
Image distance:

$$\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f}$$

$$d_I = \frac{d_O f}{d_O - f} = \frac{(50\text{ cm})(10\text{ cm})}{(50\text{ cm}) - (10\text{ cm})} = \boxed{13\text{ cm}}$$

Image height:

$$h_I = -\frac{d_I}{d_O}h_O = -\left(\frac{13\text{ cm}}{50\text{ cm}}\right)(10\text{ cm}) = \boxed{-2.5\text{ cm}}$$

The image is $\boxed{\text{real and inverted}}$.

Part d)
Image distance:

$$\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f}$$

$$d_I = \frac{d_O f}{d_O - f} = \frac{(100\text{ cm})(10\text{ cm})}{(100\text{ cm}) - (10\text{ cm})} = \boxed{11\text{ cm}}$$

Image height:

$$h_I = -\frac{d_I}{d_O}h_O = -\left(\frac{11\text{ cm}}{100\text{ cm}}\right)(10\text{ cm}) = \boxed{-1.1\text{ cm}}$$

The image is $\boxed{\text{real and inverted}}$.

REFLECT

The image should be larger, upright, and virtual if the object is within the focal point of the mirror. The image should be smaller, inverted, and real if the object distance is larger than the radius of curvature.

24.47

SET UP

The radius of curvature of a spherical concave mirror is 15 cm, which means its focal length is $f = +7.5$ cm. An object ($h_O = 20$ cm) is positioned at three different object distances: $d_I = +10$ cm, $d_I = +20$ cm, and $d_I = +100$ cm. We can use the mirror equation, $\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f}$, and its sign conventions to calculate the image distance and the image height, as well as determine whether the image is real or virtual, upright or inverted. As a reminder, a positive image distance is considered real, and a negative image height corresponds to an inverted image.

SOLVE

Image distance in general:

$$\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f}$$

$$\frac{1}{d_I} = \frac{1}{f} - \frac{1}{d_O} = \frac{d_O - f}{fd_O}$$

$$d_I = \frac{fd_O}{d_O - f}$$

Image height in general:

$$m = \frac{h_I}{h_O} = -\frac{d_I}{d_O}$$

$$h_I = -\frac{d_I}{d_O}h_O$$

Part a)

Image distance:

$$d_I = \frac{fd_O}{d_O - f} = \frac{(7.5\text{ cm})(10\text{ cm})}{(10\text{ cm}) - (7.5\text{ cm})} = \boxed{30\text{ cm}}$$

The image is $\boxed{\text{real}}$ because $d_I > 0$.

Image height:

$$h_I = -\frac{d_I}{d_O}h_O = -\left(\frac{30\text{ cm}}{10\text{ cm}}\right)(20\text{ cm}) = -60\text{ cm}$$

The image is $\boxed{\text{60 cm tall and inverted}}$ because $h_I < 0$.

Part b)

Image distance:

$$d_I = \frac{fd_O}{d_O - f} = \frac{(7.5\text{ cm})(20\text{ cm})}{(20\text{ cm}) - (7.5\text{ cm})} = \boxed{12\text{ cm}}$$

The image is $\boxed{\text{real}}$ because $d_I > 0$.

Image height:

$$h_I = -\frac{d_I}{d_O}h_O = -\left(\frac{12\text{ cm}}{20\text{ cm}}\right)(20\text{ cm}) = -12\text{ cm}$$

The image is $\boxed{\text{12 cm tall and inverted}}$ because $h_I < 0$.

Part c)

Image distance:

$$d_I = \frac{fd_O}{d_O - f} = \frac{(7.5\text{ cm})(100\text{ cm})}{(100\text{ cm}) - (7.5\text{ cm})} = \boxed{8.11\text{ cm}}$$

The image is $\boxed{\text{real}}$ because $d_I > 0$.

Image height:

$$h_I = -\frac{d_I}{d_O}h_O = -\left(\frac{8.11\text{ cm}}{100\text{ cm}}\right)(20\text{ cm}) = -1.62\text{ cm}$$

The image is $\boxed{\text{1.62 cm tall and inverted}}$ because $h_I < 0$.

REFLECT

A real object placed at a position outside of the focal point of a spherical concave mirror will always give a real image:

$$d_I = \frac{fd_O}{d_O - f} > 0, \text{ if } (d_O - f) > 0$$

Get Help: Interactive Example – Concave Mirror
P'Cast 24.4 – An Object Far from a Concave Mirror
P'Cast 24.5 – An Object Close to a Concave Mirror

24.48

SET UP

An object is positioned $d_O = +24$ cm from a spherical concave mirror of unknown focal length. We are told that an image is formed 30 cm from the mirror but not whether this corresponds to a positive or a negative image distance. Therefore, we need to calculate the focal length using the thin-lens equation for both cases. A positive image distance corresponds to a real image, whereas a negative image distance corresponds to a virtual image. Finally, the image heights are given by $h_i = -\left(\frac{d_I}{d_O}\right)h_O$.

SOLVE

Part a)

$$\frac{1}{f} = \frac{1}{d_O} + \frac{1}{d_I}$$

$$f = \frac{d_O d_I}{d_O + d_I}$$

Positive image distance:

$$f = \frac{(24.0\text{ cm})(30.0\text{ cm})}{(24.0\text{ cm}) + (30.0\text{ cm})} = \boxed{13.3\text{ cm}}$$

Negative image distance:

$$f = \frac{(24.0\text{ cm})(-30.0\text{ cm})}{(24.0\text{ cm}) + (-30.0\text{ cm})} = \boxed{-120\text{ cm}}$$

Part b) No, the answer is not unique.

Part c) A positive image distance corresponds to a real image. A negative image distance corresponds to a virtual image.

Part d)
Positive image distance:

$$h_{\mathrm{i}} = -\left(\frac{d_{\mathrm{I}}}{d_{\mathrm{O}}}\right)h_{\mathrm{O}} = -\left(\frac{30.0\ \mathrm{cm}}{24.0\ \mathrm{cm}}\right)(10\ \mathrm{cm}) = \boxed{-13\ \mathrm{cm}}$$

Negative image distance:

$$h_{\mathrm{i}} = -\left(\frac{d_{\mathrm{I}}}{d_{\mathrm{O}}}\right)h_{\mathrm{O}} = -\left(\frac{-30.0\ \mathrm{cm}}{24.0\ \mathrm{cm}}\right)(10\ \mathrm{cm}) = \boxed{13\ \mathrm{cm}}$$

REFLECT

If the image distance were positive, the final image would be inverted. If the image distance were negative, the final image would be upright.

24.49

SET UP

We are asked to construct ray trace diagrams for an object ($h_{\mathrm{O}} = 10$ cm) at various object distances ($d_{\mathrm{I}} = 5$ cm, 10 cm, 20 cm) in front of a spherical concave mirror ($R = 20$ cm). The focal length f of a spherical mirror is equal to $f = \frac{R}{2}$; in this case, $f = 10$ cm. The ray through both the top of the object and the center of curvature C reflects off the mirror back through C. The ray from the top of the object that is parallel to the optical axis is reflected through the focal point after it hits the mirror.

SOLVE

Part a)

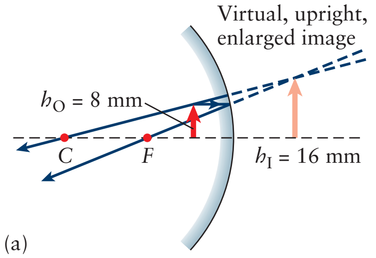


Figure 24-22 Problem 49

Part b)

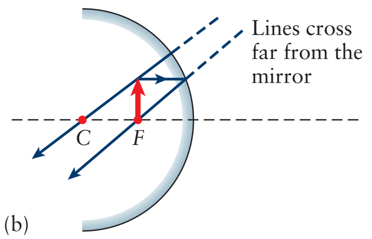


Figure 24-23 Problem 49

Part c)

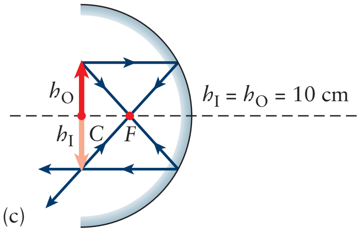


Figure 24-24 Problem 49

REFLECT

Ray trace diagrams only give quantitative information if they are drawn to scale.

24.50

SET UP

An object is a distance d_O in front of a spherical, concave mirror of radius of curvature R; the focal length of the mirror is equal to $\frac{R}{2}$. The image that is produced is upright and four times larger than the object, which means the magnification $m = 4$. The magnification is also equal to $m = -\frac{d_I}{d_O}$, where d_I is the image distance. After rearranging this to write the image distance in terms of the object distance, we can use the mirror equation to find an expression for the radius of curvature in terms of the object distance.

SOLVE

Image distance:

$$m = -\frac{d_I}{d_O}$$

$$d_I = -md_O = -4d_O$$

Radius of curvature:

$$\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f} = \frac{1}{\left(\frac{R}{2}\right)} = \frac{2}{R}$$

$$\frac{d_I + d_O}{d_O d_I} = \frac{2}{R}$$

$$R = \frac{2d_O d_I}{d_I + d_O} = \frac{2(d_O)(-4d_O)}{(-4d_O) + d_O} = \frac{-8d_O^2}{-3d_O} = \boxed{\frac{8}{3}d_O}$$

REFLECT

Because the image is larger and upright, we would expect the object distance to be smaller than the focal length, which is $\frac{4}{3}d_O$.

24.51

SET UP
We can use the ray trace diagrams from section 24-4 of the text to help determine how the images seen in a spherical convex mirror will differ when $d_O < f$ and $d_O > f$.

SOLVE
In both cases ($d_O < f$ and $d_O > f$), the images are always virtual and smaller.

REFLECT
You can prove this to yourself using a shiny spoon or holiday ornament.

24.52

SET UP
We can use the ray trace diagrams from section 24-4 of the text to help determine in what situations, if any, a spherical convex mirror produces a real image.

SOLVE
A spherical convex mirror can only produce virtual images.

REFLECT
Not only will the image always be virtual, but it will also always be smaller than the object.

24.53

SET UP
A simple mnemonic device to remember the difference in shape between a concave mirror and a convex mirror is that a concave mirror makes a shape like a cave.

SOLVE
A convex mirror is the opposite of a concave mirror. Use a phrase like "The cave goes in" to remember the shape of a concave mirror.

REFLECT
It's easier to remember that a convex mirror is opposite to a concave mirror than trying to come up with a mnemonic device using "-vex."

24.54

SET UP
A spherical convex mirror has a radius of curvature of $R = -20$ cm, which means its focal length is $f = -10$ cm. An object ($h_O = 10$ cm) is placed a distance $d_O = 5$ cm, 10 cm, and 20 cm from the mirror. We can draw a ray trace diagram of the setup to approximate the height of the image. A parallel ray from the tip of the object will reflect off of the mirror through the focal point. A ray from the tip of the object striking the center of the mirror is also easy to reflect by applying the law of reflection.

SOLVE

Part a)

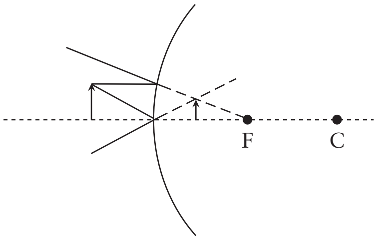


Figure 24-25 Problem 54

The height of the image is around $\boxed{7 \text{ cm}}$.

Part b)

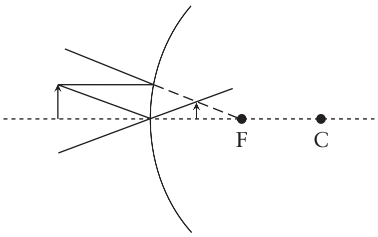


Figure 24-26 Problem 54

The height of the image is around $\boxed{5 \text{ cm}}$.

Part c)

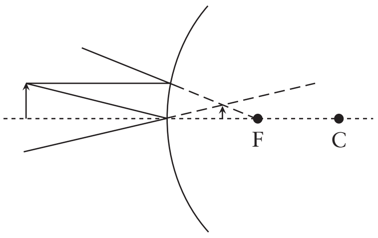


Figure 24-27 Problem 54

The height of the image is about $\boxed{3 \text{ cm}}$.

REFLECT

As expected for a spherical convex mirror, all of the images are virtual, upright, and smaller.

24.55

SET UP

The radius of curvature of a spherical convex mirror is 20 cm, which means its focal length is $f = -10$ cm. An object ($h_O = 10$ cm) is positioned at three different object distances: $d_I = +20$ cm, $d_I = +50$ cm, and $d_I = +100$ cm. We can use the mirror equation, $\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f}$, and its sign conventions to calculate the image distance and the image height, as well as determine whether the image is real or virtual, upright or inverted. As a reminder, a positive image distance is considered real, and a negative image height corresponds to an inverted image.

SOLVE

Image distance in general:

$$\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f}$$

$$\frac{1}{d_I} = \frac{1}{f} - \frac{1}{d_O} = \frac{d_O - f}{f d_O}$$

$$d_I = \frac{f d_O}{d_O - f}$$

Image height in general:

$$m = \frac{h_I}{h_O} = -\frac{d_I}{d_O}$$

$$h_I = -\frac{d_I}{d_O} h_O$$

Part a)

Image distance:

$$d_I = \frac{f d_O}{d_O - f} = \frac{(-10\text{ cm})(20\text{ cm})}{(20\text{ cm}) - (-10\text{ cm})} = \boxed{-6.67\text{ cm}}$$

The image is $\boxed{\text{virtual}}$ because $d_I < 0$.

Image height:

$$h_I = -\frac{d_I}{d_O} h_O = -\left(\frac{-6.67\text{ cm}}{20\text{ cm}}\right)(10\text{ cm}) = 3.33\text{ cm}$$

The image is $\boxed{\text{3.33 cm tall and upright}}$ because $h_I > 0$.

Part b)

Image distance:

$$d_I = \frac{f d_O}{d_O - f} = \frac{(-10\text{ cm})(50\text{ cm})}{(50\text{ cm}) - (-10\text{ cm})} = \boxed{-8.33\text{ cm}}$$

The image is $\boxed{\text{virtual}}$ because $d_I < 0$.

Image height:

$$h_I = -\frac{d_I}{d_O} h_O = -\left(\frac{-8.33\text{ cm}}{50\text{ cm}}\right)(10\text{ cm}) = 1.67\text{ cm}$$

The image is $\boxed{\text{1.67 cm tall and upright}}$ because $h_I > 0$.

Part c)
Image distance:

$$d_I = \frac{fd_O}{d_O - f} = \frac{(-10\text{ cm})(100\text{ cm})}{(100\text{ cm}) - (-10\text{ cm})} = \boxed{-9.09\text{ cm}}$$

The image is $\boxed{\text{virtual}}$ because $d_I < 0$.

Image height:

$$h_I = -\frac{d_I}{d_O}h_O = -\left(\frac{-9.09\text{ cm}}{100\text{ cm}}\right)(10\text{ cm}) = 0.909\text{ cm}$$

The image is $\boxed{\text{0.909 cm tall and upright}}$ because $h_I > 0$.

REFLECT

A real object placed *anywhere* in front of a spherical convex mirror will always give a virtual image, *i.e.*, $d_I = \dfrac{fd_O}{d_O - f} < 0$. The term $(d_O - f)$ will always be positive for a real object and a convex mirror.

Get Help: P'Cast 24.4 – An Object Far from a Convex Mirror
P'Cast 24.5 – An Object Close to a Convex Mirror

24.56

SET UP

The magnitude of the radius of curvature of a spherical convex mirror is 15 cm, which means its focal length is $f = -7.5$ cm. An object ($h_O = 20$ cm) is positioned at three different object distances: $d_I = +5.0$ cm, $d_I = +20$ cm, and $d_I = +100$ cm. We can use the mirror equation, $\dfrac{1}{d_O} + \dfrac{1}{d_I} = \dfrac{1}{f}$, and its sign conventions to calculate the image distance and the image height, as well as determine whether the image is real or virtual, upright or inverted. As a reminder, a positive image distance is considered real, and a negative image height corresponds to an inverted image.

SOLVE

Part a)
Image distance:

$$\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f}$$

$$d_I = \frac{d_O f}{d_O - f} = \frac{(5.0\text{ cm})(-7.5\text{ cm})}{(5.0\text{ cm}) - (-7.5\text{ cm})} = \boxed{-3.0\text{ cm}}$$

Image height:

$$h_I = -\frac{d_I}{d_O}h_O = -\left(\frac{-3.0\text{ cm}}{5.0\text{ cm}}\right)(20\text{ cm}) = \boxed{12\text{ cm}}$$

The image is virtual and upright.

Part b)
Image distance:

$$\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f}$$

$$d_I = \frac{d_O f}{d_O - f} = \frac{(20\text{ cm})(-7.5\text{ cm})}{(20\text{ cm}) - (-7.5\text{ cm})} = \boxed{-5.5\text{ cm}}$$

Image height:

$$h_I = -\frac{d_I}{d_O}h_O = -\left(\frac{-5.5\text{ cm}}{20\text{ cm}}\right)(20\text{ cm}) = \boxed{5.5\text{ cm}}$$

The image is $\boxed{\text{virtual and upright}}$.

Part c)
Image distance:

$$\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f}$$

$$d_I = \frac{d_O f}{d_O - f} = \frac{(100\text{ cm})(-7.5\text{ cm})}{(100\text{ cm}) - (-7.5\text{ cm})} = \boxed{-7.0\text{ cm}}$$

Image height:

$$h_I = -\frac{d_I}{d_O}h_O = -\left(\frac{-7.0\text{ cm}}{100\text{ cm}}\right)(20\text{ cm}) = \boxed{1.4\text{ cm}}$$

The image is $\boxed{\text{virtual and upright}}$.

REFLECT
A spherical convex mirror can only produce virtual, upright images.

24.57

SET UP
The rearview mirror of a car can be treated as a spherical convex mirror with a radius of curvature of magnitude $R = 15$ m, which means the focal length is $f = -7.5$ m. An object is positioned at a distance $d_O = 10$ m in front of the mirror. We can use the mirror equation, $\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f}$, and the definition of the magnification, $m = -\frac{d_I}{d_O}$, to calculate the image distance, magnification, and type of the image (that is, real or virtual, inverted or upright). We'll assume the mirror is located at the origin.

SOLVE
Image distance:

$$\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f}$$

$$d_I = \frac{d_O f}{d_O - f} = \frac{(10\text{ m})(-7.5\text{ m})}{(10\text{ m}) - (-7.5\text{ m})} = \boxed{-4.3\text{ m}}$$

Magnification:

$$m = -\frac{d_I}{d_O} = -\left(\frac{-4.3 \text{ m}}{10 \text{ m}}\right) = \boxed{0.43}$$

Since $d_I < 0$ and $m > 0$, the image is virtual and upright.

REFLECT

A spherical convex mirror can only produce virtual, upright images.

24.58

SET UP

A pencil ($h_O = 18.0$ cm) is placed in front of a convex spherical mirror ($R = -88.4$ cm). The image produced must be upright since its height is $h_I = 10.5$ cm. The magnification of the pencil is given by $m = \frac{h_I}{h_O}$. The magnification is also related to the object and image distances through $m = -\frac{d_I}{d_O}$; this equation, along with the mirror equation, will allow us to find the numerical values for the object and image distances.

SOLVE

Part a)

$$m = \frac{h_I}{h_O} = \frac{10.5 \text{ cm}}{18.0 \text{ cm}} = \boxed{0.583}$$

$$m = -\frac{d_I}{d_O}$$

$$d_I = -md_O$$

Part b)

$$\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f} = \frac{1}{\left(\frac{R}{2}\right)} = \frac{2}{R}$$

$$\frac{1}{d_O} + \frac{1}{-md_O} = \frac{2}{R}$$

$$\frac{m-1}{md_O} = \frac{2}{R}$$

$$d_O = \frac{R(m-1)}{2m} = \frac{(-88.4 \text{ cm})(0.583-1)}{2(0.583)} = \boxed{31.6 \text{ cm}}$$

Part c)

$$d_I = -md_O = -(0.583)(31.6 \text{ cm}) = \boxed{-18.4 \text{ cm}}$$

REFLECT

The object distance for a single optical element must be positive. The image distance for a spherical convex mirror should be negative since the image must be virtual.

24.59

SET UP

A shiny sphere can be treated as a spherical convex mirror with a radius of curvature $R = -25.0\text{ cm}$. A horse fly ($h_\text{O} = 1.00\text{ cm}$) is positioned at a distance $d_\text{O} = 1.00\text{ cm}$ in front of the sphere. We can use the mirror equation, $\frac{1}{d_\text{O}} + \frac{1}{d_\text{I}} = \frac{1}{f}$, and the definition of the magnification, $m = -\frac{d_\text{I}}{d_\text{O}}$, to calculate the image distance, magnification, and type of the image (that is, real or virtual, inverted or upright). We'll assume the surface of the sphere is located at the origin.

SOLVE

Image distance:

$$\frac{1}{d_\text{O}} + \frac{1}{d_\text{I}} = \frac{1}{f} = \frac{1}{\left(\frac{R}{2}\right)} = \frac{2}{R}$$

$$d_\text{I} = \frac{d_\text{O}R}{2d_\text{O} - R} = \frac{(1.00\text{ cm})(-25.0\text{ cm})}{2(1.00\text{ cm}) - (-25.0\text{ cm})} = -0.926\text{ cm}$$

The image of the fly is located $\boxed{0.926\text{ cm behind the surface of the sphere}}$.
Image height:

$$h_\text{I} = -\frac{d_\text{I}}{d_\text{O}}h_\text{O} = -\left(\frac{-0.926\text{ cm}}{1.00\text{ cm}}\right)(1.00\text{ cm}) = \boxed{0.926\text{ cm}}$$

Since $d_\text{I} < 0$ and h_I, the image is $\boxed{\text{virtual and upright}}$.

REFLECT

The image is virtual and upright as expected for a spherical convex mirror.

24.60

SET UP

A spherical convex mirror has a radius of curvature of $R = -1.85\text{ m}$. An object is positioned at a distance $d_\text{O} = 12.6\text{ m}$ in front of the mirror. We can use the mirror equation, $\frac{1}{d_\text{O}} + \frac{1}{d_\text{I}} = \frac{1}{f}$, to find the location of the image. We'll assume the mirror is located at the origin.

SOLVE

$$\frac{1}{d_\text{O}} + \frac{1}{d_\text{I}} = \frac{1}{f} = \frac{1}{\left(\frac{R}{2}\right)} = \frac{2}{R}$$

$$d_\text{I} = \frac{d_\text{O}R}{2d_\text{O} - R} = \frac{(12.6\text{ m})(-1.85\text{ m})}{2(12.6\text{ m}) - (-1.85\text{ m})} = -0.862\text{ m}$$

The image is located $\boxed{0.862\text{ m behind the mirror}}$.

REFLECT

The image will be upright and about 7% of its original size.

24.61

SET UP

In order to prove that all images produced by a spherical convex mirror are virtual, we need to show mathematically that the image distance is negative regardless of our choice of d_{O}. We can rearrange the mirror equation, solve for the image distance, and apply the necessary sign conventions, mainly that a spherical convex mirror has a negative focal length.

SOLVE

$$\frac{1}{d_{\mathrm{O}}}+\frac{1}{d_{\mathrm{I}}}=\frac{1}{f}$$

$$\frac{1}{d_{\mathrm{I}}}=\frac{1}{f}-\frac{1}{d_{\mathrm{O}}}=\frac{d_{\mathrm{O}}-f}{fd_{\mathrm{O}}}$$

$$d_{\mathrm{I}}=\frac{fd_{\mathrm{O}}}{d_{\mathrm{O}}-f}$$

The focal length of a spherical convex mirror is always negative, which means the term $(d_{\mathrm{O}}-f)$ will always be positive for a real object. Therefore,

$$d_{\mathrm{I}}=\frac{fd_{\mathrm{O}}}{d_{\mathrm{O}}-f}<0$$

REFLECT

A negative image distance means the image is virtual.

Get Help: P'Cast 24.4 – An Object Far from a Convex Mirror
P'Cast 24.5 – An Object Close to a Convex Mirror

24.62

SET UP

A spherical ornament has a radius of curvature of $R=-5.0\text{ cm}$. The image in the ornament is located 1.5 cm behind the surface of the ornament, which corresponds to an image distance of $d_{\mathrm{I}}=-1.5\text{ cm}$. We can use the mirror equation, $\frac{1}{d_{\mathrm{O}}}+\frac{1}{d_{\mathrm{I}}}=\frac{1}{f}$, to find the location of the object. We'll assume the surface of the ornament is located at the origin.

SOLVE

$$\frac{1}{d_{\mathrm{O}}}+\frac{1}{d_{\mathrm{I}}}=\frac{1}{f}=\frac{1}{\left(\frac{R}{2}\right)}=\frac{2}{R}$$

$$d_{\mathrm{O}}=\frac{d_{\mathrm{I}}R}{2d_{\mathrm{I}}-R}=\frac{(-1.5\text{ cm})(-5.0\text{ cm})}{2(-1.5\text{ cm})-(-5.0\text{ cm})}=3.8\text{ cm}$$

The child is located $\boxed{\text{3.8 cm in front of the ornament}}$.

REFLECT

The image is 40% of the size of the object.

24.63

SET UP

A squirrel ($h_O = 6.00$ cm) is a distance $d_O = 40$ cm in front of a convex spherical mirror ($R = -15$ cm). We can use the mirror equation, $\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f}$, and the definition of the magnification, $m = \frac{h_I}{h_O} = -\frac{d_I}{d_O}$, to calculate the image distance, image height, and type of the image (that is, real or virtual, inverted or upright). We'll assume the mirror is located at the origin.

SOLVE

Image distance:

$$\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f} = \frac{1}{\left(\frac{R}{2}\right)} = \frac{2}{R}$$

$$d_I = \frac{Rd_O}{2d_O - R} = \frac{(-15\text{ cm})(40\text{ cm})}{2(40\text{ cm}) - (-15\text{ cm})} = \boxed{-6.3\text{ cm}}$$

Image height:

$$h_I = -\frac{d_I}{d_O}h_O = -\left(\frac{-6.3\text{ cm}}{40\text{ cm}}\right)(6.00\text{ cm}) = \boxed{0.95\text{ cm}}$$

Because $d_I < 0$ and $h_I > 0$, the image is $\boxed{\text{virtual and upright}}$.

REFLECT

A spherical convex mirror will produce a smaller, virtual, and upright image.

24.64

SET UP

We can use the ray trace diagrams from section 24-6 of the text to help determine under what circumstances, if any, a real image will be formed by a converging or diverging lens.

SOLVE

Only converging lenses can create real images. Diverging lenses only produce virtual images. A converging lens will produce a real image when the object is outside the focal point (that is, $d_O > f$).

REFLECT

A converging lens can produce a real or virtual image depending on the location of the object relative to the focal point.

24.65

SET UP
A real image created by reflection in a spherical mirror appears in front of the mirrored surface. A real image created by refraction through a lens appears behind the lens.

SOLVE
$\boxed{\text{No}}$, a real image in a converging lens occurs on the opposite side of the lens from the object.

REFLECT
We expect light to be reflected by a mirror and transmitted by a lens.

24.66

SET UP
Refraction occurs when light travels from one medium into another medium with a different index of refraction. For a biconcave or biconvex lens, there are two air–lens interfaces where refraction will take place, but in a ray trace diagram, we only show the ray bending once. This is due to the thin-lens approximation.

SOLVE
The bending of light occurs at both surfaces of the air–lens interface. We usually draw ray trace diagrams in such a way that all of the bending occurs at the central plane of the lens. In the thin-lens approximation, we treat the lens as vanishingly thin, so the net effect of the lens must only take place at the central plane of the lens.

REFLECT
The thin-lens approximation also allows us to say all incoming parallel light rays will converge exactly at the focal point.

24.67

SET UP
A nearsighted eye focuses the image in front of the retinal plane, and a farsighted eye focuses the image in back of the retinal plane. The corrective lenses for nearsightedness need to help produce a final image that is farther behind the lens of the eye, whereas the corrective lenses for farsightedness need to help produce a final image closer to the lens. To counteract these problems, a diverging lens or a converging lens is used.

SOLVE

Part a) Nearsightedness is corrected by a diverging lens.

Part b) Farsightedness is corrected by a converging lens.

REFLECT
Nearsightedness means you can clearly see objects that are close, but objects that are far away are blurry.

24.68

SET UP
A ray trace diagram shows the location of an image by showing where all of the rays intersect. We only need to draw a minimum of two rays in our diagrams because a point is described by the intersection of two distinct lines.

SOLVE

You only need two rays to determine where an image is formed.

REFLECT

Oftentimes people draw a third ray in order to confirm their answer.

24.69

SET UP

The power of a converging lens is 5 diopters, which means its focal length is $f = \frac{1}{P} = \frac{1}{5\text{ m}^{-1}} = 0.20\text{ m} = +20\text{ cm}$. An object ($h_O = 10$ cm) is positioned at four different object distances: $d_O = +5$ cm, $d_O = +10$ cm, $d_O = +20$ cm, and $d_O = +50$ cm. We can use the lens equation, $\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f}$, and its sign conventions to calculate the image distance and the image height, as well as determine whether the image is real or virtual, upright or inverted. As a reminder, a positive image distance is considered real, and a negative image height corresponds to an inverted image. We can also draw a ray trace diagram of each situation to confirm our numerical results.

SOLVE

Image distance in general:

$$\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f}$$

$$\frac{1}{d_I} = \frac{1}{f} - \frac{1}{d_O} = \frac{d_O - f}{f d_O}$$

$$d_I = \frac{f d_O}{d_O - f}$$

Image height in general:

$$m = \frac{h_I}{h_O} = -\frac{d_I}{d_O}$$

$$h_I = -\frac{d_I}{d_O} h_O$$

Part a)

Image distance:

$$d_I = \frac{f d_O}{d_O - f} = \frac{(20\text{ cm})(5\text{ cm})}{(5\text{ cm}) - (20\text{ cm})} = \boxed{-6.7\text{ cm}}$$

The image is $\boxed{\text{virtual}}$ because $d_I < 0$.

Image height:

$$h_I = -\frac{d_I}{d_O} h_O = -\left(\frac{-6.7\text{ cm}}{5\text{ cm}}\right)(10\text{ cm}) = 13\text{ cm}$$

The image is $\boxed{\text{13 cm tall and upright}}$ because $h_I > 0$.

Ray trace diagram:

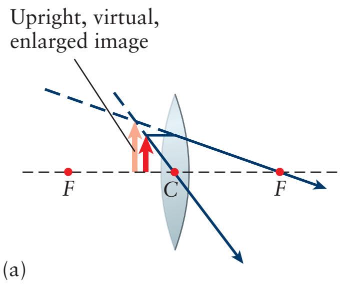


Figure 24-28 Problem 69

Part b)
Image distance:

$$d_I = \frac{fd_O}{d_O - f} = \frac{(20\text{ cm})(10\text{ cm})}{(10\text{ cm}) - (20\text{ cm})} = \boxed{-20\text{ cm}}$$

The image is $\boxed{\text{virtual}}$ because $d_I < 0$.

Image height:

$$h_I = -\frac{d_I}{d_O}h_O = -\left(\frac{-20\text{ cm}}{10\text{ cm}}\right)(10\text{ cm}) = 20\text{ cm}$$

The image is $\boxed{\text{20 cm tall and upright}}$ because $h_I > 0$.

Ray trace diagram:

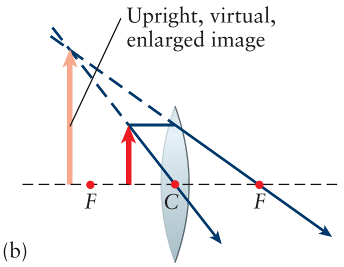


Figure 24-29 Problem 69

Part c)
Image distance:

$$d_I = \frac{fd_O}{d_O - f} = \frac{(20\text{ cm})(20\text{ cm})}{(20\text{ cm}) - (20\text{ cm})} = \infty$$

$\boxed{\text{No image is formed}}$.

Ray trace diagram:

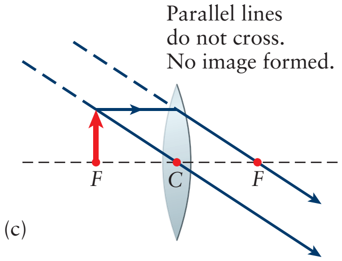


Figure 24-30 Problem 69

Part d)

Image distance:

$$d_{\mathrm{I}} = \frac{fd_{\mathrm{O}}}{d_{\mathrm{O}} - f} = \frac{(20\text{ cm})(50\text{ cm})}{(50\text{ cm}) - (20\text{ cm})} = \boxed{33\text{ cm}}$$

The image is $\boxed{\text{real}}$ because $d_{\mathrm{I}} > 0$.

Image height:

$$h_{\mathrm{I}} = -\frac{d_{\mathrm{I}}}{d_{\mathrm{O}}}h_{\mathrm{O}} = -\left(\frac{33\text{ cm}}{50\text{ cm}}\right)(10\text{ cm}) = -6.7\text{ cm}$$

The image is $\boxed{\text{6.7 cm tall and inverted}}$ because $h_{\mathrm{I}} < 0$.

Ray trace diagram:

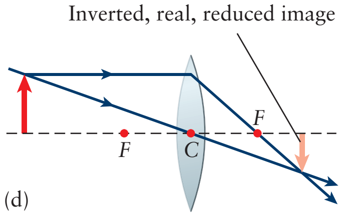


Figure 24-31 Problem 69

REFLECT

An object placed at the focal point of a lens will not create an image because the rays come out parallel and do not converge. This is the reverse situation of an object at infinity emitting parallel rays, which are converged at the focal point by a converging lens.

Get Help: Picture It – Converging Lens

24.70

SET UP

The power of a diverging lens is 5 diopters, which means its focal length is $f = \frac{1}{P} = \frac{1}{-5\text{ m}^{-1}} = -0.20\text{ m} = -20\text{ cm}$. An object ($h_{\mathrm{O}} = 10$ cm) is positioned at four different object distances: $d_{\mathrm{O}} = +5$ cm, $d_{\mathrm{O}} = +10$ cm, $d_{\mathrm{O}} = +20$ cm, and $d_{\mathrm{O}} = +50$ cm.

We can use the lens equation, $\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f}$, and its sign conventions to calculate the image distance and the image height, as well as determine whether the image is real or virtual, upright or inverted. As a reminder, a positive image distance is considered real, and a negative image height corresponds to an inverted image. We can also draw a ray trace diagram of each situation to confirm our numerical results.

SOLVE

Image distance in general:

$$\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f}$$

$$\frac{1}{d_I} = \frac{1}{f} - \frac{1}{d_O} = \frac{d_O - f}{f d_O}$$

$$d_I = \frac{f d_O}{d_O - f}$$

Image height in general:

$$m = \frac{h_I}{h_O} = -\frac{d_I}{d_O}$$

$$h_I = -\frac{d_I}{d_O} h_O$$

Part a)

Image distance:

$$d_I = \frac{f d_O}{d_O - f} = \frac{(-20\text{ cm})(5\text{ cm})}{(5\text{ cm}) - (-20\text{ cm})} = \boxed{-4\text{ cm}}$$

The image is $\boxed{\text{virtual}}$ because $d_I < 0$.

Image height:

$$h_I = -\frac{d_I}{d_O} h_O = -\left(\frac{-4\text{ cm}}{5\text{ cm}}\right)(10\text{ cm}) = 8\text{ cm}$$

The image is $\boxed{\text{8 cm tall and upright}}$ because $h_I > 0$.

Ray trace diagram:

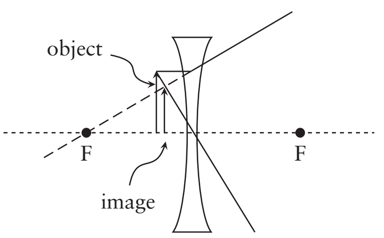


Figure 24-32 Problem 70

Part b)
Image distance:

$$d_I = \frac{fd_O}{d_O - f} = \frac{(-20\text{ cm})(10\text{ cm})}{(10\text{ cm}) - (-20\text{ cm})} = \boxed{-6.7\text{ cm}}$$

The image is $\boxed{\text{virtual}}$ because $d_I < 0$.

Image height:

$$h_I = -\frac{d_I}{d_O}h_O = -\left(\frac{-6.7\text{ cm}}{10\text{ cm}}\right)(10\text{ cm}) = 6.7\text{ cm}$$

The image is $\boxed{\text{6.7 cm tall and upright}}$ because $h_I > 0$.

Ray trace diagram:

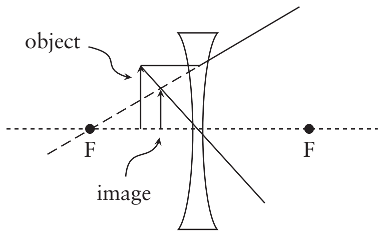


Figure 24-33 Problem 70

Part c)
Image distance:

$$d_I = \frac{fd_O}{d_O - f} = \frac{(-20\text{ cm})(20\text{ cm})}{(20\text{ cm}) - (-20\text{ cm})} = \boxed{-10\text{ cm}}$$

The image is $\boxed{\text{virtual}}$ because $d_I < 0$.

Image height:

$$h_I = -\frac{d_I}{d_O}h_O = -\left(\frac{-10\text{ cm}}{20\text{ cm}}\right)(10\text{ cm}) = 5.0\text{ cm}$$

The image is $\boxed{\text{5.0 cm tall and upright}}$ because $h_I > 0$.

Ray trace diagram:

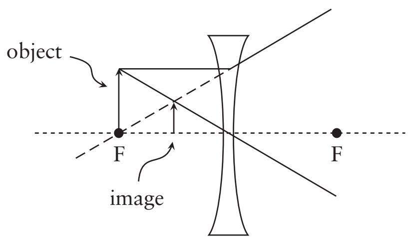


Figure 24-34 Problem 70

Part d)
Image distance:

$$d_I = \frac{fd_O}{d_O - f} = \frac{(-20\text{ cm})(50\text{ cm})}{(50\text{ cm}) - (-20\text{ cm})} = \boxed{-14\text{ cm}}$$

The image is $\boxed{\text{virtual}}$ because $d_I > 0$.

Image height:

$$h_I = -\frac{d_I}{d_O}h_O = -\left(\frac{-14\text{ cm}}{50\text{ cm}}\right)(10\text{ cm}) = 2.9\text{ cm}$$

The image is $\boxed{\text{2.9 cm tall and upright}}$ because $h_I > 0$.

Ray trace diagram:

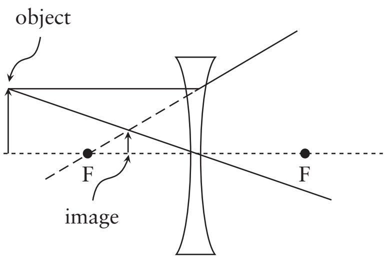


Figure 24-35 Problem 70

REFLECT
The image formed by a single diverging lens will always be virtual, upright, and smaller.

24.71

SET UP
An object ($h_I = 2.00$ cm) is located at a distance $d_O = 18.0$ cm in front of a lens ($f = 30.0$ cm). We can use the thin-lens equation, $\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f}$, to calculate the type, final location, and height of the image. We will assume the lens is located at the origin.

SOLVE

Part a)
Location of image:

$$\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f}$$

$$d_I = \frac{d_O f}{f - d_O} = \frac{(18.0\text{ cm})(30.0\text{ cm})}{(18.0\text{ cm}) - (30.0\text{ cm})} = \boxed{-45.0\text{ cm}}$$

Height of image:

$$h_I = -\frac{d_I}{d_O}h_O = -\left(\frac{-45.0\text{ cm}}{18.0\text{ cm}}\right)(2.00\text{ cm}) = \boxed{5.00\text{ cm}}$$

Because $d_I < 0$ and $h_I > 0$, the image is $\boxed{\text{virtual and upright}}$.

Part b)

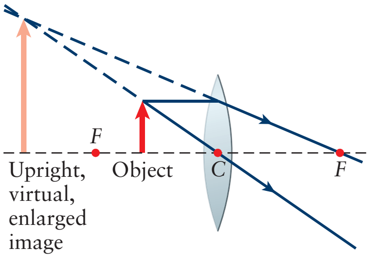


Figure 24-36 Problem 71

REFLECT

The object is within the focal point of the converging lens, so the image should be virtual, upright, and larger.

24.72

SET UP

A lens is formed from a plastic material ($n = 1.55$). With the sign conventions in mind, the radius of curvature of the front surface is $R_1 = 1.25$ m, and the radius of curvature of the back surface is $R_2 = -1.75$ m. We can calculate the power and the focal length of the lens from the lensmaker's equation, $P = \frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.

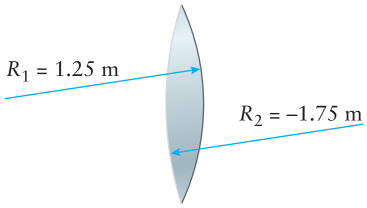


Figure 24-37 Problem 72

SOLVE

Power:

$$P = \frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.55 - 1)\left(\frac{1}{1.25\text{ m}} - \frac{1}{(-1.75\text{ m})}\right) = \boxed{0.75\text{ m}^{-1}}$$

Focal length:

$$f = \frac{1}{P} = \frac{1}{0.75\text{ m}^{-1}} = \boxed{1.3\text{ m}}$$

REFLECT

The focal length is positive, as we would expect for a converging lens.

24.73

SET UP

A plano-concave, glass lens ($n = 1.60$) has a focal length of $f = -31.8$ cm. We can calculate the radius of curvature of the concave surface from the lensmaker's equation, $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$. We'll assume that the front surface of the lens is planar, which means $R_1 = \infty$. For the lensmaker's equation, R_2 is positive for a concave surface and negative for a convex surface. Once we know the R_2 for a concave surface, we can invert its sign in order to

calculate the focal length of a plano-convex, glass lens with the same radius of curvature using the lensmaker's equation.

SOLVE

Part a)

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f(n-1)} = \frac{1}{R_1} - \frac{1}{R_2} = \frac{1}{\infty} - \frac{1}{R_2} = -\frac{1}{R_2}$$

$$R_2 = -f(n-1) = -(-31.8\text{ cm})((1.60)-1) = \boxed{19.1\text{ cm} = 0.191\text{ m}}$$

Part b)

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$f = \frac{1}{(n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)} = \frac{1}{(n-1)\left(\frac{1}{\infty} - \frac{1}{R_2}\right)} = \frac{1}{(n-1)\left(-\frac{1}{R_2}\right)}$$

$$= \frac{-R_2}{n-1} = \frac{-(-19.1\text{ cm})}{(1.60)-1} = \boxed{+31.8\text{ cm}}$$

REFLECT

It makes sense that changing the back surface of the lens from concave to convex should convert the lens from a diverging one to a converging one. Also, since we're not changing the magnitude of the radius of curvature of that surface, the magnitude of the focal length should not change.

24.74

SET UP

An object ($h_\mathrm{I} = 2.00$ cm) is located at a distance $d_\mathrm{O} = 30.0$ cm in front of a lens ($f = 18.0$ cm). We can use the thin-lens equation, $\frac{1}{d_\mathrm{O}} + \frac{1}{d_\mathrm{I}} = \frac{1}{f}$, to calculate the type, final location, and height of the image. We will draw a ray trace diagram to check our answers. We will assume the lens is located at the origin.

SOLVE

Part a)
Image distance:

$$\frac{1}{d_\mathrm{O}} + \frac{1}{d_\mathrm{I}} = \frac{1}{f}$$

$$\frac{1}{d_I} = \frac{1}{f} - \frac{1}{d_O} = \frac{d_O - f}{f d_O}$$

$$d_I = \frac{f d_O}{d_O - f} = \frac{(18.0\text{ cm})(30.0\text{ cm})}{(30.0\text{ cm}) - (18.0\text{ cm})} = \boxed{45.0\text{ cm}}$$

The image is $\boxed{\text{real}}$ because $d_I > 0$.

Image height:

$$m = \frac{h_I}{h_O} = -\frac{d_I}{d_O}$$

$$h_I = -\frac{d_I}{d_O} h_O = -\left(\frac{45.0\text{ cm}}{30.0\text{ cm}}\right)(2.00\text{ cm}) = -3.00\text{ cm}$$

The image is $\boxed{\text{3.00 cm tall and inverted}}$ because $h_I < 0$.

Part b)

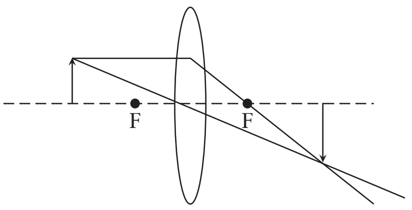


Figure 24-38 Problem 74

REFLECT

The image created by an object outside of the focal length of a converging lens should be inverted and real.

24.75

SET UP

A microscope consists of an objective lens and an eyepiece separated by $L = 18$ cm. The focal length of the objective is $f_o = 0.50$ cm, and the focal length of the eyepiece is $f_e = 2.50$ cm. The total magnification of a system of lenses is equal to the product of the magnifications due to each individual lens; in this case, $m_{total} = m_o m_e$. A microscope is constructed in such a way that a specimen located at the focal plane of the objective lens creates an image near the focal plane of the eyepiece. Since the distance between the lenses is much larger than the focal length of the lenses, the magnification of the objective lens is equal to the distance separating the two lenses divided by the focal length of the objective lens. The eyepiece takes the image from the objective as its object and creates an image at the near point of your eye ($N = 25$ cm), which means the magnification due to the eyepiece is equal to N divided by the focal length of the eyepiece. Using this information, we can solve for the total magnification of the microscope.

SOLVE

$$m_{total} = m_o m_e = \left(\frac{L}{f_o}\right)\left(\frac{N}{f_e}\right) = \frac{(18\text{ cm})(25\text{ cm})}{(0.50\text{ cm})(2.50\text{ cm})} = \boxed{360}$$

REFLECT

This is a reasonable value for the total magnification of a compound microscope. In a compound microscope consisting of two lenses, the focal length of the objective lens is always smaller than the focal length of the eyepiece.

24.76

SET UP

The walls of a barber shop are lined with plane mirrors. The width of the shop is 6.50 m. You stand 2.00 m from the north wall and see multiple images reflected in the north and south mirrors. Using the fact that the image distance is equal in magnitude to the object distance for a plane mirror, we can calculate the separation distance between the first two images in the north and south mirrors.

SOLVE

Part a) The first image in the north mirror is located 2.00 m behind the wall. The first image in the south mirror will be 4.50 m behind the south wall. This object, which is a distance of (4.50 m) + (6.50 m) = 11.00 m from the north mirror, will reflect back into the north mirror and form an image 11.00 m behind the north wall. The distance between these first two images in the north mirror is

$$\Delta x = (11.00\text{ m}) - (2.00\text{ m}) = \boxed{9.00\text{ m}}$$

Part b) As a reminder, the first image in the south mirror will be 4.50 m behind the south wall. The first image in the north mirror acts as an object for the south mirror at an object distance of (2.00 m) + (6.50 m) = 8.50 m, which means the second image will be 8.50 m behind the south wall. The distance between these first two images in the south mirror is

$$\Delta x = (8.50\text{ m}) - (4.50\text{ m}) = \boxed{4.00\text{ m}}$$

REFLECT

Each of the images in the mirrors acts as an object.

24.77

SET UP

An object is placed in front of a spherical concave mirror that has a radius of curvature of $R = 32.0$ cm, which means the focal length is $f = +16.0$ cm. The object distance is $d_O = 40.0$ cm. We can apply the mirror equation, $\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f}$, and its sign conventions to calculate the image distance. The magnification of the system is given by $m = \frac{h_I}{h_O} = -\frac{d_I}{d_O}$.

A positive image distance is considered real, and a negative magnification corresponds to an inverted image. Next, we can draw a ray trace diagram of the setup to confirm the image distance and magnification of the image. A parallel ray from the tip of the object will reflect off of the mirror through the focal point. A ray from the tip of the object striking the center of the mirror is also easy to reflect by applying the law of reflection. A person must be facing the shiny side of the mirror in order to see the image produced.

SOLVE

Part a)
Image distance:

$$\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f}$$

$$\frac{1}{d_I} = \frac{1}{f} - \frac{1}{d_O} = \frac{d_O - f}{fd_O}$$

$$d_I = \frac{fd_O}{d_O - f} = \frac{(16.0\text{ cm})(40.0\text{ cm})}{(40.0\text{ cm}) - (16.0\text{ cm})} = \boxed{26.7\text{ cm}}$$

The image is $\boxed{\text{real}}$ because $d_I > 0$.
Magnification:

$$m = \frac{h_I}{h_O} = -\frac{d_I}{d_O} = -\frac{26.7\text{ cm}}{40.0\text{ cm}} = \boxed{-0.667}$$

The image is $\boxed{\text{inverted}}$ because $m < 0$ and $\boxed{\text{smaller}}$ because $|m| < 1$.

Part b)

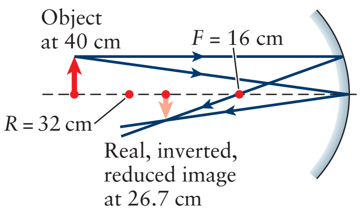


Figure 24-39 Problem 77

REFLECT

The image produced by an object outside the focal point of a spherical concave mirror should be real and inverted.

24.78

SET UP

To derive the lensmaker's equation, we will consider a ray that comes in parallel to the optical axis of a thin double convex lens with surfaces with radii of curvature R_1 and R_2 and a focal length f. Since the lens is thin, we can ignore the thickness of the lens so the distance between the focal point and either surface of the lens is equal to f. We know that parallel incoming rays pass through the far focal point. Applying Snell's law at each surface of the lens, working through the geometry, and then applying the optical sign conventions, we will arrive at the lensmaker's equation, $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.

SOLVE

Schematic with the size of the lens exaggerated to better see the geometry:

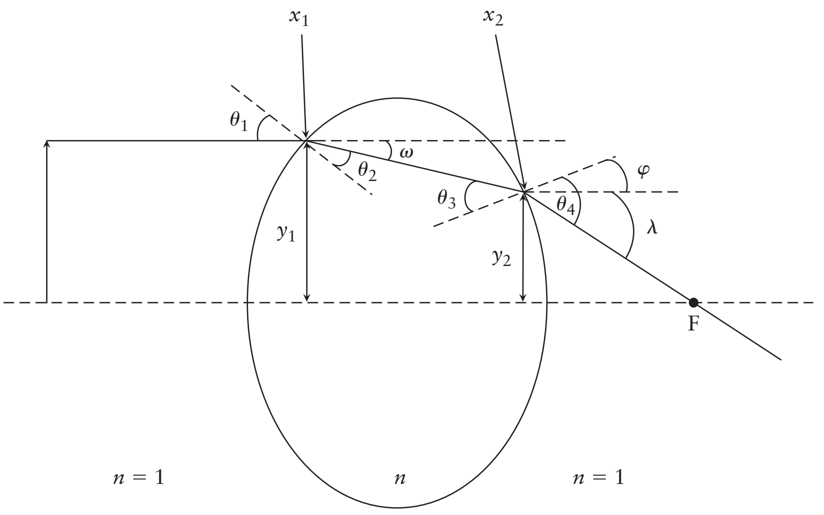


Figure 24-40 Problem 78

Resulting triangles:

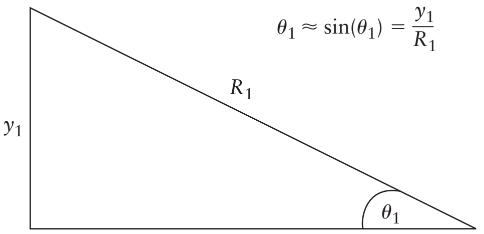


Figure 24-41 Problem 78

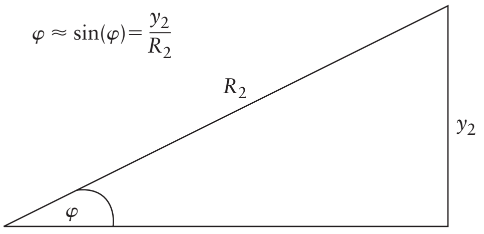


Figure 24-42 Problem 78

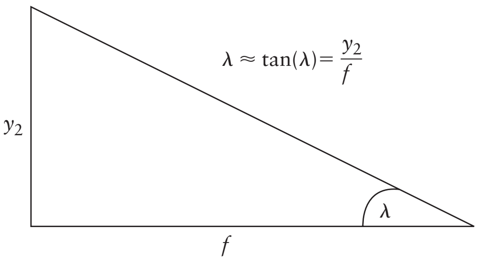


Figure 24-43 Problem 78

Snell's law:

$$1.00 \sin(\theta_1) = n \sin(\theta_2)$$

$$\theta_1 \approx n\,\theta_2$$

$$n \sin(\theta_3) = 1.00 \sin(\theta_4)$$

$$n\theta_3 \approx \theta_4$$

At point X_1:

$$\theta_1 = \theta_2 + \omega$$

$$\omega = \theta_1 - \theta_2$$

At point X_2:

$$\theta_4 = \varphi + \lambda$$

$$\theta_3 = \omega + \varphi$$

$$\varphi = \theta_3 - \omega = \left(\frac{\theta_4}{n}\right) - (\theta_1 - \theta_2)$$

$$\varphi = \left(\frac{\varphi + \lambda}{n}\right) - \theta_1 + \theta_2$$

$$\varphi = \frac{\varphi}{n} + \frac{\lambda}{n} - \theta_1 + \theta_2$$

$$\frac{y_2}{R_2} = \frac{y_2}{nR_2} + \frac{y_2}{nf} - \frac{y_1}{R_1} + \frac{y_1}{nR_1}$$

Since the lens is thin, $y_1 \approx y_2$:

$$\frac{1}{R_2} = \frac{1}{nR_2} + \frac{1}{nf} - \frac{1}{R_1} + \frac{1}{nR_1}$$

$$\frac{n}{R_2} = \frac{1}{R_2} + \frac{1}{f} - \frac{n}{R_1} + \frac{1}{R_1}$$

$$\frac{1}{R_2}(n-1) + \frac{1}{R_1}(n-1) = \frac{1}{f}$$

Invoking the sign conventions for a double convex lens:

$$-\frac{1}{R_2}(n-1) + \frac{1}{R_1}(n-1) = \frac{1}{f}$$

$$\boxed{\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

REFLECT

We were able to invoke the small angle approximation because the thickness of the lens is negligible.

24.79

SET UP

A typical human lens has a double convex shape of variable curvature and an index of refraction of $n = 1.43$. At minimum power, the radii of curvature of the lens are $R_{1,\,\text{min}} = 10.0$ mm and $R_{2,\,\text{min}} = -6.00$ mm. At maximum power, the radii of curvature of the lens are $R_{1,\,\text{max}} = 6.00$ mm and $R_{2,\,\text{min}} = -5.50$ mm. We can find the power and focal length of the lens in each case using the lensmaker's equation. Once we know the focal length at maximum power, we can use the thin-lens equation to calculate the image distance for an object located $d_O = 25$ cm in front of the lens. To determine if this image is formed on the retina, we need to compare d_I to the distance the retina is behind the lens, which is about 2.5 cm.

SOLVE

Part a)
Minimum power:

$$P_{\text{min}} = \frac{1}{f} = (n-1)\left(\frac{1}{R_{1,\,\text{min}}} - \frac{1}{R_{2,\,\text{min}}}\right) = (1.43-1)\left(\frac{1}{10.0\text{ mm}} - \frac{1}{(-6.00\text{ mm})}\right)$$

$$= 0.11\text{ mm}^{-1} = \boxed{110\text{ D}}$$

Focal length at minimum power:

$$f = \frac{1}{P_{\text{min}}} = \frac{1}{0.11\text{ mm}^{-1}} = \boxed{8.7\text{ mm}}$$

Maximum power:

$$P_{\text{max}} = \frac{1}{f} = (n-1)\left(\frac{1}{R_{1,\,\text{max}}} - \frac{1}{R_{2,\,\text{max}}}\right) = (1.43-1)\left(\frac{1}{6.00\text{ mm}} - \frac{1}{(-5.50\text{ mm})}\right)$$

$$= 0.15\text{ mm}^{-1} = \boxed{150\text{ D}}$$

Focal length at maximum power:

$$f = \frac{1}{P_{\text{min}}} = \frac{1}{0.15\text{ mm}^{-1}} = \boxed{6.7\text{ mm}}$$

Part b)

$$\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f_{\text{min}}}$$

$$d_I = \frac{d_O f}{d_O - f} = \frac{(250\text{ mm})(6.7\text{ mm})}{(250\text{ mm}) - (6.7\text{ mm})} = \boxed{6.9\text{ mm}}$$

Part c) The image distance is much shorter than the distance between the lens and the retina, so the image $\boxed{\text{would not fall on the retina}}$.

REFLECT

The focal lengths are positive as expected for a converging lens. Vision problems arise for most people later in life because the ciliary muscles can no longer stretch the lens to its maximum power.

24.80

SET UP

The retina of a person's eye is located a distance $d_I = 2.5$ cm behind the lens and cornea. The near point and far point for a person with normal vision are $d_{O,\text{ near}} = 25$ cm and $d_{O,\text{ far}} = \infty$, respectively. We can use the thin-lens equation to calculate the range of the effective focal lengths of the focusing mechanism of a typical human eye. For a person to "see" an object, the light from the object must be absorbed by the cells in the retina, so the lens and cornea must produce a real image on the retina.

SOLVE

Part a)
Near point:

$$\frac{1}{d_{O,\text{ near}}} + \frac{1}{d_I} = \frac{1}{f_{\text{near}}}$$

$$f_{\text{near}} = \frac{d_I d_{O,\text{ near}}}{d_I + d_{O,\text{ near}}} = \frac{(2.5\text{ cm})(25\text{ cm})}{(2.5\text{ cm}) + (25\text{ cm})} = 2.3\text{ cm}$$

Far point:

$$\frac{1}{d_{O,\text{ far}}} + \frac{1}{d_I} = \frac{1}{f_{\text{far}}}$$

$$\frac{1}{\infty} + \frac{1}{d_I} = 0 + \frac{1}{d_I} = \frac{1}{f_{\text{far}}}$$

$$f_{\text{far}} = d_I = 2.5\text{ cm}$$

The range of effective focal lengths for the lens plus cornea system is $\boxed{2.3\text{ cm} \le f \le 2.5\text{ cm}}$.

Part b) The focusing mechanism of the eye must focus the energy of the light onto the retinal cells, so it must form a real image on the retina. Therefore, the equivalent focusing mechanism acts as a $\boxed{\text{converging lens}}$. This is consistent with the positive focal lengths we found in part (a).

REFLECT

Photoreceptor cells in the retina, also known as "rods" and "cones," need to absorb light in order to transmit an electrical signal to the brain.

24.81

SET UP

A microscope that consists of an objective lens and an eyepiece separated by $L = 16$ cm has a total magnification of $m_{\text{total}} = 400$. The focal length of the objective is $f_{\text{o}} = 0.60$ cm. The total magnification of a system of lenses is equal to the product of the magnifications due to each individual lens; in this case, $m_{\text{total}} = m_{\text{o}}m_{\text{e}}$. A microscope is constructed in such a way that a specimen located at the focal plane of the objective lens creates an image near the focal plane of the eyepiece. Since the distance between the lenses is much larger than the focal length of the lenses, the magnification of the objective lens is equal to the distance separating the two lenses divided by the focal length of the objective lens. The eyepiece takes the image from the objective as its object and creates an image at the near point of your eye ($N = 25$ cm), which means the magnification due to the eyepiece is equal to N divided by the focal length of the eyepiece. By rearranging the expression for the total magnification, we can solve for the focal length of the eyepiece.

SOLVE

$$m_{\text{total}} = m_{\text{o}}m_{\text{e}} = \left(\frac{L}{f_{\text{o}}}\right)\left(\frac{N}{f_{\text{e}}}\right)$$

$$f_e = \frac{LN}{f_o m_{\text{total}}} = \frac{(16\text{ cm})(25\text{ cm})}{(0.60\text{ cm})(400)} = \boxed{1.7\text{ cm}}$$

REFLECT

In a compound microscope consisting of two lenses, the focal length of the objective lens is always smaller than the focal length of the eyepiece.

Get Help: Interactive Example – Two Lens System
P'Cast 24.6 – Magnification

24.82

SET UP

A thin lens ($n = 1.60$) has surfaces with radii of curvature of magnitude 12.0 mm and 18.0 mm. We can look at the different combinations of these radii and the lensmaker's equation, $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$, to calculate the possible focal lengths for the resulting lenses and then sketch the cross section of each one.

SOLVE

Part a)
General formula for the focal length:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$f = \frac{R_1R_2}{(n-1)(R_2 - R_1)}$$

Possible focal lengths assuming both radii of curvature are positive:

$$f = \frac{R_1R_2}{(n-1)(R_2-R_1)} = \frac{(12.0\text{ mm})(18.0\text{ mm})}{(1.60-1)((18.0\text{ mm})-(12.0\text{ mm}))} = \boxed{60\text{ mm}}$$

$$f = \frac{R_1R_2}{(n-1)(R_2-R_1)} = \frac{(18.0\text{ mm})(12.0\text{ mm})}{(1.60-1)((12.0\text{ mm})-(18.0\text{ mm}))} = \boxed{-60\text{ mm}}$$

Possible focal lengths assuming both radii of curvature are negative:

$$f = \frac{R_1R_2}{(n-1)(R_2-R_1)} = \frac{(-12.0\text{ mm})(-18.0\text{ mm})}{(1.60-1)((-18.0\text{ mm})-(-12.0\text{ mm}))} = \boxed{-60\text{ mm}}$$

$$f = \frac{R_1R_2}{(n-1)(R_2-R_1)} = \frac{(-18.0\text{ mm})(-12.0\text{ mm})}{(1.60-1)((-12.0\text{ mm})-(-18.0\text{ mm}))} = \boxed{60\text{ mm}}$$

Possible focal lengths assuming one radius of curvature is negative and the other is positive:

$$f = \frac{R_1R_2}{(n-1)(R_2-R_1)} = \frac{(12.0\text{ mm})(-18.0\text{ mm})}{(1.60-1)((-18.0\text{ mm})-(12.0\text{ mm}))} = \boxed{12\text{ mm}}$$

$$f = \frac{R_1R_2}{(n-1)(R_2-R_1)} = \frac{(-18.0\text{ mm})(12.0\text{ mm})}{(1.60-1)((12.0\text{ mm})-(-18.0\text{ mm}))} = \boxed{-12\text{ mm}}$$

$$f = \frac{R_1R_2}{(n-1)(R_2-R_1)} = \frac{(18.0\text{ mm})(-12.0\text{ mm})}{(1.60-1)((-12.0\text{ mm})-(18.0\text{ mm}))} = \boxed{12\text{ mm}}$$

$$f = \frac{R_1R_2}{(n-1)(R_2-R_1)} = \frac{(-12.0\text{ mm})(18.0\text{ mm})}{(1.60-1)((18.0\text{ mm})-(-12.0\text{ mm}))} = \boxed{-12\text{ mm}}$$

Part b)

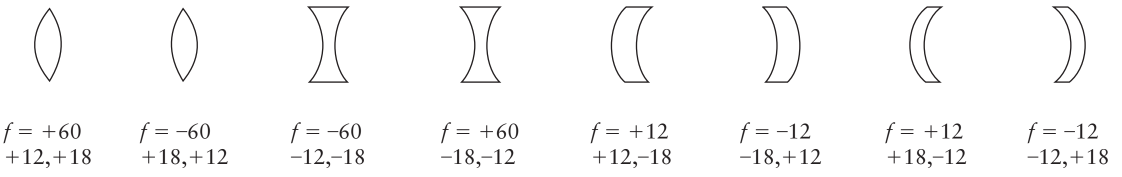


Figure 24-44 Problem 82

REFLECT

There are eight combinations because each radius can be positive or negative and each value can correspond to either R_1 or R_2.

24.83

SET UP

A converging lens made out of plastic of unknown index of refraction has radii of curvature of $R_1 = 3.50$ cm and $R_2 = -4.25$ cm. The focal length of the lens is $f = 1.65$ cm. We can use the lensmaker's equation to calculate the index of refraction of the plastic.

SOLVE

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$n = \frac{1}{f\left(\frac{1}{R_1} - \frac{1}{R_2}\right)} + 1 = \frac{1}{(1.65\text{ cm})\left(\frac{1}{3.50\text{ cm}} - \frac{1}{(-4.25\text{ cm})}\right)} + 1 = \boxed{2.16}$$

REFLECT

We would arrive at the same answer if we reversed the two pieces of plastic, such that $R_1 = 4.25$ cm and $R_2 = -3.50$ cm:

$$n = \frac{1}{(1.65\text{ cm})\left(\frac{1}{4.25\text{ cm}} - \frac{1}{(-3.50\text{ cm})}\right)} + 1 = 2.16$$

24.84

SET UP

A lens ($f_A = 15.0$ cm) is located 10.0 cm to the left of a second lens ($f_B = -15.0$ cm) with the same optical axis. An object is placed a distance $d_{O,A} = 30.0$ cm in front of the first lens. We can use the thin-lens equation to determine the location of the image produced by lens A. This image now acts as the object for the second lens (lens B), so we can use the thin-lens equation again to calculate the location of the image after it passes through both lenses. If the image distance from the final calculation is positive, then the image is real; if it's negative, the image is virtual. Whether or not the image is upright or inverted can be found by calculating the total magnification due to both lenses. A person must look through both lenses in order to see the final image created by both of them.

SOLVE

Part a)
Image formed by the first lens:

$$\frac{1}{d_{O,A}} + \frac{1}{d_{I,A}} = \frac{1}{f_A}$$

$$\frac{1}{d_{I,A}} = \frac{1}{f_A} - \frac{1}{d_{O,A}} = \frac{d_{O,A} - f_A}{f_A d_{O,A}}$$

$$d_{I,A} = \frac{f_A d_{O,A}}{d_{O,A} - f_A} = \frac{(15.0\text{ cm})(30.0\text{ cm})}{(30.0\text{ cm}) - (15.0\text{ cm})} = 30.0\text{ cm}$$

Image formed by the second lens:
The image formed by the first lens is located 30.0 cm behind the lens, which means $d_{O,B} = -20.0$ cm.

$$\frac{1}{d_{O,B}} + \frac{1}{d_{I,B}} = \frac{1}{f_B}$$

$$\frac{1}{d_{\mathrm{I,\,B}}} = \frac{1}{f_{\mathrm{B}}} - \frac{1}{d_{\mathrm{O,\,B}}} = \frac{d_{\mathrm{O,\,B}} - f_{\mathrm{B}}}{f_{\mathrm{B}} d_{\mathrm{O,\,B}}}$$

$$d_{\mathrm{I,\,B}} = \frac{f_{\mathrm{B}} d_{\mathrm{O,\,B}}}{d_{\mathrm{O,\,B}} - f_{\mathrm{B}}} = \frac{(-15.0\text{ cm})(-20.0\text{ cm})}{(-20.0\text{ cm}) - (-15.0\text{ cm})} = -60.0\text{ cm}$$

The image formed by the mirror is located $\boxed{\text{60 cm in front of the second lens}}$.

Part b) The final image is $\boxed{\text{virtual}}$ because $d_{\mathrm{I,\,B}} < 0$.

Part c)

$$m_{\text{total}} = m_{\mathrm{A}} m_{\mathrm{B}} = \left(-\frac{d_{\mathrm{I,\,A}}}{d_{\mathrm{O,\,A}}}\right)\left(-\frac{d_{\mathrm{I,\,B}}}{d_{\mathrm{O,\,B}}}\right) = \left(\frac{30.0\text{ cm}}{30.0\text{ cm}}\right)\left(\frac{-60.0\text{ cm}}{-20.0\text{ cm}}\right) = 3$$

Because $m_{\text{total}} = 3$, the final image is $\boxed{\text{upright and three times as large as the object}}$.

Part d) To see the final image, a viewer would need to look to the left through both lenses.

REFLECT

The real image from the first lens acts as a virtual object for the second lens because it is on the opposite side of the lens as the side from whence the light came.

24.85

SET UP

A very small, thin plano-concave lens ($f_{\mathrm{A}} = -45.0$ cm) has the same principal axis as a concave mirror with a radius of curvature of +60.0 cm. The focal length of a spherical mirror is equal to the radius of curvature divided in half, so $f_{\mathrm{B}} = 30.0$ cm. An object is placed $d_{\mathrm{O,\,A}} =$ 15.0 cm in front of the lens. We can use the thin lens equation to determine the location of the image produced by the lens. This image now acts as the object for the concave mirror, so we can use the mirror equation to calculate the location of the image after it was reflected by the mirror. This will be the final image because the lens is so much smaller than the mirror; essentially none of the light rays reflected by the mirror will pass through the lens. If the image distance from the mirror calculation is positive, then the image is real; if it's negative, the image is virtual. Whether or not the image is upright or inverted can be found by calculating the total magnification due to both the lens and the mirror. A convex mirror of the same magnitude radius of curvature will have a focal length of $f_{\mathrm{B}} = -30.0$ cm. We can follow the same process as the concave mirror to determine where the image is produced by this convex mirror.

SOLVE

Part a)

Image formed by the lens:

$$\frac{1}{d_{\mathrm{O,\,A}}} + \frac{1}{d_{\mathrm{I,\,A}}} = \frac{1}{f_{\mathrm{A}}}$$

$$\frac{1}{d_{\text{I, A}}} = \frac{1}{f_{\text{A}}} - \frac{1}{d_{\text{O, A}}} = \frac{d_{\text{O, A}} - f_{\text{A}}}{f_{\text{A}} d_{\text{O, A}}}$$

$$d_{\text{I, A}} = \frac{f_{\text{A}} d_{\text{O, A}}}{d_{\text{O, A}} - f_{\text{A}}} = \frac{(-45.0 \text{ cm})(15.0 \text{ cm})}{(15.0 \text{ cm}) - (-45.0 \text{ cm})} = -11.25 \text{ cm}$$

Image formed by the mirror:

The image formed by the lens is located 11.25 cm in front of the lens, which means $d_{\text{O, B}} = (11.25 \text{ cm}) + (20.0 \text{ cm}) = 31.25 \text{ cm}$.

$$\frac{1}{d_{\text{O, B}}} + \frac{1}{d_{\text{I, B}}} = \frac{1}{f_{\text{B}}}$$

$$\frac{1}{d_{\text{I, B}}} = \frac{1}{f_{\text{B}}} - \frac{1}{d_{\text{O, B}}} = \frac{d_{\text{O, B}} - f_{\text{B}}}{f_{\text{B}} d_{\text{O, B}}}$$

$$d_{\text{I, B}} = \frac{f_{\text{B}} d_{\text{O, B}}}{d_{\text{O, B}} - f_{\text{B}}} = \frac{(30.0 \text{ cm})(31.25 \text{ cm})}{(31.25 \text{ cm}) - (30.0 \text{ cm})} = 750 \text{ cm}$$

The image formed by the mirror is located $\boxed{\text{750 cm in front of the mirror}}$ (or 730 cm in front of the lens).

Part b)

The final image is $\boxed{\text{real because } d_{\text{I, B}} > 0}$.

Magnification:

$$m_{\text{total}} = m_{\text{A}} m_{\text{B}} = \left(-\frac{d_{\text{I, A}}}{d_{\text{O, A}}}\right)\left(-\frac{d_{\text{I, B}}}{d_{\text{O, B}}}\right) = \left(\frac{-11.25 \text{ cm}}{15.0 \text{ cm}}\right)\left(\frac{750 \text{ cm}}{31.25 \text{ cm}}\right) = -18$$

The image is $\boxed{\text{inverted because } m_{\text{total}} < 0}$.

Part c)

The only thing that changes is that $f_{\text{B}} = -30.0$ cm.

Therefore,

$$d_{\text{I, B}} = \frac{f_{\text{B}} d_{\text{O, B}}}{d_{\text{O, B}} - f_{\text{B}}} = \frac{(-30.0 \text{ cm})(31.25 \text{ cm})}{(31.25 \text{ cm}) - (-30.0 \text{ cm})} = -15.3 \text{ cm}$$

The image created by the mirror is $\boxed{\text{virtual because } d_{\text{I, B}} < 0}$.

Magnification:

$$m_{\text{total}} = m_{\text{A}} m_{\text{B}} = \left(-\frac{d_{\text{I, A}}}{d_{\text{O, A}}}\right)\left(-\frac{d_{\text{I, B}}}{d_{\text{O, B}}}\right) = \left(\frac{-11.25 \text{ cm}}{15.0 \text{ cm}}\right)\left(\frac{-15.3 \text{ cm}}{31.25 \text{ cm}}\right) = 0.367$$

The image is $\boxed{\text{upright because } m_{\text{total}} > 0}$.

REFLECT

If the lens were the same size as the mirror, the light reflected by the mirror will pass through the lens again, in the opposite direction as part a. The real image created by the concave mirror will act as a virtual object for the lens in this case.

Get Help: P'Cast 24.6 – Magnification

24.86

SET UP

When an object is placed a distance $d_O = 36.0$ cm to the left of a lens, an upright image is produced at a distance $|d_{I,\,\text{lens}}| = 14.0$ cm. The only way to produce an upright image with a single lens is if we use a diverging lens. Therefore, the image is virtual and $d_{I,\,\text{lens}} = -14.0$ cm. From this information, we can use the thin-lens equation to calculate the focal length of the lens. We are also told that a faint inverted image is seen at a distance $d_{I,\,1} = 13.8$ cm to the left of the lens due to the reflection off of its left face. When the lens is turned around, a faint inverted image is seen at a distance $d_{I,\,2} = 25.7$ cm to the left of the lens due to reflection off of the original right face. Treating the two faces of the lens as spherical mirrors, we can calculate their radii of curvature from the mirror equation. We can then put all of this information—the focal length of the lens and the two radii of curvature—into the lensmaker's equation to calculate the index of refraction of the material of the lens.

SOLVE

Focal length of lens:

$$\frac{1}{d_O} + \frac{1}{d_{I,\,\text{lens}}} = \frac{1}{f_{\text{lens}}}$$

$$f_{\text{lens}} = \frac{d_O d_{I,\,\text{lens}}}{d_O + d_{I,\,\text{lens}}} = \frac{(36.0\text{ cm})(-14.0\text{ cm})}{(36.0\text{ cm}) + (-14.0\text{ cm})} = -22.9\text{ cm}$$

Radius of curvature of the left side of the lens:

$$\frac{1}{d_O} + \frac{1}{d_{I,\,1}} = \frac{1}{f_1} = \frac{1}{\left(\frac{R_1}{2}\right)} = \frac{2}{R_1}$$

$$\frac{d_{I,\,1} + d_O}{d_{I,\,1} d_O} = \frac{2}{R_1}$$

$$R_1 = \frac{2d_{I,\,1} d_O}{d_{I,\,1} + d_O} = \frac{2(13.8\text{ cm})(36.0\text{ cm})}{(13.8\text{ cm}) + (36.0\text{ cm})} = 20.0\text{ cm}$$

Radius of curvature of the right side of the lens:

$$\frac{1}{d_O} + \frac{1}{d_{I,\,2}} = \frac{1}{f_2} = \frac{1}{\left(\frac{R_2}{2}\right)} = \frac{2}{R_2}$$

$$\frac{d_{I,\,1} + d_O}{d_{I,\,1} d_O} = \frac{2}{R_1}$$

$$R_2 = \frac{2d_{I,2}d_O}{d_{I,2} + d_O} = \frac{2(25.7 \text{ cm})(36.0 \text{ cm})}{(25.7 \text{ cm}) + (36.0 \text{ cm})} = 30.0 \text{ cm}$$

Index of refraction:

$$\frac{1}{f_{lens}} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$n = \frac{1}{f_{lens}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)} + 1 = \frac{1}{(-22.9 \text{ cm})\left(\frac{1}{-20.0 \text{ cm}} - \frac{1}{30.0 \text{ cm}}\right)} + 1 = \boxed{1.52}$$

REFLECT

The fact that the inverted images due to reflection show up on the left side of the lens means the faces of the lens act as spherical concave mirrors. This confirms our result that the lens is a diverging lens.

24.87

SET UP

A very small, thin double convex lens ($f_A = 25.0$ cm) is placed 1.000 m from a plane mirror on the same principal axis. A flower ($h_O = 8.50$ cm) is placed 1.450 m in front of the mirror, which means $d_{O,A} = 0.450 \text{ m} = 45.0$ cm. We can use the thin-lens equation to determine the location of the image produced by the lens. This image now acts as the object for the mirror, so we can use the mirror equation to calculate the location of the image after it was reflected by the mirror. This will be the final image because the lens is so much smaller than the mirror; essentially none of the light rays reflected by the mirror will pass back through the lens. If the image distance from the mirror calculation is positive, then the image is real; if it's negative, the image is virtual. Whether or not the image is upright or inverted can be found by calculating the total magnification due to both the lens and the mirror. A concave mirror with the same magnitude focal length will have a focal length of $f_A = -25.0$ cm. We can follow the same process as the concave mirror to determine where the image is produced in this case.

SOLVE

Part a)

Image formed by the lens:

$$\frac{1}{d_{O,A}} + \frac{1}{d_{I,A}} = \frac{1}{f_A}$$

$$\frac{1}{d_{I,A}} = \frac{1}{f_A} - \frac{1}{d_{O,A}} = \frac{d_{O,A} - f_A}{f_A d_{O,A}}$$

$$d_{I,A} = \frac{f_A d_{O,A}}{d_{O,A} - f_A} = \frac{(25.0 \text{ cm})(45.0 \text{ cm})}{(45.0 \text{ cm}) - (25.0 \text{ cm})} = 56.25 \text{ cm}$$

Image formed by the mirror:

The image formed by the lens is located 56.25 cm behind the lens, which means $d_{O,B} = (100.0 \text{ cm}) - (56.25 \text{ m}) = 43.75$ cm.

$$\frac{1}{d_{\text{O, B}}} + \frac{1}{d_{\text{I, B}}} = \frac{1}{f_{\text{B}}}$$

$$\frac{1}{d_{\text{I, B}}} = \frac{1}{f_{\text{B}}} - \frac{1}{d_{\text{O, B}}} = \frac{1}{\infty} - \frac{1}{d_{\text{O, B}}}$$

$$d_{\text{I, B}} = -d_{\text{O, B}} = \boxed{-43.8 \text{ cm}}$$

The image formed by the mirror is located $\boxed{\text{43.8 cm behind the mirror}}$. Since $d_{\text{I, B}} < 0$, the image is $\boxed{\text{virtual}}$.
Image height:

$$m_{\text{total}} = m_{\text{A}} m_{\text{B}} = \left(-\frac{d_{\text{I, A}}}{d_{\text{O, A}}}\right)\left(-\frac{d_{\text{I, B}}}{d_{\text{O, B}}}\right) = \left(\frac{56.25 \text{ cm}}{45.0 \text{ cm}}\right)\left(\frac{-43.8 \text{ cm}}{43.8 \text{ cm}}\right) = -1.25$$

$$h_{\text{I}} = m_{\text{total}} h_{\text{O}} = (-1.25)(8.40 \text{ cm}) = -10.5 \text{ cm}$$

The final image is $\boxed{\text{10.5 cm tall and inverted}}$.

Part b)
Image formed by the lens:

$$\frac{1}{d_{\text{O, A}}} + \frac{1}{d_{\text{I, A}}} = \frac{1}{f_{\text{A}}}$$

$$\frac{1}{d_{\text{I, A}}} = \frac{1}{f_{\text{A}}} - \frac{1}{d_{\text{O, A}}} = \frac{d_{\text{O, A}} - f_{\text{A}}}{f_{\text{A}} d_{\text{O, A}}}$$

$$d_{\text{I, A}} = \frac{f_{\text{A}} d_{\text{O, A}}}{d_{\text{O, A}} - f_{\text{A}}} = \frac{(-25.0 \text{ cm})(45.0 \text{ cm})}{(45.0 \text{ cm}) - (-25.0 \text{ cm})} = -16.1 \text{ cm}$$

Image formed by the mirror:
The image formed by the lens is located 16.1 cm in front of the lens, which means $d_{\text{O, B}} = (100.0 \text{ cm}) + (16.1 \text{ m}) = 116.1 \text{ cm}$.

$$\frac{1}{d_{\text{O, B}}} + \frac{1}{d_{\text{I, B}}} = \frac{1}{f_{\text{B}}}$$

$$\frac{1}{d_{\text{I, B}}} = \frac{1}{f_{\text{B}}} - \frac{1}{d_{\text{O, B}}} = \frac{1}{\infty} - \frac{1}{d_{\text{O, B}}}$$

$$d_{\text{I, B}} = -d_{\text{O, B}} = \boxed{-116 \text{ cm}}$$

The image formed by the mirror is located $\boxed{\text{116 cm behind the mirror}}$. Since $\text{d}_{\text{I, B}} < 0$, the image is virtual.
Image height:

$$m_{\text{total}} = m_{\text{A}} m_{\text{B}} = \left(-\frac{d_{\text{I, A}}}{d_{\text{O, A}}}\right)\left(-\frac{d_{\text{I, B}}}{d_{\text{O, B}}}\right) = \left(\frac{-16.1 \text{ cm}}{45.0 \text{ cm}}\right)\left(\frac{-116.1 \text{ cm}}{116.1 \text{ cm}}\right) = 0.357$$

$$h_I = m_{total}h_O = (0.357)(8.40\text{ cm}) = 3.00\text{ cm}$$

The final image is $\boxed{\text{3.00 cm tall and upright}}$.

REFLECT

If the lens were the same size as the mirror, the light reflected by the mirror will pass through the lens again, in the opposite direction as in part (a).

24.88

SET UP

A microscope that consists of an objective lens and an eyepiece separated by $L = 20.0\text{ cm}$ has a total magnification of $m_{total} = 200$. The focal length of the objective is $f_o = 0.80\text{ cm}$. The total magnification of a system of lenses is equal to the product of the magnifications due to each individual lens; in this case, $m_{total} = m_o m_e$. A microscope is constructed in such a way that a specimen located at the focal plane of the objective lens creates an image near the focal plane of the eyepiece. Since the distance between the lenses is much larger than the focal length of the lenses, the magnification of the objective lens is equal to the distance separating the two lenses divided by the focal length of the objective lens. The eyepiece takes the image from the objective as its object and creates an image at the near point of your eye ($N = 25\text{ cm}$), which means the magnification due to the eyepiece is equal to N divided by the focal length of the eyepiece. By rearranging the expression for the total magnification, we can solve for the focal length of the eyepiece. After solving for the focal length of the eyepiece, we can work backward to determine the distance an object must be placed from the objective lens if the final image distance is $d_{I,e} = \infty$.

SOLVE

Part a)

$$m_{total} = m_o m_e = \left(\frac{L}{f_o}\right)\left(\frac{N}{f_e}\right)$$

$$f_e = \frac{LN}{f_o m_{total}} = \frac{(20.0\text{ cm})(25\text{ cm})}{(0.80\text{ cm})(200)} = \boxed{3.1\text{ cm}}$$

Part b)
Final image:

$$\frac{1}{d_{O,e}} + \frac{1}{d_{I,e}} = \frac{1}{f_e}$$

$$\frac{1}{d_{O,e}} + \frac{1}{\infty} = \frac{1}{f_e}$$

$$d_{O,e} = f_e$$

Intermediate image:

$$d_{I,o} = L - d_{O,e} = L - f_e$$

$$\frac{1}{d_{O,o}} + \frac{1}{d_{I,o}} = \frac{1}{f_o}$$

$$\frac{1}{d_{O,o}} + \frac{1}{L - d_{O,e}} = \frac{1}{f_o}$$

$$\frac{1}{d_{O,o}} + \frac{1}{L - f_e} = \frac{1}{f_o}$$

$$d_{O,o} = \left[\frac{1}{f_o} - \frac{1}{L - f_e}\right]^{-1} = \left[\frac{1}{0.80\text{ cm}} - \frac{1}{(20.0\text{ cm}) - (3.1\text{ cm})}\right]^{-1} = \boxed{0.84\text{ cm}}$$

REFLECT

The object distance for the eyepiece is not equal to the image distance for the objective even though the image of one becomes the object for the other. These distances are measured relative to the optical element in question.

24.89

SET UP

The power of a lens is given by the lensmaker's equation, $\frac{1}{f} = P = (n - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$. The maximum power of the lens for a given R_2 will occur when the term $\frac{1}{R_1}$ is the largest or R_1 is the smallest. Therefore, the lens has its maximum power as the limit of R_1 approaches zero.

SOLVE

$$\lim_{R_1 \to 0} P = \lim_{R_1 \to 0} (n - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \boxed{\infty}$$

REFLECT

A planar leading surface of the lens will minimize the absolute value of the power of the lens for a fixed value of R_2.

24.90

SET UP

A typical human eye has a diameter of approximately $d_I = 2.5$ cm. A person first looks at a coin ($h_I = 2.3$ cm) a distance $d_O = 30.0$ cm from her eye and then looks at her friend ($h_I = 1.8$ m) a distance $d_O = 3.25$ m from her eye. We can use similar triangles and set up a ratio to calculate the size of the images produced on the retina. Since the person perceives the objects as right-side up, the images on the retina must be real and inverted. We can also sketch a ray trace diagram following the rays from the top and bottom of the object.

SOLVE

Part a)

$$\left|\frac{h_O}{d_O}\right| = \left|\frac{h_I}{d_I}\right|$$

$$|h_I| = \left|\frac{d_I}{d_O} h_O\right|$$

Coin:

$$|h_I| = \left(\frac{2.5 \text{ cm}}{30.0 \text{ cm}}\right)(2.3 \text{ cm}) = \boxed{0.19 \text{ cm}}$$

Friend:

$$|h_I| = \left(\frac{2.5 \text{ cm}}{3.25 \text{ m}}\right)(1.8 \text{ m}) = \boxed{1.4 \text{ cm}}$$

Part b) Both images are inverted because the ray from the top of the object forms the bottom of the image, while the ray from the bottom of the object forms the top of the image. The image is real in both cases because the rays travel through the lens and actually strike the retina.

REFLECT

The images on the retina must be real because the photoreceptor cells in the retina need to absorb the energy from the light in order for our brains to process the information.

24.91

SET UP

The *tapetum lucidum* is a highly reflective membrane just behind the retina of the eyes of cats, which have a typical diameter of $d = 1.25$ cm. We can model this membrane as a concave spherical mirror with a radius of curvature of $r = d/2 = +0.625$ cm. We will assume that the light entering the cat's eye is traveling parallel to the principal axis of the lens. Parallel incoming light rays will be focused at the focal point; the focal length f for a concave spherical mirror is equal to $r/2$. The eye is filled with fluid with a refractive index of $n = 1.4$. Although this will affect the refraction of the light rays, it has no effect on reflection.

SOLVE

Part a)

$$f = \frac{r}{2} = \frac{0.625 \text{ cm}}{2} = 0.31 \text{ cm} = 3.1 \text{ mm}$$

The light will be focused $\boxed{\text{3.1 mm in front of the retina}}$.

Part b) Reflection is not affected by the index of refraction, so the answer would be the same as in part (a).

REFLECT

We could have also calculated the image distance from the mirror equation:

$$\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{\infty} + \frac{1}{d_I} = \frac{1}{f}$$

$$d_I = f$$

24.92

SET UP

In correcting nearsightedness, a corrective lens images an object at infinity (that is, $d_O = \infty$) at the actual far point of the patient's eye. In this case, $d_I = -0.70$ m. We can use the thin-lens equation to calculate the power of the lens.

SOLVE

$$P = \frac{1}{f} = \frac{1}{d_{\mathrm{O}}} + \frac{1}{d_{\mathrm{I}}} = \frac{1}{\infty} + \frac{1}{(-0.70\ \mathrm{m})} = \boxed{-1.4\ \mathrm{D}}$$

REFLECT

When working with corrective lenses, the image distance from the lens must be negative because the light must then pass through the person's eye and form a real image on the retina.

24.93

SET UP

In correcting farsightedness, a corrective lens images an object at a close distance (for example, $d_{\mathrm{O}} = 25$ cm) at the actual near point of the patient's eye. In this case, $d_{\mathrm{I}} = -0.75$ m. We can use the thin-lens equation to calculate the power of the lens.

SOLVE

$$P = \frac{1}{f} = \frac{1}{d_{\mathrm{O}}} + \frac{1}{d_{\mathrm{I}}} = \frac{1}{(0.25\ \mathrm{m})} + \frac{1}{(-0.75\ \mathrm{m})} = \boxed{2.7\ \mathrm{D}}$$

REFLECT

A person who is farsighted can clearly see things far away, but close objects are blurry; a person who is nearsighted can clearly see things up close, but distant objects are blurry.

24.94

SET UP

In order to prove the combined focal length f_{combined} of two thin lenses (1 and 2) pressed next to one another is equal to $\frac{1}{f_{\mathrm{combined}}} = \frac{1}{f_1} + \frac{1}{f_2}$, we will assume light from an object at infinity shines through this combination of lenses as well as a single lens that has a focal length of f_{combined}. By comparing these two expressions, we can find an expression for f_{combined} in terms f_1 and f_2. This has consequences for correcting vision, as contact lenses sit directly on a person's eye, whereas eyeglasses sit about an inch in front of a person's eye.

SOLVE

Image from first lens:

$$\frac{1}{d_{\mathrm{O},1}} + \frac{1}{d_{\mathrm{I},1}} = \frac{1}{\infty} + \frac{1}{d_{\mathrm{I},1}} = \frac{1}{f_1}$$

$$d_{\mathrm{I},1} = f_1$$

Image from second lens:

$$d_{\mathrm{O},2} = -d_{\mathrm{I},1} = -f_1$$

$$\frac{1}{d_{\mathrm{O},2}} + \frac{1}{d_{\mathrm{I},2}} = \frac{1}{f_2}$$

$$\frac{1}{-f_1} + \frac{1}{d_{\text{I},2}} = \frac{1}{f_2}$$
$$\frac{1}{d_{\text{I},2}} = \frac{1}{f_1} + \frac{1}{f_2}$$

Image from equivalent lens:

$$\frac{1}{d_{\text{O, combined}}} + \frac{1}{d_{\text{I, combined}}} = \frac{1}{\infty} + \frac{1}{d_{\text{I, combined}}} = \frac{1}{f_{\text{combined}}}$$
$$\frac{1}{d_{\text{I, combined}}} = \frac{1}{f_{\text{combined}}}$$

Comparing the two expressions:

$$\boxed{\frac{1}{f_{\text{combined}}} = \frac{1}{f_1} + \frac{1}{f_2}}$$

Part b) Since the contact lens sits directly on top of the person's eye, the total combined power necessary to achieve corrected vision is equal to the power of the contact lens plus the power of the person's lens.

Part c) An eyeglass prescription cannot have the identical prescription as a contact lens because eyeglasses sit about an inch in front of the person's eye. The lenses are not pressed next to one another in this case.

REFLECT
Using eyeglasses with the same prescription as your contact lenses would result in a slightly blurry picture.

24.95

SET UP
A specific person's near point was measured to be 15.0 cm and his far point is 2.75 m. These points for normal vision are 25.0 cm and infinity, respectively. It's not considered an issue if a person's near point is "too close"; his unaided eye can see things closer than an average person can, which is fine. Therefore, he wouldn't need to correct this issue and would only require lenses with a single focal length. The corrective contact lenses would need to take an object located at infinity and produce an image at the person's uncorrected far point. Contact lenses are located on a person's eye, so the image distance would be equal to the far point distance. Plugging all of this information with the correct sign conventions into the thin-lens equation will allow us to calculate the required focal length of the contact lenses. The power of the contact lenses is equal to the reciprocal of the focal length.

SOLVE

Part a) The person's far point is too close, so he needs a single focal length lens to correct this issue.

Part b)

$$\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f}$$

$$\frac{1}{\infty} + \frac{1}{d_I} = \frac{1}{f}$$

$$f = d_I = \boxed{-2.75 \text{ m}}$$

Part c)

$$P = \frac{1}{f} = \frac{1}{-2.75 \text{ m}} = \boxed{-0.364 \text{ diopters}}$$

REFLECT

People who have trouble seeing things off in the distance are nearsighted because they can only see things near to them.

24.96

SET UP

A person who needs bifocals has a far point that is closer than infinity and a near point that is larger than 0.25 m. A particular patient who needs bifocals has a far point of 1.12 m and a near point of 0.83 m. The upper portion of the bifocals corrects the nearsightedness. In correcting nearsightedness, a corrective lens images an object at infinity (that is, $d_O = \infty$) at the actual far point of the patient's eye. The lower portion of the bifocals corrects the farsightedness. In correcting farsightedness, a corrective lens images an object at a close distance (for example, $d_O = 25$ cm) at the actual near point of the patient's eye. We can use the thin-lens equation to calculate the power of each lens necessary to correct this patient's vision.

SOLVE

Upper half of the bifocals:

$$\frac{1}{d_{O,\,\infty}} + \frac{1}{d_{I,\text{ far point}}} = \frac{1}{\infty} + \frac{1}{d_{I,\text{ far point}}} = \frac{1}{f_{\text{upper}}}$$

$$P_{\text{upper}} = \frac{1}{f_{\text{upper}}} = \frac{1}{d_{I,\text{ far point}}} = \frac{1}{-1.12 \text{ m}} = \boxed{-0.893 \text{ D}}$$

Lower half of the bifocals:

$$\frac{1}{d_{O,\text{ near}}} + \frac{1}{d_{I,\text{ near point}}} = \frac{1}{f_{\text{lower}}}$$

$$P_{\text{lower}} = \frac{1}{f_{\text{lower}}} = \frac{d_{I,\text{ near point}} + d_{O,\text{ near}}}{d_{O,\text{ near}} d_{I,\text{ near point}}} = \frac{(-0.83 \text{ m}) + (0.25 \text{ m})}{(0.25 \text{ m})(-0.83 \text{ m})} = \boxed{2.80 \text{ D}}$$

REFLECT

A person who is farsighted can clearly see things far away, but close objects are blurry; a person who is nearsighted can clearly see things up close, but distant objects are blurry.

24.97

SET UP

A zoom lens for a digital camera is first zoomed such that the focal length is $f = 200 \text{ mm} = 0.200 \text{ m}$. The camera is focused on an object that is a distance $d_O = 15.0 \text{ m}$ from the lens. We can use the lens equation to calculate the distance between the lens and the photosensor array inside the camera, *i.e.*, the image distance d_I. The width of the image is essentially the same as the image height, h_I, which we can find from the definition of the magnification. The focal length of the zoom lens is then changed to $f = 18 \text{ mm}$; since we already know what the image distance must be, we can calculate the closest an object can be for this case.

SOLVE

Part a)

$$\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f}$$

$$\frac{1}{d_I} = \frac{1}{f} - \frac{1}{d_O} = \frac{d_O - f}{f d_O}$$

$$d_I = \frac{f d_O}{d_O - f} = \frac{(0.200 \text{ m})(15.0 \text{ m})}{(15.0 \text{ m}) - (0.200 \text{ m})} = \boxed{0.203 \text{ m} = 203 \text{ mm}}$$

Part b)

$$m = \frac{h_I}{h_O} = -\frac{d_I}{d_O}$$

$$h_I = -\frac{d_I}{d_O} h_O = -\left(\frac{0.203 \text{ m}}{15.0 \text{ m}}\right)(38 \text{ cm}) = \boxed{0.51 \text{ cm}}$$

Part c)

$$\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f}$$

$$\frac{1}{d_O} = \frac{1}{f} - \frac{1}{d_I} = \frac{d_I - f}{f d_I}$$

$$d_O = \frac{f d_I}{d_I - f} = \frac{(18 \text{ mm})(52 \text{ mm})}{(52 \text{ mm}) - (18 \text{ mm})} = \boxed{28 \text{ mm}}$$

REFLECT

The image distance must remain constant since it is dictated by the dimensions and construction of the camera.

24.98

SET UP

Macro lenses for cameras are designed to take very close-range photographs of small objects. A certain macro lens has a focal length $f = 35.0 \text{ mm}$. We can use the definition of the magnification of a lens and the thin-lens equation to calculate the object distance that

magnifies an object by 1.09. Keeping the sign conventions in mind, this magnification must be negative because the image must be real to be captured by the camera. We can follow the same process to calculate the magnification of an object that is double the object distance as in part (a).

SOLVE

Part a)
Magnification:

$$m = -\frac{d_I}{d_O}$$

$$d_I = -d_O m$$

Object distance:

$$\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f}$$

$$\frac{1}{d_O} - \frac{1}{d_O m} = \frac{1}{f}$$

$$\frac{m-1}{d_O m} = \frac{1}{f}$$

$$\frac{d_O m}{m-1} = f$$

$$d_O = \frac{f(m-1)}{m} = \frac{(35.0\text{ mm})(-1.09-1)}{-1.09} = \boxed{67.1\text{ mm}}$$

Part b)
Image distance:

$$\frac{1}{d_O} + \frac{1}{d_I} = \frac{1}{f}$$

$$d_I = \frac{d_O f}{d_O - f} = \frac{2(67.1\text{ mm})(35.0\text{ mm})}{2(67.1\text{ mm}) - (35.0\text{ mm})} = 47.3\text{ mm}$$

Magnification:

$$m = -\frac{d_I}{d_O} = -\frac{47.3\text{ mm}}{2(67.1\text{ mm})} = \boxed{-0.352}$$

REFLECT

The object distance in part (a) is reasonably small, and the magnification is much larger the closer the object is to the lens, which both make sense given the point of a macro lens.

24.99

SET UP

The focal lengths of the objective lens and the eyepiece in the Lick Refractor in the Lick Observatory are $f_o = 17.37$ m and $f_e = 0.022$ m, respectively. The overall magnification of an astronomical telescope is equal to $M = -\frac{f_o}{f_e}$. The negative sign in the magnification equation tells us the final image is inverted relative to the object.

SOLVE

Part a)

$$M = -\frac{f_o}{f_e} = -\frac{17.37\ \text{m}}{0.022\ \text{m}} = -790$$

Part b) The negative sign tells us that the image is inverted when viewed in the eyepiece.

REFLECT

The "36-in" refers to the diameter of the lenses.

Chapter 25
Relativity

Conceptual Questions

25.1 There is always a great deal of resistance to change, in any context, but especially in academia. Physics in this era (c. 1900) was predicated on Newtonian theories regarding motion, gravity, and cosmology. In addition, Maxwell's theory of electromagnetism was firmly in place. These two scientists were held in the highest esteem and were "above reproach." It was heretical for anyone, much less a young, upstart physicist who was not even part of a university, to challenge these time-tested, universally accepted underpinnings of physics. On a much more pragmatic level, it is always a "tough sell" when the new theory is complicated and not intuitive.

25.2 If the frame were moving in both x and y directions, the given Galilean transformation equations would be as follows:

$$x' = x - V_x t$$

$$y' = y - V_y t$$

$$z' = z$$

$$t' = t$$

25.3 A frame of reference or reference frame is a coordinate system with respect to which we will make observations or measurements. An inertial frame is one that moves at constant speed relative to another, that is, we refer to a frame of reference attached to a nonaccelerating object as an inertial frame.

25.4 Earth's surface is a noninertial frame because Earth rotates around its axis and revolves around the Sun, both of which involve acceleration. We often approximate Earth's surface as an inertial frame because the accelerations associated with the rotational motions of Earth are small compared to the gravitational acceleration g.

25.5 It means that there is no preferential frame of reference that is "fixed in place." Even the velocity of a car relative to Earth can be rephrased as the velocity of Earth relative to the car!

25.6 The Michelson–Morley experiment was performed to determine our motion through the "luminiferous ether" (the hypothesized medium of light). In all situations, before this famous experiment, waves required some type of medium upon which to transmit their energy. So it was only natural to search for the medium upon which light would vibrate. When the null result was ascertained, it was difficult for physicists to abandon the long-held notion about waves and their media, so it was prudent to repeat the

experiment again and again to make sure there was nothing wrong with the equipment. The experiment compared travel times for two beams of light that were moving at right angles to each other. As the beams were turned 90 degrees, it was expected that a noticeable deviation would be perceived due to the "ether wind." The difference between the two travel times was related to the factor $\frac{1}{\sqrt{1-\frac{V^2}{c^2}}}$.

25.7 (1) The "ether drag theory" proposed that Earth had zero relative velocity compared to ether because we "coincidentally" drag a patch of the ether with our planet as we orbit around our Sun. (2) The speed of light is infinite. This would cause the factor mentioned in the previous question $\left(\frac{1}{\sqrt{1-\frac{V^2}{c^2}}}\right)$ to be equal to unity (and, therefore, there would be no difference between the travel times of the two beams of light). There (3) is no ether; electromagnetic waves do not require a medium upon which to "wave."

25.8 The two postulates of the theory of special relativity are (1) all laws of physics are the same in all inertial frames and (2) the speed of light in a vacuum is the same in all frames, independent of both the speed of the sources of the light and the speed of the observer.

25.9 If the object is moving relative to you, you measure its length by finding the difference between the coordinates of its end points at the same time.

25.10 Yes. Subtraction of the inverse Lorentz transformation expression for the time gives $t'_A - t'_B = \gamma\left[(t_A - t_B) - \left(\frac{v}{c^2}\right)(x_A - x_B)\right]$. Suppose in reference frame S event A occurs after event B so that $(t_A - t_B)$ is positive. The time ordering $(t'_A - t'_B)$ between the two events as determined in reference frame S' can be positive, zero, or negative depending on the spatial separation $(x_A - x_B)$ in reference frame S and the relative velocity v.

25.11 The relativistic mass is equal to or greater than the rest mass. According to Equation 25-23, the relativistic mass is a product of relativistic gamma and the object's rest mass. When V, the speed of an object or the speed of the frame from which an observation is made, equals zero, relativistic gamma equals one. For all other values of V, gamma must be greater than one. Thus, the relativistic mass is always equal to or greater than the rest mass.

25.12 When the total energy $E \gg mc^2$ or equivalently when the kinetic energy $K \gg mc^2$, you can use $p = \frac{E}{c}$ to a good approximation.

25.13 Some people prefer to describe the increase in momentum between two observers in relative motion as if the mass is the variable (in the formula $p = mv$) that solely experiences the change. That is fine; it gives the correct answer for the momentum of a particle as measured by an observer that moves relative to the particle. However, the physical quantity of mass is not one that changes with relative motion. Mass measures the amount of matter that is present in an object, which is constant.

25.14 It is impossible to accelerate an object to the speed of light. From the equation $E = \gamma m_0 c^2$, as the speed of an object approaches the speed of light, the amount of energy required to increase the speed approaches infinity. It would take an infinite amount of work or an infinitely large force to accelerate the object to the speed of light.

25.15 Writers like to use travel through higher dimensions, which could, presumably, shorten the distance traveled, making it appear that the traveler has moved faster than the speed of light. They also often employ wormholes, or connections between two points in space-time, created when the space-time "surfaces" become deformed enough to "touch" each other and become physically connected. These sorts of deformations are possible (at least in theory) where high-mass objects like black holes exist.

25.16 The equivalence of gravity and acceleration is fundamental to the theory of general relativity. This leads to a context for relativistic physics that includes noninertial (that is, accelerating) frames of reference as well as inertial frames.

25.17 One important prediction of general relativity is that light bends as it passes through a gravitational field.

Multiple-Choice Questions

25.18 **E** (Only two of the above statements are true). The laws of motion are the same in all inertial frames of reference, and there is no way to detect absolute motion.

25.19 **B** (was a failure because they didn't detect a shift in the interference pattern). They expected to see a shift in the interference pattern due to the ether wind in the luminiferous ether.

25.20 **B** (time really does pass more slowly in a frame of reference moving relative to a frame of reference at relative rest). The amount by which time slows depends on the speed of the object.

25.21 **D** ($1.00c$). The speed of light is constant in all reference frames.

25.22 **D** ($1.00c$). The speed of light is constant in all reference frames.

25.23 **D** (1.00 m). The length of the meter stick is perpendicular to its motion, so it will not experience length contraction.

25.24 **A** (greater than $2p$).

$$\frac{p_2}{p_1}=\frac{\gamma_2 m_0 v_2}{\gamma_1 m_0 v_1}=\frac{\left(\dfrac{(2v_1)}{\sqrt{1-\left(\dfrac{2v_1}{c}\right)^2}}\right)}{\left(\dfrac{v_1}{\sqrt{1-\left(\dfrac{v_1}{c}\right)^2}}\right)}=2\left(\frac{\sqrt{1-\left(\dfrac{v_1}{c}\right)^2}}{\sqrt{1-\left(\dfrac{2v_1}{c}\right)^2}}\right)>2$$

25.25 **A** (the clock in Colorado runs slow). The effect of Earth's gravitational field decreases in magnitude as you move away from Earth.

Estimation/Numerical Questions

25.26

$$\Delta t=t_{\text{proper}}(\gamma-1)=(10{,}000\ \text{s})\left(\frac{1}{\sqrt{1-\left(\dfrac{v}{c}\right)^2}}-1\right)$$

$$\approx(10{,}000\ \text{s})\left(\left(1+\frac{1}{2}\left(\frac{v}{c}\right)^2-1\right)\right)=\frac{10{,}000\ \text{s}}{2}\left(\frac{\left(300\dfrac{\text{m}}{\text{s}}\right)}{\left(3.00\times10^8\dfrac{\text{m}}{\text{s}}\right)}\right)=5\times10^{-9}\ \text{s}$$

25.27

$$t=\gamma t_{\text{proper}}$$

$$\gamma=\frac{1}{\sqrt{1-\left(\dfrac{v}{c}\right)^2}}=\frac{t}{t_{\text{proper}}}=1.1$$

$$\frac{v}{c}=\sqrt{1-\frac{1}{1.1}}=0.417$$

$$v=0.417c$$

25.28

Part a) At speeds of about 100 m/s, $\gamma=1$ within 13 decimal points, so there would be no difference between the classical and relativistic momenta.

Part b) At speeds of about $0.3c$, $\gamma=1.05$, so there is about a 5% increase in momentum.

25.29 Yes. If $\beta = 0.6$, then $\gamma = 1.67$. If $\beta = 0.8$, then $\gamma = 1.25$.

25.30 The person on Earth would say that the astronaut would be younger because time would be dilated and the rate of aging would be proportionally decreased. If the astronaut travels at an average speed of 4000 m/s for a total of 30 minutes, each blast off would add 3×10^{-9} minutes to the lifetime. If the astronaut lives for 100 years on Earth, then the space travel is supposed to increase that to 110 years: $\dfrac{10 \text{ yr}}{\left(\dfrac{3 \times 10^{-9} \text{ min}}{\text{blastoff}}\right)} = 2 \times 10^{15}$ trips. There isn't enough time in a lifetime to do this!

25.31 About 5% of the world population.

25.32

$$\Delta E = (\Delta m)c^2$$

$$\Delta m = \frac{\Delta E}{c^2} = \frac{(100 \text{ kg})\left(387 \dfrac{\text{J}}{\text{kg} \cdot {}^\circ\text{C}}\right)(100\ {}^\circ\text{C})}{\left(3 \times 10^8 \dfrac{\text{m}}{\text{s}}\right)^2} = 4 \times 10^{-11} \text{ kg}$$

25.33

β	γ	β	γ
0	1.0000	0.7	1.4003
0.1	1.0050	0.8	1.6667
0.2	1.0206	0.9	2.2942
0.3	1.0483	0.99	7.0888
0.4	1.0911	0.999	22.3663
0.5	1.1547	0.9999	70.7124
0.6	1.2500		

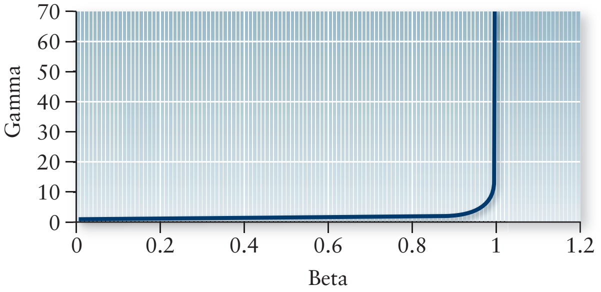


Figure 25-1 Problem 33

Part a) $\beta \approx 0.1$

Part b)

$$\frac{d\gamma}{d\beta} = \frac{d}{d\beta}\left(\frac{1}{\sqrt{1-\beta^2}}\right) = \frac{\beta}{(1-\beta^2)^{\frac{3}{2}}}$$

At $\beta = 0.1$, slope $= 0.102$.
At $\beta = 0.5$, slope $= 0.770$.
At $\beta = 0.9$, slope $= 10.9$.
At $\beta = 0.999$, slope $= 11{,}200$.

Problems

25.34

SET UP

A bicycle, car, and truck are all traveling in a straight line. In our coordinate system, we'll define $+y$ to point north. The velocity of each with respect to the ground is $\vec{v}_{\text{BG}} = \left(8\frac{\text{m}}{\text{s}}\right)\hat{y}$, $\vec{v}_{\text{CG}} = \left(25\frac{\text{m}}{\text{s}}\right)\hat{y}$, and $\vec{v}_{\text{TG}} = \left(-15\frac{\text{m}}{\text{s}}\right)\hat{y}$, respectively. We can use our knowledge of Newtonian relative motion to calculate the velocity of the car with respect to the bicycle, the velocity of the car with respect to the truck, and the velocity of the bicycle with respect to the truck.

SOLVE

Part a)

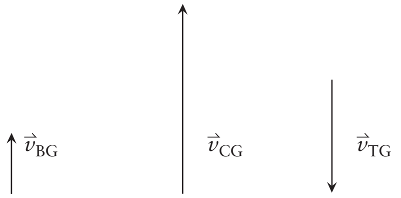


Figure 25-2 Problem 34

Part b)
Velocity of the car relative to the bicycle:

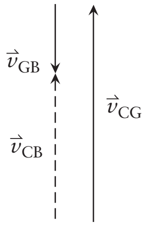


Figure 25-3 Problem 34

$$\vec{v}_{\text{CB}} = \vec{v}_{\text{CG}} + \vec{v}_{\text{GB}} = \vec{v}_{\text{CG}} - \vec{v}_{\text{BG}} = \left(25\frac{\text{m}}{\text{s}}\right)\hat{y} - \left(8\frac{\text{m}}{\text{s}}\right)\hat{y} = \boxed{\left(17\frac{\text{m}}{\text{s}}\right)\hat{y}}$$

Velocity of the car relative to the truck:

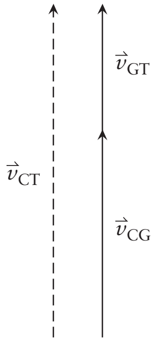


Figure 25-4 Problem 34

$$\vec{v}_{CT} = \vec{v}_{CG} + \vec{v}_{GT} = \vec{v}_{CG} - \vec{v}_{TG} = \left(25\frac{\text{m}}{\text{s}}\right)\hat{y} - \left(-15\frac{\text{m}}{\text{s}}\right)\hat{y} = \boxed{\left(40\frac{\text{m}}{\text{s}}\right)\hat{y}}$$

Velocity of the bicycle relative to the truck:

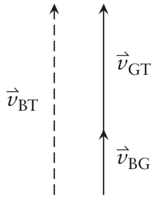


Figure 25-5 Problem 34

$$\vec{v}_{BT} = \vec{v}_{BG} + \vec{v}_{GT} = \vec{v}_{BG} - \vec{v}_{TG} = \left(8\frac{\text{m}}{\text{s}}\right)\hat{y} - \left(-15\frac{\text{m}}{\text{s}}\right)\hat{y} = \boxed{\left(23\frac{\text{m}}{\text{s}}\right)\hat{y}}$$

REFLECT

The bicycle and car are both traveling northward, so the relative speed of the car to the bicycle should be less than the speed of the car relative to the ground. The truck is traveling southward, so the relative speed of the car relative to the truck should be larger than the relative speed of the car to the ground.

25.35

SET UP

A frame of reference, S, is fixed on the surface of Earth with the x-axis pointing toward the east, y-axis pointing toward the north, and the z-axis pointing up. A second frame of reference, S', is moving at a constant speed of $V = 4$ m/s to the east. We can use the Galilean transformation to mathematically describe the relationships between x and x', y and y', and z and z'. Once we have the expressions for x', y', and z', we can calculate their values given $t = 4$ s, $x = 2$ m, $y = 1$ m, and $z = 0$ m.

SOLVE

Part a)

$$\boxed{x' = x - 4t} \text{ (SI units)}$$

$$\boxed{y' = y}$$

$$\boxed{z' = z}$$

Part b)

$$x' = x - 4t = (2) - 4(4) = \boxed{-14\text{ m}}$$

$$y' = y = \boxed{1\text{ m}}$$

$$z' = z = \boxed{0\text{ m}}$$

REFLECT

The second reference frame was moving toward the east $(+x)$, so the sign of V should be positive.

25.36

SET UP

A boat sails from a pier directly toward an island, while a kite rider heads directly away from the island toward the boat. We will use a coordinate system where positive x points along the motion toward the island. The velocity of the boat relative to the pier is $\vec{v}_{\text{bp}} = \left(4\frac{\text{m}}{\text{s}}\right)\hat{x}$; the velocity of the kite rider relative to the boat is $\vec{v}_{\text{kb}} = \left(-6\frac{\text{m}}{\text{s}}\right)\hat{x}$. The velocity of the kite rider relative to the pier, $\vec{v}_{\text{kp}}$, is equal to the vector sum $\vec{v}_{\text{kp}} = \vec{v}_{\text{kb}} + \vec{v}_{\text{bp}}$.

SOLVE

$$\vec{v}_{\text{bp}} = \left(4\frac{\text{m}}{\text{s}}\right)\hat{x}$$

$$\vec{v}_{\text{kb}} = \left(-6\frac{\text{m}}{\text{s}}\right)\hat{x}$$

$$\vec{v}_{\text{kp}} = \vec{v}_{\text{kb}} + \vec{v}_{\text{bp}} = \left(-6\frac{\text{m}}{\text{s}}\right)\hat{x} + \left(4\frac{\text{m}}{\text{s}}\right)\hat{x} = \boxed{\left(-2\frac{\text{m}}{\text{s}}\right)\hat{x}}$$

REFLECT

The kite rider is moving faster toward the boat than the boat is moving away from the pier, so we should expect the kite rider to be moving toward the pier (in the pier's reference frame).

25.37

SET UP

A remote control car (frame of reference S') is moving to the right, which we'll call positive x, at a velocity of $\vec{v}_{\text{cp}} = \left(15\frac{\text{m}}{\text{s}}\right)\hat{x}$ with respect to a parking lot (frame of reference S). A girl (frame of reference S'') is chasing after the remote control car at a velocity of $\vec{v}_{\text{gp}} = \left(4\frac{\text{m}}{\text{s}}\right)\hat{x}$. We can use the Galilean transformation to mathematically describe the relationships between the coordinates of the three reference frames. In order to write the Galilean transformation between frames S' and S'', we need to find the velocity of the car relative to the girl, $\vec{v}_{\text{cg}}$.

SOLVE

Part a)

$$\boxed{x' = x - 15t} \text{ (SI units)}$$

$$\boxed{y' = y}$$

$$\boxed{z' = z}$$

Part b)

$$\boxed{x'' = x - 4t} \text{ (SI units)}$$

$$\boxed{y'' = y}$$

$$\boxed{z'' = z}$$

Part c)
Velocity of the car relative to the girl:

$$\vec{v}_{\text{cg}} = \vec{v}_{\text{cp}} + \vec{v}_{\text{pg}} = \vec{v}_{\text{cp}} - \vec{v}_{\text{gp}} = \left(15\frac{\text{m}}{\text{s}}\right)\hat{x} - \left(4\frac{\text{m}}{\text{s}}\right)\hat{x} = 11\frac{\text{m}}{\text{s}}$$

Galilean transformation:

$$\boxed{x' = x'' - 15t} \text{ (SI units)}$$

$$\boxed{y' = y''}$$

$$\boxed{z' = z''}$$

REFLECT

Since the girl is chasing after the car, the speed of the car relative to the girl should be smaller than the speed of the car relative to the parking lot.

25.38

SET UP

The origins of two reference frames, frames S and S', coincide at $t = t' = 0$. An observer in reference frame S observes an event occurring at $x = 750$ m, $y = 250$ m, $z = 250$ m, and $t = 2.0 \times 10^{-6}$ s. We can use the Galilean transformation to calculate the coordinates of the event in S', which is moving with a velocity of $\vec{v} = (0.01c)\hat{x}$, relative to S.

SOLVE

$$x' = x - v_x t = (750 \text{ m}) - (0.01)\left(3.00 \times 10^8\frac{\text{m}}{\text{s}}\right)(2.0 \times 10^{-6} \text{ s}) = \boxed{744 \text{ m}}$$

$$\boxed{y' = y = 250 \text{ m}}$$

$$\boxed{z' = z = 250 \text{ m}}$$

$$\boxed{t' = t = 2.0 \times 10^{-6} \text{ s}}$$

REFLECT
Reference frame S' is moving toward the location of the event, so an observer in S' should experience the event at a position closer than an observer in S.

25.39

SET UP
The origins of two reference frames, frames S and S', coincide at $t = t' = 0$. The position of a particle at $t' = 4 \times 10^{-3}$ s in reference frame S' is $x' = 10$ m, $y' = 4$ m, and $z' = 6$ m. We can use the Galilean transformation to calculate the coordinates of the particle in S, which is moving with a velocity of either $\vec{v} = \left(500\frac{\text{m}}{\text{s}}\right)\hat{x}$ or $\vec{v} = \left(-500\frac{\text{m}}{\text{s}}\right)\hat{x}$, relative to S'.

SOLVE
Part a)

$$x' = x - v_x t$$

$$x = x' + v_x t = (10\text{ m}) + \left(500\frac{\text{m}}{\text{s}}\right)(4 \times 10^{-3}\text{ s}) = \boxed{12\text{ m}}$$

$$\boxed{y = y' = 4\text{ m}}$$

$$\boxed{z = z' = 6\text{ m}}$$

$$\boxed{t = t' = 4 \times 10^{-3}\text{ s}}$$

Part b)

$$x' = x - v_x t$$

$$x = x' + v_x t = (10\text{ m}) + \left(-500\frac{\text{m}}{\text{s}}\right)(4 \times 10^{-3}\text{ s}) = \boxed{8\text{ m}}$$

$$\boxed{y = y' = 4\text{ m}}$$

$$\boxed{z = z' = 6\text{ m}}$$

$$\boxed{t = t' = 4 \times 10^{-3}\text{ s}}$$

REFLECT
Since reference frame S' is moving toward $+x$ away from reference frame S in part (a), the position of the particle in S should be farther away from the origin than in S'. Since reference frame S' is moving toward $-x$ away from reference frame S in part (b), the position of the particle in S should be closer to the origin than in S'.

25.40

SET UP
The origins of two reference frames, frames S and S', coincide at $t = t' = 0$. The position of a particle at $t = 2.0 \times 10^{-6}$ s in reference frame S is $x = 100$ m, $y = 10$ m, and $z = 30$ m. We

can use the Galilean transformation to calculate the coordinates of the particle in S', which is moving with a velocity of $\vec{v} = \left(150{,}000\frac{\text{m}}{\text{s}}\right)\hat{x}$, relative to S.

SOLVE

$$x' = x - v_x t = (100\text{ m}) - \left(150{,}000\frac{\text{m}}{\text{s}}\right)(6 \times 10^{-4}\text{ s}) = \boxed{10\text{ m}}$$

$$\boxed{y' = y = 10\text{ m}}$$

$$\boxed{z' = z = 30\text{ m}}$$

$$\boxed{t' = t = 6 \times 10^{-4}\text{ s}}$$

REFLECT

Since the reference frames coincide at $t = 0$, the frames must be moving away from one another at $t = 6 \times 10^{-4}$ s.

25.41

SET UP

An airplane lands in a river during a hurricane. We will use a coordinate system where positive x points east and positive y points north. The velocity of the airplane relative to the wind is $\vec{v}_{\text{aw}} = \left(30\frac{\text{m}}{\text{s}}\right)\hat{x}$; the velocity of the wind relative to the ground is $\vec{v}_{\text{wg}} = \left(20\frac{\text{m}}{\text{s}}\right)\hat{y}$; and the velocity of the river relative to the ground is $\vec{v}_{\text{rg}} = -\left(5\frac{\text{m}}{\text{s}}\right)\hat{y}$. The velocity of the airplane relative to the river, $\vec{v}_{\text{ar}}$, is equal to the vector sum $\vec{v}_{\text{ar}} = \vec{v}_{\text{aw}} + \vec{v}_{\text{wg}} + \vec{v}_{\text{gr}}$. Recall that the sign of the velocity changes when the subscripts are reversed, for example, $\vec{v}_{\text{gr}} = -\vec{v}_{\text{rg}}$.

SOLVE

$$\vec{v}_{\text{aw}} = \left(30\frac{\text{m}}{\text{s}}\right)\hat{x}$$

$$\vec{v}_{\text{wg}} = \left(20\frac{\text{m}}{\text{s}}\right)\hat{y}$$

$$\vec{v}_{\text{rg}} = -\left(5\frac{\text{m}}{\text{s}}\right)\hat{y}$$

$$\vec{v}_{\text{ar}} = \vec{v}_{\text{aw}} + \vec{v}_{\text{wg}} + \vec{v}_{\text{gr}} = \vec{v}_{\text{aw}} + \vec{v}_{\text{wg}} - \vec{v}_{\text{rg}} = \left(30\frac{\text{m}}{\text{s}}\right)\hat{x} + \left(20\frac{\text{m}}{\text{s}}\right)\hat{y} - \left(-5\frac{\text{m}}{\text{s}}\right)\hat{y}$$

$$= \left(30\frac{\text{m}}{\text{s}}\right)\hat{x} + \left(25\frac{\text{m}}{\text{s}}\right)\hat{y}$$

Magnitude:

$$v_{\text{ar}} = \sqrt{v_{\text{ar},x}^2 + v_{\text{ar},y}^2} = \sqrt{\left(30\frac{\text{m}}{\text{s}}\right)^2 + \left(25\frac{\text{m}}{\text{s}}\right)^2} = \boxed{39.1\frac{\text{m}}{\text{s}}}$$

Direction:

$$\theta = \arctan\left(\frac{v_{\text{ar},\,y}}{v_{\text{ar},\,x}}\right) = \arctan\left(\frac{\left(25\frac{\text{m}}{\text{s}}\right)}{\left(30\frac{\text{m}}{\text{s}}\right)}\right) = \boxed{39.8^\circ \text{ north of east}}$$

REFLECT

We could have just as easily reported our answer in vector form, $\vec{v}_{\text{ar}} = \left(30\frac{\text{m}}{\text{s}}\right)\hat{x} + \left(25\frac{\text{m}}{\text{s}}\right)\hat{y}$. A good mnemonic for determining the correct order when calculating a relative velocity is that the leftmost and rightmost subscripts should be the ones found in your answer and that all neighboring, intermediate subscripts should be the same. For example, in this problem we want our answer to have the subscripts $\vec{v}_{\text{ar}}$, so the leftmost subscript should be "a" and the rightmost should be "r." Then, we can fill in the rest of the sum by matching the subscripts: $\vec{v}_{\text{ar}} = \vec{v}_{\text{aw}} + \vec{v}_{\text{wg}} + \vec{v}_{\text{gr}}$.

25.42

SET UP

A spaceship is moving at a velocity $\vec{v}_{\text{rE}} = \left(2.4 \times 10^8\frac{\text{m}}{\text{s}}\right)\hat{x}$ relative to Earth. A satellite is moving at a velocity $\vec{v}_{\text{sE}} = \left(-1.6 \times 10^8\frac{\text{m}}{\text{s}}\right)\hat{x}$ relative to Earth. We are asked to calculate the relative velocity of the satellite relative to the spaceship, $\vec{v}_{\text{sr}}$, using $\vec{v}_{\text{sr}} = \vec{v}_{\text{sE}} + \vec{v}_{\text{Er}}$.

SOLVE

$$\vec{v}_{\text{sr}} = \vec{v}_{\text{sE}} + \vec{v}_{\text{Er}} = \vec{v}_{\text{sE}} - \vec{v}_{\text{rE}} = \left(-1.6 \times 10^8\frac{\text{m}}{\text{s}}\right)\hat{x} - \left(2.4 \times 10^8\frac{\text{m}}{\text{s}}\right)\hat{x} = \boxed{\left(-4.0 \times 10^8\frac{\text{m}}{\text{s}}\right)\hat{x}}$$

This is larger than the speed of light, so the Galilean transformation cannot be used to calculate relative velocities for speeds approaching c.

REFLECT

We will learn a new method (the Lorentz transformation) for calculating relative velocities for objects traveling close to the speed of light.

25.43

SET UP

The Michelson–Morley experiment was performed on a slab of marble floating in a pool of mercury in order to minimize the effects of vibrations.

SOLVE

The massive slab of marble was used to keep any vibrations from disrupting the light as it moved through the interferometer. The slab needed to be rotated at different points in the experiment, so the mercury made it easier to move.

REFLECT

It is unlikely that such large volumes of mercury would be used so cavalierly nowadays.

25.44

SET UP

An airplane is traveling at an airspeed (that is, speed of the plane relative to the air) of $25\frac{\text{m}}{\text{s}}$ between two points separated by a distance $d = 2000 \times 10^3$ m. If there is no wind blowing, the round-trip time is simply the total distance traveled divided by the airspeed. If a wind is blowing at $10\frac{\text{m}}{\text{s}}$ perpendicular to the line between the points, we need to calculate the component of the airspeed that lies along the line connecting the points. The round-trip time in this case is the total distance traveled divided by this component of the airspeed. Finally, if a wind is blowing at $10\frac{\text{m}}{\text{s}}$ along the line connecting the points, we need to calculate the speed of the aircraft traveling with the wind and against the wind. This round-trip time is equal to the sum of the times of each leg.

SOLVE

Part a)

$$v = \frac{\Delta x}{\Delta t} = \frac{2d}{\Delta t}$$

$$\Delta t = \frac{2d}{v} = \frac{2(2000 \times 10^3\ \text{m})}{\left(25\frac{\text{m}}{\text{s}}\right)} = \boxed{1.6 \times 10^5\ \text{s}}$$

Part b)
Component of velocity along the path connecting the two points:

$$v = \sqrt{\left(25\frac{\text{m}}{\text{s}}\right)^2 - \left(10\frac{\text{m}}{\text{s}}\right)^2} = 23\frac{\text{m}}{\text{s}}$$

Time:

$$v = \frac{\Delta x}{\Delta t} = \frac{2d}{\Delta t}$$

$$\Delta t = \frac{2d}{v} = \frac{2(2000 \times 10^3\ \text{m})}{\left(23\frac{\text{m}}{\text{s}}\right)} = \boxed{1.7 \times 10^5\ \text{s}}$$

Part c)

$$\Delta t = (\Delta t)_{\text{against wind}} + (\Delta t)_{\text{with wind}} = \frac{d}{v_{\text{against wind}}} + \frac{d}{v_{\text{with wind}}}$$

$$= \frac{2000 \times 10^3\ \text{m}}{\left(25\frac{\text{m}}{\text{s}}\right) - \left(10\frac{\text{m}}{\text{s}}\right)} + \frac{2000 \times 10^3\ \text{m}}{\left(25\frac{\text{m}}{\text{s}}\right) + \left(10\frac{\text{m}}{\text{s}}\right)} = \boxed{1.9 \times 10^5\ \text{s}}$$

REFLECT

Since the speed of the plane is much less than the speed of light, we can safely ignore relativistic effects.

25.45

SET UP

The Michelson–Morley experiment attempted to measure a difference between light travel times in two perpendicular legs of an interferometer: One leg was situated parallel to the ether wind and the other perpendicular to the ether wind. For the "parallel leg" of the interferometer, we need to calculate the speed of the light traveling with the wind and against the wind; the round-trip time on this leg is equal to the sum of the times of each one-way trip. For the "perpendicular leg," we need to calculate the component of the speed of light that lies along this axis; this speed will be the same for both portions of the one-way trip. Subtracting these two expressions for the travel times—the times for the "parallel leg" and the "perpendicular leg"—will give us an expression for the time difference between the legs in terms of the speed of the ether wind. Using this, we can calculate the time difference if the speed of the ether wind were equal to the orbital speed of Earth around the Sun, $0.01c$, $0.1c$, $0.5c$, and $0.9c$.

SOLVE

Parallel time:

$$t_{\parallel} = \frac{L}{c+v} + \frac{L}{c-v} = \frac{L(c-v)+L(c+v)}{(c+v)(c-v)} = \frac{2Lc}{c^2-v^2} = \frac{2L}{c\left(1-\left(\frac{v}{c}\right)^2\right)} = \frac{2L\gamma^2}{c}$$

Perpendicular time:

$$t_{\perp} = \frac{L}{\sqrt{c^2-v^2}} + \frac{L}{\sqrt{c^2-v^2}} = \frac{2L}{\sqrt{c^2-v^2}} = \frac{2L}{c\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{2L\gamma}{c}$$

Time difference between legs:

$$t_{\parallel} - t_{\perp} = \frac{2L\gamma^2}{c} - \frac{2L\gamma}{c} = \frac{2L\gamma}{c}(\gamma-1) = \frac{2(20{,}000\text{ m})}{\left(3.00\times 10^8\frac{\text{m}}{\text{s}}\right)}\gamma(\gamma-1)$$

$$(1.33\times 10^{-4}\text{ s})\gamma(\gamma-1) = (1.33\ \mu\text{s})\gamma(\gamma-1)$$

Part a)
Speed:

$$v = \frac{2\pi(1.5\times 10^{11}\text{ m})}{\left(365\text{ days}\times\frac{24\text{ hr}}{1\text{ day}}\times\frac{3600\text{ s}}{1\text{ hr}}\right)} = 2.99\times 10^4\frac{\text{m}}{\text{s}}$$

Gamma:

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{1}{\sqrt{1-\left(\frac{\left(2.99\times 10^4\frac{\text{m}}{\text{s}}\right)}{\left(3.00\times 10^8\frac{\text{m}}{\text{s}}\right)}\right)^2}} = 1.000000005$$

Time difference between legs:

$$(133\ \mu\text{s})(1.000000005)((1.000000005)-1) = \boxed{6.61\times 10^{-7}\ \mu\text{s}}$$

Part b)
Gamma:

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{1}{\sqrt{1-\left(\frac{0.01c}{c}\right)^2}} = 1.00005$$

Time difference between legs:

$$(133\ \mu\text{s})(1.00005)((1.00005)-1) = \boxed{6.67\times 10^{-3}\ \mu\text{s}}$$

Part c)
Gamma:

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{1}{\sqrt{1-\left(\frac{0.1c}{c}\right)^2}} = 1.005$$

Time difference between legs:

$$(133\ \mu\text{s})(1.005)((1.005)-1) = \boxed{0.675\ \mu\text{s}}$$

Part d)
Gamma:

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{1}{\sqrt{1-\left(\frac{0.5c}{c}\right)^2}} = 1.155$$

Time difference between legs:

$$(133\ \mu\text{s})(1.155)((1.155)-1) = \boxed{23.8\ \mu\text{s}}$$

Part e)
Gamma:

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{1}{\sqrt{1-\left(\frac{0.9c}{c}\right)^2}} = 2.294$$

Time difference between legs:

$$(133\ \mu\text{s})(2.294)((2.294)-1) = \boxed{396\ \mu\text{s}}$$

REFLECT

Michelson and Morley expected to see a shift in the interference pattern between the legs due to the difference in travel times. As we know, they saw no definitive shift.

25.46

SET UP

An observer in reference frame S observes a time difference of $\Delta t = 10^{-4}$ s between two lightning bolts that are $\Delta x = 1.50 \times 10^5$ m apart on the x-axis. The Lorentz transformation is used to find the time difference between the lightning bolts determined by an observer in a reference frame S' traveling at $V = 0.8c$.

SOLVE

$$\Delta t = \gamma \Delta t_{\text{proper}} = \frac{\Delta t_{\text{proper}}}{\sqrt{1-\left(\frac{V}{c}\right)^2}} = \frac{(10^{-4}\ \text{s})}{\sqrt{1-\left(\frac{0.8c}{c}\right)^2}} = \boxed{1.7 \times 10^{-4}\ \text{s}}$$

$$\Delta t' = \gamma\left(\Delta t - \frac{V}{c^2}\Delta x\right) = \frac{\left(\Delta t - \frac{V}{c^2}\Delta x\right)}{\sqrt{1-\left(\frac{V}{c}\right)^2}} = \frac{\left(\Delta t - \frac{0.8c}{c^2}\Delta x\right)}{\sqrt{1-\left(\frac{0.8c}{c}\right)^2}}$$

$$= \frac{\left((10^{-4}\ \text{s}) - \left(\frac{0.8}{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)}\right)(1.50 \times 10^5\ \text{m})\right)}{\sqrt{1-(0.8)^2}} = \boxed{-5 \times 10^{-4}\ \text{s}}$$

REFLECT

The observer in frame S' detects lightning bolt B before lightning bolt A. The time difference is fairly small but different than the time difference as calculated in the S reference frame, as expected.

25.47

SET UP

A particle is traveling at a speed $V = 0.80c$ relative to observers in the laboratory. The half-life t of the particle in the laboratory frame compared to half-life t_{proper} of the particle in its proper frame is equal to relativistic gamma.

SOLVE

$$t = \gamma t_{\text{proper}}$$

$$\frac{t}{t_{\text{proper}}} = \gamma = \frac{1}{\sqrt{1 - \left(\frac{V}{c}\right)^2}} = \frac{1}{\sqrt{1 - \left(\frac{0.80c}{c}\right)^2}} = \boxed{1.67}$$

REFLECT

The particle is moving relative to the observers, so the observers will measure a longer lifetime in their frame.

25.48

SET UP

The average lifetime of muons at rest was measured to be $t_{\text{proper}} = 2.2\ \mu\text{s}$. If the muons were traveling at $V = 0.99c$ relative to the laboratory frame, then their average lifetime would be equal to the $t = \gamma t_{\text{proper}}$.

SOLVE

$$t = \gamma t_{\text{proper}} = \frac{t_{\text{proper}}}{\sqrt{1 - \left(\frac{V}{c}\right)^2}} = \frac{2.2\ \mu\text{s}}{\sqrt{1 - \left(\frac{0.99c}{c}\right)^2}} = \boxed{16\ \mu\text{s}}$$

REFLECT

The lifetime of the moving particles measured in the laboratory frame should be longer than the proper frame.

25.49

SET UP

The average lifetime of muons traveling at a speed of $V = 0.98c$ was measured to be $\Delta t = 11\ \mu\text{s}$. If the muons were at rest, then their average lifetime would be equal to the proper time $\Delta t_{\text{proper}} = \Delta t\sqrt{1 - \frac{V^2}{c^2}}$.

SOLVE

$$\Delta t_{\text{proper}} = \Delta t\sqrt{1 - \frac{V^2}{c^2}} = (11\ \mu\text{s})\sqrt{1 - \frac{(0.98c)^2}{c^2}} = (11\ \mu\text{s})\sqrt{1 - (0.98)^2} = \boxed{2.19\ \mu\text{s}}$$

REFLECT

The proper time is a time measurement made at rest with respect to the reference frame. We expect the lifetime measured for the moving muons to be longer than the ones at rest due to time dilation.

Get Help: P'Cast 25.3 – Moving Clock

25.50

SET UP

A relative difference of $\Delta t = \dfrac{t - t_p}{t_p} = 0.0001$ was measured in the laboratory. Using the relationship between the measured time and the proper time, we can calculate the relative speed.

SOLVE

$$\Delta t = \frac{t - t_p}{t_p} = \frac{\gamma t_p - t_p}{t_p} = \gamma - 1$$

$$\gamma = \Delta t + 1 = 0.0001 + 1 = 1.0001$$

$$\beta = \frac{v}{c} = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \frac{1}{(1.0001)^2}} = 0.01414$$

$$v = 0.01414c = 0.01414\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right) = \boxed{4.24 \times 10^6 \frac{\text{m}}{\text{s}}}$$

REFLECT

The relative speed is about 1.4% of the speed of light, so we need to consider relativistic effects.

25.51

SET UP

The Andromeda galaxy is a distance $\Delta x = 2.54 \times 10^6$ ly from Earth. The required speed for a spaceship to travel in order to deliver a person to the galaxy within $\Delta t_{\text{proper}} = 80$ yr is given by $v = \dfrac{\Delta x}{\Delta t}$, where $\Delta t = \gamma \Delta t_{\text{proper}}$. After calculating the required speed, we can determine whether or not this speed is actually attainable.

SOLVE

$$v = \frac{\Delta x}{\Delta t} = \frac{(2.54 \times 10^6 \text{ yr})c}{\gamma(80 \text{ yr})} = \frac{31{,}750c}{\gamma}$$

$$\frac{v\gamma}{c} = \beta\gamma = \frac{\beta}{\sqrt{1 - \beta^2}} = 31{,}750$$

$$\frac{\beta^2}{1 - \beta^2} = 1.008 \times 10^9$$

$$\beta^2 = 1.008 \times 10^9(1 - \beta^2)$$

$$\beta = \sqrt{\frac{1.008 \times 10^9}{1.008 \times 10^9 + 1}} = 0.9999999995$$

$$\boxed{v = 0.9999999995c}$$

Yes, it is $\boxed{\text{possible}}$ for a human being to reach the Andromeda galaxy in a lifetime, but this speed is unattainably high.

REFLECT

The Andromeda galaxy is 1.5×10^{19} miles from Earth!

25.52

SET UP

The star Proxima Centauri is $\Delta x = 4.24$ ly from Earth. A spaceship is traveling toward Proxima Centauri at a speed of $v = 0.75c$. The time required for a one-way trip to the star, according to a stationary observer on Earth, is related to $v = \dfrac{\Delta x}{\Delta t_{\text{proper}}}$, whereas the time required for the trip according to the captain of the spaceship is given by $v = \dfrac{\Delta x}{\Delta t} = \dfrac{\Delta x}{\gamma \Delta t_{\text{proper}}}$.

SOLVE

Part a)

$$v = \frac{\Delta x}{\Delta t_{\text{proper}}}$$

$$\Delta t_{\text{proper}} = \frac{\Delta x}{v} = \frac{(4.24\ \text{yr})c}{0.75\ \text{c}} = \boxed{5.7\ \text{yr}}$$

Part b)

$$v = \frac{\Delta x}{\Delta t} = \frac{\Delta x}{\gamma \Delta t_{\text{proper}}}$$

$$\Delta t_{\text{proper}} = \frac{\Delta x}{\gamma v} = \frac{\Delta x}{v}\sqrt{1 - \left(\frac{v}{c}\right)^2} = \frac{(4.24\ \text{yr})c}{0.75c}\sqrt{1 - \left(\frac{0.75c}{c}\right)^2} = \boxed{3.7\ \text{yr}}$$

REFLECT

The time measured on Earth should be longer than the time measured in the moving ship.

25.53

SET UP

The observed lifetime of a decaying subatomic particle is $t = 30$ ns. The proper lifetime of the particle is given by $t = \gamma t_{\text{proper}}$, where $\gamma = 20$.

SOLVE

$$t = \gamma t_{\text{proper}}$$

$$t_{\text{proper}} = \frac{t}{\gamma} = \frac{30\ \text{ns}}{20} = \boxed{1.5\ \text{ns}}$$

REFLECT

The proper lifetime of the particle is the lifetime of the particle as measured in the frame of the particle itself.

25.54

SET UP

Alpha Centauri is $\Delta x = 4.37$ ly from Earth. A spaceship is traveling toward Alpha Centauri at a speed of $v = 0.95c$. The time required for a one-way trip to the star, according to a stationary observer on Earth, is related to $v = \dfrac{\Delta x}{\Delta t_{\text{proper}}}$, whereas the time required for the trip according to the crew of the spaceship is given by $v = \dfrac{\Delta x}{\Delta t} = \dfrac{\Delta x}{\gamma \Delta t_{\text{proper}}}$.

SOLVE

Part a)

$$v = \frac{\Delta x}{\Delta t} = \frac{\Delta x}{\gamma \Delta t_{\text{proper}}}$$

$$\Delta t_{\text{proper}} = \frac{\Delta x}{\gamma v} = \frac{\Delta x}{v}\sqrt{1 - \left(\frac{v}{c}\right)^2} = \frac{(4.37\ \text{yr})c}{0.95c}\sqrt{1 - \left(\frac{0.95c}{c}\right)^2} = \boxed{1.4\ \text{yr}}$$

Part b)

$$v = \frac{\Delta x}{\Delta t_{\text{proper}}}$$

$$\Delta t_{\text{proper}} = \frac{\Delta x}{v} = \frac{(4.37\ \text{yr})c}{0.95\ \text{c}} = \boxed{4.6\ \text{yr}}$$

REFLECT

The time measured with respect to Earth is longer than that measured on the spaceship as expected.

25.55

SET UP

A car, with a proper length of $L_{\text{proper}} = 3.20$ m, is moving in the x direction in the reference frame S. The reference frame S' moves at a speed of $V = 0.80c$ toward positive x. The length of the car according to observers in S' is equal to $L = \dfrac{1}{\gamma}L_{\text{proper}}$, where $\gamma = \dfrac{1}{\sqrt{1 - \dfrac{V^2}{c^2}}}$.

SOLVE

$$L = \frac{1}{\gamma}L_{\text{proper}} = \left(\sqrt{1 - \frac{V^2}{c^2}}\right)L_{\text{proper}} = \left(\sqrt{1 - \frac{(0.80c)^2}{c^2}}\right)(3.20\ \text{m})$$

$$= \left(\sqrt{1 - (0.80)^2}\right)(3.20\ \text{m}) = \boxed{1.92\ \text{m}}$$

REFLECT

The observed length of the car in the moving reference frame should be smaller than the proper length of the car due to length contraction.

Get Help: P'Cast 25.5 – Meter Stick
P'Cast 25.8 – A Galactic Competition
P'Cast 25.9 – A Galactic Competition, Reconsidered

25.56

SET UP

The relative length L of a moving spaceship was measured to be $L = \frac{2}{3}L_{\text{proper}}$. The relative length of the spaceship according to observers in S' is also equal to $L = \frac{1}{\gamma}L_{\text{proper}}$, where $\gamma = \frac{1}{\sqrt{1-\frac{V^2}{c^2}}}$. We can rearrange this expression to solve for the speed of the spaceship.

SOLVE

$$L = \frac{1}{\gamma}L_{\text{proper}} = \left(\sqrt{1-\frac{V^2}{c^2}}\right)L_{\text{proper}}$$

$$\sqrt{1-\frac{V^2}{c^2}} = \frac{L}{L_{\text{proper}}}$$

$$1-\frac{V^2}{c^2} = \left(\frac{L}{L_{\text{proper}}}\right)^2$$

$$\frac{V^2}{c^2} = 1-\left(\frac{L}{L_{\text{proper}}}\right)^2$$

$$V = c\sqrt{1-\left(\frac{L}{L_{\text{proper}}}\right)^2} = c\sqrt{1-\left(\frac{\left(\frac{2}{3}L_{\text{proper}}\right)}{L_{\text{proper}}}\right)^2} = \boxed{0.75c}$$

REFLECT

The faster the spaceship is moving, the shorter the spaceship appears to be to an outside observer.

25.57

SET UP

A stick that is oriented parallel to its direction of motion is traveling with a speed $V = 0.44c$. An observer measures the length of the stick to be $L = 0.88$ m. The proper length of the stick can be found using $L = \frac{1}{\gamma}L_{\text{proper}}$.

SOLVE

$$L = \frac{1}{\gamma}L_{\text{proper}} = \left(\sqrt{1 - \frac{V^2}{c^2}}\right)L_{\text{proper}}$$

$$L_{\text{proper}} = \frac{L}{\sqrt{1 - \frac{V^2}{c^2}}} = \frac{0.88\ \text{m}}{\sqrt{1 - \left(\frac{0.44c}{c}\right)^2}} = \boxed{0.98\ \text{m}}$$

REFLECT

The observed length of the stick should be smaller than its proper length due to length contraction.

25.58

SET UP

A standard domino is 1.5 in wide by 2.5 in long. If the measured dimensions of the domino are 1.5 in wide by 1.5 in long, the domino is traveling at a high speed in a direction parallel to its length. The speed of the domino can be found using $L = \frac{1}{\gamma}L_{\text{proper}}$, where $L = 1.5$ in and $L_{\text{proper}} = 2.5$ in.

SOLVE

$$L = \frac{1}{\gamma}L_{\text{proper}} = \left(\sqrt{1 - \frac{V^2}{c^2}}\right)L_{\text{proper}}$$

$$\sqrt{1 - \frac{V^2}{c^2}} = \frac{L}{L_{\text{proper}}}$$

$$1 - \frac{V^2}{c^2} = \left(\frac{L}{L_{\text{proper}}}\right)^2$$

$$\frac{V^2}{c^2} = 1 - \left(\frac{L}{L_{\text{proper}}}\right)^2$$

$$V = c\sqrt{1 - \left(\frac{L}{L_{\text{proper}}}\right)^2} = c\sqrt{1 - \left(\frac{1.5\ \text{in}}{2.5\ \text{in}}\right)^2} = 0.80c$$

The domino should $\boxed{\text{move at a speed } V = 0.8c \text{ and in a direction parallel to its length}}$.

REFLECT

The domino must be traveling parallel to its length because that is the only dimension that experiences a contraction.

25.59

SET UP

We need to determine the speed of a pion that travels $D_{\text{proper}} = 100$ m before it decays. The average lifetime, at rest, of a pion is $t_{\text{proper}} = 2.60 \times 10^{-8}$ s. The speed we're interested in is

equal to the distance in the frame of the pion D divided by the lifetime of the pion t_{proper}. Since the pions are moving, the distance they travel will be contracted by a factor of $\gamma = \dfrac{1}{\sqrt{1 - \dfrac{V^2}{c^2}}}$.

We can then solve for V to calculate the speed of the pions.

SOLVE

$$V = \frac{D}{t_{\text{proper}}} = \frac{\left(\frac{1}{\gamma}D_{\text{proper}}\right)}{t_{\text{proper}}} = \frac{D_{\text{proper}}}{\gamma t_{\text{proper}}}$$

$$\gamma V = \frac{D_{\text{proper}}}{t_{\text{proper}}}$$

$$\frac{\gamma V}{c} = \frac{D_{\text{proper}}}{t_{\text{proper}}c}$$

$$\left(\frac{\gamma V}{c}\right)^2 = \left(\frac{D_{\text{proper}}}{t_{\text{proper}}c}\right)^2$$

$$\frac{\left(\frac{V}{c}\right)^2}{\left(1 - \left(\frac{V}{\text{c}}\right)^2\right)} = \left(\frac{D_{\text{proper}}}{t_{\text{proper}}c}\right)^2$$

$$\left(\frac{V}{c}\right)^2 = \frac{\left(\frac{D_{\text{proper}}}{t_{\text{proper}}c}\right)^2}{1 + \left(\frac{D_{\text{proper}}}{t_{\text{proper}}c}\right)^2}$$

$$\frac{V}{c} = \sqrt{\frac{\left(\frac{D_{\text{proper}}}{t_{\text{proper}}c}\right)^2}{1 + \left(\frac{D_{\text{proper}}}{t_{\text{proper}}c}\right)^2}} = \sqrt{\frac{\left(\frac{100\text{ m}}{(2.60 \times 10^{-8}\text{ s})\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)}\right)^2}{1 + \left(\frac{100\text{ m}}{(2.60 \times 10^{-8}\text{ s})\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)}\right)^2}} = \boxed{0.997}$$

$$V = 0.997c = 0.997\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right) = \boxed{2.99 \times 10^8 \frac{\text{m}}{\text{s}}}$$

REFLECT

This solution is equivalent to transforming the measurement of the average lifetime of the pions in the moving frame, $V = \frac{D_{\text{proper}}}{t} = \frac{D_{\text{proper}}}{\gamma t_{\text{proper}}}$.

Get Help: P'Cast 25.5 – Meter Stick
P'Cast 25.8 – A Galactic Competition
P'Cast 25.9 – A Galactic Competition, Reconsidered

25.60

SET UP

Spaceship A moves at a velocity relative to Earth of $\vec{v}_{\text{AE}} = (0.8c)\hat{x}$, where the positive x-axis points toward the right. Spaceship B moves at a velocity of $\vec{v}_{\text{BE}} = (-0.7c)\hat{x}$ relative to Earth. The velocities of Earth relative to each spaceship have the same magnitude but point in the opposite directions. To find the velocity of spaceship A relative to spaceship B, we will need to invoke the Lorentz transformation, $v_{\text{AB},\,x} = \frac{v_{\text{AE},\,x} - v_{\text{EB},\,x}}{\left(1 - \frac{v_{\text{AE},\,x} v_{\text{EB},\,x}}{c^2}\right)}$.

SOLVE

Part a)

$$\vec{v}_{\text{EA}} = -\vec{v}_{\text{AE}} = \boxed{-(0.8c)\hat{x}}$$

Part b)

$$\vec{v}_{\text{EB}} = -\vec{v}_{\text{BE}} = \boxed{(0.7c)\hat{x}}$$

Part c)

$$v_{\text{AB},\,x} = \frac{v_{\text{AE},\,x} - v_{\text{EB},\,x}}{\left(1 - \frac{v_{\text{AE},\,x} v_{\text{EB},\,x}}{c^2}\right)} = \frac{(0.8c) - (-0.7c)}{\left(1 - \frac{(0.8c)(-0.7c)}{c^2}\right)} = \frac{1.5c}{1.56} = 0.96c$$

$$\boxed{\vec{v}_{\text{AB}} = (0.96c)\hat{x}}$$

REFLECT

If we didn't apply the Lorentz transformation, the relative speed between the two spaceships would be $1.5c$, which is larger than the speed of light and, thus, not valid.

25.61

SET UP

A car is moving toward $+x$ past an observer at a velocity $\vec{v}_{\text{CO}} = (0.35c)\hat{x}$. A truck also moves past an observer toward $+x$ at a velocity $\vec{v}_{\text{TO}} = (0.25c)\hat{x}$. To find the velocity of the car relative to the truck, we will need to invoke the Lorentz transformation, $v_{\text{CT},\,x} = \frac{v_{\text{CO},\,x} - v_{\text{OT},\,x}}{\left(1 - \frac{v_{\text{CO},\,x} v_{\text{OT},\,x}}{c^2}\right)}$. The velocity of the truck relative to the car will be equal to $\vec{v}_{\text{TC}} = -\vec{v}_{\text{CT}}$.

SOLVE

Velocity of the car relative to the truck:

$$v_{\mathrm{CT},x} = \frac{v_{\mathrm{CO},x} + v_{\mathrm{OT},x}}{\left(1 + \frac{v_{\mathrm{CO},x}v_{\mathrm{OT},x}}{c^2}\right)} = \frac{v_{\mathrm{CO},x} - v_{\mathrm{TO},x}}{\left(1 + \frac{v_{\mathrm{CO},x}(-v_{\mathrm{TO},x})}{c^2}\right)} = \frac{(0.35c) - (0.25c)}{\left(1 - \frac{(0.35c)(0.25c)}{c^2}\right)} = \frac{0.10c}{0.9125} = 0.11c$$

$$\boxed{\vec{v}_{\mathrm{CT}} = (0.11c)\hat{x}}$$

Velocity of the truck relative to the car:

$$\vec{v}_{\mathrm{TC}} = -\vec{v}_{\mathrm{CT}} = \boxed{-(0.11c)\hat{x}}$$

REFLECT

The car and the truck are moving in the same direction, so their relative speed should be less than their individual speeds.

25.62

SET UP

A spaceship is moving past Earth at a velocity $\vec{v}_{\mathrm{SE}} = (0.92c)\hat{x}$ when it fires a rocket at a velocity $\vec{v}_{\mathrm{RS}} = (0.75c)\hat{x}$ relative to the spaceship. To find the velocity of the rocket relative to Earth, we will need to invoke the Lorentz transformation, $v_{\mathrm{RE},x} = \frac{v_{\mathrm{RS},x} + v_{\mathrm{SE},x}}{\left(1 + \frac{v_{\mathrm{RS},x}v_{\mathrm{SE},x}}{c^2}\right)}$. The velocity of the truck relative to the car will be equal to $\vec{v}_{\mathrm{TC}} = -\vec{v}_{\mathrm{CT}}$.

SOLVE

$$v_{\mathrm{RE},x} = \frac{v_{\mathrm{RS},x} + v_{\mathrm{SE},x}}{\left(1 + \frac{v_{\mathrm{RS},x}v_{\mathrm{SE},x}}{c^2}\right)} = \frac{(0.75c) + (0.92c)}{\left(1 + \frac{(0.75c)(0.92c)}{c^2}\right)} = \frac{1.67c}{1.69} = \boxed{0.988c}$$

REFLECT

The rocket is moving in the $+x$ direction relative to the spaceship, which means it must be moving faster than the spaceship in that direction.

25.63

SET UP

A spaceship is traveling past Earth when it fires a rocket in the backward direction relative to the spaceship, and we want to know the velocity of the rocket relative to Earth. We will attach a stationary reference frame S to Earth and reference frame S' to the spaceship. The spaceship is traveling past Earth at a speed of $V = 0.92c$. The speed of the rocket relative to the spaceship is $v'_x = -0.75c$; the negative sign means the rocket is shot backward relative to the spaceship. We can rearrange the expression for the Lorentz transformation (Equation 25-21) in order to solve for the velocity of the rocket relative to Earth v_x.

SOLVE

$$v'_x = \frac{v_x - V}{1 - \left(\frac{V}{c^2}v_x\right)}$$

$$v'_x\left(1 - \left(\frac{V}{c^2}v_x\right)\right) = v_x - V$$

$$v'_x + V = v_x + \left(\frac{V}{c^2}v_x\right)v'_x$$

$$v_x = \frac{v'_x + V}{1 + \left(\frac{V}{c^2}v_x\right)} = \frac{(-0.75c) + (0.92c)}{1 + \left(\frac{(0.92c)}{c^2}(-0.75c)\right)} = \frac{0.17c}{1 - 0.69} = \boxed{0.548c}$$

REFLECT

It makes sense that a rocket shot backward from a spaceship should be moving at a slower speed relative to Earth than the speed of the rocket relative to Earth.

25.64

SET UP

A spaceship is moving past Earth at a velocity $\vec{v}_{\text{SE}} = (0.92c)\hat{x}$ when it fires a laser beam. The velocity of the laser beam relative to the spaceship must be $\vec{v}_{\text{LS}} = (c)\hat{x}$. To prove that the speed of the laser relative to Earth is also c, we will need to invoke the Lorentz transformation,
$v_{\text{LE},\,x} = \dfrac{v_{\text{LS},\,x} + v_{\text{SE},\,x}}{\left(1 + \dfrac{v_{\text{LS},\,x}v_{\text{SE},\,x}}{c^2}\right)}$.

SOLVE

$$v_{\text{LE},\,x} = \frac{v_{\text{LS},\,x} + v_{\text{SE},\,x}}{\left(1 + \frac{v_{\text{LS},\,x}v_{\text{SE},\,x}}{c^2}\right)} = \frac{(c) + (0.92c)}{\left(1 + \frac{(c)(0.92c)}{c^2}\right)} = \frac{1.92c}{1.92} = \boxed{c}$$

REFLECT

The speed of light in a vacuum is the same in all frames.

25.65

SET UP

A proton travels at a velocity $\vec{v}_{\text{PL}} = (0.99999954c)\hat{x}$ relative to the laboratory when it collides with an antiproton traveling at a velocity $\vec{v}_{\text{AL}} = (-0.99999954c)\hat{x}$ relative to the laboratory. The relative velocity between the two particles can be found using the Lorentz transformation,
$v_{\text{LE},\,x} = \dfrac{v_{\text{LS},\,x} + v_{\text{SE},\,x}}{\left(1 + \dfrac{v_{\text{LS},\,x}v_{\text{SE},\,x}}{c^2}\right)}$.

SOLVE

$$v_{\text{PA},\,x} = \frac{v_{\text{PL},\,x} - v_{\text{AL},\,x}}{\left(1 - \frac{v_{\text{PL},\,x}v_{\text{AL},\,x}}{c^2}\right)} = \frac{(0.99999954c) - (-0.99999954c)}{\left(1 + \frac{(0.99999954c)(-0.99999954c)}{c^2}\right)} = \frac{1.9999908c}{1.99999080002} \approx c$$

To the level of precision that the proton and antiproton speeds are given, the relative velocity of the two particles is $\boxed{\text{approximately } c}$.

REFLECT

It's reasonable that the relative velocities of particles in an accelerator approach the speed of light.

25.66

SET UP

An object ($m_0 = 2.00$ kg) is traveling at a speed $v = 400{,}000$ m/s. The classical momentum of the object is equal to $p = m_0 v$; the relativistic momentum of the object is equal to $p = \gamma m_0 v$. Since the object is traveling at a high speed, the relativistic momentum is more accurate. The percent difference between the two results is given by $\%\ \text{error} = \dfrac{p_{\text{Relativistic}} - p_{\text{Newtonian}}}{p_{\text{Relativistic}}} \times 100\%$.

SOLVE

Part a)

$$p_{\text{classical}} = m_0 v = (2.00\ \text{kg})\left(400{,}000\frac{\text{m}}{\text{s}}\right) = \boxed{800{,}000\frac{\text{kg}\cdot\text{m}}{\text{s}}}$$

Part b)

$$p_{\text{relativistic}} = \gamma m_0 v = \frac{m_0 v}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{(2.00\ \text{kg})\left(400{,}000\frac{\text{m}}{\text{s}}\right)}{\sqrt{1-\left(\frac{\left(400{,}000\frac{\text{m}}{\text{s}}\right)}{\left(3.00\times10^8\frac{\text{m}}{\text{s}}\right)}\right)^2}} = \boxed{800{,}001\frac{\text{kg}\cdot\text{m}}{\text{s}}}$$

Part c) Although they are virtually the same, the $\boxed{\text{relativistic version}}$ is more accurate.
Percent error:

$$\%\text{error} = \frac{p_{\text{Relativistic}} - p_{\text{Newtonian}}}{p_{\text{Relativistic}}} \times 100\%$$

$$= \frac{\left(800{,}001\frac{\text{kg}\cdot\text{m}}{\text{s}}\right) - \left(800{,}000\frac{\text{kg}\cdot\text{m}}{\text{s}}\right)}{800{,}001\frac{\text{kg}\cdot\text{m}}{\text{s}}} \times 100 = \boxed{0.000125}$$

REFLECT

The object is traveling at a little over 0.1% of the speed of light.

25.67

SET UP

An electron (rest mass $m_e = 9.11 \times 10^{-31}$ kg) travels at $v = 0.444c$. The magnitude of the relativistic momentum is $p = \gamma m_0 v$, where $\gamma = \dfrac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$. The total energy E of the electron

is the sum of its relativistic kinetic energy K and the energy not associated with its motion, *i.e.*, its rest mass energy, E_0. The rest energy is equal to $E_0 = m_0c^2$, and its total energy is equal to $E = \gamma m_0 c^2$.

SOLVE

Relativistic gamma:

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{1}{\sqrt{1-\left(\frac{0.444c}{c}\right)^2}} = \frac{1}{\sqrt{1-0.197}} = \frac{1}{\sqrt{0.803}} = 1.116$$

Part a)

$$p = \gamma m_0 v = \gamma m_e(0.444c)$$

$$= (1.116)(9.11 \times 10^{-31}\text{ kg})(0.444)\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right) = \boxed{1.35 \times 10^{-22} \frac{\text{kg} \cdot \text{m}}{\text{s}}}$$

Part b)

$$K + E_0 = E$$

$$K = E - E_0 = \gamma m_0 c^2 - m_0 c^2 = (\gamma - 1)m_0 c^2 = ((1.116) - 1)m_e c^2$$

$$= (0.116)(9.11 \times 10^{-31}\text{ kg})\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)^2 = \boxed{9.51 \times 10^{-15}\text{ J}}$$

Part c)

$$E_0 = m_0 c^2 = m_e c^2 = (9.11 \times 10^{-31}\text{ kg})\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}^2}\right)^2 = \boxed{8.20 \times 10^{-14}\text{ J}}$$

Part d)

$$E = \gamma m_0 c^2 = (1.116)m_e c^2 = (1.116)(9.11 \times 10^{-31}\text{ kg})\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}^2}\right)^2 = \boxed{9.15 \times 10^{-14}\text{ J}}$$

REFLECT

The work done on an object required to increase the speed approaches infinity since relativistic gamma approaches infinity as the object's speed approaches c.

Get Help: P'Cast 25.11 – Rest Energy

25.68

SET UP

A proton ($m_0 = 1.673 \times 10^{-27}$ kg) is traveling at $v = 0.5c$. The relativistic momentum and relativistic kinetic energy of the particle are $p = \gamma m_0 v$ and $K = (\gamma - 1)m_0c^2$, respectively.

SOLVE

Part a)

$$p = \gamma m_0 v = \frac{m_0 v}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{(1.673 \times 10^{-27}\ \text{kg})(0.5)\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)}{\sqrt{1 - \left(\frac{0.5c}{c}\right)^2}} = \boxed{2.90 \times 10^{-19} \frac{\text{kg} \cdot \text{m}}{\text{s}}}$$

Part b)

$$K = (\gamma - 1) m_0 c^2 = \left(\frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} - 1 \right) m_0 c^2$$

$$= \left(\frac{1}{\sqrt{1 - \left(\frac{0.5c}{c}\right)^2}} - 1 \right) (1.673 \times 10^{-27}\ \text{kg}) \left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)^2 = \boxed{2.33 \times 10^{-11}\ \text{J}}$$

REFLECT

Gamma is equal to about 1.15, so the relativistic momentum is about 15% larger than the classical momentum.

25.69

SET UP

The relativistic energy ($E = \gamma m_0 c^2$) of an object is twice its rest energy ($E_0 = m_0 c^2$). We can find the speed of the object by taking the ratio of these two expressions and solving for γ and then v.

SOLVE

$$\frac{E}{E_0} = \frac{\gamma m_0 c^2}{m_0 c^2} = \gamma = \frac{1}{\sqrt{1 - \beta^2}} = 2$$

$$\frac{1}{1 - \beta^2} = 4$$

$$1 - \beta^2 = \frac{1}{4}$$

$$\beta = \sqrt{\frac{3}{4}} = 0.866$$

$$\boxed{v = 0.866c}$$

REFLECT

This is a reasonable speed.

25.70

SET UP

A particle is moving along the x-axis under the influence of a constant force F_x. We can find an expression for the acceleration of the particle by explicitly calculating the derivative in Newton's second law $\left(F_x = \dfrac{dp_x}{dt}\right)$ using the definition of the relativistic momentum $(p_x = \gamma m_0 v_x)$.

SOLVE

$$F_x = \frac{dp_x}{dt} = \frac{d}{dt}(\gamma m_0 v_x) = m_0 v_x \frac{d\gamma}{dt} + m_0 \gamma \frac{dv_x}{dt} = m_0\left[v_x\left(\frac{d\gamma}{dv_x}\right)\left(\frac{dv_x}{dt}\right) + \gamma \frac{dv_x}{dt}\right]$$

$$= m_0 \frac{dv_x}{dt}\left[-\frac{v_x\left(-\dfrac{2v_x}{c^2}\right)}{2\left(1-\left(\dfrac{v_x}{c}\right)^2\right)^{\frac{3}{2}}} + \gamma\right] = m_0 a_x\left[\frac{\left(\dfrac{v_x^2}{c^2}\right)}{\left(1-\left(\dfrac{v_x}{c}\right)^2\right)^{\frac{3}{2}}} + \frac{1}{\left(1-\left(\dfrac{v_x}{c}\right)^2\right)^{\frac{1}{2}}}\right]$$

$$= m_0 a_x\left[\frac{1}{\left(1-\left(\dfrac{v_x}{c}\right)^2\right)^{\frac{3}{2}}}\right]$$

$$\boxed{a_x = \frac{F_x}{m_0}\left(1-\left(\frac{v_x}{c}\right)^2\right)^{\frac{3}{2}}}$$

REFLECT

Since the term $\dfrac{v_x}{c}$ is dimensionless, our expression does indeed have dimensions of acceleration.

25.71

SET UP

By integrating our solution to Problem 25.70, we can derive an expression for the x component of the velocity of the particle for any time t. We'll assume the initial speed of the particle is zero at $t = 0$.

SOLVE

$$a_x = \frac{dv_x}{dt} = \frac{F_x}{m_0}\left(1-\left(\frac{v}{c}\right)^2\right)^{\frac{3}{2}}$$

$$\int_0^{v_x(t)} \frac{dv_x}{\left(1-\left(\dfrac{v}{c}\right)^2\right)^{\frac{3}{2}}} = \int_0^t \frac{F_x}{m_0}dt$$

$$\left[\frac{v}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\right]_0^{v_x(t)} = \frac{F_x}{m_0}[t]_0^t$$

$$\left[\frac{v_x(t)}{\sqrt{1-\left(\frac{v_x(t)}{c}\right)^2}} - 0\right] = \frac{F_x}{m_0}[t-0]$$

$$\gamma v_x(t) = \frac{F_x}{m_0}t$$

$$\gamma\beta = \frac{F_x t}{m_0 c}$$

$$\gamma^2\beta^2 = \left(\frac{F_x t}{m_0 c}\right)^2$$

$$\frac{\beta^2}{1-\beta^2} = \left(\frac{F_x t}{m_0 c}\right)^2$$

$$\beta^2\left(1+\left(\frac{F_x t}{m_0 c}\right)^2\right) = \left(\frac{F_x t}{m_0 c}\right)^2$$

$$\beta = \sqrt{\frac{\left(\frac{F_x t}{m_0 c}\right)^2}{1+\left(\frac{F_x t}{m_0 c}\right)^2}}$$

$$\boxed{v_x(t) = c\sqrt{\frac{\left(\frac{F_x t}{m_0 c}\right)^2}{1+\left(\frac{F_x t}{m_0 c}\right)^2}}}$$

REFLECT

Our answer to Problem 25.71 is an example of a separable differential equation, which is how we can simply solve it by separating the variables by relegating them to opposite sides of the equation.

25.72

SET UP

A particle has a rest energy of $E_0 = 5.34 \times 10^{-13}$ J and a total relativistic energy of $E = 9.61 \times 10^{-13}$ J. First we'll derive an expression for the total relativistic energy in terms of the momentum and then calculate the momentum of the particle.

SOLVE

A useful relationship:

$$E = mc^2 = \gamma m_0 c^2 = \frac{m_0 c^2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

$$(mc^2)^2 = \frac{(m_0 c^2)^2}{\left(1 - \left(\frac{v}{c}\right)^2\right)}$$

$$(mc^2)^2\left(1 - \left(\frac{v}{c}\right)^2\right) = (m_0 c^2)^2$$

$$(mc^2)^2 - m^2 v^2 c^2 = (m_0 c^2)^2$$

$$(mc^2)^2 = m^2 v^2 c^2 + (m_0 c^2)^2$$

$$E^2 = (pc)^2 + E_0^2$$

Solving for the momentum:

$$p = \frac{\sqrt{E^2 - E_0^2}}{c} = \frac{\sqrt{(9.61 \times 10^{-13}\ \text{J}) - (5.34 \times 10^{-13}\ \text{J})}}{\left(3.00 \times 10^{8}\frac{\text{m}}{\text{s}}\right)} = \boxed{2.18 \times 10^{-15}\frac{\text{kg} \cdot \text{m}}{\text{s}}}$$

REFLECT

If the momentum is small, the total relativistic energy is approximately equal to $E \approx \frac{p^2}{2m_0} + m_0 c^2$.

25.73

SET UP

A proton has a rest energy of $E_0 = m_0 c^2 = 1.50 \times 10^{-10}$ J and a momentum of $p = \gamma m_0 v = 1.067 \times 10^{-19}\frac{\text{kg} \cdot \text{m}}{\text{s}}$. In order to find the speed of the proton, we first need to derive an expression for the relativistic energy of the proton in terms of its momentum. This will allow us to easily solve for γ (actually, γ^2) in terms of our known quantities, the rest energy and the momentum. The expressions for the rest energy, momentum, and relativistic energy are $E_0 = m_0 c^2$, $p = \gamma m_0 v$, and $E = mc^2 = \gamma E_0$, respectively. Finally, using the definition of γ we can solve for the speed of the proton.

SOLVE

Total energy in terms of momentum:

$$E = mc^2 = \gamma m_0 c^2 = \frac{m_0 c^2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

$$m^2c^4 = \frac{m_0^2c^4}{1 - \left(\frac{v}{c}\right)^2}$$

$$m^2c^4 - m^2v^2c^2 = m_0^2c^4$$

$$m^2c^4 = E^2 = m^2v^2c^2 + m_0^2c^4 = (\gamma m_0)^2v^2c^2 + m_0^2c^4 = p^2c^2 + m_0^2c^4$$

$$E^2 = p^2c^2 + m_0^2c^4$$

Solving for gamma:

$$E^2 = (\gamma m_0c^2)^2 = p^2c^2 + m_0^2c^4$$

$$\gamma^2 = \frac{p^2c^2 + (m_0c^2)^2}{(m_0c^2)^2}$$

$$= \frac{\left(1.067 \times 10^{-19}\frac{\text{kg} \cdot \text{m}}{\text{s}}\right)^2\left(3.00 \times 10^8\frac{\text{m}}{\text{s}}\right)^2 + (1.50 \times 10^{-10}\ \text{J})^2}{(1.50 \times 10^{-10}\ \text{J})^2} = 1.0455$$

Solving for the speed:

$$\gamma^2 = \frac{1}{1 - \left(\frac{v}{c}\right)^2}$$

$$\left(\frac{v}{c}\right)^2 = \frac{\gamma^2 - 1}{\gamma^2}$$

$$v = c\sqrt{\frac{\gamma^2 - 1}{\gamma^2}} = c\sqrt{\frac{1.0455 - 1}{1.0455}} = \boxed{0.209c}$$

REFLECT

We couldn't have simply divided the momentum by γm_0 to solve for v because γ is also a function of v.

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25.74

SET UP

An elevator near Earth's surface is accelerating downward at 8.0 m/s^2. We can use the general theory of relativity to determine the free-fall acceleration an observer inside the elevator would experience.

SOLVE

$$a = \left(-9.8\frac{\text{m}}{\text{s}^2}\right) - \left(-8.0\frac{\text{m}}{\text{s}^2}\right) = 1.8\frac{\text{m}}{\text{s}^2}$$

The observer in the elevator would measure the free-fall acceleration as $\boxed{1.8 \text{ m/s}^2 \text{ in the downward direction}}$.

REFLECT

The acceleration due to gravity downward is larger than the acceleration of the elevator downward, so the measured acceleration should still be in the downward direction.

25.75

SET UP

An elevator near Earth's surface is accelerating downward at 18 m/s^2. We can use the general theory of relativity to determine the free-fall acceleration an observer inside the elevator would experience.

SOLVE

$$a = \left(-9.8\frac{\text{m}}{\text{s}^2}\right) - \left(-18\frac{\text{m}}{\text{s}^2}\right) = 8.2\frac{\text{m}}{\text{s}^2}$$

The observer in the elevator would measure the free-fall acceleration as $\boxed{8.2 \text{ m/s}^2 \text{ in the upward direction}}$.

REFLECT

We can do our own thought experiment to confirm our answer. The observer inside the elevator throws a ball horizontally. If the elevator car were stationary in free space, the ball would go in a straight line until it hit the wall. If the elevator car were accelerating downward, the ball would hit the wall at a location *higher* than the previous case because the entire elevator would have shifted downward due to its acceleration.

25.76

SET UP

An outside observer measures the time it takes a rocket car to travel $\Delta x = 2.40 \times 10^5$ m to be $\Delta t = 10^{-3}$ s. The speed of the rocket car must be the same in both the frame of the car and the frame of the observer and equal to $V = \frac{\Delta x}{\Delta t}$. The time elapsed in the rocket car during the trip is equal to the proper time interval, where $\Delta t = \gamma \Delta t_{\text{proper}}$. The distance traveled during the trip, according to the driver of the rocket car, would be the observed length L, where $L = \frac{1}{\gamma}L_{\text{proper}}$ and $L_{\text{proper}} = \Delta x$.

SOLVE

Part a)
Speed of the car as measured by the observer:

$$V = \frac{\Delta x}{\Delta t} = \frac{2.40 \times 10^5 \text{ m}}{10^{-3} \text{ s}} = 2.40 \times 10^8 \frac{\text{m}}{\text{s}}$$

Proper time:

$$\Delta t = \gamma \Delta t_{\text{proper}}$$

$$\Delta t_{\text{proper}} = \frac{\Delta t}{\gamma} = \Delta t\sqrt{1 - \left(\frac{V}{c}\right)^2} = (10^{-3}\ \text{s})\sqrt{1 - \left(\frac{\left(2.40 \times 10^8 \frac{\text{m}}{\text{s}}\right)}{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)}\right)^2} = \boxed{6 \times 10^{-4}\ \text{s}}$$

Part b)

$$L = \frac{1}{\gamma}L_{\text{proper}} = L_{\text{proper}}\sqrt{1 - \left(\frac{V}{c}\right)^2} = (2.40 \times 10^5\ \text{m})\sqrt{1 - \left(\frac{\left(2.40 \times 10^8 \frac{\text{m}}{\text{s}}\right)}{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)}\right)^2}$$

$$= \boxed{1.44 \times 10^5\ \text{m}}$$

REFLECT

The distance determined by the driver of the car must be shorter than the distance measured by the observer because the car's speed is the same in both frames. Since less time elapses in the driver's frame, the driver must travel a shorter distance.

25.77

SET UP

Observers in a reference frame S see one explosion at $x_1 = 580$ m and then a second explosion $\Delta t = 4.5\ \mu$s later at $x_2 = 1500$ m. Reference frame S' is moving along the positive x-axis at a speed v. We can calculate v from the data measured by those in S. Observers in S' see the explosions occur at the same point in space. Since the observers in S' see the explosions occur at the same point in space, we know that the proper time is the time separation measured in S'. The time between explosions as measured in S' is related to the time interval measured in S through relativistic gamma, $\Delta t = \gamma \Delta t_{\text{proper}}$.

SOLVE

Speed

$$v = \frac{x_2 - x_1}{\Delta t} = \frac{(1500\ \text{m}) - (580\ \text{m})}{4.5 \times 10^{-6}\ \text{s}} = 2.044 \times 10^8 \frac{\text{m}}{\text{s}}$$

Time separation

$$\Delta t = \gamma \Delta t_{\text{proper}}$$

$$\Delta t_{\text{proper}} = \frac{\Delta t}{\gamma} = \Delta t\sqrt{1 - \left(\frac{v}{c}\right)^2} = (4.5\ \mu\text{s})\sqrt{1 - \left(\frac{\left(2.044 \times 10^8 \frac{\text{m}}{\text{s}}\right)}{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)}\right)^2} = \boxed{3.29\ \mu\text{s}}$$

REFLECT

The proper time interval should be smaller than the time interval observed by those in S due to the effects of time dilation.

25.78

SET UP

Alpha Centauri is $\Delta x = 4.37$ ly from Earth. A spaceship is traveling toward Alpha Centauri at a speed of $v = 0.77c$. The time required for a one-way trip to the star, according to the crew of the spaceship, is given by $v = \dfrac{\Delta x}{\Delta t} = \dfrac{\Delta x}{\gamma \Delta t_{\text{proper}}}$, whereas the time required for the trip according to a stationary observer on Earth is found using $v = \dfrac{\Delta x}{\Delta t_{\text{proper}}}$.

SOLVE

Part a)

$$v = \frac{\Delta x}{\Delta t} = \frac{\Delta x}{\gamma \Delta t_{\text{proper}}}$$

$$\Delta t_{\text{proper}} = \frac{\Delta x}{\gamma v} = \frac{\Delta x}{v}\sqrt{1 - \left(\frac{v}{c}\right)^2} = \frac{(4.37 \text{ yr})c}{0.77c}\sqrt{1 - \left(\frac{0.77c}{c}\right)^2} = \boxed{3.6 \text{ yr}}$$

Part b)

$$v = \frac{\Delta x}{\Delta t_{\text{proper}}}$$

$$\Delta t_{\text{proper}} = \frac{\Delta x}{v} = \frac{(4.37 \text{ yr})c}{0.77 \text{ c}} = \boxed{5.8 \text{ yr}}$$

REFLECT

The time measured with respect to Earth is longer than that measured on the spaceship as expected.

25.79

SET UP

In part (a), a radioactive nucleus is traveling at a velocity of $\vec{v}_{\text{NO}} = (0.8c)\hat{x}$ relative to a stationary observer when it emits an electron with a velocity of $\vec{v}_{\text{EN}} = (0.6c)\hat{x}$ relative to the nucleus. The speed of the electron relative to the observer is given by $v_{\text{EO},x} = \dfrac{v_{\text{EN},x} + v_{\text{NO},x}}{\left(1 + \dfrac{v_{\text{EN},x}v_{\text{NO},x}}{c^2}\right)}$. In part (b), rocket A travels at a velocity of $\vec{v}_{\text{AE}} = (-0.6c)\hat{x}$ relative to Earth and rocket B travels at a velocity $\vec{v}_{\text{BE}} = (0.8c)\hat{x}$ relative to Earth. The speed of rocket B relative to rocket A is given by $v_{\text{BA},x} = \dfrac{v_{\text{BE},x} - v_{\text{AE},x}}{\left(1 - \dfrac{v_{\text{BE},x}v_{\text{AE},x}}{c^2}\right)}$. Once we get numerical

answers for parts (a) and (b) we can make deeper connections to find similarities between the two problems and figure out why those similarities exist.

SOLVE

Part a)

$$v_{\mathrm{EO},\,x} = \frac{v_{\mathrm{EN},\,x} + v_{\mathrm{NO},\,x}}{\left(1 + \dfrac{v_{\mathrm{EN},\,x}v_{\mathrm{NO},\,x}}{c^2}\right)} = \frac{(0.8c) + (0.6c)}{\left(1 + \dfrac{(0.8c)(0.6c)}{c^2}\right)} = \frac{1.4c}{1.48} = \boxed{0.946c}$$

Part b)

$$v_{\mathrm{BA},\,x} = \frac{v_{\mathrm{BE},\,x} - v_{\mathrm{AE},\,x}}{\left(1 - \dfrac{v_{\mathrm{BE},\,x}v_{\mathrm{AE},\,x}}{c^2}\right)} = \frac{(0.8c) - (-0.6c)}{\left(1 - \dfrac{(0.8c)(-0.6c)}{c^2}\right)} = \frac{1.4c}{1.48} = \boxed{0.946c}$$

Part c) If we consider the nucleus in part (a) to be the "Earth," then the laboratory goes by at a speed of $0.8c$, which corresponds to rocket B in part (b). The nucleus emits an electron in the forward direction, at a speed of $0.6c$, and this electron corresponds to rocket A in part (b). In part (a) we determined the speed of the electron relative to the laboratory frame, which is then equivalent to the speed of rocket A relative to rocket B in part (b). Since two objects must always have the same speed relative to each other, our calculation of the speed of rocket B relative to rocket A should give the same answer.

REFLECT

Since rocket B is moving away from rocket A, the relative speed of B to A should be larger than the relative speed of B to Earth.

25.80

SET UP

The lifetime of a beam of pions traveling at a speed $V = 0.88c$ is measured to be $\Delta t = 2.6 \times 10^{-8}$ s in the reference frame of the laboratory. The lifetime in the frame of the pions is the proper time, $\Delta t = \gamma \Delta t_{\mathrm{proper}}$. Once we have the lifetime of the pions in the moving frame, the distance Δx the laboratory travels relative to the pions is related to the speed of the pions and the lifetime of the pions in the moving frame, $V = \dfrac{\Delta x}{\Delta t_{\mathrm{proper}}}$.

SOLVE

Proper time:

$$\Delta t = \gamma \Delta t_{\mathrm{proper}}$$

$$\Delta t_{\mathrm{proper}} = \frac{\Delta t}{\gamma} = \Delta t\sqrt{1 - \left(\frac{V}{c}\right)^2} = (2.6 \times 10^{-8}\ \mathrm{s})\sqrt{1 - \left(\frac{0.88c}{c}\right)^2} = 1.2 \times 10^{-8}\ \mathrm{s}$$

Length traveled:

$$V = \frac{\Delta x}{\Delta t_{\mathrm{proper}}}$$

$$\Delta x = V\Delta t_{\mathrm{proper}} = 0.88\left(3.00 \times 10^8 \frac{\mathrm{m}}{\mathrm{s}}\right)(1.2 \times 10^{-8}\ \mathrm{s}) = \boxed{3.3\ \mathrm{m}}$$

REFLECT

It makes sense that the lifetime of the pions in the moving frame is less than the lifetime measured in the laboratory frame.

25.81

SET UP

Muons have a proper lifetime of $\Delta t_{\text{proper}} = 2.20 \times 10^{-6}$ s. A muon is formed a distance of 3000 m above the surface of Earth and travels straight downward with a speed $V = 0.950c$. To determine whether or not the muon reaches Earth's surface before it decays, we have to calculate how far the muon travels in 2.20×10^{-6} s relative to both the reference frame of the muon and the reference frame of Earth. The distance the muon travels will be contracted in the muon's reference frame, but the observed time will be longer in Earth's reference frame. In either case, the distance the muon travels must be at least 3000 m if it is to reach Earth's surface before decaying. The minimum necessary speed for the muon to just reach Earth's surface before decaying can be calculated from $V = \frac{L_{\text{proper}}}{\Delta t} = \frac{L_{\text{proper}}}{\gamma \Delta t_{\text{proper}}}$, where $L_{\text{proper}} = 3000$ m.

SOLVE

Part a)

Length contraction in the muon's reference frame:

$$L = \frac{1}{\gamma} L_{\text{proper}} = L_{\text{proper}} \sqrt{1 - \left(\frac{V}{c}\right)^2} = (3000 \text{ m}) \sqrt{1 - \left(\frac{0.950c}{c}\right)^2} = 937 \text{ m}$$

Distance traveled by muon in the muon's reference frame:

$$V = \frac{L}{\Delta t_{\text{proper}}}$$

$$L = V \Delta t_{\text{proper}} = 0.950\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)(2.20 \times 10^{-6} \text{ s}) = 627 \text{ m}$$

Because 627 m < 937 m, the $\boxed{\text{muon does not make it to Earth}}$.

Time dilation in Earth's reference frame:

$$\Delta t = \gamma \Delta t_{\text{proper}} = \frac{\Delta t_{\text{proper}}}{\sqrt{1 - \left(\frac{V}{c}\right)^2}} = \frac{2.20 \times 10^{-6} \text{ s}}{\sqrt{1 - \left(\frac{0.950c}{c}\right)^2}} = 7.05 \times 10^{-6} \text{ s}$$

Distance traveled in Earth's reference frame:

$$V = \frac{L_{\text{proper}}}{\Delta t}$$

$$L_{\text{proper}} = V \Delta t = 0.950\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)(7.05 \times 10^{-6} \text{ s}) = 2008 \text{ m}$$

Because 2008 m < 3000 m, the $\boxed{\text{muon does not make it to Earth}}$.

Part b)

$$V = \frac{L_{\text{proper}}}{\Delta t} = \frac{L_{\text{proper}}}{\gamma \Delta t_{\text{proper}}}$$

$$\gamma V = \frac{L_{\text{proper}}}{\Delta t_{\text{proper}}}$$

$$\gamma\beta = \frac{L_{\text{proper}}}{c\Delta t_{\text{proper}}} = \frac{3000 \text{ m}}{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)(2.2 \times 10^{-6} \text{ s})} = 4.55$$

$$\gamma^2\beta^2 = \frac{\beta^2}{1-\beta^2} = 20.7$$

$$\beta^2 = 20.7(1-\beta^2)$$

$$\beta = \sqrt{\frac{20.7}{21.7}} = 0.977$$

$$\boxed{V = 0.977c}$$

REFLECT

Since the muon decays before reaching Earth's surface, it makes sense that the actual speed of the muon is smaller than the minimum required speed to just reach Earth's surface.

25.82

SET UP

Allison and Annabelle are the same age when Allison hops aboard a flying saucer traveling at a speed $V = 0.800c$ relative to Earth. According to Allison, she was on the saucer for $\Delta t_{\text{proper}} = 20$ yr before returning to Earth. The time interval according to Annabelle on Earth is $\Delta t = \gamma \Delta t_{\text{proper}}$. According to an observer on Earth, Allison must be younger because she experiences less time passing than Annabelle.

SOLVE

Time elapsed according to Annabelle on Earth:

$$\Delta t = \gamma \Delta t_{\text{proper}} = \frac{\Delta t_{\text{proper}}}{\sqrt{1-\left(\frac{V}{c}\right)^2}} = \frac{20 \text{ yr}}{\sqrt{1-\left(\frac{0.800c}{c}\right)^2}} = 33 \text{ yr}$$

According to Annabelle, $\boxed{\text{Allison is 13 years younger}}$.

REFLECT

From Allison's perspective, Annabelle is younger.

25.83

SET UP

A jet plane flies at $v = 300$ m/s relative to an observer on the ground. There is a clock aboard the plane, as well as one on the ground; the two were synchronized at the start. We want to know the distance (as measured by the observer on the ground) the plane must fly such that the clock on the plane is 10 s behind the clock on the ground. This time difference is the difference between the dilated time experienced on the plane and the proper time experienced on the ground. From this, we can calculate the proper time; the distance the plane must fly is equal to the plane's speed relative to the ground multiplied by the proper time. When calculating relativistic gamma, it will be helpful to use the approximation for $(1-x)^{-\frac{1}{2}}$ when x is much smaller than 1: $(1-x)^{-\frac{1}{2}} \approx 1 + \frac{1}{2}x$.

SOLVE

Relativistic gamma:

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{1}{\sqrt{1-\left(\frac{\left(300\frac{\text{m}}{\text{s}}\right)}{\left(3.00 \times 10^8\frac{\text{m}}{\text{s}}\right)}\right)^2}} = \frac{1}{\sqrt{1-(1 \times 10^{-12})}} = [1-(1 \times 10^{-12})]^{-\frac{1}{2}}$$

$$\approx 1 + \frac{1}{2}(1 \times 10^{-12})$$

Time observed on the ground:

$$t - t_{\text{proper}} = \gamma t_{\text{proper}} - t_{\text{proper}} = (\gamma - 1)t_{\text{proper}} = 10 \text{ s}$$

$$t_{\text{proper}} = \frac{10 \text{ s}}{(\gamma - 1)}$$

Distance of the flight as measured by an observer on the ground:

$$d = vt_{\text{proper}} = v\left(\frac{10 \text{ s}}{\gamma - 1}\right) \approx \left(300\frac{\text{m}}{\text{s}}\right)\frac{(10 \text{ s})}{\left(1 + \frac{1}{2}(1 \times 10^{-12})\right) - 1} = \left(300\frac{\text{m}}{\text{s}}\right)\frac{2(10 \text{ s})}{1 \times 10^{-12}}$$

$$= \boxed{6 \times 10^{15} \text{ m}}$$

REFLECT

This is approximately 0.6 light years. For comparison, the distance between Earth and the Sun is about 1.5×10^{11} m; the distance the jet has to travel is 40,000 times larger than that.

25.84

SET UP

The speed of an electron is $V = 0.99999999995c$. The value of relativistic gamma for this electron is given by $\gamma = \frac{1}{\sqrt{1-\left(\frac{V}{c}\right)^2}}$. A time interval of $\Delta t_{\text{proper}} = 1.66 \times 10^{-6}$ s as measured

relative to the electron frame is related to the time interval Δt as measured relative to the laboratory frame through$\Delta t = \gamma \Delta t_{\text{proper}}$.

SOLVE

Part a)

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{V}{c}\right)^2}} = \frac{1}{\sqrt{1 - \left(\frac{0.99999999995c}{c}\right)^2}} = \boxed{100{,}000}$$

Part b)

$$\Delta t = \gamma \Delta t_{\text{proper}} = (100{,}000)(1.66 \times 10^{-6}\ \text{s}) = \boxed{0.166\ \text{s}}$$

REFLECT

The time interval in the laboratory frame must be longer than the time interval in the electron frame due to time dilation.

25.85

SET UP

The proper half-life of a subatomic particle is $\Delta t_{\text{proper}} = 1 \times 10^{-8}$ s. The observed half-life of a moving beam of these particles is $\Delta t = 6 \times 10^{-8}$ s. The speed V of the particles in the beam can be found through$\Delta t = \gamma \Delta t_{\text{proper}}$, where $\gamma = \frac{1}{\sqrt{1 - \left(\frac{V}{c}\right)^2}}$.

SOLVE

$$\Delta t = \gamma \Delta t_{\text{proper}} = \frac{\Delta t_{\text{proper}}}{\sqrt{1 - \left(\frac{V}{c}\right)^2}}$$

$$\sqrt{1 - \left(\frac{V}{c}\right)^2} = \frac{\Delta t_{\text{proper}}}{\Delta t}$$

$$1 - \left(\frac{V}{c}\right)^2 = \left(\frac{\Delta t_{\text{proper}}}{\Delta t}\right)^2$$

$$V = c\sqrt{1 - \left(\frac{\Delta t_{\text{proper}}}{\Delta t}\right)^2} = c\sqrt{1 - \left(\frac{1 \times 10^{-8}\ \text{s}}{6 \times 10^{-8}\ \text{s}}\right)^2} = \boxed{0.986c}$$

REFLECT

The observed lifetime is six times longer than the proper time interval, so the speed of the particles should be approaching the speed of light.

25.86

SET UP

At rest, the half-life of a certain radioisotope is $\Delta t_{\text{proper}} = 2.25 \times 10^{-6}$ s. The half-life of the same radioisotope traveling in a high-speed spaceship as measured from Earth is $\Delta t = 3.15 \times 10^{-6}$ s. Since the radioisotope is at rest relative to the spaceship, an astronaut in the ship will measure the same lifetime as the radioisotope at rest on Earth. The speed of the spaceship can be calculated from the proper and observed lifetimes, $\Delta t = \gamma \Delta t_{\text{proper}}$, where $\gamma = \dfrac{1}{\sqrt{1 - \left(\frac{V}{c}\right)^2}}$.

SOLVE

Part a)

$$\boxed{\Delta t_{\text{proper}} = 2.25 \times 10^{-6}\ \text{s}}$$

Part b)

$$\Delta t = \gamma \Delta t_{\text{proper}} = \frac{\Delta t_{\text{proper}}}{\sqrt{1 - \left(\frac{V}{c}\right)^2}}$$

$$\sqrt{1 - \left(\frac{V}{c}\right)^2} = \frac{\Delta t_{\text{proper}}}{\Delta t}$$

$$1 - \left(\frac{V}{c}\right)^2 = \left(\frac{\Delta t_{\text{proper}}}{\Delta t}\right)^2$$

$$V = c\sqrt{1 - \left(\frac{\Delta t_{\text{proper}}}{\Delta t}\right)^2} = c\sqrt{1 - \left(\frac{2.25 \times 10^{-6}\ \text{s}}{3.15 \times 10^{-6}\ \text{s}}\right)^2} = \boxed{0.700c}$$

REFLECT

The half-life of the isotope in the spaceship as measured by the astronaut in the ship is the proper time because she is at rest relative to the sample.

25.87

SET UP

In one month, a household uses 411 kWh of electrical energy and 201 therms of gas heating (1.0 therm = 29.3 kWh). After converting these values into joules, we can use $E = mc^2$ to calculate the mass necessary to meet the monthly energy needs for this house.

SOLVE

Conversions:

$$411\text{ kWh} \times \frac{3600\text{ s}}{1\text{ hr}} \times \frac{1000\text{ W}}{1\text{ kW}} = 1.480 \times 10^{9}\text{ J}$$

$$201\text{ therms} \times \frac{29.3\text{ kWh}}{1.0\text{ therm}} \times \frac{3600\text{ s}}{1\text{ hr}} \times \frac{1000\text{ W}}{1\text{ kW}} = 2.120 \times 10^{10}\text{ J}$$

Total energy:

$$E = (2.120 \times 10^{10}\text{ J}) + (1.480 \times 10^{9}\text{ J}) = 2.268 \times 10^{10}\text{ J}$$

Mass:

$$E = mc^2$$

$$m = \frac{E}{c^2} = \frac{2.268 \times 10^{10}\text{ J}}{\left(3.00 \times 10^{8}\frac{\text{m}}{\text{s}}\right)^2} = 2.52 \times 10^{-7}\text{ kg} \times \frac{10^{6}\text{ mg}}{1\text{ kg}} = \boxed{0.252\text{ mg}}$$

REFLECT

This would be the mass necessary if we were harvesting the energy directly from the atoms that make up the molecules.

Get Help: P'Cast 25.11 – Rest Energy
P'Cast 25.12 – The Sun

25.88

SET UP

Jessica and Marsha are twins. Jessica hops aboard a spaceship headed for Jupiter ($d = 6.28 \times 10^{11}$ m). She flies to Jupiter and back to Earth at a constant speed $V = 0.75c$ relative to Earth. The time interval according to Marsha on Earth is related to the total distance traveled and the speed of the spaceship, $V = \frac{\Delta x}{\Delta t}$, where $\Delta x = 2d$. The time interval according to Jessica in the spaceship, on the other hand, is related to the proper time, $\Delta t = \gamma \Delta t_{\text{proper}}$. After calculating the time experienced by each twin for the trip, the difference in their ages is equal to the difference in these experienced time intervals. According to an observer on Earth, Jessica must be younger because she experiences less time passing than Marsha.

SOLVE

Time elapsed according to Marsha on Earth:

$$V = \frac{\Delta x}{\Delta t}$$

$$\Delta t = \frac{\Delta x}{V} = \frac{2d}{0.75c} = \frac{2(6.28 \times 10^{11}\text{ m})}{0.75\left(3.00 \times 10^{8}\frac{\text{m}}{\text{s}}\right)} = 5582.2\text{ s}$$

Time elapsed according to Jessica in the spaceship:

$$\Delta t = \gamma \Delta t_{\text{proper}}$$

$$\Delta t_{\text{proper}} = \frac{\Delta t}{\gamma} = \Delta t\sqrt{1-\left(\frac{V}{c}\right)^2} = (5582.2\text{ s})\sqrt{1-\left(\frac{0.75c}{c}\right)^2} = 3692.3\text{ s}$$

Difference in age:

$$\Delta t - \Delta t_{\text{proper}} = (5582.2\text{ s}) - (3692.3\text{ s}) = 1889.9\text{ s} \times \frac{1\text{ min}}{60\text{ s}} = 31.5\text{ min}$$

$\boxed{\text{Marsha is older than Jessica by 31.5 min}}$.

REFLECT

We could have also calculated the time of Jessica's trip by considering the length contraction of the distance she travels rather than calculating the proper time.

25.89

SET UP

A cosmic ray particle is created $L_{\text{proper}} = 25.0$ km above the surface of Earth and travels downward at a speed $V = 0.900c$. The proper time is measured in the particle's reference frame, whereas the proper length is measured in Earth's reference frame. We can calculate the time it takes the particle to travel to Earth relative to the reference frame of the particle and the reference frame of Earth. We will need to calculate the contracted length when finding the time relative to the particle's reference frame. Finally, we can make sure the times are consistent with time dilation by explicitly calculating the time relative to Earth from the time relative to the particle.

SOLVE

Part a) The proper time is the time measured by the particle because the two events occur at the same location in that reference frame. The proper length is in Earth's reference frame because the atmosphere and Earth are at rest in that frame.

Part b)
Length contraction:

$$L = \frac{L_{\text{proper}}}{\gamma} = L_{\text{proper}}\sqrt{1-\left(\frac{V}{c}\right)^2} = (25.0\text{ km})\sqrt{1-\left(\frac{0.900c}{c}\right)^2} = 10.9\text{ km}$$

Time to travel this distance:

$$V = \frac{L}{\Delta t_{\text{particle}}}$$

$$\Delta t_{\text{particle}} = \frac{L}{V} = \frac{10.9 \times 10^3\text{ m}}{0.900\left(3.00 \times 10^8\,\frac{\text{m}}{\text{s}}\right)} = \boxed{4.04 \times 10^{-5}\text{ s} = 40.4\ \mu\text{s}}$$

Part c)

$$V = \frac{L_{\text{proper}}}{\Delta t_{\text{Earth}}}$$

$$\Delta t_{\text{Earth}} = \frac{L_{\text{proper}}}{V} = \frac{25.0 \times 10^3 \text{ m}}{0.900\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)} = \boxed{9.26 \times 10^{-5} \text{ s} = 92.6 \ \mu\text{s}}$$

Part d)

$$\Delta t_{\text{Earth}} \stackrel{?}{=} \gamma \Delta t_{\text{particle}} = \frac{\Delta t_{\text{particle}}}{\sqrt{1 - \left(\frac{V}{c}\right)^2}} = \frac{40.4 \ \mu\text{s}}{\sqrt{1 - \left(\frac{0.900c}{c}\right)^2}} = 92.6 \ \mu\text{s}$$

$\boxed{\text{Yes, the results are consistent}}$ with time dilation.

REFLECT

We could have also solved parts (b) and (c) using time dilation and part (d) using length contraction. All of the results must be self-consistent.

25.90

SET UP

A rocket that is 642 m long is traveling parallel to Earth's surface at a speed $V = 0.5c$ from left to right. At $t = 0$, a light flashes for an instant at the center of the rocket. Detectors at opposite ends of the rocket detect when the light arrives—event A is when the light strikes the left side of the rocket, and event B is when the light strikes the right side of the rocket. The events are recorded by observers at rest in the rocket and observers at rest on Earth. Since the speed of light must be constant in all inertial reference frames, the speed of the light measured by both sets of observers must be the same and equal to c. In the observations made on the rocket, the light travels an equal distance to the detectors on the left and right ends of the ship. Events A and B will take place at a time equal to half of the length of the rocket divided by the speed of light. Both length contraction of the rocket and the relative motion of the rocket and the light need to be considered in the measurements made on Earth. The light for event A must travel half of the contracted length of the rocket minus the distance the rocket travels during that time. The light for event B must travel half of the contracted length of the rocket plus the distance the rocket travels during that time. We can also perform this calculation using time dilation—the proper time is the time measured on the rocket and the observed time interval is the time difference between events A and B as measured in Earth's frame.

SOLVE

Part a) The speed of light is equal to c in all inertial reference frames, so it is the same for both the observer at rest in the rocket and the observer at rest on Earth.

Part b)
i) Observer in the rocket

$$t_{\text{A}} = t_{\text{B}} = \frac{x}{c} = \frac{321 \text{ m}}{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)} = \boxed{1.07 \times 10^{-6} \text{ s}}$$

ii) Observer on Earth
Event A:

$$ct_{\mathrm{A}} = x_{\mathrm{A}} = x\sqrt{1-\left(\frac{V}{c}\right)^2} - Vt_{\mathrm{A}}$$

$$t_{\mathrm{A}} = \frac{x\sqrt{1-\left(\frac{V}{c}\right)^2}}{c+V} = \frac{x\sqrt{1-\left(\frac{0.5c}{c}\right)^2}}{c+(0.5c)} = \frac{(321\ \mathrm{m})\sqrt{1-(0.5)^2}}{1.5\left(3.00\times 10^8\frac{\mathrm{m}}{\mathrm{s}}\right)} = \boxed{6.178\times 10^{-7}\ \mathrm{s}}$$

Event B:

$$ct_{\mathrm{B}} = x_{\mathrm{B}} = x\sqrt{1-\left(\frac{V}{c}\right)^2} + Vt_{\mathrm{B}}$$

$$t_{\mathrm{B}} = \frac{x\sqrt{1-\left(\frac{V}{c}\right)^2}}{c-V} = \frac{x\sqrt{1-\left(\frac{0.5c}{c}\right)^2}}{c-(0.5c)} = \frac{(321\ \mathrm{m})\sqrt{1-(0.5)^2}}{0.5\left(3.00\times 10^8\frac{\mathrm{m}}{\mathrm{s}}\right)} = \boxed{1.853\times 10^{-6}\ \mathrm{s}}$$

Part c)

$$t_{\mathrm{B}} - t_{\mathrm{A}} \stackrel{?}{=} \gamma\Delta \mathrm{t}_{\mathrm{proper}} = \frac{\Delta t_{\mathrm{proper}}}{\sqrt{1-\left(\frac{V}{c}\right)^2}}$$

$$(1.853\times 10^{-6}\ \mathrm{s}) - (6.178\times 10^{-7}\ \mathrm{s}) \stackrel{?}{=} \frac{1.07\times 10^{-6}\ \mathrm{s}}{\sqrt{1-\left(\frac{0.5c}{c}\right)^2}}$$

$$(1.853\times 10^{-6}\ \mathrm{s}) - (6.178\times 10^{-7}\ \mathrm{s}) \stackrel{?}{=} \frac{1.07\times 10^{-6}\ \mathrm{s}}{\sqrt{1-\left(\frac{0.5c}{c}\right)^2}}$$

$$1.235\times 10^{-6}\ \mathrm{s} = 1.235\times 10^{-6}\ \mathrm{s}$$

This is consistent with time dilation.

REFLECT

To the observer on the rocket, events A and B are simultaneous. To the observer on Earth, event A occurs before event B.

25.91

SET UP

Harry and Larry have the same pulse rates on Earth, $70\frac{\text{beats}}{\text{min}}$. Larry is placed in a spaceship traveling at a speed $V = 0.75c$, while Harry remains on Earth. Larry's pulse rate as measured by Harry will change due to time dilation. We can represent the time dilation relationship in terms of frequency to calculate Larry's pulse rate as measured by Harry. Larry's motion will not affect his own time, so his pulse rate as measured by a doctor on the rocket will remain the same as on Earth.

SOLVE

Part a)

$$t = \gamma t_{\text{proper}}$$

$$\frac{1}{f} = \gamma \frac{1}{f_{\text{proper}}}$$

$$f = \frac{f_{\text{proper}}}{\gamma} = f_{\text{proper}}\sqrt{1 - \left(\frac{V}{c}\right)^2} = \left(70\frac{\text{beats}}{\text{min}}\right)\sqrt{1 - \left(\frac{0.75c}{c}\right)^2} = \boxed{46.3\frac{\text{beats}}{\text{min}}}$$

Part b)

$$\boxed{70\frac{\text{beats}}{\text{min}}}$$

REFLECT

The same number of heartbeats takes place in a longer amount of time, so Larry's pulse rate (as measured on Earth) decreases.

25.92

SET UP

A rocket is traveling at a velocity $\vec{v}_{\text{RE}} = v\hat{x}$ relative to Earth. Inside the rocket, two laser beams are turned on: one pointing in the forward direction (relative to the rocket's velocity), the other pointing in the backward direction (relative to the rocket's velocity). Since the speed of light must be constant in all inertial reference frames, the speed of the light measured by both sets of observers must be the same and equal to c. The velocity of the forward laser beam relative to the spaceship must be $\vec{v}_{\text{FR}} = (c)\hat{x}$, and the velocity of the backward laser beam relative to the spaceship must be $\vec{v}_{\text{BR}} = (-c)\hat{x}$. The Lorentz transformation can be used to find the velocity of each laser beam as measured by an observer at rest on Earth. Once we know the velocities of each laser beam relative to Earth, we can then find the speed at which the beams are separating from each other as viewed by a stationary observer on Earth.

SOLVE

Part a) Both beams travel at the speed of light.

Part b)
Forward beam:

$$v_{\text{FE},x} = \frac{v_{\text{FR},x} + v_{\text{RE},x}}{\left(1 + \dfrac{v_{\text{FR},x}v_{\text{RE},x}}{c^2}\right)} = \frac{(c) + (v)}{\left(1 + \dfrac{(c)(v)}{c^2}\right)} = \frac{c+v}{1 + \left(\dfrac{v}{c}\right)} = \frac{c+v}{\left(\dfrac{c+v}{c}\right)} = \boxed{c}$$

Backward beam:

$$v_{\text{BE},x} = \frac{v_{\text{BR},x} + v_{\text{RE},x}}{\left(1 + \dfrac{v_{\text{BR},x}v_{\text{RE},x}}{c^2}\right)} = \frac{(-c) + (v)}{\left(1 + \dfrac{(-c)(v)}{c^2}\right)} = \frac{v-c}{1 - \left(\dfrac{v}{c}\right)} = \frac{v-c}{\left(\dfrac{c-v}{c}\right)} = \boxed{-c}$$

Part c) Since the two beams are going in opposite directions with speed c, an observer on Earth will see them separate at twice the speed of light, $\boxed{2c}$.

REFLECT

The observer on Earth will still see each beam move at c relative to the source, so we haven't broken any physical laws.

25.93

SET UP

We can derive an expression for the relativistic kinetic energy of a particle from the fact that total relativistic energy is not only equal to $E = mc^2$ but also that it is equal to the sum of the relativistic kinetic energy and the rest energy. Once we have an expression for the relativistic kinetic energy, we can calculate both the relativistic and Newtonian kinetic energies for a rocket ($m_0 = 1000$ kg) traveling at a speed $V = 0.90c$. The percent error if we use the Newtonian formula is given by % error $= \dfrac{K_{\text{Relativistic}} - K_{\text{Newtonian}}}{K_{\text{Relativistic}}} \times 100\%$. If the percent error is positive, that means the Newtonian formula underestimates the actual kinetic energy.

SOLVE

Part a)

$$E_{\text{total}} = K + E_0$$

$$K = E_{\text{total}} - E_0 = mc^2 - m_0c^2 = \gamma m_0 c^2 - m_0 c^2 = \boxed{(\gamma - 1)m_0c^2}$$

Part b)
Relativistic kinetic energy:

$$K_{\text{Relativistic}} = (\gamma - 1)m_0c^2 = \left(\frac{1}{\sqrt{1 - \left(\dfrac{V}{c}\right)^2}} - 1\right)m_0c^2$$

$$= \left(\frac{1}{\sqrt{1 - \left(\dfrac{0.90c}{c}\right)^2}} - 1\right)(1000\text{ kg})\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)^2 = \boxed{1.1647 \times 10^{20}\text{ J}}$$

Newtonian kinetic energy:

$$K_{\text{Newtonian}} = \frac{1}{2}m_0v^2 = \frac{1}{2}(1000\text{ kg})(0.90)^2\left(3.00 \times 10^8\frac{\text{m}}{\text{s}}\right)^2 = \boxed{3.645 \times 10^{19}\text{ J}}$$

Part c)

$$\%\text{ error} = \frac{K_{\text{Relativistic}} - K_{\text{Newtonian}}}{K_{\text{Relativistic}}} \times 100\%$$

$$= \frac{(1.1647 \times 10^{20}\text{ J}) - (3.645 \times 10^{19}\text{ J})}{1.1647 \times 10^{20}\text{ J}} \times 100 = \boxed{69}$$

Part d) The $\boxed{\text{Newtonian formula underestimates}}$ the kinetic energy.

REFLECT

The Newtonian formula for the kinetic energy is not valid for speeds approaching the speed of light.

25.94

SET UP

A kilogram of TNT yields an energy of 4.2×10^6 J. We can use this as a conversion factor along with $E_0 = m_0c^2$ to determine the rest mass required to create an explosion equivalent to 1.8×10^9 kg of TNT.

SOLVE

$$E_0 = m_0c^2$$

$$m_0 = \frac{E_0}{c^2} = \frac{\left(1.8 \times 10^9\text{ kg TNT} \times \frac{4.2 \times 10^6\text{ J}}{1\text{ kg TNT}}\right)}{\left(3.00 \times 10^8\frac{\text{m}}{\text{s}}\right)^2} = \boxed{8.4 \times 10^{-2}\text{ kg} = 84\text{ g}}$$

REFLECT

This example illustrates the sheer energies that exist in the creation of matter.

25.95

SET UP

A 25-year-old captain pilots a spaceship to a planet that is $\Delta x = 42$ ly from Earth. The captain needs to be no more than 60 years old when she arrives at the planet, which means the proper time interval, as observed by the spaceship, is $\Delta t_{\text{proper}} = 35$ y. The speed of the spaceship relative to Earth v is equal to Δx divided by the time interval as observed by Earth Δt, where $\Delta t = \gamma\Delta t_{\text{proper}}$. Using the definition of relativistic gamma, $\gamma = \dfrac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$, we can solve for v. In order to make the math easier, we'll use the fact that 1 ly = (1 y)c. The captain sends a radio signal back to Earth as soon as it reaches the planet. The total time after the launch when the signal arrives is equal to the time it takes the ship to reach the planet plus the time

it takes the signal to travel back to Earth. We can relate these times to the distance between Earth and the planet as well as the speeds of the spaceship and the speed of light.

SOLVE

Part a)

$$\Delta t = \gamma \Delta t_{\text{proper}}$$

$$v = \frac{\Delta x}{\Delta t} = \frac{\Delta x}{\gamma \Delta t_{\text{proper}}} = \frac{\Delta x}{\Delta t_{\text{proper}}}\sqrt{1 - \left(\frac{v}{c}\right)^2}$$

$$v^2 = \left(\frac{\Delta x}{\Delta t_{\text{proper}}}\right)^2 - \left(\frac{\Delta x}{\Delta t_{\text{proper}}}\right)^2\left(\frac{v}{c}\right)^2$$

$$v = \frac{\Delta x}{\Delta t_{\text{proper}}\sqrt{1 + \left(\frac{\Delta x}{c\Delta t_{\text{proper}}}\right)^2}} = \frac{c\Delta x}{\sqrt{(c\Delta t_{\text{proper}})^2 + (\Delta x)^2}} = \frac{c(42\text{ ly})}{\sqrt{(c(35\text{ y}))^2 + (42\text{ ly})^2}}$$

$$= \frac{c((42\text{ y})c)}{\sqrt{(c(35\text{ y}))^2 + ((42\text{ y})c)^2}} = \frac{c(42\text{ y})}{\sqrt{(35\text{ y})^2 + (42\text{ y})^2}} = \boxed{0.77c}$$

Part b)

$$\Delta t_{\text{total}} = \Delta t_{\text{spaceship}} + \Delta t_{\text{signal}} = \frac{\Delta x}{v} + \frac{\Delta x}{c} = \Delta x\left(\frac{1}{v} + \frac{1}{c}\right) = \Delta x\left(\frac{1}{0.77c} + \frac{1}{c}\right) = \frac{\Delta x}{c}\left(\frac{1}{0.77} + 1\right)$$

$$= \frac{(42\text{ ly})}{c}(2.30) = \frac{(42\text{ y})c}{c}(2.30) = \boxed{96.7\text{ y}}$$

REFLECT

A distance of 42 ly is around 4×10^{17} m!

25.96

SET UP

A cylindrical piston in a spaceship traveling at a speed $v = 0.50c$ is oriented parallel to the ship's motion relative to Earth. The piston is 50.0 cm long and 4.50 cm in diameter and contains $n = 1.25$ mol of an ideal gas at a pressure and temperature of $P = 2.20$ atm and $T = 298$ K, respectively. The particle density $\left(\frac{n}{V}\right)$ of the gas in the cylinder, as measured by a scientist in the ship, is given directly by the ideal gas law, $PV = nRT$. A scientist on Earth will observe a Lorentz contraction along the length of the cylinder, thus decreasing the volume by a factor of $\frac{1}{\gamma}$.

SOLVE

Part a)

$$PV = nRT$$

$$\left(\frac{n}{V}\right)_{\text{spaceship}} = \frac{P}{RT} = \frac{\left(2.20\text{ atm} \times \dfrac{1.01 \times 10^5\text{ Pa}}{1\text{ atm}}\right)}{\left(8.31\dfrac{\text{J}}{\text{mol}\cdot\text{K}}\right)(298\text{ K})}$$

$$= 89.7\frac{\text{mol}}{\text{m}^3} \times \frac{6.02 \times 10^{23}\text{ molecules}}{1\text{ mol}} = \boxed{5.40 \times 10^{25}\frac{\text{molecules}}{\text{m}^3}}$$

Part b)
Gamma:

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{1}{\sqrt{1 - \left(\frac{0.50c}{c}\right)^2}} = 1.15$$

Particle density:

$$\left(\frac{n}{V}\right)_{\text{Earth}} = \left(\frac{n}{\left(\frac{V}{\gamma}\right)}\right)_{\text{spaceship}} = \gamma\left(\frac{n}{V}\right)_{\text{spaceship}}$$

$$= 1.15\left(5.40 \times 10^{25}\frac{\text{molecules}}{\text{m}^3}\right) = \boxed{6.24 \times 10^{25}\frac{\text{molecules}}{\text{m}^3}}$$

REFLECT

The apparent density increase can be reconciled. The gas constant R and the temperature T should be the same for both observers, so the difference must come from the pressure P. The pressure is larger by a factor of γ because the surface area of the inside of the cylinder has decreased by that same factor due to the Lorentz contraction.

25.97

SET UP

A metric tensor is written in the form $ds^2 = g_{xx}dx^2 + g_{xy}dxdy + g_{xz}dxdz + \ldots + g_{zy}dzdy + g_{zz}dz^2$. We can compare this general form with Einstein's theories of relativity as stated in a spherical coordinate system, $c^2d\tau^2 = c^2dt^2 - dr^2 - r^2d\theta^2 - r^2sin^2(\theta)d\varphi^2$, to determine the values for the 16 tensor elements.

SOLVE

$$c^2d\tau^2 = c^2dt^2 - dr^2 - r^2d\theta^2 - r^2sin^2(\theta)d\varphi^2$$

The only terms that are nonzero are the ones with subscripts corresponding to the differentials in the above equation:

$$g_{tt} = c^2$$
$$g_{rr} = 1$$
$$g_{\theta\theta} = r^2$$
$$g_{\varphi\varphi} = r^2 sin^2(\theta)$$
$$g_{\text{tr}} = g_{\text{t}\theta} = g_{\text{t}\varphi} = g_{\text{rt}} = g_{\text{r}\theta} = g_{\text{r}\varphi} = g_{\theta\text{t}} = g_{\theta\text{r}} = g_{\theta\varphi} = g_{\varphi\text{t}} = g_{\varphi\text{r}} = g_{\varphi\theta} = 0$$

REFLECT

Matrices are a simple mathematical way of writing down equations and operations.

25.98

SET UP

We can derive the expression for the relativistic kinetic energy, $K = (\gamma - 1)m_0c^2$, starting from the expression for the work–energy theorem for an object with rest mass m_0 being acted upon by a net force F in the x direction. To convert the integral from one with respect to x to one with respect to speed v, we need to invoke the definition of force $\left(F = \frac{dp}{dt}\right)$ and then multiply by $\left(\frac{dv}{dv}\right)$. We will assume the initial speed of the object is zero.

SOLVE

$$\Delta K = W = \int F dx$$

$$\Delta K = K_\text{f} - K_0 = K_\text{f} - 0 = \int\left(\frac{dp}{dt}\right)dx = \int_0^v\left(\frac{dp}{dv}\right)\left(\frac{dx}{dt}\right)dv = \int_0^v\left(\frac{dp}{dv}\right)v dv = \int_0^v\left(\frac{d}{dv}[\gamma m_0 v]\right)v dv$$

$$= \int_0^v\left(m_0 v\frac{d\gamma}{dv} + \gamma m_0\right)v dv = m_0\int_0^v\left(v\left(-\frac{\left(-\frac{2v}{c^2}\right)}{2\left(1-\left(\frac{v}{c}\right)^2\right)^{\frac{3}{2}}}\right) + \frac{1}{\left(1-\left(\frac{v}{c}\right)^2\right)^{\frac{1}{2}}}\right)v dv$$

$$= m_0\int_0^v\left(\frac{\left(\frac{v^2}{c^2}\right) + \left(1-\left(\frac{v}{c}\right)^2\right)}{\left(1-\left(\frac{v}{c}\right)^2\right)^{\frac{3}{2}}}\right)v dv = m_0\int_0^v\left(\frac{1}{\left(1-\left(\frac{v}{c}\right)^2\right)^{\frac{3}{2}}}\right)v dv$$

$$= m_0\left[\frac{c^2}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\right]_0^v = m_0c^2\left[\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} - 1\right] = \boxed{(\gamma - 1)m_0c^2}$$

REFLECT

The total relativistic energy is equal to the sum of the rest energy and the relativistic kinetic energy.

Chapter 26
Modern and Atomic Physics

Conceptual Questions

26.1 The notion that the amount of energy that was flowing out of a hot oven was approaching infinity as the wavelength decreased was extremely perplexing, to say the least! Physics has made many great discoveries of both phenomena and ideas, but we are still very limited when our theories predict that an infinite amount of energy will emerge from an otherwise finite system. It was catastrophic and not easily "swept under the rug"!

26.2 Intensity is proportional to the fourth power of the temperature (Stefan–Boltzmann law), so a small increase in temperature produces a large increase in radiated power. The spectrum is shifted toward shorter wavelengths (higher frequencies) approximately in proportion to the change in absolute temperature.

26.3 The electron in the Bohr atom would be constantly accelerating, and so it would be constantly radiating. As the electron radiates, it would lose energy and slowly spiral into the nucleus.

26.4 Yes; electric charge is the most obvious example, but all the mechanical properties of standing waves on a string would qualify, too.

26.5 The shortest wavelength is the wavelength that is emitted when a free electron is captured into the lowest energy state, which is 91.2 nm.

26.6 If the energy from electromagnetic radiation (that is, light) continuously varied from zero to infinity, there would be a gradual and smooth transition between the absence of the emitted photoelectrons and their eventual presence. The cutoff frequency implies that there is a discrete "moment" at which the light becomes energetic enough to knock the electrons from the metal; this leads us to accept that light is more like a particle.

26.7 Part a) As long as the radiation has a frequency above the cutoff value, the greater the intensity and the larger the flux of photoelectrons will be that flow off of the metal plate. If the frequency of the light is below the cutoff, it does not matter how much the intensity is increased. No photoelectrons will be emitted.

Part b) If the wavelength is increased, the energy of the radiation will decrease. If it slips below the work function of the metal plate, the photoelectric effect will cease.

Part c) If the work function of the metal is increased, it will eventually grow past the energy of the incident photons and the effect will cease.

26.8 Yes, it is possible for photoelectrons to be emitted from a metal at relativistic speeds, as long as the photon energy is sufficiently high.

26.9 The exact amount of energy required to free an electron from a surface depends on a variety of factors, such as the distance the electron is from the surface and the electron's motion at the moment it is struck by a photon. The kinetic energy of any electron is the difference between the energy of a photon and the varying energy required to liberate the electron.

26.10 Increasing the intensity means delivering photons to the surface of the metal at a greater rate; correspondingly, the photocurrent—the rate at which the electrons are emitted—increases.

26.11 Visible light can Compton scatter. However, because the Compton wavelength (0.0024 nm) is so small compared to the wavelength of visible light (400−750 nm), the change in the wavelength of a visible photon would be negligibly small.

26.12 The Compton wavelength for electrons is about 1800 times larger than the Compton wavelength for protons. So, if you use electrons, you will not have to produce electromagnetic radiation with such high frequency, and it will be easier to demonstrate the particle nature of light.

26.13 The electron has the longer wavelength.

26.14 Since the electron and the photon have the same total energy, their wavelengths will be identical.

26.15 Hydrogen is the first, most elemental, simplest element in the periodic table. Any cogent theory of atoms had better describe hydrogen first and then work its way up to the larger, more complicated elements. In addition, because hydrogen only has one electron, it presents no complications to the orbit of an electron due to the interaction with other electrons.

26.16 Just as Kepler's three laws of planetary motion are still taught, there is much about our world that can be explained with this approximate description. Also it is much easier to visualize and far easier to relate to mechanical models and analogues. Bohr's model is the first step toward understanding the more complete and accurate theory of quantum mechanics. However, the four quantum numbers that quantum theory introduced to ultimately describe all the atoms in the periodic table still use Bohr's model as the primary description of the energy levels.

26.17 Because the rest mass of a macroscopic object is so large, the de Broglie wavelength is too small to observe. The wavelength of everyday objects is many orders of magnitude less than the radius of an atom. This is far too small for us to observe.

26.18 The de Broglie wavelength of a particle is inversely proportional to its kinetic energy. Therefore, the de Broglie wavelength decreases as the kinetic energy increases.

Multiple-Choice Questions

26.19 **D** (absorbs all the energy that strikes it and emits all the energy it generates). A perfect absorber reflects no energy, so its surface appears black. It emits all of the energy as it reaches thermal equilibrium with its surroundings.

26.20 **D** (the temperature of the object). Thomas Wedgwood observed the color of the light emitted by a hot object depends only on the temperature of the object.

26.21 **A** (a photon of ultraviolet radiation). Out of all of the choices, ultraviolet light has the highest frequency and, thus, the highest energy.

26.22 **A** (be greater). A smaller wavelength corresponds to a larger photon energy. Since the work function of the metal is constant, the maximum kinetic energy of the ejected electrons will be larger.

26.23 **A** (the scattering angle). The change in the photon's wavelength is proportional to $(1 - \cos(\theta))$.

26.24 **E** (180°). The change in the photon's wavelength is proportional to $(1 - \cos(\theta))$, so the largest change occurs when $\cos(\theta) = -1$ or $\theta = 180°$.

26.25 **C** (decreases). The increase in wavelength in the Compton effect as a function of wavelength is $\Delta\lambda \propto (1 - \cos(\theta))$. The energy is inversely proportional to the wavelength.

Get Help: Picture It – Compton Scattering

26.26 **D** (the particle's momentum only). The de Broglie wavelength of a particle is given by Planck's constant divided by the magnitude of the particle's momentum.

26.27 **C** (is unique to that element). The emission spectrum of an atom can be used to identify which element it is.

Estimation/Numerical Questions

26.28 An atom is about 1 angstrom, or 0.1 nm, in diameter.

26.29 Your body radiates about 500−1000 W.

26.30 The frequency of an electron in the first Bohr orbit is given by $f_1 = \dfrac{v_1}{2\pi r_1} = \dfrac{\left(2.18 \times 10^6 \dfrac{\text{m}}{\text{s}}\right)}{2\pi(5.29 \times 10^{-11}\ \text{m})} = 7 \times 10^{15}$ Hz. Therefore, the number of revolutions in 10^{-8} s would be about 7×10^7 rev.

26.31 Around 0.5 eV to less than 10 eV.

26.32 Ionization energies are on the order of 1–10 eV. Nuclear binding energies are about one million times larger.

26.33 The electrostatic force is 10^{39} times greater than the gravitational force.

26.34 The mass of the proton is 1836 times larger than the mass of the electron.

26.35 An electron must travel at 7.3×10^{6} m/s for its de Broglie wavelength to be about the size of an atom (about 10^{-10} m).

Get Help: P'Cast 26.6 – A Slow Hummingbird

26.36

	$\Delta I/\Delta\lambda$ (W/nm^3)	$\Delta I/\Delta\lambda$ (W/nm^3)	$\Delta I/\Delta\lambda$ (W/nm^3)	$\Delta I/\Delta\lambda$ (W/nm^3)
Wavelength (nm)	T = 1000 K	T = 1200 K	T = 1300 K	T = 1400 K
100	9.82E–44	2.65E–33	2.72E–29	7.46E–26
1000	2.07E+08	2.28E+09	5.75E+09	1.27E+10
1500	3.32E+09	1.65E+10	3.05E+10	5.17E+10
2000	8.71E+09	2.90E+10	4.61E+10	6.86E+10
2500	1.21E+10	3.18E+10	4.61E+10	6.36E+10
3000	1.28E+10	2.87E+10	3.93E+10	5.16E+10
3500	1.18E+10	2.39E+10	3.14E+10	3.98E+10
4000	1.03E+10	1.91E+10	2.44E+10	3.02E+10
4500	8.61E+09	1.51E+10	1.89E+10	2.30E+10
5000	7.12E+09	1.20E+10	1.47E+10	1.76E+10
5500	5.85E+09	9.46E+09	1.15E+10	1.36E+10
6000	4.80E+09	7.53E+09	9.02E+09	1.06E+10
6500	3.95E+09	6.05E+09	7.18E+09	8.35E+09
7000	3.26E+09	4.89E+09	5.76E+09	6.66E+09
7500	2.71E+09	3.99E+09	4.67E+09	5.37E+09
8000	2.26E+09	3.28E+09	3.82E+09	4.37E+09
8500	1.90E+09	2.72E+09	3.15E+09	3.59E+09
9000	1.60E+09	2.27E+09	2.62E+09	2.97E+09
9500	1.36E+09	1.91E+09	2.19E+09	2.48E+09
10,000	1.16E+09	1.61E+09	1.85E+09	2.08E+09
11,000	8.60E+08	1.18E+09	1.34E+09	1.50E+09
12,000	6.49E+08	8.76E+08	9.92E+08	1.11E+09
13,000	4.98E+08	6.65E+08	7.50E+08	8.36E+08
14,000	3.88E+08	5.13E+08	5.77E+08	6.42E+08
15,000	3.06E+08	4.02E+08	4.51E+08	5.01E+08
16,000	2.45E+08	3.20E+08	3.58E+08	3.96E+08

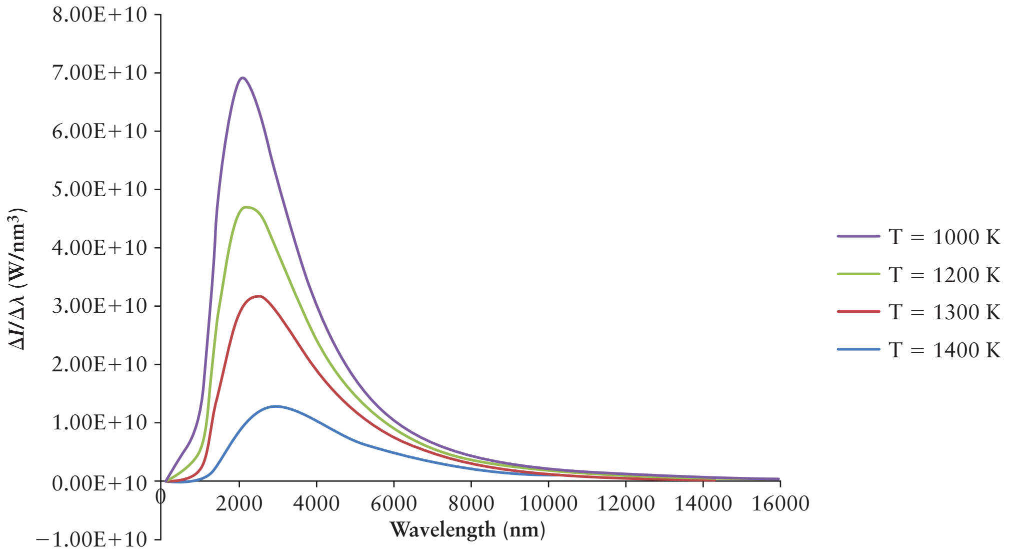

Figure 26-1 Problem 36

26.37

The line of best fit is $K = (4.26 \times 10^{-15}\ \text{eV} \cdot \text{s})f - (3.50 \times 10^{-19}\ \text{eV})$. Therefore, our experimental value for Planck's constant is $h = 4.26 \times 10^{-15}\ \text{eV} \cdot \text{s} = 6.82 \times 10^{-34}\ \text{J} \cdot \text{s}$. The accepted value is $h = 4.14 \times 10^{-15}\ \text{eV} \cdot \text{s} = 6.626 \times 10^{-34}\ \text{J} \cdot \text{s}$.

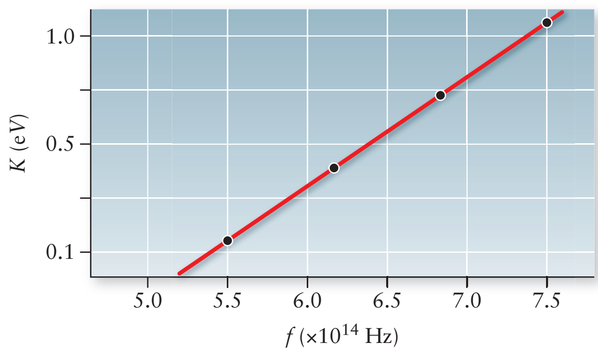

Figure 26-2 Problem 37

Problems

26.38

SET UP

A blackbody at $T_1 = 400$ K radiates just enough heat to boil a sample of water in $t_1 = 15$ min. The intensity of the blackbody radiation is inversely proportional to the time required to boil the water but directly proportional to the temperature to the fourth power. We can set up a ratio between the time required to boil the water and the temperature of the blackbody to determine how long it would take a blackbody at $T_2 = 500$ K to boil the water.

SOLVE

$$\frac{t_2}{t_1} = \frac{T_1^4}{T_2^4}$$

$$t_2 = \left(\frac{T_1}{T_2}\right)^4 t_1 = \left(\frac{400\ \text{K}}{500\ \text{K}}\right)^4 (15\ \text{min}) = \boxed{6.1\ \text{min}}$$

REFLECT
A blackbody with a higher temperature should boil water faster.

26.39

SET UP
Wien's displacement law, $\lambda_{max} = \frac{0.290\text{ K}\cdot\text{cm}}{T}$, tells us the wavelength at which a blackbody emits the maximum power. We can directly plug in $T = 673$ K to find λ_{max} for a blackbody at 400 degrees C.

SOLVE
$$\lambda_{max} = \frac{0.290\text{ K}\cdot\text{cm}}{T} = \frac{0.290\text{ K}\cdot\text{cm}}{673\text{ K}} = \boxed{4.31\times10^{-4}\text{ cm} = 4310\text{ nm}}$$

REFLECT
This is infrared radiation, which makes sense for a blackbody with a temperature under 1000 K.

26.40

SET UP
A metal inert gas (MIG) welder and a tungsten inert gas (TIG) welder can be treated as blackbodies at temperatures of $T_{MIG} = 4000$ K and $T_{TIG} = 6000$ K, respectively. We can use Wien's displacement law, $\lambda_{max} = \frac{0.290\text{ K}\cdot\text{cm}}{T}$, to calculate the wavelength for peak radiation.

SOLVE
MIG:
$$\lambda_{max} = \frac{0.290\text{ K}\cdot\text{cm}}{T_{MIG}} = \frac{0.290\text{ K}\cdot\text{cm}}{4000\text{ K}} = \boxed{7.25\times10^{-5}\text{ cm} = 725\text{ nm}}$$
TIG:
$$\lambda_{max} = \frac{0.290\text{ K}\cdot\text{cm}}{T_{TIG}} = \frac{0.290\text{ K}\cdot\text{cm}}{6000\text{ K}} = \boxed{4.83\times10^{-5}\text{ cm} = 483\text{ nm}}$$

REFLECT
Both wavelengths are in the visible spectrum, but the MIG radiation is red, whereas the TIG radiation is blue.

26.41

SET UP
A blackbody at $T = 300$ K radiates heat into its immediate surroundings. Wien's displacement law, $\lambda_{max} = \frac{0.290\text{ K}\cdot\text{cm}}{T}$, tells us the wavelength at which the maximum intensity is emitted.

SOLVE
$$\lambda_{max} = \frac{0.290\text{ K}\cdot\text{cm}}{300\text{ K}} = \boxed{9.67\times10^{-4}\text{ cm} = 9670\text{ nm}}$$

REFLECT

This corresponds to infrared light, which makes sense for a blackbody at a relatively low temperature.

26.42

SET UP

We can use Wien's displacement law, $\lambda_{\text{max}} = \dfrac{0.290\ \text{K}\cdot\text{cm}}{T}$, to calculate the wavelength for peak radiation for the polar ice cap ($T_{\text{ice}} = 233$ K), burning ethyl alcohol ($T_{\text{ethanol}} = 638$ K), and deep space ($T_{\text{space}} = 2.7$ K).

SOLVE

Part a)

$$\lambda_{\text{max}} = \frac{0.290\ \text{K}\cdot\text{cm}}{T_{\text{ice}}} = \frac{0.290\ \text{K}\cdot\text{cm}}{233\ \text{K}} = \boxed{1.24 \times 10^{-3}\ \text{cm}}$$

Part b)

$$\lambda_{\text{max}} = \frac{0.290\ \text{K}\cdot\text{cm}}{T_{\text{ethanol}}} = \frac{0.290\ \text{K}\cdot\text{cm}}{638\ \text{K}} = \boxed{4.55 \times 10^{-4}\ \text{cm}}$$

Part c)

$$\lambda_{\text{max}} = \frac{0.290\ \text{K}\cdot\text{cm}}{T_{\text{space}}} = \frac{0.290\ \text{K}\cdot\text{cm}}{2.7\ \text{K}} = \boxed{0.11\ \text{cm}}$$

REFLECT

The peak wavelength should decrease as the temperature of the blackbody increases since energy is inversely proportional to the wavelength.

26.43

SET UP

The Sun emits its maximum power at a wavelength of $\lambda_{\text{max}} = 4.75 \times 10^{-5}$ cm. If we assume it acts as a blackbody, we can use Wien's displacement law, $\lambda_{\text{max}} = \dfrac{0.290\ \text{K}\cdot\text{cm}}{T}$, to calculate the temperature of the outer layer of the Sun.

SOLVE

$$\lambda_{\text{max}} = \frac{0.290\ \text{K}\cdot\text{cm}}{T}$$

$$T = \frac{0.290\ \text{K}\cdot\text{cm}}{\lambda_{\text{max}}} = \frac{0.290\ \text{K}\cdot\text{cm}}{4.75 \times 10^{-5}\ \text{cm}} = \boxed{6110\ \text{K}}$$

REFLECT

This crude calculation is not far off from the actual surface temperature of the Sun, which is around 5800 K.

26.44

SET UP

If we treat the human body as a blackbody radiating at a temperature of $T = 304\ \text{K}$, we can use Wien's displacement law, $\lambda_{\text{max}} = \dfrac{0.290\ \text{K}\cdot\text{cm}}{T}$, to calculate the wavelength of the peak emitted radiation.

SOLVE

Part a)

$$\lambda_{\text{max}} = \frac{0.290\ \text{K}\cdot\text{cm}}{T} = \frac{0.290\ \text{K}\cdot\text{cm}}{304\ \text{K}} = \boxed{9.54 \times 10^{-4}\ \text{cm}}$$

Part b) This corresponds to $\boxed{\text{infrared light}}$.

REFLECT

Emitted radiation in the infrared is reasonable for a blackbody under 1000 K.

26.45

SET UP

We are asked to calculate the range of frequencies and energies in the visible spectrum of light, which is approximately from $\lambda_{\text{violet}} = 380 \times 10^{-9}\ \text{m}$ to $\lambda_{\text{red}} = 750 \times 10^{-9}\ \text{m}$. The frequency of a photon is inversely related to its wavelength, $f = \dfrac{c}{\lambda}$; the energy is directly proportional to the frequency, $E = hf$.

SOLVE

Frequencies:

$$f_{\text{violet}} = \frac{c}{\lambda_{\text{violet}}} = \frac{\left(3.00 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{380 \times 10^{-9}\ \text{m}} = \boxed{7.89 \times 10^{14}\ \text{Hz}}$$

$$f_{\text{red}} = \frac{c}{\lambda_{\text{red}}} = \frac{\left(3.00 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{750 \times 10^{-9}\ \text{m}} = \boxed{4.00 \times 10^{14}\ \text{Hz}}$$

Energies:

$$E_{\text{violet}} = hf_{\text{violet}} = (6.63 \times 10^{-34}\ \text{J}\cdot\text{s})(7.89 \times 10^{14}\ \text{Hz}) = 5.23 \times 10^{-19}\ \text{J}$$

$$5.23 \times 10^{-19}\ \text{J} \times \frac{1\ \text{eV}}{1.6 \times 10^{-19}\ \text{J}} = \boxed{3.27\ \text{eV}}$$

$$E_{\text{red}} = hf_{\text{red}} = (6.63 \times 10^{-34}\ \text{J}\cdot\text{s})(4.00 \times 10^{14}\ \text{Hz}) = 2.65 \times 10^{-19}\ \text{J}$$

$$2.65 \times 10^{-19}\ \text{J} \times \frac{1\ \text{eV}}{1.6 \times 10^{-19}\ \text{J}} = \boxed{1.66\ \text{eV}}$$

REFLECT
Keep in mind that the wavelength is inversely proportional to the energy; red light has a longer wavelength than violet light, but is less energetic. We could have also solved this in eV directly using $h = 4.1357 \times 10^{-15}\ \text{eV} \cdot \text{s}$.

26.46

SET UP
As we saw in Chapter 22, the energy of a photon ($f = 2000$ Hz) is given by $E = hf$.

SOLVE

$$E = hf = (6.63 \times 10^{-34}\ \text{J} \cdot \text{s})(2000\ \text{Hz}) = \boxed{1.33 \times 10^{-30}\ \text{J}}$$

REFLECT
This corresponds to 8.28×10^{-12} eV.

26.47

SET UP
The energy of a photon with a wavelength of $\lambda = 0.200 \times 10^{-9}$ m is given by $E = \dfrac{hc}{\lambda}$. The conversion between joules and electron volts is $1\ \text{eV} = 1.60 \times 10^{-19}$ J.

SOLVE

$$E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}\ \text{J} \cdot \text{s})\left(3.00 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{0.200 \times 10^{-9}\ \text{m}} = \boxed{9.93 \times 10^{-16}\ \text{J}} \times \frac{1\ \text{eV}}{1.60 \times 10^{-19}\ \text{J}} = \boxed{6200\ \text{eV}}$$

REFLECT
This corresponds to x-ray radiation.

26.48

SET UP
The wavelength and frequency of a 2.48-eV photon are given by $E = \dfrac{hc}{\lambda}$ and $E = hf$, respectively. The conversion between joules and electron volts is $1\ \text{eV} = 1.60 \times 10^{-19}$ J.

SOLVE
Conversion to joules:

$$2.48\ \text{eV} \times \frac{1.60 \times 10^{-19}\ \text{J}}{1\ \text{eV}} = 3.97 \times 10^{-19}\ \text{J}$$

Wavelength:

$$E = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34}\ \text{J} \cdot \text{s})\left(3.00 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{3.97 \times 10^{-19}\ \text{J}} = \boxed{5.00 \times 10^{-7}\ \text{m} = 500\ \text{nm}}$$

Frequency:

$$E = hf$$

$$f = \frac{E}{h} = \frac{3.97 \times 10^{-19}\ \text{J}}{6.63 \times 10^{-34}\ \text{J} \cdot \text{s}} = \boxed{6.00 \times 10^{14}\ \text{Hz}}$$

REFLECT

This corresponds to blue-green visible light.

26.49

SET UP

To find the total energy absorbed by the eye when 100 photons hit the photoreceptors at the back of the eye, each with a wavelength of $\lambda = 550 \times 10^{-9}$ m, we first need to use $E = \frac{hc}{\lambda}$ to find the energy of one photon of this wavelength, $E_{1\ \text{photon}}$. The total energy of 100 photons is $E_{\text{total}} = 100E_{1\ \text{photon}}$. The conversion between joules and electron volts is $1\ \text{eV} = 1.60 \times 10^{-19}$ J.

SOLVE

Energy of one photon:

$$E_{1\ \text{photon}} = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}\ \text{J} \cdot \text{s})\left(3.00 \times 10^{8} \frac{\text{m}}{\text{s}}\right)}{550 \times 10^{-9}\ \text{m}} = 3.61 \times 10^{-19}\ \text{J}$$

Energy of 100 photons:

$$E_{\text{total}} = 100E_{1\ \text{photon}} = 100(3.61 \times 10^{-19}\ \text{J}) = \boxed{3.61 \times 10^{-17}\ \text{J}} \times \frac{1\ \text{eV}}{1.60 \times 10^{-19}\ \text{J}} = \boxed{225\ \text{eV}}$$

REFLECT

The total energy of a bunch of photons is just the sum of component energies.

26.50

SET UP

The threshold wavelength for silver is $\lambda_0 = 262 \times 10^{-9}$ m. The work function for silver is given by $\Phi_0 = \frac{hc}{\lambda_0}$. Once we know the work function, we can calculate the stopping potential V_0 for light incident on a silver sample with a wavelength of $\lambda = 222 \times 10^{-9}$ m through $V_0 = \frac{hc}{e\lambda} - \frac{\Phi_0}{e}$.

SOLVE

Part a)

$$\Phi_0 = \frac{hc}{\lambda_0} = \frac{(6.63 \times 10^{-34}\ \text{J} \cdot \text{s})\left(3.00 \times 10^{8} \frac{\text{m}}{\text{s}}\right)}{262 \times 10^{-9}\ \text{m}} = \boxed{7.59 \times 10^{-19}\ \text{J}}$$

Part b)

$$V_0 = \frac{hc}{e\lambda} - \frac{\Phi_0}{e} = \frac{(6.63 \times 10^{-34}\ \mathrm{J \cdot s})\left(3.00 \times 10^8 \frac{\mathrm{m}}{\mathrm{s}}\right)}{(1.60 \times 10^{-19}\ \mathrm{C})(222 \times 10^{-9}\ \mathrm{m})} - \frac{7.59 \times 10^{-19}\ \mathrm{J}}{1.60 \times 10^{-19}\ \mathrm{C}} = \boxed{0.86\ \mathrm{V}}$$

REFLECT

We would expect the stopping potential to be rather small since the incidence light has a wavelength that is similar to the threshold wavelength.

26.51

SET UP

Light ($\lambda = 195 \times 10^{-9}$ m) strikes a metal surface and photoelectrons are produced moving with a maximum speed of $v_{\max} = 0.004c$. The expression relating the maximum kinetic energy of the photoelectrons to the work function is $K_{\max} = hf - \Phi_0$. The threshold wavelength for the metal corresponds to the wavelength absorbed when the kinetic energy of the photoelectrons is equal to zero. Once we know the numerical value of the work function, we can search the Internet for a list of work functions to determine which metal our sample may be.

SOLVE

Part a)

$$K_{\max} = hf - \Phi_0$$

$$\Phi_0 = hf - K_{\max} = hf - \frac{1}{2}m_e v_{\max}^2 = \frac{hc}{\lambda} - \frac{1}{2}m_e(0.004c)^2$$

$$= \frac{(6.63 \times 10^{-34}\ \mathrm{J \cdot s})\left(3.00 \times 10^8 \frac{\mathrm{m}}{\mathrm{s}}\right)}{195 \times 10^{-9}\ \mathrm{m}} - \frac{1}{2}(9.11 \times 10^{-31}\ \mathrm{kg})(0.004)^2\left(3.00 \times 10^8 \frac{\mathrm{m}}{\mathrm{s}}\right)^2$$

$$= 3.63 \times 10^{-19}\ \mathrm{J} \times \frac{1\ \mathrm{eV}}{1.6 \times 10^{-19}\ \mathrm{J}} = \boxed{2.27\ \mathrm{eV}}$$

Part b)

$$K_{\max} = 0 = hf_{\min} - \Phi_0 = \frac{hc}{\lambda_{\max}} - \Phi_0$$

$$\lambda_{\max} = \frac{hc}{\Phi_0} = \frac{(6.63 \times 10^{-34}\ \mathrm{J \cdot s})\left(3.00 \times 10^8 \frac{\mathrm{m}}{\mathrm{s}}\right)}{3.63 \times 10^{-19}\ \mathrm{J}} = \boxed{5.46 \times 10^{-7}\ \mathrm{m} = 546\ \mathrm{nm}}$$

Part c) The metal could be either sodium ($\Phi_0 = 2.28$ eV) or potassium ($\Phi_0 = 2.3$ eV).

REFLECT

We can't know the exact type of metal without more information and a more precise measurement.

Get Help: P'Cast 26.2 – Photoelectric Effect with Cesium

26.52

SET UP

The work function of a given metal is $\Phi_0 = 6.53 \times 10^{-19}$ J. The minimum frequency of light required to eject electrons from this metal is given by $f_0 = \frac{\Phi_0}{h}$.

SOLVE

$$\Phi_0 = hf_0$$

$$f_0 = \frac{\Phi_0}{h} = \frac{6.53 \times 10^{-19}\ \text{J}}{6.63 \times 10^{-34}\ \text{J} \cdot \text{s}} = \boxed{9.86 \times 10^{14}\ \text{Hz}}$$

REFLECT

This is ultraviolet light.

26.53

SET UP

The work functions for aluminum, calcium, potassium, and cesium are 4.08 eV, 2.90 eV, 2.23 eV, and 2.10 eV, respectively. To determine which metals will emit photoelectrons if irradiated with visible light, we first need to determine the maximum possible energy of a visible photon. We will take a 380-nm violet photon to be the most energetic visible photon. If the energy of this photon is larger than the work function of the metal, electrons will be emitted.

SOLVE

Energy of 380-nm violet light:

$$E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}\ \text{J} \cdot \text{s})\left(3.00 \times 10^{8} \frac{\text{m}}{\text{s}}\right)}{380 \times 10^{-9}\ \text{m}} = 5.23 \times 10^{-19}\ \text{J} \times \frac{1\ \text{eV}}{1.60 \times 10^{-19}\ \text{J}} = 3.26\ \text{eV}$$

Electrons will be ejected if the work function of the metal is less than 3.26 eV. Therefore, photoelectrons will be emitted when $\boxed{\text{calcium, potassium, and cesium}}$ are illuminated with visible light.

REFLECT

The energy of the incident photon must be larger than the work function for electrons to be ejected from the metal.

26.54

SET UP

The current in a photoelectric effect experiment decreases to zero when the stopping potential is $V_0 = 2.27$ V. This means the maximum kinetic energy of the photoelectrons is equal to eV_0. Using this along with the definition of the kinetic energy, we can calculate the maximum speed of the emitted photoelectrons. To express our answer in terms of β, we need to divide this maximum speed by the speed of light c.

SOLVE

$$K_{max} = \frac{1}{2}m_e v_{max}^2 = eV_0$$

$$v_{max} = \sqrt{\frac{2eV_0}{m_e}} = \sqrt{\frac{2(1.60 \times 10^{-19}\ \text{C})(2.27\ \text{V})}{9.11 \times 10^{-31}\ \text{kg}}} = 8.93 \times 10^5 \frac{\text{m}}{\text{s}}$$

$$\beta = \frac{v_{max}}{c} = \frac{\left(8.93 \times 10^5\ \frac{\text{m}}{\text{s}}\right)}{\left(3.00 \times 10^8\ \frac{\text{m}}{\text{s}}\right)} = \boxed{0.00298}$$

REFLECT

The maximum speed of the electrons is about 0.3% of the speed of light, which seems reasonable.

26.55

SET UP

The momentum of a photon is equal to the energy of the photon divided by the speed of light. By using the expression for the energy in terms of the wavelength of the light, we can easily calculate the momenta of photons with wavelengths of 550×10^{-9} m and 0.0711×10^{-9} m.

SOLVE

Relating momentum to wavelength:

$$p = \frac{E}{c} = \left(\frac{hc}{\lambda}\right)\left(\frac{1}{c}\right) = \frac{h}{\lambda}$$

Part a)

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}\ \text{J} \cdot \text{s}}{550 \times 10^{-9}\ \text{m}} = \boxed{1.21 \times 10^{-27} \frac{\text{kg} \cdot \text{m}}{\text{s}}}$$

Part b)

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}\ \text{J} \cdot \text{s}}{0.0711 \times 10^{-9}\ \text{m}} = \boxed{9.32 \times 10^{-24} \frac{\text{kg} \cdot \text{m}}{\text{s}}}$$

REFLECT

We can also use the fact that $hc = 1240\ \text{eV} \cdot \text{nm}$ to represent the momentum in units of eV/c. Our answers in these units would be 2.25 eV/c and 17,400 eV/c, respectively.

Get Help: Picture It – Compton Scattering

26.56

SET UP

X-rays with wavelength $\lambda_i = 0.1250 \times 10^{-9}$ m are scattered off free electrons at an angle of $\theta = 30.00°$. The wavelength of the scattered radiation λ_f is given by $\Delta\lambda = \lambda_f - \lambda_i = \lambda_C(1 - \cos(\theta))$. After finding λ_f we can find the fractional wavelength change of the radiation, $\frac{\Delta\lambda}{\lambda_E} = \frac{\Delta\lambda}{\lambda_i}$.

SOLVE

Part a)

$$\Delta\lambda = \lambda_f - \lambda_i = \lambda_C(1 - \cos(\theta))$$

$$\lambda_f = \lambda_i + \lambda_C(1 - \cos(\theta)) = \lambda_i + \frac{h}{m_e c}(1 - \cos(\theta))$$

$$= (0.1250 \times 10^{-9}\ \text{m}) + \frac{6.63 \times 10^{-34}\ \text{J}\cdot\text{s}}{(9.11 \times 10^{-31}\ \text{kg})\left(3.00 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}(1 - \cos(30.00°))$$

$$= \boxed{0.1253 \times 10^{-9}\ \text{m} = 0.1253\ \text{nm}}$$

Part b)
Change in wavelength:

$$\Delta\lambda = \lambda_C(1 - \cos(\theta)) = \frac{h}{m_e c}(1 - \cos(\theta))$$

$$= \frac{6.63 \times 10^{-34}\ \text{J}\cdot\text{s}}{(9.11 \times 10^{-31}\ \text{kg})\left(3.00 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}(1 - \cos(30.00°)) = 3.25 \times 10^{-13}\ \text{m}$$

Fractional wavelength change:

$$\frac{\Delta\lambda}{\lambda_E} = \frac{\Delta\lambda}{\lambda_i} = \frac{3.25 \times 10^{-13}\ \text{m}}{0.1253 \times 10^{-9}\ \text{m}} = \boxed{0.00260}$$

REFLECT

The scattered wavelength only changes by about 0.3%, which is reasonable.

26.57

SET UP

A photon experiences a fractional wavelength change of $\frac{\Delta\lambda}{\lambda_E} = 0.0725$. We can calculate the angle of the scattered photons from the definition of the fractional wavelength change, $\frac{\Delta\lambda}{\lambda_E} = \frac{\Delta\lambda}{\lambda_i} = \frac{\lambda_C(1 - \cos(\theta))}{\lambda_i}$, given $\lambda_i = 0.00335 \times 10^{-9}$ m.

SOLVE

$$\frac{\Delta\lambda}{\lambda_E} = \frac{\Delta\lambda}{\lambda_i} = \frac{\lambda_C(1 - \cos(\theta))}{\lambda_i}$$

$$\cos(\theta) = 1 - \left(\frac{\Delta\lambda}{\lambda_E}\right)\left(\frac{\lambda_i}{\lambda_C}\right)$$

$$\theta = \arccos\left[1 - \left(\frac{\Delta\lambda}{\lambda_E}\right)\left(\frac{\lambda_i}{\lambda_C}\right)\right] = \arccos\left[1 - \left(\frac{\Delta\lambda}{\lambda_E}\right)\left(\frac{\lambda_i}{\left(\frac{h}{m_e c}\right)}\right)\right] = \arccos\left[1 - \left(\frac{\Delta\lambda}{\lambda_E}\right)\left(\frac{\lambda_i m_e c}{h}\right)\right]$$

$$= \arccos\left[1 - (0.0725)\frac{(0.00335 \times 10^{-9}\ \text{m})(9.11 \times 10^{-31}\ \text{kg})\left(3.00 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{6.63 \times 10^{-34}\ \text{J}\cdot\text{s}}\right]$$

$$= \boxed{25.9°}$$

REFLECT

A scattering angle of about 26 degrees is reasonable.

26.58

SET UP

Photons that have a wavelength of $\lambda_i = 0.00225$ nm are Compton scattered at an angle of 45 degrees. We can use $\Delta\lambda = \lambda_f - \lambda_i = \lambda_C(1 - \cos(\theta))$, where $\lambda_C = 0.00243$ nm, to calculate the wavelength of the scattered photons λ_f. The energy of the scattered photons is then $E_f = \frac{hc}{\lambda_f}$. The conversion between joules and electron volts is $1\ \text{eV} = 1.60 \times 10^{-19}$ J.

SOLVE

Wavelength of scattered photons:

$$\Delta\lambda = \lambda_f - \lambda_i = \lambda_C(1 - \cos(\theta))$$

$$\lambda_f = \lambda_i + \lambda_C(1 - \cos(\theta))$$

$$= (0.00225\ \text{nm}) + (0.00243\ \text{nm})(1 - \cos(45°)) = 0.00296\ \text{nm}$$

Energy of scattered photons:

$$E_f = \frac{hc}{\lambda_f} = \frac{(6.63 \times 10^{-34}\ \text{J}\cdot\text{s})\left(3.00 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{0.00296 \times 10^{-9}\ \text{m}}$$

$$= 6.72 \times 10^{-14}\ \text{J} \times \frac{1\ \text{eV}}{1.60 \times 10^{-19}\ \text{J}} = \boxed{4.20 \times 10^{5}\ \text{eV}}$$

REFLECT

The energy of the photons scattered at 45 degrees should be less than the incoming photons, so our answer makes sense.

26.59

SET UP

In order to find the Compton scattering angle that gives the maximum value for the shift in wavelength of a scattered photon, we need to differentiate the expression for the wavelength shift with respect to θ, set it equal to zero, and solve for θ.

SOLVE

$$\Delta\lambda = \lambda_C(1 - \cos(\theta))$$

$$\frac{d(\Delta\lambda)}{d\theta} = \frac{d}{d\theta}[\lambda_C(1 - \cos(\theta))] = 0$$

$$\lambda_C \sin(\theta) = 0$$

$$\sin(\theta) = 0$$

$$\theta = \arcsin(0) = n\pi$$

The maximum shift occurs for $\boxed{\theta = \pi = 180°}$.

REFLECT

The minimum shift occurs for $\theta = 0$.

26.60

SET UP

The Compton wavelength is given by $\lambda_C = \dfrac{h}{mc}$. We can use the mass of the electron ($m_e = 9.11 \times 10^{-31}$ kg), the mass of the proton ($m_p = 1.67 \times 10^{-27}$ kg), and the mass of the pi meson ($m_\pi = 140 \times 10^6 \dfrac{\text{eV}}{c^2}$) to determine the numerical value for the Compton wavelength in each case.

SOLVE

Part a)

$$\lambda_C = \frac{h}{m_e c} = \frac{6.63 \times 10^{-34}\ \text{J} \cdot \text{s}}{(9.11 \times 10^{-31}\ \text{kg})\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)} = \boxed{2.43 \times 10^{-12}\ \text{m}}$$

Part b)

$$\lambda_C = \frac{h}{m_p c} = \frac{6.63 \times 10^{-34}\ \text{J} \cdot \text{s}}{(1.67 \times 10^{-27}\ \text{kg})\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)} = \boxed{1.32 \times 10^{-15}\ \text{m}}$$

Part c)

$$\lambda_C = \frac{h}{m_\pi c} = \frac{6.63 \times 10^{-34}\ \text{J} \cdot \text{s}}{\left(\frac{140 \times 10^6\ \text{eV}}{c^2}\right)c} = \frac{6.63 \times 10^{-34}\ \text{J} \cdot \text{s}}{\left(\frac{140 \times 10^6\ \text{eV}}{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)} \times \frac{1.60 \times 10^{-19}\ \text{J}}{1\ \text{eV}}\right)} = \boxed{8.88 \times 10^{-15}\ \text{m}}$$

REFLECT

The Compton wavelength is inversely proportional to the mass of the particle.

26.61

SET UP

Photons with a wavelength of $\lambda_i = 0.1400$ nm are scattered at varying angles—0 degrees, 30 degrees, 45 degrees, 60 degrees, 90 degrees, and 180 degrees—off of carbon atoms. The shift in wavelength of the Compton scattered photons is given by $\Delta\lambda = \lambda_C(1 - \cos(\theta))$, where $\lambda_C = 0.00243$ nm; the wavelength of the scattered photons can be calculated from this relationship. The kinetic energy of the scattered electrons is equal to the difference between the energy of the initial photons and the energy of the scattered photons.

SOLVE

Wavelength of scattered photon:

$$\Delta\lambda = \lambda_f - \lambda_i = \lambda_C(1 - \cos(\theta))$$

$$\lambda_f = \lambda_i + \lambda_C(1 - \cos(\theta))$$

Kinetic energy of scattered electrons:

$$K = \frac{hc}{\lambda_i} - \frac{hc}{\lambda_f} = hc\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_f}\right)$$

Part a)

$$\lambda_f = \lambda_i + \lambda_C(1 - \cos(\theta)) = (0.1400\text{ nm}) + (0.00243\text{ nm})(1 - \cos(0°)) = \boxed{0.1400\text{ nm}}$$

$$K = hc\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_f}\right) = (1240\text{ eV}\cdot\text{nm})\left(\frac{1}{0.1400\text{ nm}} - \frac{1}{0.1400\text{ nm}}\right) = \boxed{0}$$

Part b)

$$\lambda_f = \lambda_i + \lambda_C(1 - \cos(\theta)) = (0.1400\text{ nm}) + (0.00243\text{ nm})(1 - \cos(30°)) = \boxed{0.14033\text{ nm}}$$

$$K = hc\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_f}\right) = (1240\text{ eV}\cdot\text{nm})\left(\frac{1}{0.1400\text{ nm}} - \frac{1}{0.14033\text{ nm}}\right) = \boxed{20.8\text{ eV}}$$

Part c)

$$\lambda_f = \lambda_i + \lambda_C(1 - \cos(\theta)) = (0.1400\text{ nm}) + (0.00243\text{ nm})(1 - \cos(45°)) = \boxed{0.14071\text{ nm}}$$

$$K = hc\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_f}\right) = (1240\text{ eV}\cdot\text{nm})\left(\frac{1}{0.1400\text{ nm}} - \frac{1}{0.14071\text{ nm}}\right) = \boxed{44.7\text{ eV}}$$

Part d)

$$\lambda_f = \lambda_i + \lambda_C(1 - \cos(\theta)) = (0.1400\text{ nm}) + (0.00243\text{ nm})(1 - \cos(60°)) = \boxed{0.14122\text{ nm}}$$

$$K = hc\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_f}\right) = (1240\text{ eV}\cdot\text{nm})\left(\frac{1}{0.1400\text{ nm}} - \frac{1}{0.14122\text{ nm}}\right) = \boxed{75.3\text{ eV}}$$

Part e)

$$\lambda_f = \lambda_i + \lambda_C(1 - \cos(\theta)) = (0.1400\text{ nm}) + (0.00243\text{ nm})(1 - \cos(90°)) = \boxed{0.14243\text{ nm}}$$

$$K = hc\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_f}\right) = (1240\ \text{eV}\cdot\text{nm})\left(\frac{1}{0.1400\ \text{nm}} - \frac{1}{0.14243\ \text{nm}}\right) = \boxed{151\ \text{eV}}$$

Part f)

$$\lambda_f = \lambda_i + \lambda_C(1-\cos(\theta)) = (0.1400\ \text{nm}) + (0.00243\ \text{nm})(1-\cos(180°)) = \boxed{0.14486\ \text{nm}}$$

$$K = hc\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_f}\right) = (1240\ \text{eV}\cdot\text{nm})\left(\frac{1}{0.1400\ \text{nm}} - \frac{1}{0.14486\ \text{nm}}\right) = \boxed{297\ \text{eV}}$$

REFLECT

Since the wavelength of a photon is inversely proportional to its energy, the maximum wavelength of the scattered photon and the maximum kinetic energy of the scattered electrons both occur when the scattering angle is 180 degrees.

Get Help: Picture It – Compton Scattering

26.62

SET UP

A photon ($\lambda_i = 0.075$ nm) Compton scatters off of a stationary electron. The maximum speed of the scattered electron occurs when the photon backscatters, that is, $\theta = \pi$. The shift in wavelength of the Compton scattered photons is given by $\Delta\lambda = \lambda_C(1-\cos(\theta))$, where $\lambda_C = 0.00243$ nm. The maximum speed of the scattered electron is related to its maximum kinetic energy, which is equal to the difference in the initial and final energies of the scattered photons.

SOLVE

Scattered wavelength:

$$\Delta\lambda = \lambda_f - \lambda_i = \lambda_C(1-\cos(\theta))$$

$$\lambda_f = \lambda_i + \lambda_C(1-\cos(\theta))$$

Maximum speed:

$$K_{max} = \frac{hc}{\lambda_i} - \frac{hc}{\lambda_f}$$

$$\frac{1}{2}m_e v_{max}^2 = \frac{hc}{\lambda_i} - \frac{hc}{\lambda_i + \lambda_C(1-\cos(\pi))}$$

$$(m_e c^2)\frac{v_{max}^2}{c^2} = 2hc\left[\frac{1}{\lambda_i} - \frac{1}{\lambda_i + \lambda_C(1-(-1))}\right]$$

$$v_{max} = c\sqrt{\frac{2hc}{m_e c^2}\left[\frac{1}{\lambda_i} - \frac{1}{\lambda_i + 2\lambda_C}\right]}$$

$$= \left(3.00\times10^8\frac{\text{m}}{\text{s}}\right)\sqrt{\frac{2(1240\ \text{eV}\cdot\text{nm})}{(0.511\times10^6\ \text{eV})}\left[\frac{1}{0.075\ \text{nm}} - \frac{1}{(0.075\ \text{nm}) + 2(0.00243\ \text{nm})}\right]}$$

$$= \boxed{1.9\times10^7\frac{\text{m}}{\text{s}}}$$

REFLECT

This is approximately 6% of the speed of light.

26.63

SET UP

A photon Compton scatters off of a stationary electron at an angle of 60 degrees. The electron moves off with a kinetic energy $K = 80\ \text{eV}$. The kinetic energy of the electron is equal to the change in the energy of the photon. Using this along with the expression for the shift in wavelength due to Compton scattering, we can calculate the initial wavelength of the photon.

SOLVE

Scattered wavelength:

$$\Delta\lambda = \lambda_{\text{f}} - \lambda_{\text{i}} = \lambda_{\text{C}}(1 - \cos(\theta))$$

$$\lambda_{\text{f}} = \lambda_{\text{i}} + \lambda_{\text{C}}(1 - \cos(\theta))$$

Initial wavelength:

$$K = \frac{hc}{\lambda_{\text{i}}} - \frac{hc}{\lambda_{\text{f}}}$$

$$K = \frac{hc}{\lambda_{\text{i}}} - \frac{hc}{\lambda_{\text{i}} + \lambda_{\text{C}}(1 - \cos(\theta))}$$

$$\frac{K}{hc} = \frac{\lambda_{\text{i}} + \lambda_{\text{C}}(1 - \cos(\theta)) - \lambda_{\text{i}}}{\lambda_{\text{i}}(\lambda_{\text{i}} + \lambda_{\text{C}}(1 - \cos(\theta)))}$$

$$\frac{K}{hc} = \frac{\lambda_{\text{C}}(1 - \cos(\theta))}{\lambda_{\text{i}}^2 + \lambda_{\text{i}}\lambda_{\text{C}}(1 - \cos(\theta))}$$

$$\frac{80\ \text{eV}}{1240\ \text{eV} \cdot \text{nm}} = \frac{(0.00243\ \text{nm})(1 - \cos(60°))}{\lambda_{\text{i}}^2 + \lambda_{\text{i}}(0.00243\ \text{nm})(1 - \cos(60°))}$$

Simplifying and dropping units for now:

$$0.064 = \frac{0.00122}{\lambda_{\text{i}}^2 + 0.00122\lambda_{\text{i}}}$$

$$\lambda_{\text{i}}^2 + 0.00122\lambda_{\text{i}} - 0.019 = 0$$

Solving the quadratic equation and taking the positive solution:

$$\lambda = \frac{-0.00122 \pm \sqrt{(0.00122)^2 - 4(1)(-0.019)}}{2(1)} = \boxed{0.137\ \text{nm}}$$

REFLECT

We had to reject the negative solution from the quadratic equation because wavelengths must be positive.

26.64

SET UP

An x-ray source is incident on a collection of stationary electrons. The electrons are scattered at a speed of $v = 4.50 \times 10^5 \frac{\text{m}}{\text{s}}$, and the photon is scattered at an angle of 60 degrees. The kinetic energy of the electron, $K = \frac{1}{2} m_e v^2$, is equal to the change in the energy of the photon. Using this along with the expression for the shift in wavelength due to Compton scattering, we can calculate the initial wavelength of the photon.

SOLVE

Scattered wavelength:

$$\Delta\lambda = \lambda_f - \lambda_i = \lambda_C(1 - \cos(\theta))$$

$$\lambda_f = \lambda_i + \lambda_C(1 - \cos(\theta))$$

Initial wavelength:

$$K = \frac{hc}{\lambda_i} - \frac{hc}{\lambda_f}$$

$$\frac{1}{2} m_e v^2 = \frac{hc}{\lambda_i} - \frac{hc}{\lambda_i + \lambda_C(1 - \cos(\theta))}$$

$$\frac{m_e v^2}{2hc} = \frac{\lambda_i + \lambda_C(1 - \cos(\theta)) - \lambda_i}{\lambda_i(\lambda_i + \lambda_C(1 - \cos(\theta)))}$$

$$\frac{m_e v^2}{2hc} = \frac{\lambda_C(1 - \cos(\theta))}{\lambda_i^2 + \lambda_i \lambda_C(1 - \cos(\theta))}$$

$$\frac{\left[(9.11 \times 10^{-31}\ \text{kg})\left(4.50 \times 10^5 \frac{\text{m}}{\text{s}}\right)^2\right]\left(\frac{1\ \text{eV}}{1.60 \times 10^{-19}\ \text{J}}\right)}{2(1240\ \text{eV} \cdot \text{nm})} = \frac{(0.00243\ \text{nm})(1 - \cos(60°))}{\lambda_i^2 + \lambda_i(0.00243\ \text{nm})(1 - \cos(60°))}$$

Simplifying and dropping units for now:

$$0.000465 = \frac{0.00122}{\lambda_i^2 + 0.00122\lambda_i}$$

$$\lambda_i^2 + 0.00122\lambda_i - 2.62 = 0$$

Solving the quadratic equation and taking the positive solution:

$$\lambda = \frac{-0.00122 \pm \sqrt{(0.00122)^2 - 4(1)(-2.62)}}{2(1)} = \boxed{1.62\ \text{nm}}$$

REFLECT

A wavelength of around 1 nm falls within the x-ray region of the spectrum, so our answer is reasonable.

26.65

SET UP

Compton's famous scattering experiment used photons of wavelength $\lambda_i = 0.0711$ nm. We can find the frequency and energy of these photons from $\lambda f = c$ and $E = \frac{hc}{\lambda}$, respectively. The wavelength of the photons that are scattered at $\theta = 90°$ is given by $\Delta\lambda = \lambda_f - \lambda_i = \lambda_C(1 - \cos(\theta))$. Once we have the wavelength of the scattered photons, we can calculate their energy directly. Finally, the kinetic energy of the electrons that recoil from the scattering process is equal to the change in the energy of the photons.

SOLVE

Part a)
Frequency:

$$f = \frac{c}{\lambda} = \frac{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)}{0.0711 \times 10^{-9}\ \text{m}} = \boxed{4.22 \times 10^{18}\ \text{Hz}}$$

Energy:

$$E = \frac{hc}{\lambda} = \frac{1240\ \text{eV} \cdot \text{nm}}{0.0711\ \text{nm}} = \boxed{17{,}440\ \text{eV}}$$

Part b)

$$\Delta\lambda = \lambda_f - \lambda_i = \lambda_C(1 - \cos(\theta))$$

$$\lambda_f = \lambda_i + \lambda_C(1 - \cos(\theta)) = (0.0711\ \text{nm}) + (0.00243\ \text{nm})(1 - \cos(90°)) = \boxed{0.0735\ \text{nm}}$$

Part c)

$$E = \frac{hc}{\lambda_f} = \frac{1240\ \text{eV} \cdot \text{nm}}{0.0735\ \text{nm}} = \boxed{16{,}900\ \text{eV} = 16.9\ \text{keV}}$$

Part d)

$$K = \frac{hc}{\lambda_i} - \frac{hc}{\lambda_f} = hc\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_f}\right) = (1240\ \text{eV} \cdot \text{nm})\left(\frac{1}{0.0711\ \text{nm}} - \frac{1}{0.0735\ \text{nm}}\right) = \boxed{576\ \text{eV}}$$

REFLECT

The wavelength of the photons does not change by much, so we would expect the kinetic energy of the electrons to be much smaller than the energy of the photons.

26.66

SET UP

A ball ($m = 0.150$ kg) has a speed of $v = 40$ m/s. Its de Broglie wavelength is equal to Planck's constant divided by its momentum.

SOLVE

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63 \times 10^{-34}\ \text{J}\cdot\text{s}}{(0.150\ \text{kg})\left(40\frac{\text{m}}{\text{s}}\right)} = \boxed{1.1 \times 10^{-34}\ \text{m}}$$

The de Broglie wavelength is negligible compared with the size of the ball, so we can safely treat the ball as a particle.

REFLECT

This is the reason why we usually ignore the wave nature of macroscopic objects.

26.67

SET UP

An electron ($m_e = 9.11 \times 10^{-31}$ kg) has a speed of $v = 0.00730c$. Its de Broglie wavelength is equal to Planck's constant divided by its momentum.

SOLVE

$$\lambda = \frac{h}{p} = \frac{h}{m_e v} = \frac{6.63 \times 10^{-34}\ \text{J}\cdot\text{s}}{(9.11 \times 10^{-31}\ \text{kg})(0.00730)\left(3.00 \times 10^{8}\frac{\text{m}}{\text{s}}\right)} = \boxed{3.32 \times 10^{-10}\ \text{m}}$$

REFLECT

A distance of 3 angstroms (1 angstrom = 0.1 nm) is a little larger than a bond length in a molecule; this is why electrons are commonly used in microscopy to probe molecular structure. The exact de Broglie wavelength of the electron can be tuned by changing its speed.

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26.68

SET UP

A proton ($m_p = 1.67 \times 10^{-27}$ kg) has a speed of $v = 400{,}000$ m/s. Its de Broglie wavelength is given by $\lambda = \frac{h}{p} = \frac{h}{m_p v}$.

SOLVE

$$\lambda = \frac{h}{p} = \frac{h}{m_p v} = \frac{6.63 \times 10^{-34}\ \text{J}\cdot\text{s}}{(1.67 \times 10^{-27}\ \text{kg})\left(400{,}000\frac{\text{m}}{\text{s}}\right)} = \boxed{9.93 \times 10^{-13}\ \text{m}}$$

REFLECT

The de Broglie wavelength of a proton going a certain speed will always be smaller than that of an electron going the same speed since the wavelength is inversely proportional to the mass of the particle.

26.69

SET UP

In order to restate the de Broglie wavelength for particles at relativistic speeds, we need to replace the classical momentum with the relativistic momentum, $p = \gamma m v$, where $\gamma = \frac{1}{\sqrt{1-\beta^2}}$.

SOLVE

$$\lambda = \frac{h}{p} = \frac{h}{\gamma mv} = \frac{h\sqrt{1-\beta^2}}{mv} = \frac{h\sqrt{1-\left(\frac{v}{c}\right)^2}}{mv}$$

REFLECT

We can also write the relativistic de Broglie wavelength as $\lambda = \frac{h\sqrt{1-\beta^2}}{mv} = \frac{h\sqrt{1-\left(\frac{v}{c}\right)^2}}{mv}$, which is useful depending on the information provided in a given problem.

26.70

SET UP

A proton ($mc^2 = 938 \times 10^6$ eV) has a de Broglie wavelength of $\lambda = 1.2 \times 10^{-6}$ nm. We can use the expression for the relativistic de Broglie wavelength, $\lambda = \frac{h}{\gamma mv}$, to calculate the speed of the proton.

SOLVE

$$\lambda = \frac{h}{\gamma mv} = \frac{h}{\gamma mv}\left(\frac{c^2}{c^2}\right) = \frac{hc}{\gamma mc^2 \beta}$$

$$\gamma\beta = \frac{hc}{\lambda mc^2}$$

$$\gamma^2\beta^2 = \left(\frac{hc}{\lambda mc^2}\right)^2$$

$$\left(\frac{1}{1-\beta^2}\right)\beta^2 = \left(\frac{hc}{\lambda mc^2}\right)^2$$

$$\beta^2 = \left(\frac{hc}{\lambda mc^2}\right)^2(1-\beta^2)$$

$$\left(1+\left(\frac{hc}{\lambda mc^2}\right)^2\right)\beta^2 = \left(\frac{hc}{\lambda mc^2}\right)^2$$

$$\beta = \sqrt{\frac{\left(\frac{hc}{\lambda(mc^2)}\right)^2}{\left(1+\left(\frac{hc}{\lambda(mc^2)}\right)^2\right)}} = \sqrt{\frac{\left(\frac{1240 \text{ eV}\cdot\text{nm}}{(1.2\times10^{-6}\text{ nm})(938\times10^6\text{ eV})}\right)^2}{\left(1+\left(\frac{1240\text{ eV}\cdot\text{nm}}{(1.2\times10^{-6}\text{ nm})(938\times10^6\text{ eV})}\right)^2\right)}} = 0.740$$

$$\boxed{v = 0.740c}$$

REFLECT

Rearranging the equation and invoking the fact that $hc = 1240$ eV · nm made solving for β much easier.

26.71

SET UP

We are the given the kinetic energies of six different electrons—1 eV, 10 eV, 100 eV, 1000 eV, 1×10^6 eV, and 1×10^9 eV—and asked to find the de Broglie wavelength, $\lambda = \frac{h}{p}$, of each one. First, we need to determine whether or not we need to use the expression for the classical or relativistic momentum; relativistic effects should be included if the speed of the particle is about 10% the speed of light. If the speed we calculate from the kinetic energy is less than $0.1c$, then we can use the classical expressions for the kinetic energy $\left(K = \frac{1}{2}m_e v^2\right)$ and momentum ($p = m_e v$). If the speed is larger than $0.1c$, we need to use the relativistic expressions for the kinetic energy, $K = (\gamma - 1)m_e c^2$, and momentum, $p = \gamma m_e v$.

SOLVE

Part a)

Speed:

$$K = \frac{1}{2}m_e v^2$$

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2\left(1 \text{ eV} \times \frac{1.6 \times 10^{-19} \text{ J}}{1 \text{ eV}}\right)}{9.11 \times 10^{-31} \text{ kg}}} = 5.93 \times 10^5 \frac{\text{m}}{\text{s}}$$

Ratio of v to c:

$$\frac{v}{c} = \frac{\left(5.93 \times 10^5 \frac{\text{m}}{\text{s}}\right)}{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)} = 1.98 \times 10^{-3}, \text{ so we can safely ignore relativistic effects.}$$

de Broglie wavelength:

$$\lambda = \frac{h}{p} = \frac{h}{m_e v} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(9.11 \times 10^{-31} \text{ kg})\left(5.93 \times 10^5 \frac{\text{m}}{\text{s}}\right)} = \boxed{1.23 \times 10^{-9} \text{ m}}$$

Part b)

Speed:

$$K = \frac{1}{2}m_e v^2$$

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2\left(10 \text{ eV} \times \frac{1.6 \times 10^{-19} \text{ J}}{1 \text{ eV}}\right)}{9.11 \times 10^{-31} \text{ kg}}} = 1.87 \times 10^6 \frac{\text{m}}{\text{s}}$$

Ratio of v to c:

$$\frac{v}{c} = \frac{\left(1.87 \times 10^6 \frac{\text{m}}{\text{s}}\right)}{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)} = 6.23 \times 10^{-3}, \text{ so we can safely ignore relativistic effects.}$$

de Broglie wavelength:

$$\lambda = \frac{h}{p} = \frac{h}{m_e v} = \frac{6.63 \times 10^{-34}\ \text{J} \cdot \text{s}}{(9.11 \times 10^{-31}\ \text{kg})\left(1.87 \times 10^6 \frac{\text{m}}{\text{s}}\right)} = \boxed{3.88 \times 10^{-10}\ \text{m}}$$

Part c)

Speed:

$$K = \frac{1}{2} m_e v^2$$

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2\left(100\ \text{eV} \times \frac{1.6 \times 10^{-19}\ \text{J}}{1\ \text{eV}}\right)}{9.11 \times 10^{-31}\ \text{kg}}} = 5.93 \times 10^6 \frac{\text{m}}{\text{s}}$$

Ratio of v to c:

$$\frac{v}{c} = \frac{\left(5.93 \times 10^6 \frac{\text{m}}{\text{s}}\right)}{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)} = 1.98 \times 10^{-2}, \text{ so we can safely ignore relativistic effects.}$$

de Broglie wavelength:

$$\lambda = \frac{h}{p} = \frac{h}{m_e v} = \frac{6.63 \times 10^{-34}\ \text{J} \cdot \text{s}}{(9.11 \times 10^{-31}\ \text{kg})\left(5.93 \times 10^6 \frac{\text{m}}{\text{s}}\right)} = \boxed{1.23 \times 10^{-10}\ \text{m}}$$

Part d)

Speed:

$$K = \frac{1}{2} m_e v^2$$

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2\left(1000\ \text{eV} \times \frac{1.6 \times 10^{-19}\ \text{J}}{1\ \text{eV}}\right)}{9.11 \times 10^{-31}\ \text{kg}}} = 1.87 \times 10^7 \frac{\text{m}}{\text{s}}$$

Ratio of v to c:

$$\frac{v}{c} = \frac{\left(1.87 \times 10^7 \frac{\text{m}}{\text{s}}\right)}{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)} = 6.23 \times 10^{-2}.$$

This speed is around 6% of the speed of light. For higher kinetic energies, we should start including relativistic effects in our calculations.

de Broglie wavelength:

$$\lambda = \frac{h}{p} = \frac{h}{m_e v} = \frac{6.63 \times 10^{-34}\ \text{J}\cdot\text{s}}{(9.11 \times 10^{-31}\ \text{kg})\left(1.87 \times 10^{7}\frac{\text{m}}{\text{s}}\right)} = \boxed{3.88 \times 10^{-11}\ \text{m}}$$

Part e)

Speed from the relativistic kinetic energy:

$$K = (\gamma - 1)m_e c^2 = \left(\frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} - 1\right)m_e c^2$$

$$K + m_e c^2 = \frac{m_e c^2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

$$\sqrt{1 - \left(\frac{v}{c}\right)^2} = \frac{m_e c^2}{K + m_e c^2}$$

$$1 - \left(\frac{v}{c}\right)^2 = \left(\frac{m_e c^2}{K + m_e c^2}\right)^2$$

$$v = c\sqrt{1 - \left(\frac{m_e c^2}{K + m_e c^2}\right)^2}$$

$$= \left(3.00 \times 10^{8}\frac{\text{m}}{\text{s}}\right)\sqrt{1 - \left(\frac{(9.11 \times 10^{-31}\ \text{kg})\left(3.00 \times 10^{8}\frac{\text{m}}{\text{s}}\right)^2}{\left(1 \times 10^{6}\ \text{eV} \times \frac{1.6 \times 10^{-19}\ \text{J}}{1\ \text{eV}}\right) + (9.11 \times 10^{-31}\ \text{kg})\left(3.00 \times 10^{8}\frac{\text{m}}{\text{s}}\right)^2}\right)^2}$$

$$= 2.82 \times 10^{8}\frac{\text{m}}{\text{s}}$$

Ratio of v to c:

$$\frac{v}{c} = \frac{\left(2.82 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{\left(3.00 \times 10^{8}\frac{\text{m}}{\text{s}}\right)} = 0.94$$

Relativistic gamma:

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{1}{\sqrt{1 - (0.94)^2}} = 2.93$$

de Broglie wavelength:

$$\lambda = \frac{h}{p} = \frac{h}{\gamma m_e v} = \frac{6.63 \times 10^{-34}\ \mathrm{J \cdot s}}{(2.93)(9.11 \times 10^{-31}\ \mathrm{kg})\left(2.82 \times 10^{8}\frac{\mathrm{m}}{\mathrm{s}}\right)} = \boxed{8.80 \times 10^{-13}\ \mathrm{m}}$$

Part f)

Speed:

$$v = c\sqrt{1 - \left(\frac{m_e c^2}{K + m_e c^2}\right)^2}$$

$$= \left(3.00 \times 10^{8}\frac{\mathrm{m}}{\mathrm{s}}\right)\sqrt{1 - \left(\frac{(9.11 \times 10^{-31}\ \mathrm{kg})\left(3.00 \times 10^{8}\frac{\mathrm{m}}{\mathrm{s}}\right)^2}{\left(1 \times 10^{9}\ \mathrm{eV} \times \frac{1.6 \times 10^{-19}\ \mathrm{J}}{1\ \mathrm{eV}}\right) + (9.11 \times 10^{-31}\ \mathrm{kg})\left(3.00 \times 10^{8}\frac{\mathrm{m}}{\mathrm{s}}\right)^2}\right)^2}$$

$$= 2.9999996 \times 10^{8}\frac{\mathrm{m}}{\mathrm{s}}$$

Ratio of v to c:

$$\frac{v}{c} = \frac{\left(2.9999996 \times 10^{8}\frac{\mathrm{m}}{\mathrm{s}}\right)}{\left(3.00 \times 10^{8}\frac{\mathrm{m}}{\mathrm{s}}\right)} = 0.99999987$$

Relativistic gamma:

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{1}{\sqrt{1 - (0.99999987)^2}} = 1961$$

de Broglie wavelength:

$$\lambda = \frac{h}{p} = \frac{h}{\gamma m_e v} = \frac{6.63 \times 10^{-34}\ \mathrm{J \cdot s}}{(1961)(9.11 \times 10^{-31}\ \mathrm{kg})\left(2.9999996 \times 10^{8}\frac{\mathrm{m}}{\mathrm{s}}\right)} = \boxed{1.24 \times 10^{-14}\ \mathrm{m}}$$

REFLECT

The de Broglie wavelength is inversely proportional to the speed of the particle; the kinetic energy is proportional to the speed of the particle. Therefore, as the kinetic energy increases, the de Broglie wavelength of the particle decreases.

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26.72

SET UP

We are the given the kinetic energies of three different alpha particles—1 eV, 5 eV, and 10 eV—and asked to find the de Broglie wavelength, $\lambda = \frac{h}{p}$, of each one. We first need to use

the relativistic expressions for the kinetic energy, $K = (\gamma - 1)m_\alpha c^2$, and momentum, $p = \gamma m_\alpha v$, in order to calculate relativistic gamma and the speed of the particle. The mass of an alpha particle is $m_\alpha = 6.64 \times 10^{-27}\ \text{kg} = 3730\frac{\text{MeV}}{c^2}$.

SOLVE

Speed from the relativistic kinetic energy:

$$K = (\gamma - 1)m_\alpha c^2 = \left(\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} - 1\right)m_\alpha c^2$$

$$K + m_\alpha c^2 = \frac{m_\alpha c^2}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

$$\sqrt{1-\left(\frac{v}{c}\right)^2} = \frac{m_\alpha c^2}{K + m_\alpha c^2}$$

$$1-\left(\frac{v}{c}\right)^2 = \left(\frac{m_\alpha c^2}{K + m_\alpha c^2}\right)^2$$

$$v = c\sqrt{1-\left(\frac{m_\alpha c^2}{K + m_\alpha c^2}\right)^2}$$

Part a)

Speed:

$$v = \left(3.00 \times 10^8\frac{\text{m}}{\text{s}}\right)\sqrt{1-\left(\frac{3730\ \text{MeV}}{(1\ \text{MeV}) + (3730\ \text{MeV})}\right)^2} = 6.95 \times 10^6\frac{\text{m}}{\text{s}}$$

Ratio of v to c:

$$\frac{v}{c} = \frac{\left(6.95 \times 10^6\frac{\text{m}}{\text{s}}\right)}{\left(3.00 \times 10^8\frac{\text{m}}{\text{s}}\right)} = 0.0232$$

Relativistic gamma:

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{1}{\sqrt{1-(0.0232)^2}} = 1.0003$$

De Broglie wavelength:

$$\lambda = \frac{h}{p} = \frac{h}{\gamma m_\alpha v} = \frac{6.63 \times 10^{-34}\ \text{J}\cdot\text{s}}{(1.0003)(6.64 \times 10^{-27}\ \text{kg})\left(6.95 \times 10^6\frac{\text{m}}{\text{s}}\right)} = \boxed{1.44 \times 10^{-14}\ \text{m}}$$

Part b)
Speed:

$$v = \left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)\sqrt{1 - \left(\frac{3730 \text{ MeV}}{(5 \text{ MeV}) + (3730 \text{ MeV})}\right)^2} = 1.55 \times 10^7 \frac{\text{m}}{\text{s}}$$

Ratio of v to c:

$$\frac{v}{c} = \frac{\left(1.55 \times 10^7 \frac{\text{m}}{\text{s}}\right)}{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)} = 0.0517$$

Relativistic gamma:

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{1}{\sqrt{1 - (0.0517)^2}} = 1.0013$$

De Broglie wavelength:

$$\lambda = \frac{h}{p} = \frac{h}{\gamma m_\alpha v} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(1.0013)(6.64 \times 10^{-27} \text{ kg})\left(1.55 \times 10^6 \frac{\text{m}}{\text{s}}\right)} = \boxed{6.42 \times 10^{-15} \text{ m}}$$

Part c)
Speed:

$$v = \left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)\sqrt{1 - \left(\frac{3730 \text{ MeV}}{(10 \text{ MeV}) + (3730 \text{ MeV})}\right)^2} = 2.19 \times 10^7 \frac{\text{m}}{\text{s}}$$

Ratio of v to c:

$$\frac{v}{c} = \frac{\left(1.55 \times 10^7 \frac{\text{m}}{\text{s}}\right)}{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)} = 0.0731$$

Relativistic gamma:

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{1}{\sqrt{1 - (0.0731)^2}} = 1.0027$$

De Broglie wavelength:

$$\lambda = \frac{h}{p} = \frac{h}{\gamma m_\alpha v} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(1.0027)(6.64 \times 10^{-27} \text{ kg})\left(2.19 \times 10^6 \frac{\text{m}}{\text{s}}\right)} = \boxed{4.54 \times 10^{-15} \text{ m}}$$

REFLECT

The de Broglie wavelength is inversely proportional to the speed, which means it should decrease as the kinetic energy increases.

26.73

SET UP

A neutron ($m_n = 1.675 \times 10^{-27}$ kg) has a kinetic energy $K = 0.0400$ eV. Its de Broglie wavelength is given by $\lambda = \frac{h}{p} = \frac{h}{m_n v}$, where the speed v can be found from the kinetic energy.

SOLVE

Speed:

$$K = \frac{1}{2}m_n v^2$$

$$v = \sqrt{\frac{2K}{m_n}}$$

De Broglie wavelength:

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{m_n}\sqrt{\frac{m_n}{2K}} = h\sqrt{\frac{1}{2Km_n}}$$

$$= (6.63 \times 10^{-34}\ \text{J}\cdot\text{s})\sqrt{\frac{1}{2\left(0.0400\ \text{eV} \times \frac{1.60 \times 10^{-19}\ \text{J}}{1\ \text{eV}}\right)(1.675 \times 10^{-27}\ \text{kg})}} = \boxed{1.43 \times 10^{-10}\ \text{m}}$$

REFLECT

The speed of the neutron is around 3×10^3 m/s, which is much smaller than the speed of light.

26.74

SET UP

A relativistic electron has a de Broglie wavelength of $\lambda = 346 \times 10^{-15}$ m. We can use the expression for the relativistic de Broglie wavelength, $\lambda = \frac{h}{\gamma m v}$, to calculate the speed of the electron.

SOLVE

$$\lambda = \frac{h}{\gamma m v}$$

$$\gamma v = \frac{h}{m\lambda}$$

$$\frac{\gamma v}{c} = \frac{h}{m\lambda c}$$

$$\gamma\beta = \frac{h}{m\lambda c}$$

$$\gamma^2\beta^2 = \left(\frac{h}{m\lambda c}\right)^2$$

$$\frac{\beta^2}{1-\beta^2} = \left(\frac{h}{m\lambda c}\right)^2$$

$$\beta^2 = \left(\frac{h}{m\lambda c}\right)^2(1-\beta^2)$$

$$\beta^2\left(1 + \left(\frac{h}{m\lambda c}\right)^2\right) = \left(\frac{h}{m\lambda c}\right)^2$$

$$\beta^2 = \frac{\left(\frac{h}{m\lambda c}\right)^2}{1 + \left(\frac{h}{m\lambda c}\right)^2}$$

$$\beta = \sqrt{\frac{\left(\frac{h}{m\lambda c}\right)^2}{1 + \left(\frac{h}{m\lambda c}\right)^2}}$$

$$v = c\sqrt{\frac{\left(\frac{h}{m\lambda c}\right)^2}{1 + \left(\frac{h}{m\lambda c}\right)^2}}$$

$$= \left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)\sqrt{\frac{\left(\frac{6.63 \times 10^{-34}\ \text{J}\cdot\text{s}}{(9.11 \times 10^{-31}\ \text{kg})(346 \times 10^{-15}\ \text{m})\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)}\right)^2}{1 + \left(\frac{6.63 \times 10^{-34}\ \text{J}\cdot\text{s}}{(9.11 \times 10^{-31}\ \text{kg})(346 \times 10^{-15}\ \text{m})\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)}\right)^2}}$$

$$= \boxed{2.97 \times 10^8 \frac{\text{m}}{\text{s}}}$$

REFLECT

The electron is traveling at 99% of the speed of light.

26.75

SET UP

In order to write an expression for the de Broglie wavelength of a nonrelativistic particle in terms of the classical kinetic energy, we can rearrange the definition of the kinetic energy for the speed and plug it into the expression for its de Broglie wavelength.

SOLVE

Speed:

$$K = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2K}{m}}$$

De Broglie wavelength:

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{m}\sqrt{\frac{m}{2K}} = \boxed{h\sqrt{\frac{1}{2Km}}}$$

REFLECT

The kinetic energy of a particle in terms of its momentum is $K = \dfrac{p^2}{2m}$.

26.76

SET UP

The Balmer formula is used to calculate the emission wavelengths for transitions to the $n = 2$ level in the hydrogen atom. To calculate the first four wavelengths, we need to plug in $m = 3$, 4, 5, and 6.

SOLVE

Balmer formula:

$$\lambda = (364.56\text{ nm})\left(\frac{m^2}{m^2 - 4}\right)$$

$m = 3$:

$$\lambda = (364.56\text{ nm})\left(\frac{3^2}{3^2 - 4}\right) = \boxed{656.21\text{ nm}}$$

$m = 4$:

$$\lambda = (364.56\text{ nm})\left(\frac{4^2}{4^2 - 4}\right) = \boxed{486.08\text{ nm}}$$

$m = 5$:

$$\lambda = (364.56\text{ nm})\left(\frac{5^2}{5^2 - 4}\right) = \boxed{434.00\text{ nm}}$$

$m = 6$:

$$\lambda = (364.56\text{ nm})\left(\frac{6^2}{6^2 - 4}\right) = \boxed{410.13\text{ nm}}$$

REFLECT

These are the four visible lines of the hydrogen emission spectrum. The colors of these lines are red, blue-green, blue, and violet, respectively.

26.77

SET UP

The wavelength emitted by an electron flipping spin states in the ground state of hydrogen is $\lambda = 0.21\text{ m}$. The energy difference between energy states is equal to $\Delta E = \frac{hc}{\lambda}$.

SOLVE

$$\Delta E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}\text{ J}\cdot\text{s})\left(3.00 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{0.21\text{ m}}$$

$$= 9.47 \times 10^{-21}\text{ J} \times \frac{1\text{ eV}}{1.6 \times 10^{-19}\text{ J}} = \boxed{5.90 \times 10^{-6}\text{ eV}}$$

REFLECT

Light with a wavelength of 21 cm is in the microwave region of the spectrum.

26.78

SET UP

We can show that the Balmer formula, $\lambda = b\left(\frac{m^2}{m^2 - 4}\right)$, is simply a special case of the Rydberg formula, $\frac{1}{\lambda} = R_{\text{H}}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$, by plugging in $n = 2$ and rearraranging. The numerical values for the Rydberg constant and the Balmer constant are $R_{\text{H}} = 1.09737 \times 10^7\text{ m}^{-1}$ and $b = 364.56\text{ nm}$, respectively.

SOLVE

Rydberg formula with $n = 2$:

$$\frac{1}{\lambda} = R_{\text{H}}\left(\frac{1}{2^2} - \frac{1}{m^2}\right) = R_{\text{H}}\left(\frac{1}{4} - \frac{1}{m^2}\right) = R_{\text{H}}\left(\frac{m^2 - 4}{4m^2}\right)$$

$$\lambda = \frac{1}{R_{\text{H}}}\left(\frac{4m^2}{m^2 - 4}\right) = \frac{4}{1.09737 \times 10^7\text{ m}^{-1}}\left(\frac{m^2}{m^2 - 4}\right) = (3.6456 \times 10^{-7}\text{ m})\left(\frac{m^2}{m^2 - 4}\right)$$

$$= (364.56\text{ nm})\left(\frac{m^2}{m^2 - 4}\right) = b\left(\frac{m^2}{m^2 - 4}\right)$$

REFLECT

The set of visible lines in the hydrogen emission spectrum is called the Balmer series.

26.79

SET UP

In one case, a hypothetical atom has four unequally spaced energy levels, while it has ten unequally spaced energy levels in another case. Since the energy levels are unequally spaced, each transition between states will result in a unique line in the emission spectrum.

SOLVE

Part a) There are six combinations: $n = 4 \to n = 3$, $n = 4 \to n = 2$, $n = 4 \to n = 1$, $n = 3 \to n = 2$, $n = 3 \to n = 1$, and $n = 2 \to n = 1$.

Part b) There are 45 combinations: $n = 10 \to n = 9$, $n = 10 \to n = 8$, $n = 10 \to n = 7$, $n = 10 \to n = 6$, $n = 10 \to n = 5$, $n = 10 \to n = 4$, $n = 10 \to n = 3$, $n = 10 \to n = 2$, $n = 10 \to n = 1$, $n = 9 \to n = 8$, $n = 9 \to n = 7$, $n = 9 \to n = 6$, $n = 9 \to n = 5$, $n = 9 \to n = 4$, $n = 9 \to n = 3$, $n = 9 \to n = 2$, $n = 9 \to n = 1$, $n = 8 \to n = 7$, $n = 8 \to n = 6$, $n = 8 \to n = 5$, $n = 8 \to n = 4$, $n = 8 \to n = 3$, $n = 8 \to n = 2$, $n = 8 \to n = 1$, $n = 7 \to n = 6$, $n = 7 \to n = 5$, $n = 7 \to n = 4$, $n = 7 \to n = 3$, $n = 7 \to n = 2$, $n = 7 \to n = 1$, $n = 6 \to n = 5$, $n = 6 \to n = 4$, $n = 6 \to n = 3$, $n = 6 \to n = 2$, $n = 6 \to n = 1$, $n = 5 \to n = 4$, $n = 5 \to n = 3$, $n = 5 \to n = 2$, $n = 5 \to n = 1$, $n = 4 \to n = 3$, $n = 4 \to n = 2$, $n = 4 \to n = 1$, $n = 3 \to n = 2$, $n = 3 \to n = 1$, and $n = 2 \to n = 1$.

REFLECT

Drawing a diagram of all of the levels and the possible transitions is extremely helpful when solving this problem.

26.80

SET UP

The Lyman series results from transitions of electrons in hydrogen from higher energy states to the $n = 1$ level. The Rydberg formula, $\frac{1}{\lambda} = R_{\mathrm{H}}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$, where $R_{\mathrm{H}} = 1.09737 \times 10^7\ \mathrm{m}^{-1}$, gives the wavelengths of the photons associated with a transition from the mth to the nth energy level.

SOLVE

Rydberg formula:

$$\frac{1}{\lambda} = R_{\mathrm{H}}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

From $m = 2$ to $n = 1$:

$$\lambda = \left[R_{\mathrm{H}}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)\right]^{-1} = \left[(1.09737 \times 10^7\ \mathrm{m}^{-1})\left(\frac{1}{(1)^2} - \frac{1}{(2)^2}\right)\right]^{-1}$$
$$= \boxed{1.22 \times 10^{-7}\ \mathrm{m} = 122\ \mathrm{nm}}$$

From $m = 3$ to $n = 1$:

$$\lambda = \left[R_{\mathrm{H}}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)\right]^{-1} = \left[(1.09737 \times 10^7\ \mathrm{m}^{-1})\left(\frac{1}{(1)^2} - \frac{1}{(3)^2}\right)\right]^{-1}$$
$$= \boxed{1.03 \times 10^{-7}\ \mathrm{m} = 103\ \mathrm{nm}}$$

From $m = 4$ to $n = 1$:

$$\lambda = \left[R_{\mathrm{H}}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)\right]^{-1} = \left[(1.09737 \times 10^7\ \mathrm{m}^{-1})\left(\frac{1}{(1)^2} - \frac{1}{(4)^2}\right)\right]^{-1}$$
$$= \boxed{9.72 \times 10^{-8}\ \mathrm{m} = 97.2\ \mathrm{nm}}$$

From $m = 5$ to $n = 1$:

$$\lambda = \left[R_{\mathrm{H}}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)\right]^{-1} = \left[(1.09737 \times 10^7\ \mathrm{m}^{-1})\left(\frac{1}{(1)^2} - \frac{1}{(5)^2}\right)\right]^{-1}$$

$$= \boxed{9.49 \times 10^{-8}\ \mathrm{m} = 94.9\ \mathrm{nm}}$$

From $m = 6$ to $n = 1$:

$$\lambda = \left[R_{\mathrm{H}}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)\right]^{-1} = \left[(1.09737 \times 10^7\ \mathrm{m}^{-1})\left(\frac{1}{(1)^2} - \frac{1}{(6)^2}\right)\right]^{-1}$$

$$= \boxed{9.37 \times 10^{-8}\ \mathrm{m} = 93.7\ \mathrm{nm}}$$

REFLECT

The emitted light in the Lyman series is in the ultraviolet region of the electromagnetic spectrum.

26.81

SET UP

The Balmer series results from transitions of electrons in hydrogen from higher energy states to the $n = 2$ level. The Rydberg formula, $\frac{1}{\lambda} = R_{\mathrm{H}}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$, where $R_{\mathrm{H}} = 1.09737 \times 10^7\ \mathrm{m}^{-1}$, gives the wavelengths of the photons associated with a transition from the mth to the nth energy level.

SOLVE

Rydberg formula:

$$\frac{1}{\lambda} = R_{\mathrm{H}}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

From $m = 3$ to $n = 2$:

$$\lambda = \left[R_{\mathrm{H}}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)\right]^{-1} = \left[(1.09737 \times 10^7\ \mathrm{m}^{-1})\left(\frac{1}{(2)^2} - \frac{1}{(3)^2}\right)\right]^{-1}$$

$$= \boxed{6.56 \times 10^{-7}\ \mathrm{m} = 656\ \mathrm{nm}}$$

From $m = 4$ to $n = 2$:

$$\lambda = \left[R_{\mathrm{H}}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)\right]^{-1} = \left[(1.09737 \times 10^7\ \mathrm{m}^{-1})\left(\frac{1}{(2)^2} - \frac{1}{(4)^2}\right)\right]^{-1}$$

$$= \boxed{4.86 \times 10^{-7}\ \mathrm{m} = 486\ \mathrm{nm}}$$

From $m = 5$ to $n = 2$:

$$\lambda = \left[R_{\mathrm{H}}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)\right]^{-1} = \left[(1.09737 \times 10^7\ \mathrm{m}^{-1})\left(\frac{1}{(2)^2} - \frac{1}{(5)^2}\right)\right]^{-1}$$

$$= \boxed{4.34 \times 10^{-7}\ \mathrm{m} = 434\ \mathrm{nm}}$$

From $m = 6$ to $n = 2$:

$$\lambda = \left[R_H\left(\frac{1}{n^2} - \frac{1}{m^2}\right)\right]^{-1} = \left[(1.09737 \times 10^7\ \text{m}^{-1})\left(\frac{1}{(2)^2} - \frac{1}{(6)^2}\right)\right]^{-1}$$
$$= \boxed{4.10 \times 10^{-7}\ \text{m} = 410\ \text{nm}}$$

REFLECT

The first four lines in the Balmer series are in the visible range, ranging from red to blue to violet; the remaining ones are in the ultraviolet region.

26.82

SET UP

The Paschen series results from transitions of electrons in hydrogen from higher energy states to the $n = 3$ level. The Rydberg formula, $\frac{1}{\lambda} = R_H\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$, where $R_H = 1.09737 \times 10^7\ \text{m}^{-1}$, gives the wavelengths of the photons associated with a transition from the mth to the nth energy level.

SOLVE

Rydberg formula:

$$\frac{1}{\lambda} = R_H\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

From $m = 4$ to $n = 3$:

$$\lambda = \left[R_H\left(\frac{1}{n^2} - \frac{1}{m^2}\right)\right]^{-1} = \left[(1.09737 \times 10^7\ \text{m}^{-1})\left(\frac{1}{(3)^2} - \frac{1}{(4)^2}\right)\right]^{-1}$$
$$= \boxed{1.87 \times 10^{-6}\ \text{m} = 1870\ \text{nm}}$$

From $m = 5$ to $n = 3$:

$$\lambda = \left[R_H\left(\frac{1}{n^2} - \frac{1}{m^2}\right)\right]^{-1} = \left[(1.09737 \times 10^7\ \text{m}^{-1})\left(\frac{1}{(3)^2} - \frac{1}{(5)^2}\right)\right]^{-1}$$
$$= \boxed{1.28 \times 10^{-6}\ \text{m} = 1280\ \text{nm}}$$

From $m = 6$ to $n = 3$:

$$\lambda = \left[R_H\left(\frac{1}{n^2} - \frac{1}{m^2}\right)\right]^{-1} = \left[(1.09737 \times 10^7\ \text{m}^{-1})\left(\frac{1}{(3)^2} - \frac{1}{(6)^2}\right)\right]^{-1}$$
$$= \boxed{1.09 \times 10^{-6}\ \text{m} = 1090\ \text{nm}}$$

REFLECT

The emitted light in the Paschen series is in the infrared region of the electromagnetic spectrum.

26.83

SET UP

The Rydberg formula, $\frac{1}{\lambda} = R_{\mathrm{H}}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$, where $R_{\mathrm{H}} = 1.09737 \times 10^7\ \mathrm{m}^{-1}$, gives the wavelengths of the photons associated with a transition from the mth to the nth energy level. The shortest wavelength (that is, highest energy) photon emitted in the Lyman ($n = 1$), Balmer ($n = 2$), and Paschen ($n = 3$) series corresponds to a transition from $m = \infty$.

SOLVE

Part a)

$$\lambda = \left[R_{\mathrm{H}}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)\right]^{-1} = \left[(1.09737 \times 10^7\ \mathrm{m}^{-1})\left(\frac{1}{(1)^2} - \frac{1}{(\infty)^2}\right)\right]^{-1}$$
$$= \boxed{9.11 \times 10^{-8}\ \mathrm{m} = 91.1\ \mathrm{nm}}$$

Part b)

$$\lambda = \left[R_{\mathrm{H}}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)\right]^{-1} = \left[(1.09737 \times 10^7\ \mathrm{m}^{-1})\left(\frac{1}{(2)^2} - \frac{1}{(\infty)^2}\right)\right]^{-1}$$
$$= \boxed{3.65 \times 10^{-7}\ \mathrm{m} = 365\ \mathrm{nm}}$$

Part c)

$$\lambda = \left[R_{\mathrm{H}}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)\right]^{-1} = \left[(1.09737 \times 10^7\ \mathrm{m}^{-1})\left(\frac{1}{(3)^2} - \frac{1}{(\infty)^2}\right)\right]^{-1}$$
$$= \boxed{8.20 \times 10^{-7}\ \mathrm{m} = 820\ \mathrm{nm}}$$

REFLECT

If these photons were to be absorbed by an electron in the $n = 1, 2,$ or 3 state, respectively, the atom would be ionized.

26.84

SET UP

The Balmer formula is $\lambda = (364.56\ \mathrm{nm})\left(\frac{m^2}{m^2 - 4}\right)$. In order to express this in terms of the frequency of the emitted photons, we need to use $f = \frac{c}{\lambda}$.

SOLVE

$$\lambda = (364.56\ \mathrm{nm})\left(\frac{m^2}{m^2 - 4}\right)$$

$$f = \frac{c}{\lambda} = \frac{\left(2.9979 \times 10^8 \frac{\mathrm{m}}{\mathrm{s}}\right)}{\left(364.56\ \mathrm{nm} \times \frac{1\ \mathrm{m}}{10^9\ \mathrm{nm}}\right)\left(\frac{m^2}{m^2 - 4}\right)} = \boxed{(8.2233\ \mathrm{Hz})\left(\frac{m^2 - 4}{m^2}\right)}$$

REFLECT

To achieve five significant figures in our answer, we must use $c = 2.9979 \times 10^8 \frac{\text{m}}{\text{s}}$.

26.85

SET UP

The standard form of the Rydberg formula gives the wavelength of the emitted photons. Using $c = \lambda f$ we can express the Rydberg formula in terms of the frequency of the emitted photons.

SOLVE

$$\frac{1}{\lambda} = R_{\text{H}}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

$$f = \frac{c}{\lambda} = cR_{\text{H}}\left(\frac{1}{n^2} - \frac{1}{m^2}\right) = \left(2.9979 \times 10^8 \frac{\text{m}}{\text{s}}\right)(1.09737 \times 10^7 \text{ m}^{-1})\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

$$\boxed{f = (3.2898 \times 10^{15} \text{ Hz})\left(\frac{1}{n^2} - \frac{1}{m^2}\right)}$$

REFLECT

Comparing the frequency of photons is a bit more intuitive than comparing their wavelengths since the energy is directly proportional to the frequency, but inversely proportional to the wavelength.

26.86

SET UP

We can consult section 26-6 of the text to find expressions for the angular momentum, orbital radius, kinetic energy, total energy, and speed of an electron in the nth state of hydrogen.

SOLVE

Part a)

$$L_n = n\hbar$$

Part b)

$$r_n = \frac{n^2\hbar^2}{m_{\text{e}}ke^2}$$

Part c)

$$K_n = \frac{m_{\text{e}}(ke^2)^2}{2n^2\hbar^2}$$

Part d)

$$E_n = -K_n = -\frac{m_{\text{e}}(ke^2)^2}{2n^2\hbar^2}$$

Part e)

$$v_n = \frac{ke^2}{n\hbar}$$

REFLECT

All of these values are quantized.

26.87

SET UP

Using the expressions from problem 86, we can calculate the values for the angular momentum, orbital radius, kinetic energy, total energy, and speed of an electron in the $n = 1, 2, 3, 4,$ and 5 states of hydrogen.

SOLVE

n	L_n (J · s)	r_n (m)	K_n (J)	E_n (J)	v_n(m/s)
1	1.06×10^{-34}	5.29×10^{-11}	2.18×10^{-18}	-2.18×10^{-18}	2.16×10^{-6}
2	2.11×10^{-34}	2.11×10^{-10}	5.44×10^{-19}	-5.44×10^{-19}	1.09×10^{-6}
3	3.16×10^{-34}	4.76×10^{-10}	2.42×10^{-19}	-2.42×10^{-19}	7.29×10^{-5}
4	4.22×10^{-34}	8.46×10^{-10}	1.36×10^{-19}	-1.36×10^{-19}	5.47×10^{-5}
5	5.27×10^{-34}	1.32×10^{-9}	8.70×10^{-20}	-8.70×10^{-20}	4.38×10^{-5}

REFLECT

The angular momentum and orbital radius increase with n. The magnitudes of the kinetic energy, total energy, and speed all decrease as n increases.

26.88

SET UP

The angular momentum of an electron in the nth Bohr orbit is equal to $L_n = n\hbar$. Knowing this, we can find an expression for the speed of the electron in terms of n using $L_n = r_n m_e v_n$.

SOLVE

Part a)

$$\boxed{L_n = n\hbar}$$

Part b)

$$L_n = r_n m_e v_n$$

$$v_n = \frac{L_n}{r_n m_e} = \frac{n\hbar}{\left(\frac{n^2}{Z}a_0\right)m_e} = \boxed{\frac{Z\hbar}{n a_0 m_e}}$$

REFLECT

The speed decreases as n gets larger, while the angular momentum increases.

26.89

SET UP

The angular momentum of an electron in the nth Bohr orbit is $L_n = n\hbar$, where $\hbar = \frac{h}{2\pi} = 1.055 \times 10^{-34}\ \mathrm{J \cdot s}$. The radius of the nth Bohr orbit of hydrogen is given by $r_n = n^2 a_0$, where $a_0 = 0.529 \times 10^{-10}$ m. The speed of an electron can be found by dividing its angular momentum by its mass and the radius of its orbit; therefore, the speed of an electron in the nth Bohr orbit of hydrogen is $v_n = \frac{L_n}{m_e r_n}$. To find these values for an electron in the tenth Bohr orbit, we should plug in $n = 10$ and solve.

SOLVE

Angular momentum of an electron in the tenth Bohr orbit:

$$L_n = n\hbar$$

$$L_{10} = 10\hbar = 10(1.055 \times 10^{-34}\ \mathrm{J \cdot s}) = \boxed{1.055 \times 10^{-33}\ \mathrm{J \cdot s}}$$

Radius of the tenth Bohr orbit of hydrogen:

$$r_n = n^2 a_0$$

$$r_{10} = (10)^2(0.529 \times 10^{-10}\ \mathrm{m}) = 5.29 \times 10^{-9}\ \mathrm{m}$$

Speed of an electron in the tenth Bohr orbit:

$$v_n = \frac{L_n}{m_e r_n}$$

$$v_{10} = \frac{L_{10}}{m_e r_{10}} = \frac{1.066 \times 10^{-33}\ \mathrm{J \cdot s}}{(9.11 \times 10^{-31}\ \mathrm{kg})(5.29 \times 10^{-9}\ \mathrm{m})} = \boxed{2.19 \times 10^5 \frac{\mathrm{m}}{\mathrm{s}}}$$

REFLECT

The general expression for the quantized radii of electron orbits is $r_n = \frac{n^2 a_0}{Z}$, where Z is the atomic number; for hydrogen, $Z = 1$.

26.90

SET UP

The wavelength and frequency of the transition from $m = 273$ to $n = 272$ in hydrogen can be calculated from the Rydberg equation, $\frac{1}{\lambda} = R_\mathrm{H}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$, and $f = \frac{c}{\lambda}$, respectively.

SOLVE

Wavelength:

$$\lambda = \left[R_\mathrm{H}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)\right]^{-1} = \left[(1.09737 \times 10^7\ \mathrm{m}^{-1})\left(\frac{1}{(272)^2} - \frac{1}{(273)^2}\right)\right]^{-1} = \boxed{0.922\ \mathrm{m}}$$

Frequency:

$$f = \frac{c}{\lambda} = \frac{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)}{0.922 \text{ m}} = \boxed{3.25 \times 10^8 \text{ Hz}}$$

REFLECT

The transition from $m = 273$ to $n = 272$ should be low in energy, so a frequency in the radio wave region of the spectrum is reasonable.

26.91

SET UP

The frequencies of the spectral lines resulting from a single electron transition in a carbon atom are increased over those for hydrogen by the ratio of the Rydberg constants, $\frac{R_C}{R_H} = \frac{\left(1 - \frac{m_e}{m_C}\right)}{\left(1 - \frac{m_e}{m_H}\right)}$. The mass of a hydrogen atom is 1837 times larger than that of the electron; the mass of a carbon atom is 12 times that of a hydrogen atom. Using this information and the frequency of the 272α line in hydrogen, we can calculate the frequency of the 272α line in carbon. The frequency of the 272α transition in hydrogen, from problem 90, is $f_H = 325$ MHz.

SOLVE

Ratio of the Rydberg constants:

$$\frac{R_C}{R_H} = \frac{\left(1 - \frac{m_e}{m_C}\right)}{\left(1 - \frac{m_e}{m_H}\right)} = \frac{\left(1 - \frac{m_e}{12m_H}\right)}{\left(1 - \frac{m_e}{m_H}\right)} = \frac{\left(1 - \frac{m_e}{12(1837m_e)}\right)}{\left(1 - \frac{m_e}{1837m_e}\right)} = \frac{\left(1 - \frac{1}{12(1837)}\right)}{\left(1 - \frac{1}{1837}\right)} = 1.0005$$

Frequency shift of carbon 272α transition:

$$\frac{f_C}{f_H} = \frac{R_C}{R_H}$$

$$f_C = \left(\frac{R_C}{R_H}\right)f_H = 1.0005f_H = f_H + 0.0005f_H$$

$$f_C - f_H = 0.0005(325 \text{ MHz}) = \boxed{0.2 \text{ MHz}}$$

REFLECT

Even though the frequencies in the carbon spectrum are always larger than the frequencies in the hydrogen spectrum by this ratio, the absolute shift in the frequency depends on the numerical value of the frequency of the hydrogen line.

26.92

SET UP

The Rydberg formula, $\frac{1}{\lambda} = R_{\mathrm{H}}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$, where $R_{\mathrm{H}} = 1.09737 \times 10^7\ \mathrm{m}^{-1}$, gives the wavelengths of the photons associated with a transition from the mth to the nth energy level. The shortest wavelength (that is, highest energy) photon emitted in the Brackett ($n = 4$) and Pfund ($n = 5$) series corresponds to a transition from $m = \infty$. The longest wavelength (that is, lowest energy) photon emitted in the Brackett ($n = 4$) and Pfund ($n = 5$) series corresponds to a transition from $m = 5$ and $m = 6$, respectively.

SOLVE

Part a)
Shortest:

$$\lambda = \left[R_{\mathrm{H}}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)\right]^{-1} = \left[(1.09737 \times 10^7\ \mathrm{m}^{-1})\left(\frac{1}{(4)^2} - \frac{1}{(\infty)^2}\right)\right]^{-1}$$

$$= \boxed{1.458 \times 10^{-6}\ \mathrm{m} = 1458\ \mathrm{nm}}$$

Longest:

$$\lambda = \left[R_{\mathrm{H}}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)\right]^{-1} = \left[(1.09737 \times 10^7\ \mathrm{m}^{-1})\left(\frac{1}{(4)^2} - \frac{1}{(5)^2}\right)\right]^{-1}$$

$$= \boxed{4.050 \times 10^{-6}\ \mathrm{m} = 4050\ \mathrm{nm}}$$

Part b)
Shortest:

$$\lambda = \left[R_{\mathrm{H}}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)\right]^{-1} = \left[(1.09737 \times 10^7\ \mathrm{m}^{-1})\left(\frac{1}{(5)^2} - \frac{1}{(\infty)^2}\right)\right]^{-1}$$

$$= \boxed{2.278 \times 10^{-6}\ \mathrm{m} = 2278\ \mathrm{nm}}$$

Longest:

$$\lambda = \left[R_{\mathrm{H}}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)\right]^{-1} = \left[(1.09737 \times 10^7\ \mathrm{m}^{-1})\left(\frac{1}{(5)^2} - \frac{1}{(6)^2}\right)\right]^{-1}$$

$$= \boxed{7.456 \times 10^{-6}\ \mathrm{m} = 7456\ \mathrm{nm}}$$

REFLECT

The smallest energy gap corresponds to consecutive energy levels.

26.93

SET UP

A hydrogen atom has an electron in the $n = 2$ state when it absorbs a photon, which promotes the electron to the $n = 4$ state. This electron will then emit light and relax back

down to the $n = 1$ state. We can use the Rydberg equation to calculate the wavelength of the absorbed photon, as well as the wavelengths of the possible emitted photons. Since the Rydberg equation is used to calculate the wavelength of the *emitted* photon (*i.e.*, $n > m$), we need to either take the absolute value or reverse the terms to correctly calculate the wavelength of the *absorbed* photon.

SOLVE

Part a)

$$\frac{1}{\lambda} = R_{\mathrm{H}}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

$$\lambda = \left[R_{\mathrm{H}}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)\right]^{-1} = \left|\left[(1.09737 \times 10^7\ \mathrm{m}^{-1})\left(\frac{1}{(4)^2} - \frac{1}{(2)^2}\right)\right]^{-1}\right| =$$

$$= \boxed{4.86 \times 10^{-7}\ \mathrm{m} = 486\ \mathrm{nm}}$$

Part b)

Emission process #1: $n = 4 \rightarrow n = 3 \rightarrow n = 2 \rightarrow n = 1$

$$\lambda_{4\rightarrow 3} = \left[(1.09737 \times 10^7\ \mathrm{m}^{-1})\left(\frac{1}{3^2} - \frac{1}{4^2}\right)\right]^{-1} = 1.875 \times 10^{-6}\ \mathrm{m} = \boxed{1875\ \mathrm{nm}}$$

$$\lambda_{3\rightarrow 2} = \left[(1.09737 \times 10^7\ \mathrm{m}^{-1})\left(\frac{1}{2^2} - \frac{1}{3^2}\right)\right]^{-1} = 6.56 \times 10^{-7}\ \mathrm{m} = \boxed{656\ \mathrm{nm}}$$

$$\lambda_{2\rightarrow 1} = \left[(1.09737 \times 10^7\ \mathrm{m}^{-1})\left(\frac{1}{1^2} - \frac{1}{2^2}\right)\right]^{-1} = 1.22 \times 10^{-7}\ \mathrm{m} = \boxed{122\ \mathrm{nm}}$$

Emission process #2: $n = 4 \rightarrow n = 2 \rightarrow n = 1$

$$\lambda_{4\rightarrow 2} = \left[(1.09737 \times 10^7\ \mathrm{m}^{-1})\left(\frac{1}{2^2} - \frac{1}{4^2}\right)\right]^{-1} = 4.86 \times 10^{-6}\ \mathrm{m} = \boxed{486\ \mathrm{nm}}$$

$$\lambda_{2\rightarrow 1} = 122\ \mathrm{nm}$$

Emission process #3: $n = 4 \rightarrow n = 3 \rightarrow n = 1$

$$\lambda_{4\rightarrow 3} = 1875\ \mathrm{nm}$$

$$\lambda_{3\rightarrow 1} = \left[(1.09737 \times 10^7\ \mathrm{m}^{-1})\left(\frac{1}{1^2} - \frac{1}{3^2}\right)\right]^{-1} = 1.03 \times 10^{-7}\ \mathrm{m} = \boxed{103\ \mathrm{nm}}$$

Emission process #4: $n = 4 \rightarrow n = 1$

$$\lambda_{4\rightarrow 2} = \left[(1.09737 \times 10^7\ \mathrm{m}^{-1})\left(\frac{1}{1^2} - \frac{1}{4^2}\right)\right]^{-1} = 9.72 \times 10^{-8}\ \mathrm{m} = \boxed{97.2\ \mathrm{nm}}$$

REFLECT

We don't know ahead of time which relaxation process the electron will undergo on its way back to the $n = 1$ state; all are possible, but some are more probable than others.

26.94

SET UP

The electron of a hydrogen atom starts in the $n = 3$ energy level. The magnitude of the energy required to ionize this atom can be found through $|\Delta E| = \left|\left(\frac{1}{n^2} - \frac{1}{m^2}\right)E_0\right|$, where the initial energy level is $m = 3$, the final energy level is $n = \infty$, and $E_0 = 13.6 \text{ eV}$.

SOLVE

$$\Delta E = \left(\frac{1}{n^2} - \frac{1}{m^2}\right)E_0$$

$$|\Delta E| = \left|\left(\frac{1}{\infty^2} - \frac{1}{3^2}\right)(13.6 \text{ eV})\right| = \boxed{1.51 \text{ eV}}$$

REFLECT

Removing an electron from an atom is equivalent to saying the electron is infinitely far away from the nucleus.

26.95

SET UP

We can use $E_n = -\frac{E_0}{Z^2n^2}$, where $E_0 = 13.6 \text{ eV}$, to find the energies of the first ten energy levels of an He^+ ion ($Z = 2$).

SOLVE

Energy levels (starting with $n = 10$):

$$E_{10} = -\frac{13.6 \text{ eV}}{(2)^2(10)^2} = \boxed{-0.0340 \text{ eV}}$$

$$E_9 = -\frac{13.6 \text{ eV}}{(2)^2(9)^2} = \boxed{-0.0420 \text{ eV}}$$

$$E_8 = -\frac{13.6 \text{ eV}}{(2)^2(8)^2} = \boxed{-0.0531 \text{ eV}}$$

$$E_7 = -\frac{13.6 \text{ eV}}{(2)^2(7)^2} = \boxed{-0.0694 \text{ eV}}$$

$$E_6 = -\frac{13.6 \text{ eV}}{(2)^2(6)^2} = \boxed{-0.0944 \text{ eV}}$$

$$E_5 = -\frac{13.6 \text{ eV}}{(2)^2(5)^2} = \boxed{-0.136 \text{ eV}}$$

$$E_4 = -\frac{13.6 \text{ eV}}{(2)^2(4)^2} = \boxed{-0.213 \text{ eV}}$$

$$E_3 = -\frac{13.6 \text{ eV}}{(2)^2(3)^2} = \boxed{-0.378 \text{ eV}}$$

$$E_2 = -\frac{13.6\text{ eV}}{(2)^2(2)^2} = \boxed{-0.850\text{ eV}}$$

$$E_1 = -\frac{13.6\text{ eV}}{(2)^2(1)^2} = \boxed{-3.40\text{ eV}}$$

Energy level diagram:

$$E_{10} = -0.0340\text{ eV}$$

$$E_9 = -0.0420\text{ eV}$$

$$E_8 = -0.0531\text{ eV}$$

$$E_7 = -0.0944\text{ eV}$$

$$E_6 = -0.0944\text{ eV}$$

$$E_5 = -0.136\text{ eV}$$

$$E_4 = -0.213\text{ eV}$$

$$E_3 = -0.378\text{ eV}$$

$$E_2 = -0.850\text{ eV}$$

$$E_1 = -3.40\text{ eV}$$

REFLECT

The formula $E_n = -\dfrac{E_0}{Z^2n^2}$ works for any hydrogenic species, which means any species that has only one electron, such as H, He^+, Li^{2+}, Be^{3+}, and so on.

26.96

SET UP

A parameter x is allowed to have any value from $x = 0$ to $x = 1$. The probability of observing any one specific value is proportional to the cube of the value, $P(x) = 4x^3$. Since we are able to observe *any* value between $x = 0$ and $x = 1$, the integral over that interval of $P(x)$ with respect to x must be equal to 1 (that is, 100%). To find the average value of x and x^2 over this interval, we need to integrate the products $xP(x)$ and $x^2P(x)$, respectively, with respect to x.

SOLVE

Part a)

$$P = \int_0^1 P(x)dx = \int_0^1 4x^3dx = [x^4]_0^1 = (1)^4 - 0 = \boxed{1}$$

Part b)

$$\bar{x} = \int_0^1 xP(x)dx = \int_0^1 x(4x^3)dx = 4\int_0^1 x^4 dx = 4\left[\frac{1}{5}x^5\right]_0^1 = \frac{4}{5}[(1)^4 - 0] = \boxed{\frac{4}{5}}$$

Part c)

$$\overline{(x^2)} = \int_0^1 x^2 P(x)dx = \int_0^1 x^2(4x^3)dx = 4\int_0^1 x^5 dx = 4\left[\frac{1}{6}x^6\right]_0^1 = \frac{2}{3}[(1)^6 - 0] = \boxed{\frac{2}{3}}$$

REFLECT

The average value of x should be closer to $x = 1$ due to the larger values of the function in this region. The average value of x^2 should be smaller than that of x because a number less than 1 becomes smaller when you square it.

26.97

SET UP

The energy probability of gas molecules inside a blackbody cavity is $P(E) = \frac{1}{kT}e^{-\frac{E}{kT}}$; the energy can have any value from $E = 0$ to $E = \infty$. Since we are able to observe *any* value between $E = 0$ to $E = \infty$., the integral over that interval of $P(E)$ with respect to E must be equal to 1 (that is, 100%). To find the average value of E and E^2 over this interval, we need to integrate the products $EP(E)$ and $E^2P(E)$, respectively, with respect to E. We will need to integrate by parts to solve these integrals: $\int_a^b udv = uv|_a^b - \int_a^b vdu$.

SOLVE

Part a)

$$P = \int_0^\infty P(E)dE = \int_0^\infty \frac{1}{kT}e^{-\frac{E}{kT}}\,dE = \frac{1}{kT}\left[-kTe^{-\frac{E}{kT}}\right]_0^\infty = -\left[e^{-\frac{\infty}{kT}} - e^0\right] = -[0 - 1] = \boxed{1}$$

Part b)

$$\begin{aligned}\bar{E} &= \int_0^\infty EP(E)dE = \int_0^\infty \frac{E}{kT}e^{-\frac{E}{kT}}\,dE = \frac{1}{kT}\left[\left(-kTEe^{-\frac{E}{kT}}\right)_0^\infty - \int_0^\infty (-kT)e^{-\frac{E}{kT}}\,dE\right] \\ &= \left(-(\infty)e^{-\frac{\infty}{kT}} + (0)e^{-\frac{0}{kT}}\right) + \left[-kTe^{-\frac{E}{kT}}\right]_0^\infty = (0+0) - kT\left[e^{-\frac{\infty}{kT}} - e^{-\frac{0}{kT}}\right] \\ &= -kT[0-1] = \boxed{kT}\end{aligned}$$

Part c)

$$\begin{aligned}\overline{(E^2)} &= \int_0^\infty E^2P(E)dE = \int_0^\infty \frac{E^2}{kT}e^{-\frac{E}{kT}}\,dE = \frac{1}{kT}\left[\left(-kTE^2e^{-\frac{E}{kT}}\right)_0^\infty - \int_0^\infty (-2kT)Ee^{-\frac{E}{kT}}\,dE\right] \\ &= \left(-(\infty)^2e^{-\frac{\infty}{kT}} + (0)^2e^{-\frac{0}{kT}}\right) + 2kT\int_0^\infty \frac{1}{kT}Ee^{-\frac{E}{kT}}\,dE = (0+0) + 2kT[kT] = \boxed{2k^2T^2}\end{aligned}$$

REFLECT

Rather than re-solve the integral, we invoked our result from part (b) since the same integral arose in our solution in part (c).

26.98

SET UP

We are asked to calculate the energy of UVB rays having wavelengths of 270×10^{-9} m and 300×10^{-9} m in both joules and electron volts. The energy of a photon in terms of its wavelength is $E = \frac{hc}{\lambda}$. The conversion between joules and electron volts is $1 \text{ eV} = 1.60 \times 10^{-19}$ J.

SOLVE

270 nm:

$$E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}\ \text{J} \cdot \text{s})\left(3.00 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{270 \times 10^{-9}\ \text{m}} = \boxed{7.36 \times 10^{-19}\ \text{J}} \times \frac{1\ \text{eV}}{1.60 \times 10^{-19}\ \text{J}} = \boxed{4.59\ \text{eV}}$$

300 nm:

$$E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}\ \text{J} \cdot \text{s})\left(3.00 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{300 \times 10^{-9}\ \text{m}} = \boxed{6.62 \times 10^{-19}\ \text{J}} \times \frac{1\ \text{eV}}{1.60 \times 10^{-19}\ \text{J}} = \boxed{4.13\ \text{eV}}$$

REFLECT

Since energy is inversely proportional to wavelength, the smaller the wavelength, the larger the energy.

26.99

SET UP

A HeNe laser emits red photons with a wavelength of $\lambda = 632.8 \times 10^{-9}$ m and operates at a power of 0.500×10^{-3} W. The number of photons delivered by the laser per second is equal to the power divided by the energy of one photon, $E = \frac{hc}{\lambda}$. Since all of the photons are absorbed by the target, the magnitude of the total linear momentum transferred to the target is given by the number of photons per second multiplied by the magnitude of the linear momentum for a single photon, $p = \frac{h}{\lambda}$.

SOLVE

Part a)
Energy of one photon:

$$E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}\ \text{J} \cdot \text{s})\left(3.00 \times 10^{8}\frac{\text{m}}{\text{s}}\right)}{632.8 \times 10^{-9}} = 3.14 \times 10^{-19}\ \text{J}$$

Number of photons per second:

$$\frac{\Delta n_{\text{photon}}}{\Delta t} = \frac{0.500 \times 10^{-3}\ \text{J}}{1\ \text{s}} \times \frac{1\ \text{photon}}{3.14 \times 10^{-19}\ \text{J}} = \boxed{1.59 \times 10^{15} \frac{\text{photons}}{\text{s}}}$$

Part b)

$$\frac{\Delta p}{\Delta t} = \frac{\left(\frac{\Delta n_{\text{photon}}}{\Delta t}\right)h}{\lambda} = \frac{\left(1.59 \times 10^{15} \frac{\text{photons}}{\text{s}}\right)(6.63 \times 10^{-34}\ \text{J} \cdot \text{s})}{632.8 \times 10^{-9}\ \text{m}} = \boxed{1.67 \times 10^{-12} \frac{\text{kg} \cdot \text{m}}{\text{s}}}$$

REFLECT

The fact that photons carry momentum means that electromagnetic radiation can exert a pressure on a surface; this is referred to as "radiation pressure."

26.100

SET UP

We are asked to find the wavelengths associated with photons of energy $E_{700\ \text{GeV}} = 700 \times 10^9$ eV and $E_{5\ \text{TeV}} = 5.00 \times 10^{12}$ eV; the energy of a photon is equal to $E = \frac{hc}{\lambda}$. We can use this same expression to calculate the ratio of the energy of a photon of wavelength 500×10^{-9} m to the energy of a photon of energy $E_{5\ \text{TeV}}$.

SOLVE

Part a)
Wavelength of 700 GeV:

$$E_{700\ \text{GeV}} = \frac{hc}{\lambda_{700\ \text{GeV}}}$$

$$\lambda_{700\ \text{GeV}} = \frac{hc}{E_{700\ \text{GeV}}} = \frac{1240\ \text{eV} \cdot \text{nm}}{700 \times 10^9\ \text{eV}} = \boxed{1.77 \times 10^{-9}\ \text{nm} = 1.77 \times 10^{-18}\ \text{m}}$$

Wavelength of 5.00 TeV:

$$E_{5\ \text{TeV}} = \frac{hc}{\lambda_{5\ \text{TeV}}}$$

$$\lambda_{5\ \text{TeV}} = \frac{hc}{E_{5\ \text{TeV}}} = \frac{1240\ \text{eV} \cdot \text{nm}}{5.00 \times 10^{12}\ \text{eV}} = \boxed{2.48 \times 10^{-10}\ \text{nm} = 2.48 \times 10^{-19}\ \text{m}}$$

Part b)

$$\frac{E_{5\ \text{TeV}}}{E_{500\ \text{nm}}} = \frac{\left(\frac{hc}{\lambda_{5\ \text{TeV}}}\right)}{\left(\frac{hc}{\lambda_{500\ \text{nm}}}\right)} = \frac{\lambda_{500\ \text{nm}}}{\lambda_{5\ \text{TeV}}} = \frac{500 \times 10^{-9}\ \text{m}}{2.48 \times 10^{-19}\ \text{m}} = \boxed{2.02 \times 10^{12}}$$

REFLECT

The wavelength for a 5-TeV photon is 12 orders of magnitude smaller than the wavelength of a 500-nm photon, so the energy of the 5-TeV should be 12 orders of magnitude larger.

26.101

SET UP

The Compton scattering formula, $\Delta\lambda = \lambda_f - \lambda_i = \frac{h}{cm_e}(1 - \cos(\theta))$, can be derived by treating the collision between the incoming photon and the initially stationary electron as an elastic collision, which means both momentum and energy are conserved during the process. We will assume the photon has an initial wavelength of λ_i and is initially traveling in the $+x$ direction toward the stationary electron, which has a rest energy of $m_e c^2$. After the collision, the photon has a final wavelength of λ_i and moves off at an angle λ above the $+x$-axis, whereas the electron moves off with some momentum p_e at an angle of λ below the $+x$-axis. We can then rearrange the three conditions—conservation of momentum in the x direction, conservation of momentum in the y direction, and conservation of energy—to arrive at the Compton scattering formula.

SOLVE

Conservation of linear momentum, x direction:

$$\frac{h}{\lambda_i} = \frac{h}{\lambda_f}\cos(\theta) + p_e\cos(\phi)$$

$$p_e\cos(\phi) = \frac{h}{\lambda_i} - \frac{h}{\lambda_f}\cos(\theta)$$

Conservation of linear momentum, y direction:

$$0 = \frac{h}{\lambda_f}\sin(\theta) - p_e\sin(\phi)$$

$$\frac{h}{\lambda_f}\sin(\theta) = p_e\sin(\phi)$$

Conservation of energy:

$$\frac{hc}{\lambda_i} + m_e c^2 = \frac{hc}{\lambda_f} + \sqrt{p_e^2c^2 + m_e^2c^4}$$

$$\sqrt{p_e^2c^2 + m_e^2c^4} = \frac{hc}{\lambda_i} + m_e c^2 - \frac{hc}{\lambda_f}$$

Finding the total momentum:

$$p_e^2(\cos^2(\phi) + \sin^2(\phi)) = h^2\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_f}\cos(\theta)\right)^2 + \frac{1}{\lambda_f^2}\sin^2(\theta)$$

$$p_e^2 = h^2\left(\frac{1}{\lambda_i^2} - \frac{2}{\lambda_i\lambda_f}\cos(\theta) + \frac{1}{\lambda_f^2}\right)$$

Squaring the conservation of energy equation:

$$p_e^2c^2 + m_e^2c^4 = \frac{h^2c^2}{\lambda_i^2} - \frac{2h^2c^2}{\lambda_i\lambda_f} + \frac{h^2c^2}{\lambda_f^2} + 2\left(\frac{hc}{\lambda_i} - \frac{hc}{\lambda_f}\right)m_e c^2 + m_e^2c^4$$

$$p_e^2 = \frac{h^2}{\lambda_i^2} - \frac{2h^2}{\lambda_i\lambda_f} + \frac{h^2}{\lambda_f^2} + 2\left(\frac{hc}{\lambda_i} - \frac{hc}{\lambda_f}\right)m_e$$

Setting the two expressions for the momentum squared equal:

$$h^2\left(\frac{1}{\lambda_i^2} - \frac{2}{\lambda_i\lambda_f}\cos(\theta) + \frac{1}{\lambda_f^2}\right) = \frac{h^2}{\lambda_i^2} - \frac{2h^2}{\lambda_i\lambda_f} + \frac{h^2}{\lambda_f^2} + 2\left(\frac{hc}{\lambda_i} - \frac{hc}{\lambda_f}\right)m_e$$

$$h^2\left(-\frac{2}{\lambda_i\lambda_f}\cos(\theta)\right) = -\frac{2h^2}{\lambda_i\lambda_f} + 2\left(\frac{hc}{\lambda_i} - \frac{hc}{\lambda_f}\right)m_e$$

$$h\left(-\frac{1}{\lambda_i\lambda_f}\cos(\theta)\right) = -\frac{h}{\lambda_i\lambda_f} + \left(\frac{c}{\lambda_i} - \frac{c}{\lambda_f}\right)m_e$$

$$-h\cos(\theta) = -h + cm_e\lambda_i\lambda_f\left(\frac{\lambda_f - \lambda_i}{\lambda_i\lambda_f}\right)$$

$$1 - \cos(\theta) = \frac{cm_e}{h}(\lambda_f - \lambda_i)$$

$$\boxed{\lambda_f - \lambda_i = \frac{h}{cm_e}(1 - \cos(\theta))}$$

REFLECT

We must consider relativistic effects in order to derive the Compton scattering formula.

26.102

SET UP

A photon ($f_i = 4.81 \times 10^{19}$ Hz) Compton scatters off of a stationary electron at an angle of $\theta = 125°$. The energy gained by the electron is equal to the change in the energy of the photon after the scattering process. The percent energy loss for the photon is equal to the energy gained by the electron divided by the initial energy of the photon multiplied by 100%.

SOLVE

Part a)

Initial wavelength of the photon:

$$\lambda_i = \frac{c}{f_i} = \frac{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)}{4.81 \times 10^{19}\ \text{Hz}} = 6.24 \times 10^{-12}\ \text{m} = 0.00624\ \text{nm}$$

Final wavelength of the photon:

$$\Delta\lambda = \lambda_f - \lambda_i = \lambda_C(1 - \cos(\theta))$$

$$\lambda_f = \lambda_i + \lambda_C(1 - \cos(\theta)) = (0.00624\ \text{nm}) + (0.00243\ \text{nm})(1 - \cos(125°)) = 0.01006\ \text{nm}$$

Energy gained by the electron:

$$\Delta E = \frac{hc}{\lambda_i} - \frac{hc}{\lambda_f} = hc\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_f}\right)$$

$$= (6.63 \times 10^{-34}\ \text{J}\cdot\text{s})\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)\left(\frac{1}{6.24 \times 10^{-12}\ \text{m}} - \frac{1}{1.006 \times 10^{-11}\ \text{m}}\right)$$

$$= \boxed{1.21 \times 10^{-14}\ \text{J}}$$

Part b)

$$\frac{\Delta E}{E_\mathrm{i}} = \frac{\Delta E}{\left(\frac{hc}{\lambda_\mathrm{i}}\right)} = \frac{\lambda_\mathrm{i}(\Delta E)}{hc} = \frac{(6.24 \times 10^{-12}\ \mathrm{m})(1.21 \times 10^{-14}\ \mathrm{J})}{(6.63 \times 10^{-34}\ \mathrm{J \cdot s})\left(3.00 \times 10^{8} \frac{\mathrm{m}}{\mathrm{s}}\right)} = 0.380 = \boxed{38.0\%}$$

REFLECT

We would need to invoke relativity to calculate the speed of the electron after the collision.

26.103

SET UP

The speed of an electron in hydrogen traveling in a circular orbit of radius r_n is equal to $v_n = \frac{2\pi r_n}{T_n}$. From Equations 26-26 and 26-33, the radius and speed are also equal to $r_n = \frac{n^2\hbar^2}{ke^2 m_\mathrm{e}}$ and $v_n = \frac{ke^2}{n\hbar}$. Combining these with the fact that $f_n = \frac{1}{T_n}$ will allow us to derive an expression for the frequency of an electron revolving in the nth orbit of a hydrogenic atom.

SOLVE

$$v_n = \frac{2\pi r_n}{T_n}$$

$$f_n = \frac{1}{T_n} = \frac{v_n}{2\pi r_n} = \left(\frac{ke^2}{n\hbar}\right)\left(\frac{1}{2\pi}\right)\left(\frac{ke^2 m_\mathrm{e}}{n^2\hbar^2}\right) = \boxed{\frac{k^2e^4 m_\mathrm{e}}{2\pi n^3\hbar^3}}$$

REFLECT

We can also represent the frequency in terms of the Bohr radius: $f_n = \frac{ke^2}{2\pi n^3 \hbar a_0}$

26.104

SET UP

The electron in a hydrogen atom makes a transition from $m = 5$ to $n = 1$. The energy of the emitted photon is equal to $\Delta E = \left(\frac{1}{n^2} - \frac{1}{m^2}\right)E_0$, where $E_0 = 13.6$ eV. This photon strikes a silicon ($\Phi_0 = 4.8$ eV) surface. To determine if a photoelectron is ejected from the silicon, we need to compare the energy of the photon to the work function. If the energy of the photon is larger than the work function, then an electron will be ejected from the metal. If this is the case, the maximum kinetic energy of the electron is equal to the difference in energy between the absorbed photon and the work function.

SOLVE

Energy of the emitted photon:

$$\Delta E = \left(\frac{1}{n^2} - \frac{1}{m^2}\right)E_0 = \left(\frac{1}{(1)^2} - \frac{1}{(5)^2}\right)(13.6\ \mathrm{eV}) = 13.1\ \mathrm{eV}$$

This is larger than the work function of silicon, so a photoelectron will be ejected from the silicon.
Maximum kinetic energy;

$$K_{\text{max}} = E - \Phi_0 = (13.1\text{ eV}) - (4.8\text{ eV}) = \boxed{8.3\text{ eV}}$$

REFLECT
Since the electron relaxes back to the ground state, this emitted photon would correspond to a transition in the Lyman series.

26.105

SET UP
We can compare the algebraic expressions for the electrostatic force and the gravitational force to see what would happen to the expressions for the radius and energy of an electron in hydrogen if it were bound to the proton by gravity rather than electrostatics. Once we derive new expressions for these, we can plug in $Z = 1$ and $n = 1$ to find the values for the radius and energy of the first orbit.

SOLVE
Magnitude of the electrostatic force:

$$F = \frac{ke^2}{r^2}$$

Magnitude of the gravitational force:

$$F = \frac{Gm_e m_p}{r^2}$$

By comparing the two functional forms, we see that we should replace ke^2 with $Gm_e m_p$ in our expressions for the radius and energy of the first orbit if we replace the electrostatic force with the gravitational force.

Part a)
Radius:

$$r_n = \frac{n^2\hbar^2}{Zm_e(ke^2)} \rightarrow r_n = \frac{n^2\hbar^2}{Zm_e(Gm_e m_p)} = \frac{n^2\hbar^2}{ZGm_e^2 m_p}$$

Radius for $n = 1$ of hydrogen:

$$r_1 = \frac{(1)^2(1.055 \times 10^{-34}\text{ J}\cdot\text{s})^2}{(1)\left(6.67 \times 10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(9.11 \times 10^{-31}\text{ kg})^2(1.67 \times 10^{-27}\text{ kg})} = \boxed{1.20 \times 10^{29}\text{ m}}$$

Part b)
Energy:

$$E_n = -\frac{Z^2 m_e(ke^2)^2}{2n^2\hbar^2} \rightarrow E_n = -\frac{Z^2 m_e(Gm_e m_p)^2}{2n^2\hbar^2} = -\frac{Z^2G^2m_e^3m_p^2}{2n^2\hbar^2}$$

Energy for $n = 1$ of hydrogen:

$$E_1 = -\frac{(1)^2\left(6.67 \times 10^{-11}\frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}\right)^2(9.11 \times 10^{-31}\ \mathrm{kg})^3(1.67 \times 10^{-27}\ \mathrm{kg})^2}{2(1)^2(1.055 \times 10^{-34}\ \mathrm{J \cdot s})^2} = \boxed{-4.21 \times 10^{-97}\ \mathrm{J}}$$

REFLECT

Our answer for the radius of the first Bohr orbit is clearly unreasonable, which means our assumption that the electron is bound to the atom by gravity is wrong.

26.106

SET UP

A bacterium is about 2.0×10^{-6} m long. We want to study it with photons of this wavelength or electrons having that de Broglie wavelength. The energy of a photon is given by $E_{\mathrm{photon}} = \frac{hc}{\lambda}$. The kinetic energy of an electron is related to its momentum through the de Broglie wavelength. Ideally, the particle we use to study the bacterium should not affect or damage the sample.

SOLVE

Part a)
Photon energy:

$$E_{\mathrm{photon}} = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}\ \mathrm{J \cdot s})\left(3.00 \times 10^8 \frac{\mathrm{m}}{\mathrm{s}}\right)}{2.0 \times 10^{-6}\ \mathrm{m}} = \boxed{9.9 \times 10^{-20}\ \mathrm{J}}$$

Electron energy:

$$\lambda = \frac{h}{p} = \frac{h}{m_{\mathrm{e}}v}$$

$$v = \frac{h}{m_{\mathrm{e}}\lambda}$$

$$K = \frac{1}{2}m_{\mathrm{e}}v^2 = \frac{1}{2}m_{\mathrm{e}}\left(\frac{h}{m_{\mathrm{e}}\lambda}\right)^2 = \frac{h^2}{2m_{\mathrm{e}}\lambda^2} = \frac{(6.63 \times 10^{-34}\ \mathrm{J \cdot s})^2}{2(9.11 \times 10^{-31}\ \mathrm{kg})(2.0 \times 10^{-6}\ \mathrm{m})^2} = \boxed{6.0 \times 10^{-26}\ \mathrm{J}}$$

Part b) The photon energy is over a million times greater than that of the electron. Therefore, the photon would be much more likely to damage (or destroy) the bacterium in the process of studying it. Thus, it would be better to use the $\boxed{\text{electron}}$.

REFLECT

The wavelength of the light or electrons should be on the order of the size scale we are interested in probing.

26.107

SET UP

A laboratory oven contains hydrogen molecules (mass $m_{\mathrm{H_2}}$) and oxygen molecules (mass $m_{\mathrm{O_2}} = 16m_{\mathrm{H_2}}$) at a constant temperature. Since the temperature of the molecules is the same, the mean kinetic energy of the two species must also be the same. After writing the kinetic energies in terms of the respective momenta, we can determine the ratio of the de Broglie wavelength of the hydrogen molecules to the de Broglie wavelength of the oxygen molecules.

SOLVE

$$K_{\mathrm{H_2}} = K_{\mathrm{O_2}}$$

$$\frac{p_{\mathrm{H_2}}^2}{2m_{\mathrm{H_2}}} = \frac{p_{\mathrm{O_2}}^2}{2m_{\mathrm{O_2}}}$$

$$\frac{\left(\frac{h}{\lambda_{\mathrm{H_2}}}\right)^2}{m_{\mathrm{H_2}}} = \frac{\left(\frac{h}{\lambda_{\mathrm{O_2}}}\right)^2}{m_{\mathrm{O_2}}}$$

$$\frac{1}{m_{\mathrm{H_2}}\lambda_{\mathrm{H_2}}^2} = \frac{1}{m_{\mathrm{O_2}}\lambda_{\mathrm{O_2}}^2}$$

$$\frac{\lambda_{\mathrm{H_2}}^2}{\lambda_{\mathrm{O_2}}^2} = \frac{m_{\mathrm{O_2}}}{m_{\mathrm{H_2}}} = \frac{16m_{\mathrm{H_2}}}{m_{\mathrm{H_2}}} = 16$$

$$\boxed{\frac{\lambda_{\mathrm{H_2}}}{\lambda_{\mathrm{O_2}}} = 4}$$

REFLECT

Since the hydrogen molecules and oxygen molecules each are in thermal equilibrium with the oven, they must be in thermal equilibrium with themselves. This is referred to as the zeroth law of thermodynamics.

26.108

SET UP

We need to show Bohr's hypothesis, $L_n = n\hbar$, by fitting circular standing waves into the orbits of the Bohr model. The circumference of the first Bohr orbit ($n = 1$) fits exactly one de Broglie wavelength; the circumference of the second Bohr orbit ($n = 2$) fits exactly two de Broglie wavelengths, and so on. Following this pattern, we see that there are n de Broglie wavelengths that fit in the circumference of the nth Bohr orbit. Invoking the definitions of the de Broglie wavelength and the angular momentum, we can rearrange our expression to arrive at Bohr's hypothesis.

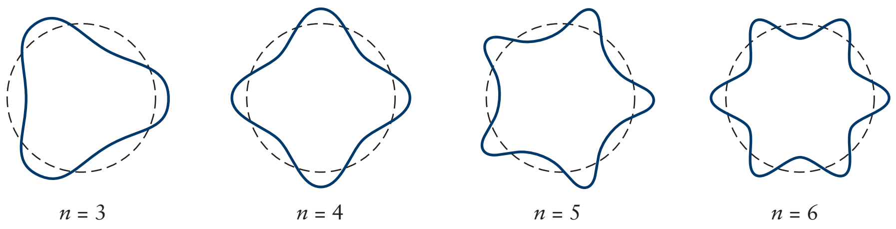

Figure 26-3 Problem 108

SOLVE

$n = 1$:

$$\lambda_1 = 2\pi r_1$$

$n = 2$:

$$2\lambda_2 = 2\pi r_2$$

$n = 3$:

$$3\lambda_3 = 2\pi r_3$$

In general:

$$n\lambda_n = 2\pi r_n$$

$$n\left(\frac{h}{mv_n}\right) = 2\pi r_n$$

$$mv_n r_n = n\left(\frac{h}{2\pi}\right)$$

$$\boxed{L_n = n\hbar}$$

REFLECT

The quantization in the system arises from the fact that only integral values of the de Broglie wavelength can match up with each circumference.

26.109

SET UP

A gamma ray is converted into an electron and a positron, each of which have a mass $m_e = 9.11 \times 10^{-31}$ kg. The maximum wavelength of a gamma ray is the same as asking, "What's the least energetic gamma ray that can produce an electron and positron, each at rest?" The energy of the gamma ray is completely converted into the mass of the electron and the positron. We can set the sum of the rest energies of the electron and the positron equal to the energy of the gamma ray in order to solve for the maximum possible wavelength.

SOLVE

Energy of the gamma ray:

$$E = \frac{hc}{\lambda}$$

Minimum energy to create an electron and a positron:

$$E = m_e c^2 + m_p c^2 = m_e c^2 + m_e c^2 = 2m_e c^2$$

Minimum wavelength for the gamma ray:

$$\frac{hc}{\lambda} = 2m_e c^2$$

$$\lambda = \frac{h}{2m_e c} = \frac{6.63 \times 10^{-34}\ \mathrm{J \cdot s}}{2(9.11 \times 10^{-31}\ \mathrm{kg})\left(3.00 \times 10^{8} \frac{\mathrm{m}}{\mathrm{s}}\right)} = \boxed{1.21 \times 10^{-12}\ \mathrm{m} = 1.21\ \mathrm{pm}}$$

REFLECT

In the electromagnetic spectrum, gamma rays have a wavelength less than 10 pm, so our result is reasonable.

Get Help: P'Cast 26.9 – A Mercury Line

Chapter 27
Nuclear Physics

Conceptual Questions

27.1 Isotopes are nuclides with the same atomic number, Z, but a different number of neutrons, N.

27.2 The atomic number is equal to the number of protons in the nucleus. The mass number is equal to the number of protons plus neutrons in the nucleus.

27.3 The volume of the spherical nucleus is directly proportional to the number of nucleons inside; that is, $\frac{4}{3}\pi r^3$ is directly proportional to A. Thus, $r \propto A^{\frac{1}{3}}$.

27.4 Part a) "Neutron rich" refers to the fact that heavier nuclei (larger than $A \sim 40$) possess a greater number of neutrons than protons.

Part b) Neutrons are important in order to balance out the increasing, repulsive Coulomb force between the positive protons. Since the neutrons are neutral, they do not repel one another. However, they do contribute to the attractive nuclear forces between the nucleons.

27.5 Binding energy is a measure of how difficult it is to break the nucleus into its constituent "pieces," similar to the way that the work function measures the difficulty in removing an electron from the surface of a conductor. Also, the binding energy is different for different isotopes, similar to the way that the work function is tabulated for different conductors that are studied in the photoelectric effects.

The work function is usually a few eV (maybe $\sim$ 10 eV at the maximum) while the binding energy in the nucleus is on the order of MeV. There is really no definitive means of predicting the work function. It is measured empirically, whereas for the binding energy we can use Einstein's equation ($E = \Delta mc^2$) to calculate the expected values.

27.6 The sum of the individual parts (the protons and neutrons) adds up to be a larger mass than the mass of the combined nucleus. In this way, the binding energy is always a positive quantity.

27.7 Using Figure 27-4 from the text, we see that Fe-56 is the most stable isotope.

27.8 The nuclear force is charge independent, so it doesn't have attractive or repulsive components like the electrostatic Coulomb force. In addition, it is very short-ranged. Outside of a few femtometers, the nuclear force quickly goes to zero. Inside about 0.7 fm, the force is repulsive between nucleons, so the radius of the

nucleus will never shrink to zero. The major competition inside the nucleus comes from the repulsive Coulomb force between the tightly packed, positively charged protons.

27.9 Certainly the central ideas of binding energy and "Q values" of nuclear reactions would not have been possible to develop without the $E = mc^2$ concept. The Manhattan project in World War II, where nuclear weapons were designed and successfully tested, would not have been possible had it not been for Einstein's theory of the equivalence of mass and energy (as well as his letter to President Roosevelt endorsing this project in the early 1940s).

27.10 In fission, a heavy nucleus splits, usually by absorbing a neutron, into two lighter nuclei with a corresponding release of neutrons and energy. In fusion, two light nuclei fuse to form a heavier nucleus with a corresponding release of energy.

27.11 Part a) Elements with a higher atomic number than iron are more likely to undergo fission.

Part b) Elements with a lower atomic number than iron are more likely to undergo fusion.

27.12 Part a) Our Sun, like most stars in the universe, is made of hydrogen. The hydrogen is being burned in extremely hot, high-pressure environments, and it fuses to helium. This process releases energy and repeats in a cyclic fashion.

Part b) Ultimately, the hydrogen will be depleted and the delicate balance between the outward tendency of the Sun to explode due to all this fusion energy being emitted and the inward pull of gravity will become unequal. Gravity will "win" and cause the Sun to radically reduce in size, which can lead to a nova, a supernova, or even a black hole. In hundreds of billions of years our solar system will cease to exist as we know it, and our star will evolve into a much more unstable central figure.

27.13 Consider the typical β decay of a neutron: $\mathrm{n} \to \mathrm{p} + \beta^- + \bar{\nu}_e$. If the electron ($\beta^-$) were the only decay product, application of conservation of energy and momentum to the two-body decay would require that the β^- particle be ejected with a single unique energy. Instead, we observe experimentally that β^- particles are produced with energies that range from zero to a maximum value. Further, because the original neutron had spin 1/2, conservation of angular momentum would be violated if the final decay products consisted of only the two particles p and β^-, each with spin 1/2.

27.14 The decay constant is equal to $\lambda = \dfrac{\ln(2)}{\tau_{1/2}}$. If this constant changed with temperature, there would be temperature-dependent half-lives! If this were the case, it would be very difficult to make predictions about the decay of radioactive isotopes.

27.15 As uranium begins to decay, the daughter nuclei are themselves often radioactive, decaying by various paths (which all lead down to a stable isotope in the periodic table).

So, at any instant there are isotopes from all the many possible decay modes of all the radioactive progeny of the parent nuclei.

27.16 Part a) Every carbon-based life form (for example, trees, people, animals) contains a small fraction of the isotope carbon-14, which is a beta emitter. In most living tissue (either plant or animal) the ratio between carbon-14 and carbon-12 is very small ($\sim 1.3 \times 10^{-12}$). The half-life of this carbon-14 is 5730 years. When a plant or animal dies, it ceases to respire and then stops exchanging and replenishing its carbon. Whatever carbon isotopes exist in the object at its death will begin to decay until it is studied by scientists at a much later time. Since the beta count rate can be measured very accurately, we can predict the amount of carbon-14 that is present today in the artifact, the amount it started with (knowing the activity that is associated with a particular-sized object), and then we can use the radioactive decay law to predict the time required for this decay to have occurred.

Part b) Most carbon-based life forms that are younger than about 25,000 years can be accurately dated.

27.17 Because the atomic masses include Z electron masses in their values, it is already taken into account. There is 1 electron mass included in the atomic mass of the proton. There is no need to add in another one to account for the beta particle.

27.18 Alpha particles are not very penetrating and not very harmful as long as they are external to the body. If they are inhaled, ingested, or put into the bloodstream, they are very dangerous. Beta particles are more penetrating than alpha particles and a bit more harmful but not terribly so. Gamma particles are very penetrating—they can only be reduced, not stopped—and very harmful.

Multiple-Choice Questions

27.19 **D** (neutrons and protons). The nuclear force is an attractive force between nucleons and is responsible for holding the atomic nucleus together.

27.20 **A** (always less than). The nucleus is lower in energy and, therefore, more stable than the individual separate nucleons. Mass and energy are equivalent through $E = mc^2$.

27.21 **D** (Both fusion and fission release energy). Energy is released in either process in order to form more stable products.

27.22 **E** (The total number of protons and the total number of neutrons both remain the same). Mass must be conserved.

27.23 **B** (less than). Energy is released in a spontaneous fission process. Mass and energy are equivalent through $E = mc^2$.

27.24 **C** (fusion reactions). The proton-proton cycle, which begins with two protons fusing together, is the set of fusion reactions describing the source of the Sun’s energy.

27.25 **B** (less than). Energy is released in a spontaneous fusion process. Mass and energy are equivalent through $E = mc^2$.

27.26 **C** (the half-life). The decay constant λ is equal to $\lambda = \dfrac{\ln(2)}{\tau_{1/2}}$.

27.27 **C** (decreases exponentially with time). The number of atoms in a radioactive sample as a function of time is given by $N(t) = N_0 e^{-\lambda t}$.

27.28 **C** (decreases exponentially with time). The decay rate for a radioactive sample as a function of time is given by $R(t) = R_0 e^{-\lambda t}$.

Estimation/Numerical Questions

27.29 The nuclear force is about 100 times larger than the electrostatic force.

27.30 The size of a nucleus is on the order of 10^{-15} m, whereas the size of an atom is on the order of 10^{-9} m; an atom is about a million times larger.

27.31 An atomic nucleus is about 5×10^{17} times more dense than an atom.

27.32 Energies associated with nuclear reactions are on the order of 10–100 MeV, whereas energies associated with chemical reactions are on the order of 1–10 eV.

27.33 The force is about 40 times greater when the nucleons are closer together. In some models, the force is actually repulsive for small separation distances (< 0.7 fm) and attractive for larger separation distances.

27.34 After 3 half-lives, 12.5% of the initial sample remains. After 7 half-lives, 0.8% of the original sample remains. After 10 half-lives, 0.098% of the original sample remains.

27.35 The basketball would have a mass of around 2×10^{16} kg.

27.36 Assuming the elemental composition of the body is 63% H, 26% O, 10% C, and 1% N, there would be about 2×10^{28} nuclei in a 50-kg body.

27.37

$$\frac{N(t)}{N_0} = \left[\frac{1}{2}\right]^{\frac{t}{\tau_{1/2}}}$$

$$t = \tau_{1/2} \frac{\ln\left(\dfrac{N(t)}{N_0}\right)}{\ln\left(\dfrac{1}{2}\right)}$$

Part a)

$$t = (1\text{ day})\frac{\ln\left(\frac{1}{2}\right)}{\ln(0.625)} = 0.678\text{ days}$$

Part b)

$$t = (1\text{ day})\frac{\ln(0.0625)}{\ln\left(\frac{1}{2}\right)} = 4\text{ days}$$

27.38

Binding energy:

$$E_B = (Nm_n + Zm_{^1H} - m_{atom})c^2$$

Part a)

Isotope	Mass (u)	Binding Energy (MeV)	BE/nucleon (MeV/nucleon)
hydrogen-1	1.007825	0	0
hydrogen-2	2.014102	2.224364	1.112182
hydrogen-3	3.016049	8.482097	2.827366
helium-3	3.016029	7.718315	2.572772
helium-4	4.002602	28.296837	7.074209
helium-6	6.018886	29.271092	4.878515
helium-8	8.033922	31.407851	3.925981
lithium-6	6.015121	31.995756	5.332626
lithium-7	7.016003	39.245530	5.606504
lithium-8	8.022486	41.278006	5.159751
lithium-9	9.026789	45.341139	5.037904
lithium-11	11.043897	45.547843	4.140713
beryllium-7	7.016928	37.601487	5.371641
beryllium-9	9.012174	58.172512	6.463612
beryllium-10	10.013534	64.977032	6.497703
beryllium-11	11.021657	65.481858	5.952896
beryllium-12	12.026921	68.649825	5.720819
beryllium-14	14.024866	86.706749	6.193339
boron-8	8.024605	37.739348	4.717418
boron-10	10.012936	64.751655	6.475165
boron-11	11.009305	76.205261	6.927751
boron-12	12.014352	79.575362	6.631280
boron-13	13.017780	84.453552	6.496427
boron-14	14.025404	85.423194	6.101657
boron-15	15.031100	88.188756	5.879250

Part b)

Isotope	Mass (u)	Binding Energy (MeV)	BE/nucleon (MeV/ nucleon)
radium-221	221.01391	1701.959454	7.701174
radium-223	223.018499	1713.827531	7.685325
radium-224	224.020187	1720.326521	7.680029
radium-226	226.025402	1731.611483	7.661998
radium-228	228.031064	1742.480067	7.642456
actinium-227	227.027749	1736.714207	7.650723
actinium-228	228.031015	802.177948	3.518324
thorium-227	227.027701	1735.976508	7.647474
thorium-228	228.028716	1743.102393	7.645186
thorium-229	229.031757	1748.341071	7.634677
thorium-230	230.033127	1755.136276	7.631027
thorium-231	231.036299	1760.252929	7.620143
thorium-232	232.038051	1766.692303	7.615053
thorium-234	234.043593	1777.672666	7.596892
protactinium-231	231.035880	1759.860814	7.618445
protactinium-234	234.043300	1777.163183	7.594714
uranium-231	231.036264	1758.720709	7.613510
uranium-232	232.037131	1765.984455	7.612002
uranium-233	233.039630	1771.728003	7.603983
uranium-234	234.040946	1778.573509	7.600741
uranium-235	235.043924	1783.870871	7.590940
uranium-236	236.045562	1790.416436	7.586510
uranium-238	238.050784	1801.694877	7.570147
uranium-239	239.054290	1806.500411	7.558579

Part c) The lighter elements have E_B/nucleon values that vary radically and stay much less than the maximum of 8.79 MeV/nucleon at ${}^{56}_{26}Fe$. The heavier elements have E_B/nucleon values that are very stable and stay right near the maximum value of 8.79 MeV/nucleon.

27.39

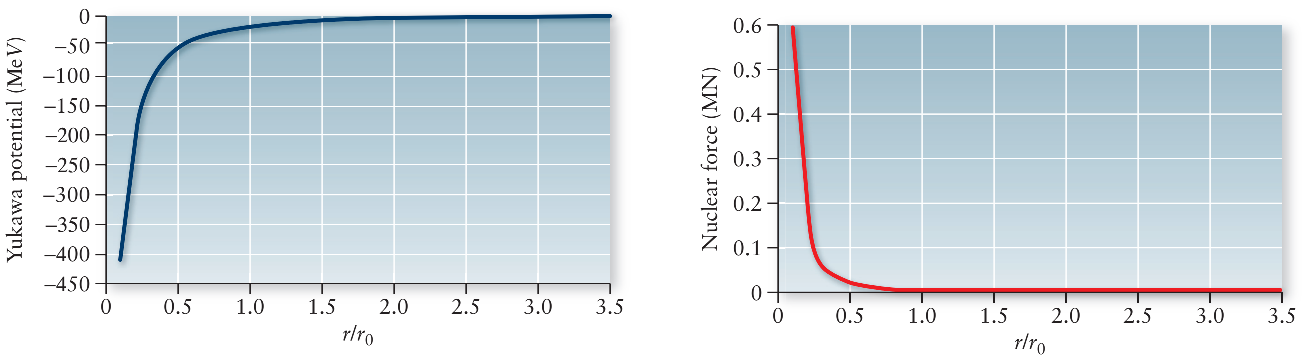

Figure 27-1 Problem 39

Problems

27.40

SET UP
We are given the atomic symbols for eight elements and asked to identify the element, the number of protons Z, the number of neutrons N, and the mass number A for each. A periodic table will tell us not only the name of the element but also Z, as this is equal to the atomic number. The number to the top left of the atomic symbol is equal to A, which is also equal to the sum $Z + N$.

SOLVE

A) hydrogen, $Z = 1, N = 1, A = 2$

B) helium, $Z = 2, N = 2, A = 4$

C) lithium, $Z = 3, N = 3, A = 6$

D) carbon, $Z = 6, N = 6, A = 12$

E) iron, $Z = 26, N = 30, A = 56$

F) strontium, $Z = 38, N = 52, A = 90$

G) iodine, $Z = 53, N = 78, A = 131$

H) uranium, $Z = 92, N = 143, A = 235$

REFLECT
Even though the atomic number Z is often listed to the lower left of the atomic symbol, this information is redundant since each atomic number is a unique element.

27.41

SET UP
We are given eight isotopes and asked to provide the atomic symbol for the element, the number of protons Z, the number of neutrons N, and the mass number A for each. A periodic table will tell us not only the atomic symbol but also Z, as this is equal to the atomic number. The number listed after the name of the element is equal to A, which is also equal to the sum $Z + N$.

SOLVE

A) ^{3}H, $Z = 1, N = 2, A = 3$

B) ^{8}Be, $Z = 4, N = 4, A = 8$

C) ^{26}Al, $Z = 13, N = 13, A = 26$

D) ^{197}Au, $Z = 79, N = 118, A = 197$

E) ^{100}Tc, $Z = 43, N = 57, A = 100$

F) ^{184}W, $Z = 74, N = 110, A = 184$

G) ^{190}Os, $Z = 76$, $N = 114$, $A = 190$

H) ^{239}Pu, $Z = 94$, $N = 145$, $A = 239$

REFLECT

Hydrogen-3 is also known as "tritium." The mass number is equal to the number of neutrons N plus the number of protons Z.

27.42

SET UP

An atomic nucleus is approximately spherical with radius $r = r_0 A^{\frac{1}{3}}$, where $r_0 = 1.2 \times 10^{-15}$ m and A is the mass number. We can refer back to our solution to Problem 27.40 for the mass numbers in order to find the radii of those nuclei.

SOLVE

A) hydrogen-2: $r = r_0 A^{\frac{1}{3}} = (1.2\ \text{fm})(2)^{\frac{1}{3}} = \boxed{1.5\ \text{fm}}$

B) helium-4: $r = r_0 A^{\frac{1}{3}} = (1.2\ \text{fm})(4)^{\frac{1}{3}} = \boxed{1.9\ \text{fm}}$

C) lithium-6: $r = r_0 A^{\frac{1}{3}} = (1.2\ \text{fm})(6)^{\frac{1}{3}} = \boxed{2.2\ \text{fm}}$

D) carbon-12: $r = r_0 A^{\frac{1}{3}} = (1.2\ \text{fm})(12)^{\frac{1}{3}} = \boxed{2.7\ \text{fm}}$

E) iron-56: $r = r_0 A^{\frac{1}{3}} = (1.2\ \text{fm})(56)^{\frac{1}{3}} = \boxed{4.6\ \text{fm}}$

F) strontium-90: $r = r_0 A^{\frac{1}{3}} = (1.2\ \text{fm})(90)^{\frac{1}{3}} = \boxed{5.4\ \text{fm}}$

G) iodine-131: $r = r_0 A^{\frac{1}{3}} = (1.2\ \text{fm})(131)^{\frac{1}{3}} = \boxed{6.1\ \text{fm}}$

H) uranium-235: $r = r_0 A^{\frac{1}{3}} = (1.2\ \text{fm})(235)^{\frac{1}{3}} = \boxed{7.4\ \text{fm}}$

REFLECT

The nuclear radius should increase as the mass number increases since there are more nucleons.

27.43

SET UP

A nucleus is approximately spherical with a radius $r = r_0 A^{\frac{1}{3}}$, where $r_0 = 1.2 \times 10^{-15}$ m and A is the mass number. The mass density of the nucleus is equal to its mass divided by its volume. We can approximate the total mass of the nucleus as A multiplied by the average mass of a nucleon. The mass of a proton is $m_p = 1.6726 \times 10^{-27}$ kg, and the mass of a neutron is $m_n = 1.6749 \times 10^{-27}$ kg.

SOLVE

$$\rho = \frac{m}{V} = \frac{Am_{\text{avg}}}{\left(\frac{4}{3}\pi r^3\right)} = \frac{3A\left(\frac{m_{\text{p}} + m_{\text{n}}}{2}\right)}{4\pi\left(r_0 A^{\frac{1}{3}}\right)^3} = \frac{3(m_{\text{p}} + m_{\text{n}})}{8\pi r_0^3}$$

$$= \frac{3((1.6726 \times 10^{-27}\text{ kg}) + (1.6749 \times 10^{-27}\text{ kg}))}{8\pi(1.2 \times 10^{-15}\text{ m})^3} = \boxed{2.3 \times 10^{17}\frac{\text{kg}}{\text{m}^3}}$$

$$2.3 \times 10^{17}\frac{\text{kg}}{\text{m}^3} \times \frac{2.2\text{ lb}}{1\text{ kg}} \times \frac{1\text{ ton}}{2000\text{ lb}} \times \left(\frac{1\text{ m}}{100\text{ cm}}\right)^3 \times \left(\frac{2.54\text{ cm}}{1\text{ in}}\right)^3 = \boxed{4.2 \times 10^{9}\frac{\text{tons}}{\text{in}^3}}$$

REFLECT

The nucleus is extremely dense, so an answer on the order of 10^{17} is reasonable. Note that the density did not depend upon the mass number.

Get Help: P'Cast 27.2 – Nuclear Density

27.44

SET UP

A nucleus is approximately spherical with radius $r = r_0 A^{\frac{1}{3}}$, where $r_0 = 1.2 \times 10^{-15}$ m and A is the mass number. The mass density of the nucleus is equal to its mass divided by its volume. We can use the atomic masses listed in Appendix C of the textbook to approximate the mass of the nucleus.

SOLVE

Density:

$$\rho = \frac{m}{V} = \frac{m}{\left(\frac{4}{3}\pi r^3\right)} = \frac{3m}{4\pi\left(r_0 A^{\frac{1}{3}}\right)^3} = \frac{3m}{4\pi r_0^3 A}$$

A) $$\rho = \frac{3m}{4\pi r_0^3 A} = \frac{3(3.016049\text{ u})}{4\pi(1.2\text{ fm})^3(3)}$$

$$= 0.14\frac{\text{u}}{\text{fm}^3} \times \left(\frac{10^{15}\text{ fm}}{1\text{ m}}\right)^3 \times \frac{1.661 \times 10^{-27}\text{ kg}}{1\text{ u}} = \boxed{2.3 \times 10^{17}\frac{\text{kg}}{\text{m}^3}}$$

B) $$\rho = \frac{3m}{4\pi r_0^3 A} = \frac{3(9.012174\text{ u})}{4\pi(1.2\text{ fm})^3(9)}$$

$$= 0.14\frac{\text{u}}{\text{fm}^3} \times \left(\frac{10^{15}\text{ fm}}{1\text{ m}}\right)^3 \times \frac{1.661 \times 10^{-27}\text{ kg}}{1\text{ u}} = \boxed{2.3 \times 10^{17}\frac{\text{kg}}{\text{m}^3}}$$

C) $\rho = \dfrac{3m}{4\pi r_0^3 A} = \dfrac{3(15.994915\text{ u})}{4\pi(1.2\text{ fm})^3(16)}$

$$= 0.14\frac{\text{u}}{\text{fm}^3} \times \left(\frac{10^{15}\text{ fm}}{1\text{ m}}\right)^3 \times \frac{1.661 \times 10^{-27}\text{ kg}}{1\text{ u}} = \boxed{2.3 \times 10^{17}\frac{\text{kg}}{\text{m}^3}}$$

D) $\rho = \dfrac{3m}{4\pi r_0^3 A} = \dfrac{3(55.934940\text{ u})}{4\pi(1.2\text{ fm})^3(56)}$

$$= 0.14\frac{\text{u}}{\text{fm}^3} \times \left(\frac{10^{15}\text{ fm}}{1\text{ m}}\right)^3 \times \frac{1.661 \times 10^{-27}\text{ kg}}{1\text{ u}} = \boxed{2.3 \times 10^{17}\frac{\text{kg}}{\text{m}^3}}$$

E) $\rho = \dfrac{3m}{4\pi r_0^3 A} = \dfrac{3(62.929959\text{ u})}{4\pi(1.2\text{ fm})^3(63)}$

$$= 0.14\frac{\text{u}}{\text{fm}^3} \times \left(\frac{10^{15}\text{ fm}}{1\text{ m}}\right)^3 \times \frac{1.661 \times 10^{-27}\text{ kg}}{1\text{ u}} = \boxed{2.3 \times 10^{17}\frac{\text{kg}}{\text{m}^3}}$$

F) $\rho = \dfrac{3m}{4\pi r_0^3 A} = \dfrac{3(226.025402\text{ u})}{4\pi(1.2\text{ fm})^3(226)}$

$$= 0.14\frac{\text{u}}{\text{fm}^3} \times \left(\frac{10^{15}\text{ fm}}{1\text{ m}}\right)^3 \times \frac{1.661 \times 10^{-27}\text{ kg}}{1\text{ u}} = \boxed{2.3 \times 10^{17}\frac{\text{kg}}{\text{m}^3}}$$

G) $\rho = \dfrac{3m}{4\pi r_0^3 A} = \dfrac{3(208.982405\text{ u})}{4\pi(1.2\text{ fm})^3(209)}$

$$= 0.14\frac{\text{u}}{\text{fm}^3} \times \left(\frac{10^{15}\text{ fm}}{1\text{ m}}\right)^3 \times \frac{1.661 \times 10^{-27}\text{ kg}}{1\text{ u}} = \boxed{2.3 \times 10^{17}\frac{\text{kg}}{\text{m}^3}}$$

H) $\rho = \dfrac{3m}{4\pi r_0^3 A} = \dfrac{3(238.050784\text{ u})}{4\pi(1.2\text{ fm})^3(238)}$

$$= 0.14\frac{\text{u}}{\text{fm}^3} \times \left(\frac{10^{15}\text{ fm}}{1\text{ m}}\right)^3 \times \frac{1.661 \times 10^{-27}\text{ kg}}{1\text{ u}} = \boxed{2.3 \times 10^{17}\frac{\text{kg}}{\text{m}^3}}$$

REFLECT

The density of the nucleus is very large and approximately constant across the periodic table!

27.45

SET UP

The density of a nucleus is approximately $\rho_{\text{nucleus}} = 2.3 \times 10^{17}\frac{\text{kg}}{\text{m}^3}$. Treating the Sun as a sphere of mass $m_{\text{Sun}} = 1.99 \times 10^{30}$ kg, we can calculate the radius of the collapsed Sun with the same density as a nucleus.

SOLVE

$$\rho_{\text{Sun}} = \rho_{\text{nucleus}}$$

$$\frac{m_{\text{Sun}}}{V_{\text{Sun}}} = \frac{m_{\text{Sun}}}{\left(\frac{4}{3}\pi r^3\right)} = \rho_{\text{nucleus}}$$

$$r^3 = \frac{3m_{\text{Sun}}}{4\pi\rho_{\text{nucleus}}}$$

$$r = \sqrt[3]{\frac{3m_{\text{Sun}}}{4\pi\rho_{\text{nucleus}}}} = \sqrt[3]{\frac{3(1.99 \times 10^{30}\ \text{kg})}{4\pi\left(2.3 \times 10^{17}\frac{\text{kg}}{\text{m}^3}\right)}} = \boxed{1.3 \times 10^4\ \text{m}}$$

REFLECT

The radius of the Sun should decrease if it were to collapse to be the same density as a nucleus.

27.46

SET UP

The atomic mass (in amu) of each given isotope can be read directly from Appendix C of the textbook. The conversion factor between amu and grams is 1 u = 1.661×10^{-24} g.

SOLVE

A) $m_{^1\text{H}} = \boxed{1.007825\ \text{u}} \times \frac{1.661 \times 10^{-24}\ \text{g}}{1\ \text{u}} = \boxed{1.674 \times 10^{-24}\ \text{g}}$

B) $m_{^4\text{He}} = \boxed{4.002602\ \text{u}} \times \frac{1.661 \times 10^{-24}\ \text{g}}{1\ \text{u}} = \boxed{6.648 \times 10^{-24}\ \text{g}}$

C) $m_{^9\text{Be}} = \boxed{9.012182\ \text{u}} \times \frac{1.661 \times 10^{-24}\ \text{g}}{1\ \text{u}} = \boxed{1.497 \times 10^{-23}\ \text{g}}$

D) $m_{^{12}\text{C}} = \boxed{12.000000\ \text{u}} \times \frac{1.661 \times 10^{-24}\ \text{g}}{1\ \text{u}} = \boxed{1.993 \times 10^{-23}\ \text{g}}$

E) $m_{^{56}\text{Fe}} = \boxed{55.924942\ \text{u}} \times \frac{1.661 \times 10^{-24}\ \text{g}}{1\ \text{u}} = \boxed{9.289 \times 10^{-23}\ \text{g}}$

F) $m_{^{90}\text{Sr}} = \boxed{89.907737\ \text{u}} \times \frac{1.661 \times 10^{-24}\ \text{g}}{1\ \text{u}} = \boxed{1.493 \times 10^{-22}\ \text{g}}$

G) $m_{^{127}\text{I}} = \boxed{126.904474\ \text{u}} \times \frac{1.661 \times 10^{-24}\ \text{g}}{1\ \text{u}} = \boxed{2.108 \times 10^{-22}\ \text{g}}$

H) $m_{^{238}\text{U}} = \boxed{238.050784\ \text{u}} \times \frac{1.661 \times 10^{-24}\ \text{g}}{1\ \text{u}} = \boxed{3.954 \times 10^{-22}\ \text{g}}$

REFLECT

The mass listed in the periodic table is a weighted average of the masses of all known isotopes, weighted by their relative abundances.

27.47

SET UP

Carbon-12 has a mass of $m_{\text{atom}} = 12.000000\text{ u}$ and is made up of 6 neutrons, 6 protons, and 6 electrons. The binding energy of the atom is the difference in energy between the component parts of the atom and the atom itself, $E_B = (Nm_n + Zm_{^1H} - m_{\text{atom}})c^2$, where $m_n = 1.008665\text{ u}$ and $m_{^1H} = 1.007825\text{ u}$. The conversion between u and MeV is $1\text{ u} = 931.494\text{ MeV}/c^2$.

SOLVE

$$E_B = (Nm_n + Zm_{^1H} - m_{\text{atom}})c^2$$

$$E_B = (6(1.008665\text{ u}) + 6(1.007825\text{ u}) - (12.000000\text{ u}))c^2$$

$$= (0.09894\text{ u})c^2 \times \frac{\left(931.494\frac{\text{MeV}}{c^2}\right)}{1\text{ u}} = \boxed{92.2\text{ MeV}}$$

REFLECT

The larger the binding energy is, the more stable the nucleus.

27.48

SET UP

The binding energy of an atom is the difference in energy between the component parts of the atom and the atom itself, $E_B = (Nm_n + Zm_{^1H} - m_{\text{atom}})c^2$, where $m_n = 1.008665\text{ u}$ and $m_{^1H} = 1.007825\text{ u}$. We can use Appendix C in the textbook to find the mass of each atom. The conversion between u and MeV is $1\text{ u} = 931.494\text{ MeV}/c^2$. After we calculate the binding energy, we will need to divide by the mass number to get the binding energy per nucleon.

SOLVE

Binding energy:

$$E_B = (Nm_n + Zm_{^1H} - m_{\text{atom}})c^2$$

A)

Binding energy:

$$E_B = (1(1.008665\text{ u}) + 1(1.007825\text{ u}) - (2.014102\text{ u}))c^2$$

$$= (0.002388\text{ u})c^2 \times \frac{\left(931.494\frac{\text{MeV}}{c^2}\right)}{1\text{ u}} = 2.224\text{ MeV}$$

Binding energy per nucleon:

$$\frac{2.224\text{ MeV}}{2} = \boxed{1.112\frac{\text{MeV}}{\text{nucleon}}}$$

B)
Binding energy:

$$E_B = (2(1.008665\text{ u}) + 2(1.007825\text{ u}) - (4.002602\text{ u}))c^2$$

$$= (0.030378\text{ u})c^2 \times \frac{\left(931.494\frac{\text{MeV}}{c^2}\right)}{1\text{ u}} = 28.297\text{ MeV}$$

Binding energy per nucleon:

$$\frac{28.297\text{ MeV}}{4} = \boxed{7.0742\frac{\text{MeV}}{\text{nucleon}}}$$

C)
Binding energy:

$$E_B = (3(1.008665\text{ u}) + 3(1.007825\text{ u}) - (6.015121\text{ u}))c^2$$

$$= (0.034349\text{ u})c^2 \times \frac{\left(931.494\frac{\text{MeV}}{c^2}\right)}{1\text{ u}} = 31.996\text{ MeV}$$

Binding energy per nucleon:

$$\frac{31.996\text{ MeV}}{6} = \boxed{5.3326\frac{\text{MeV}}{\text{nucleon}}}$$

D)
Binding energy:

$$E_B = (6(1.008665\text{ u}) + 6(1.007825\text{ u}) - (12.000000\text{ u}))c^2$$

$$= (0.098940\text{ u})c^2 \times \frac{\left(931.494\frac{\text{MeV}}{c^2}\right)}{1\text{ u}} = 92.162\text{ MeV}$$

Binding energy per nucleon:

$$\frac{92.162\text{ MeV}}{12} = \boxed{7.6802\frac{\text{MeV}}{\text{nucleon}}}$$

E)
Binding energy:

$$E_B = (30(1.008665\text{ u}) + 26(1.007825\text{ u}) - (55.934940\text{ u}))c^2$$

$$= (0.52846\text{ u})c^2 \times \frac{\left(931.494\frac{\text{MeV}}{c^2}\right)}{1\text{ u}} = 492.26\text{ MeV}$$

Binding energy per nucleon:

$$\frac{492.26 \text{ MeV}}{56} = \boxed{8.7903 \frac{\text{MeV}}{\text{nucleon}}}$$

F)
Binding energy:

$$E_\text{B} = (52(1.008665 \text{ u}) + 38(1.007825 \text{ u}) - (89.907737 \text{ u}))c^2$$

$$= (0.84019 \text{ u})c^2 \times \frac{\left(931.494 \frac{\text{MeV}}{c^2}\right)}{1 \text{ u}} = 782.63 \text{ MeV}$$

Binding energy per nucleon:

$$\frac{782.63 \text{ MeV}}{90} = \boxed{8.6959 \frac{\text{MeV}}{\text{nucleon}}}$$

G)
Binding energy:

$$E_\text{B} = (76(1.008665 \text{ u}) + 53(1.007825 \text{ u}) - (128.904984 \text{ u}))c^2$$

$$= (1.16828 \text{ u})c^2 \times \frac{\left(931.494 \frac{\text{MeV}}{c^2}\right)}{1 \text{ u}} = 1088.25 \text{ MeV}$$

Binding energy per nucleon:

$$\frac{1088.25 \text{ MeV}}{129} = \boxed{8.4360 \frac{\text{MeV}}{\text{nucleon}}}$$

H)
Binding energy:

$$E_\text{B} = (143(1.008665 \text{ u}) + 92(1.007825 \text{ u}) - (235.043924 \text{ u}))c^2$$

$$= (1.915 \text{ u})c^2 \times \frac{\left(931.494 \frac{\text{MeV}}{c^2}\right)}{1 \text{ u}} = 1784 \text{ MeV}$$

Binding energy per nucleon:

$$\frac{1784 \text{ MeV}}{235} = \boxed{7.591 \frac{\text{MeV}}{\text{nucleon}}}$$

REFLECT

All of the values are smaller than the binding energy per nucleon for iron-56, which is the most stable nucleus.

27.49

SET UP

The binding energy of a nucleus is given by $E_B = (Nm_n + Zm_{^1H} - m_{atom})c^2$. After calculating the binding energy and dividing it by the number of nucleons for each of the given nuclei, we can make a sketch of the binding energy per nucleon versus the mass number.

SOLVE

	Nucleus	BE/Nucleon
(a)	^1_1H	0
(b)	^2_1H	1.11 MeV/nucleon
(c)	^4_2He	7.07 MeV/nucleon
(d)	^6_3Li	5.33 MeV/nucleon
(e)	$^{12}_6C$	7.68 MeV/nucleon
(f)	$^{32}_{16}S$	8.49 MeV/nucleon
(g)	$^{56}_{26}F$	8.79 MeV/nucleon
(h)	$^{197}_{79}Au$	7.92 MeV/nucleon
(i)	$^{207}_{82}Pb$	7.87 MeV/nucleon

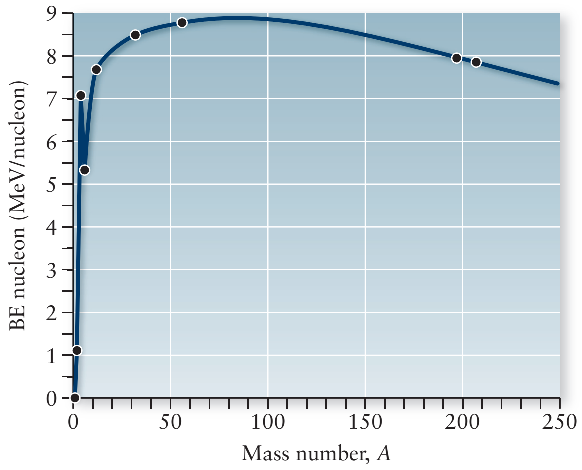

Figure 27-2 Problem 49

REFLECT

Iron-56 is the most stable nucleus.

27.50

SET UP

We can use the plot from our solution to Problem 27.50 to approximate the binding energy per nucleon for boron-10.

SOLVE

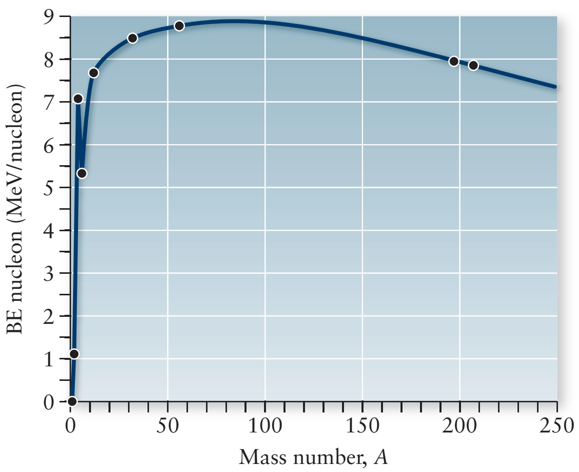

Figure 27-3 Problem 50

The binding energy per nucleon for boron-10 is approximately $\boxed{7\frac{\text{MeV}}{\text{nucleon}}}$.

REFLECT

The actual binding energy per nucleon for boron-10 is $6.48\frac{\text{MeV}}{\text{nucleon}}$.

27.51

SET UP

The energy needed to remove a neutron from calcium-40 to form calcium-39 is related to the difference in mass between a calcium-40 atom and the sum of the masses of a calcium-39 atom and a neutron.

SOLVE

$$^{40}\text{Ca} \rightarrow \text{n} + {}^{39}\text{Ca}$$

$$\Delta E = ((m_{\text{n}} + m_{^{39}\text{Ca}}) - m_{^{40}\text{Ca}})c^2 = ((1.008665\ \text{u}) + (38.97071972\ \text{u}) - (39.96259098\ \text{u}))c^2$$

$$= (0.016794\ \text{u})c^2 \times \frac{\left(931.494\frac{\text{MeV}}{c^2}\right)}{1\ \text{u}} = \boxed{15.643\ \text{MeV}}$$

REFLECT

Energy should be required to remove a neutron from a stable nucleus.

27.52

SET UP

The binding energy of the last neutron of carbon-13 is related to the difference in mass between a carbon-13 atom and the sum of the masses of a carbon-12 atom and a neutron.

SOLVE

$$n + {}^{12}C \rightarrow {}^{13}C$$

$$\Delta E = (m_{^{13}C} - (m_n + m_{^{12}C}))c^2 = ((13.003355\ u) - ((1.008665\ u) + (12.000000\ u)))c^2$$

$$= (-0.005310\ u)c^2 \times \frac{\left(931.494\frac{MeV}{c^2}\right)}{1\ u} = -4.946\ MeV$$

The binding energy of the last neutron of carbon-13 is $\boxed{4.949\ MeV}$.

REFLECT

This is the energy released upon adding the final neutron.

27.53

SET UP

The binding energy of iodine-131 is the difference in energy between the component parts of the atom and the atom itself, $E_B = (Nm_n + Zm_{^1H} - m_{atom})c^2$, where $m_n = 1.008665$ u, $m_{^1H} = 1.007825$ u, and $m_{atom} = 130.906124$ u. The conversion between u and MeV is 1 u = 931.494 MeV/c^2. After we calculate the binding energy, we will need to divide by the mass number to get the binding energy per nucleon.

SOLVE

Binding energy:

$$E_B = (78(1.008665\ u) + 53(1.007825\ u) - (130.906124\ u))c^2$$

$$= (1.18447\ u)c^2 \times \frac{\left(931.494\frac{MeV}{c^2}\right)}{1\ u} = \boxed{1103.33\ MeV}$$

Binding energy per nucleon:

$$\frac{1103.33\ MeV}{131} = \boxed{8.42235\frac{MeV}{nucleon}}$$

REFLECT

Iodine-129 ($\tau_{1/2} \sim 10^7$ yr) is more stable than iodine-131 ($\tau_{1/2} \sim 8$ days).

27.54

SET UP

Plutonium-239 undergoes the following nuclear fission reaction: ${}^{239}Pu + n \rightarrow {}^{98}Tc + {}^{138}Sb + 4n$. The energy released by the reaction is equal to the total energy of the reactants minus the total energy of the products. The atomic masses for the various atoms are plutonium-239 = 239.052157 u, a neutron = 1.008665 u, technetium-98 = 97.907215 u, and antimony-138 = 137.940793 u.

SOLVE

$$E_{\text{released}} = [(239.052157\text{ u}) + (1.008665\text{ u}) - ((97.907215\text{ u}) + (137.940793\text{ u}) + 4(1.008665\text{ u}))]c^2$$

$$= (0.178154\text{ u})c^2 \times \frac{\left(931.494\dfrac{\text{MeV}}{c^2}\right)}{1\text{ u}} = \boxed{165.949\text{ MeV}}$$

REFLECT

The products of a fission reaction are more stable than the reactants.

27.55

SET UP

Thorium-232 undergoes the following nuclear fission reaction: $^{232}\text{Th} + \text{n} \rightarrow {}^{99}\text{Kr} + {}^{124}\text{Xe} + \_\_$. We can determine the missing product by calculating its expected atomic number Z and mass number A. The sum of the atomic numbers of the products must equal the sum of the atomic numbers of the reactants; the same must be true for the mass numbers. The atomic numbers of thorium, krypton, and xenon are 90, 36, and 54, respectively. The mass numbers of each species are to the top left of the atomic symbol; a neutron has a mass number of 1. The energy Q released by the reaction is equal to the total energy of the reactants minus the total energy of the products. The atomic masses are thorium-232 = 232.038051 u, a neutron = 1.008665 u, krypton-99 = 98.957606 u, and xenon-124 = 123.905894 u.

SOLVE

Atomic number:

$$Z = 90 - (36 + 54) = 0$$

Since $Z = 0$, the missing product will be a neutron.

Mass number:

$$A = 232 + 1 - (99 + 124) = 10$$

The total mass number of the missing must be 10. We already know it is a neutron, which has a mass number of 1, so the missing product must be 10 neutrons, or $\boxed{10n}$.

Energy released:

$$Q = [(232.038051\text{ u}) + (1.008665\text{ u}) - ((98.957606\text{ u}) + (123.905894\text{ u}) + 10(1.008665\text{ u}))]c^2$$

$$= (0.096566\text{ u})c^2 \times \frac{\left(931.494\dfrac{\text{MeV}}{c^2}\right)}{1\text{ u}} = \boxed{89.95\text{ MeV}}$$

REFLECT

The total number of protons and the total number of neutrons must remain constant.

Get Help: P'Cast 27.5 – Fission to Xenon and Beyond

27.56

SET UP

We need to balance the provided fission reactions by determining the missing species by calculating its expected atomic number Z and mass number A. The sum of the atomic numbers of the products must equal the sum of the atomic numbers of the reactants; the same must be true for the mass numbers.

SOLVE

A)

$$^{235}\mathrm{U} + \mathrm{n} \rightarrow {}^{128}\mathrm{Sb} + {}^{101}\mathrm{Nb} + \_\_$$

Atomic number:

$$Z = 92 - (51 + 41) = 0$$

Since $Z = 0$, the missing product will be a neutron.
Mass number:

$$A = (235 + 1) - (128 + 101) = 7$$

The total mass number of the missing species must be 7. We already know it is a neutron, which has a mass number of 1, so the missing product must be 7 neutrons, or $\boxed{7n}$.

B)

$$^{235}\mathrm{U} + \mathrm{n} \rightarrow \_\_ + {}^{116}\mathrm{Pd} + 4\mathrm{n}$$

Atomic number:

$$Z = 92 - 46 = 46$$

Since $Z = 46$, the missing product will be a palladium nucleus.

Mass number:

$$A = (235 + 1) - (116 + 4(1)) = 116$$

The missing product will be a palladium-116 nucleus, $\boxed{^{116}\mathrm{Pd}}$.

C)

$$^{238}\mathrm{U} + \mathrm{n} \rightarrow {}^{99}\mathrm{Kr} + \_\_ + 11\mathrm{n}$$

Atomic number:

$$Z = 92 - 36 = 56$$

Since $Z = 56$, the missing product will be a barium nucleus.

Mass number:

$$A = (238 + 1) - (99 + 11(1)) = 129$$

The missing product will be a barium-129 nucleus, $\boxed{^{129}\mathrm{Ba}}$.

D)

$$\_\_ + \mathrm{n} \rightarrow {}^{101}\mathrm{Rb} + {}^{130}\mathrm{Cs} + 8\mathrm{n}$$

Atomic number:

$$Z = 37 + 55 = 92$$

Since $Z = 92$, the missing product will be a uranium nucleus.

Mass number:

$$A = (101 + 130 + 8(1)) - 1 = 238$$

The missing product will be a uranium-238 nucleus, $\boxed{^{238}\mathrm{U}}$.

REFLECT

The nuclei of the products should be smaller than the nuclei of the reactant in fission reactions.

27.57

SET UP

We need to balance the provided fission reactions by determining the missing species by calculating its expected atomic number Z and mass number A. The sum of the atomic numbers of the products must equal the sum of the atomic numbers of the reactants; the same must be true for the mass numbers.

SOLVE

A)

$$^{242}\mathrm{Am} + \ \rightarrow {}^{90}\mathrm{Sr} + {}^{149}\mathrm{La} + 4\mathrm{n}$$

Atomic number:

$$Z = 38 + 57 - 95 = 0$$

Since $Z = 0$, the missing product will be a neutron.

Mass number:

$$A = 90 + 149 + 4(1) - 242 = 1$$

The missing product must be 1 neutron, or $\boxed{n}$.

B)

$$^{244}\mathrm{Pa} + \mathrm{n} \rightarrow \_\_ + {}^{131}\mathrm{Sb} + 12\mathrm{n}$$

Atomic number:

$$Z = 91 - 51 = 40$$

Since $Z = 40$, the missing product will be a zirconium nucleus.

Mass number:

$$A = (244 + 1) - (131 + 12(1)) = 102$$

The missing product will be a zirconium-102 nucleus, $\boxed{^{102}\mathrm{Zr}}$.

C)

$$\_\_ + \text{n} \rightarrow {}^{92}\text{Se} + {}^{153}\text{Sm} + 6\text{n}$$

Atomic number:

$$Z = 34 + 62 = 96$$

Since $Z = 96$, the missing product will be a curium nucleus.

Mass number:

$$A = 92 + 153 + 6(1) - 1 = 250$$

The missing product will be a curium-250 nucleus, $\boxed{{}^{250}\text{Cm}}$.

D)

$$^{262}\text{Fm} + \text{n} \rightarrow {}^{112}\text{Rh} + \_\_ + 9\text{n}$$

Atomic number:

$$Z = 100 - 45 = 55$$

Since $Z = 55$, the missing product will be a cesium nucleus.
Mass number:

$$A = (262 + 1) - (112 + 9(1)) = 142$$

The missing product will be a cesium-142 nucleus, $\boxed{{}^{142}\text{Cs}}$.

REFLECT

The nuclei of the products should be smaller than the nuclei of the reactant in fission reactions.

27.58

SET UP

Americium-242 undergoes the following nuclear fission reaction: $^{242}\text{Am} + \_\_ \rightarrow {}^{90}\text{Sr} + {}^{149}\text{La} + 4\text{n}$. We can determine the missing product by calculating its expected atomic number Z and mass number A. The sum of the atomic numbers of the products must equal the sum of the atomic numbers of the reactants; the same must be true for the mass numbers. The atomic numbers of americium, strontium, and lanthanum are 95, 38, and 57, respectively. The mass numbers of each species are to the top left of the atomic symbol; a neutron has a mass number of 1. The energy released by the reaction is equal to the total energy of the reactants minus the total energy of the products. The atomic masses are americium-242 = 242.059549 u, a neutron = 1.008665 u, strontium-90 = 89.9077387 u, and lanthanum-149 = 148.934733 u.

SOLVE

Atomic number:

$$Z = 38 + 57 - 95 = 0$$

Since $Z = 0$, the missing product will be a neutron.

Mass number:

$$A = 90 + 149 + 4(1) - 242 = 1$$

The missing product must be 1 neutron, or $\boxed{n}$.

Energy released:

$$E_{\text{released}} = [(242.059549\text{ u}) + (1.008665\text{ u}) - ((89.9077387\text{ u}) + (148.934733\text{ u}) + 4(1.008665\text{ u}))]c^2$$

$$= (0.191082\text{ u})c^2 \times \frac{\left(931.494\frac{\text{MeV}}{c^2}\right)}{1\text{ u}} = \boxed{177.992\text{ MeV}}$$

REFLECT

Relatively large amounts of energy will be released in a nuclear reaction.

27.59

SET UP

A neutron loses half of its energy after each collision with an atom of the moderator. In order to determine how many collisions are required to slow a neutron with an initial energy of $E_{\text{i}} = 200$ MeV to a final energy of $E_{\text{f}} = 0.04$ eV, we can set a ratio of the energies equal to $\left(\frac{1}{2}\right)^n$, where n is the number of collisions.

SOLVE

$$E_{\text{f}} = \left(\frac{1}{2}\right)^n E_{\text{i}}$$

$$\ln\left(\frac{E_{\text{f}}}{E_{\text{i}}}\right) = n\ln\left(\frac{1}{2}\right) = -n\ln(2)$$

$$n = -\frac{\ln\left(\frac{E_{\text{f}}}{E_{\text{i}}}\right)}{\ln(2)} = \frac{\ln\left(\frac{0.04\text{ eV}}{200 \times 10^6\text{ eV}}\right)}{\ln(2)} = 32.2$$

It will require $\boxed{33\text{ collisions}}$ to slow the neutron.

REFLECT

The energy of the neutron decreases by 10 orders of magnitude, so 33 collisions seems reasonable.

27.60

SET UP

The binding energy per nucleon in uranium-235 is about $7.6\frac{\text{MeV}}{\text{nucleon}}$, whereas the binding energy for typical fission fragments is $8.5\frac{\text{MeV}}{\text{nucleon}}$. The average energy release per reaction is equal to the difference between the total final energy and the total initial energy; uranium-235 has 235 nucleons.

SOLVE

$$\Delta E_{\text{avg}} = E_{\text{f, avg}} - E_{\text{i, avg}}$$

$$= (235\text{ nucleons})\left(8.5\frac{\text{MeV}}{\text{nucleon}}\right) - (235\text{ nucleons})\left(7.6\frac{\text{MeV}}{\text{nucleon}}\right) = \boxed{200\text{ MeV}}$$

REFLECT

An average energy of around 200 MeV seems reasonable for a fission reaction.

27.61

SET UP

The fission of uranium-235 releases 185×10^6 eV of energy. In order to determine how many kilograms of uranium-235 are necessary to produce 1000×10^6 W of power continuously for one year, we must first determine the total amount of energy required by multiplying the power by the time interval. Each fission reaction uses 1 nucleus; we can use Avogadro's number and the molar mass of uranium-235 in order to calculate the mass.

SOLVE

Total energy required:

$$E = P\Delta t = (1000 \times 10^6\text{ W})\left(1\text{ yr} \times \frac{365.25\text{ days}}{1\text{ yr}} \times \frac{24\text{ hr}}{1\text{ day}} \times \frac{3600\text{ s}}{1\text{ hr}}\right) = 3.1558 \times 10^{16}\text{ J}$$

Total number of fission reactions necessary:

$$\frac{3.1558 \times 10^{16}\text{ J}}{185 \times 10^6\text{ eV}} \times \frac{1\text{ eV}}{1.6 \times 10^{-19}\text{ J}} = 1.066 \times 10^{27}\text{ reactions}$$

Mass of uranium-235 required:

$$1.066 \times 10^{27}\text{ reactions} \times \frac{1\text{ nucleus U} - 235}{1\text{ reaction}} \times \frac{235\text{ g}}{6.02 \times 10^{23}\text{ nuclei}} = \boxed{4.16 \times 10^5\text{ g} = 416\text{ kg}}$$

REFLECT

We've assumed that the fission reaction is 100% efficient, which means our answer is the *minimum* mass required.

27.62

SET UP

The fission of a uranium-235 releases 185×10^6 eV of energy. In order to determine how many kilograms of uranium-235 are required to operate a reactor at 30% efficiency, producing 1000×10^6 W of power continuously for one year, we must first determine the total amount of energy required by multiplying the power by the time interval. Each fission reaction uses 1 nucleus; we can use Avogadro's number and the molar mass of uranium-235 in order to calculate the mass. An efficiency of 30% means for every 100 J of energy put in only 30 J of energy comes out.

SOLVE

Total energy required:

$$E = P\Delta t = (1000 \times 10^6 \text{ W})\left(1 \text{ yr} \times \frac{365 \text{ days}}{1 \text{ yr}} \times \frac{24 \text{ hr}}{1 \text{ day}} \times \frac{3600 \text{ s}}{1 \text{ hr}}\right)$$

$$= 3.1558 \times 10^{16} \text{ J out} \times \frac{100 \text{ J in}}{30 \text{ J out}} = 1.0519 \times 10^{17} \text{ J}$$

Total number of fission reactions necessary:

$$\frac{1.0519 \times 10^{17} \text{ J}}{185 \times 10^6 \text{ eV}} \times \frac{1 \text{ eV}}{1.602 \times 10^{-19} \text{ J}} = 3.549 \times 10^{27} \text{ reactions}$$

Mass of uranium-235 required:

$$3.549 \times 10^{27} \text{ reactions} \times \frac{1 \text{ nucleus U} - 235}{1 \text{ reaction}} \times \frac{235 \text{ g}}{6.02 \times 10^{23} \text{ nuclei}} = \boxed{1.39 \times 10^6 \text{ g} = 1390 \text{ kg}}$$

REFLECT

This is a larger mass than what we calculated in Problem 27.61, which makes sense since not all of the energy from the fission reaction goes directly into usable energy.

27.63

SET UP

A given fission reaction releases 200 MeV. The number of fission reactions per second in a 1000×10^6 W reactor is equal to the total power divided by the energy released per reaction.

SOLVE

$$1000 \times 10^6 \frac{\text{J}}{\text{s}} \times \frac{1 \text{ eV}}{1.602 \times 10^{-19} \text{ J}} \times \frac{1 \text{ MeV}}{10^6 \text{ eV}} \times \frac{1 \text{ reaction}}{200 \text{ MeV}} = \boxed{3.12 \times 10^{19} \frac{\text{reactions}}{\text{s}}}$$

REFLECT

An energy of 200 MeV is approximately 3×10^{-11} J, so 10^{19} reactions is a reasonable solution.

27.64

SET UP

We need to balance the provided fusion reactions by determining the missing species by calculating its expected atomic number Z and mass number A. The sum of the atomic numbers of the products must equal the sum of the atomic numbers of the reactants; the same must be true for the mass numbers.

SOLVE

A)

$$^2\text{H} + {}^3\text{H} \rightarrow {}^4\text{He} + \_\_$$

Atomic number:

$$Z = 1 + 1 - 2 = 0$$

Since $Z = 0$, the missing product will be a neutron.

Mass number:

$$A = 2 + 3 - 4 = 1$$

The missing product must be 1 neutron, or $\boxed{n}$.

B)

$$^{4}\text{He} + {}^{4}\text{He} \rightarrow {}^{7}\text{Be} + \_\_$$

Atomic number:

$$Z = 2 + 2 - 4 = 0$$

Since $Z = 0$, the missing product will be a neutron.

Mass number:

$$A = 4 + 4 - 7 = 1$$

The missing product must be 1 neutron, or $\boxed{n}$.

C)

$$^{2}\text{H} + {}^{2}\text{H} \rightarrow {}^{3}\text{He} + \_\_$$

Atomic number:

$$Z = 1 + 1 - 2 = 0$$

Since $Z = 0$, the missing product will be a neutron.
Mass number:

$$A = 2 + 2 - 3 = 1$$

The missing product must be 1 neutron, or $\boxed{n}$.

D)

$$^{2}\text{H} + {}^{1}\text{H} \rightarrow {}^{0}\gamma + \_\_$$

Atomic number:

$$Z = 1 + 1 - 0 = 2$$

Since $Z = 2$, the missing product will be a helium nucleus.

Mass number:

$$A = 2 + 1 - 0 = 3$$

The missing product must be a helium-3 nucleus, or $\boxed{^{3}\text{He}}$.

E)

$$^{2}\text{H} + {}^{2}\text{H} \rightarrow {}^{3}\text{H} + \_\_$$

Atomic number:

$$Z = 1 + 1 - 1 = 1$$

Since $Z = 1$, the missing product will be a hydrogen nucleus.

Mass number:

$$A = 2 + 2 - 3 = 1$$

The missing product must be a hydrogen-1 nucleus, or $\boxed{^1\text{H}}$.

REFLECT

A hydrogen-1 nucleus is the same as a proton.

27.65

SET UP

The energy released by the reaction is equal to the total energy of the reactants minus the total energy of the products.

SOLVE

A)

$$^2\text{H} + {}^3\text{H} \rightarrow {}^4\text{He} + \text{n}$$

$$\begin{aligned} E_{\text{released}} &= (m_{^2\text{H}} + m_{^3\text{H}} - (m_{^4\text{He}} + m_{\text{n}}))c^2 \\ &= ((2.014102\ \text{u}) + (3.016049\ \text{u}) - ((4.002602\ \text{u}) + (1.008665\ \text{u})))c^2 \\ &= (0.018884\ \text{u})c^2 \times \frac{\left(931.494\dfrac{\text{MeV}}{c^2}\right)}{1\ \text{u}} = \boxed{17.590\ \text{MeV}} \end{aligned}$$

B)

$$^4\text{He} + {}^4\text{He} \rightarrow {}^7\text{Be} + \text{n}$$

$$\begin{aligned} E_{\text{released}} &= (2m_{^4\text{He}} - (m_{^7\text{Be}} + m_{\text{n}}))c^2 \\ &= (2(4.002602\ \text{u}) - ((7.016928\ \text{u}) + (1.008665\ \text{u})))c^2 \\ &= (-0.020389\ \text{u})c^2 \times \frac{\left(931.494\dfrac{\text{MeV}}{c^2}\right)}{1\ \text{u}} = -18.992\ \text{MeV} \end{aligned}$$

The energy required to initiate this reaction is $\boxed{18.992\ \text{MeV}}$.

C)

$$^{2}\text{H} + {}^{2}\text{H} \rightarrow {}^{3}\text{He} + \text{n}$$

$$\begin{aligned} E_{\text{released}} &= (2m_{^2\text{H}} - (m_{^3\text{He}} + m_{\text{n}}))c^2 \\ &= (2(2.014102\ \text{u}) - ((3.016029\ \text{u}) + (1.008665\ \text{u})))c^2 \\ &= (0.003510\ \text{u})c^2 \times \frac{\left(931.494\dfrac{\text{MeV}}{c^2}\right)}{1\ \text{u}} = \boxed{3.270\ \text{MeV}} \end{aligned}$$

D)

$$^{2}\text{H} + {}^{1}\text{H} \rightarrow {}^{0}\gamma + {}^{3}\text{He}$$

$$\begin{aligned} E_{\text{released}} &= (m_{^2\text{H}} + m_{^1\text{H}} - m_{^3\text{He}})c^2 \\ &= ((2.014102\ \text{u}) + (1.007825\ \text{u}) - (3.016029\ \text{u}))c^2 \\ &= (0.005898\ \text{u})c^2 \times \frac{\left(931.494\dfrac{\text{MeV}}{c^2}\right)}{1\ \text{u}} = \boxed{5.494\ \text{MeV}} \end{aligned}$$

E)

$$^{2}\text{H} + {}^{2}\text{H} \rightarrow {}^{3}\text{H} + {}^{1}\text{H}$$

$$\begin{aligned} E_{\text{released}} &= (2m_{^2\text{H}} - (m_{^3\text{H}} + m_{^1\text{H}}))c^2 \\ &= (2(2.014102\ \text{u}) - ((3.016049\ \text{u}) + (1.007825\ \text{u})))c^2 \\ &= (0.004330\ \text{u})c^2 \times \frac{\left(931.494\dfrac{\text{MeV}}{c^2}\right)}{1\ \text{u}} = \boxed{4.033\ \text{MeV}} \end{aligned}$$

REFLECT

In reaction B, the energy "released" is negative, which means the reactants are lower in energy than the products, so it takes energy to fuse the two together. In reaction D, this energy corresponds to the energy of the gamma ray.

27.66

SET UP

The proton–proton cycle that occurs in most stars consists of three steps. In the first step, two hydrogen-1 nuclei fuse to form a hydrogen-2 nucleus, a positron, and a neutrino. This positron then annihilates an electron. Since we're only interested in the energy released in the fusion of two nuclei, we need to subtract the rest energy of an electron from the rest energy of the hydrogen-1 atom. The electron–positron pair annihilation releases $2m_ec^2$. The energy calculations for steps 2 and 3 are relatively straightforward, as we are looking at the energy

difference between the products and reactants. Both of the first two steps must occur twice before the third step can occur; therefore, we must multiply the energy released in each of these steps by 2 when calculating the overall energy released from the fusion process as a whole.

SOLVE

Step 1

Fusion:

$$^{1}\text{H} + {}^{1}\text{H} \rightarrow {}^{2}\text{H} + \text{e}^{+} + \nu_{\text{e}}$$

$$\begin{aligned} E_{\text{released}} &= (2(m_{^1\text{H}} - m_{\text{e}}) - ((m_{^2\text{H}} - m_{\text{e}}) + m_{\text{e}^+}))c^2 \\ &= (2((1.007825\ \text{u}) - (0.000549\ \text{u})) - (((2.014102\ \text{u}) - (0.000549\ \text{u})) \\ &\quad + (0.000549\ \text{u})))c^2 \\ &= (0.000450\ \text{u})c^2 \times \frac{\left(931.494\dfrac{\text{MeV}}{c^2}\right)}{1\ \text{u}} = 0.419\ \text{MeV} \end{aligned}$$

Pair annihilation:

$$m_{\text{e}}c^2 + m_{\text{e}}c^2 = 2m_{\text{e}}c^2 = 2(0.000549\ \text{u})c^2 \times \frac{931.494\dfrac{\text{MeV}}{c^2}}{1\ \text{u}} = 1.02\ \text{MeV}$$

Total energy released for step 1:

$$(0.419\ \text{MeV}) + (1.02\ \text{MeV}) = 1.44\ \text{MeV}$$

Step 2:

$$^{2}\text{H} + {}^{1}\text{H} \rightarrow {}^{3}\text{He} + {}^{0}\gamma$$

$$\begin{aligned} E_{\text{released}} &= (m_{^2\text{H}} + m_{^1\text{H}} - m_{^3\text{He}})c^2 \\ &= ((2.014102\ \text{u}) + (1.007825\ \text{u}) - (3.016029\ \text{u}))c^2 \\ &= (0.005898\ \text{u})c^2 \times \frac{\left(931.494\dfrac{\text{MeV}}{c^2}\right)}{1\ \text{u}} = 5.494\ \text{MeV} \end{aligned}$$

Step 3:

$$^{3}\text{He} + {}^{3}\text{He} \rightarrow {}^{4}\text{He} + 2\,{}^{1}\text{H} + {}^{0}\gamma$$

$$\begin{aligned} E_{\text{released}} &= (2m_{^3\text{He}} - (m_{^4\text{He}} + 2m_{^1\text{H}}))c^2 \\ &= (2(3.016029\ \text{u}) - ((4.002602\ \text{u}) + 2(1.007825\ \text{u})))c^2 \\ &= (0.013806\ \text{u})c^2 \times \frac{\left(931.494\dfrac{\text{MeV}}{c^2}\right)}{1\ \text{u}} = 12.860\ \text{MeV} \end{aligned}$$

Net energy released for all three steps:

$$E_{\text{released, net}} = 2(1.44\ \text{MeV}) + 2(5.494\ \text{MeV}) + (12.860\ \text{MeV}) = \boxed{26.73\ \text{MeV}}$$

REFLECT

Technically, we should have subtracted 13.6 eV for the ionization energy of hydrogen in step 1 when removing the electron. We ignored this because the energies from the fusion process are orders of magnitude larger than this.

27.67

SET UP

The deuterium–tritium (D–T) fusion reaction forms helium-4 and a neutron from deuterium and tritium: $D + T \rightarrow {}^4He + n$; the reaction releases about 20×10^6 eV of energy. We can use this information, along with Avogadro's number and the molar mass of tritium (1 mol = 3 g) in order to calculate the amount of tritium necessary to create 10^{14} J of energy.

SOLVE

$$10^{14}\ \text{J} \times \frac{1\ \text{eV}}{1.6 \times 10^{-19}\ \text{J}} \times \frac{1\ \text{nucleus T}}{20 \times 10^6\ \text{eV}} \times \frac{3\ \text{g}}{6.02 \times 10^{23}\ \text{nuclei T}} = \boxed{156\ \text{g}}$$

REFLECT

This reaction is the simplest fusion reaction to perform on Earth and, because of this, has great promise for potential future sources of nuclear energy.

27.68

SET UP

The deuterium−tritium (D−T) fusion reaction forms helium-4 and a neutron from deuterium and tritium: $D + T \rightarrow {}^4He + n$; the reaction releases about 20×10^6 eV of energy. We can use this information in order to calculate the number of reactions per second that must be sustained to operate a D−T fusion reactor operating at 33% efficiency.

SOLVE

$$1000 \times 10^6 \frac{\text{J out}}{\text{s}} \times \frac{1\ \text{eV out}}{1.602 \times 10^{-19}\ \text{J out}} \times \frac{100\ \text{eV in}}{33\ \text{eV out}} \times \frac{1\ \text{reaction}}{20 \times 10^6\ \text{eV in}} = \boxed{9.5 \times 10^{20} \frac{\text{reactions}}{\text{s}}}$$

REFLECT

An efficiency of 33% means for every 100 J of energy put in only 33 J of energy comes out.

27.69

SET UP

We are asked to convert between common units related to radioactivity. The necessary conversions are $1\ \text{Ci} = 3.7 \times 10^{10}$ Bq and $1\ \frac{\text{count}}{\text{s}} = 1$ Bq.

SOLVE

A) $100\ \mu\text{Ci} \times \frac{1\ \text{Ci}}{10^6\ \mu\text{Ci}} \times \frac{3.7 \times 10^{10}\ \text{Bq}}{1\ \text{Ci}} = \boxed{3.70 \times 10^6\ \text{Bq}}$

B) $1500 \frac{\text{counts}}{\text{min}} \times \frac{1\ \text{min}}{60\ \text{s}} = 25 \frac{\text{counts}}{\text{s}} = \boxed{25\ \text{Bq}}$

C) $16{,}500\ \text{Bq} \times \frac{1\ \text{Ci}}{3.7 \times 10^{10}\ \text{Bq}} = \boxed{4.4595 \times 10^{-7}\ \text{Ci}}$

D) $7.55 \times 10^{10}\ \text{Bq} = 7.55 \times 10^{10} \frac{\text{counts}}{\text{s}} \times \frac{60\ \text{s}}{1\ \text{min}} = \boxed{4.53 \times 10^{12}\ \text{cpm}}$

REFLECT

The SI unit for decay rate is the becquerel (Bq).

27.70

SET UP

The curie, which is approximately the rate at which radiation is emitted from 1.00 g of radium, is defined as 1 Ci = 3.7×10^{10} Bq. The decay constant λ for this sample is not only related to this rate and the number of atoms present, $R_0 = \lambda N_0$, but also the half-life, $\lambda = \frac{\ln(2)}{\tau_{1/2}}$. The actual half-life of radium-226 is 1600 yr. If the calculated half-life is smaller than this value, the actual radiation emission rate of radium is less than 1 Ci; if the calculated half-life is larger than this value, the actual radiation emission rate of radium is more than 1 Ci.

SOLVE

Part a)
Number of atoms:

$$1.00\ \text{g} \times \frac{1\ \text{mol}}{226\ \text{g}} \times \frac{6.02 \times 10^{23}\ \text{atoms}}{1\ \text{mol}} = 2.66 \times 10^{21}\ \text{atoms}$$

Half-life:

$$R_0 = \lambda N_0 = \left(\frac{\ln(2)}{\tau_{1/2}}\right) N_0$$

$$\tau_{1/2} = \frac{N_0 \ln(2)}{R_0} = \frac{(2.66 \times 10^{21}\ \text{atoms}) \ln(2)}{3.7 \times 10^{10}\ \text{Bq}}$$

$$= 4.99 \times 10^{10}\ \text{s} \times \frac{1\ \text{hr}}{3600\ \text{s}} \times \frac{1\ \text{day}}{24\ \text{hr}} \times \frac{1\ \text{yr}}{365\ \text{days}} = \boxed{1580\ \text{yr}}$$

Part b) The radiation emission rate is slightly less than 1 Ci.

REFLECT

All of the isotopes of radium are radioactive, but radium-226 has the longest half-life.

27.71

SET UP

In order to derive the radioactive decay formula, $N(t) = N_0e^{-\lambda t}$, we need to solve the separable differential equation describing the time rate of change for a sample of N radioactive nuclei, $\frac{dN}{dt} = -\lambda N$. The number of nuclei at time $t = 0$ is N_0, and the number of nuclei at any given time t is $N(t)$.

SOLVE

$$\frac{dN}{dt} = -\lambda N$$

$$\int_{N_0}^{N(t)} \frac{dN}{N} = -\int_0^t \lambda dt$$

$$[\ln(N)]_{N_0}^{N(t)} = [-\lambda t]_0^t$$

$$\ln\left(\frac{N(t)}{N_0}\right) = -\lambda t$$

$$\frac{N(t)}{N_0} = e^{-\lambda t}$$

$$\boxed{N(t) = N_0e^{-\lambda t}}$$

REFLECT

In terms of the half-life, the equation becomes $N(t) = N_0\left(\frac{1}{2}\right)^{\frac{t}{\tau_{1/2}}}$.

27.72

SET UP

The half-life of tritium is $\tau_{1/2} = 12.3$ yr. The decay constant is given by $\lambda = \frac{\ln(2)}{\tau_{1/2}}$.

SOLVE

$$\lambda = \frac{\ln(2)}{\tau_{1/2}} = \frac{\ln(2)}{12.3 \text{ yr}} = \boxed{0.0564 \text{ yr}^{-1}} \times \frac{1 \text{ yr}}{365 \text{ days}} \times \frac{1 \text{ day}}{24 \text{ hr}} \times \frac{1 \text{ hr}}{3600 \text{ s}} = \boxed{1.79 \times 10^{-9} \text{ s}^{-1}}$$

REFLECT

A large half-life corresponds to a small decay constant since they are inversely proportional to one another.

27.73

SET UP

A certain radioisotope has a decay constant of $\lambda = 0.00334 \text{ s}^{-1}$. The half-life is equal to $\tau_{1/2} = \frac{\ln(2)}{\lambda}$.

SOLVE

$$\tau_{1/2} = \frac{\ln(2)}{\lambda} = \frac{\ln(2)}{0.00334\ \mathrm{s}^{-1}} = \boxed{208\ \mathrm{s}} \times \frac{1\ \mathrm{hr}}{3600\ \mathrm{s}} \times \frac{1\ \mathrm{day}}{24\ \mathrm{hr}} = \boxed{0.00240\ \mathrm{days}}$$

REFLECT

This is about 3.5 min.

27.74

SET UP

A radioactive sample initially has a decay rate of $R_0 = 5640$ cpm. After $t = 12$ hr, the decay rate is $R = 1410$ cpm. We can use $R = R_0 e^{-\lambda t}$ and $\tau_{1/2} = \dfrac{\ln(2)}{\lambda}$ to calculate the decay constant of the sample and its half-life, respectively.

SOLVE

Part a)

$$R = R_0 e^{-\lambda t}$$

$$\ln\left(\frac{R}{R_0}\right) = -\lambda t$$

$$\lambda = \frac{\ln\left(\frac{R_0}{R}\right)}{t} = \frac{\ln\left(\frac{5640\ \mathrm{cpm}}{1410\ \mathrm{cpm}}\right)}{12\ \mathrm{hr}} = \boxed{0.12\ \mathrm{hr}^{-1}}$$

Part b)

$$\tau_{1/2} = \frac{\ln(2)}{\lambda} = \frac{\ln(2)}{0.12\ \mathrm{hr}^{-1}} = \boxed{6.0\ \mathrm{hr}}$$

REFLECT

Twelve hours is equal to two half-lives; therefore, the decay rate should have decreased by a factor of 4:

$$\frac{5640\ \mathrm{cpm}}{1410\ \mathrm{cpm}} = 4$$

27.75

SET UP

The half-life of phosphorus-32 is $\tau_{1/2} = 14.3$ days. The fraction of the sample remaining after 4 months (or 120 days) is given by $\dfrac{N(t)}{N_0} = \left(\dfrac{1}{2}\right)^{\frac{t}{\tau_{1/2}}}$.

SOLVE

$$\frac{N(t)}{N_0} = \left(\frac{1}{2}\right)^{\frac{t}{\tau_{1/2}}} = \left(\frac{1}{2}\right)^{\left(\frac{120 \text{ days}}{14.3 \text{ days}}\right)} = \boxed{0.00298 = 0.298\%}$$

REFLECT

A period of 120 days is about 8.4 half-lives of phosphorus-32.

27.76

SET UP

The fraction of the sample remaining after a time t is given by $\frac{N(t)}{N_0} = \left(\frac{1}{2}\right)^{\frac{t}{\tau_{1/2}}}$. We can plug in $t = 6\tau_{1/2}$ and $t = 7.5\tau_{1/2}$ to calculate the fraction of the sample left after 6 and 7.5 half-lives, respectively.

SOLVE

Part a)

$$\frac{N(t)}{N_0} = \left(\frac{1}{2}\right)^{\frac{t}{\tau_{1/2}}} = \left(\frac{1}{2}\right)^{\frac{6\tau_{1/2}}{\tau_{1/2}}} = \left(\frac{1}{2}\right)^{6} = \boxed{0.015625 = 1.5625\%}$$

Part b)

$$\frac{N(t)}{N_0} = \left(\frac{1}{2}\right)^{\frac{t}{\tau_{1/2}}} = \left(\frac{1}{2}\right)^{\frac{7.5\tau_{1/2}}{\tau_{1/2}}} = \left(\frac{1}{2}\right)^{7.5} = \boxed{0.0055 = 0.55\%}$$

REFLECT

The longer the time, the less of the original radioactive sample there is.

27.77

SET UP

A patient is injected with 7.88 μCi of iodine-131 that has a half-life of $\tau_{1/2} = 8.02$ days. We want to calculate the expected decay rate in the patient's thyroid after 30 days. The decay rate exponentially decays with time, $R = R_0 e^{-\lambda t}$, where R_0 is the initial rate and $\lambda = \frac{\ln(2)}{\tau_{1/2}}$. Only 90% of the initial dose makes its way to the thyroid, which means $R_0 = (0.90)(7.88\ \mu\text{Ci})$.

SOLVE

Decay constant:

$$\lambda = \frac{\ln(2)}{\tau_{1/2}} = \frac{\ln(2)}{8.02 \text{ days}} = 0.0864 \text{ days}^{-1}$$

Decay rate after 30 days:

$$R = R_0 e^{-\lambda t} = (0.90)(7.88\ \mu\text{Ci})e^{-(0.0864 \text{ days}^{-1})(30 \text{ days})} = \boxed{0.531\ \mu\text{Ci}}$$

REFLECT

The number of decays per second depends upon the number of atoms present; if the number of atoms decreases exponentially with time, so too should the decay rate.

27.78

SET UP

The ratio of carbon-14 to carbon-12 in living wood is 1.3×10^{-12}. The number of decays per second in a block of wood ($m_{\text{wood}} = 550$ g) is equal to the product of the decay constant and the number of carbon-14 nuclei present in the sample. The half-life of carbon-14 is 5730 yr.

SOLVE

Number of carbon-14 nuclei:

$$m_{\text{carbon-14}} = (1.3 \times 10^{-12})m_{\text{carbon-12}} \approx (1.3 \times 10^{-12})m_{\text{wood}} = (1.3 \times 10^{-12})(550 \text{ g})$$

$$= 7.1 \times 10^{-10} \text{ g} \times \frac{1 \text{ mol}}{14.003242 \text{ g}} \times \frac{6.02 \times 10^{23} \text{ nuclei}}{1 \text{ mol}} = 3.1 \times 10^{13} \text{ nuclei}$$

Decays per second:

$$R = \lambda N = \left(\frac{\ln(2)}{\tau_{1/2}}\right)N = \frac{(3.1 \times 10^{13} \text{ nuclei})\ln(2)}{\left(5730 \text{ yr} \times \dfrac{365 \text{ days}}{1 \text{ yr}} \times \dfrac{24 \text{ hr}}{1 \text{ day}} \times \dfrac{3600 \text{ s}}{1 \text{ hr}}\right)} = \boxed{120 \text{ Bq}}$$

REFLECT

Since the number of carbon-14 nuclei is much less than the number of carbon-12 nuclei in living wood, we can approximate the block of wood as being made completely of carbon-12.

27.79

SET UP

A totem pole contains 225 g of carbon and has a beta activity of $R = 150$ cpm. Using the fact that the ratio of carbon-14 to carbon-12 in living wood is 1.3×10^{-12}, we can calculate the initial number of carbon-14 nuclei in the totem pole and then the initial decay rate of the sample. The age of the totem pole can be found using $R = R_0 e^{-\lambda t}$, where $\lambda = \dfrac{\ln(2)}{\tau_{1/2}}$; the half-life of carbon-14 is $\tau_{1/2} = 5730$ yr.

SOLVE

Initial number of carbon-14 nuclei:

$$225 \text{ g C} \times \frac{1 \text{ mol } ^{12}\text{C}}{12.0 \text{ g } ^{12}\text{C}} \times \frac{6.02 \times 10^{23} \text{ nuclei } ^{12}\text{C}}{1 \text{ mol } ^{12}\text{C}} \times \frac{1.3 \times 10^{-12} \text{ nuclei } ^{14}\text{C}}{1 \text{ nuclei } ^{12}\text{C}} = 1.5 \times 10^{13} \text{ nuclei } ^{14}\text{C}$$

Initial rate:

$$R_0 = \lambda N_0 = \left(\frac{\ln(2)}{\tau_{1/2}}\right)N_0 = \frac{(1.5 \times 10^{13}\text{ nuclei})\ln(2)}{\left(5730\text{ yr} \times \dfrac{365\text{ days}}{1\text{ yr}} \times \dfrac{24\text{ hr}}{1\text{ day}} \times \dfrac{3600\text{ s}}{1\text{ hr}}\right)}$$

$$= 56\text{ Bq} = 56\frac{\text{counts}}{\text{s}} \times \frac{60\text{ s}}{1\text{ min}} = 3400\text{ cpm}$$

Age of the pole:

$$R = R_0 e^{-\lambda t}$$

$$\ln\left(\frac{R}{R_0}\right) = -\lambda t$$

$$t = \frac{\ln\left(\dfrac{R_0}{R}\right)}{\lambda} = \frac{\ln\left(\dfrac{R_0}{R}\right)}{\left(\dfrac{\ln(2)}{\tau_{1/2}}\right)} = \frac{\tau_{1/2}\ln\left(\dfrac{R_0}{R}\right)}{\ln(2)} = \frac{(5730\text{ yr})\ln\left(\dfrac{3400\text{ cpm}}{150\text{ cpm}}\right)}{\ln(2)} = \boxed{2.6 \times 10^4\text{ yr}}$$

REFLECT

Since the number of carbon-14 nuclei is much less than the number of carbon-12 nuclei in living wood, we can approximate the molar mass of carbon as the molar mass of carbon-12.

27.80

SET UP

A tree limb contains 500 g of carbon. Using the fact that the ratio of carbon-14 to carbon-12 in living wood is 1.3×10^{-12}, we can calculate the initial number of carbon-14 nuclei in the totem pole and then the initial decay rate of the sample. The decay rate in the tree limb after 1200 yr can be found using $R = R_0\left(\frac{1}{2}\right)^{\frac{t}{\tau_{1/2}}}$; the half-life of carbon-14 is $\tau_{1/2} = 5730$ yr.

SOLVE

Initial number of carbon-14 nuclei:

$$500\text{ g C} \times \frac{1\text{ mol }^{12}\text{C}}{12.0\text{ g }^{12}\text{C}} \times \frac{6.02 \times 10^{23}\text{ nuclei }^{12}\text{C}}{1\text{ mol }^{12}\text{C}} \times \frac{1.3 \times 10^{-12}\text{ nuclei }^{14}\text{C}}{1\text{ nuclei }^{12}\text{C}} = 3.26 \times 10^{13}\text{ nuclei }^{14}\text{C}$$

Initial rate:

$$R_0 = \lambda N_0 = \left(\frac{\ln(2)}{\tau_{1/2}}\right)N_0 = \frac{(3.26 \times 10^{13}\text{ nuclei})\ln(2)}{\left(5730\text{ yr} \times \dfrac{365\text{ days}}{1\text{ yr}} \times \dfrac{24\text{ hr}}{1\text{ day}} \times \dfrac{3600\text{ s}}{1\text{ hr}}\right)} = 125\text{ Bq}$$

Decay rate after 12 centuries:

$$R = R_0\left(\frac{1}{2}\right)^{\frac{t}{\tau_{1/2}}} = (125\text{ Bq})\left(\frac{1}{2}\right)^{\frac{1200\text{ yr}}{5430\text{ yr}}} = \boxed{107\text{ Bq}}$$

REFLECT

Since the number of carbon-14 nuclei is much less than the number of carbon-12 nuclei in living wood, we can approximate the molar mass of carbon as the molar mass of carbon-12.

27.81

SET UP

A sample of radon-222 has a decay rate of $R = 485$ counts/min. The decay rate of the sample is equal to the product of the decay constant λ and the number of nuclei N. The half-life of radon-222 from Appendix C is $\tau_{1/2} = 3.823$ days. After finding λ from the half-life, we can divide the decay rate by it in order to calculate the number of nuclei in the sample.

SOLVE

Decay constant:

$$\lambda = \frac{\ln(2)}{\tau_{1/2}} = \frac{\ln(2)}{\left(3.823 \text{ days} \times \frac{24 \text{ hr}}{1 \text{ day}} \times \frac{3600 \text{ s}}{1 \text{ hr}}\right)} = 2.0985 \times 10^{-6} \text{ s}^{-1}$$

Number of nuclei:

$$R = \lambda N$$

$$N = \frac{R}{\lambda} = \frac{\left(485 \frac{\text{counts}}{\text{min}} \times \frac{1 \text{ min}}{60 \text{ s}}\right)}{2.0985 \times 10^{-6} \text{ s}^{-1}} = \boxed{3.85 \times 10^{6} \text{ nuclei}}$$

REFLECT

The units of "counts/second" are equivalent to becquerels, as long as the detector detects 100% of the radioactive decays.

27.82

SET UP

Rubidium-87 undergoes beta decay to form strontium-87 with a half-life of $\tau_{1/2} = 4.75 \times 10^{10}$ yr. An ancient sample contains a ratio of strontium-87 to rubidium-87 of $\frac{N_{\text{Sr}}}{N_{\text{Rb}}} = 0.0225$. If the sample originally contained no strontium, then that means all of the strontium that exists had to have come from the beta decay of rubidium. The amount of rubidium that decayed in a time interval Δt is $-\Delta N_{\text{Rb}} = \lambda_{\text{Rb}} N_{\text{Rb}} \Delta t$, which is also equal to the amount of strontium present. Rearranging this equation, we can solve for the age of the sample.

SOLVE

$$N_{\text{Sr}} = -\Delta N_{\text{Rb}} = \lambda_{\text{Rb}} N_{\text{Rb}} \Delta t$$

$$\Delta t = \left(\frac{N_{\text{Sr}}}{N_{\text{Rb}}}\right)\frac{1}{\lambda_{\text{Rb}}} = \left(\frac{N_{\text{Sr}}}{N_{\text{Rb}}}\right)\frac{1}{\left(\frac{\ln(2)}{\tau_{1/2}}\right)} = \left(\frac{N_{\text{Sr}}}{N_{\text{Rb}}}\right)\frac{\tau_{1/2}}{\ln(2)}$$

$$= (0.0225)\left(\frac{4.75 \times 10^{10} \text{ yr}}{\ln(2)}\right) = \boxed{1.54 \times 10^{9} \text{ yr}}$$

REFLECT

The age of the sample is less than one-tenth of one half-life, so it makes sense that there would not be that much strontium present.

27.83

SET UP

We need to balance the provided alpha decay reactions by determining the missing species by calculating its expected atomic number Z and mass number A. The sum of the atomic numbers of the products must equal the sum of the atomic numbers of the reactants; the same must be true for the mass numbers.

SOLVE

A)

$$^{238}\mathrm{U} \rightarrow {}^{4}\alpha + \_\_$$

Atomic number:

$$Z = 92 - 2 = 90$$

Since $Z = 90$, the missing product will be a thorium nucleus.

Mass number:

$$A = 238 - 4 = 234$$

The missing product will be a thorium-234 nucleus, $\boxed{^{234}\mathrm{Th}}$.

B)

$$^{234}\mathrm{Th} \rightarrow \_\_ + {}^{?}\mathrm{Ra}$$

Since this is an alpha decay, one of the products will be an alpha particle, $\boxed{^{4}\alpha}$.

Mass number of radium:

$$A = 234 - 4 = 230$$

The missing product will be a radium-230 nucleus, $\boxed{^{230}\mathrm{Ra}}$.

C)

$$\_\_ \rightarrow {}^{4}\alpha + {}^{236}\mathrm{U}$$

Atomic number:

$$Z = 92 + 2 = 94$$

Since $Z = 94$, the missing reactant will be a plutonium nucleus.

Mass number:

$$A = 236 + 4 = 240$$

The missing product will be a plutonium-240 nucleus, $\boxed{^{240}\mathrm{Pu}}$.

D)

$$^{214}\text{Bi} \rightarrow {}^{4}\alpha + \_\_$$

Atomic number:

$$Z = 83 - 2 = 81$$

Since $Z = 81$, the missing product will be a thallium nucleus.

Mass number:

$$A = 214 - 4 = 210$$

The missing product will be a thallium-210 nucleus, $\boxed{^{210}\text{Tl}}$.

REFLECT

An alpha particle is the same as a helium-4 nucleus.

27.84

SET UP

We need to balance the provided beta decay reactions by determining the missing species by calculating its expected atomic number Z and mass number A. The sum of the atomic numbers of the products must equal the sum of the atomic numbers of the reactants; the same must be true for the mass numbers.

SOLVE

A)

$$^{14}\text{C} \rightarrow \text{e}^- + \bar{\nu} + \_\_$$

Atomic number:

$$Z = 6 - (-1) = 7$$

Since $Z = 7$, the missing product will be a nitrogen nucleus.

Mass number:

$$A = 14 - 0 = 14$$

The missing product will be a nitrogen-14 nucleus, $\boxed{^{14}\text{N}}$.

B)

$$^{239}\text{Np} \rightarrow \text{e}^- + \bar{\nu} + \_\_$$

Atomic number:

$$Z = 93 - (-1) = 94$$

Since $Z = 94$, the missing product will be a plutonium nucleus.

Mass number:

$$A = 239 - 0 = 239$$

The missing product will be a plutonium-239 nucleus, $\boxed{^{239}\mathrm{Pu}}$.

C)

$$\_\_ \rightarrow \mathrm{e}^- + \bar{\nu} + {}^{60}\mathrm{Ni}$$

Atomic number:

$$Z = 28 + (-1) = 27$$

Since $Z = 27$, the missing product will be a cobalt nucleus.

Mass number:

$$A = 60 - 0 = 60$$

The missing product will be a cobalt-60 nucleus, $\boxed{^{60}\mathrm{Co}}$.

D)

$$^{3}\mathrm{H} \rightarrow \mathrm{e}^- + \bar{\nu} + \_\_$$

Atomic number:

$$Z = 1 - (-1) = 2$$

Since $Z = 2$, the missing product will be a helium nucleus.

Mass number:

$$A = 3 - 0 = 3$$

The missing product will be a helium-3 nucleus, $\boxed{^{3}\mathrm{He}}$.

E)

$$^{13}\mathrm{N} \rightarrow \mathrm{e}^+ + \_\_ + \_\_$$

Since this is a β^+ decay, one of the products must be a neutrino, $\boxed{\nu_\mathrm{e}}$.

Atomic number of the daughter nucleus:

$$Z = 7 - 1 = 6$$

Since $Z = 6$, the other missing product will be a carbon nucleus.

Mass number:

$$A = 13 - 0 = 13$$

The other missing product will be a carbon-13 nucleus, $\boxed{^{13}\mathrm{C}}$.

REFLECT

A β^- decay releases an electron and an antineutrino; a β^+ decay releases a positron and a neutrino.

27.85

SET UP

We need to balance the provided gamma decay reactions by determining the missing species. Gamma radiation does not change the atomic number or the mass number of a nucleus; it only affects the energy state of the nucleus.

SOLVE

A)

$$^{131}\mathrm{I}^* \rightarrow \gamma + \boxed{^{131}\mathrm{I}}$$

B)

$$^{145}\mathrm{Pm}^* \rightarrow \gamma + \boxed{^{145}\mathrm{Pm}}$$

C)

$$\boxed{^{24}\mathrm{Na}^*} \rightarrow \gamma + {}^{24}\mathrm{Na}$$

REFLECT

Gamma radiation is emitted when a nucleus in an excited state decays to a lower energy state that is about 1 MeV less in energy.

27.86

SET UP

An excited state of nickel-64 is $E_\gamma = 1.34$ MeV higher in energy than the ground state. The atomic mass of the ground state is $m_{\mathrm{Ni}} = 63.927967$ u. The mass of the excited state is equal to the mass of the ground state plus the mass equivalent of the energy difference. The wavelength of the gamma ray emitted when the nucleus decays to the ground state is given by $E_\gamma = \dfrac{hc}{\lambda_\gamma}$.

SOLVE

Part a)

$$m_{\mathrm{Ni}^*} = m_{\mathrm{Ni}} + m_\gamma = m_{\mathrm{Ni}} + \frac{E_\gamma}{c^2}$$

$$= (63.927967\ \mathrm{u}) + \frac{\left(1.34\ \mathrm{MeV} \times \dfrac{1\ \mathrm{u}}{\left(931.494\dfrac{\mathrm{MeV}}{c^2}\right)}\right)}{c^2} = \boxed{63.92941\ \mathrm{u}}$$

Part b)

$$E_\gamma = \frac{hc}{\lambda_\gamma}$$

$$\lambda_\gamma = \frac{hc}{E_\gamma} = \frac{1240 \text{ eV} \cdot \text{nm}}{1.34 \times 10^6 \text{ eV}} = \boxed{9.25 \times 10^{-4} \text{ nm}}$$

REFLECT

An increase in energy of about 1 MeV only increases the mass of the excited state by about 1×10^{-3} u.

27.87

SET UP

The approximate radius of a uranium-238 nucleus is given by $r = r_0 A^{\frac{1}{3}}$, where $r_0 = 1.2$ fm and A is the mass number. The magnitude of the force between two protons located at opposite ends of the nucleus can be calculated from Coulomb's law. If this were the only force acting on the protons, we can calculate their resulting acceleration from Newton's second law. Finally, the nucleus is held together by the strong nuclear force, which allows the nucleons to fuse together to form nuclei.

SOLVE

Part a)

$$r = r_0 A^{\frac{1}{3}} = (1.2 \text{ fm})(238)^{\frac{1}{3}} = \boxed{7.4 \text{ fm}}$$

Part b)

$$F = \frac{k(e)(e)}{r^2} = \frac{\left(9.0 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(1.6 \times 10^{-19} \text{ C})^2}{(2(7.4 \times 10^{-15} \text{ m}))^2} = \boxed{1.0 \text{ N}}$$

Part c)

$$\sum F = F = m_\text{p} a$$

$$a = \frac{F}{m_\text{p}} = \frac{1.0 \text{ N}}{(1.67 \times 10^{-27} \text{ kg})} = \boxed{6.2 \times 10^{26} \frac{\text{m}}{\text{s}^2}}$$

Part d) The strong nuclear force holds them together.

REFLECT

The weight of a proton is about 10^{-26} N, which means the electrostatic force between the two farthest protons in the nucleus is 26 orders of magnitude larger! For a comparison, the mass of Earth is "only" 23 orders of magnitude larger than the mass of a person.

Get Help: P'Cast 27.1 – Nuclear Radii

27.88

SET UP

We are given a semiempirical formula to calculate the binding energy of fermium-253 and asked to compare the resulting binding energy per nucleon to the binding energy per nucleon calculated using the standard common expression for binding energy, $E_B = (Nm_n + Zm_{^1H} - m_{atom})c^2$. Fermium-253 has an atomic number of $Z = 100$, $A = 253$, and $N = 153$.

SOLVE

Semiempirical binding energy per nucleon:

$$E_B = (15.8\text{ MeV})A - (17.8\text{ MeV})A^{\frac{2}{3}} - (0.71\text{ MeV})\frac{Z(Z-1)}{A^{\frac{1}{3}}} - (23.7\text{ MeV})\frac{(N-Z)^2}{A}$$

$$= (15.8\text{ MeV})(253) - (17.8\text{ MeV})(253)^{\frac{2}{3}} - (0.71\text{ MeV})\frac{100(100-1)}{(253)^{\frac{1}{3}}}$$

$$- (23.7\text{ MeV})\frac{(153-100)^2}{253}$$

$$= 1900\text{ MeV}$$

$$\frac{1900\text{ MeV}}{253\text{ nucleons}} = \boxed{7.6\frac{\text{MeV}}{\text{nucleon}}}$$

Standard binding energy per nucleon:

$$E_B = (Nm_n + Zm_{^1H} - m_{atom})c^2$$

$$= (153(1.008665\text{ u}) + 100(1.007825\text{ u}) - (253.085173\text{ u}))c^2$$

$$= (2.0231\text{ u})c^2 \times \frac{\left(931.494\frac{\text{MeV}}{c^2}\right)}{1\text{ u}} = 1884.5\text{ MeV}$$

$$\frac{1884.5\text{ MeV}}{253\text{ nucleons}} = \boxed{7.4485\frac{\text{MeV}}{\text{nucleon}}}$$

REFLECT

Experimental data and observations are used to create a semiempirical formula. The parameters are changed until the fit from the equation matches the experimental data.

27.89

SET UP

The fissionability parameter of a given nucleus is defined as $\frac{Z^2}{A}$, where Z is the atomic number and A is the mass number. The atomic number of uranium is $Z = 92$; the atomic number of plutonium is $Z = 94$; and the atomic number of californium is $Z = 98$.

SOLVE

$$\text{fissionability parameter} = \frac{Z^2}{A}$$

Part a)

$$\frac{Z^2}{A} = \frac{(92)^2}{235} = \boxed{36}$$

Part b)

$$\frac{Z^2}{A} = \frac{(92)^2}{238} = \boxed{36}$$

Part c)

$$\frac{Z^2}{A} = \frac{(94)^2}{239} = \boxed{37}$$

Part d)

$$\frac{Z^2}{A} = \frac{(94)^2}{240} = \boxed{37}$$

Part e)

$$\frac{Z^2}{A} = \frac{(98)^2}{246} = \boxed{39}$$

Part f)

$$\frac{Z^2}{A} = \frac{(98)^2}{254} = \boxed{38}$$

REFLECT

Since all of the fissionability parameters are less than 44, none of these nuclei will undergo spontaneous fission.

27.90

SET UP

Sodium-23 is a stable nucleus, while sodium-22 and sodium-24 are unstable. This tells us that sodium-22 has too few neutrons, whereas sodium-24 has too many. Therefore, sodium-22 will undergo β^+ decay in order to decrease the number of protons, and sodium-24 will undergo β^- decay to increase the number of protons.

SOLVE

Part a) Sodium-22 will undergo β^+ decay:

$$^{22}\text{Na} \rightarrow \text{e}^+ + \nu + {}^{22}\text{Ne}$$

Part b) Sodium-24 will undergo β^- decay:

$$^{24}\text{Na} \rightarrow \text{e}^- + \overline{\nu} + {}^{24}\text{Mg}$$

REFLECT

Although the mass number remains constant, the number of neutrons and the number of protons change.

27.91

SET UP

An isotope of element 117 was created that had 176 neutrons and a half-life of $\tau_{1/2} = 14 \times 10^{-3}$ s. The mass number A of the isotope is the sum of the atomic number and the number of neutrons. The approximate radius of the nucleus is given by $r = r_0 A^{\frac{1}{3}}$, where $r_0 = 1.2$ fm. The fraction of the sample that remains as a function of time is related to the half-life by $\frac{N(t)}{N_0} = \left[\frac{1}{2}\right]^{\frac{t}{\tau_{1/2}}}$; we can use this expression to calculate the percent of the created isotope left after 1.0 s.

SOLVE

Part a)

Mass number:

$$A = N + Z = 176 + 117 = 293$$

Radius of the nucleus:

$$r = r_0 A^{\frac{1}{3}} = (1.2 \text{ fm})(293)^{\frac{1}{3}} = \boxed{8.0 \text{ fm}}$$

Part b)

$$\frac{N(1.0 \text{ s})}{N_0} = \left[\frac{1}{2}\right]^{\frac{1.0 \text{ s}}{14 \times 10^{-3} \text{ s}}} = 3.1 \times 10^{-22} = \boxed{3.1 \times 10^{-20} \ \%}$$

REFLECT

A time interval of 1.0 s is a little over 71 half-lifes of the new element, so we should expect the amount of the element created to be extremely small.

27.92

SET UP

Two of the isotopes in natural uranium are uranium-235 ($\tau_{1/2,\,235} = 7.04 \times 10^8$ yr) and uranium-238 ($\tau_{1/2,\,238} = 4.47 \times 10^9$ yr). Assuming the two uranium isotopes were created in equal amounts at the same time Earth was formed, we can estimate the age of Earth by

calculating the time necessary for a sample of each isotope to decay to its current amounts. The percent abundance of uranium-235 and uranium-238 are 0.72% and 99.28%, respectively. The number of nuclei as a function of time is described by $N(t) = N_0 e^{-\lambda t}$, where $\lambda = \dfrac{\ln(2)}{\tau_{1/2}}$.

SOLVE

$$N_{0_{235}} = N_{0_{238}}$$

$$\frac{N_{235}}{e^{-\lambda_{235}t}} = \frac{N_{238}}{e^{-\lambda_{238}t}}$$

$$\frac{N_{235}}{N_{238}} = \frac{e^{-\lambda_{235}t}}{e^{-\lambda_{238}t}} = e^{-(\lambda_{235}-\lambda_{238})t}$$

$$\ln\left(\frac{N_{235}}{N_{238}}\right) = -(\lambda_{235} - \lambda_{238})t$$

$$t = \frac{\ln\left(\frac{N_{238}}{N_{235}}\right)}{\lambda_{235} - \lambda_{238}} = \frac{\ln\left(\frac{N_{238}}{N_{235}}\right)}{\left(\frac{\ln(2)}{\tau_{1/2,\,235}}\right) - \left(\frac{\ln(2)}{\tau_{1/2,\,238}}\right)} = \frac{\ln\left(\frac{N_{238}}{N_{235}}\right)}{\ln(2)\left(\left(\frac{1}{\tau_{1/2,\,235}}\right) - \left(\frac{1}{\tau_{1/2,\,238}}\right)\right)}$$

$$= \frac{\ln\left(\frac{0.9928}{0.0072}\right)}{\ln(2)\left(\left(\frac{1}{7.04 \times 10^8\ \text{yr}}\right) - \left(\frac{1}{4.47 \times 10^9\ \text{yr}}\right)\right)} = \boxed{5.94 \times 10^9\ \text{yr}}$$

REFLECT

The accepted age of Earth is around 4.5×10^9 yr, so our answer is a bit high but on the same order of magnitude.

27.93

SET UP

The spectral lines of technetium-98 ($\tau_{1/2} = 4.2 \times 10^6$ yr) have been detected in stars near the end of their lifetimes. We can calculate the amount of technetium-98 remaining after a time period of either 4.5×10^9 yr or 10×10^9 yr using $\dfrac{N}{N_0} = \left(\dfrac{1}{2}\right)^{\frac{t}{\tau_{1/2}}}$. If there is an appreciable amount left, this could explain the existence of the technetium spectral lines; if not, technetium must have been produced since the formation of Earth.

SOLVE

Part a)
Amount of technetium-98 remaining:

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{t}{\tau_{1/2}}} = 2^{\frac{t}{\tau_{1/2}}} = 2^{-\left(\frac{4.5\times10^9\ \text{yr}}{4.2\times10^6\ \text{yr}}\right)} = 2^{-1100}$$

Expressing the answer in scientific notation:

$$2^{-1100} = 10^x$$

$$-1100\log(2) = x\log(10)$$

$$x = -322$$

$$\frac{N}{N_0} = 10^{-322} = \boxed{10^{-320}\%}$$

Part b)
Amount of technetium-98 remaining:

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{t}{\tau_{1/2}}} = 2^{-\frac{t}{\tau_{1/2}}} = 2^{-\left(\frac{1.0\times10^9\ \text{yr}}{4.2\times10^6\ \text{yr}}\right)} = 2^{-2400}$$

Expressing the answer in scientific notation:

$$2^{-2400} = 10^x$$

$$-2400\log(2) = x\log(10)$$

$$x = -717$$

$$\frac{N}{N_0} = 10^{-717} = \boxed{10^{-715}\%}$$

Part c) Almost none of the original technetium would remain at the end of the star's lifetime, so the technetium seen in the spectral lines of the star must have been produced more recently. Therefore, the star is manufacturing the element.

REFLECT

After about 1100 half-lives, we would expect there to be essentially no technetium remaining.

27.94

SET UP

Radiocarbon dating was used to determine the age of a wood sample found in a bowl. The results showed that the level of carbon-14 present was one-quarter of the level in a present-day wooden sample. The fraction of the carbon-14 nuclei that remains as a function of time is related to the half-life by $\frac{N(t)}{N_0} = \left[\frac{1}{2}\right]^{\frac{t}{\tau_{1/2}}}$; we can use this expression to calculate the age of the bowl. The half-life of carbon-14 from Appendix C is $\tau_{1/2} = 5730$ yr.

SOLVE

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{t}{\tau_{1/2}}}$$

$$t = \frac{\tau_{1/2}\ln\left(\frac{N}{N_0}\right)}{\ln\left(\frac{1}{2}\right)} = \frac{\tau_{1/2}\ln\left(\frac{N_0}{N}\right)}{\ln(2)} = \frac{\tau_{1/2}\ln\left(\frac{N_0}{\left(\frac{N_0}{4}\right)}\right)}{\ln(2)} = \frac{\tau_{1/2}\ln(4)}{\ln(2)} = \frac{2\tau_{1/2}\ln(2)}{\ln(2)}$$

$$= 2\tau_{1/2} = 2(5730\text{ yr}) = \boxed{11{,}460\text{ yr}}$$

REFLECT

The amount of carbon-14 decreased by 75%, which is exactly what we would expect after two half-lives.

27.95

SET UP

Radiocarbon dating was used to determine the age of a bone sample found in a cave. The results showed that the level of carbon-14 present was 2.35% of its present-day level. The fraction of the sample that remains as a function of time is related to the half-life by $\frac{N(t)}{N_0} = \left[\frac{1}{2}\right]^{\frac{t}{\tau_{1/2}}}$; we can use this expression to calculate the age of the bones. The half-life of carbon-14 from Appendix C is $\tau_{1/2} = 5730$ yr.

SOLVE

$$\frac{N(t)}{N_0} = \left[\frac{1}{2}\right]^{\frac{t}{\tau_{1/2}}} = 2^{-\frac{t}{\tau_{1/2}}}$$

$$\ln\left(\frac{N(t)}{N_0}\right) = -\frac{t}{\tau_{1/2}}\ln(2)$$

$$t = -\tau_{1/2}\frac{\ln\left(\frac{N(t)}{N_0}\right)}{\ln(2)} = -(5730\text{ yr})\frac{\ln(0.0235)}{\ln(2)} = \boxed{31{,}000\text{ yr}}$$

REFLECT

Radiocarbon dating is commonly used to date organic samples. Plants exchange carbon with the atmosphere through photosynthesis and will, therefore, contain the various isotopes of carbon in the same ratio as the atmosphere. As long as the plant is alive, the ratio should remain relatively constant. When the plant dies, photosynthesis no longer takes place, and the amount of carbon-14 will decrease with time because it is radioactive. Some animals eat plants, and other animals will eat these animals, which is how carbon-14 incorporates itself into animals.

27.96

SET UP

Americium-241 decays via alpha emission with a half-life of $\tau_{1/2} = 433$ yr. First, we need to determine the daughter nucleus by balancing the alpha decay reaction of americium-241 by calculating its expected atomic number Z and mass number A. The sum of the atomic numbers of the products must equal the sum of the atomic numbers of the reactants; the same must be true for the mass numbers. Next, we can use $\frac{R}{R_0} = \left(\frac{1}{2}\right)^{\frac{t}{\tau_{1/2}}}$ to calculate by how much the alpha particle current, which is directly related to the decay rate, decreases after 1.0 yr and 50 yr. Finally, assuming an initial decay rate of $R_0 = 690$ Bq, we can calculate the initial number of americium-241 nuclei in the sample with $R_0 = \lambda N_0$, where $\lambda = \frac{\ln(2)}{\tau_{1/2}}$. From this, we can calculate the initial mass of americium-241 using the molar mass of americium-241, which is $241\frac{\text{g}}{\text{mol}}$.

SOLVE

Part a)

$$^{241}\text{Am} \rightarrow {}^{4}\alpha + \_\_$$

Atomic number:

$$Z = 95 - 2 = 93$$

Since $Z = 93$, the missing product will be a neptunium nucleus.

Mass number:

$$A = 241 - 4 = 237$$

The missing product will be a neptunium-237 nucleus, $\boxed{^{237}\text{Np}}$.

Part b)
After 1.0 yr:

$$\frac{R}{R_0} = \left(\frac{1}{2}\right)^{\frac{t}{\tau_{1/2}}} = \left(\frac{1}{2}\right)^{\frac{1.0\text{ yr}}{433\text{ yr}}} = 0.998$$

After 1.0 yr, the alpha particle current decreases by $\boxed{0.2\%}$.

After 50 yr:

$$\frac{R}{R_0} = \left(\frac{1}{2}\right)^{\frac{t}{\tau_{1/2}}} = \left(\frac{1}{2}\right)^{\frac{50\text{ yr}}{433\text{ yr}}} = 0.923$$

After 50 yr, the alpha particle current decreases by $\boxed{7.7\%}$.

Part c)

$$R_0 = \lambda N_0$$

$$N_0 = \frac{R_0}{\lambda} = \frac{R_0}{\left(\frac{\ln(2)}{\tau_{1/2}}\right)} = \frac{\tau_{1/2} R_0}{\ln(2)} = \frac{\left(433 \text{ yr} \times \frac{365 \text{ days}}{1 \text{ yr}} \times \frac{24 \text{ hr}}{1 \text{ day}} \times \frac{3600 \text{ s}}{1 \text{ hr}}\right)\left(690 \frac{\text{decays}}{\text{s}}\right)}{\ln(2)}$$

$$= 1.36 \times 10^{13} \text{ atoms} \times \frac{1 \text{ mol}}{6.02 \times 10^{23} \text{ atoms}} \times \frac{241 \text{ g}}{1 \text{ mol}} = \boxed{5.44 \times 10^{-9} \text{ g}}$$

REFLECT

The decay rate of americium-241 in a household smoke detector should not decrease that much over the average lifetime of the detector or it would not be reliable to use.

27.97

SET UP

Cesium-137 undergoes β^- decay and has a half-life of $\tau_{1/2} = 30$ yr. First, we need to determine the daughter nucleus by balancing the beta decay reaction of cesium-137 by calculating its expected atomic number Z and mass number A. The sum of the atomic numbers of the products must equal the sum of the atomic numbers of the reactants; the same must be true for the mass numbers. The decay rate as a function of time is given by $R = R_0 e^{-\lambda t}$. We can rearrange this equation and write the decay constant in terms of the half-life in order to calculate the time required for the decay rate to decrease by 99%, that is, $R = 0.01R_0$.

SOLVE

Part a)

$$^{137}\text{Cs} \rightarrow \text{e}^- + \underline{\quad}$$

Atomic number:

$$Z = 55 - (-1) = 56$$

Since $Z = 56$, the missing product will be a barium nucleus.
Mass number:

$$A = 137 - 0 = 137$$

The missing product will be a barium-137 nucleus, $\boxed{^{137}\text{Ba}}$.

Part b)

$$R = R_0 e^{-\lambda t}$$

$$\ln\left(\frac{R}{R_0}\right) = -\lambda t$$

$$t = \frac{\ln\left(\frac{R_0}{R}\right)}{\lambda} = \frac{\ln\left(\frac{R_0}{R}\right)}{\left(\frac{\ln(2)}{\tau_{1/2}}\right)} = \frac{\tau_{1/2} \ln\left(\frac{R_0}{0.01R_0}\right)}{\ln(2)} = \frac{(30 \text{ yr}) \ln(100)}{\ln(2)} = \boxed{200 \text{ yr}}$$

REFLECT

After six half-lives, the rate is about 1.5% of its initial value, so a period of 200 yr is reasonable to achieve a decay rate that is 1% of its initial value.

27.98

SET UP

The binding energy of an atom is the difference in energy between the component parts of the atom and the atom itself, $E_B = (Nm_n + Zm_{^1H} - m_{atom})c^2$, where $m_n = 1.008665$ u and $m_{^1H} = 1.007825$ u. We are given the masses of three isotopes of aluminum: aluminum-26, aluminum-27, and aluminum-28. After we calculate the binding energy, we will need to divide by the mass number to get the binding energy per nucleon. We are told that aluminum-27 is stable, while the other two isotopes are not. Aluminum-26 has fewer neutrons than aluminum-27, so it will most likely undergo β^+ decay in order to decrease the number of protons in the nucleus. Aluminum-28 has more neutrons than aluminum-27, so it will most likely undergo β^- decay in order to increase the number of protons in the nucleus.

SOLVE

Binding energy:

$$E_B = (Nm_n + Zm_{^1H} - m_{atom})c^2$$

Binding energy of aluminum-26:

$$E_B = (13(1.008665 \text{ u}) + 13(1.007825 \text{ u}) - (25.986892 \text{ u}))c^2$$

$$= (0.22746 \text{ u})c^2 \times \frac{\left(931.494\frac{\text{MeV}}{c^2}\right)}{1 \text{ u}} = 211.88 \text{ MeV}$$

Binding energy per nucleon of aluminum-26:

$$\frac{211.88 \text{ MeV}}{26} = \boxed{8.1493\frac{\text{MeV}}{\text{nucleon}}}$$

Decay process for aluminum-26:

$$\boxed{^{26}\text{Al} \rightarrow \text{e}^+ + \nu + {}^{26}\text{Mg}}$$

Binding energy of aluminum-27:

$$E_B = (14(1.008665 \text{ u}) + 13(1.007825 \text{ u}) - (26.981538 \text{ u}))c^2$$

$$= (0.24148 \text{ u})c^2 \times \frac{\left(931.494\frac{\text{MeV}}{c^2}\right)}{1 \text{ u}} = 224.94 \text{ MeV}$$

Binding energy per nucleon of aluminum-27:

$$\frac{224.94\ \text{MeV}}{27} = \boxed{8.3311\frac{\text{MeV}}{\text{nucleon}}}$$

Binding energy of aluminum-28:

$$E_B = (15(1.008665\ \text{u}) + 13(1.007825\ \text{u}) - (27.981910\ \text{u}))c^2$$

$$= (0.24977\ \text{u})c^2 \times \frac{\left(931.494\frac{\text{MeV}}{c^2}\right)}{1\ \text{u}} = 232.66\ \text{MeV}$$

Binding energy per nucleon of aluminum-28:

$$\frac{232.66\ \text{MeV}}{28} = \boxed{8.3094\frac{\text{MeV}}{\text{nucleon}}}$$

Decay process for aluminum-28:

$$\boxed{{}^{28}\text{Al} \rightarrow \text{e}^- + \bar{\nu} + {}^{28}\text{Si}}$$

REFLECT

The binding energy per nucleon for both aluminum-26 and aluminum-28 is smaller than that of aluminum-27, which means aluminum-27 will be more stable than the other two isotopes.

27.99

SET UP

Tritium, which is the same as hydrogen-3, undergoes β^- decay with a half-life of $\tau_{1/2} = 12.3$ yr. The number of protons in tritium is equal to the atomic number of the element. The number of neutrons is equal to the difference between the mass number and the atomic number. In order to write the balanced beta decay reaction we need to determine the missing species by calculating its expected atomic number Z and mass number A. The sum of the atomic numbers of the products must equal the sum of the atomic numbers of the reactants; the same must be true for the mass numbers. Initially, the decay rate of a sample of tritium was $R_0 = 70{,}500\frac{\text{pCi}}{\text{L}}$. The time required for the decay rate to decrease to $R = 20{,}000\frac{\text{pCi}}{\text{L}}$ can be found using $R = R_0e^{-\lambda t}$, where $\lambda = \frac{\ln(2)}{\tau_{1/2}}$.

SOLVE

Part a) Tritium contains $\boxed{\text{one proton and two neutrons}}$.

Part b)

$${}^{3}\text{H} \rightarrow \text{e}^- + \underline{\quad}$$

Atomic number:

$$Z = 1 - (-1) = 2$$

Since $Z = 2$, the missing product will be a helium nucleus.

Mass number:

$$A = 3 - 0 = 3$$

The missing product will be a helium-3 nucleus, $\boxed{^3\text{He}}$.

Part c)

$$R = R_0 e^{-\lambda t}$$

$$\ln\left(\frac{R}{R_0}\right) = -\lambda t = -\left(\frac{\ln(2)}{\tau_{1/2}}\right)t$$

$$t = \frac{\tau_{1/2}\ln\left(\frac{R_0}{R}\right)}{\ln(2)} = \frac{(12.3\text{ yr})\ln\left(\frac{\left(70{,}500\frac{\text{pCi}}{\text{L}}\right)}{\left(20{,}000\frac{\text{pCi}}{\text{L}}\right)}\right)}{\ln(2)} = \boxed{22.4\text{ yr}}$$

REFLECT

The rate needs to decrease by a little less than 75%, which would correspond to two half-lives, or 24.6 yr, so our answer is reasonable.

27.100

SET UP

The half-life of iodine-125 is $\tau_{1/2} = 59.4$ days. The initial decay rate of a sample of iodine-125 is $R_0 = 525\ \mu\text{Ci}$. The decay rate after 365 days is given by $R = R_0\left(\frac{1}{2}\right)^{\frac{t}{\tau_{1/2}}}$. This equation can be rearranged to calculate the length of time required for the decay rate to decrease by 90%, that is, $R = 0.10R_0$.

SOLVE

Part a)

$$R = R_0\left(\frac{1}{2}\right)^{\frac{t}{\tau_{1/2}}} = (525\ \mu\text{Ci})\left(\frac{1}{2}\right)^{\left(\frac{365\text{ days}}{59.4\text{ days}}\right)} = \boxed{7.41\ \mu\text{Ci}}$$

Part b)

$$\frac{R}{R_0} = \left(\frac{1}{2}\right)^{\frac{t}{\tau_{1/2}}}$$

$$\ln\left(\frac{R}{R_0}\right) = \frac{t}{\tau_{1/2}} \ln\left(\frac{1}{2}\right)$$

$$t = \frac{\tau_{1/2} \ln\left(\frac{R}{R_0}\right)}{\ln\left(\frac{1}{2}\right)} = \frac{\tau_{1/2} \ln\left(\frac{R_0}{R}\right)}{\ln(2)} = \frac{\tau_{1/2} \ln\left(\frac{R_0}{0.10R_0}\right)}{\ln(2)} = \frac{(59.4 \text{ days}) \ln(10)}{\ln(2)} = \boxed{197 \text{ days}}$$

REFLECT

After three half-lives, the decay rate will be 12.5% of its initial rate. A time period of 197 days, or about 6.5 months, is reasonable.

27.101

SET UP

Ruthenium-106 undergoes β^- decay with a half-life of $\tau_{1/2} = 373.59$ days. The number of protons in ruthenium-106 is equal to the atomic number of the element. The number of neutrons is equal to the difference between the mass number and the atomic number. Since the half-life of ruthenium-106 is about a year, we would not expect to find any of it in rocks mined from the Earth, as these are millions of years old. The balanced beta decay reaction can be determined by calculating the expected atomic number Z and mass number A of the daughter nucleus. The sum of the atomic numbers of the products must equal the sum of the atomic numbers of the reactants; the same must be true for the mass numbers. Finally, the equation $\frac{R}{R_0} = \left(\frac{1}{2}\right)^{\frac{t}{\tau_{1/2}}}$ can be rearranged to calculate the length of time required for the decay rate to decrease by 75%, that is, $R = 0.25R_0$.

SOLVE

Part a) There are $\boxed{\text{44 protons and 62 neutrons}}$ in a ruthenium-106 nucleus.

Part b) We would not find any ruthenium-106 in mined ores. Such ores are many millions of years old at the least, so if any ruthenium-106 were present initially, it would have decayed to essentially zero.

Part c)

$$^{106}\text{Ru} \rightarrow \text{e}^- + \_\_$$

Atomic number:

$$Z = 44 - (-1) = 45$$

Since $Z = 45$, the missing product will be a rhodium nucleus.
Mass number:

$$A = 106 - 0 = 106$$

The missing product will be a rhodium-106 nucleus, $\boxed{^{106}\text{Rh}}$.

Part d)

$$\frac{R}{R_0} = \left(\frac{1}{2}\right)^{\frac{t}{\tau_{1/2}}}$$

$$\ln\left(\frac{R}{R_0}\right) = \frac{t}{\tau_{1/2}}\ln\left(\frac{1}{2}\right)$$

$$t = \frac{\tau_{1/2}\ln\left(\frac{R}{R_0}\right)}{\ln\left(\frac{1}{2}\right)} = \frac{\tau_{1/2}\ln\left(\frac{R_0}{R}\right)}{\ln(2)} = \frac{\tau_{1/2}\ln\left(\frac{R_0}{0.25R_0}\right)}{\ln(2)} = \frac{(373.59\text{ days})\ln(4.0)}{\ln(2)}$$

$$= 747\text{ days} \times \frac{1\text{ yr}}{365\text{ days}} = \boxed{2.05\text{ yr}}$$

REFLECT

The number of nuclei in a sample of ruthenium-106 decreases by 75% after about 2 years, which bolsters our argument that there will be no ruthenium-106 available in mined ores.

27.102

SET UP

Ruthenium-106 has a molar mass of $106\frac{\text{g}}{\text{mol}}$ and a density of $\rho = 12.45\frac{\text{g}}{\text{cm}^3}$. The half-life of ruthenium-106 undergoing beta decay is $\tau_{1/2} = 373.59$ days. In order to find the mass of a sample of ruthenium-106 with an activity of 125 μCi, we first need to determine the number of nuclei present and then use the molar mass to find the mass. Assuming the sample is spherical, we can use this mass along with the density to calculate the radius of the sphere.

SOLVE

Part a)

$$R_0 = \lambda N_0$$

$$N_0 = \frac{R_0}{\lambda} = \frac{R_0}{\left(\frac{\ln(2)}{\tau_{1/2}}\right)} = \frac{\tau_{1/2}R_0}{\ln(2)}$$

$$= \frac{\left(373.59\text{ days} \times \frac{24\text{ hr}}{1\text{ day}} \times \frac{3600\text{ s}}{1\text{ hr}}\right)\left(125\ \mu\text{Ci} \times \frac{1\text{ Ci}}{10^6\ \mu\text{Ci}} \times \frac{3.7 \times 10^{10}\text{ Bq}}{1\text{ Ci}}\right)}{\ln(2)}$$

$$= 2.15 \times 10^{14}\text{ atoms} \times \frac{1\text{ mol}}{6.02 \times 10^{23}\text{ atoms}} \times \frac{106\text{ g}}{1\text{ mol}} = \boxed{3.79 \times 10^{-8}\text{ g}}$$

Part b)

$$\rho = \frac{m}{V} = \frac{m}{\left(\frac{4}{3}\pi R^3\right)} = \frac{3m}{4\pi R^3}$$

$$R = \sqrt[3]{\frac{3m}{4\pi\rho}} = \sqrt[3]{\frac{3(3.79 \times 10^{-8}\text{ g})}{4\pi\left(12.45\frac{\text{g}}{\text{cm}^3}\right)}} = \boxed{8.99 \times 10^{-4}\text{ cm} = 8.99\ \mu\text{m}}$$

REFLECT

A sphere with a radius of around 10 μm seems reasonable if its mass were around 40 ng.

27.103

SET UP

We are told that electron capture by a proton is not allowed in nature, which means the reaction $e^- + p \rightarrow n + \nu$, where ν represents a particle of negligible mass called the neutrino, does not occur. We can calculate the energy Q released by this process in order to better understand why this process does not take place. The masses of a neutron, proton, and electron in amu are $m_{\text{n}} = 1.008665$ u, $m_{\text{p}} = 1.007277$ u, and $m_{\text{e}} = 0.005486$ u.

SOLVE

$$Q = [(0.0005486\text{ u}) + (1.007277\text{ u}) - (1.008665\text{ u})]c^2$$

$$= (-0.0008934\text{ u})c^2 \times \frac{\left(931.494\,\frac{\text{MeV}}{c^2}\right)}{1\text{ u}} = -0.7819\text{ MeV}$$

Since the energy released we calculated is negative, this reaction will not proceed.

REFLECT

If this reaction were to occur, all low energy electrons could react with protons and turn them into neutrons. This would destroy chemistry as we know it!

27.104

SET UP

Uranium-235 ($m_{^{235}\text{U}} = 235.043924$ u) undergoes alpha decay: $^{235}\text{U} \rightarrow {}^{4}\alpha + {}^{231}\text{Th}$. The total energy released by the decay is equal to the sum of the kinetic energies of the alpha particle ($m_\alpha = 4.002602$ u) and the daughter nucleus ($m_{^{231}\text{Th}} = 231.036299$ u). The kinetic energy of the daughter nucleus is equal to $K_{\text{D}} = \left(\frac{m_\alpha}{m_{\text{D}}}\right)K_\alpha$. Using this relationship, we can calculate the kinetic energy of the alpha particle.

SOLVE

$$^{235}\text{U} \rightarrow {}^{4}\alpha + {}^{231}\text{Th}$$

Energy released:

$$E = (m_{^{231}\text{Th}} + m_{^4\alpha} - m_{^{235}\text{U}})c^2$$
$$= ((231.036299\text{ u}) + (4.002602\text{ u}) - (235.043924\text{ u}))c^2$$
$$= (-0.005023\text{ u})c^2 \times \frac{\left(931.494\dfrac{\text{MeV}}{c^2}\right)}{1\text{ u}} = -4.679\text{ MeV}$$

The energy released in the decay is 4.679 MeV.

Kinetic energy of the alpha particle:

$$E_{\text{released}} = K_{\text{D}} + K_\alpha$$

$$E_{\text{released}} = \left(\frac{m_\alpha}{m_{\text{D}}}\right)K_\alpha + K_\alpha$$

$$E_{\text{released}} = \left(\frac{m_\alpha + m_{\text{D}}}{m_{\text{D}}}\right)K_\alpha$$

$$K_\alpha = \left(\frac{m_{\text{D}}}{m_\alpha + m_{\text{D}}}\right)E_{\text{released}} = \left(\frac{231.036299\text{ u}}{(4.002602\text{ u}) + (231.036299\text{ u})}\right)(4.679\text{ MeV}) = \boxed{4.599\text{ MeV}}$$

REFLECT

The alpha particle is much lighter than the daughter nucleus, so, through conservation of momentum and energy, we would expect the alpha particle to be traveling much faster than the daughter nucleus.

27.105

SET UP

We can use the given decay modes for unstable nuclei in Appendix C to determine the decay pathway for each of the given decay series. For the neptunium decay series, we will need the main decay mode of bismuth-213, which is beta decay.

SOLVE

A)

$$^{232}\text{Th} \xrightarrow{\alpha} {}^{228}\text{Ra} \xrightarrow{\beta-} {}^{228}\text{Ac} \xrightarrow{\beta-} {}^{228}\text{Th} \xrightarrow{\alpha} {}^{224}\text{Ra} \xrightarrow{\alpha} {}^{220}\text{Rn} \xrightarrow{\alpha} {}^{216}\text{Po} \xrightarrow{\alpha}$$
$$^{212}\text{Pb} \xrightarrow{\beta-} {}^{212}\text{Bi} \xrightarrow{\beta-} {}^{212}\text{Po} \xrightarrow{\alpha} {}^{208}\text{Pb}$$

or

$$^{232}\text{Th} \xrightarrow{\alpha} {}^{228}\text{Ra} \xrightarrow{\beta-} {}^{228}\text{Ac} \xrightarrow{\beta-} {}^{228}\text{Th} \xrightarrow{\alpha} {}^{224}\text{Ra} \xrightarrow{\alpha} {}^{220}\text{Rn} \xrightarrow{\alpha} {}^{216}\text{Po} \xrightarrow{\alpha}$$
$$^{212}\text{Pb} \xrightarrow{\beta-} {}^{212}\text{Bi} \xrightarrow{\alpha} {}^{208}\text{Ti} \xrightarrow{\beta-} {}^{208}\text{Pb}$$

Particles emitted: $\boxed{6\ \alpha,\ 4\ \beta}$.

B)

$$^{238}\text{U} \xrightarrow{\alpha} {}^{234}\text{Th} \xrightarrow{\beta-} {}^{234}\text{Pa} \xrightarrow{\beta-} {}^{234}\text{U} \xrightarrow{\alpha} {}^{230}\text{Th} \xrightarrow{\alpha} {}^{226}\text{Ra} \xrightarrow{\alpha} {}^{222}\text{Rn} \xrightarrow{\alpha} {}^{218}\text{Po} \xrightarrow{\alpha}$$
$$^{214}\text{Pb} \xrightarrow{\beta-} {}^{214}\text{Bi} \xrightarrow{\beta-} {}^{214}\text{Po} \xrightarrow{\alpha} {}^{210}\text{Pb} \xrightarrow{\beta-} {}^{210}\text{Bi} \xrightarrow{\beta-} {}^{210}\text{Po} \xrightarrow{\alpha} {}^{206}\text{Pb}$$

Particles emitted: $\boxed{8\ \alpha,\ 6\ \beta.}$

C)

$$^{235}\text{U} \xrightarrow{\alpha} {}^{231}\text{Th} \xrightarrow{\beta-} {}^{231}\text{Pa} \xrightarrow{\alpha} {}^{227}\text{Ac} \xrightarrow{\beta-} {}^{227}\text{Th} \xrightarrow{\alpha} {}^{223}\text{Ra} \xrightarrow{\alpha} {}^{219}\text{Rn} \xrightarrow{\alpha} {}^{215}\text{Po} \xrightarrow{\alpha}$$
$$^{211}\text{Pb} \xrightarrow{\beta-} {}^{211}\text{Bi} \xrightarrow{\alpha} {}^{207}\text{Tl} \xrightarrow{\beta-} {}^{207}\text{Pb}$$

Particles emitted: $\boxed{7\ \alpha,\ 4\ \beta.}$

D)

$$^{237}\text{Np} \xrightarrow{\alpha} {}^{233}\text{Pa} \xrightarrow{\beta-} {}^{233}\text{U} \xrightarrow{\alpha} {}^{229}\text{Th} \xrightarrow{\alpha} {}^{225}\text{Ra} \xrightarrow{\beta-} {}^{225}\text{Ac} \xrightarrow{\alpha} {}^{221}\text{Fr} \xrightarrow{\alpha} {}^{217}\text{At} \xrightarrow{\alpha}$$
$$^{213}\text{Bi} \xrightarrow{\beta-} {}^{213}\text{Po} \xrightarrow{\alpha} {}^{209}\text{Pb} \xrightarrow{\beta-} {}^{209}\text{Bi} \xrightarrow{\alpha} {}^{205}\text{Tl}$$

Particles emitted: $\boxed{8\ \alpha,\ 4\ \beta.}$

REFLECT

Lead-208 is the heaviest stable isotope.

27.106

SET UP

Natural uranium contains three isotopes: uranium-234, uranium-235, and uranium-238. The activity of 1 g of natural uranium is 25,280 Bq. A sample of natural uranium has an activity of 4.48 μCi. Dividing the activity of the sample by the activity per gram of natural uranium will tell us the mass of the sample. The contribution of each isotope to the activity of the sample is equal to the fractional contribution of each isotope to the activity of natural uranium multiplied by the activity of the sample.

SOLVE

Part a)

Activity of the sample:

$$4.48\ \mu\text{Ci} \times \frac{1\ \text{Ci}}{10^6\ \mu\text{Ci}} \times \frac{3.7 \times 10^{10}\ \text{Bq}}{1\ \text{Ci}} = 1.66 \times 10^5\ \text{Bq}$$

Mass of the sample:

$$m = \frac{\text{activity of sample}}{\text{activity per gram}} = \frac{1.66 \times 10^5\ \text{Bq}}{\left(25{,}280\frac{\text{Bq}}{\text{g}}\right)} = \boxed{6.56\ \text{g}}$$

Part b)

$$R_{^{234}\text{U}} = (0.489)(4.48\ \mu\text{Ci}) = \boxed{2.19\ \mu\text{Ci}}$$

$$R_{^{235}\text{U}} = (0.022)(4.48\ \mu\text{Ci}) = \boxed{0.099\ \mu\text{Ci}}$$

$$R_{^{238}\text{U}} = (0.489)(4.48\ \mu\text{Ci}) = \boxed{2.19\ \mu\text{Ci}}$$

REFLECT

Even though the relative abundance of uranium-234 is much less than the other isotopes, its specific activity is much larger.

27.107

SET UP

The fusion reaction between neodymium-157 and germanium-80 is considered: $^{157}\text{Nd} + {}^{80}\text{Ge} \rightarrow {}^{235}\text{U} + 2\text{n}$. We can calculate the difference in energy between the products and the reactants in order to determine whether or not energy needs to be added to the system for the reaction to proceed.

SOLVE

$$\begin{aligned}
\Delta E &= ((m_{^{235}\text{U}} + 2m_{\text{n}}) - (m_{^{157}\text{Nd}} + m_{^{80}\text{Ge}}))c^2 \\
&= (((235.043924\ \text{u}) + 2(1.008665\ \text{u})) - ((156.939032\ \text{u}) + (79.925373\ \text{u})))c^2 \\
&= (0.196849\ \text{u})c^2 \times \frac{\left(931.494\dfrac{\text{MeV}}{c^2}\right)}{1\ \text{u}} = 183.364\ \text{MeV}
\end{aligned}$$

The products are higher in energy by about 183 MeV, which means this reaction will only occur if 183 MeV of energy is added to the system. This means the fusion reaction consumes energy rather than produces it.

REFLECT

The reverse reaction—the fission of uranium-235—would *release* 183 MeV of energy.

27.108

SET UP

A radioactive nucleus is being produced in a nuclear reactor at a constant rate of R_{P}. At the same time it is decaying at a rate of $R_{\text{D}} = \lambda N$. The time rate of change of the number of nuclei in the sample is equal to the rate at which nuclei are being produced minus the rate at which the nuclei decay. We can solve this first-order differential equation with the initial condition $N(0) = 0$ to show that $N(t) = \frac{R_{\text{P}}}{\lambda}(1 - e^{-\lambda t})$.

SOLVE

$$\frac{dN}{dt} = R(t) = R_{\mathrm{P}} - R_{\mathrm{D}} = R_{\mathrm{P}} - \lambda N$$

$$\frac{dN}{dt} + \lambda N = R_{\mathrm{P}}$$

Integrating factor:

$$\mu = e^{\int \lambda dt} = e^{\lambda t}$$

$$e^{\lambda t}\frac{dN}{dt} + \lambda N e^{\lambda t} = R_{\mathrm{P}} e^{\lambda t}$$

$$\frac{d}{dt}[Ne^{\lambda t}] = R_{\mathrm{P}} e^{\lambda t}$$

$$Ne^{\lambda t} = \frac{R_{\mathrm{P}}}{\lambda} e^{\lambda t} + C$$

$$N(t) = \frac{R_{\mathrm{P}}}{\lambda} + Ce^{-\lambda t}$$

Applying the initial condition, $N(0) = 0$:

$$0 = \frac{R_{\mathrm{P}}}{\lambda} + C$$

$$C = -\frac{R_{\mathrm{P}}}{\lambda}$$

Plugging in C:

$$N(t) = \frac{R_{\mathrm{P}}}{\lambda} - \frac{R_{\mathrm{P}}}{\lambda} e^{-\lambda t} = \boxed{\frac{R_{\mathrm{P}}}{\lambda}(1 - e^{-\lambda t})}$$

REFLECT

As time goes to infinity, the number of radioactive nuclei will approach $\frac{R_{\mathrm{P}}}{\lambda}$.

27.109

SET UP

A radioactive parent nucleus decays with a decay constant λ_{P}. At the same time the daughter nucleus is decaying with a decay constant λ_{D}. The time rate of change of the number of daughter nuclei in the sample is equal to the rate at which nuclei are being produced minus the rate at which the nuclei decay. We can solve this first-order differential equation with the initial conditions $N_{\mathrm{D}}(0) = 0$ and $N_{\mathrm{P}}(0) = N_0$ to find an expression for the number of daughter nuclei as a function of time.

SOLVE

$$\frac{dN_D}{dt} = \lambda_P N_P - \lambda_D N_D$$

$$\frac{dN_D}{dt} + \lambda_D N_D = \lambda_P N_P = \lambda_P (N_0 e^{-\lambda_P t})$$

Integrating factor:

$$\mu = e^{\int \lambda_D dt} = e^{\lambda_D t}$$

$$e^{\lambda_D t}\frac{dN_D}{dt} + \lambda_D N_D e^{\lambda_D t} = \lambda_P N_0 e^{-\lambda_P t} e^{\lambda_D t}$$

$$\frac{d}{dt}[N_D e^{\lambda_D t}] = \lambda_P N_0 e^{(\lambda_D - \lambda_P)t}$$

$$N_D e^{\lambda_D t} = \frac{\lambda_P N_0}{\lambda_D - \lambda_P} e^{(\lambda_D - \lambda_P)t} + C$$

$$N_D(t) = \frac{\lambda_P N_0}{\lambda_D - \lambda_P} e^{-\lambda_P t} + C e^{-\lambda_D t}$$

Applying the initial condition, $N_D(0) = 0$:

$$0 = \frac{\lambda_P N_0}{\lambda_D - \lambda_P} + C$$

$$C = -\frac{\lambda_P N_0}{\lambda_D - \lambda_P}$$

Plugging in C:

$$N_D(t) = \frac{\lambda_P N_0}{\lambda_D - \lambda_P} e^{-\lambda_P t} - \frac{\lambda_P N_0}{\lambda_D - \lambda_P} e^{-\lambda_D t} = \boxed{\left(\frac{\lambda_P N_0}{\lambda_D - \lambda_P}\right)(e^{-\lambda_P t} - e^{-\lambda_D t})}$$

REFLECT

If the daughter nucleus is stable (that is, $\lambda_D = 0$), then our expression reduces to the same result we found in Problem 27.108.

27.110

SET UP

The number of daughter nuclei if both the parent and daughter nuclei decay as a function of time is given by $N_D(t) = \left(\frac{\lambda_P N_0}{\lambda_D - \lambda_P}\right)(e^{-\lambda_P t} - e^{-\lambda_D t})$, where the parent nucleus decays with decay constant λ_P and the daughter nucleus decays with decay constant λ_D. To show that the maximum number of daughter nuclei occurs at $t_{max} = \left(\frac{1}{\lambda_P - \lambda_D}\right) \ln\left(\frac{\lambda_P}{\lambda_D}\right)$, we need to differentiate $N_D(t)$ with respect to time, evaluate it at $t = t_{max}$, set it equal to zero, and solve for t_{max}.

SOLVE

$$N_{\mathrm{D}}(t) = \left(\frac{\lambda_{\mathrm{P}} N_0}{\lambda_{\mathrm{D}} - \lambda_{\mathrm{P}}}\right)(e^{-\lambda_{\mathrm{P}} t} - e^{-\lambda_{\mathrm{D}} t})$$

$$\frac{dN_{\mathrm{D}}}{dt} = \frac{d}{dt}\left(\frac{\lambda_{\mathrm{P}} N_0}{\lambda_{\mathrm{D}} - \lambda_{\mathrm{P}}}\right)(e^{-\lambda_{\mathrm{P}} t} - e^{-\lambda_{\mathrm{D}} t}) = \left(\frac{\lambda_{\mathrm{P}} N_0}{\lambda_{\mathrm{D}} - \lambda_{\mathrm{P}}}\right)\frac{d}{dt}(e^{-\lambda_{\mathrm{P}} t} - e^{-\lambda_{\mathrm{D}} t}) = 0$$

$$-\lambda_{\mathrm{P}} e^{-\lambda_{\mathrm{P}} t_{\mathrm{max}}} + \lambda_{\mathrm{D}} e^{-\lambda_{\mathrm{D}} t_{\mathrm{max}}} = 0$$

$$\ln(\lambda_{\mathrm{D}} e^{-\lambda_{\mathrm{D}} t_{\mathrm{max}}}) = \ln(\lambda_{\mathrm{P}} e^{-\lambda_{\mathrm{P}} t_{\mathrm{max}}})$$

$$\ln(\lambda_{\mathrm{D}}) - \lambda_{\mathrm{D}} t_{\mathrm{max}} = \ln(\lambda_{\mathrm{P}}) - \lambda_{\mathrm{P}} t_{\mathrm{max}}$$

$$\lambda_{\mathrm{P}} t_{\mathrm{max}} - \lambda_{\mathrm{D}} t_{\mathrm{max}} = \ln(\lambda_{\mathrm{P}}) - \ln(\lambda_{\mathrm{D}})$$

$$\boxed{t_{\mathrm{max}} = \left(\frac{1}{\lambda_{\mathrm{P}} - \lambda_{\mathrm{D}}}\right)\ln\left(\frac{\lambda_{\mathrm{P}}}{\lambda_{\mathrm{D}}}\right)}$$

REFLECT

If the daughter nucleus is stable (that is, $\lambda_{\mathrm{D}} = 0$), then the maximum number of daughter nuclei occurs when $t \to \infty$.

Chapter 28
Particle Physics

Conceptual Questions

28.1 Part a) Baryons are hadrons made up of three quarks.

Part b) Mesons are hadrons make up of quark–antiquark pairs.

Part c) Quarks are fractionally charged particles that make up hadronic material (baryons and/or mesons).

Part d) Leptons are particles that are driven by the electroweak force and mediated with photons and W and Z bosons.

Part e) Antiparticles are the negative-energy counterparts to every particle. All particles possess antiparticles that are equal in mass but opposite in charge.

28.2 Since there is such a vast number of neutrinos in the universe, even a very small mass (on the order of 1 eV) will lead to a significant impact on the theories of the dark matter that is part of the universe. Therefore, neutrinos must be massless in the electroweak theory.

28.3 The photon and the neutrino are both neutral in charge and possess no baryon quantum number. There are some early models in which the electron neutrino is massless; in that case it would travel at the speed of light. These two particles are driven by the electroweak force.

28.4 Part a) e^+

Part b) $\overline{n}$

Part c) $\overline{p}$

Part d) π^-

Part e) $\overline{K}^0$

28.5 There are 10 combinations: uud, udd, ddd, uuu, uus, dds, uds, uss, dss, and sss.

28.6 Once formed, a positron quickly meets an electron from the abundant supply of electrons in the matter in our universe and they annihilate one another.

28.7 If only one photon were created, linear momentum could not be conserved because a zero-momentum photon is not possible.

28.8 The strong force will be involved as well as gravity. The electroweak force will not.

28.9 When two particles interact through one of the four fundamental forces (strong, weak, electromagnetic, or gravity), the interaction is mediated by the creation of an exchange particle that is transferred between the two interacting particles.

28.10 The Standard Model is the current explanation of the fundamental particles and the forces that govern them. It is hypothesized that there are six quarks (up, down, charm, strange, top, bottom), six leptons (electron, electron neutrino, muon, muon neutrino, tau, tau neutrino), plus all of their corresponding antiparticles for a total of 24 quarks, leptons, antiquarks, and antileptons. In addition, there are exchange particles that mediate these particles in the strong nuclear force (gluons), the electroweak force (W^+, W^-, Z^0 bosons for the weak force; photons for the electromagnetic force), and the force of gravity (gravitons). The interactions between quarks are determined by the quantum field theory quantum chromodynamics (QCD) and the interaction between neutrinos and leptons is explained by quantum electrodynamics (QED).

Multiple-Choice Questions

28.11 **B** (meson). A particle composed of two quarks is classified as a meson.

28.12 **A** (baryon). A particle composed of three quarks is classified as a baryon.

28.13 **D** ($\overline{\text{uud}}$). The quark composition of a proton is uud, so an antiproton must be $\overline{\text{uud}}$.

28.14 **D** (weak force). The weak force is at the heart of the interaction that governs beta decay.

28.15 **A** (strong). Quarks are attracted to each other through the strong force.

28.16 **B** (electromagnetic). The photon acts as the mediator of the electromagnetic force.

28.17 **D** (quarks). Quarks interact via the strong force.

28.18 **C** (dark energy). Physicists estimate that most of the composition of the universe is in the form of dark energy.

28.19 **D** (dark energy). The bulk of the energy in the expanding universe has not been measured. "Dark" suggests we can't see the energy (yet).

Estimation/Numerical Questions

28.20 $mc^2 = \dfrac{\hbar c}{R} = \dfrac{1240\ \text{eV}\cdot\text{nm}}{2\pi(1.5\times 10^{-6}\ \text{nm})} = 130\ \text{MeV}$

28.21 The ratios of the meson masses are K:π = 3.6, D:K = 3.8, B:D = 2.8. The ratio of the quark masses, from Table 28-1, are s:d = 20, c:s = 13, b:c = 3.8. No, the ratios of the meson masses do not follow the ratio of the quark masses. The mass of the quarks alone does not account for the mass of the mesons they comprise.

28.22 $R = \dfrac{\hbar c}{mc^2} = \dfrac{1240\ \text{eV}\cdot\text{nm}}{2\pi(80\times 10^{9}\ \text{eV})} = 2.5\times 10^{-9}\ \text{nm}$

Problems

28.23

SET UP

We can determine whether or not the reaction $n \to \pi^+ + \pi^- + \mu^+ + \mu^-$ is possible by seeing if the charge, baryon number, and the lepton number are conserved. Every matter baryon has a baryon number of +1, and particles that are not baryons have a baryon number of 0. A positron has an electron-lepton number of −1, and a particle that is not a lepton has an electron–lepton number of 0.

SOLVE

Charge: A neutron has a charge of 0, π^+ and μ^+ each have a charge of $+e$, and π^- and μ^- each have a charge of $-e$. Charge is conserved.

Baryon number: A neutron has a baryon number of +1. Pions and muons each have a baryon number of 0, as they are not baryons. The baryon number is not conserved.

Lepton number: Neither neutrons nor pions are leptons, so they have a lepton number of 0. A muon has a muon–lepton number equal to +1, and an antimuon has a muon–lepton number of −1. The lepton number is conserved.

No, the reaction is $\boxed{\text{not possible}}$ because the baryon number is not conserved.

REFLECT

Charge, baryon number, and lepton number must all be conserved in a valid reaction.

28.24

SET UP

We can determine whether or not the reaction $e^- + p \to n + \bar{\nu}_e$ is possible by seeing if the charge, baryon number, and the lepton number are conserved. Every matter baryon has a baryon number of +1, and particles that are not baryons have a baryon number of 0. An electron has an electron–lepton number of +1, an antimatter electron neutrino has an electron–lepton number of −1, and a particle that is not a lepton has an electron–lepton number of 0.

SOLVE

Charge: A neutron and an antimatter electron neutrino have a charge of 0, a proton has a charge of $+e$, and an electron has a charge of $-e$. Charge is conserved.

Baryon number: A neutron and a proton each have a baryon number of +1. Electrons and neutrinos each have a baryon number of 0, as they are not baryons. The baryon number is conserved.

Electron–lepton number: Neither neutrons nor protons are leptons, so they have an electron–lepton number of 0. An electron has an electron–lepton number equal to +1, and an antimatter electron neutrino has an electron–lepton number of −1. The lepton number is not conserved.

No, the reaction is $\boxed{\text{not possible}}$ because the lepton number is not conserved.

REFLECT

Charge, baryon number, and lepton number must all be conserved in a valid reaction.

28.25

SET UP

We can determine whether or not the reaction $p \rightarrow e^{+} + \gamma$ is possible by seeing if the charge, baryon number, and the lepton number are conserved. Every matter baryon has a baryon number of $+1$, and particles that are not baryons have a baryon number of 0. A positron has an electron–lepton number of -1, and a particle that is not a lepton has an electron–lepton number of 0.

SOLVE

Charge: A proton and a positron each have a charge of $+e$. A photon is uncharged. Charge is conserved.

Baryon number: A proton has a baryon number of $+1$. A positron and a photon each have a baryon number of 0. The baryon number is not conserved.

Electron–lepton number: A proton and a photon each have an electron–lepton number of 0. A positron has an electron–lepton number of -1. The electron–lepton number is not conserved.

No, the reaction is $\boxed{\text{not possible}}$ because neither the baryon number nor the electron–lepton number is conserved.

REFLECT

The sum of the baryon numbers and lepton numbers of the particles that participate in the process must equal the sum of the values of the particles present at the end of the process.

28.26

SET UP

In order to determine a possible identity for X in the reaction $p + \pi^{-} \rightarrow K^{0} + X$ we need to invoke conservation of charge, baryon number, and lepton number—the charge, baryon number, and lepton number must remain constant throughout the reaction.

SOLVE

Charge: A proton has a charge of $+e$, π^{-} has a charge of $-e$, and K^{0} has a charge of 0. For charge to be conserved, X must have a charge of 0.

Baryon number: A proton has a baryon number of $+1$, whereas π^{-} and K^{0} each have a baryon number of 0. For baryon number to be conserved, X must have a baryon number of $+1$.

Lepton number: Protons, pions, and kaons are not leptons, so they all have lepton numbers equal to 0. For lepton number to be conserved, X must also have a lepton number of 0.

Based on all of this information the unknown particle could be a $\boxed{\text{neutron}}$.

REFLECT

If we had information related to the energies and momenta of the products, we could be more confident in the identity of the unknown particle.

28.27

SET UP

We can use Figure 28-1 in the text to determine the particle that is composed of the quark combination uss. This answer would change if the quark combination were changed to uds as down quarks and strange quarks are different and have different properties.

SOLVE

Part a) According to Figure 28-1 in the text, a $\boxed{\Xi^0}$ particle has a quark structure of uss.

Part b) Yes, uds would be different from uss. Although they are both neutral particles, the properties of the d and s quarks are different, so some of the properties of a uds baryon would be different from those of a Ξ^0 baryon.

REFLECT

An up quark has a charge of $+\frac{2}{3}e$. A down quark and a strange quark each have a charge of $-\frac{1}{3}e$, so the total charge of a particle with a quark structure of uds will be 0. A Λ^0 particle has a quark combination of uds.

28.28

SET UP

The quark composition of a π^- meson is given in Got the Concept 28-1.

SOLVE

A π^- meson has a quark composition of $\boxed{\overline{\mathrm{u}}\mathrm{d}}$.

REFLECT

A meson is made up of two quarks.

28.29

SET UP

We are asked to determine the quark structure of a K^0 meson; a meson must be composed of two quarks. We are told that the K meson is the lowest mass strange particle. Therefore, it must contain an s or an s-bar quark. A K^0 meson will be made up of quark–antiquark pair because it is neutrally charged. The lowest mass quarks are u (around 2 MeV/c^2) and d (around 5 MeV/c^2). Both an s quark and a d quark have a charge of $-1/3e$, whereas a u quark has a charge of $+2/3e$. In order to satisfy charge neutrality, the K^0 meson must contain either a d or d-bar quark. It turns out that the K^0 meson is made up of a d quark and an s-bar quark.

SOLVE

The quark structure of a K^0 meson is $\boxed{\mathrm{d}\overline{\mathrm{s}}}$.

REFLECT

The antiparticle of the K^0 meson is the $\overline{K}^0$ meson and is composed of a d-bar quark and an s quark, $\overline{\mathrm{d}}\mathrm{s}$.

28.30

SET UP

We are asked to determine which of the following reactions is possible for the weak decay of a sigma particle, which is a baryon: $\Sigma^- \rightarrow \pi^- + \mathrm{p}$ or $\Sigma^- \rightarrow \pi^- + \mathrm{n}$. For a reaction to be valid, the charge, baryon number, and lepton number must be conserved.

SOLVE

Part a) The sigma particle and the pion each have a charge of $-e$, whereas the proton has a charge of $+e$. Charge is not conserved in this reaction, so this is $\boxed{\text{not possible}}$.

Part b) The sigma particle and the pion each have a charge of $-e$, whereas the neutron has a charge of 0. Charge and baryon number are both conserved in this reaction, so this is $\boxed{\text{possible}}$.

REFLECT

None of the particles listed are leptons, so all of the particles have lepton numbers equal to 0.

28.31

SET UP

We are asked to determine which of the following reactions are possible for the collision of two protons to generate new particles: $\mathrm{p} + \mathrm{p} \rightarrow \mathrm{p} + \mathrm{p} + \mathrm{p} + \overline{\mathrm{p}}$, $\mathrm{p} + \mathrm{p} \rightarrow \mathrm{p} + \mathrm{p} + \mathrm{n} + \overline{\mathrm{n}}$, or $\mathrm{p} + \mathrm{p} \rightarrow \mathrm{p} + \mathrm{K}^+$. For a reaction to be valid, the charge, baryon number, and lepton number must be conserved.

SOLVE

Part a) Protons have a baryon number of $+1$ and a charge of $+e$. Antiprotons have a baryon number of -1 and a charge of $-e$. In this reaction, charge and baryon number are both conserved, so this reaction is $\boxed{\text{possible}}$.

Part b) Protons have a baryon number of $+1$ and a charge of $+e$. Neutrons have a baryon number of $+1$ and a charge of 0. Antineutrons have a baryon number of -1 and a charge of 0. In this reaction, charge and baryon number are both conserved, so this reaction is $\boxed{\text{possible}}$.

Part c) Protons have a baryon number of $+1$ and a charge of $+e$. The K^+ particle has a baryon number of 0 and a charge of $+e$. In this reaction, charge is conserved, but baryon number is not, so this reaction is $\boxed{\text{not possible}}$.

REFLECT

None of the particles listed are leptons, so all of the particles have lepton numbers equal to 0.

28.32

SET UP

We can determine whether or not the reactions are possible by seeing if the charge, baryon number, and the lepton number are conserved. Every matter baryon has a baryon number of $+1$, and particles that are not baryons have a baryon number of 0. Matter leptons have a lepton number of $+1$, while antimatter leptons have a lepton number of -1. Particles that are not leptons have a lepton number of 0.

SOLVE

Part a)

$$\Lambda^0 \rightarrow \mathrm{p} + \pi^-$$

Charge: The charge on Λ^0 is 0, the charge on a proton is $+e$, and the charge on π^- is $-e$. The charge is conserved.

Baryon number: A proton and a Λ^0 particle are both baryons and have baryon numbers of $+1$. A π^- meson has a baryon number of 0. The baryon number is conserved.

Lepton number: None of these particles are leptons, so they have lepton numbers of zero. Lepton number is conserved.

This reaction $\boxed{\text{is possible}}$.

Part b)

$$\Delta^+ \rightarrow \Sigma^+ + \pi^0 + \gamma$$

Charge: The charge on Δ^+ and Σ^+ is $+e$, and the charge on π^0 is 0. The charge is conserved.

Baryon number: The particles Δ^+ and Σ^+ are both baryons and have baryon numbers of $+1$. A π^- meson has a baryon number of 0. The baryon number is conserved.

Lepton number: None of these particles are leptons, so they have lepton numbers of zero. Lepton number is conserved.

This reaction $\boxed{\text{is possible}}$.

Part c)

$$\pi^0 \rightarrow \mu^- + \bar{\nu}_\mu$$

Charge: The charges on π^0 and $\bar{\nu}_\mu$ are zero. The charge on μ^- is $-e$. The charge is not conserved.

Baryon number: None of the particles are baryons, so they each have a baryon number of 0. The baryon number is conserved.

Lepton number: A π^0 meson is not a lepton, so it has a lepton number of 0. A μ^- has a muon–lepton number of $+1$, and a $\bar{\nu}_\mu$ particle has a muon–lepton number of -1. Muon–lepton number is conserved.

This reaction $\boxed{\text{is not possible because charge is not conserved}}$.

REFLECT

All conservation laws must be obeyed for a reaction to be valid.

28.33

SET UP

We can determine whether or not the reactions are possible by seeing if the charge, baryon number, and the lepton number are conserved. Every matter baryon has a baryon number of $+1$, and particles that are not baryons have a baryon number of 0. Matter leptons have a

lepton number of $+1$, while antimatter leptons have a lepton number of -1. Particles that are not leptons have a lepton number of 0.

SOLVE

Part a)

$$\mathrm{n} \rightarrow \mathrm{p} + \mathrm{e}^- + \bar{\nu}_\mathrm{e}$$

Charge: A neutron and an antimatter neutrino each have a charge of 0. A proton has a charge of $+e$, and an electron has a charge of $-e$. Charge is conserved.

Baryon number: A neutron and proton each have a baryon number of $+1$. An electron and an antimatter electron neutrino each have a baryon number of 0. The baryon number is conserved.

Electron–lepton number: A neutron and a proton each have an electron–lepton number of 0. An electron has an electron–lepton number of $+1$. An antimatter electron neutrino has an electron–lepton number of -1. The electron–lepton number is conserved.

This reaction is $\boxed{\text{possible}}$.

Part b)

$$\mu^- \rightarrow \mathrm{e}^- + \bar{\nu}_\mathrm{e} + \nu_\mu$$

Charge: A neutrino and an antimatter neutrino each have a charge of 0. A muon and an electron each have a charge of $-e$. Charge is conserved.

Baryon number: All of the particles involved have a baryon number of 0. The baryon number is conserved.

Lepton number: An electron has an electron–lepton number of $+1$. An antimatter electron neutrino has an electron–lepton number of -1. The electron–lepton number is conserved. A muon and a muon neutrino each have a muon–lepton number of $+1$. The muon–lepton number is conserved.

This reaction is $\boxed{\text{possible}}$.

Part c)

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$

Charge: A negatively charged pion and a muon each have a charge of $-e$. An antimatter neutrino has a charge of 0. Charge is conserved.

Baryon number: All of the particles involved have a baryon number of 0. The baryon number is conserved.

Lepton number: A pion has a lepton number of 0. A muon has a muon–lepton number of $+1$. An antimatter muon neutrino has a muon–lepton number of -1. The muon–lepton number is conserved.

This reaction is $\boxed{\text{possible}}$.

REFLECT

All three reactions conserve charge, baryon number, and lepton number.

28.34

SET UP

We know the following reaction takes place: $\Delta^0 \rightarrow \mathrm{p} + \pi^-$. A Δ^0 particle is composed of udd, the proton is composed of uud, and π^- is composed of $\bar{\mathrm{u}}$d. One of the down quarks from the Δ^0 particle is converted into an up quark to form the proton. This can occur through a weak process, similar to that shown in Figure 28-9 of the textbook.

SOLVE

Since Δ^0 is composed of udd and the proton is composed of uud, one d in the Δ^0 particle needs to "become" a u. This can proceed via a weak process $\mathrm{u} \rightarrow \mathrm{dW}^-$ and then $\mathrm{W}^- \rightarrow \bar{\mathrm{u}}\mathrm{d}$, which is the quark composition for π^-.

REFLECT

The weak force is mediated by W^+, W^-, and Z^0 particles.

28.35

SET UP

We know the following reaction takes place: $\rho^0 \rightarrow \pi^+ + \pi^-$. A ρ^0 particle is composed of u$\bar{\mathrm{u}}$, π^+ is composed of u$\bar{\mathrm{d}}$, and π^- is composed of $\bar{\mathrm{u}}$d. One way the reaction can proceed is by the $\bar{\mathrm{u}}$ of ρ^0 emitting a gluon.

SOLVE

The $\bar{\mathrm{u}}$ of ρ^0 emits a gluon, which becomes d$\bar{\mathrm{d}}$. The d from the gluon and $\bar{\mathrm{u}}$ from the ρ^0 combine to form π^-; the $\bar{\mathrm{d}}$ from the gluon and the u from the ρ^0 combine to form π^+.

REFLECT

The strong force, through which quarks are attracted to one another, is mediated by the neutrally charged gluon. The gluon couples primarily to a quark and its antimatter partner, like d$\bar{\mathrm{d}}$ in this case.

28.36

SET UP

A photon can decay into a proton–antiproton pair, $\gamma \rightarrow \mathrm{p} + \bar{\mathrm{p}}$. To determine the maximum wavelength of such a photon, we need to determine the minimum energy required to form the proton–antiproton pair. The mass of a proton or antiproton is equal to $m_{\mathrm{p}} = m_{\bar{\mathrm{p}}} = 938\frac{\mathrm{MeV}}{c^2}$. The energy of this photon is related to its wavelength by $E_\gamma = \frac{hc}{\lambda}$.

SOLVE

Energy of photon:

$$m_{\mathrm{p}} = m_{\bar{\mathrm{p}}} = 938\frac{\mathrm{MeV}}{c^2}$$

$$E_\gamma = (m_{\bar{\mathrm{p}}})c^2 = 2\left(938\frac{\mathrm{MeV}}{c^2}\right)c^2 = 1880\ \mathrm{MeV}$$

Wavelength of photon:

$$E_\gamma = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{E_\gamma} = \frac{1240 \text{ eV} \cdot \text{nm}}{1880 \times 10^6 \text{ eV}} = \boxed{6.61 \times 10^{-7} \text{ nm}}$$

REFLECT

Since the wavelength and the energy of a photon are inversely proportional to one another, the maximum wavelength corresponds to the minimum energy necessary to form the proton–antiproton pair. Wavelengths smaller than this value correspond to higher energies. Excess energy can also be released in the decay.

28.37

SET UP

A neutral η particle $\left(m_\eta = 547\frac{\text{MeV}}{c^2}\right)$ decays into two gamma rays, $\eta^0 \to \gamma + \gamma$. By applying conservation of energy, we can calculate the energy, wavelength, and the magnitude of the momentum of each of the photons.

SOLVE

Energy:

$$2E_\gamma = E_\eta$$

$$E_\gamma = \frac{E_\eta}{2} = \frac{m_\eta c^2}{2} = \frac{\left(547\frac{\text{MeV}}{c^2}\right)c^2}{2} = \boxed{274 \text{ MeV}}$$

Wavelength:

$$E_\gamma = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{E_\gamma} = \frac{1240 \text{ eV} \cdot \text{nm}}{274 \times 10^6 \text{ eV}} = \boxed{4.53 \times 10^{-6} \text{ nm}}$$

Momentum:

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{4.53 \times 10^{-15} \text{ m}} = \boxed{1.46 \times 10^{-19}\frac{\text{kg} \cdot \text{m}}{\text{s}}}$$

REFLECT

A wavelength on the order of 10^{-15} m is reasonable for a gamma ray.

28.38

SET UP

A neutral particle at rest decays into two photons, each with an energy of $E_\gamma = 67.5$ MeV. We can apply conservation of energy to calculate the rest mass and then the identity of the unknown particle.

SOLVE

$$E_? = 2E_\gamma = 2(67.5\ \text{MeV}) = 135\ \text{MeV}$$

$$m_? = 135\frac{\text{MeV}}{c^2}$$

The $\boxed{\text{neutral pion } \pi^0}$ has a rest mass of $135\frac{\text{MeV}}{c^2}$.

REFLECT

The unknown particle had to be neutral due to charge conservation.

28.39

SET UP

A high-energy photon in the vicinity of a nucleus can create an electron–positron pair by pair production: $\gamma \rightarrow e^- + e^+$. The minimum energy required for this process is equal to the rest energy of an electron and a positron. The mass of an electron and the mass of a positron are both equal to $m_e = 9.11 \times 10^{-31}$ kg. Since the electron and positron are created at rest, the nucleus is necessary to conserve momentum.

SOLVE

Part a)

$$E = m_e c^2 + m_e c^2 = 2m_e c^2 = 2(9.11 \times 10^{-31}\ \text{kg})\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)^2$$

$$= \boxed{1.64 \times 10^{-13}\ \text{J}} \times \frac{1\ \text{eV}}{1.6 \times 10^{-19}\ \text{J}} = 1.02 \times 10^6\ \text{eV} = \boxed{1.02\ \text{MeV}}$$

Part b) The nucleus is required to absorb the momentum so that conservation of momentum is not violated.

REFLECT

The excess energy of a photon with an energy higher than 1.02 MeV undergoing pair production goes into the kinetic energy of the electron and the positron.

28.40

SET UP

A neutral pion is created when two protons, each with the same kinetic energy K_p, collide: $p + p \rightarrow p + p + \pi^0$. We can apply energy conservation to calculate the minimum value of K_p. The rest mass of a neutral pion is $m_{\pi^0} = 135.0\frac{\text{MeV}}{c^2}$.

SOLVE

$$2K_p = E_{\pi^0}$$

$$K_p = \frac{E_{\pi^0}}{2} = \frac{m_{\pi^0}c^2}{2} = \frac{\left(135.0\frac{\text{MeV}}{c^2}\right)c^2}{2} = \boxed{67.50\ \text{MeV}}$$

REFLECT
The rest energy due to the protons is the same on both sides of the reaction, so we only need to consider the kinetic energy.

28.41

SET UP
A neutrino has an energy $E = 150 \times 10^6$ eV. Assuming it is massless, the wavelength, frequency, and the magnitude of the momentum are given by $E = \frac{hc}{\lambda}$, $f = \frac{c}{\lambda}$, and $p = \frac{h}{\lambda}$, respectively.

SOLVE
Wavelength:

$$E = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{E} = \frac{1240\text{ eV}\cdot\text{nm}}{150 \times 10^6\text{ eV}} = \boxed{8.27 \times 10^{-6}\text{ nm}}$$

Frequency:

$$f = \frac{c}{\lambda} = \frac{\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)}{8.27 \times 10^{-15}\text{ m}} = \boxed{3.63 \times 10^{22}\text{ Hz}}$$

Momentum:

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}\text{ J}\cdot\text{s}}{8.27 \times 10^{-15}\text{ m}} = \boxed{8.02 \times 10^{-20} \frac{\text{kg}\cdot\text{m}}{\text{s}}}$$

REFLECT
These seem like reasonable numerical values for a neutrino.

28.42

SET UP
Energy in the form of gamma rays is released when a positron ($E_{e^+} = 34$ MeV) and an electron ($E_{e^-} = 16$ MeV) annihilate one another. The total energy released is equal to the sum of the initial energies of the particles and their rest energies. The rest energy of a positron, which is the same as the rest energy of an electron, is 0.511 MeV.

SOLVE

$$E_{\text{total}} = (m_{e^-} + m_{e^+})c^2 + E_{e^+} + E_{e^-} = 2(0.511\text{ MeV}) + (34\text{ MeV}) + (16\text{ MeV}) = \boxed{51\text{ MeV}}$$

REFLECT
Photons are massless, and energy must be conserved in the reaction.

28.43

SET UP

A proton–antiproton annihilation takes place and the resulting photons have a total energy of 2.5×10^9 eV. The energy of the photons is equal to the sum of the rest energies and the kinetic energies of the proton and antiproton. The mass of a proton and the mass of an antiproton are both equal to $m_p = 1.67 \times 10^{-27}$ kg. Using energy conservation, we can calculate the kinetic energy of the proton K_p and the kinetic energy of the antiproton $K_{\bar{p}}$ if they have the same kinetic energy or if $K_p = 1.25K_{\bar{p}}$.

SOLVE

Mass of proton in MeV/c^2:

$$E = m_p c^2 = (1.67 \times 10^{-27}\text{ kg})\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}^2}\right)$$

$$= 1.503 \times 10^{-10}\text{ J} \times \frac{1\text{ eV}}{1.6 \times 10^{-19}\text{ J}} = 9.38 \times 10^8\text{ eV}$$

$$m_p = 9.38 \times 10^8 \frac{\text{eV}}{c^2}$$

Part a)

$$E = m_p c^2 + m_{\bar{p}} c^2 + K_p + K_{\bar{p}} = 2m_p c^2 + 2K_p$$

$$K_p = \frac{E - 2m_p c^2}{2} = \frac{(2.5 \times 10^9\text{ eV}) - 2\left(9.38 \times 10^8 \frac{\text{eV}}{c^2}\right)c^2}{2} = \boxed{3.12 \times 10^8\text{ eV} = 0.312\text{ GeV}}$$

$$\boxed{K_p = K_{\bar{p}} = 0.312\text{ GeV}}$$

Part b)

$$E = m_p c^2 + m_{\bar{p}} c^2 + K_p + K_{\bar{p}} = 2m_p c^2 + 1.25K_{\bar{p}} + K_{\bar{p}}$$

$$K_{\bar{p}} = \frac{E - 2m_p c^2}{2.25} = \frac{(2.5 \times 10^9\text{ eV}) - 2\left(9.38 \times 10^8 \frac{\text{eV}}{c^2}\right)c^2}{2.25} = \boxed{2.77 \times 10^8\text{ eV} = 0.277\text{ GeV}}$$

$$\boxed{K_p = 1.25K_{\bar{p}} = 1.25(0.277\text{ GeV}) = 0.347\text{ GeV}}$$

REFLECT

Since the proton and antiproton annihilate one another, both their kinetic energy *and* rest energy are converted into photons.

28.44

SET UP

A pion, which has a rest energy of 139.6 MeV, decays into a neutrino and a muon, which has a rest energy of 105.7 MeV. The pion has a kinetic energy of $K_{\pi^+} = 860$ MeV. We can use

conservation of energy and momentum to calculate the kinetic energies of the neutrino and the muon.

SOLVE

Conservation of energy:

$$E_{\text{total}} = E_{0,\,\pi} + K_\pi = (860\text{ MeV}) + (139.6\text{ MeV}) = 999.6\text{ MeV}$$

$$E_{\text{total}} = E_\mu + E_\nu$$

Conservation of momentum:

$$p_\mu = p_\nu$$

$$\sqrt{\frac{E_\mu^2 - E_{0,\,\mu}^2}{c^2}} = \frac{E_\nu}{c}$$

$$E_\mu^2 - E_{0,\,\mu}^2 = E_\nu^2$$

But $E_\mu = E_{\text{total}} - E_\nu$:

$$(E_{\text{total}} - E_\nu)^2 - E_{0,\,\mu}^2 = E_\nu^2$$

$$E_{\text{total}}^2 - 2E_{\text{total}}\,E_\nu + E_\nu^2 - E_{0,\,\mu}^2 = E_\nu^2$$

$$E_{\text{total}}^2 - 2E_{\text{total}}\,E_\nu - E_{0,\,\mu}^2 = 0$$

$$E_\nu = \frac{E_{\text{total}}^2 - E_{0,\,\mu}^2}{2E_{\text{total}}} = \frac{(999.6\text{ MeV})^2 - (105.7\text{ MeV})^2}{2(999.6\text{ MeV})} = 494\text{ MeV}$$

Since the neutrino is massless, this is all kinetic energy: $\boxed{K_\nu = 494\text{ MeV}}$.

Kinetic energy of the muon:

$$E_\mu = E_{\text{total}} - E_\nu = (999.6\text{ MeV}) - (494\text{ MeV}) = 505\text{ MeV}$$

$$E_\mu = K_\mu + E_{0,\,\mu}$$

$$K_\mu = E_\mu - E_{0,\,\mu} = (505\text{ MeV}) - (105.7\text{ MeV}) = \boxed{400\text{ MeV}}$$

REFLECT

Since a muon has a smaller rest energy than a pion, the total kinetic energy of the products must be larger than the initial kinetic energy.

28.45

SET UP

A neutron changes into a proton, an electron, and an antineutrino via beta-minus decay. One of the d quarks in the neutron decays to a u quark through the weak interaction, which is mediated by a W^- particle. The W^- particle then decays to an electron and an antineutrino. The proton, electron, and antineutrino are represented by straight lines, whereas the W^- particle is represented by a wavy line.

SOLVE

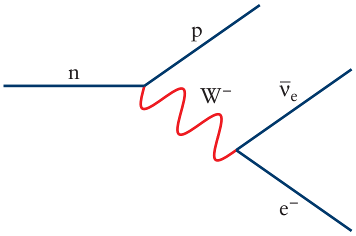

Figure 28-1 Problem 45

REFLECT

Each vertex in the Feynman diagram corresponds to a decay process. The W^- particle is a mediator particle for the weak force.

28.46

SET UP

An electron and a positron are created in a pair production. The Feynman diagram should have a wavy line representing the photon. At the end of the wavy line, two straight lines representing the electron and positron should head off toward the right.

SOLVE

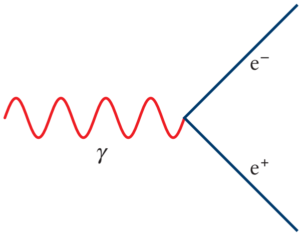

Figure 28-2 Problem 46

REFLECT

Time runs from left to right in a Feynman diagram, so the process starts with a photon, which becomes two particles, as expected for a pair production process.

28.47

SET UP

An electron and a positron undergo pair annihilation. The Feynman diagram should have two straight lines on the left representing the electron and positron that meet at a vertex. A wavy line representing the photon should then leave the vertex headed towards the right.

SOLVE

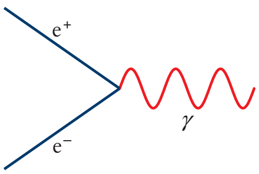

Figure 28-3 Problem 47

REFLECT
Time runs from left to right in a Feynman diagram, so the process starts with two particles and ends with a photon, as expected for an annihilation process.

28.48

SET UP
An antineutron decays into an antiproton, a positron, and a neutrino via beta-plus decay. One of the $\overline{\mathrm{d}}$ quarks in the neutron decays to a $\overline{\mathrm{u}}$ quark through the weak interaction, which is mediated by a W^+ particle. The W^+ particle then decays to a positron and a neutrino. The antiproton, positron, and neutrino are represented by straight lines, whereas the W^+ particle is represented by a wavy line.

SOLVE

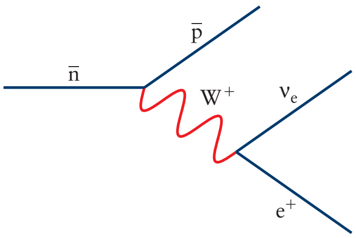

Figure 28-4 Problem 48

REFLECT
This is an analogous process to the beta-minus decay of a neutron from Problem 28.45.

28.49

SET UP
A new fundamental force is discovered having a range about equal to the radius of a hydrogen atom, 0.0529 nm. The range R of the force is on the order of the distance the corresponding mediating particle can travel in a time Δt allowed by the Heisenberg uncertainty principle. The fastest a particle can travel is the speed of light in a vacuum. The energy spread in the uncertainty principle should be equal to the rest energy of the mediating particle, which will allow us to calculate the mass of the mediating particle. We can then compare the mass of the mediating particle to the mass of the electron, $m_e = 0.511\dfrac{\mathrm{MeV}}{c^2}$.

SOLVE
Mass of the mediating particle:

$$R < c\Delta t$$

$$R < c\left(\frac{\hbar}{2\Delta E}\right)$$

$$R < \frac{\hbar c}{2mc^2}$$

$$m \sim \frac{\hbar c}{2Rc^2} = \frac{hc}{4\pi Rc^2} = \frac{1240\ \mathrm{eV}\cdot\mathrm{nm}}{4\pi(0.0529\ \mathrm{nm})c^2} = 1870\frac{\mathrm{eV}}{c^2} = \boxed{1.87\times10^{-3}\frac{\mathrm{MeV}}{c^2}}$$

Mass of the mediating particle relative to the mass of the electron:

$$\frac{m_e}{m} = \frac{\left(0.511\frac{\text{MeV}}{c^2}\right)}{\left(1.87 \times 10^{-3}\frac{\text{MeV}}{c^2}\right)} = \boxed{274}$$

REFLECT

The range of this new fundamental force is about 7 orders of magnitude larger than the weak force.

28.50

SET UP

An electron is confined to a nanotube that has a width $\Delta x = 1.2 \times 10^{-9}$ m. We can use the Heisenberg uncertainty principle to calculate the minimum uncertainty in the speed of the electron.

SOLVE

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

$$\Delta x(m_e \Delta v) \geq \frac{\hbar}{2}$$

$$\Delta v \geq \frac{\hbar}{2m_e \Delta x} = \frac{(1.055 \times 10^{-34}\ \text{J} \cdot \text{s})}{2(9.11 \times 10^{-31}\ \text{kg})(1.2 \times 10^{-9}\ \text{m})} = \boxed{4.8 \times 10^4 \frac{\text{m}}{\text{s}}}$$

REFLECT

Since the spread in the position is relatively small, the spread in the speed must be large since they are inversely proportional to one another.

28.51

SET UP

Sodium-22 decays by emitting a positron. In order to write out the decay, we need to apply the conservation of baryon number and electron–lepton number. The resulting nucleus needs to have a mass number of 22 in order to conserve baryon number, but the atomic number of the nucleus should decrease by 1 due to the emitted positron. Sodium has an atomic number of 11, which means the product nucleus should have an atomic number 10, corresponding to neon. Finally, the electron–lepton number of the positron is -1, so we need to also produce an electron neutrino in order to conserve lepton number. Initially, there are 5×10^{23} sodium-22 nuclei. The number of positrons emitted per second is equal to the number of sodium-22 decays per second, which is the activity of the sample. The decay constant is related to the half-life of sodium-22, $\tau_{1/2} = 2.60$ yr.

SOLVE

Decay:

$$^{22}\text{Na} \rightarrow \text{e}^+ + {}^{22}\text{Ne} + \nu_e$$

Emitted positrons per second:

$$\text{activity} = \lambda N = \left(\frac{\ln(2)}{\tau_{1/2}}\right)N = \frac{\ln(2)}{\left(2.60\text{ yr} \times \dfrac{365.25\text{ days}}{1\text{ yr}} \times \dfrac{24\text{ hr}}{1\text{ day}} \times \dfrac{3600\text{ s}}{1\text{ hr}}\right)}(5 \times 10^{23}\ {}^{22}\text{Na})$$

$$= 4.22 \times 10^{15}\frac{{}^{22}\text{Na}}{\text{s}} \times \frac{1\text{ e}^+}{1\ {}^{22}\text{Na}} = \boxed{4.22 \times 10^{15}\frac{\text{e}^+}{\text{s}}}$$

REFLECT

Remember that the mass number is the total number of protons *and* neutrons. Sodium-22 has 11 protons and 11 neutrons, while neon-22 has 10 protons and 12 neutrons.

28.52

SET UP

Copper-59 decays by emitting a positron. In order to write out the decay, we need to apply the conservation of baryon number and electron–lepton number. The resulting nucleus needs to have a mass number of 59 in order to conserve baryon number, but the atomic number of the nucleus should decrease by 1 due to the emitted positron. Copper has an atomic number of 29, which means the product nucleus should have an atomic number of 28, corresponding to nickel. Finally, the electron–lepton number of the positron is -1, so we need to also produce an electron neutrino in order to conserve lepton number. The decay rate of a sample of copper-59 is $R = 2\text{ s}^{-1}$. The number of positrons emitted per second is equal to the number of copper-59 decays per second, which is the activity of the sample, λN_0, where λ is the decay constant and N_0 is the number of copper-59 nuclei initially present in the sample, which is what we are interested in. The decay constant is related to the half-life of copper-59, $\tau_{1/2} = 81.5$ s.

SOLVE

Part a)

$${}^{59}\text{Cu} \rightarrow \text{e}^+ + {}^{59}\text{Ni} + \nu_\text{e}$$

Part b)

$$R = \lambda N_0 = \left(\frac{\ln(2)}{\tau_{1/2}}\right)N_0$$

$$N_0 = \frac{\tau_{1/2}R}{\ln(2)} = \frac{(81.5\text{ s})(2\text{ s}^{-1})}{\ln(2)} = \boxed{235\text{ nuclei}}$$

REFLECT

In positron decay, a proton decays into a neutron. Remember that the mass number is the total number of protons *and* neutrons. Copper-59 has 29 protons and 30 neutrons, whereas nickel-59 has 28 protons and 31 neutrons.

28.53

SET UP

Potassium-40 is a positron emitter and has a half-life of $\tau_{1/2} = 1.28 \times 10^9$ yr. The decay rate is equal to the activity of the sample, λN_0, where λ is the decay constant and N_0 is the number

of potassium-40 nuclei initially present in the 400-g sample. The atomic mass of potassium is $39.0983\frac{\text{g}}{\text{mol}}$.

SOLVE

Decay process:

$$^{40}\text{K} \rightarrow \text{e}^{+} + {}^{40}\text{Ar} + \nu_{\text{e}}$$

Half-life:

$$\tau_{1/2} = 1.28 \times 10^{9}\ \text{yr} \times \frac{365\ \text{days}}{1\ \text{yr}} \times \frac{24\ \text{hr}}{1\ \text{day}} \times \frac{3600\ \text{s}}{1\ \text{hr}} = 4.04 \times 10^{16}\ \text{s}$$

Decay constant:

$$\lambda = \frac{\ln(2)}{\tau_{1/2}} = \frac{\ln(2)}{4.04 \times 10^{16}\ \text{s}} = 1.72 \times 10^{-17}\ \text{s}^{-1}$$

Decay rate:

$$R = \lambda N_0$$

$$= (1.72 \times 10^{-17}\ \text{s}^{-1})\left(400\ \text{g K} \times \frac{1\ \text{mol K}}{39.0983\ \text{g K}} \times \frac{6.02 \times 10^{23}\ \text{K nuclei}}{1\ \text{mol K}} \times \frac{1\ \text{e}^{+}}{1\ \text{K nucleus}}\right)$$

$$= \boxed{1.06 \times 10^{8}\frac{\text{e}^{+}}{\text{s}}}$$

REFLECT

Even though a decay rate on the order of $10^{8}\ \text{s}^{-1}$ seems very large, there are about 6×10^{24} potassium-40 nuclei initially present.

28.54

SET UP

The minimum energy released in a particle–antiparticle annihilation is equal to the sum of the rest energies of the particles. Because the rest energy of an antiparticle is equal to the rest energy of its corresponding matter particle, the minimum energy released is twice the rest energy of the particle. The rest energies of an electron, muon, tau particle, and a proton are 0.511 MeV, 106 MeV, 1777 MeV, and 938 MeV, respectively.

SOLVE

Part a)

$$E = 2(0.511\ \text{MeV}) = \boxed{1.02\ \text{MeV}}$$

Part b)

$$E = 2(106\ \text{MeV}) = \boxed{212\ \text{MeV}}$$

Part c)

$$E = 2(1777\text{ MeV}) = \boxed{3554\text{ MeV}}$$

Part d)

$$E = 2(938\text{ MeV}) = \boxed{1880\text{ MeV}}$$

REFLECT

These are the minimum possible energies since we've ignored any kinetic energy of the starting particles.

28.55

SET UP

The strong force acts over distance of approximately 1 fm. We're interested in the nucleus that has a diameter equal to ten times this, or with a radius of $r = 5.0$ fm. We can use $r = r_0 A^{\frac{1}{3}}$, where $r_0 = 1.2$ fm, to calculate the mass number of this nucleus, and then consult a periodic table (or Appendix C) to see which element has an atomic mass similar to this. As an approximation, we can treat the outer proton of the nucleus as being repelled by all of the inner protons located at the center of the nucleus and use Coulomb's law to calculate the magnitude of this repulsion.

SOLVE

Part a)

$$r = r_0 A^{\frac{1}{3}}$$

$$A = \left(\frac{r}{r_0}\right)^3 = \left(\frac{5.0\text{ fm}}{1.2\text{ fm}}\right)^3 = 72.3$$

Consulting the periodic table, germanium (Ge) has an atomic weight of 72.6, so a $\boxed{\text{germanium}}$ nucleus would have a diameter equal to about 10 times the range of the strong force.

Part b)

Germanium has an atomic number of 32. We will treat the outer proton as being repelled by the 31 inner protons located at the center of the nucleus:

$$F = \frac{k(e)(31e)}{r^2} = \frac{\left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(31)(1.6 \times 10^{-19}\text{ C})^2}{(5.0 \times 10^{-15}\text{ m})^2} = \boxed{290\text{ N}}$$

REFLECT

The strong force opposes this electrical repulsion and keeps the nucleus together.

28.56

SET UP

A fictitious fundamental force is mediated by electrons and positrons. The range R of a force is on the order of the distance the corresponding mediating particle can travel in a time Δt allowed by the Heisenberg uncertainty principle. The fastest a particle can travel is the speed of light in a vacuum. The energy spread in the uncertainty principle should be equal to the rest energy of the mediating particle, which will allow us to calculate the range of the new force.

SOLVE

$$R \sim c\Delta t = c\left(\frac{\hbar}{2\Delta E}\right) = \frac{\hbar c}{2m_e c^2} = \frac{hc}{4\pi m_e c^2}$$

$$= \frac{1240\ \text{eV}\cdot\text{nm}}{4\pi\left(0.511\times 10^6\frac{\text{eV}}{c^2}\right)c^2} = \boxed{1.93\times 10^{-4}\ \text{nm} = 193\ \text{fm}}$$

REFLECT

This is about 5 orders of magnitude larger than the range of the weak force.

28.57

SET UP

An electron is located in a region that has a width of $\Delta x = 75\times 10^{-9}$ m. We can use the Heisenberg uncertainty principle to calculate the minimum speed of the electron, which is when the speed is equal to the uncertainty in the speed. We can then relate the speed to the maximum time the electron could remain in this region using $v = \frac{\Delta x}{\Delta t}$.

SOLVE

$$\Delta x\Delta p \geq \frac{\hbar}{2}$$

$$\Delta x(m_e\Delta v) \geq \frac{\hbar}{2}$$

$$\Delta x(m_e v_{\text{min}}) = \frac{\hbar}{2}$$

$$\Delta x m_e\left(\frac{\Delta x}{\Delta t_{\text{max}}}\right) = \frac{\hbar}{2}$$

$$\Delta t_{\text{max}} = \frac{2(\Delta x)^2 m_e}{\hbar} = \frac{2(75\times 10^{-9}\ \text{m})^2(9.11\times 10^{-31}\ \text{kg})}{1.055\times 10^{-34}\ \text{J}\cdot\text{s}} = \boxed{9.7\times 10^{-11}\ \text{s} = 97\ \text{ps}}$$

REFLECT

Since it's in the denominator, the maximum time it takes the electron to cross the dot will correspond to the minimum speed of the particle.

28.58

SET UP

Sulfur-31 is a positron emitter. A positron–electron annihilation releases about 1.022 MeV of energy. To determine the mass of sulfur-31 required per second to produce 1.00×10^{-6} W of power, we must first determine the number of annihilations per second by dividing this power by the energy per annihilation. Every annihilation corresponds to one decay of a sulfur-31 nucleus, so the mass necessary is related to the molar mass of sulfur, $32.065\frac{\text{g}}{\text{mol}}$.

SOLVE

Number of annihilations per second to produce 1.00×10^{-6} W:

$$1.00 \times 10^{-6}\frac{\text{J}}{\text{s}} \times \frac{1\text{ eV}}{1.602 \times 10^{-19}\text{ J}} \times \frac{1\text{ MeV}}{10^6\text{ eV}} = 6.24 \times 10^6\frac{\text{MeV}}{\text{s}}$$

$$\frac{\left(6.24 \times 10^6\frac{\text{MeV}}{\text{s}}\right)}{\left(1.022\frac{\text{MeV}}{\text{annihilation}}\right)} = 6.11 \times 10^6\frac{\text{annihilations}}{\text{s}}$$

Mass of sulfur-31 decaying per second:

$$6.11 \times 10^6\frac{\text{annihilations}}{\text{s}} = 6.11 \times 10^6\frac{\text{nuclei}}{\text{s}} \times \frac{1\text{ mol}}{6.02 \times 10^{23}\text{ nuclei}} \times \frac{32.065\text{ g}}{1\text{ mol}} = \boxed{3.25 \times 10^{-16}\frac{\text{g}}{\text{s}}}$$

REFLECT

An energy of 1.022 MeV corresponds to about 1.6×10^{-13} J, so we would need about 10^7 nuclei to decay per second. It makes sense that the mass we would need per second should be very small.

28.59

SET UP

Muonic hydrogen is a hydrogen atom where the electron has been replaced by a muon. We can calculate the Bohr radius and ground-state ionization energy of muonic hydrogen by setting up a ratio with the values for regular hydrogen ($a_0 = 5.29 \times 10^{-11}$ m, $E_0 = 13.6$ eV). The algebraic expressions for the Bohr radius and the ground-state energy of hydrogen (also known as the Rydberg energy) are $\frac{\hbar^2}{mke^2}$ and $\frac{m(ke^2)^2}{2\hbar^2}$, respectively; the mass is either the mass of the electron ($m_e = 0.5110\text{ MeV}/c^2$) or the mass of the muon ($m_\mu = 105.7\text{ MeV}/c^2$). The big difference in ionization energies arises from the difference in electrostatic attraction between the muon and the nucleus and the electron and the nucleus.

SOLVE

Part a)

$$\frac{a_0(\mu)}{a_0(\mathrm{e})} = \frac{\left(\dfrac{\hbar^2}{m_\mu ke^2}\right)}{\left(\dfrac{\hbar^2}{m_\mathrm{e} ke^2}\right)} = \frac{m_\mathrm{e}}{m_\mu} = \frac{\left(0.5110\dfrac{\mathrm{MeV}}{c^2}\right)}{\left(105.7\dfrac{\mathrm{MeV}}{c^2}\right)} = 0.00483$$

The Bohr radius in muonic hydrogen is $\boxed{4.83 \times 10^{-3} \text{ times smaller}}$ than the Bohr radius of ordinary hydrogen.

$$a_0(\mu) = (0.00483)a_0(\mathrm{e}) = 0.00483(5.29 \times 10^{-11}\ \mathrm{m}) = \boxed{2.56 \times 10^{-13}\ \mathrm{m} = 2.56 \times 10^{-4}\ \mathrm{nm}}$$

Part b)

$$\frac{E_0(\mu)}{E_0(\mathrm{e})} = \frac{\left(\dfrac{m_\mu (ke^2)^2}{2\hbar^2}\right)}{\left(\dfrac{m_\mathrm{e} (ke^2)^2}{2\hbar^2}\right)} = \frac{m_\mu}{m_\mathrm{e}} = \frac{\left(105.7\dfrac{\mathrm{MeV}}{c^2}\right)}{\left(0.5110\dfrac{\mathrm{MeV}}{c^2}\right)} = 207$$

The minimum ionization energy of ground-state muonic hydrogen is $\boxed{207 \text{ times}}$ larger than the ionization energy of ordinary ground-state hydrogen.

$$E_0(\mu) = (207)E_0(\mathrm{e}) = 207(13.6\ \mathrm{eV}) = \boxed{2810\ \mathrm{eV}}$$

Part c) The muon has the same charge as the electron but it is much closer to the nucleus than the electron in ordinary hydrogen. Therefore the muon feels a much stronger electrical attraction from the nucleus, so it takes much more energy to remove it from the atom than is required for the electron in ordinary hydrogen.

REFLECT

The muon is a lepton, just like the electron, so it is not affected by the strong force.

28.60

SET UP

Using our results from Problem 28.59, we can use the Rydberg equation to calculate the three longest wavelengths in the Balmer series ($n = 2$) of muonic hydrogen. The time it takes for 90% of the original muonic hydrogen to be used up can be found from $N = 0.10N_0 = N_0 2^{-\frac{t}{\tau_{1/2}}}$, where the half-life of a muon is $\tau_{1/2} = 1.56\ \mu\mathrm{s}$.

SOLVE

Part a)

$$\frac{1}{\lambda} = \frac{E_0(\mu)}{hc}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

$$\lambda = \frac{hc}{E_0(\mu)}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)^{-1}$$

$m = 3 \rightarrow n = 2$:

$$\lambda_{3\rightarrow 2} = \frac{1240\ \text{eV}\cdot\text{nm}}{2810\ \text{eV}}\left(\frac{1}{2^2} - \frac{1}{3^2}\right)^{-1} = \boxed{3.178\ \text{nm}}$$

$m = 4 \rightarrow n = 2$:

$$\lambda_{4\rightarrow 2} = \frac{1240\ \text{eV}\cdot\text{nm}}{2810\ \text{eV}}\left(\frac{1}{2^2} - \frac{1}{4^2}\right)^{-1} = \boxed{2.353\ \text{nm}}$$

$m = 5 \rightarrow n = 2$:

$$\lambda_{5\rightarrow 2} = \frac{1240\ \text{eV}\cdot\text{nm}}{2810\ \text{eV}}\left(\frac{1}{2^2} - \frac{1}{5^2}\right)^{-1} = \boxed{2.101\ \text{nm}}$$

Part b)

$$N = 0.10N_0 = N_0 2^{-\frac{t}{\tau_{1/2}}}$$

$$t = \frac{-\tau_{1/2}\ln(0.10)}{\ln(2)} = \frac{-(1.56\ \mu\text{s})\ln(0.10)}{\ln(2)} = \boxed{5.18\ \mu\text{s}}$$

REFLECT

You would need to work very quickly when performing an experiment with muonic hydrogen!

28.61

SET UP

A neutrino and an antineutrino, both of which are barely moving, annihilate one another. A photon is emitted in the process. We can use conservation of energy to calculate the minimum wavelength of the photon emitted by the annihilation. The minimum wavelength of the photon will occur for a maximum mass of the neutrino, which is around $2.0\frac{\text{eV}}{c^2}$.

SOLVE

$$\frac{hc}{\lambda} = 2(m_\nu c^2)$$

$$\lambda = \frac{hc}{2m_\nu c^2} = \frac{1240\ \text{eV}\cdot\text{nm}}{2\left(2.0\frac{\text{eV}}{c^2}\right)c^2} = \boxed{310\ \text{nm}}$$

This is in the ultraviolet region of the electromagnetic spectrum and $\boxed{\text{will not be visible}}$.

REFLECT

The rest mass of a neutrino is relatively small, so the energy of the emitted photon should also be relatively small. For comparison, usually energies on the order of gamma rays are released from annihilation events.

28.62

SET UP

Each quark also has a "color" quantum number associated with it: red (R), green (G), or blue (B). Baryons are "color neutral," which means they consist of one red quark, one green quark, and one blue quark. With this in mind, we can write down all possible quark color combinations for an ordinary proton, which contains two up quarks and one down quark.

SOLVE

Possible color quantum states for an ordinary proton: $\mathrm{u_R u_G d_B}$, $\mathrm{u_R u_B d_G}$, $\mathrm{u_G u_R d_B}$, $\mathrm{u_G u_B d_R}$, $\mathrm{u_B u_G d_R}$, $\mathrm{u_B u_R d_G}$.

REFLECT

These are the only combinations that contain exactly one green, one red, and one blue quark.

28.63

SET UP

Each quark also has a "color" quantum number associated with it: red (R), green (G), or blue (B). Mesons are "color neutral," which means they consist of one red quark and one red antiquark, or one green quark and one green antiquark, or one blue quark and one blue antiquark. With this in mind, we can write down all possible quark color combinations for the three pions: π^+ $(\mathrm{u\overline{d}})$, π^0 $(\mathrm{u\overline{u}})$, and π^- $(\mathrm{d\overline{u}})$.

SOLVE

Possible color quantum states for π^+: $\mathrm{u_R\overline{d_R}}$, $\mathrm{u_B\overline{d_B}}$, and $\mathrm{u_G\overline{d_G}}$.
Possible color quantum states for π^0: $\mathrm{u_R\overline{u_R}}$, $\mathrm{u_B\overline{u_B}}$, and $\mathrm{u_G\overline{u_G}}$.
Possible color quantum states for π^-: $\mathrm{d_R\overline{u_R}}$, $\mathrm{d_B\overline{u_B}}$, and $\mathrm{d_G\overline{u_G}}$.

REFLECT

Unlike baryons, which require one of each color quark, color neutrality for mesons requires that the quark and antiquark have the same color.

28.64

SET UP

The United States used approximately 4.12×10^{12} kWh of electrical energy during 2010. We can use $E = mc^2$ to calculate the total mass of matter and antimatter that has this same amount of rest energy. If we were to generate this much energy through annihilation, half of this total mass corresponds to the mass of matter and the other half to antimatter. We can use the density of iron and anti-iron (both $\rho = 7800\dfrac{\text{kg}}{\text{m}^3}$) to find the dimensions of a cube of fuel that would provide this much energy.

SOLVE

Part a)
Converting from kWh:

$$4.12 \times 10^{12}\ \text{kWh} \times \frac{1000\ \text{W}}{1\ \text{kW}} \times \frac{3600\ \text{s}}{1\ \text{hr}} = 1.48 \times 10^{19}\ \text{J}$$

Mass necessary to produce this much energy:

$$E = mc^2$$

$$m = \frac{E}{c^2} = \frac{1.48 \times 10^{19}\ \mathrm{J}}{\left(3.00 \times 10^8 \frac{\mathrm{m}}{\mathrm{s}}\right)^2} = \boxed{165\ \mathrm{kg}}$$

Part b)

$$\rho = \frac{\left(\frac{m}{2}\right)}{V} = \frac{\left(\frac{m}{2}\right)}{L^3}$$

$$L = \sqrt[3]{\frac{m}{2\rho}} = \sqrt[3]{\frac{165\ \mathrm{kg}}{2\left(7800 \frac{\mathrm{kg}}{\mathrm{m}^3}\right)}} = \boxed{0.219\ \mathrm{m} = 21.9\ \mathrm{cm}}$$

REFLECT

This example illustrates the sheer magnitudes involved when working with rest energies.

28.65

SET UP

A Ξ_b baryon consists of a strange quark, an up quark, and a bottom quark. The total charge of the Ξ_b baryon is equal to the sum of the charges of the component quarks. The Ξ_b baryon travels a distance of 0.5×10^{-3} before it decays. We can estimate the lifetime of the particle using this information, assuming the baryon is moving close to the speed of light.

SOLVE

Part a)

$$Q_{\Xi_b} = Q_s + Q_u + Q_b = \left(-\frac{1}{3}e\right) + \left(\frac{2}{3}e\right) + \left(-\frac{1}{3}e\right) = \boxed{0}$$

Part b)

$$c = \frac{\Delta x}{\Delta t}$$

$$\Delta t = \frac{\Delta x}{c} = \frac{0.5 \times 10^{-3}\ \mathrm{m}}{\left(3.00 \times 10^8 \frac{\mathrm{m}}{\mathrm{s}}\right)} = \boxed{2 \times 10^{-12}\ \mathrm{s} = 2\ \mathrm{ps}}$$

REFLECT

The Ξ_b baryon is about six times heavier than a proton or neutron.

28.66

SET UP

The Planck time, t_P, is related to G, c, and $\hbar$. We can use dimensional analysis to determine the proper combination of the three fundamental constants that gives dimensions of time. If we

assume the proportionality constant from dimensional analysis is equal to 1, then we can plug in the values for G, c, and $\hbar$ to calculate the numerical value for t_P.

SOLVE

Dimensional analysis:

$$[t_P] = [G]^x[c]^y[\hbar]^z$$

$$[T] = \left[\frac{L^3}{MT^2}\right]^x\left[\frac{L}{T}\right]^y\left[\frac{ML^2}{T}\right]^z$$

$$[T] = [L]^{3x+y+2z}[M]^{z-x}[T]^{-2x-y-z}$$

Setting up the system of equations:

$$3x + y + 2z = 0$$

$$-2x - y - z = 1$$

$$z - x = 0$$

Solving the system of equations:

$$z = x$$

$$3x + y + 2z = 3x + y + 2x = 5x + y = 0$$

$$y = -5x$$

$$-2x - y - z = -2x - y - x = -3x - y = 1$$

$$-3x - y = -3x - (-5x) = 2x = 1$$

$$x = z = \frac{1}{2}$$

$$y = -\frac{5}{2}$$

Planck time:

$$\boxed{t_P \propto \sqrt{\frac{G\hbar}{c^5}}}$$

Numerical value of the Planck time, assuming a proportionality constant of 1:

$$t_P = \sqrt{\frac{G\hbar}{c^5}} = \sqrt{\frac{\left(6.6738 \times 10^{-11}\frac{\mathrm{N\cdot m^2}}{\mathrm{kg^2}}\right)(1.0546 \times 10^{-34}\ \mathrm{J\cdot s})}{\left(2.9979 \times 10^{8}\frac{\mathrm{m}}{\mathrm{s}}\right)^5}} = \boxed{5.3912 \times 10^{-44}\ \mathrm{s}}$$

REFLECT

We are told the Planck Epoch took place about 10^{-43} s after the Big Bang, so a Planck time of around 5×10^{-44} s is reasonable.

28.67

SET UP

Two stationary nucleons can be described with a potential function V according to $\nabla^2 V(r) = K^2 V(r)$, where $\nabla^2 V(r) = \frac{1}{r^2}\frac{d}{dr}\left[r^2 \frac{dV(r)}{dr}\right]$. We can show that $V(r) = g^2 \frac{e^{-Kr}}{r}$ is a solution to the above differential equation by explicitly calculating the necessary derivatives and showing that the left-hand and right-hand sides of the equation are equal. The constant K is equal to both $\frac{mc}{\hbar}$, where m is the mass of the exchanged particle, and $\frac{1}{R} = \frac{1}{1.5 \times 10^{-15}\text{ m}}$. Combining these equations, we can show that the mass is equal to that of a pion. Finally, using the mass of the exchanged particle, we can use the Heisenberg uncertainty principle, $\Delta E \Delta t \geq \frac{\hbar}{2}$, to estimate the lifetime of the pion.

SOLVE

Part a)
Left-hand side:

$$\frac{dV(r)}{dr} = \frac{d}{dr}\left[g^2 \frac{e^{-Kr}}{r}\right] = g^2\left[\frac{-Ke^{-Kr}}{r} - \frac{e^{-Kr}}{r^2}\right] = -\frac{g^2 e^{-Kr}}{r^2}[Kr + 1]$$

$$r^2 \frac{dV(r)}{dr} = -g^2 e^{-Kr}[Kr + 1]$$

$$\frac{d}{dr}\left[r^2 \frac{dV(r)}{dr}\right] = \frac{d}{dr}[-g^2 e^{-Kr}[Kr + 1]] = -g^2[-Ke^{-Kr}[Kr + 1] + Ke^{-Kr}]$$

$$= g^2 K e^{-Kr}[[Kr + 1] - 1] = g^2 K^2 r e^{-Kr}$$

$$\frac{1}{r^2}\frac{d}{dr}\left[r^2 \frac{dV(r)}{dr}\right] = \frac{g^2 K^2 e^{-Kr}}{r}$$

Right-hand side:

$$K^2 V(r) = K^2\left[g^2 \frac{e^{-Kr}}{r}\right] = \frac{g^2 K^2 e^{-Kr}}{r}$$

Part b)

$$K = \frac{mc}{\hbar} = \frac{1}{1.5 \times 10^{-15}\text{ m}}$$

$$m = \frac{\hbar}{(1.5 \times 10^{-15}\text{ m})c} = \frac{hc}{2\pi(1.5 \times 10^{-6}\text{ nm})c^2} = \frac{1240\text{ eV} \cdot \text{nm}}{2\pi(1.5 \times 10^{-6}\text{ nm})c^2}$$

$$= 1.3 \times 10^8 \frac{\text{eV}}{c^2} = 130 \frac{\text{MeV}}{c^2}$$

This is the mass of the π^0 meson to two significant digits.

Part c)

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

$$\Delta t \sim \frac{\hbar}{2\Delta E} = \frac{\hbar}{2mc^2} = \frac{1.055 \times 10^{-34}\ \text{J} \cdot \text{s}}{2\left(130\dfrac{\text{MeV}}{c^2} \times \dfrac{10^6\ \text{eV}}{1\ \text{MeV}} \times \dfrac{1.602 \times 10^{-19}\ \text{J}}{1\ \text{eV}}\right)c^2} = \boxed{2.5 \times 10^{-24}\ \text{s}}$$

REFLECT

The given range of the interaction is around 1 fm; this corresponds to the approximate range of the strong force, which is the force responsible for holding nucleons together.